Game Tree Searching by Min / Max Approximation

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Background

In 1988, Scientists at MIT investigated a suitable method for generalized mean-value functions by approximating min and max functions for decision tree applications in games. The researchers attempt to use a min/max approximation to locate/isolate the key roots that have the most weight during decision making, while minimizing the cost of the system. The goal was to optimize these specifications in continuous systems in which the derivative could easily be taken (exponential functions for example) in order to formulate and model approximations that allowed for the improved usage of min/max functions with a generalized mean representation. The results and proofs implied novel considerations to how the leaf of a decision tree can have a value that the root heavily depends on.

When the cost function (C) is small, the entire decision tree can be explored as expected. This is not true for larger games (decision trees) that require more stochastic considerations, as parameters are dependent on time, cost of path, and memory. Implementing an approximate value allows the branches to be traversed more quickly and efficiently, though it introduces some errors in precision. A depth (d) restriction is placed to bind the resources used in depth searches using a static evaluator and backing up the information to a minimax value. A technique known as "iterative deepening" is used that allows for incremental depth search iterations with a given time limit. Once the time allowed for each node is expire, a value is returned or the node is forfeited.

An important note from the documentation is reminds the reader that increasing the depth often leads in a decrease in accuracy (for pathological games), so implementation of an approximated min/max function is even more beneficial in this case. While some of the selections that were made are arbitrary (viewed as a separate but related question), the researches do intend to report which expandable tip should be selected, expanded and ancestors (to the root) updated.

Results

The game selection for the experiment, Connect Four, was played 980 times. Half of the games were time constrained and the other half were bounded by the number of movements allowed. The win-lose (W/L) ratio is almost a perfect inverse.

Upon cursory investigation, its appears that alpha-beta (AB) moving was optimal, since the initial tests were with respect to time (between one and five seconds). In these tests, AB won or tied approximately 62% of the time (13% ties, with min/max (MM) methods only winning 38% of the time. As the time was increased incrementally, it was more likely to lose.

However, whenever boundaries were placed on the number of moves, the results are reverse. MM won or tied 61% of the time (10% tie) with AB losing 39% of the time. This shows the different situations in which AB or MM techniques should be used.

The MM with approximated with a generalized mean is given the label of a "penalty-based scheme" whereas AB with pruning is classified as an "iterative scheme." There could be a program that is written with update-able probabilistic criterion that allows for the implementation of both algorithms using conditional statements. However, heuristics usually follow one method, so this might simply be a non-pragmatic (or inefficient) concept at this time.

https://people.csail.mit.edu/rivest/pubs/Riv87c.pdf