UNESCO-IHE

(Delft, the Netherlands)

Exams (with answers) of module Transient Groundwater Flow from 2006 onwards

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Closed book reexamination Transient Flow 2015-2016

1.1 Question 1

1. *Explain what barometer efficiency (BE) is and how it physically works.*
2. *Explain in words what the charateristic (half) time of a groundwater system is. What does is say about the behavior of the system?*
3. *For which of the parameters L (system width), kD (transmissivity) and S_y (specific yield) would an increase make the characteristic system time smaller?*
4. *Explain whey in hydrological logic you think that this is the case.*
5. *If you see a close-up of two grains held together by a small amount of water at their point of contact. What then is the pressure in that water? Explain why that is so.*

1)
Barometer efficiency is the head decline due to barometer pressure increase, both expressed in pressure of head

$$BE = -\rho g \frac{\Delta \phi}{\Delta p_a}$$

2)
Characteristic half time of a groundwater system is the time during which the head above the fixed boundary is halved due to natural drainage alone. The characteristic time can be expressed as

$$T = \frac{L^2 S}{kD}$$

3)
Hence it is 4 times as long for a system with a double width; it is proportional to the storage coefficient and inversely proportional to the transmissivity of the groundwater system.
4)
This is due to the volume in the system relative to the drainage rate.
5)
The curvature of the free water surface between the grains shows that the pressure must be negative, which causes the cohesion between the grains.

Question 2

Consider an aquifer in direct contact with the ocean. The tide of the ocean has an amplitude $A = 1.0m$ and the cycle time is $T = 0.5d$ (one full tide in 12h). The aquifer is confined. It consists of two parts. The first part reaches from the ocean to 500 m inland, the second part is present at more than 500 m from the ocean. The first part of the aquifer has the following properties: transmissivity $kD = 900 m^2/d$ and storage coefficient $S = 0.002$. The second part of the aquifer has the following properties: $kD = 1800 m^2/d$ and storage coefficient $S = 0.001$. Because we consider the fluctuation of the head to be superposed on the mean head, we are only interested in the head s relative to the mean head at every location, that is, $s(x,t) = h(x,t) - \overline{h}(x)$. This head fluctuation s obeys following expression:

$$s = Ae^{-ax}\cos(\omega t - ax), \quad a = \sqrt{\frac{\omega S}{2kD}}, \ \omega = \frac{2\pi}{T}$$

Notice that the storage coefficient is capital S and the head relative to the mean head is lowercase s.

1. *Explain the parameters in the expression and given their dimension*

s = drawdown [L]
A = amplitude [L]
x = distance to where amplitude is given [L]
a = damping [L^{-1}]
ω = angular velocity [radians/T] = [T^{-1}]
T = cycle time [T]

2. *What is the amplitude of the groundwater head fluctuation, that is, the amplitude of s, in the aquifer at 500 m and at 1000 m from the ocean?*

The amplitude only considers the damping, not the cosine,

$$s = Ae^{-ax}$$

For the first part use aquifer properties of that part, yielding a_1 and fill in $x = L_1$
Hence

$$s_1 = Ae^{-a_1 L_1}$$

The amplitude at $L = L_2$ can be computed relative to that at $L = L_1$

$$s_2 = \left(Ae^{-a_1 L_1}\right)e^{-a_2(L_2 - L_1)}$$
$$= Ae^{-(a_1 L_1 + a_2(L_2 - L_1))}$$

3. *What is the delay of the head wave at 500 m and 1000 m relative to the ocean tide? Hint: compute the velocity of the tidal wave in the aquifer and then the time until the peak of the wave starting at the ocean reaches $x = 500$ m and $x = 1000$ m.*

Delay requires knowledge of the wave velocity. The velocity is obtained by realizing that the argument of the cosine must be constant when following the wave at its own speed

$$\omega t - ax = const$$

take the derivative with respect to time

$$\omega - a\frac{dx}{dt} = 0$$

or

$$\frac{dx}{dt} = \frac{\omega}{a}$$

The delay then is

$$\Delta t = \frac{a}{\omega} \Delta x$$

The delay at $x = L$ is obtained by setting $\Delta x = L$
For the point at $L = L_1 = 500m$ we have

$$\Delta t_1 = \frac{a_1}{\omega} L_1$$

and for the point at $L = L_2 = L_1 + (L_2 - L_1)$ just take the delay from point L_1, hence

$$\Delta t_2 = \Delta t_1 + \frac{a_2}{\omega}(L_2 - L_1)$$

where a_1 and a_2 are the damping coefficients computed for the two parts of the aquifer that have different properties. ω is, of course, the same for both parts of the aquifer.

Question 3

Consider a well in an infinite water table (phreatic) aquifer. Drawdowns are considered small compared to the thickness of the aquifer, so that $kD = 900\, m^2 / d$ may be considered constant. The specific yield, $S_y = 0.15$, is also constant. As there are no boundary conditions, the drawdown by the well follows the Theis equation. An approximation of which is

$$s = \frac{2.3Q}{4\pi kD} \log\left(\frac{2.25kDt}{r^2 S}\right)$$

1. *Derive a mathematical expression for the so-called radius of influence, that is, the distance beyond which the drawdown can be neglected at a given time t after the well was first switched on.*

The mathematical expression for the radius of influence is obtained by computing the radius for which the drawdown computed by this simplified formula is zero. That is for which the argument of the logarithm is 1.

$$\frac{2.25kDt}{r^2 S} = 1$$

so that

$$r = \sqrt{\frac{2.25kDt}{S}}$$

2. *As you can see, the drawdown is a logarithmic function in time. Derive a mathematical expression of the increase of the drawdown per log cycle, that is, between for instance t=6d and t=60d, or t=2d and t=20d.*

Simply subtract the drawdown at t from that at $10t$

$$s_{10t} - s_t = \frac{2.3Q}{4\pi kD}\left\{\log\left(\frac{2.25kD(10t)}{r^2S}\right) - \log\left(\frac{2.25kDt}{r^2S}\right)\right\}$$
$$= \frac{2.3Q}{4\pi kD}\log 10$$

3. *Assume the well has been continuously pumping for time $t = t_1$, after which the extraction was stopped. What is the drawdown at distance r_0 at time is $t = t_1 + \Delta t$, where Δt is any time passed since t_1.*

The drawdown at $t_1 + \Delta t$ is obtained by superposition of an extraction Q for the entire period from $t = 0$ to $t + \Delta t$ and an extraction of -Q for the period between $t = t_1$ and $t = t_1 + \Delta t$. Hence

$$s = \frac{2.3Q}{4\pi kD}\left\{\log\left(\frac{2.25kD(t_1 + \Delta t)}{r_0^{\ 2}S}\right) - \log\left(\frac{2.25kD\Delta t}{r_0^{\ 2}S}\right)\right\}$$
$$= \frac{2.3Q}{4\pi kD}\log\frac{t_1 + \Delta t}{\Delta t}$$

Under the condition that the logarithmic expression is valid for the considered point r_0 and times $t \geq t_1$, otherwise we have to apply the regular Theis equation, in which case we cannot simply combine two logarithms.

Exam Hydrogeology Module: Transient Groundwater Flow Course 2015/2016

Written exam held on Feb. 1, 2016

Question 1: (16 points)

1. *Explain loading efficiency, LE*

   LE is the ratio of the pressure increase of the water in the (confined) aquifer and the pressure of the load on ground surface

2. *Explain the barometer efficiency, BE*

   The BE is the ratio of the head decline measurable in a piezometer in the confined aquifer over the pressure increase of the barometer, both expressed in pressure units [N/m2].

3. *What is the difference registered by a pressure gauge in a confined aquifer measuring absolute pressure, given on the one hand a uniform mass placed at ground surface of weight Δp N/m^2 and on the other hand a barometer increase of the same value of Δp N/m^2 ?*

   The absolute pressure in the confined aquifer increases by the same amount in both cases, i.e. $LE \times \Delta p$. Of course, the head in the piezometer declines due to an increase of the barometer, but not the absolute pressure in the aquifer. (only one student had this right).

4. *What is the origin of delayed yield?*

   Delayed yield stems from delayed drawdown due to unconfined storage that manifests itself after the drawdown due to elastic, which is must faster due to the elastic storage coefficient that is about two orders of magnitude smaller than the phreatic storage or specific yield. Delayed yield may be encountered both in a phreatic aquifer and in semi-confined aquifers with an overlaying phreatic water table than cannot be maintained when it is affected by downward leakage into the pumped semi-confined aquifer below.

5. *In which case does the influence of tide reach further inland into an aquifer?*

   1. The case with the higher or with the lower frequency?
      The lower the frequency the further the reach, with zero frequency (that is, steady-state), the reach is theoretically infinite. In reverse, with infinite frequency, the reach is zero, obviously.

2. The case with the larger or the smaller transmissivity *kD*?
   The larger the *kD* the farther the inland reach of the tide. With infinite *kD*, the reach is infinite, with zero *kD* the reach is zero, obviously.

3. The case with the larger or with the smaller storage coefficient *S*?
   The smaller the storage coefficient, the farther the tidal fluctuation reaches inland. With zero storage coefficient the reach is infinite, and with infinite storage coefficient, the reach is zero, obviously.

6. *What is the difference between the situations with the wells that were studied by Theis and by Hantush?*

   Theis studies confined aquifers, including unconfined ones with constant transmissivity, in general, aquifers without external sources, in which all extracted water stems form storage alone.
   Hantush studied semi-confined aquifers, aquifers in which the extracted water stems both from storage and from leakage from an overlying layer with constant head.

7. *Does the Theis case have a final equilibrium drawdown? Explain your answer.*

   Because all water in the Theis case comes from storage there is no steady-stage drawdown possible.

8. *Does the Hantush case have a final, steady-state drawdown? Explain your answer.*

   Because the part of the water from the overlaying layer is proportional to drawdown, there will be equilibrium in the end. So yes, there exists a steady-state solution in the situation that was studied by Hantush.

Question 2: (14 points)

1. *Explain what is the radius of influence of an extraction well in an aquifer of constant transmissivity and storage coefficient?*

   The radius of influence is the radius beyond which the (transient) drawdown is negligible.

2. *The simplified Theis solution is as follows:*

   $$s(r,t) \approx \frac{Q}{4\pi kD} \ln\left(\frac{2.25 kDt}{r^2 S}\right)$$

   From it derive an expression of the radius of influence.

   Set the argument of the log to 1 so that the log and with it the drawdown becomes zero. Then

   $$\frac{2.25 kDt}{r^2 S} = 1 \rightarrow r = \sqrt{\frac{2.35 kDt}{S}}$$

3. *Also show what is the drawdown difference per log cycle of time, that is, between time is t and time is 10t.*

   The drawdown difference per log cycle (not ratio as some of you assumed) is

   $$s_{10t} - s_t = \frac{Q}{4\pi kD} \ln\left(\frac{2.25 kD}{r^2 S} 10t\right) - \frac{Q}{4\pi kD} \ln\left(\frac{2.25 kD}{r^2 S} t\right) = \frac{Q}{4\pi kD} \ln 10 \approx \frac{2.3 Q}{4\pi kD}$$

4. *4 Consider a well in a water table aquifer at 300 m from an impervious wall that reaches to the bottom of the aquifer. The aquifer has $kD = 600$ m^2/d and the specific yield of $S_y = 0.2$. The pumping rate is Q=1200 m2/d.*

   Assume that the approximation of the Theis equation that is given in this question is applicable. Compute the head change of the groundwater at the wall closest to the well.

   Due to the presence of an impervious wall we have to use a mirror well that guarantees that there is no flow perpendicular to the wall. This mirror well must be placed on the other side of the wall at the same distance and its flow must be equal in both quantity and sign as that of the real well. The resulting drawdown is then the superposition of that of the well and its mirror well.

$$s = \frac{Q}{4\pi kD}\ln\frac{2.25kDt}{r_1^2 S_y} + \frac{Q}{4\pi kD}\ln\frac{2.25kDt}{r_2^2 S_y}$$

$$s = \frac{Q}{4\pi kD}\ln\left(\left[\frac{2.25kDt}{r_1 r_2 S_y}\right]^2\right)$$

$$s = \frac{Q}{2\pi kD}\ln\left(\frac{2.25kDt}{r_1 r_2 S_y}\right)$$

with $r = r_1 = r_2 = 300$ m

$$s = \frac{Q}{2\pi kD}\ln\left(\frac{2.25kDt}{r^2 S_y}\right)$$

Just fill in the provided numbers to get the numerical answer.

Question 3 ?

Exam Hydrogeology, Module Transient Groundwater Flow Course 2014/2015

Question 3:

1. *What types of storage or storage coefficients are associated with transient groundwater flow? And explain short how they physically work.*
   A) Elastic storage, from expansion and shrinking of the volume of the water ands the porous medium under changes of water and grain pressure.
   B) Storage from decline of the water table, i.e. drainage of pores, drainage from the unsaturated zone
2. *Explain the relation between capillary rise and pore diameter*

   Capillary rise is the net effect of attraction between water and grain surface (cohesion) and gravity. It may be expressed as upward attraction equals downward gravity force:

   $$2\pi r\sigma\cos\gamma = \pi r^2\rho g h$$

   Hence the capillary rise is inversely proportional to the effective pore radius *r*

   $$h = \frac{2\pi r\sigma\cos\gamma}{\pi\ r^2\rho g} = \frac{1}{r}\frac{2\sigma\cos\gamma}{\rho g}$$

3. *Explain the general shape of the moisture curve in the unsaturated zone. Describe where the water comes from when the water table is lowered.*

   The general shape of the moisture cuve is straight at porosity upward from the water table to the top of the capillary zone and then declining upward as more and more pores fall dry depending on their pore size. At each elevation only the pore with radius smaller than elevations h above the water table will contain water, where h is computed with the previous formula. The actual shape varies with time as plants transpire from the root zone and downward or upward flow through the unsaturated zone also affect the exact shape of the moisture curve at any time.
4. *Explain the difference between the loading efficiency (LE) and the barometer efficiency (BE)?*

   The loading efficiency is the increase in water pressure (or head) in a confined aquifer due to and relative to a load placed uniformly on ground surface. The barometric efficiency is same caused by an increase of the barometric pressure. However, due to the barometric pressure also working on the water surface in the piezometer, the head in the piezometer declines. The decline is such that LE+BE=1.
5. *When you see animal holes in the field, like rabbit, rat and worm holes, how much do you think these holes may contribute to the infiltration of*

rainwater during and after showers, to what extent are the animals living in those holes affected by heavy rains, and , finally, what would it take to swim them out of their holes? Explain your answer from your insight in how water in the subsurface behaves.

