

Exams Transient Groundwater FLOW - from 2006-2022

Prof.dr.ir.T.N.Olsthorn

June 2, 2022

Contents

| | |
|--|-----------|
| Open-book exam (1h), Feb 7, 2022 | 3 |
| Question 1: | 3 |
| Question 2: | 4 |
| Closed-book exam (1h), Feb 23, 2021 | 5 |
| Question 1: | 5 |
| Question 2: | 6 |
| Question 3: | 6 |
| Question 4: | 6 |
| Closed book exam (1h), Feb 4, 2020 | 8 |
| Question 1: | 8 |
| Question 2: | 8 |
| Question 3: | 9 |
| Question 4: | 9 |
| Question 5: | 10 |
| Closed book reexam (1h), March 2018 | 10 |
| Question 1 | 10 |
| Question 2 | 11 |
| Question 3 | 11 |
| Closed book exam (1h), Feb 7, 2017 | 12 |
| Question 1 | 12 |
| Question 2 | 13 |
| Question 3 | 14 |
| Closed book reexam (1h), 2016 | 15 |
| Question 1 | 15 |
| Question 2 | 15 |
| Question 3 | 16 |

| | |
|--|-----------|
| Closed-book exam (1h), Feb 1, 2016 | 16 |
| Question 1: (16 points) | 16 |
| Question 2: (14 points) | 17 |
| Closed-book exam (1h), Feb 2015 | 17 |
| Question 1 | 17 |
| Question 2 | 18 |
| Question 3 | 18 |
| Question 4 | 19 |
| Closed-book reexam (1h), March 2015 | 19 |
| Question 1 | 19 |
| Question 2 | 20 |
| Question 3 | 20 |
| Closed book exam, Feb 2014 | 21 |
| Question 1 | 21 |
| Question 2 | 21 |
| Question 3 | 22 |
| Question 4 | 22 |
| Closed-book exam (1h), Feb 3, 2011 | 23 |
| Question 1: Pressure in confined aquifer | 23 |
| Question 2 | 24 |
| Question 3 | 24 |
| Question 4: wells | 24 |
| Closed-book exam (1h), Feb 2010 | 25 |
| Question 1 | 25 |
| Question 2 | 25 |
| Question 3 | 26 |
| Question 4 | 26 |
| Closed-book exam (3h), Feb 2009 | 27 |
| Question 1 | 27 |
| Question 2 | 27 |
| Question 3 | 28 |
| Question 4 | 28 |
| Question 4 | 29 |
| Question 5 | 29 |
| Question 6 | 30 |
| Closed-book exam (3h), Feb 2007 | 31 |
| Question 1: General | 31 |
| Question 2: Diffusion equation | 31 |
| Question 3: Fluctuation groundwater | 31 |
| Question 4: Flow to an extraction canal | 32 |

| | |
|--|----|
| Question 4: Well in semi-confined aquifer | 32 |
| Question 6: Drawdown due to a pumping station in an unconfined aquifer | 32 |

Closed-book exam (3h), Feb 2006 33

| | |
|--|----|
| Question 1: Conceptual | 33 |
| Question 2: Characteristic time of groundwater basin | 34 |
| Question 3: Tides in groundwater | 35 |
| Question 4: Aquifer with river | 35 |
| Question 5: Well in a confined aquifer | 36 |
| Question 6: Well in a leaky aquifer | 37 |
| Question 7: Well in an unconfined aquifer | 37 |

Open-book exam (1h), Feb 7, 2022

Question 1:

A well is installed at 250 m distance from a river. The well fully penetrates the aquifer. The aquifer is unconfined, it has a conductivity of $k = 30$ m/d and a representative thickness of 40 m, which may be assumed constant. It further has a storage coefficient (specific yield) of $S = 0.20$. The well extracts water at a rate of $Q = 1200$ m³/d during exactly 7 days, after which it stops. The questions refer to the drawdown in point p at 50 m from the well on the line through the well perpendicular to the river as shown in the figure.

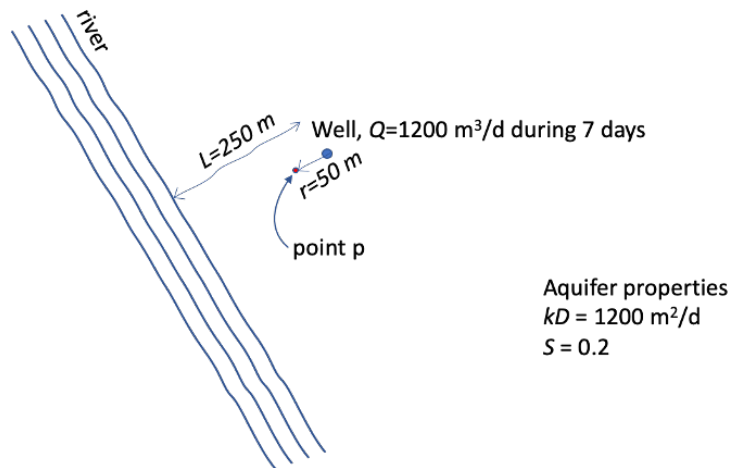


Figure 1: Well adjacent to a straight fully penetrating river. The aquifer is homogeneous and extends to infinity. This is clearly a well in an aquifer with constant transmissivity, for which the well-known Theis solution is applicable. To obtain values for the Theis well function, you can make use of the Theis type curve shown below.

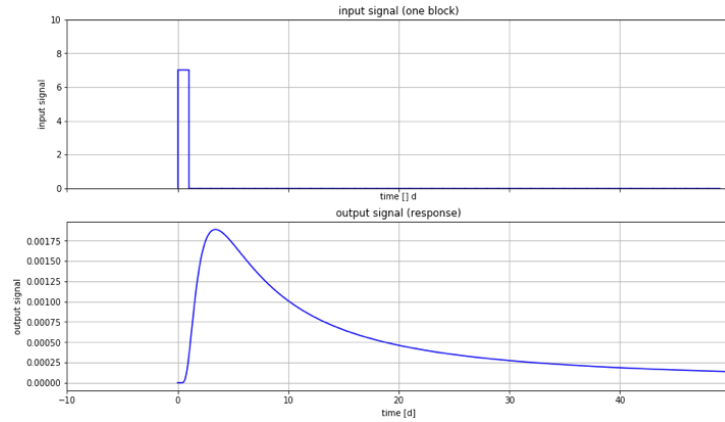


Figure 2: Theis type curve, i.e., the Theis well function as a function of $1/u$

1. What will be the drawdown at point p at $t = 7$ d after the start of the extraction?
2. What will be the drawdown at point p 14 days after the start of the extraction, i.e., 7 days after the well has started pumping?

Question 2:

The picture shows an aquifer bounded by a fully penetrating river at $x=0$. The aquifer is unbounded to the right and has a transmissivity and a specific yield as indicated in the picture. Note that the transmissivity may be considered constant. The river water level varies continuously according to a sinewave with a cycle time of $T=1$ d and an amplitude of $A=1.2$ m.



Figure 3: Aquifer bounded by fully penetrating water body with fluctuating water level at $x=0$. The aquifer extends at the right to infinity. Shown is the water table (or head) at an arbitrary time.

1. What is the maximum and minimum head at $x = 25$ m and at $x = 100$ m?
2. What is the delay of the wave at $x = 100$ m relative to the wave at $x = 0$ m?

- By how much (i.e., by how large a factor) does this delay change if the storage coefficient would be 100 times as small as the given value, i.e., if it would be $S = 0.001$ instead of $S = 0.1$?

The picture below shows an aquifer of limited lateral extent. To the left, at $x = 0$, it is bounded by a fully penetrating surface water body, such as a lake. To the right, at $x = L$, it is bounded by an impervious land mass as shown. The aquifer properties are shown in the picture, but you don't need them to answer the questions. The water level of the lake and the groundwater table are initially flat at a level equal to $h=0$ m as indicated by the horizontal blue line. At $t = 0$, the water level of the lake suddenly changes upward by an amount A as indicated. Ignore other hydrological features like rain and evapotranspiration. Only the effect of the sudden change of the lake level is considered.

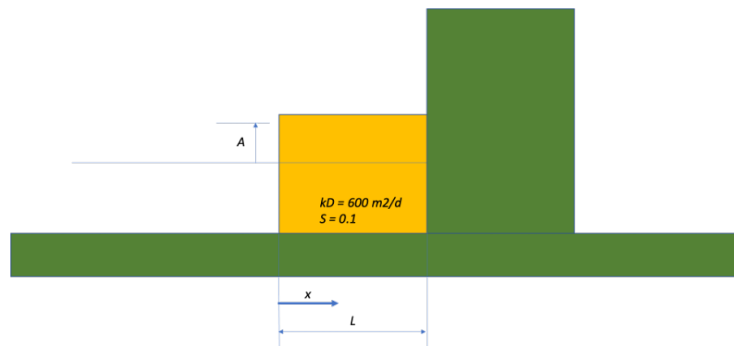


Figure 4: Picture of the aquifer with fully penetrating water body at $x = 0$ and impervious mass at $x = L$

- What are the boundary conditions at $x=0$ and $x=L$?
- Describe how the head in the aquifer will develop over time due to the sudden change at $x=0$ and $t=0$. Your description must include the situation at $t = 0$ and at $t = \infty$.

