Comments on 'A Correspondence Principle for the Theory of Leaky Aquifers' by Ismael Herrera and Germán E. Figueroa V.

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In the introduction to their paper *Herrera and Figueroa* [1969] state that '. . . the application of existing theories [of flow in leaky aquifers] to regional problems is difficult, leading to rather complicated situations that are not supported in practice by . . . the existing field measurements.' They attempt to correct this situation by advancing a simplified theory that would enable one to replace any leaky system by an equivalent one consisting only of nonleaky aquifers. In their development, the authors adopt Hantush's modified approach to nonsteady flow in leaky aquifers as expressed by equations 1, 2, and 3.

One of the difficulties in solving the initial boundary value problem defined by these equations stems from the fact that the equations are coupled and must be solved simultaneously. The authors introduce a very interesting method of uncoupling the original equations through the use of (6), (7), and (8). The result is an integrodifferential equation (14) involving only one unknown function s.

Equation 14 does not necessarily represent a simplification of the original problem. The authors therefore attempt to reduce (14) to a simpler form, and their entire treatment is based on equation 15. They state that because s and $\partial s/\partial t$ vary slowly at large values of time, one is justified in writing

$$\int_0^t \frac{\partial}{\partial t} s(x, y, \tau)F(t - \tau)\, d\tau \simeq \frac{\partial}{\partial t} s(x, y, t) \int_0^t F(t - \tau)\, d\tau$$

What this approximation really implies, however, is that the variation of s with time is nearly linear over the entire period 0 to t. Otherwise, $\partial s/\partial t$ could not be treated as a constant, and the authors would not be justified in removing it from under the integral sign.

We respectfully disagree with the authors' use of this kind of approximation because in our view, its validity has not been properly demonstrated. The well-known fact that at early time the variation of s is far from linear means that $\partial s/\partial t$ changes significantly with time (cf. Figures 2–6 [*Neuman and Witherspoon*, 1969*a*]). If one assumes that after a sufficiently large value of time $t > t_1$ the variation of s is practically linear, then it would seem to us that a more appropriate approximation for the integral in (14) would be

$$\int_0^t \frac{\partial}{\partial t} s(x, y, \tau)F(t - \tau)\, d\tau \simeq \int_0^{t_1} \frac{\partial}{\partial t} s(x, y, \tau)F(t - \tau)\, d\tau + \frac{\partial}{\partial t} s(x, y, t) \int_{t_1}^t F(t - \tau)\, d\tau$$

This approximation, however, would not necessarily lead to the final result given by the authors in (15). By neglecting the variation of $\partial s/\partial t$ at $t > t_1$ and further by taking the upper limit of the integral to infinity, the authors are limiting the applicability of their correspondence principle to those values of time where the early transients no longer have any effect on the integral in (14), i.e., the steady state or at best a quasi-steady state.

Indeed the example cited by Herrera and Figueroa (equation 26), which is equivalent to a solution previously obtained by *Hantush* [1960] for large values of time, is restricted largely to the steady state. This restriction can be seen by examining Figures 2, 3, and 4 of our recent paper [*Neuman and Witherspoon*, 1969*b*]. One may note that our equation 5, which is the same as the authors' equation 26, corresponds almost exclusively to the horizontal (i.e., steady state) portion of the curves on these figures.

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(Manuscript received October 24, 1969.)