Animal holes stay dry as long as the zone they are in is unsaturated, which means that the pore pressure is negative, i.e. below the atmospheric pressure in the animal holes. Hence animals are not affected by rain, they are only affected when the water table rises above their bottom and the pore pressure becomes zero or positive. In that case the holes may collapse, as they become filled up with water and the animals are driven out of their homes.

Question 4:

Consider a confined aquifer in direct contact with the ocean in which the head fluctuates along with the tide of the ocean. The daily solar tide, with cycle time T=12 h or, equivalently, $T = 0.5$ d, has amplitude $A = 2.5$ m and the 4 weekly moon tide, with cycle time T=1/28 d, has amplitude A=1 m. The groundwater head in the aquifer relative to the mean value at time t and distance x from the ocean obeys to the following expression:

$$s = A e^{-ax} \cos(\omega t - ax), \quad a = \sqrt{\frac{\omega S}{2kD}}$$

If T is the time required for a complete cycle, then the angular velocity $\omega = 2\pi / T$. Further, kD and S are known to be 900 m^2/d and 0.001 respectively.

1. *Explain the parameters in the expression and give their dimension.*

   x distance [L]

   t time [T]

   s drawdown [L] or difference from average

   A amplitude of head wave [L]

   ω angle velocity [/T] or [radians/T]

   S storage coefficient [-]

   kD transmissivity [L^2/T]

2. *What is the amplitude of the groundwater fluctuation due to both tides individually at 500 and 2000 m from the coast? So the twice-a-day tide amplitude at 500 m and at 2000 m and the 28-day tide amplitude at 500 m and 2000 m?*

   The wave amplitude is given by the envelope: $s_{\max, x} = A_x = A\ e^{-ax}$

   The maximum amplitude for the daily tide at 500 and 2000 m and an amplitude A=2.5 m then becomes 0.67 and 0.013 m respectively and that due to the moon-tide with its much lower frequency at the same distances and an amplitude of A=1 m becomes 0.83 and 0.49 m respectively.

3. *How much are the waves of both tides delayed at 500 m from the coast?*

The delay time follows from the velocity of the wave obtained by setting $\omega t = ax$

From which follows $v = \frac{x}{t} = \frac{\omega}{a}$ or, equivalently for the delay $t = x / v = \frac{ax}{\omega}$

4. *Over what distance does the maximum tide-induced amplitude in the groundwater declines by a factor of two in both cases?*

   The distance over which the maximum amplitude declines by a factor 2 is readily obtained from

$$A e^{-a(x+\Delta x)} = 0.5 A e^{-ax}$$

$$a(x+\Delta x) = \ln 2 + ax$$

$$\Delta x = \frac{\ln 2}{a}$$

Question 5:

A groundwater table rise after it was agitated by a sudden recharge N [m] will decay over the thereafter. For a system of bounded by two parallel water courses at L mutual distance, this decline after some time can be approximated by the following expression:

$$s = A \frac{4}{\pi} \cos\left(\pi \frac{x}{L}\right) \exp\left(-\left(\frac{\pi}{L}\right)^2 \frac{kD}{S_y} t\right)$$

1. *Describe the parameters and given their dimension.*

   See syllabus

2. *Give an expression for the sudden rise A caused by a sudden recharge amount equal to N [m]:*

$$A = \frac{N}{S_y}$$

3. *Describe in a few words what this expression is and does, so what does its graph look like and how does it behave over time.*

   The expression shows a cosine shaped water table spanned between the boundaries at $x = \pm L/2$ that declines exponentially over time.

4. *Give an expression of what can be called characteristic time of this system.*

   We can write the argument of the exponent as t/T with T the characteristic time. Doing so, we have

$$T = \left(\frac{L}{\pi}\right)^2 \frac{S_y}{kD}$$

yielding

$$s = A\frac{4}{\pi}\cos\left(\pi\frac{x}{L}\right)\exp\left(-\frac{t}{T}\right)$$

5. *Derive an expression of the half time of this system.*

   If time proceeds by one half time then the head declines by a factor of two

$$\exp\left(-\frac{t+\Delta t_{50\%}}{T}\right) = 0.5\exp\left(-\frac{t}{T}\right)$$

$$\Delta t_{50\%} = \ln 2T \approx 0.69T$$

6. *Derive an expression for the discharge of this system.*

   The discharge equals

$$Q_x = -kD\frac{\partial s}{\partial x} = kD\,A\frac{4}{L}\sin\left(\pi\frac{x}{L}\right)\exp\left(-\left(\frac{\pi}{2}\right)^2\frac{t}{T}\right)$$

   The total discharge is obtained by multiplying by 2 the discharge at $x = L/2$:

$$Q_x = 2kD\,A\frac{4}{L}\sin\left(\frac{\pi}{2}\right)\exp\left(-\left(\frac{\pi}{2}\right)^2\frac{t}{T}\right)$$

$$Q_x = \frac{8A}{L}kD\exp\left(-\left(\frac{\pi}{2}\right)^2\frac{t}{T}\right)$$

Question 6:

A 300 m deep well in Jordan with borehole radius *r=0.25* m was drilled in a limestone aquifer to serve a refugee camp. The well was recently test pumped during one day at a rate of *Q=60* m3/h. The head at 0, 0.01, 0.1 and 1 d after the start of the pump was 100, 135, 147 and 159 m below ground surface respectively. The pump is installed at 200 m below ground surface.

Further assume:
The estimated specific yield of this aquifer is 0.01.
The unknown transmissivity is constant.
You may use the simplified expression of transient drawdown in an infinite aquifer

$$s = \frac{Q}{4\pi kD}\ln\left(\frac{2.25kDt}{r^2S_y}\right)$$

1. *Estimate the transmissivity of this aquifer.*

   The transmissivity can be computed using the measured drawdowns, for instance

$$s_3 - s_2 = \frac{Q}{4\pi kD}\left\{\ln\left(\frac{2.25kDt_3}{r^2S_y}\right) - \ln\left(\frac{2.25kDt_2}{r^2S_y}\right)\right\}$$

   or

$$s_3 - s_2 = \frac{Q}{4\pi kD}\ln\left(\frac{t_3}{t_2}\right) = \frac{Q}{4\pi kD}\ln(10) = 2.3\frac{Q}{4\pi kD}$$

hence

$$kD = \frac{2.3Q}{4\pi(s_3 - s_2)} = \frac{2.3\times 24\times 60}{4\pi(159-147)} = 22.0\ \mathrm{m^2/d}$$

2. *How much will be the drawdown after 3 years (1000 d)? Is the pump at 200 m below ground surface (i.e. 100 m below the initial water table) still deep enough to pump the water up?*

$$s = \frac{Q}{4\pi kD}\ln\left(\frac{2.25kDt}{r^2 S_y}\right)$$

$$s_{2700} = \frac{24\times 60}{4\pi 22.0}\ln\left(\frac{2.25\times 22.0\times 1000}{0.25^2\times 0.01}\right) = 95\,\mathrm{m}$$

This means that the well can be pumped continuously at the given rate. But also that other wells in the camp may prevent that because they lower the head at this well too.

3. *Another well of equal size, depth and flow rate is planned at a second location in the camp at 2 km distance. How much will be the drawdown in each well after 3 years (1000 days) in this case? Assume that both wells pump for the same period. How deep should the pumps be installed to allow pumping both wells at he given rate for 3 years?*

$$s = \frac{Q}{4\pi kD}\left[\ln\left(\frac{2.25kDt}{r_0^2 S_y}\right) + \ln\left(\frac{2.25kDt}{r_1^2 S_y}\right)\right]$$

$$s = 2\frac{Q}{4\pi kD}\ln\left(\frac{2.25kDt}{r_0 r_1 S_y}\right)$$

$$s = 2\frac{24\times 60}{4\pi 22.0}\ln\left(\frac{2.25\times 22.0\times 1000}{0.25\times 2000\times 0.01}\right) = 96\ \mathrm{m}$$

This means that the pumps are still submersed after 1000 days. Notice that if we compute the drawdown using the Theis equation instead of the logarithmic simplification, the computed drawdown would be 2 m more. After about 1500 days the drawdown would cause the pump to fall dry. This can be seen if a graph of the drawdown versus time is made.

Re-exam Hydrogeology, Module Transient Groundwater Flow Course 2014/2015

Question 1:

1. *Explain what barometer efficiency (BE) is and how it physically works?*
2. *When you see animal holes in the field, like rabbit, rat and worm holes, how much do you think these holes may contribute to the infiltration of rainwater during and after showers, to what extent are the animals living in those holes affected by heavy rains, and , finally, what would it take to swim them out of their holes? Explain your answer from your insight in how water in the subsurface behaves.*

Question 2:

Consider an aquifer in direct contact with the ocean. The tide of the ocean has an amplitude $A = 1.0$ m and cycle time is $T = 0.5$ d (one full tide in 12h).
The aquifer is confined. It consists of two parts. The first part reaches from the ocean to 500 m in land, the second part is present at more than 500 m from the ocean.
The first part of the aquifer has the following properties: transmissivity $kD = 900$ m^2/d and storage coefficient $S = 0.002$.
The second part of the aquifer has the following properties, $kD = 1800$ m^2/d and storage coefficient $S = 0.001$.
Because we consider the fluctuation of the head to be superposed on the mean head, we are only interested in the head s relative to the mean head at every location, that is $s(x,t) = h(x,t) - \overline{h}(x)$. This head fluctuation s obeys following expression:

$$s = A e^{-ax} \cos(\omega t - ax), \quad a = \sqrt{\frac{\omega S}{2kD}}, \quad \omega = \frac{2\pi}{T}$$

Notice that the storage coefficient is capital S and the head relative to the mean head is lower case s.

1. *Explain the parameters in the expression and give their dimension.*
2. *What is the amplitude of the groundwater head fluctuation, that is, the amplitude of s, in the aquifer at 500 m and at 1000 m from the ocean?*

   The amplitude at distance $0 \le x \le 500$ m is

   $$A_{x \le L} = A_0 \exp(-a_1 x)$$

   $$A_{x > L} = A_0 \exp(-a_1 L) \exp(-a_2 (x - L)), \quad x \ge L$$

3. *What is the delay of the head wave at 500 and 1000 m relative to the ocean tide? Hint: compute the velocity of the tidal wave in the aquifer and then the time until the peak of the wave starting at the ocean reaches $x = 500$ m and $x = 1000$ m.*

The velocity of the wave follows from $\omega t - ax = \text{constant}$. We can take $\text{constant} = 0$ and say $v = \frac{x}{t} = \frac{\omega}{a}$, or more formally

$\omega \frac{dt}{dt} - a\frac{dx}{dt} = 0 \rightarrow v = \frac{dx}{dt} = \frac{\omega}{a}$, so that $v = \frac{\omega}{a} = \sqrt{2\omega \frac{kD}{S_y}}$

The time t_1 for the wave peak to reach $x = L$ equals $t_1 = \frac{L}{v_1}$

The time t_2 for the wave to reach any point $x > L$ then is

$t_2 = \frac{L}{v_1} + \frac{(x-L)}{v_2}$, where $v_1 = \frac{\omega}{a_1}$, $v_2 = \frac{\omega}{a_2}$.

Question 3:

Consider a well in an infinite water-table (phreatic) aquifer. Drawdowns are considered small compared to the thickness of the aquifer, so that $kD = 900$ m^2/d may be considered constant. The specific yield, $S_y = 0.15$, is also constant. As there are no boundary conditions, the drawdown by the well follows the Theis equation. An approximation of which is

$$s = \frac{2.3\,Q}{4\pi kD}\log\left(\frac{2.25kDt}{r^2 S_y}\right)$$

1. *Derive a mathematical expression for the so-called radius of influence, that is, the distance beyond which the drawdown can be neglected at a given time t after the well was first switched on.*

   We just have to set $s = 0$, that is, the argument of the logarithm is 1

$$\frac{2.25kDt}{r^2 S_y} = 1 \;\rightarrow\; r = \sqrt{\frac{2.25kDt}{S_y}}$$

2. *As you can see the drawdown is a logarithmic in time. Derive a mathematical expression for the increase of the drawdown per log cycle, that is between for instance t=6 days and t=60 days, or t=2 days and t=20 days.*

   In this case we compare the drawdown after $t = 10\Delta t$ with $t = \Delta t$

$$s_{10\Delta t} - s_{\Delta t} = \frac{2.3\,Q}{4\pi kD}\left[\log\left(\frac{2.25kD10\Delta t}{r^2 S_y}\right) - \log\left(\frac{2.25kD\Delta t}{r^2 S_y}\right)\right]$$

$$s_{10\Delta t} - s_{\Delta t} = \frac{2.3\,Q}{4\pi kD}\left[\log\left(\frac{10\Delta t}{\Delta t}\right)\right] = \frac{2.3\,Q}{4\pi kD}$$

3. *Assume the well has been continuously pumping for time* $t = t_1$*, after which the extraction was stopped. What is the drawdown at distance* r_0 *at time* $t = t_1 + \Delta t$*, where* Δt *is any time passed since* t_1*.*

$$s_{t_1+\Delta t} - s_{t_1} = \frac{2.3\,Q}{4\pi kD}\left[\log\left(\frac{t_1+\Delta t}{t_1}\right)\right]$$

Exam Hydrogeology, Module Transient Groundwater Flow Course 2013/2014

Question 1:

1. *What types of storage or storage coefficients are associated with transient groundwater flow?*
2. *Explain how these types of storage physically work.*

   A) Storage due to drainage of pores.