Closed-book exam (1h), Feb 23, 2021

Question 1:

- When pumping from a confined aquifer, all extracted water comes from storage. But what is the precise physical mechanism that causes the release of water from this type of aquifer? Explain.
- What is the so-called air-entry pressure and how does it relate to the capillary fringe? Explain.
- A confined aquifer system has a loading efficiency of $LE = 0.6$. If the barometer pressure increases with the equivalent of 40 cm water column,

by how much does the pressure in the aquifer change? By how much does the head (water level in a piezometer in this aquifer) change? Explain and show.

4. What is the difference between a Theis and a Hantush situation? The answer must contain the difference between the two situations as and what the physical origin is of the water pumped from a well in both these situations.

Question 2:

The solution for the head in a confined aquifer driven by surface water that varies according to a sine at $x=0$ is given by

$$h_{x,t} = Ae^{-ax} \sin(\omega t - ax), \text{ with } a = \sqrt{\frac{\omega S}{2kD}}$$

1. Explain what the parameters are with their dimension.
2. What is the velocity of the wave in the subsurface? Explain mathematically.
3. What are the so-called envelopes? Explain and show them mathematically.

Question 3:

The dynamic change of head in a strip of land of limited width like the one that is shown below can be computed using the simple formula for a half-infinite aquifer, but then we must apply superposition using so-called mirror ditches. In the figure below the water level at the left-hand side has just jumped up by A m and that at the right-hand side by B m. The head for $t=0.29$ d is shown. The lower picture shows the applied mirror/superposition scheme.

1. Is the shown mirror/superposition scheme used for the superposition correct? Clearly motivate your answer, a simple yes or no is not accepted.

Question 4:

During a pumping test with an extraction of $Q = 650 \text{ m}^3/\text{d}$, the drawdown is measured in an observation well at $r = 50$ m distance from the well, sufficient to ignore any influence of partial penetration on the measurements. The measured drawdown in this piezometer is shown graphically versus the log of time in days.

The formula for the drawdown that is expected to fit the data for sufficiently large times is

$$s_{r,t} = \frac{Q}{4\pi kD} \ln \left(\frac{2.25kDt}{r^2 S} \right)$$

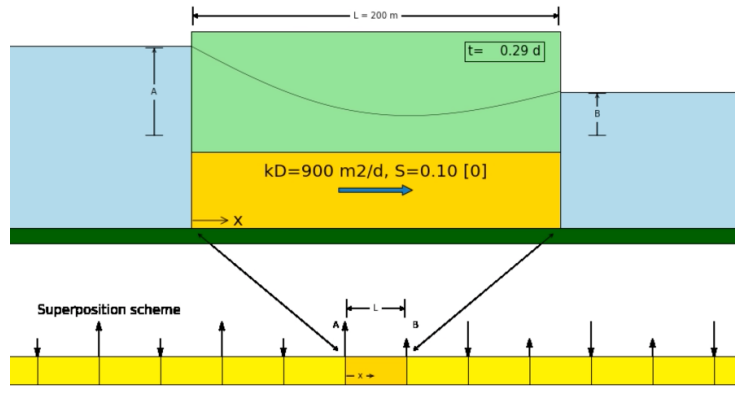


Figure 5: Strip of land bounded by fully penetrating surface water (top) and superposition scheme (bottom).

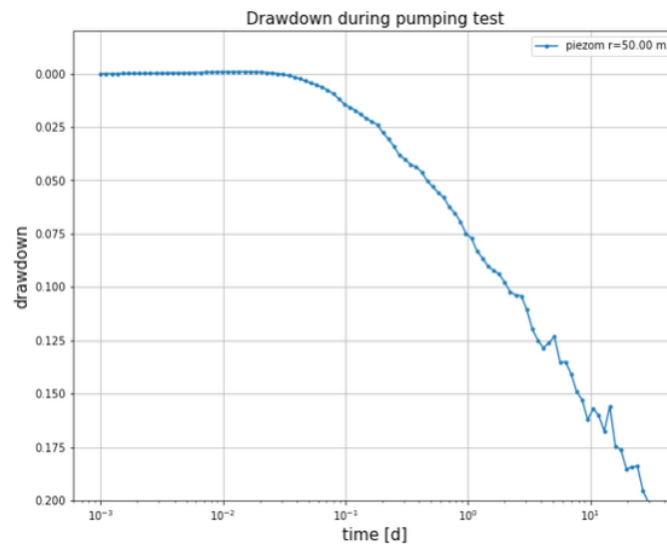


Figure 6: Measured drawdown during pumping test.

1. Do these data represent a Theis (confined/unconfined) or a Hantush (semi-confined) situation? Motivate your answer (a single yes or no is not accepted).
2. Determine the transmissivity of the aquifer
3. Determine the storage coefficient of the aquifer
4. What will be the radius of influence of this pumping test for $t = 5$ days?

Closed book exam (1h), Feb 4, 2020

Question 1:

1. Explain what is meant by air-entry pressure, and how you interpret it in terms of groundwater?
2. What happens to the water level in a piezometer installed in a confined aquifer if suddenly a load equivalent to a pressure increase Δp is placed on ground surface?
3. What happens to the water level in a piezometer if the barometer pressure suddenly change by an amount Δp ?
4. Explain what causes the difference between the answers to questions 2. And 3.
5. If a pressure transducer is fixed in a piezometer, below the water level at a given elevation, then what changes would it register in the two situations described in questions 2 and 3? (A pressure transducer measures and registers the absolute pressure, i.e. water + air).

Question 2:

Let the time-dependent change of head in a strip of land with width L [m] between two ditches be caused by a sudden change of water level equal to A [m] at the left ditch and equal to B [m] at the right ditch. We know that this can be computed using the formula that is valid for a half-infinite aquifer (that is an aquifer for which $x \geq 0$) bounded by surface water at $x = 0$, if we apply superposition. The formula for the half-infinite aquifer is

$$s(x, t) = A \operatorname{erfc} \left(x \sqrt{\frac{S}{4kDt}} \right)$$

In preparation of the superposition, a superposition scheme is drawn (see figure below), which shows the strip of land in dark yellow and the first few of the infinite series of mirror ditches. The arrows indicate the direction and size of the change of head at each ditch.

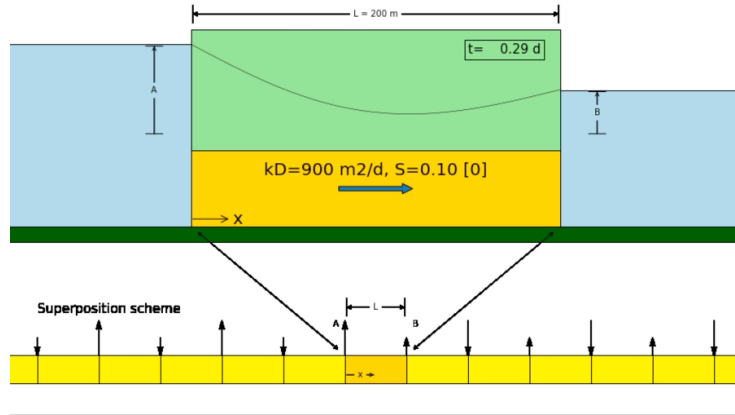


Figure 7: Strip of land bounded by fully penetrating surface water (top) and superposition scheme (bottom).

1. Is this scheme correct? Explain why or why not that is the case.

Question 3:

The first term of formula describing the drainage of a strip of land of with $L = 2b$, the head at $t = 0$ is uniform and equal to A [m] above the ditches on either side, is given by

$$s(x, t) = A \frac{4}{\pi} \cos\left(\frac{\pi x}{2b}\right) \exp\left(-\left(\frac{\pi}{2}\right)^2 \frac{t}{T}\right)$$

with

$$T = \frac{b^2 S}{kD}$$

1. What does this equation tell you? What's happening here? What name would you give to T ? Also explain why.
2. What is the halftime of this drainage process? Explain, and show it mathematically.

Question 4:

How would you compare the rate of drainage of a desert that is 500 km wide between surface -water boundaries and an arable field of 100 m wide between ditches, if both have the same aquifer properties?

Question 5:

The simplified Theis solution for the drawdown due to a pumping well in a (un)confined aquifer reads

$$s(r, t) = \frac{2.3Q}{4\pi kD} \log \left(\frac{2.25kDt}{r^2 S} \right)$$

A pumping test was carried out with an extraction of $Q = 2400 \text{ m}^3/\text{d}$. The drawdown was measured in 3 observation wells.

The figure shows the measured drawdown s in the observation wells as a function of t/r^2 on logarithmic scale.

Answer the following questions

1. What is the transmissivity? Explain and compute it.
2. What is the storage coefficient? Explain and compute it.
3. If you had only the drawdown in the well itself instead of in observation wells? What could you and what could you not determine, and why?
4. What is the radius of influence? Explain and show it mathematically.

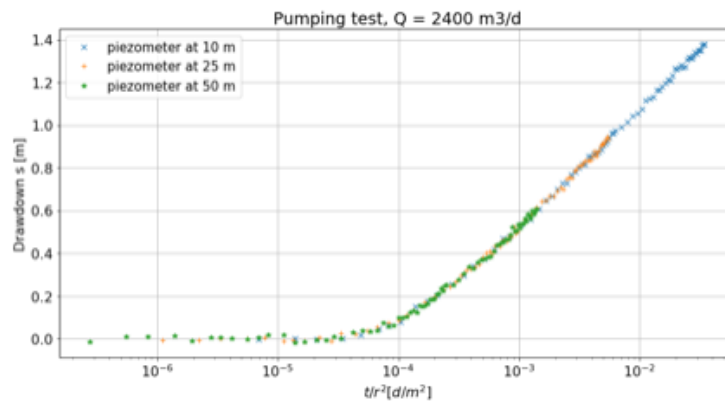


Figure 8: Measured drawdown of all piezometers versus t/r^2

Closed book reexam (1h), March 2018

Question 1

1. Explain what barometer efficiency (BE) is and how it physically works.
2. Explain in words what the characteristic halftime of an aquifer system says about the behavior of the system?