   B) Storage due to elasticity of water and the porous medium
3. *Explain the relation between capillary fringe and air entry pressure.*

   The capillary fringe and the air-entry pressure are equivalent. It is the pressure required to flow air through the largest pores down to the water table. It is also the height over which water is sucked up into the largest pores from the water table.
4. *Explain the difference between the loading efficiency (LE) and the barometer efficiency (BE)?*

   The loading efficiency is the increase in water pressure (or head) in a confined aquifer due to and relative to a load placed uniformly on ground surface. The barometric efficiency is same caused by an increase of the barometric pressure. However, due to the barometric pressure also working on the water surface in the piezometer, the head in the piezometer declines. The decline is such that LE+BE=1.
5. *Why does the specific yield of unconfined aquifers with a shallow groundwater table depend on the depth of the water table?*

   This is due to the moisture profile above the water table. When the water table is shallow, part of this profile is above ground surface and will no longer contribute to drainage and therefore, reduces the specific yield.
6. *What is halftime when considering decay of a water mound between rivers? How would you describe it?*

   The halftime is the time in which de difference between the head or the elevation of the water table between the river is reduced by 50%. The halftime is determined by L, kD and S
7. *The halftime is determined by the parameters L (distance between the rivers), kD (transmissivity) and the storage coefficient S (or specific yield S_y in the case of unconfined flow). Give a formula for the half time and explain which parameters increase the halftime and which parameters decrease the halftime.*

   A formula for the half time is $T=L^2S/(4kD)$. The formula shows that *L* and *S* increase the halftime and *kD* decreases the halftime.

Question 2:

Consider a confined aquifer in direct contact with the ocean in which the head fluctuates along with the tide of the ocean. The tide has an amplitude of $A = 2.5$ m. The groundwater head in the aquifer at time *t* and distance *x* from the ocean obeys to the following expression:

$$s = A e^{-ax} \cos(\omega t - ax), \quad a = \sqrt{\frac{\omega S}{2kD}}$$

The frequency f of the tide is one complete cycle per 24 hours, i.e. $f = 1$/d, with, of course, $\omega = 2\pi / f$.

We don't know the value of *kD* and *S*, but we have measured the amplitude of the groundwater head fluctuation at $x = 500$ m. This amplitude is 25 cm, one tenth of that of the ocean.

1. *Explain the parameters in the expression and give their dimension.*

   x distance [L]

   t time [T]

   s drawdown [L] or difference from average

   A amplitude of head wave [L]

   ω angle velocity [/T] or [radians/T]

   S storage coefficient [-]

   kD transmissivity [L^2/T]

2. *Give an expression for the amplitude at distance x from the ocean.*

   The amplitude is independent of the cosine. Hence

   $s = a\, e^{-ax}$

3. *With the given information, compute parameter* a*, and the diffusivity of the aquifer, i.e. the ratio* kD/S.

   The amplitude, $s_0 = 2.5\,m$, is given at distance $x = 0$ at see and as $s_1 = 0.1 s_0$ at $x = x_1 = 500\,m$

   $s_1 = s_0 e^{-ax_1}$, therefore, $ax_1 = \ln\left(\frac{s_0}{s_1}\right) \rightarrow a = \frac{\ln(10)}{500} \approx \frac{1}{220}$

   The diffusivity then follows from $a^2 = \frac{\omega S}{2kD}$, therefore,

   $\frac{kD}{S} = \frac{\omega}{2a^2} = \frac{2\pi/1}{2} 220^2 = 1.5 \times 10^5$ [m^2/T]

4. *Give an expression for the velocity of the wave of the groundwater-head in the subsurface? How much is this velocity?*

   The velocity of the wave of the head is found from the argument under the cosine, it must be constant, or zero for convenience

$$\omega t - ax = 0 \rightarrow \frac{x}{t} = v = \frac{\omega}{a} = (2\pi/1)\times 220 \approx 1380\, m/d = 58\, m/h$$

Question 3:

The solution for the decay of the groundwater head, *s,* that is initially uniform and at a height *A* above two open water channels at $x = -L/2$ and $x = L/2$ that are in direct contact with an aquifer is given by the following series expression:

$$s = A\frac{4}{\pi}\sum_{j=1}^{\infty}\left\{\frac{(-1)^{j-1}}{2j-1}\cos\left[(2j-1)\pi\frac{x}{L}\right]\exp\left[-(2j-1)^2\frac{\pi^2 kD}{L^2 S}t\right]\right\}$$

We can simplify this equation for larger times, when the time-dependent terms for indices *j* greater than 1 have become negligible. If we accept that this is true when the second term (i.e. for *j*=2) is less than 0.01 times the first term, in an absolute sense, then we may compute this “critical” time from:

$$\exp\left[-\frac{\pi^2 kD}{L^2 S}t\right] > 100\times\exp\left[-9\frac{\pi^2 kD}{L^2 S}t\right]$$

1. *Derive after which time this is true. In doing so, express the final result for t in terms of a characteristic time, which you obtain from a combination of parameters used in the expression.*

   We may define the characteristic time as

$$T = \left(\frac{L}{\pi}\right)^2\frac{S}{kD}$$

   Hence, with this characteristic time and after taking the natural logarithm on both sides the given comparison expression reduces to

$$-\frac{t}{T} > \ln 100 - 9\frac{t}{T}$$

$$\frac{t}{T} > \frac{\ln 100}{8\left(\frac{\pi}{2}\right)^2} = \frac{\ln 100}{2\pi^2} = 0.233$$

$$t > 0.233T$$

2. *What is the simplified expression when only the first term is taken into account?*

$$s = A\frac{4}{\pi}\cos\left(\pi\frac{x}{L}\right)\exp\left(-\frac{t}{T}\right)$$

3. *With that expression derive the so-called half-time. Express this half time also in terms of the characteristic time of this groundwater system.*

The halftime is the time in which the drawdown s is reduced to $0.5s$.

Hence

$$A\frac{4}{\pi}\cos\left(\pi\frac{x}{L}\right)\exp\left(-\frac{t+\Delta t_{0.5}}{T}\right)=0.5\ A\frac{4}{\pi}\cos\left(\pi\frac{x}{L}\right)\exp\left(-\frac{t}{T}\right)$$

$$\exp\left(-\frac{t+\Delta t_{0.5}}{T}\right)=0.5\ \exp\left(-\frac{t}{T}\right)$$

$$-\frac{t+\Delta t_{0.5}}{T}=\ln 0.5-\frac{t}{T}$$

$$\frac{t+\Delta t_{0.5}}{T}=\ln 2+\frac{t}{T}$$

$$\frac{\Delta t_{0.5}}{T}=\ln 2$$

$$\Delta t_{0.5}\approx 0.69T$$

Question 4:

Consider an unconfined aquifer with conductivity $k=10$ m/d, a specific yield of $S_y=0.1$ and an initial thickness $h=20$ m. A well is located in this aquifer on each of the four corners of a square with sides of $L=200$ m. The wells start pumping at $t=0$. They pump with a rate of $Q=120$ m^3/d for 1d, after which they stop.

The drawdown according to Theis is

$$s=\frac{Q}{4\pi kD}W(u),\qquad u=\frac{r^2S}{4kDt}$$

The Theis well function is graphically given in Figure 1 below.

1. *Compute the drawdown in the center of the square after $t=2$ d. You may neglect the change of the transmissivity caused by the change of the water depth in the aquifer.*

The drawdown in the center of the square is due to 4 wells that are all at the same distance $R=500/\sqrt{2}\approx 354\,m$. Therefore, the drawdown in the center of the square can be expressed as

$$s=4\frac{Q}{4\pi kD}W(u),\ u=\frac{R^2S}{4kDt}$$

The four wells pump during 1 day while de remaining drawdown is requested after two days. Therefore, we have to apply superposition such that after the 2 days, 4 wells have been pumping for 2 days and 4 wells have been pumping with negative flows during 1 day:

$$s_{t=2d} = 4\frac{Q}{4\pi kD}\left(W\left(u_{t=2}\right) - W\left(u_{t=1}\right)\right),\ u = \frac{R^2 S}{4kDt}$$

$$\frac{R^2 S}{4kh} = T = \frac{141^2 \times 0.1}{4 \times 10 \times 20} \approx 2.5\,d, \qquad u_2 = \frac{T}{t_2} = \frac{2.5}{2} = 1.25, u_1 = \frac{T}{t_1} = \frac{2.5}{1} = 2.5$$

To use the Theis' type curve which is W(u) versus 1/u, we have

$1/u_2 = 0.8 \rightarrow W\left(u_2\right) \approx 0.14$

$1/u_1 = 0.4 \rightarrow W\left(u_1\right) \approx 0.03$

Hence

$$s_{t=2d} = 4\frac{120}{4\pi 200}\left(0.14 - 0.03\right) = \frac{60}{\pi}0.11 = 2.10\,m$$

Question 5: (bonus question, i.e. gives extra points)

Given that the well function can be computed by the following infinite series

$$W\left(u\right) = -\gamma - \ln u + u - \frac{u^2}{2\times 2!} + \frac{u^3}{3\times 3!} - \frac{u^4}{4\times 4!} + \ldots$$

with $\gamma = 0.577216$,

1. *What would be a good approximation of the drawdown for small values of u?*

   For sufficiently small value of u, W(u) can be approximated to

   $$W\left(u\right) \approx -\gamma - \ln u$$

   This can be evaluated into

   $$W\left(u\right) \approx -\ln e^{\gamma} - \ln u = -\ln\left(e^{\gamma} u\right) = \ln\left(\frac{1}{e^{\gamma} u}\right) = \ln\left(\frac{4kDt}{e^{\gamma} r^2 S}\right)$$

   with $4/e^{\gamma} \approx 2.25$ we obtain

   $$W\left(u\right) \approx \ln\left(\frac{2.25kDt}{r^2 S}\right)$$

2. *How could you define the radius of influence of the drawdown?*

   The radius of influence may be set to the radius that makes this approximate drawdown zero, i.e. to that which makes the argument of the logarithm 1:

   $$\frac{2.25kDt}{r^2 S} = 1 \rightarrow r_{\text{influence}} = \sqrt{\frac{2.25kDt}{S}}$$

   which grows with the square root of time.

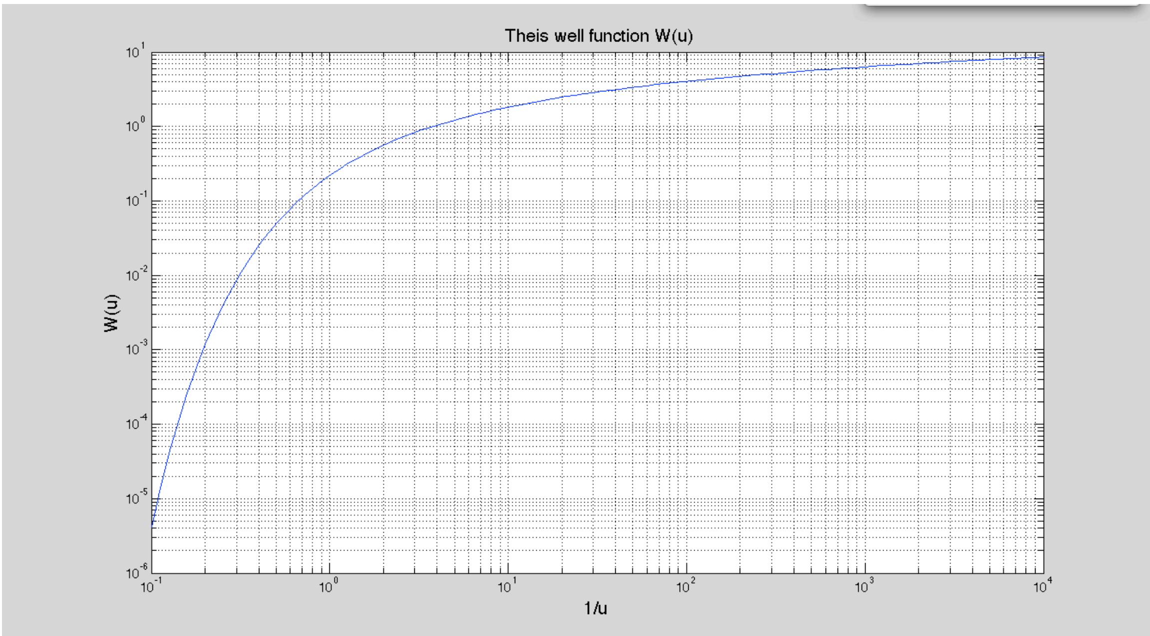


Figure 1: Theis' well function W(*u*) versus 1/*u*

Exam 2013

Only available in a separate pdf file, could not be transferred to Microsoft Word format

Exam 2012

Only available in a separate pdf file, could not be transferred to Microsoft Word format

Module Hydrogeology, Exam Transient Groundwater Flow, Thursday February 3 2011

Time 1 hour

Question 1: Pressure in confined aquifer

A water level in a piezometer in a confined aquifer is affected if the weight on a ground surface is suddenly changed. Compare two situations a) Sudden change by a load placed on the ground, such as sand or flooding by water and b) Sudden increase of the barometric pressure.

Case a: --- a load is placed on ground surface

1. *How does the water pressure change in the piezometer? (up? down? Not?)*

Up

2. *How does the head change in the piezometer? (up? down? Not?)*

Up

Case b: --- barometer pressure increased

1. *How does the water pressure change in the piezometer? (up? down? Not?*

Up

2. *How does the head change in the piezometer? (up? down? Not?)*

Down

General:

1. *If there is a difference between the two cases, then why is that?*

Although the water pressure in increased by the same amount in both cases, the water level in the piezometer goes down in the case of the increased barometric pressure, because the air pressues works for 100% on the water level inside the piezometer which is greater than the increase of water pressure in the aquifer due to the air pressure on ground surface. In the case of an increased load there is no increase of pressure on the water level in the piezometer, and so the change of water level in the piezometer fully reflects the change of water pressure in the aquifer, which is some fraction of the increase of pressure on ground surface.

Question 2: Tidal waves

The groundwater head variation in a confined aquifer due to a tidal wave at *x=0* can be expressed mathematically as follows

$$s(x,t)=\phi(x,t)-\phi_0 = A\exp(-\alpha x)\sin(\omega t - a x)$$

in which $\alpha = \sqrt{\dfrac{\omega S}{2kD}}$ and, of course, $\omega T = 2\pi$ with *T* the period of the wave.

1. *What is the amplitude of the wave at distance **x**?*

$$s(x,t) = \phi(x,t) - \phi_0 = A\exp(-\alpha x)$$

2. *What is the velocity of the wave?*

$$\omega t - ax = \text{constant} \rightarrow \omega - a\frac{dx}{dt} = 0 \rightarrow v_{wave} = \frac{dx}{dt} = \frac{\omega}{\alpha} = \sqrt{2\omega\frac{kD}{S}}$$

3. *If the wave would be just observable in a piezometer at **x**=1000 m from the coast, then at what distance would the wave be just observable on another spot along the coast where the storage coefficient is be 100 times greater than at the current spot and the transmissivity is the same?*

So how to get the same amplitude of the amplitudes at x=0 are the same for both waves?

$$\frac{s_1(x,t)}{s_2(x,t)} = 1 = \frac{\exp(-\alpha_1 x_1)}{\exp(-\alpha_2 x_2)} \rightarrow \frac{\alpha_1 x_1}{\alpha_1 x_1} = \frac{\sqrt{\frac{\omega_1 S_1 x_1^2}{2kD_1}}}{\sqrt{\frac{\omega_2 S_2 x_2^2}{2kD_2}}} \rightarrow \frac{\omega_1 S_1 x_1^2}{2kD_1} = \frac{\omega_2 S_2 x_2^2}{2kD_2}$$

so

$$\frac{\omega S_1 x_1^2}{2kD} = \frac{\omega S_2 x_2^2}{2kD} \rightarrow S_1 x_1^2 = S_2 x_2^2 \rightarrow x_2 = \sqrt{\frac{S_1}{S_2}}x_1 = \sqrt{\frac{1}{100}}1000 = 100m$$

4. *A tidal wave occuring daily is just observable in the aquifer at a distance of **x**=500 m from shore, where **x**=0. At what distance from the shore will a 14-day wave be just observable occuring due to the monthly moon cycle? Assume the same amplitude for both waves.*

$$\frac{s_1(x,t)}{s_2(x,t)} = 1 = \frac{\exp(-\alpha_1 x_1)}{\exp(-\alpha_2 x_2)} \rightarrow \frac{\alpha_1 x_1}{\alpha_1 x_1} = \frac{\sqrt{\frac{\omega_1 S_1 x_1^2}{2kD_1}}}{\sqrt{\frac{\omega_2 S_2 x_2^2}{2kD_2}}} \rightarrow \frac{\omega_1 S_1 x_1^2}{2kD_1} = \frac{\omega_2 S_2 x_2^2}{2kD_2}$$

$$\frac{\omega_1 S x_1^2}{2kD} = \frac{\omega_2 S x_2^2}{2kD} \rightarrow \omega_1 x_1^2 = \omega_2 x_2^2 \rightarrow x_2 = \sqrt{\frac{\omega_1}{\omega_2}}x_1 = \sqrt{\frac{1}{(1/14)}}500 = 500\sqrt{14} = 1800m$$

Question 3: Characteristic time of groundwater systems

In class we discussed the somewhat complicated solution by series expansion of the evolution of the head after a sudden rain shower of P [m] in a strip of land of width L [m] between parallel fixed-head boundaries with water level ϕ_0 [m]. We have seen that after some time t [d], only the first term matters, which is

$$s(x,t)=\phi(x,t)-\phi_0=\frac{P}{S_y}\frac{4}{\pi}\cos\left(\pi\frac{x}{L}\right)\exp\left(-\pi^2\frac{kD}{L^2S_y}t\right)$$

Whenever possible express your answers mathematically:

1. *What is the shape of the head?*

A cosine

a What is the maximum head, take ϕ_0=0?

$$s_{\max}(x,t)=s(0,t)=\frac{P}{S_y}\frac{4}{\pi}\exp\left(-\pi^2\frac{kD}{L^2S_y}t\right)$$

2. *What would you consider the characteristic time of this system?*

Write $\exp\left(-\pi^2\frac{kD}{L^2S_y}t\right)$ as $\exp\left(-\pi^2\frac{t}{T}\right)$ or $\exp\left(-\frac{t}{T}\right)$ so that $T=\frac{L^2S_y}{kD}$ or $T=\frac{L^2S_y}{\pi^2kD}$, it is a matter of definition.

3. *What would be the half-time of this groundwater system?*

The half-time is the time it takes for something to become half as large. For exponential declining systems this is a constant. This is the case here with the head relative to its final value. Let the half-time be Δt, then we can formally express the half-time as the time it takes such that the ratio of the head at time $t+\Delta t$ is half the head at time t :

$$\frac{s_{\max}(x,t+\Delta t)}{s_{\max}(x,t)}=\frac{\exp\left(-\pi^2\frac{kD}{L^2S_y}(t+\Delta t)\right)}{\exp\left(-\pi^2\frac{kD}{L^2S_y}t\right)}=\exp\left(-\pi^2\frac{kD}{L^2S_y}(t+\Delta t)\right)+\exp\left(\pi^2\frac{kD}{L^2S_y}t\right)=\exp\left(-\pi^2\frac{kD}{L^2S_y}\Delta t\right)=0.5$$

therefore (note the plus and the 0.5 changing into 2:

$$\exp\left(+\pi^2\frac{kD}{L^2S_y}\Delta t\right)=2\rightarrow\pi^2\frac{kD}{L^2S_y}\Delta t=\ln(2)\rightarrow\Delta t=\ln(2)\frac{L^2S}{\pi^2kD}$$

Question 4: Wells

The solution by Theis is given by

$$s(r,t)=\phi_0-\phi(r,t)=\frac{Q_0}{4\pi kD}W(u),\qquad u=\frac{r^2S}{4kDt}$$

1. *What flow conditions are described by Theis' well solution?*

The Theis solution describes the drawdown due to a well in a confined (constant *kD*) aquifer extracting a flow Q_0 from $t=0$.

2. *What are its parameters and what are their dimensions?*

We have s [L] drawdown; ϕ [m] head relative to fixed datum, Q_0 [L^3/T] is the constant extraction from the well , kD [L^2/T] aquifer transmissivity, u [-] argument

of well function, r [m] distance from well, S [-] storage coefficient of aquifer S_y or S_S*D and t [T] time since the well started its extraction.
As you know, the function W(u) is the exponential integral, which may be written as a series expansion :

$$W(u) = -0.577216 - \ln(u) + u - \frac{u^2}{2\times 2!} + \frac{u^3}{3\times 3!} - \frac{u^4}{4\times 4!} + \ldots$$

3. *How can you mathematically approximate the Theis' solution for very small values of u given that* $-0.577216 = \ln(0.5615)$ *?*

$$s(r,t) = \frac{Q_0}{4\pi kD}\ln\left(\frac{0.5615}{u}\right) = \frac{Q_0}{4\pi kD}\ln\left(\frac{0.5615\times 4kDt}{r^2 S}\right) = \frac{Q_0}{4\pi kD}\ln\left(\frac{2.25kDt}{r^2 S}\right)$$

4. *With this approximation, mathematically give the difference between the drawdown obtained at time **t** and the drawdown at time 10**t** in a piezometer at some arbitrary distance **r** from the well.*

You may use the fact that $\ln(10) = 2.3$.

$$s(r_2,t_2) - s(r_1,t_1) = \frac{Q_0}{4\pi kD}\left\{\ln\left(\frac{2.25kDt_2}{r_2^2 S}\right) - \ln\left(\frac{2.25kDt_1}{r_1^2 S}\right)\right\} = \frac{Q_0}{4\pi kD}\ln\left(\frac{t_2}{r_2^2}\frac{r_1^2}{t_1}\right)$$

$$s(r,10t_1) - s(r,t_1) = \frac{Q_0}{4\pi kD}\ln(10) = \frac{Q_0}{4\pi kD}2.3$$

5. *Also give the difference between the drawdown in a piezometer at distance **r** from the well and in a piezometer at distance 10**r** from the well, both at the same time.*

$$s(10r,t) - s(r,t) = \frac{Q_0}{4\pi kD}\ln\left(\left(\frac{10r}{r}\right)^2\right) = \frac{Q_0}{2\pi kD}\ln(10)$$

Exam: Transient Groundwater Flow, Exam 5 Feb 2010

Question 1:

1. *What is liquefaction?*

   Loosely packed fine sand may become liquefied and turn into quicksand by a shock (earthquake) by which grains become untached, the matrix tries to resettle into a more compact configuration but the water cannot escape immediately due to the small conductivity of the fine sand.

2. *What is the difference between specific yield and elastic storage?*

   Water table storage versus storage due to the elastic properties of both the water and the matrix.

3. *How does the specific yield change if an already shallow water table rises?*

   In a phreatic aquifer with a shallow water table, specific yield gets smaller the shallower the water table. This is due to the moister profile intersecting ground surface. See syllabus

4. *Why does this happen (make a sketch and explain)*

   See syllabus page 12

5. *Which of the two materials, gravel and fine sand, has the highest specific yield and why (assume both have the same porosity).*

   The coarser material has the highest specific yield because it has the lowest specific retention, because less water can adhere against gravity in the unsaturated zone because of the much smaller specific surface in a given volume of coarse material than the same volume of fine material.

Question 2:

The head in an aquifer connected to the ocean fluctuates due to tide. This fluctuation is given by the following formula, in which s expresses the head variation caused by the tide as a function of time t and the inland distance from the shore x:

$$s(x,t) = A\exp(-\alpha x)\sin(\omega t - \alpha x)$$

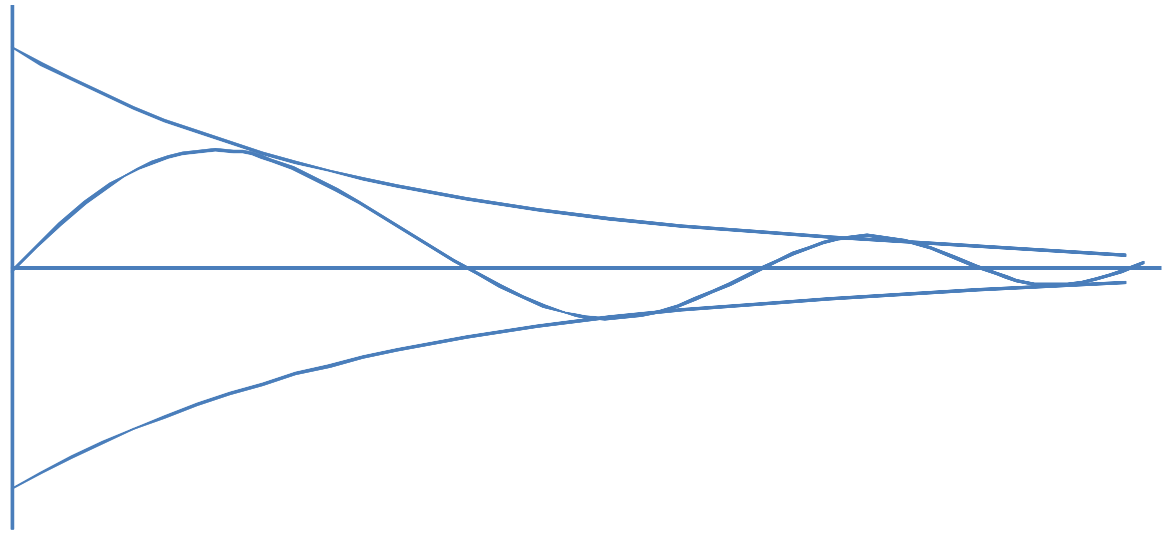
With $\alpha = \sqrt{\dfrac{\omega S}{2kH}}$

1. *What is the expression for the maximum head fluctuation as a function of x?*

   This expression is just the part of the tide formula without the sin:

$$s(x,t) = A\exp(-\alpha x)$$

2. *Sketch the head change s as a function of x at time t=0 and sketch also the envelope (maximum and minimum value of s as a function of x)*



3. *Which parameters increase the inland penetration of the tide and which parameters decrease this inland penetration?*

   Parameters increasing the penetration depth of the tide wave are those that reduce the value of the damping alfa, that is a lower frequency omega, a lower storage coefficient and a higher transmissivity.

Question 3:

Consider an extraction canal in direct contact with an aquifer of infinite extent. The aquifer has transmissivity kH=400 m^2/d and specific yield Sy =0.1. As long as t<0, the head in the aquifer is everywhere 0 m (we take the initial water level as our reference level).
At time t=0 d, the water level in the canal suddenly changes to 2 m. Then, at time t=2 d, the water level in the canal suddenly changes back to its original value of 0 m and remains constant afterwards.
The head change and the head-change gradient are:

$$s = s_o \operatorname{erfc}\left(\sqrt{\frac{x^2 S}{4kDt}}\right) \text{ and } \frac{\partial s}{\partial x} = -s_o\sqrt{\frac{S}{\pi kDt}}\exp\left(-\frac{x^2 S}{4kDt}\right)$$

To obtain values for the **erfc** function, use the graph below.
Answer the following two questions.