3. Consider the parameters L (system width), kD (transmissivity) and S_y (specific yield), for each of these three parameters, does an increase make the characteristic time larger or smaller?
4. What is capillary rise and what has capillary rise to do with air-entry pressure?
5. When we extract water from a well in an infinitely extended aquifer, from where does all this extracted water come? Explain your answer.

Question 2

Consider an aquifer in direct contact with the ocean. The tide of the ocean has an amplitude of $A = 1.0$ m and the cycle time is $T = 0.5$ d (one full tide in 12h). The aquifer is confined. The aquifer has the following properties: transmissivity $kD = 900 \text{ m}^2/\text{d}$ and storage coefficient $S_y = 0.002$. We are only interested in the effect of the tide land-inwards. The effect of the tidal fluctuation on the groundwater head land-inward, s , obeys the following expression:

$$s = A e^{-ax} \cos(\omega t - ax), \text{ where } a = \sqrt{\frac{\omega S}{2kD}}, \text{ with } \omega = \frac{2\pi}{T}$$

Notice that the difference between the uppercase S and lowercase s .

1. Explain the parameters in the expression and given their dimension
2. What is the amplitude of the groundwater head fluctuation at 750 m from the ocean? Explain your answer in a few words and first show it mathematically.
3. What is the delay of the head wave at 750 m from the ocean with respect to the tide? Hint: compute the velocity of the tidal wave in the aquifer and then the time until the peak of the wave starting at the ocean reaches $x = 750$ m. Explain in a few words your approach and start with showing your answer mathematically.

Question 3

Consider a well in an infinite water-table (phreatic) aquifer. Drawdowns are small compared to the thickness of the aquifer, so that $kD = 900 \text{ m}^2/\text{d}$ may be considered constant. The specific yield, $S_y = 0.15$, is also constant. As there are no boundary conditions, the drawdown by the well follows the Theis equation. An approximation of which is

$$s = \frac{Q}{4\pi kD} \ln \left(\frac{2.25kDt}{r^2 S} \right)$$

1. Derive a mathematical expression for the so-called radius of influence, that is, the distance beyond which the drawdown can be neglected at a given time t . Notice that $\ln(\dots) = 2.3 \log(\dots)$.
2. As you can see from the equation, the drawdown is (approximately) a logarithmic function in time. Derive a mathematical expression of the increase of the drawdown per log cycle of time, that is, between time t and time $10t$.
3. The figure below shows an example of an actual drawdown measured at a piezometer at $r = 100$ m from the well extracting $Q = 1200 \text{ m}^3/\text{d}$. Determine the transmissivity of the aquifer.
- 4) Bonus question (extra points): Determine the storage coefficient.

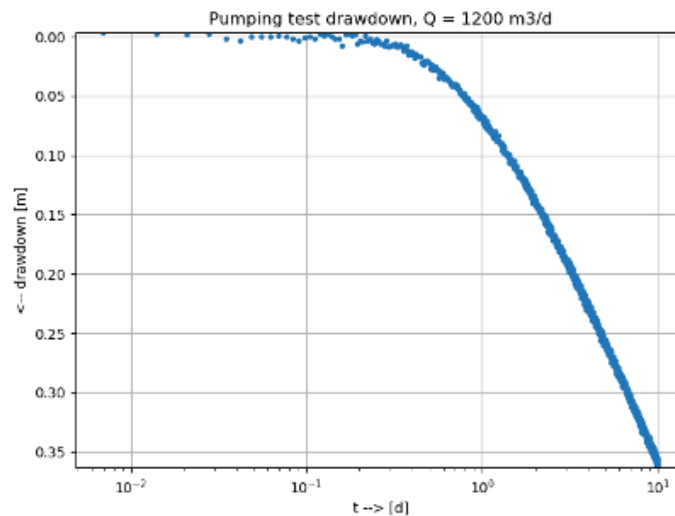


Figure 9: Measured drawdown in piezometer at $r = 100$ m from well extracting $Q = 1200 \text{ m}^3/\text{d}$

Closed book exam (1h), Feb 7, 2017

Question 1

1. Someone says the barometer efficiency of the piezometer in his garden is 25%. What does that mean? Explain your answer telling how this phenomenon physically works.
2. A pressure logger that is installed in this piezometer measures absolute pressure. What is absolute pressure? And what does this pressure gauge

see when the barometer rises by the equivalent of 40 cm of water column, given the barometric efficiency of 25%?

3. What two properties determine the value of the specific (elastic) storage coefficient?
4. What does the air-entry value have to do with the thickness of the capillary fringe/zone? Explain your answer.

Question 2

The transient drawdown of a well with a constant extraction Q in the case without any head boundary condition is mathematically described by the Theis well drawdown:

$$s = \frac{Q}{4\pi kD} W\left(\frac{r^2 S}{4kDt}\right)$$

that can be approximated by

$$s \approx \frac{Q}{4\pi kD} \ln\left(\frac{2.25kDt}{r^2 S}\right)$$

1. Sketch both graphs such that s is on a linear scale (downward positive) and time on a logarithmic horizontal scale. What's the difference between the two?
2. What is the drawdown per log-cycle of time, assuming that the approximation is valid?
3. What does "radius of influence" mean; how could you derive it from the above approximation?
4. Does the Theis drawdown reach a steady-state situation in the long run? Explain your answer.

We know the total discharge (flow) across a ring at fixed distance r from the well in a confined aquifer is given by

$$Q_{r,t} = Q_0 e^{-u}, \text{ with } u = \frac{r^2 S}{4kDt}$$

1. If you assume you are at a fixed distance r from the well, could you then formulate a characteristic time for the transient phenomenon $Q(r,t)$? Explain your answer.

Below, we observe a hydrologist interpreting a transient pumping test in a confined aquifer. He/she plotted the drawdown data on a double log graph with drawdown s vertically upward and t/r^2 horizontally. The data of this graph with the measurements was then shifted over the Theis type-curve until the best

possible match was obtained. This match is shown in the figure. Given that the extraction during the pumping test was $1200 \text{ m}^3/\text{d}$.

1. Determine the transmissivity kD and the storage coefficient S .

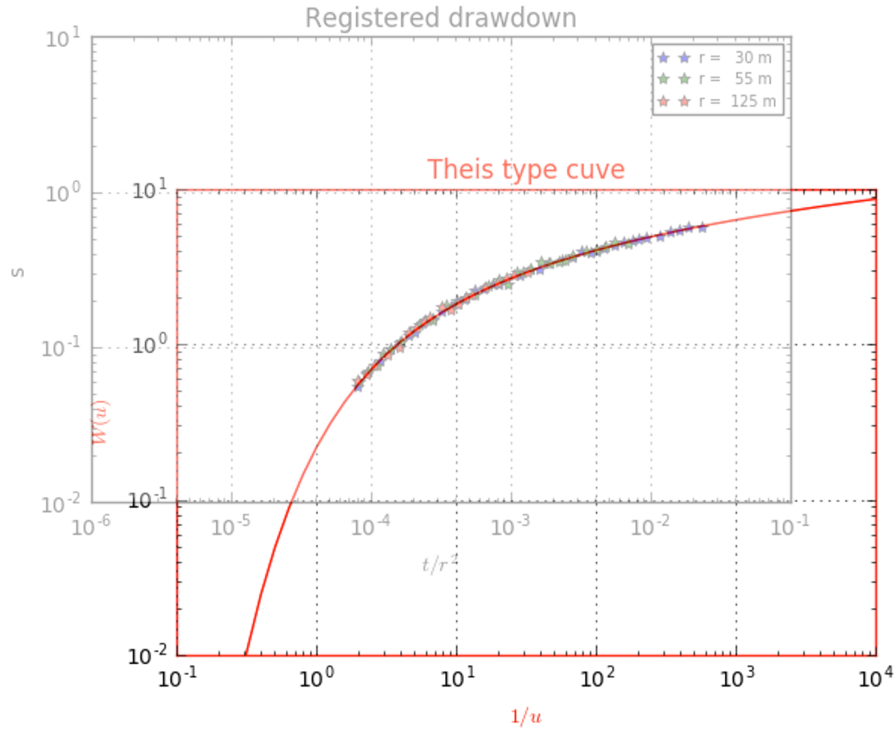


Figure 10: Measured drawdown curve matched with Theis type curve.

Question 3

Imagine the sea tide acting on a shore that has a confined aquifer inland with a constant kD and S in good vertical contact with the sea. The tide waves, which are characterized by A and ω , therefore, penetrate the aquifer; they are mathematically described by:

$$s_{x,t} = A e^{-ax} \sin(\omega t - ax)$$

1. As can be seen, this equation describes two simultaneous phenomena. Which are these two phenomena?

The factor a in the equation was derived to be

$$a = \sqrt{\frac{\omega S}{2kD}}$$

1. What are the parameters with their dimensions?
2. A tidal wave has a frequency ω of two cycles per day of 24 hours, or a cycle time T of 12 hours. Large wind waves, however, have a cycle time of only about 12 seconds. How far does the influence of these wind waves penetrate into the aquifer compared to the influence of the tide waves? Give their ratio and sketch the envelope of both to show this difference (the sketch does not have to be on scale).