1. *Compute the head at x=100 m at t=3 d. Show the formula you use and include the dimension in your answer!*

   This problem is solved by superposition in time:

$$s = s_o \operatorname{erfc}\left(\sqrt{\frac{x^2 S}{4kDt}}\right) - s_o \operatorname{erfc}\left(\sqrt{\frac{x^2 S}{4kD(t-2)}}\right)$$
$$s = 2\operatorname{erfc}\left(100\sqrt{\frac{0.1}{4\times400\times3}}\right) - 2\operatorname{erfc}\left(100\sqrt{\frac{0.1}{4\times400\times2}}\right)0$$
$$= 2\times\{\operatorname{erfc}(0.46) - \operatorname{erfc}(0.79)\} = 2\{0.5-0.2\} = 0.6\text{m}$$

2. *Compute the discharge at x=0 at t=3 d. Show the formula you use and include the dimension in your answer?*

The discharge at x=0 as a function of time equals

$$q(0,t) = -kH\frac{\partial s}{\partial x} = kHs_o\sqrt{\frac{S}{\pi kDt}}$$

So

$$q(0,3) = s_o\sqrt{\frac{kHS}{\pi}}\left(\sqrt{\frac{1}{t}} - \sqrt{\frac{1}{t-2}}\right) = 2\sqrt{\frac{400\times0.1}{\pi}}\left(\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{3-1}}\right) = -3.02\text{m}^2/\text{d}$$

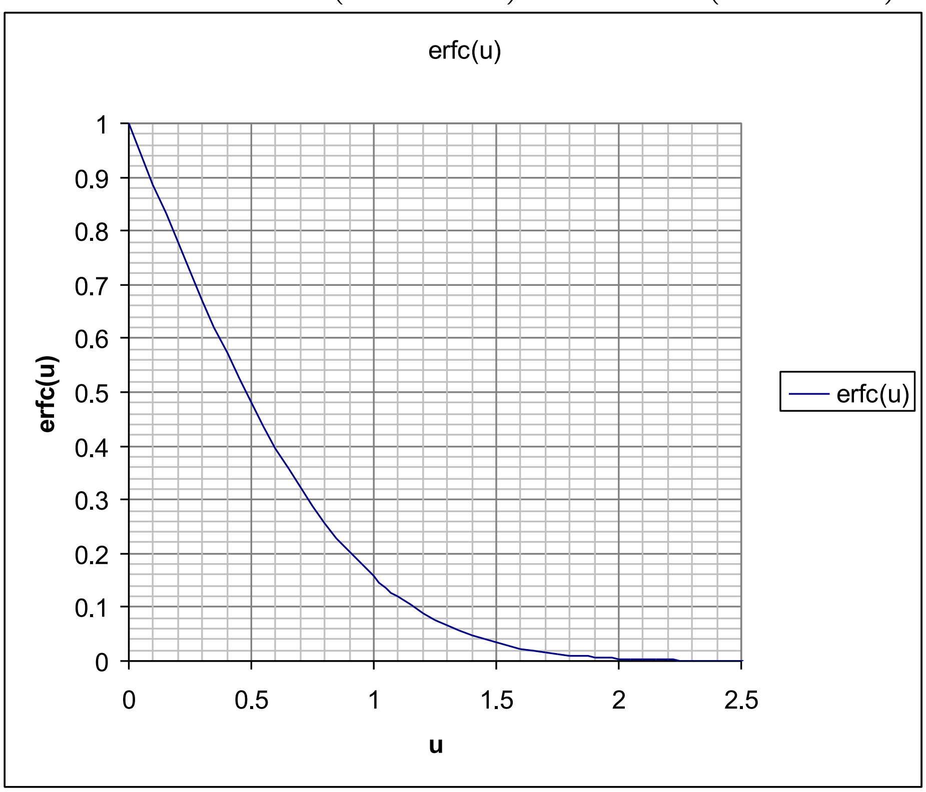


Figure 2: The function erfc(u)

Question 4:

Consider a well in a system of infinite extent which starts extracting at time t=0. We know that Theis' formula applies:

$$s(r,t) = \frac{Q}{4\pi kH} \mathrm{W}(u), \qquad u = \frac{r^2 S}{4kHt}$$

We also know that for small values of u, the well function, W(u), can be approximated by a straight line on log-t scale, which is given by:

$$\mathrm{W}(u) \approx 2.3 \log\left(\frac{0.5625}{u}\right) = 2.3 \log\left(\frac{2.25kHt}{r^2 S}\right)$$

Consider a pumping test on this well, starting the constant extraction Q=800 m^3/d at t=0. The drawdown is measured over a number of days at an observation well at 25 m distance. The measured drawdowns are shown in the figure below, which clearly reveals the straight portion of the drawdown that we expect from the expression above for large-enough values of time.

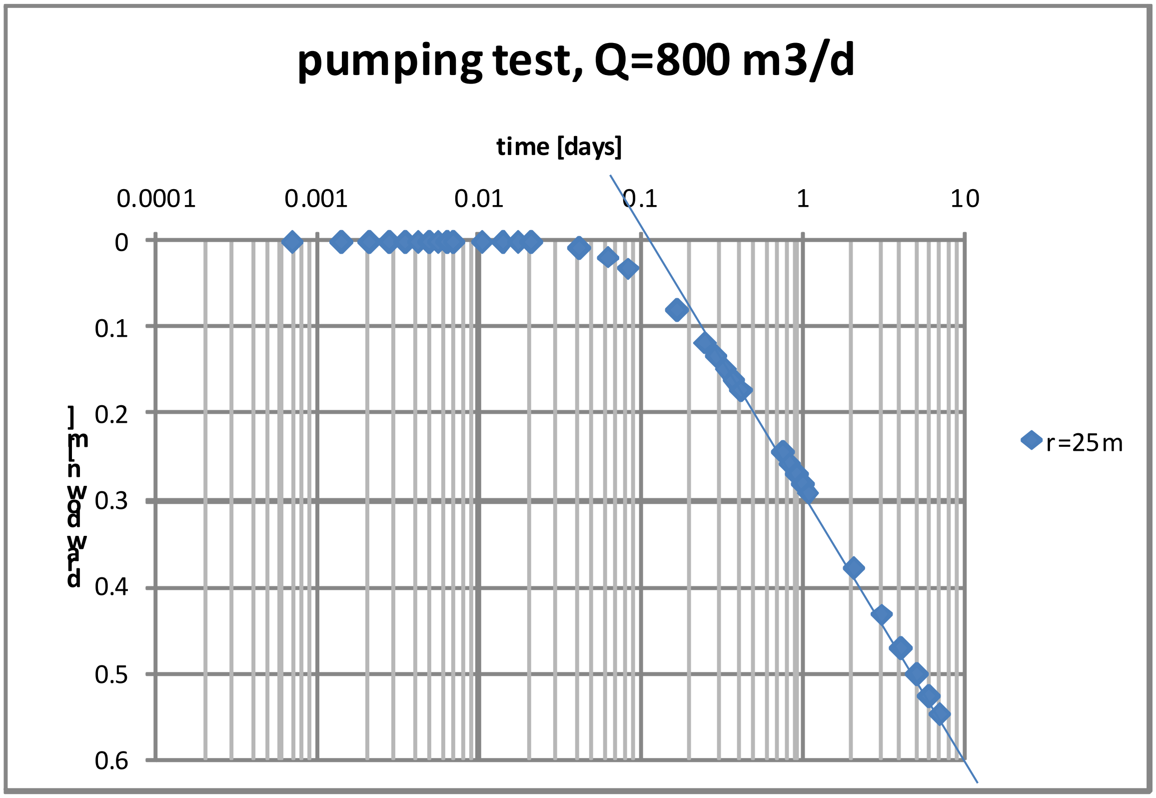
1. *Using the straight line through the measured data, compute the transmissivity kH and the storage coefficient S of this aquifer*

   The drawdown difference per log cycle is the most convenient way to compute the transmissivity. First draw a line through the straight portion of the graph (see figure below) and then measure the drawdown increase per log cycle, which is about 0.32 m. Then with log(10)=1, compute the transmissivity from

   $$s_{10t} - s_t \approx \frac{2.3Q}{4\pi kH} k \Rightarrow kH = \frac{2.3Q}{4\pi\left(s_{10t} - s_t\right)} = \frac{2.3 \times 800}{4\pi 0.32};\ 460\mathrm{m}^2/\mathrm{d}$$

   Next we get the storage coefficient from the point where the straight drawdown line of the above expression is zero, which is at about 0.12 days:

   $$\frac{2.25kHt}{r^2 S} = 1 \Rightarrow S = \frac{2.25kHt}{r^2} = \frac{2.25 \times 460 \times 0.12}{25^2} \approx 0.2$$


pumping test, Q=800 m3/d
time [days]
0.0001
0.001
0.01
0.1
1
10
0
0.1
0.2
0.3
0.4
0.5
0.6
drawdown [m]
r=25m

Exam: Transient Groundwater Flow, Exam Feb 2009

Question 1:

1. *What is the difference between specific yield and elastic storage*

   Storage coefficient in respectively unconfined and (semi)-confined aquifers

   1. How does the specific yield change if an already shallow water table rises further and becomes even shallower?

   The specific yield becomes smaller

2. *Why does this happen (make a sketch and explain)*

   Part of the unsaturated profile that would store or yield water now extends above ground surface, and no longer exists so that it cannot contribute to storage

3. *Which of the two materials, gravel and fine sand, has the highest specific yield and why (assume both have the same porosity).*

   Fine sand, due to its much larger surface area and number contact points between grains

Question 2:

1. *What do we mean by Loading Efficiency (LE) and what do we mean by Barometric Efficiency (BE)?*

   LE is the portion of the increased total pressure supported by the water and increasing the head.

   The BE is the portion of the barometric pressure change causing change of head

   Both LE and BE only work in (semi)-confined aquifers

2. *What is the difference in terms of head change if we compare a loading on land surface with an equal increase of the barometer pressure? And why?*

   In case of a load change, the head changes in the same direction ad the load, with barometric change it changes in the opposite direction. An increase of barometric pressure caused a decrease in head.

   The difference is due to the face that the barometric pressure also works in the piezometer, while a load increase does not.

Question 3:

Tidal flow in a confined aquifer may be described mathematically by

$s = A e^{-\alpha x} \sin\left(\omega t - \alpha x\right)$, where $\alpha = \sqrt{\frac{\omega}{2}\frac{S}{kD}}$

1. *What are the different quantities in these expressions and what are their dimensions?*

   A [m] is amplitude of tide, α is factor, no dimension, ω is tidal frequency[radians/day], S [-] is storage coefficient, kD [m^2/d] is transmissivity.

2. *By what expression is the envelope given (the envelope describes the maximum amplitude as a function of x?)*

   $s = A e^{-\alpha x}$

3. *How does the envelope change if the frequency of the tide would double?*

   It is reduced. Less penetraton.

4. *How will the envelope change if the transmissivity would be two times less and the storage coefficient 100 times less?*

   α is then $\sqrt{50}$ times larger, so the envelope is reduced accordingly and penetration depth of the tide is therefore much farther

Question 4:

The picture below shows a strip of land of width L bounded by two canals. Both the strip and the canals run perpendicular to the paper (so the picture is a cross section). Suddenly the water level in the left canal is raised by A m as is indicated in the figure. This causes the head to change in the strip. At the right hand side the water level is unchanged. There exists an expression which mathematically describes the effect of a sudden level rise in a strip that is unbounded on one side. We want to use this expression to computed the head in the strip. We can do this by means of mirror canals.

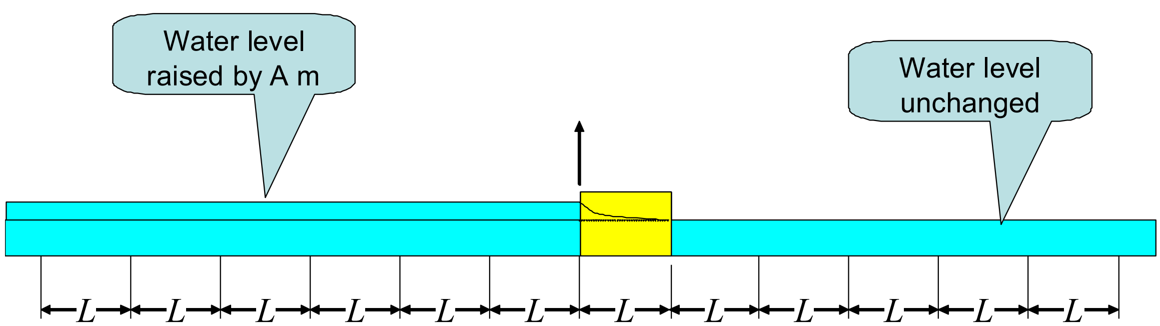
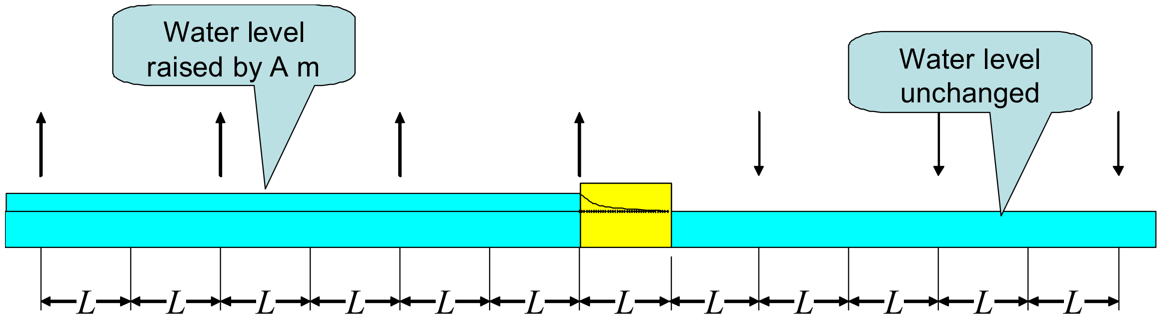


Figure 3: Strip of land of width *L* bounded by open water. The water level at the left hand side was suddenly raised by *A* m. This causes the head in the aquifer of the strip to change dynamically.

1. *Irrespective of what the mathematical looks like, where would you put the mirror canals and which are positive and which are negative? Just draw an arrow respectively up or down (see figure) at the locations where you would put the mirror canal.*



Question 5:

The characteristic dynamics of a groundwater systems (i.e. the time it takes for the head of a groundwater system to reach equilibrium) is related to the argument of transient groundwater flow solutions, This argument is $\sqrt{\frac{x^2S}{4kDt}}$ in solutions for one-dimensional flow and $\frac{r^2S}{4kDt}$ for radial flow such as in the well functions of Theis and Hantush.

1. *Explain how the characteristic dynamics relate to these arguments?*

   In a system of given width L or radius R the factor $T=\frac{L^2S}{4kD}$ has dimension time and is directly related to the dynamics of a groundwater basin. It can be considered the characteristic time of the basin/system.

2. *Compare the characteristic dynamics of two systems. System two is twice as wide as system one and its transmissivity is 3 times as large and its storage coefficient 100 times as small as that of system one. How do the dynamics of these two systems relate to each other, that is: how many times faster or slower is system two compared to system one in reaching piezometric equilibrium?*

   The factor $T=\frac{L^2S}{4kD}$ with dimensions time is a measure for the characteristic time of the groundwater system. System two thus has a characteristic time that is $2^2\times 0.01/3=0.013=1/75$ times larger or 75 times smaller than system one.

Question 6:

Consider a well in a semi-confined aquifer with kD=900 m^2/d, S=0.001 and c=400 d that is pumped at a discharge Q of 2400 m^3/d.

1. *How long does it take before the drawdown at 60 m distance form the well becomes stationary?*

   $\lambda=\sqrt{kDc}=\sqrt{900\times 400}=600,\quad r/\lambda=60/600=0.1$

See where the Hantush type curve for $r/\lambda = 0.1$ becomes horizontal (stationary). Read the 1/u value, which is about 1000, and compute the time.

$$\frac{1}{u} = \frac{4kDt}{r^2 S} \rightarrow t = \frac{r^2 S}{4kDu} = \frac{60^2 \times 0.001}{4 \times 900} \times 1000 = \frac{3600}{4 \times 900} = 1\,\mathrm{d}$$

2. *What is the final drawdown?*

This drawdown, because it is steady state can be computed either by the De Glee formula (with the Bessel Function) or with the Hantush formula

$$s = \frac{Q}{2\pi kD} \mathrm{K_o}\left(\frac{r}{\lambda}\right) = \frac{Q}{4\pi kD} \mathrm{W}\left(u, \frac{r}{\lambda}\right)$$ (because the flow is steady state)

Using the type curves given we apply Hantush which yields with 1/u=1000 and r/L=0.1 W=1.9, so

$$s = \frac{2400}{4\pi 900} 1.9 \approx 0.4\,\mathrm{m}$$

Question 7:

A pumping test has been carried out in a confined aquifer. The drawdown and the Theis type curves are given in the graphs below. Theses graphs have been drawn on the same type of double logarithmic paper. The extraction of the well during the test was 1000 m^3/d. Determine the transmissivity and the storage coefficient of this groundwater system.

Fold the paper and tear off the lower graph. Shift the two graphs over each other until they match (keep axes parallel). Then choose a "match point" and read the combined value of *s* and *W* to obtain *kD* from

$$s = \frac{Q}{4\pi kD} W \rightarrow kD = \frac{Q}{4\pi s} W$$

With numbers read from both graphs once overlaid (W=1 and s=0.1)

$$kD = \frac{Q}{4\pi s} W = \frac{1000}{4\pi \times 0.1} \times 1 = 795\,\mathrm{m^2/d}$$

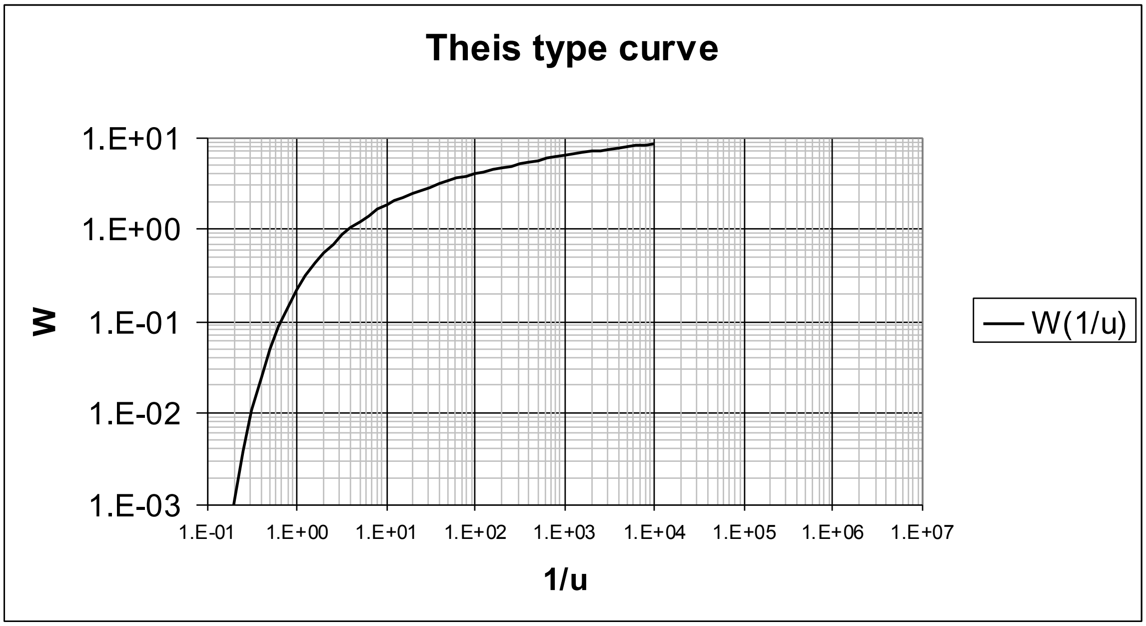
Then read the combined values of $1/u$ and t/r^2 from the graphs and determine *S*/*kD* from

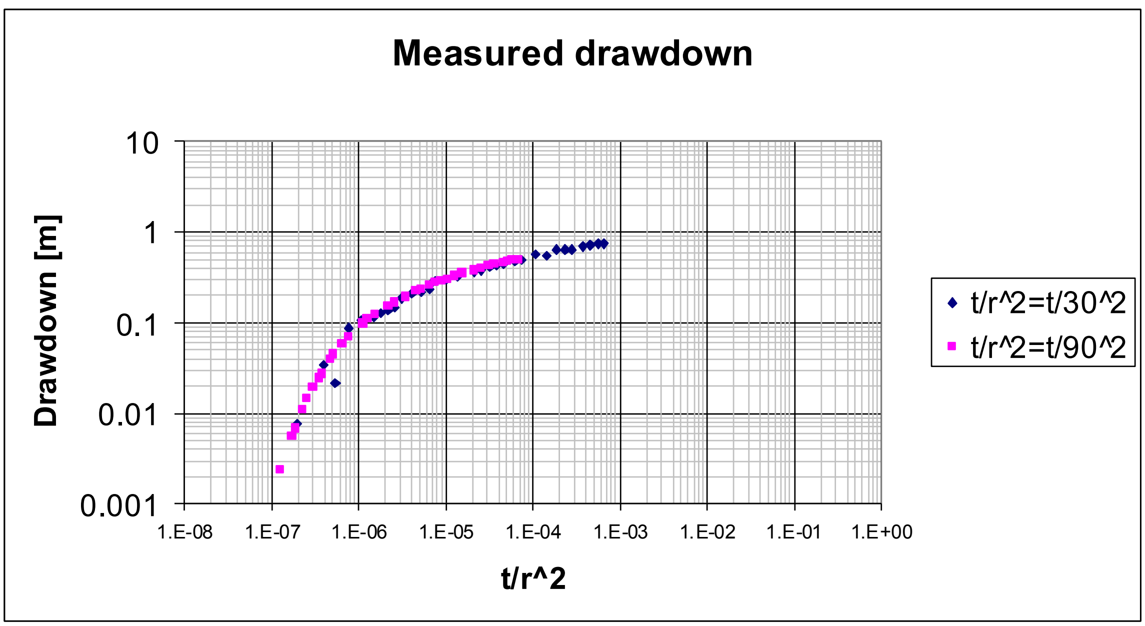
$$\frac{1}{u} = \frac{4kD}{S} \frac{t}{r^2} \rightarrow \frac{S}{kD} = 4u \frac{t}{r^2}$$

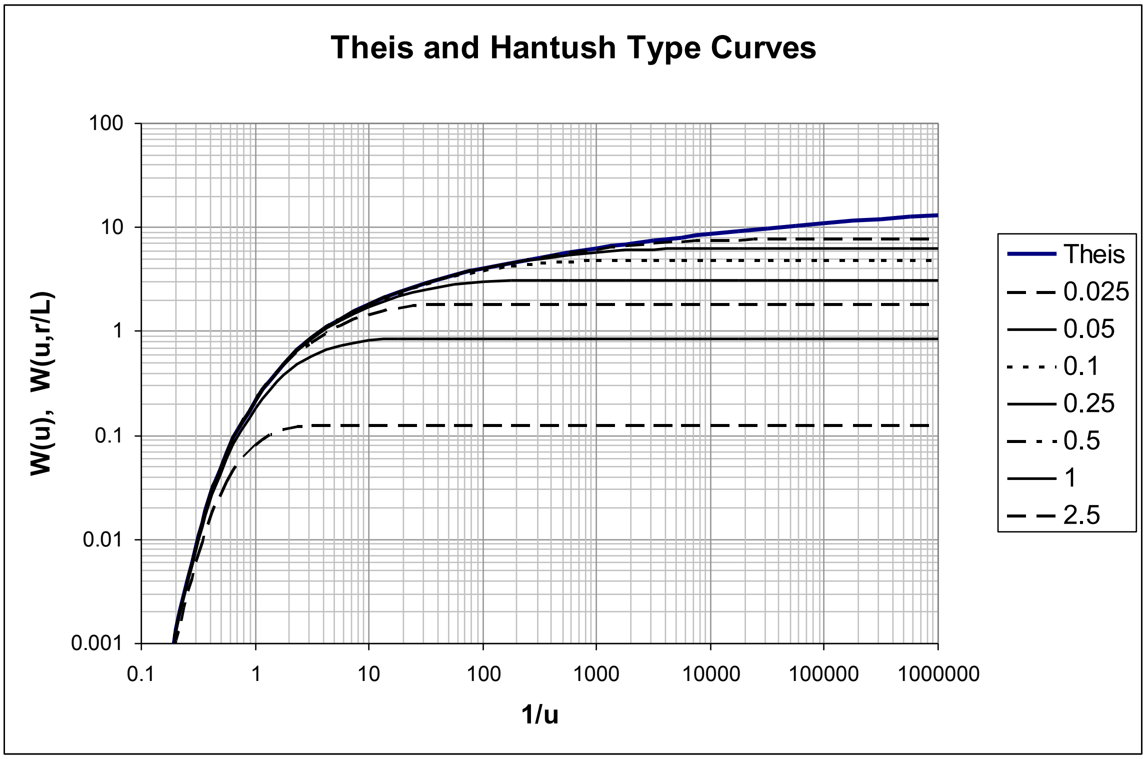
In numbers with $1/u = 1$ and $t/r^2 = 3 \times 10^{-7}$ we get

$$\frac{S}{kD} = 4u \frac{t}{r^2} = 4 \times 1 \times 3 \times 10^{-7} = 1.2 \times 10^{-6} [\mathrm{d/m^2}]$$

Finally compute $S = 1.2 \times 10^{-6} \times kD = 1.2 \times 10^{-6} \times 795 = 0.95 \times 10^{-3} \approx 10^{-3}$.


Theis type curve
W
1/u
W(1/u)
1.E+01
1.E+00
1.E-01
1.E-02
1.E-03
1.E-01
1.E+00
1.E+01
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07


Measured drawdown
Drawdown [m]
t/r^2
t/r^2=t/30^2
t/r^2=t/90^2
10
1
0.1
0.01
0.001
1.E-08
1.E-07
1.E-06
1.E-05
1.E-04
1.E-03
1.E-02
1.E-01
1.E+00


Theis and Hantush Type Curves
100
10
1
0.1
0.01
0.001
W(u), W(u,r/L)
0.1
1
10
100
1000
10000
100000
1000000
1/u
Theis
0.025
0.05
0.1
0.25
0.5
1
2.5

Exam: Transient Groundwater Flow, Exam Feb 2008

Question 1:

1. *What is the difference between specific yield and elastic storage*

   Storage coefficient in respectively unconfined and (semi)-confined aquifers

2. *How does the specific yield change if an already shallow water table rises further and becomes even shallower?*

   The specific yield becomes smaller

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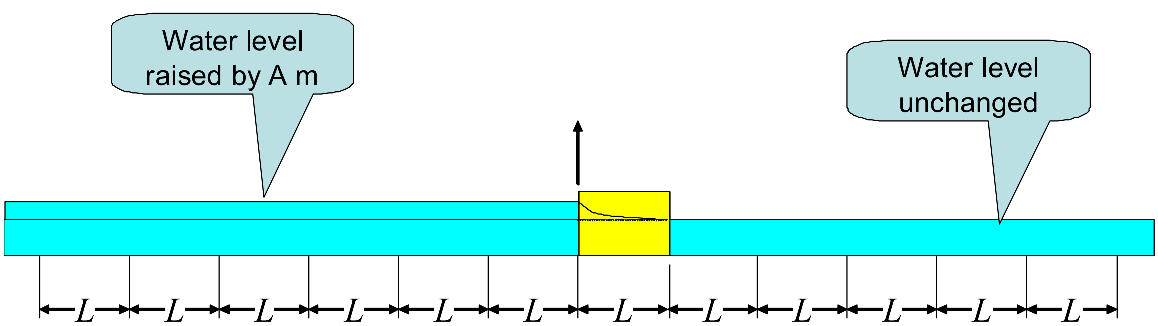