Closed book reexam (1h), 2016

Question 1

1. Explain what barometer efficiency (BE) is and how it physically works.
2. Explain in words what the characteristic (half) time of a groundwater system is. What does it say about the behavior of the system?
3. For which of the parameters L (system width), kD (transmissivity) and S_y (specific yield) would an increase make the characteristic system time smaller?
4. Explain why in hydrological logic you think that this is the case.
5. If you see a close-up of two grains held together by a small amount of water at their point of contact. What then is the pressure in that water? Explain why that is so.

Question 2

Consider an aquifer in direct contact with the ocean. The tide of the ocean has an amplitude $A = 1.0$ m and the cycle time is $T = 0.5$ d (one full tide in 12h). The aquifer is confined. It consists of two parts. The first part reaches from the ocean to 500 m inland, the second part is present at more than 500 m from the ocean. The first part of the aquifer has the following properties: transmissivity $kD = 900 \text{ m}^2/\text{d}$ and storage coefficient $S = 0.002$. The second part of the aquifer has the following properties: $kD = 1800 \text{ m}^2/\text{d}$ and storage coefficient $S = 0.001$. Because we consider the fluctuation of the head to be superposed on the mean head, we are only interested in the head s relative to the mean head at every location, that is, in $s(x, t) = h(x, t) - h(x)$. This head fluctuation, s , obeys following expression:

$$s = A e^{-ax} \cos(\omega t - ax), \text{ where } a = \sqrt{\frac{\omega S}{2kD}} \text{ and } \omega = \frac{2\pi}{T}$$

Notice that the storage coefficient is capital S and the head relative to the mean head is lowercase s .

1. Explain the parameters in the expression and given their dimension
2. What is the amplitude of the groundwater head fluctuation, that is, the amplitude of s , in the aquifer at 500 m and at 1000 m from the ocean?
3. What is the delay of the head wave at 500 m and 1000 m relative to the ocean tide? Hint: compute the velocity of the tidal wave in the aquifer and then the time until the peak of the wave starting at the ocean reaches $x = 500$ m and $x = 1000$ m.

Question 3

Consider a well in an infinite water table (phreatic) aquifer. Drawdowns are considered small compared to the thickness of the aquifer, so that $kD = 900 \text{ m}^2\text{d}$ may be considered constant. The specific yield, $S_y = 0.15$, is also constant. As there are no boundary conditions, the drawdown by the well follows the Theis equation. An approximation of which is

$$s \approx \frac{Q}{4\pi kD} \frac{2.3Q}{4\pi kD} \log \left(\frac{2.25kDt}{r^2 S} \right)$$

1. Derive a mathematical expression for the so-called radius of influence, that is, the distance beyond which the drawdown can be neglected at a given time t after the well was first switched on.
2. As you can see, the drawdown is a logarithmic function in time. Derive a mathematical expression of the increase of the drawdown per log cycle, that is, between for instance $t = 6$ d and $t = 60$ d, or $t = 2$ d and $t = 20$ d.
3. Assume the well has been continuously pumping for time $t = t_1$, after which the extraction was stopped. What is the drawdown at distance r_0 at time is $t = t_1 + \Delta t$, where Δt is any time passed since t_1 .

Closed-book exam (1h), Feb 1, 2016

Question 1: (16 points)

1. Explain loading efficiency, LE .
2. Explain the barometer efficiency, BE .
3. What is the difference registered by a pressure gauge in a confined aquifer measuring absolute pressure, given on the one hand a uniform mass placed at ground surface of weight $\Delta p \text{ N/m}^2$ and on the other hand a barometer increase of the same value of $\Delta p \text{ N/m}^2$?
4. What is the origin of delayed yield?

5. In which case does the influence of tide reach further inland into an aquifer?
 - (a) The case with the higher or with the lower frequency?
 - (b) The case with the larger or the smaller transmissivity kD ?
 - (c) The case with the larger or with the smaller storage coefficient S ?
6. What is the difference between the situations with the wells that were studied by Theis and by Hantush?
7. Does the Theis case have a final equilibrium drawdown? Explain your answer?
8. Does the Hantush case have a final, steady-state drawdown? Explain your answer?

Question 2: (14 points)

1. Explain what is the radius of influence of an extraction well in an aquifer of constant transmissivity and storage coefficient?
2. The simplified Theis solution is as follows:

$$s(r, t) \approx \frac{Q}{4\pi kD} \ln \left(\frac{2.25kDt}{r^2 S} \right)$$

From it derive an expression of the radius of influence.

1. Also show what is the drawdown difference per log cycle of time, that is, between time is t and time is $10t$.
2. Consider a well in a water table aquifer at 300 m from an impervious wall that reaches to the bottom of the aquifer. The aquifer has $kD = 600 \text{ m}^2/\text{d}$ and the specific yield of $S_y = 0.2$. The pumping rate is $Q = 1200 \text{ m}^3/\text{d}$. Assume that the approximation of the Theis equation that is given in this question is applicable. Compute the head change of the groundwater at the wall closest to the well.

Closed-book exam (1h), Feb 2015

Question 1

1. What types of storage or storage coefficients are associated with transient groundwater flow? And explain short how they physically work.
2. Explain the relation between capillary rise and pore diameter
3. Explain the general shape of the moisture curve in the unsaturated zone. Describe where the water comes from when the water table is lowered.

4. Explain the difference between the loading efficiency (LE) and the barometer efficiency (BE)?
5. When you see animal holes in the field, like rabbit, rat and worm holes, how much do you think these holes may contribute to the infiltration of rainwater during and after showers, to what extent are the animals living in those holes affected by heavy rains, and , finally, what would it take to swim them out of their holes? Explain your answer from your insight in how water in the subsurface behaves.

Question 2

Consider a confined aquifer in direct contact with the ocean in which the head fluctuates along with the tide of the ocean. The daily solar tide, with cycle time $T = 12$ h or, equivalently, $T = 0.5$ d, has amplitude $A = 2.5$ m and the 4 weekly moon tide, with cycle time $T = 1/28$ d, has amplitude or $A = 1$ m. The groundwater head in the aquifer relative to the mean value at time t and distance x from the ocean obeys to the following expression:

$$s = Ae^{-ax} \cos(\omega t - ax), \text{ where } a = \sqrt{\frac{\omega S}{2kD}}$$

If T is the time required for a complete cycle, then the angular velocity $\omega = 2\pi/T$.

Further, $kD = 900 \text{ m}^2/\text{d}$ and $S = 0.001$.

1. Explain the parameters in the expression and give their dimension.
2. What is the amplitude of the groundwater fluctuation due to both tides individually at 500 and 2000 m from the coast? So the twice-a-day tide amplitude at 500 m and at 2000 m and the 28-day tide amplitude at 500 m and 2000 m?
3. How much are the waves of both tides delayed at 500 m from the coast?
4. Over what distance does the maximum tide-induced amplitude in the groundwater declines by a factor of two in both cases?

Question 3

A groundwater table rise after it was agitated by a sudden recharge N [m] will decay over the thereafter. For a system of bounded by two parallel water courses at L mutual distance, this decline after some time can be approximated by the following expression:

$$s = A \frac{4}{\pi} \cos\left(\pi \frac{x}{L}\right) \exp\left(-\left(\frac{\pi}{L}\right)^2 \frac{kD}{S_y} t\right)$$

1. Describe the parameters and given their dimension.

2. Give an expression for the sudden rise A caused by a sudden recharge amount equal to N [m]:
3. Describe in a few words what this expression is and does, so what does its graph look like and how does it behave over time.
4. Give an expression of what can be called characteristic time of this system.
5. Derive an expression of the half time of this system.
6. Derive an expression for the discharge of this system.

Question 4

A 300 m deep well in Jordan with borehole radius $r = 0.25$ m was drilled in a limestone aquifer to serve a refugee camp. The well was recently test pumped during one day at a rate of $Q = 60 \text{ m}^3/\text{h}$. The head at 0, 0.01, 0.1 and 1 d after the start of the pump was 100, 135, 147 and 159 m below ground surface respectively. The pump is installed at 200 m below ground surface.

Further assume:

The estimated specific yield of this aquifer is 0.01.

The unknown transmissivity is constant.

You may use the simplified expression of transient drawdown in an infinite aquifer

$$s \approx \frac{Q}{4\pi kD} \ln \left(\frac{2.25kDt}{r^2 S} \right)$$

1. Estimate the transmissivity of this aquifer.
2. How much will be the drawdown after 3 years (1000 d)? Is the pump at 200 m below ground surface (i.e. 100 m below the initial water table) still deep enough to pump the water up?
3. Another well of equal size, depth and flow rate is planned at a second location in the camp at 2 km distance. How much will be the drawdown in each well after 3 years (1000 days) in this case? Assume that both wells pump for the same period. How deep should the pumps be installed to allow pumping both wells at the given rate for 3 years?

Closed-book reexam (1h), March 2015

Question 1

1. Explain what barometer efficiency (BE) is and how it physically works?
2. When you see animal holes in the field, like rabbit, rat and worm holes, how much do you think these holes may contribute to the infiltration of

rainwater during and after showers, to what extent are the animals living in those holes affected by heavy rains, and , finally, what would it take to swim them out of their holes? Explain your answer from your insight in how water in the subsurface behaves.

Question 2

Consider an aquifer in direct contact with the ocean. The tide of the ocean has an amplitude $A = 1$ mm and cycle time is $T = 0.5$ d (one full tide in 12h).

The aquifer is confined. It consists of two parts. The first part reaches from the ocean to 500 m in land, the second part is present at more than 500 m from the ocean.

The first part of the aquifer has the following properties: transmissivity $kD = 900 \text{ m}^2/\text{d}$ and storage coefficient $S = 0.002$.

The second part of the aquifer has the following properties, $kD = 1800 \text{ m}^2/\text{d}$ and storage coefficient $S = 0.001$.

Because we consider the fluctuation of the head to be superposed on the mean head, we are only interested in the head, s , relative to the mean head at every location, that is $s(x, t) = h(x, t) - h(x)$. This head fluctuation, s , obeys following expression:

$$s = A e^{-ax} \cos(\omega t - ax), \text{ where } a = \sqrt{\frac{\omega S}{2kD}} \text{ and } \omega = \frac{2\pi}{T}$$

Notice that the storage coefficient is capital S and the head relative to the mean head is lower case s .

1. Explain the parameters in the expression and give their dimension.
2. What is the amplitude of the groundwater head fluctuation, that is, the amplitude of s , in the aquifer at 500 m and at 1000 m from the ocean?
3. What is the delay of the head wave at 500 and 1000 m relative to the ocean tide? Hint: compute the velocity of the tidal wave in the aquifer and then the time until the peak of the wave starting at the ocean reaches $x = 500$ m and $x = 1000$ m?

Question 3

Consider a well in an infinite water-table (phreatic) aquifer. Drawdowns are considered small compared to the thickness of the aquifer, so that $kD = 900 \text{ m}^2/\text{d}$ may be considered constant. The specific yield, $S_y = 0.15$, is also constant. As there are no boundary conditions, the drawdown by the well follows the Theis equation. An approximation of which is

$$s \approx \frac{2.3Q}{4\pi kD} \log \left(\frac{2.25kDt}{r^2 S} \right)$$

1. Derive a mathematical expression for the so-called radius of influence, that is, the distance beyond which the drawdown can be neglected at a given time t after the well was first switched on.
2. As you can see the drawdown is logarithmic in time. Derive a mathematical expression for the increase of the drawdown per log cycle, that is between for instance $t = 6$ d and $t = 60$ d days, or $t = 2$ d days and $t = 20$ d.
3. Assume the well has been continuously pumping for time $t = t_1$, after which the extraction was stopped. What is the drawdown at distance r_0 at time $t = t_1 + \Delta t$, where Δt is any time passed since t_1 .

Closed book exam, Feb 2014

Question 1

1. What types of storage or storage coefficients are associated with transient groundwater flow?
2. Explain how these types of storage physically work.
3. Explain the relation between capillary fringe and air entry pressure.
4. Explain the difference between the loading efficiency (LE) and the barometer efficiency (BE)?
5. Why does the specific yield of unconfined aquifers with a shallow groundwater table depend on the depth of the water table?
6. What is halftime when considering decay of a water mound between rivers? How would you describe it?
7. Assume the well has been continuously pumping for time $t = t_1$, after which the extraction was stopped. What is the drawdown at distance r_0 at time $t = t_1 + \Delta t$, where Δt is any time passed since t_1 .

Question 2

Consider a confined aquifer in direct contact with the ocean in which the head fluctuates along with the tide of the ocean. The tide has an amplitude of $a = 2.5$ m. The groundwater head in the aquifer at time t and distance x from the ocean obeys to the following expression:

$$s = a e^{-ax} \cos(\omega t - ax), \text{ where } a = \sqrt{\frac{\omega S}{2kD}}$$

The frequency f of the tide is one complete cycle per 24 hours, i.e. $f = 1/d$, with, of course, $\omega = 2\pi f$.

We don't know the value of kD and S , but we have measured the amplitude of the groundwater head fluctuation at . This amplitude is 25 cm, one tenth of that of the ocean.

1. Explain the parameters in the expression and give their dimension.
2. Give an expression for the amplitude at distance x from the ocean.
3. With the given information, compute parameter a , and the diffusivity of the aquifer, i.e. the ratio kD/S .
4. Give an expression for the velocity of the wave of the groundwater-head in the subsurface? How much is this velocity?

Question 3

Consider an unconfined aquifer with conductivity $k = 10 \text{ m/d}$, a specific yield of $S_y = 0.1$ and an initial thickness . A well is located in this aquifer on each of the four corners of a square with sides of $L = 200 \text{ m}$. The wells start pumping at $t = 0$. They pump with a rate of $Q = 120 \text{ m}^3/\text{d}$ for 1d, after which they stop.

The drawdown according to Theis is

$$s = \frac{Q}{4\pi kD} W(u), \text{ where } u = \frac{r^2 S}{4kDt}$$

The Theis well function is graphically given in Figure 1 below.

1. Compute the drawdown in the center of the square after $t = 2 \text{ d}$. You may neglect the change of the transmissivity caused by the change of the water depth in the aquifer.

Question 4

Given that the well function can be computed by the following infinite series

$$W(u) = -\gamma - \ln u + u - \frac{u^2}{2 \times 2!} + \frac{u^3}{3 \times 3!} - \frac{u^4}{4 \times 4!} + \dots$$

with $\gamma = 0.577216 \dots$ and $u = r^2 S / (4kDt)$

1. What would be a good approximation of the drawdown for small values of u ? (Assume for instance that $u < 0.01$). Notice for mathematical convenience, that $\gamma = \ln(e^\gamma)$.

2. How could you define the radius of influence of the drawdown? Use the formula for the drawdown from the previous question together with the approximation from the previous question.

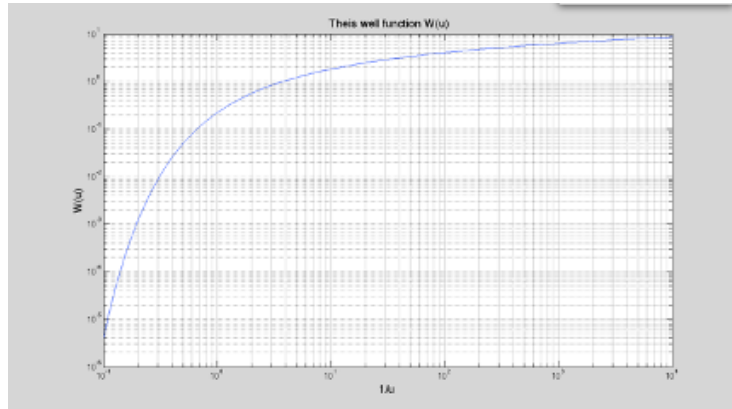


Figure 11: Theis well function type curve, ie, $W(u)$ versus $1/u$.

Closed-book exam (1h), Feb 3, 2011

Question 1: Pressure in confined aquifer

Question 1: Pressure in confined aquifer

A water level in a piezometer in a confined aquifer is affected if the weight on a ground surface is suddenly changed. Compare two situations a) Sudden change by a load placed on the ground, such as sand or flooding by water and b) Sudden increase of the barometric pressure.

Case a: — a load is placed on ground surface

1. How does the water pressure change in the piezometer? (Up? Down? Not?)
2. How does the head change in the piezometer? (Up? Down? Not?)

Case b: — barometer pressure increased

1. How does the water pressure change in the piezometer? (Up? Down? Not?)
2. How does the head change in the piezometer? (Up? Down? Not?)

General:

1. If there is a difference between the two cases, then why is that?

Question 2

The groundwater head variation in a confined aquifer due to a tidal wave at $x=0$ can be expressed mathematically as follows:

$$s(x, t) = \phi(x, t) - \phi_0 + A \exp(-ax) \sin(\omega t - ax), \text{ in which } a = \sqrt{\frac{\omega S}{2kD}}$$

and $\omega = \frac{2\pi}{T}$ with T the period of the wave.

1. What is the amplitude of the wave at distance x ?
2. What is the velocity of the wave?
3. If the wave would be just observable in a piezometer at $x = 1000$ m from the coast, then at what distance would the wave be just observable on another spot along the coast where the storage coefficient is 100 times greater than at the current spot and the transmissivity is the same?
4. A tidal wave occurring daily is just observable in the aquifer at a distance of $x = 500$ m from shore, where $x = 0$. At what distance from the shore will a 14-day wave be just observable occurring due to the monthly moon cycle? Assume the same amplitude for both waves.

Question 3

In class we discussed the somewhat complicated solution by series expansion of the evolution of the head after a sudden rain shower of P [m] in a strip of land of width L [m] between parallel fixed-head boundaries with water level ϕ_0 [m]. We have seen that after some time t [d], only the first term matters, which is

$$s(x, t) = \phi(x, t) - \phi_0 = \frac{P}{S_y} \frac{4}{\pi} \cos\left(\pi \frac{x}{L}\right) \exp\left(-\pi^2 \frac{kD}{L^2 S_y} t\right)$$

Whenever possible express your answers mathematically:

1. What is the shape of the head?
2. What is the maximum head, take $\phi_0 = 0$?
3. What would you consider the characteristic time of this system?
4. What would be the half-time of this groundwater system?