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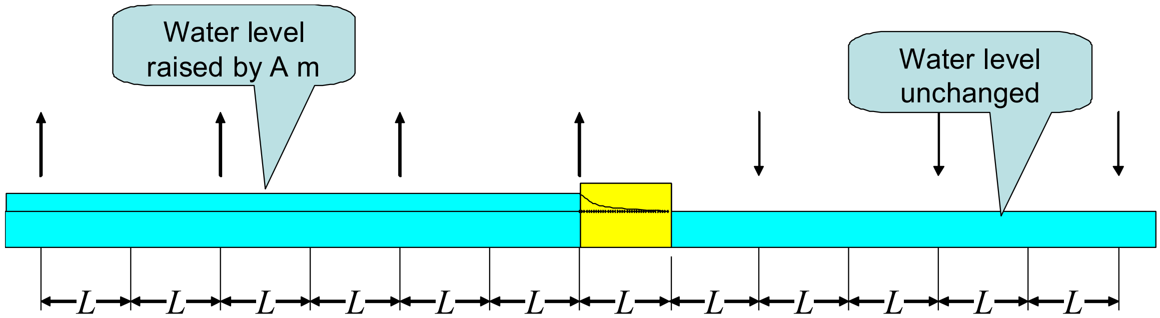

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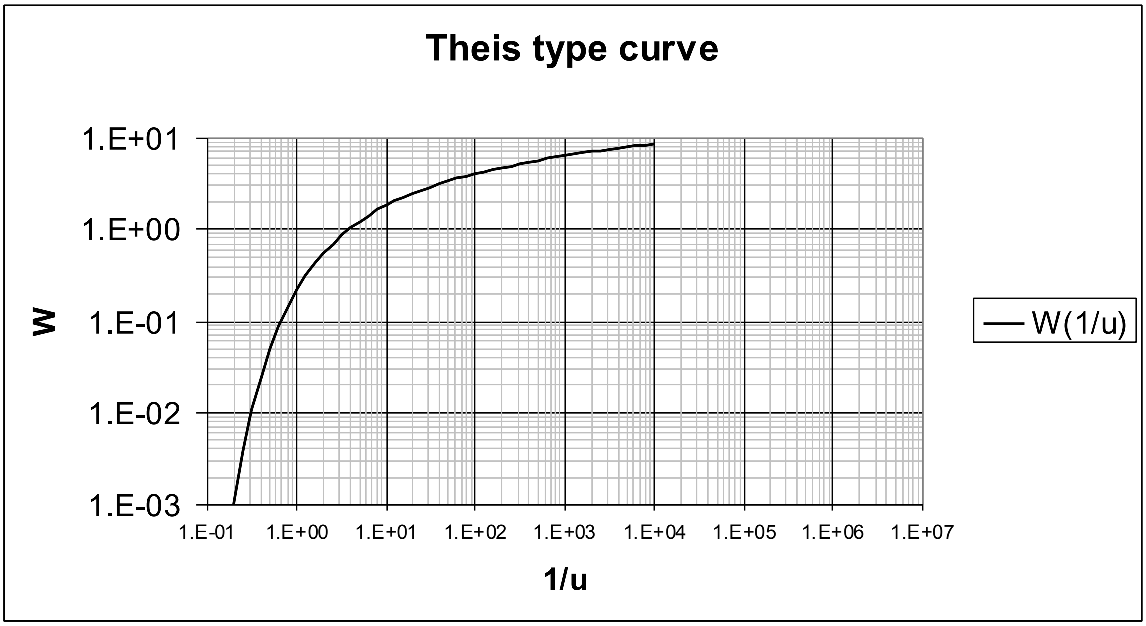
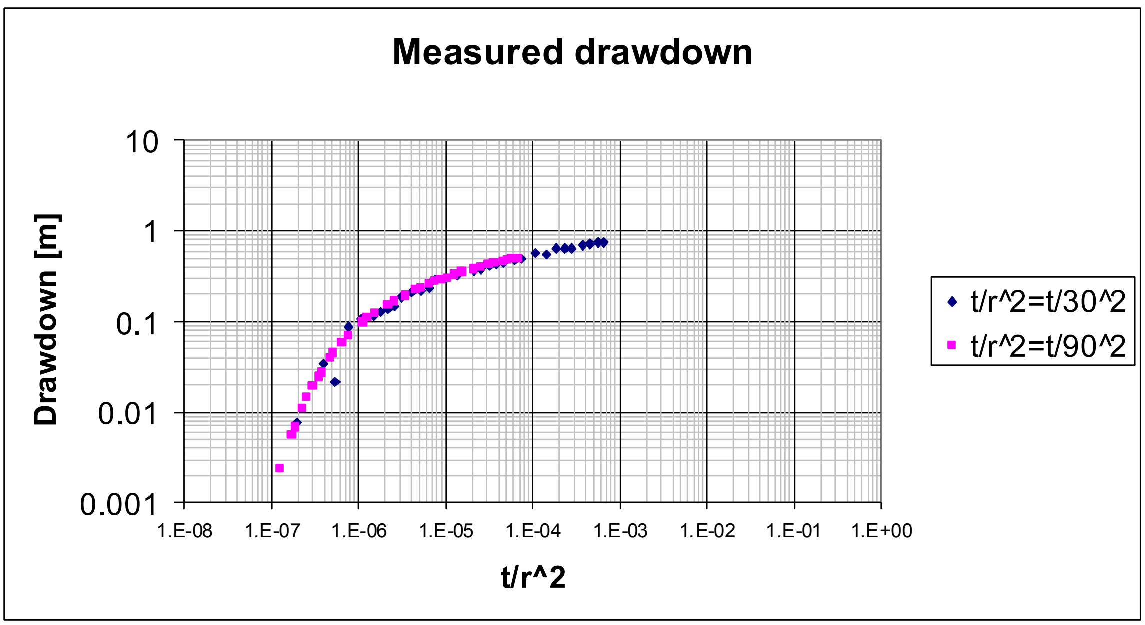

Theis type curve
1.E+01
1.E+00
1.E-01
1.E-02
1.E-03
W
1.E-01
1.E+00
1.E+01
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
1/u
W(1/u)


Measured drawdown
10
1
0.1
0.01
0.001
Drawdown [m]
1.E-08
1.E-07
1.E-06
1.E-05
1.E-04
1.E-03
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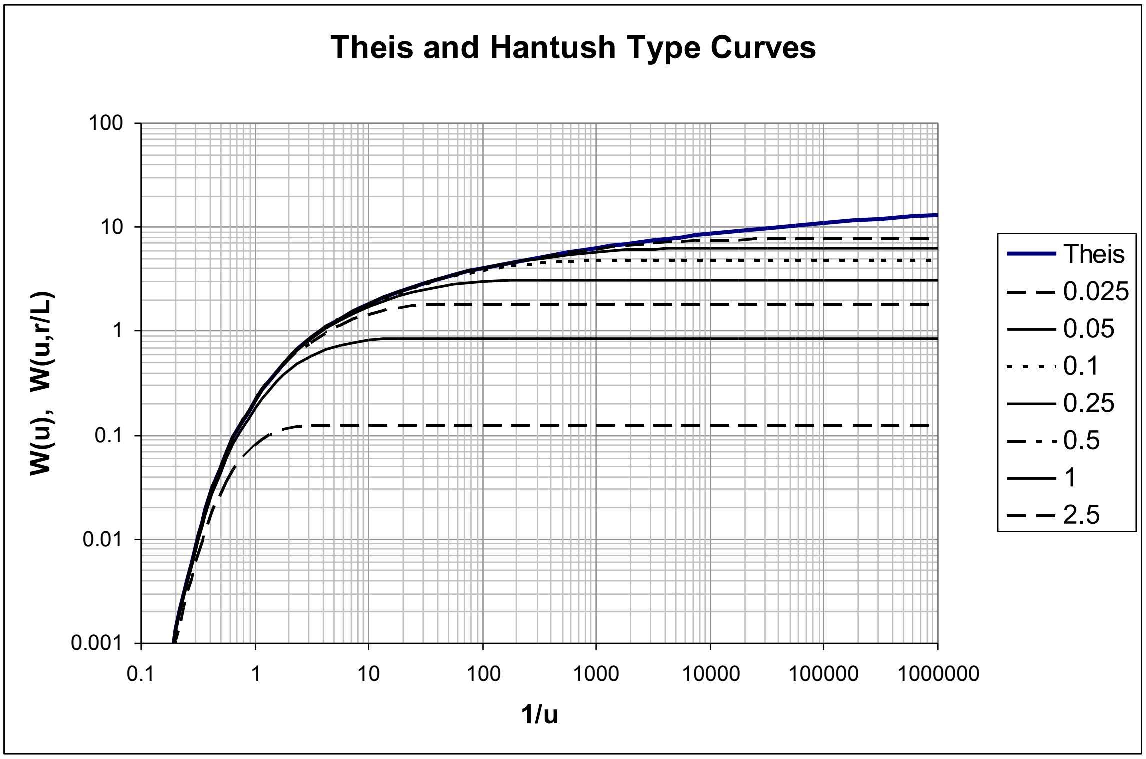
t/r^2
t/r^2=t/30^2
t/r^2=t/90^2


Theis and Hantush Type Curves
100
10
1
0.1
0.01
0.001
0.1
1
10
100
1000
10000
100000
1000000
1/u
W(u), W(u,r/L)
Theis
0.025
0.05
0.1
0.25
0.5
1
2.5

Exam February 2007

Question 1: General questions

1. *What is specific yield?*
2. *How does specific yield depend on the distance of the water table below ground level?*
3. *What happens to the water table in a piezometer in a confined aquifer when the barometer pressure goes up, why?*

Question 2: Diffusion equation

The diffusion equation for transient flow in one dimension is $D\frac{\partial^2 s}{\partial x^2} = \frac{\partial s}{\partial t}$

1. *What is the dimension of the diffusivity D?*
2. *What is diffusivity D in the case of groundwater flow?*
3. *What is diffusivity D in the case of heat flow?*

Question 3: Fluctuation groundwater

In the case of a tidal fluctuation in a river in direct contact with an aquifer having transmissivity the fluctuation of the head may be described by

$s = s_o \exp(-\alpha x)\sin(\omega t - \alpha x)$, with $\alpha = \sqrt{\frac{\omega S}{2kD}}$

Question 4: What is s and what does this function look? Make a sketch of s as a function of x, and show its envelopes. (The envelope is the curve of the values between which the function fluctuates, as a function of x).

1. *In the case of a double-day tide,* $\omega = \frac{4\pi}{24}[h^{-1}]$. *What would be the speed of the wave into the aquifer if S=0.001 and kD=500 m^2/d? (Notice the dimensions!)*
2. *At what distance from the river is the amplitude of the head fluctuation still only half of that in the river at x=0?*
3. *What happens to this distance in case the transmissivity would be 9 times a big?*

Question 5: Flow to an extraction canal

Consider an extraction canal in direct full contact with an aquifer with transmissivity kD=400 m^2/d and specific yield S_y =0.1. The water level in the canal suddenly changes by 2 m downward. The head and gradient are given by:

$$s = s_o \operatorname{erfc}\left(\sqrt{\frac{x^2 S}{4kDt}}\right), \quad \frac{\partial s}{\partial x} = -s_o \sqrt{\frac{S}{\pi kDt}} \exp\left(-\frac{x^2 S}{4kDt}\right)$$

1. *Compute the discharge into the canal after 1d. Show the formula you use and include the dimension in your answer!*
2. *What is the head change s at 100 m from the canal after 1 and after 2 d? (Use erfc-curve further down).*
3. *What is the head change at 100 m from the canal after 2 days if the head in the river would change back by 2 m at t=1d?*

Question 6: Well in semi-confined aquifer

Consider a transient well in a semi0-confined aquifer so that Hantush's solution is valid:

$s = \frac{Q}{4\pi kD} \mathrm{W}\left(u, \frac{r}{\lambda}\right)$, $u = \frac{r^2 S}{4kDt}$, $\lambda = \sqrt{kDc}$ with kD=600 m^3/d, c=900 d, S=0.001; pumping at a rate Q=2400 m^3/d.

1. *How long does it take before steady state is reached for a point at r=300 m from the well (why)? Use Hantush type curves (see graphic at the end of this exam).*

Question 7: Drawdown due to a pumping station in an unconfined aquifer

A well is situated at 100 m from an impermeable infinitely long wall. The well is pumping at a rate of 2400 m^3/d. Even though the aquifer is unconfined, the transmissivity kD may be taken as a constant equal to 600 m^2/d, while the specific yield S_y equals 0.2. The well bore has a radius of 0.25m

1. *What is the drawdown at the well bore after 10 days of pumping ?*
2. *A well in a confined aquifer of infinite extent, with kD=1000 m^2/d and S=0.001, is pumping at a rate of 24000 m^3/d. How far would the radius of influence of this well after 100 years? The radius of influence is the radius beyond which the drawdown is considered negligible. You may exploit the logarithmic approximation of the Theis well function for large times:*

$W(u) \approx \ln\left(\frac{0.5625}{u}\right)$, $u < 0.1$ with $u = \frac{r^2 S}{4kDt}$ by making it zero.