Question 4: wells

The solution by Theis is given by

$$s(r, t) = \phi_0 - \phi(r, t) = \frac{Q_0}{4\pi kD} W(u), \text{ where } u = \frac{r^2 S}{4kDt}$$

1. What flow conditions are described by Theis' well solution?
2. What are its parameters and what are their dimensions?

As you know, the function $W(u)$ is the exponential integral, which may be written as a series expansion :

$$W(u) = -0.577316 - \ln u + u - \frac{u^2}{2 \times 2!} - \frac{u^3}{3 \times 3!} + \frac{u^4}{4 \times 4!} \dots$$

1. How can you mathematically approximate the Theis' solution for very small values of u given that $-0.577216 \approx \ln(0.5615)$?
2. With this approximation, mathematically give the difference between the drawdown obtained at time t and the drawdown at time $10t$ in a piezometer at some arbitrary distance r from the well.

You may use the fact that $\ln 10 \approx 2.3$.

Also give the difference between the drawdown in a piezometer at distance r from the well and in a piezometer at distance $10r$ from the well, both at the same time.

Closed-book exam (1h), Feb 2010

Question 1

1. What is liquefaction?
2. What is the difference between specific yield and elastic storage?
3. How does the specific yield change if an already shallow water table rises?
4. Why does this happen (make a sketch and explain)
5. Which of the two materials, gravel and fine sand, has the highest specific yield and why (assume both have the same porosity)?

Question 2

The head in an aquifer connected to the ocean fluctuates due to tide. This fluctuation is given by the following formula, in which s expresses the head variation caused by the tide as a function of time t and the inland distance from the shore x :

$$s(x, t) = A \exp(-ax) \sin(\omega t - ax), \text{ with } a = \sqrt{\frac{\omega S}{2kD}}$$

1. What is the expression for the maximum head fluctuation as a function of x ?
2. Sketch the head change s as a function of x at time $t=0$ and sketch also the envelope (maximum and minimum value of s as a function of x)
3. Which parameters increase the inland penetration of the tide and which parameters decrease this inland penetration?

Question 3

Consider an extraction canal in direct contact with an aquifer of infinite extent. The aquifer has transmissivity $kD = 400 \text{ m}^2/\text{d}$ and specific yield $S_y = 0.1$. As long as $t < 0$, the head in the aquifer is everywhere 0 m (we take the initial water level as our reference level).

At time $t = 0$ d, the water level in the canal suddenly changes to 2 m. Then, at time $t = 2$ d, the water level in the canal suddenly changes back to its original value of 0 m and remains constant afterwards.

The head change and the head-change gradient are:

$$s = s_0 \operatorname{erfc} \left(\sqrt{\frac{x^2 S}{4kDt}} \right) \text{ and } \frac{\partial s}{\partial x} = -s_0 \sqrt{\frac{S}{\pi kDt}} \exp \left(-\frac{x^2 S}{4kDt} \right)$$

To obtain values for the erfc function, use the graph below.

Answer the following two questions.

1. Compute the head at $x = 100$ m at $t = 3$ d. Show the formula you use and include the dimension in your answer!
2. Compute the discharge at $x = 0$ at $t = 3$ d. Show the formula you use and include the dimension in your answer?

Question 4

Consider a well in a system of infinite extent which starts extracting at time $t=0$. We know that Theis' formula applies:

$$s(r, t) = \frac{Q}{4\pi kD} W(u), \text{ where } u = \frac{r^2 S}{4kDt}$$

We also know that for small values of u , the well function, $W(u)$, can be approximated by a straight line on log-t scale, which is given by:

$$W(u) \approx 2.3 \log \left(\frac{0.5625}{u} \right) = 2.3 \log \left(\frac{2.25kDt}{r^2 S} \right)$$

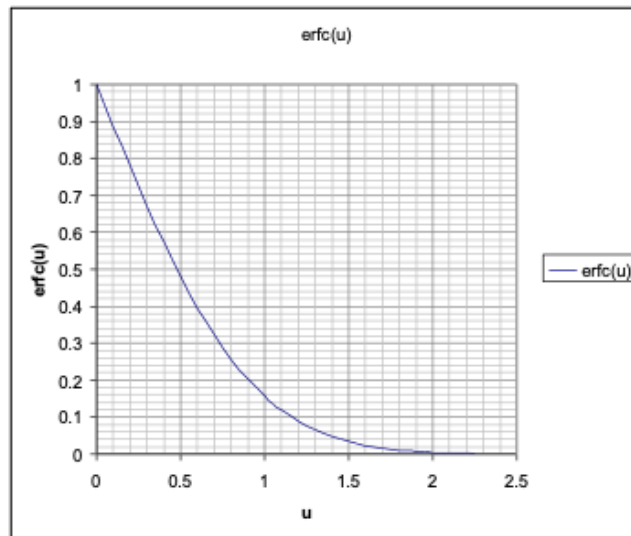


Figure 12: Function $\text{erfc}(u)$

Consider a pumping test on this well, starting the constant extraction $Q = 800 \text{ m}^3/\text{d}$ at $t = 0$. The drawdown is measured over a number of days at an observation well at 25 m distance. The measured drawdowns are shown in the figure below, which clearly reveals the straight portion of the drawdown that we expect from the expression above for large-enough values of time.

1. Using the straight line through the measured data, compute the transmissivity kD and the storage coefficient S of this aquifer.

Closed-book exam (3h), Feb 2009

Question 1

1. What is the difference between specific yield and elastic storage?
2. How does the specific yield change if an already shallow water table rises further and becomes even shallower?
3. Why does this happen (make a sketch and explain)
4. Which of the two materials, gravel and fine sand, has the highest specific yield and why (assume both have the same porosity)?

Question 2

1. What do we mean by Loading Efficiency (LE) and what do we mean by Barometric Efficiency (BE)?

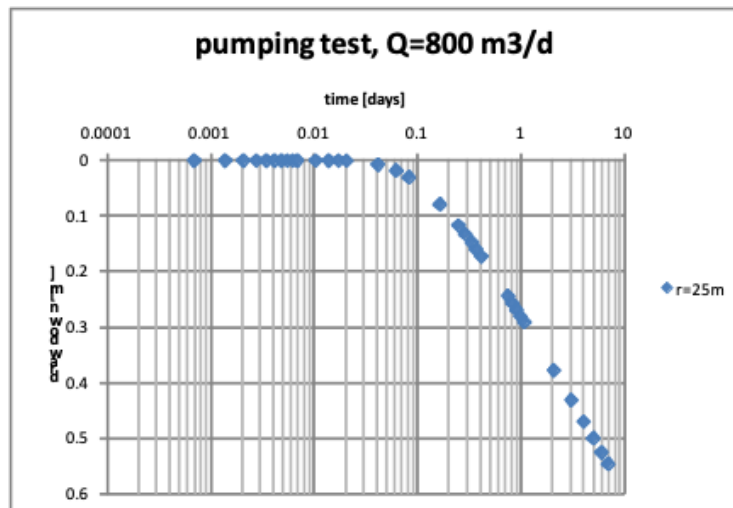


Figure 13: Measured drawdown during pumping test.

2. What is the difference in terms of head change if we compare a loading on land surface with an equal increase of the barometer pressure? And why?

Question 3

Tidal flow in a confined aquifer may be described mathematically by

$$s = A e^{-ax} \sin(\omega t - ax), \text{ where } a = \sqrt{\frac{\omega S}{2kD}}$$

1. What are the different quantities in these expressions and what are their dimensions?
2. By what expression is the envelope given (the envelope describes the maximum amplitude as a function of x ?)
3. How does the envelope change if the frequency of the tide would double?
4. How will the envelope change if the transmissivity would be two times less and the storage coefficient 100 times less?

Question 4

The picture below shows a strip of land of width L bounded by two canals. Both the strip and the canals run perpendicular to the paper (so the picture is a cross section). Suddenly the water level in the left canal is raised by A m as is indicated in the figure. This causes the head to change in the strip. At the right hand side the water level is unchanged. There exists an expression,

which mathematically describes the effect of a sudden level rise in a strip that is unbounded on one side. We want to use this expression to compute the head in the strip. We can do this by means of mirror canals.

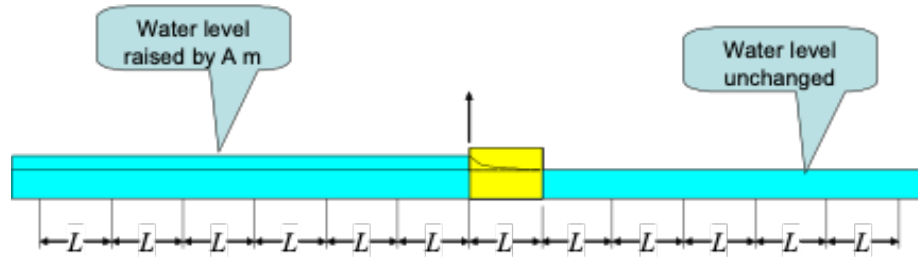


Figure 14: Strip of land of width L bounded by open water. The water level at the left hand side was suddenly raised by A m. This causes the head in the aquifer of the strip to change dynamically.

1. Irrespective of what the mathematical looks like, where would you put the mirror canals and which are positive and which are negative? Just draw an arrow respectively up or down (see figure) at the locations where you would put the mirror canal.

Question 4

The characteristic dynamics of a groundwater systems (i.e., the time it takes for the head of a groundwater system to reach equilibrium) is related to the argument of transient groundwater flow solutions, This argument is $\sqrt{\frac{x^2 S}{4kDt}}$ in solutions for one-dimensional flow and $\frac{r^2 S}{4kDt}$ for radial flow such as in the well functions of Theis and Hantush.

1. Explain how the characteristic dynamics relate to these arguments?
2. Compare the characteristic dynamics of two systems. System two is twice as wide as system one and its transmissivity is 3 times as large and its storage coefficient 100 times as small as that of system one. How do the dynamics of these two systems relate to each other, that is: how many times faster or slower is system two compared to system one in reaching piezometric equilibrium?

Question 5

Consider a well in a semi-confined aquifer with $kD = 900 \text{ m}^2/\text{d}$, $S = 0.001$ and $c = 400 \text{ d}$ that is pumped at a discharge $Q = 2400 \text{ m}^3/\text{d}$.

1. How long does it take before the drawdown at 60 m distance from the well becomes stationary?

2. What is the final drawdown?

Question 6

A pumping test has been carried out in a confined aquifer. The drawdown and the Theis type curves are given in the graphs below. These graphs have been drawn on the same type of double logarithmic paper. The extraction of the well during the test was $1000 \text{ m}^3/\text{d}$. Determine the transmissivity and the storage coefficient of this groundwater system.

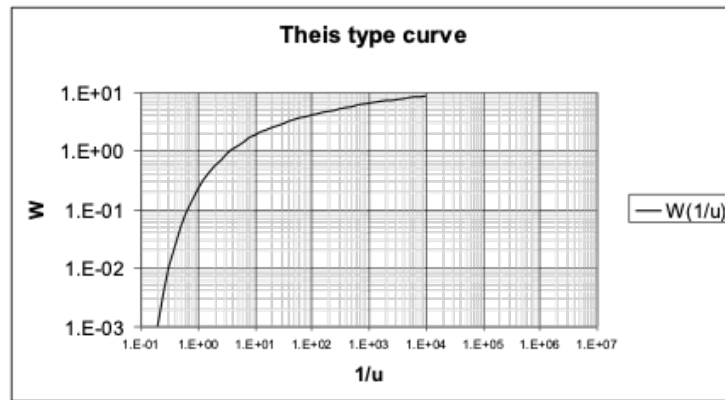


Figure 15: Theis well function Type curve.

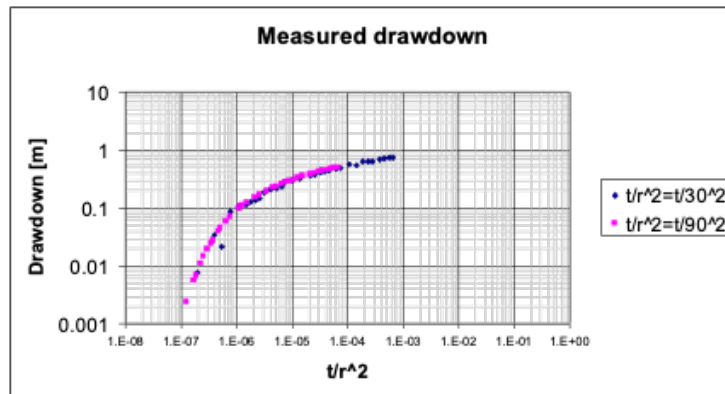


Figure 16: Measured drawdown versus t/r^2 .

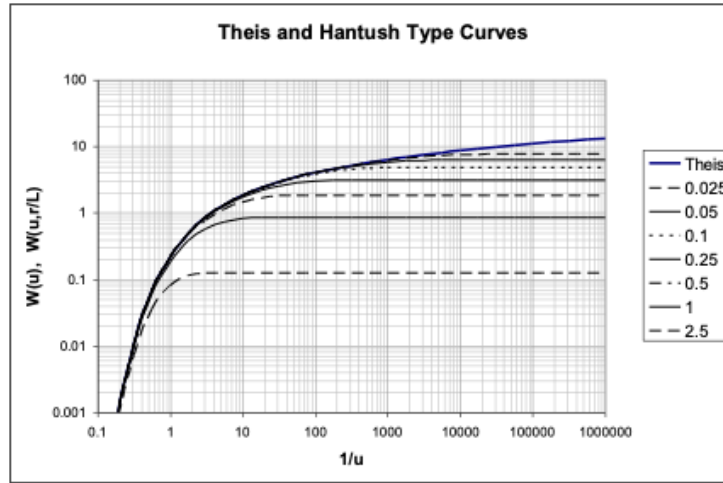


Figure 17: Theis and Hantush well function type curves.

Closed-book exam (3h), Feb 2007

Question 1: General

1. What is specific yield?
2. How does specific yield depend on the distance of the water table below ground level?
3. What happens to the water table in a piezometer in a confined aquifer when the barometer pressure goes up, why?

Question 2: Diffusion equation

The diffusion equation for transient flow in one dimension is $D \frac{\partial^2 s}{\partial x^2} = \frac{\partial s}{\partial t}$

1. What is the dimension of the diffusivity D ?
2. What is diffusivity D in the case of groundwater flow?
3. What is diffusivity D in the case of heat flow?

Question 3: Fluctuation groundwater

In the case of a tidal fluctuation in a river in direct contact with an aquifer having transmissivity the fluctuation of the head may be described by

$$s = s_0 \exp(-ax) \sin(\omega t - ax), \text{ with } a = \sqrt{\frac{\omega S}{2kD}}$$

1. What is s and what does this function look like? Make a sketch of s as a function of x , and show its envelopes. (The envelope is the curve of the values between which the function fluctuates, as a function of x).
2. In the case of a double-day tide, $\omega = \frac{4\pi}{24} \text{ h}^{-1}$, what would be the speed of the wave into the aquifer if $S = 0.001$ and $kD = 500 \text{ m}^2/\text{d}$? (Notice the dimensions!)
3. What happens to this distance in case the transmissivity would be 9 times a big?

Question 4: Flow to an extraction canal

Consider an extraction canal in direct full contact with an aquifer with transmissivity $kD = 400 \text{ m}^2/\text{d}$ and specific yield $S_y = 0.1$. The water level in the canal suddenly changes by 2 m downward. The head and gradient are given by:

$$s = s_0 \operatorname{erfc} \left(\sqrt{\frac{x^2 S}{4kDt}} \right) \text{ and } \frac{\partial s}{\partial x} = -s_0 \sqrt{\frac{S}{\pi kDt}} \exp \left(-\frac{x^2 S}{4kDt} \right)$$

1. Compute the discharge into the canal after 1d. Show the formula you use and include the dimension in your answer!
2. What is the head change s at 100 m from the canal after 1 and after 2 d? (Use erfc-curve further down).
3. What is the head change at 100 m from the canal after 2 days if the head in the river would change back by 2 m at $t=1\text{d}$?

Question 4: Well in semi-confined aquifer

Consider a transient well in a semi0-confined aquifer so that Hantush's solution is valid, hence,

$$s = \frac{Q}{4\pi kD} W \left(u, \frac{r}{\lambda} \right), \text{ with } u = \frac{r^2 S}{4kDt} \text{ and } \lambda = \sqrt{kDc}$$

with $kD = 600 \text{ m}^3/\text{d}$, $c = 900 \text{ d}$, $S = 0.001$ and pumping at a rate $Q = 2400 \text{ m}^3/\text{d}$.

1. How long does it take before steady state is reached for a point at $r=300$ m from the well (why)? Use Hantush type curves (see graphic at the end of this exam).

Question 6: Drawdown due to a pumping station in an unconfined aquifer

A well is situated at 100 m from an impermeable infinitely long wall. The well is pumping at a rate of $2400 \text{ m}^3/\text{d}$. Even though the aquifer is unconfined, the

transmissivity kD may be taken as a constant equal to $600 \text{ m}^2/\text{d}$, while the specific yield S_y equals 0.2. The well bore has a radius of $r_0 = 0.25 \text{ m}$.

1. What is the drawdown at the well bore after 10 days of pumping ?
2. A well in a confined aquifer of infinite extent, with $kD = 1000 \text{ m}^2/\text{d}$ and $S = 0.001$, is pumping at a rate of $Q = 24000 \text{ m}^3/\text{d}$. How far would the radius of influence of this well after 100 years? The radius of influence is the radius beyond which the drawdown is considered negligible. You may exploit the logarithmic approximation of the Theis well function for large times:

$$W(u) \approx \ln\left(\frac{0.5625}{u}\right), \text{ with } u < 0.1 \text{ and } u = \frac{r^2 S}{4kDt}$$

by making it zero.