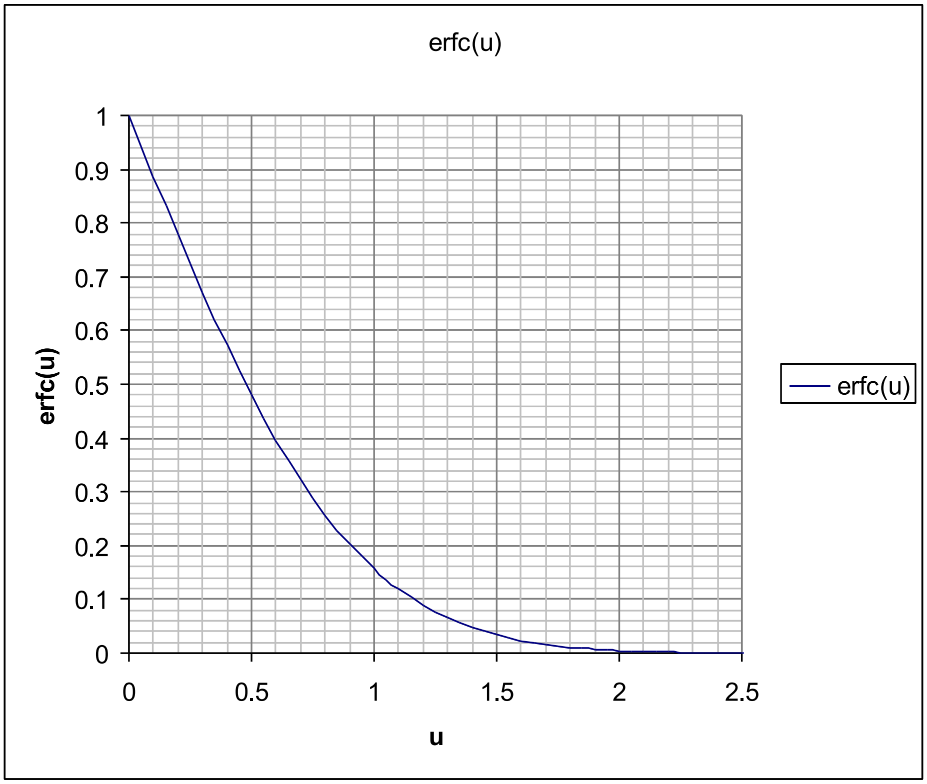


Figure 5: The function erfc(u)

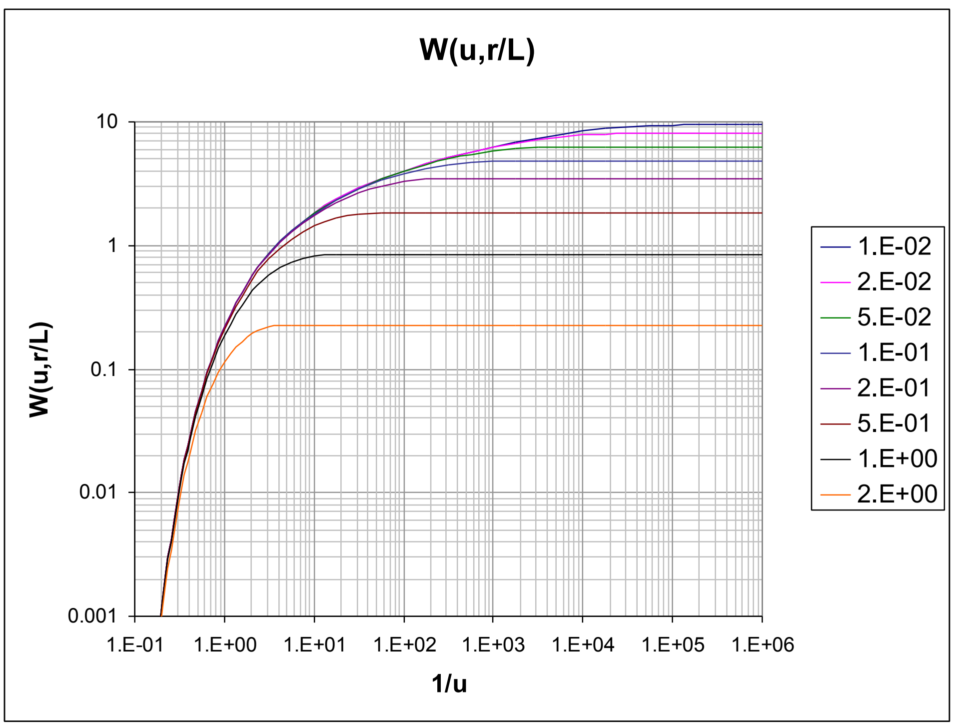


Figure 6: Theis and Hantush type curves. In case this graph is copied in black and white only, note that the lowest type curve is for the highest value of r/λ. Note that the L in the title and left axis of this figure stands for $\lambda = \sqrt{kDc}$ value

Exam module transient flow 2006

Question 1: Conceptual

1. *What types of reversible storage do you know in aquifer systems, explain how it works*
2. *What values may you expect for the respective storage coefficients?*
3. *What is barometric efficiency, explain how it works.*
4. *When the barometric pressure increases, does the head (water table in a piezometer) in the confined aquifer rise or fall?*
5. *Between what values may the barometric efficiency vary?*
6. *What happens in a confined aquifer with the head if a load is suddenly placed on ground surface, such as a train stopping near a piezometer? What happens when it leaves? Sketch a graph showing the head versus time that you would expect in that case.*

Question 2: Characteristic time of groundwater basin

Characteristic time of groundwater basin, the partial differential equation of which reads

$$kD\frac{\partial \phi^2}{\partial x^2} = S\frac{\partial \phi}{\partial t}$$

1. *What is a characteristic time of a groundwater basin that may be considered as one-dimensional of characteristic size L? (hint: Make partial differential equation dimensionless by $\xi = \frac{x}{L}, \tau = \frac{t}{T}$ and see what T is.*
2. *To reach equilibrium, how many times slower is a large basin compared to a small one with the same transmissivity and storage coefficient?*
3. *Compute the characteristic time for the following cases:*
    i. Large basin: kD=500 m^2/d, system width L=100km, storage coefficient S=0.2,
    ii. Small basin: kD=100 m^2/d, system width L=100m , storage coefficient S=0.1

Question 3: Tides in groundwater

Given: The tidal fluctuation in an aquifer in a point at distance x from the sea due to the water level fluctuation at sea with amplitude A is described by the following formula

$$s(x,t) = A\exp(-\alpha x)\sin(\omega t - \alpha x)$$

in which the damping factor is as follows $\alpha = \sqrt{\frac{\omega S}{2kD}}$, where ω is the angle velocity in radians/time or $\omega = \frac{2\pi}{T}$ where T is the time of a complete wave cycle.

Are the following expressions true or false?

b. The wave in the aquifer has a different frequency than the tide itself

c. The amplitude of the wave at a given distance from the sea becomes greater when

  i. the frequency of the tide is reduced

  ii. the storage coefficient is reduced

  iii. then the transmissivity is reduced

Question 4: Aquifer with river

Consider an aquifer of infinite extent bounded by a fully penetrating river at x=0. At t=0 the river level suddenly changes by a height A. The change of head $s(x,t)$ in the aquifer equals in this case:

$s(x,t) = A\,erfc\left(\sqrt{\frac{x^2 S}{4kDt}}\right)$ with erfc(u) as shown in the picture below

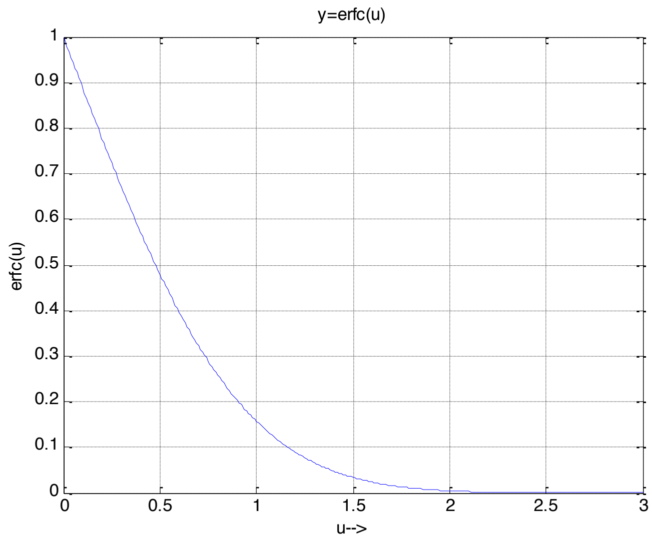


Figure: Erfc(u) versus u

1. *What is the final value of the head change (the value reached after infinite time,* $s(x,\infty)$*?*
2. *What value has the argument of erfc(-), i.e.* $\sqrt{\frac{x^2 S}{4kDt}}$ *when the head change is half the final value?*
3. *If kD=400 m²/d, S=0.1 and x=100m, after how much time is this the change of head equal to 0.5A?*
4. *What would be the formula if the head change occurred on time* t_1 *instead of time t=0?*
5. *How could you compute the head change at point x if the there was a sudden change of the river level of* A_1 *at time* $t=t_1$ *and another of* A_2 *at* $t=t_2$*?*

Question 5: Well in a confined aquifer

Consider a well in a confined aquifer starting an extraction of 1200 m^3/d at t=0. kD=1000 m^2/d, and S=0.001. For this case the Theis solution applies: $s = \frac{Q}{4\pi kD} W(u),\ u = \frac{r^2 S}{4kDt}$

(The type curve of Theis is given on a separate page).

d. Compute the head at r=20m after t=1 day.

e. The pump is switched off after 1 day. What is the head after 1.1 days at r=20 m?

Question 6: Well ina leaky aquifer

Consider a transient well in a leaky aquifer. kD=400 m^2/d, c=400 d, S=0.001, so that the groundwater behaves according to Hantush's transient well formula $s = \frac{Q}{4\pi kD} W\left(u, \frac{r}{L}\right)$ with $L = \sqrt{kDc}$.

f. How long does it take until the head at r=40 m becomes steady state or virtually steady state? (Hint: look at the type curves to get u for which this is the case, note).

Question 7: Well in an unconfined aquifer

Consider a well in an unconfined aquifer for which the Theis-solution applies (see type curve hereafter. Further given a pumping test with an extraction of 600 m^3/d during which drawdown measurements were made (see the graph with the small circles).

Interpret the test (that is: compute kD and S).

(Hint: if you can't see through the paper make the type curve thicker using a pen and hold both curves up against a light or in the direction of a window).

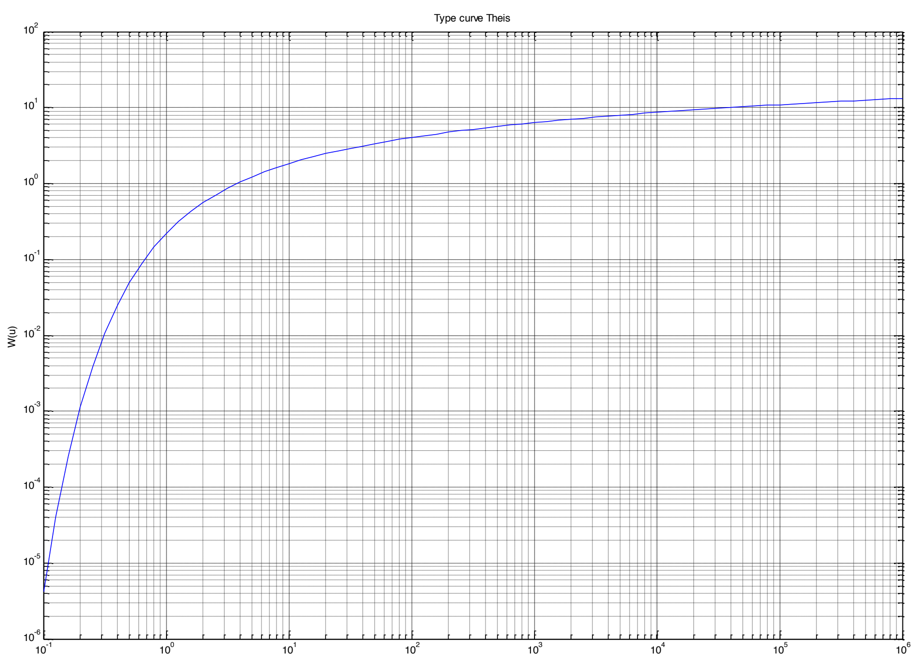


Figure: Theis type curve

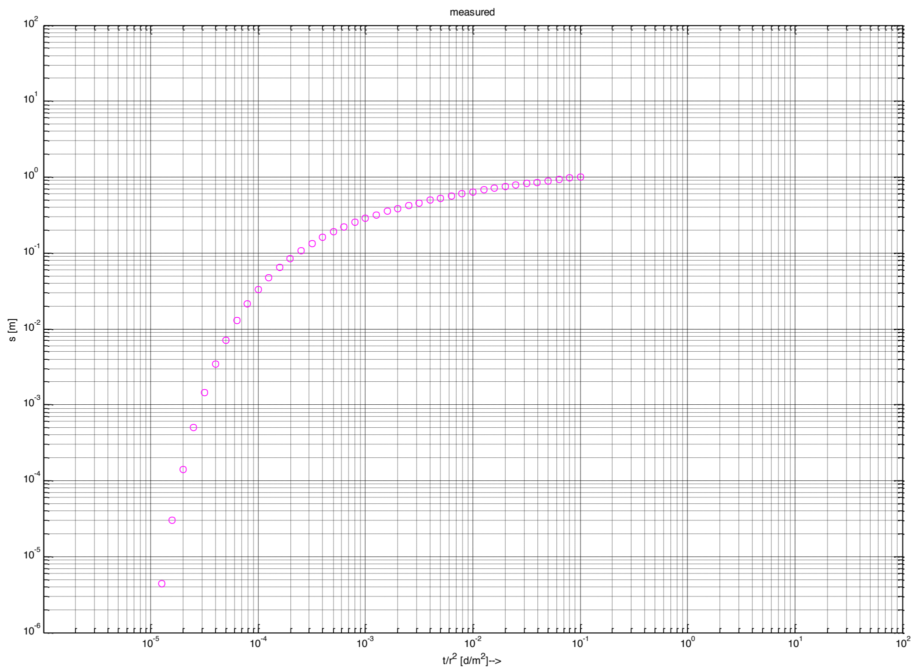


Figure: Measured data of pumping test (drawdown versus t/r^2)