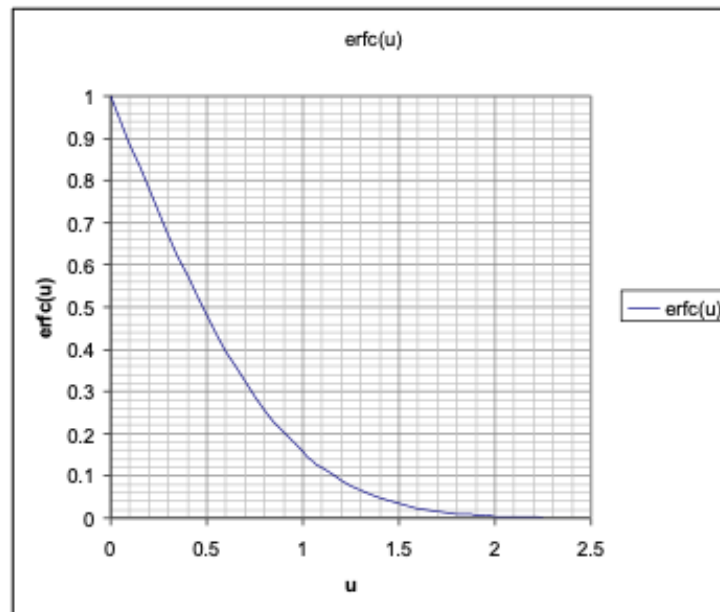


Figure 18: $\text{erfc}(u)$ function.

Closed-book exam (3h), Feb 2006

Question 1: Conceptual

1. What types of reversible storage do you know in aquifer systems, explain how it works
2. What values may you expect for the respective storage coefficients?

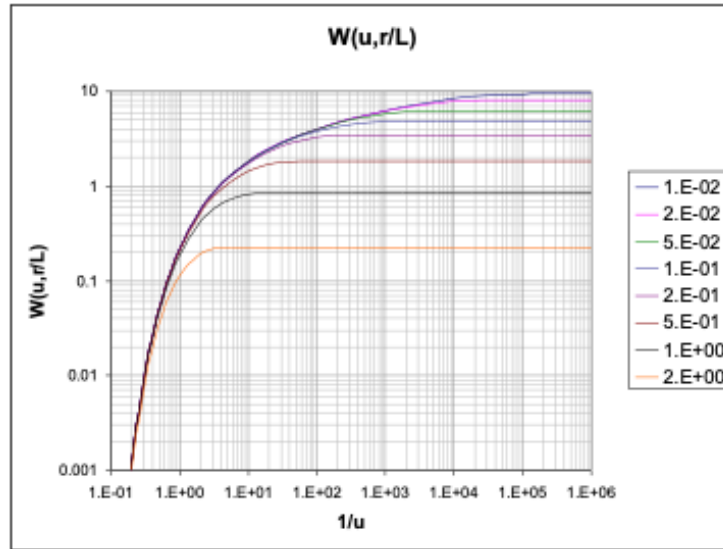


Figure 19: Theis and Hantush type curves. In case this graph is copied in black and white only, note that the lowest type curve is for the highest value of r/λ . Note that the L in the title and left axis of this figure stands for $\lambda = \sqrt{kDc}$ value

3. What is barometric efficiency, explain how it works.
4. When the barometric pressure increases, does the head (water table in a piezometer) in the confined aquifer rise or fall?
5. Between what values may the barometric efficiency vary?
6. What happens in a confined aquifer with the head if a load is suddenly placed on ground surface, such as a train stopping near a piezometer? What happens when it leaves? Sketch a graph showing the head versus time that you would expect in that case.

Question 2: Characteristic time of groundwater basin

Characteristic time of groundwater basin, the partial differential equation of which reads

$$kD \frac{\partial^2 \phi}{\partial x^2} = S \frac{\partial \phi}{\partial t}$$

1. What is a characteristic time of a groundwater basin that may be considered as one-dimensional of characteristic size L ? (hint: Make partial differential equation dimensionless by setting $\xi = \frac{x}{L}$, $\tau = \frac{t}{T}$ and see what T is.

2. To reach equilibrium, how many times slower is a large basin compared to a small one with the same transmissivity and storage coefficient?
3. Compute the characteristic time for the following cases:
 - (a) Large basin: $kD = 500 \text{ m}^2/\text{s}$, system width $L = 100 \text{ km}$, storage coefficient $S = 0.2$,
 - (b) Small basin: $kD = 100 \text{ m}^2/\text{d}$, system width $L = 100 \text{ m}$, storage coefficient $S = 0.1$.

Question 3: Tides in groundwater

Given: The tidal fluctuation in an aquifer in a point at distance x from the sea due to the water level fluctuation at sea with amplitude A is described by the following formula

$$s(x, t) = A \exp(-ax) \sin(\omega t - ax)$$

in which the damping factor is as follows $a = \sqrt{\frac{\omega S}{2kD}}$, where ω is the angle velocity in radians/time or $\omega = \frac{2\pi}{T}$ where T is the time of a complete wave cycle.

Are the following expressions true or false?

1. The wave in the aquifer has a different frequency than the tide itself.
2. The amplitude of the wave at a given distance from the sea becomes greater when,
 - (a) the frequency of the tide is reduced
 - (b) the storage coefficient is reduced
 - (c) then the transmissivity is reduced

Question 4: Aquifer with river

Consider an aquifer of infinite extent bounded by a fully penetrating river at $x = 0$. At $t = 0$ the river level suddenly changes by a height A . The change of head $s(x, t)$ in the aquifer equals in this case:

$$s(x, t) = A \operatorname{erfc} \left(\sqrt{\frac{x^2 S}{4kDt}} \right)$$

with $\operatorname{erfc}(u)$ as shown in the picture below

1. What is the final value of the head change (the value reached after infinite time, $s(x, \infty)$)?

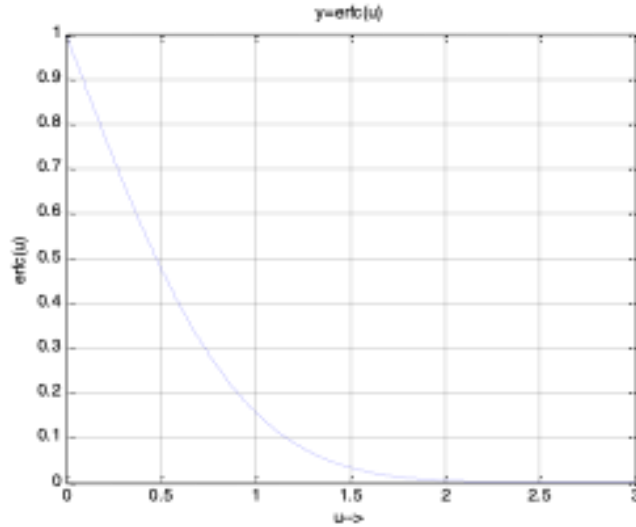


Figure 20: $\text{erfc}(u)$ versus u

2. What value has the argument of $\text{erfc}(\dots)$, i.e., $\sqrt{\frac{x^2 S}{4kDt}}$ when the head change is half the final value?
3. If $kD = 400 \text{ m}^2/\text{d}$, $S = 0.1$ and $x = 100 \text{ m}$, after how much time is this the change of head equal to $0.5A$?
4. What would be the formula if the head change occurred on time t_1 instead of time $t = 0$?
5. How could you compute the head change at point x if there was a sudden change of the river level of A_1 at time $t = t_1$ and another of A_2 at $t = t_2$?

Question 5: Well in a confined aquifer

Consider a well in a confined aquifer starting an extraction of $Q = 1200 \text{ m}^3/\text{d}$ at $t = 0$. $kD = 1000 \text{ m}^2/\text{d}$, and $S = 0.001$. For this case the Theis solution applies:

$$s = \frac{Q}{4\pi kD} W(u), \text{ with } u = \frac{r^2 S}{4kDt}$$

(See the type curve of Theis well function on a separate page).

1. Compute the head at $r = 20 \text{ m}$ after $t = 1 \text{ d}$.
2. The pump is switched off after 1 day. What is the head after 1.1 days at $r = 20 \text{ m}$?

Question 6: Well in a leaky aquifer

Consider a transient well in a leaky aquifer. $kD = 400 \text{ m}^2/\text{d}$, $c = 400 \text{ d}$, $S = 0.001$, so that the groundwater behaves according to Hantush's transient well formula

$$s = \frac{Q}{4\pi kD} W\left(u, \frac{r}{\lambda}\right), \text{ with } \lambda = \sqrt{kDc}$$

1. How long does it take until the head at $r = 40 \text{ m}$ becomes steady state or virtually steady state? (Hint: look at the type curves to get u for which this is the case, note).

Question 7: Well in an unconfined aquifer

Consider a well in an unconfined aquifer for which the Theis-solution applies (see type curve hereafter. Further given a pumping test with an extraction of $Q = 600 \text{ m}^3/\text{d}$ during which drawdown measurements were made (see the graph with the small circles).

Interpret the test (that is: compute kD and S).

(Hint: if you can't see through the paper make the type curve thicker using a pen and hold both curves up against a light or in the direction of a window).

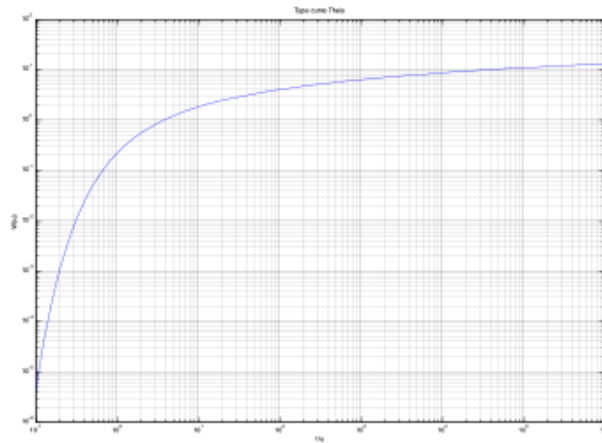


Figure 21: Theis type curve, $W(u)$ versus $1/u$

