

A-1]

* Title : Fuzzy Relations* Problem Statement :- Implement Union, Intersection, Complement and Difference operations on fuzzy sets. Also create fuzzy relation by Cartesian product of any two fuzzy sets and perform max min composition on any two fuzzy relation.* Objective :- 1. To study and implement fuzzy sets and perform different operations.

2. To study and implement fuzzy set cartesian product, max min composition on fuzzy relation.

* Outcome :- 1. I successfully studied and implemented fuzzy set operations on fuzzy set.

2. I implemented cartesian product, max min composition of fuzzy set.

* Date of Execution :- 28/01/2021Date of Submission :- 04/02/2021Theory :-Fuzzy logic was introduced in year 1965 by Lotfi A Zadeh. It offers a ~~compet~~ soft computing paradigm.

Fuzzy set operates on concept of membership. Degree of membership of any particular element of fuzzy set expresses the degree of compatibility of element.

A is a fuzzy set, containing an object x to degree $\mu_A(x)$
i.e. $\mu_A(x) = \text{Degree}(x \in A)$ * $\mu_A: X \rightarrow [\text{Membership Degree}]$ called set or membership function

A fuzzy set A in universe of discourse U is set of ordered pairs

$$A = \{ (x, \mu_A(x)) \mid x \in U \}$$

$\mu_A(x)$ - degree of membership of x in A .

$$\mu_A(x) \in [0, 1].$$

A) Union :-

$$\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)].$$

B) Intersection :-

$$\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)].$$

C) Complement :-

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x).$$

D) Difference :-

$$A \setminus B = A \cap \bar{B}$$

$$B \setminus A = B \cap \bar{A}$$

eg. $A = \left\{ \frac{1}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0.2}{8} \right\}$, $B = \left\{ \frac{0.5}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{1}{8} \right\}$

→

A) Union → $A \cup B = \left\{ \frac{1}{2} + \frac{0.4}{4} + \frac{0.5}{6} + \frac{1}{8} \right\}$.

B) Intersection → $A \cap B = \left\{ \frac{0.5}{2} + \frac{0.3}{4} + \frac{0.1}{6} + \frac{0.2}{8} \right\}$

C) Complement → $\bar{A} = 1 - \mu_A(x) = \left\{ \frac{0}{2} + \frac{0.7}{4} + \frac{0.5}{6} + \frac{0.8}{8} \right\}$
 $\bar{B} = 1 - \mu_B(x) = \left\{ \frac{0.5}{2} + \frac{0.6}{4} + \frac{0.9}{6} + \frac{0}{8} \right\}$.

D) Difference → $A \setminus B = A \cap \bar{B} = \left\{ \frac{0.5}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0}{8} \right\}$

$$B \setminus A = B \cap \bar{A} = \left\{ \frac{0}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{0.8}{8} \right\}$$

defined

e). Cartesian product -

$$R = A \times B = \min [\mu_A(x), \mu_B(y)]$$

$$\text{eg. } A = \left\{ \frac{1}{\text{low}} + \frac{0.2}{\text{medium}} + \frac{0.5}{\text{high}} \right\}, \quad B = \left\{ \frac{0.9}{\text{positive}} + \frac{0.4}{\text{zero}} + \frac{0.9}{\text{negative}} \right\}$$

$$R = A \times B = \begin{matrix} & \begin{matrix} \text{positive} & \text{zero} & \text{negative} \end{matrix} \\ \begin{matrix} \text{low} \\ \text{medium} \\ \text{high} \end{matrix} & \begin{bmatrix} 0.9 & 0.4 & 0.9 \\ 0.2 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.5 \end{bmatrix} \end{matrix}$$

f) Max Min Composition:-

$$C = \left\{ \frac{0.1}{\text{low}} + \frac{0.2}{\text{medium}} + \frac{0.7}{\text{high}} \right\}$$

$$C \cdot R = [0.1 \ 0.2 \ 0.7]_{1 \times 3} \begin{bmatrix} 0.9 & 0.4 & 0.9 \\ 0.2 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.5 \end{bmatrix}$$

$$\begin{aligned} \mu_C \cdot R(x, y) &= \max [\min(0.1, 0.9), \min(0.2, 0.2), \min(0.7, 0.5)] \\ &= \max [0.1, 0.2, 0.5] = 0.5 \end{aligned}$$

Similarly,

$$C \cdot R = [0.5 \ 0.4 \ 0.5]$$

* Pseudo code:-

1) Union_fuzz:-

- 1) for (i ranges from 0 \rightarrow n)
- 2) if set1[i] \geq set2[i]
- 3) add set1[i] to union.
- 4) else
- 5) add set2[i] to union.

2) Intersec_fuzz:-

- 1) for (i ranges from 0 \rightarrow n)
- 2) if set1[i] \geq set2[i]
- 3) add set2[i] to intersection
- 4) else
- 5) add set1[i] to intersection.

3) Compl-fuzz

- 1) for (i ranges from 0 to n)
- 2) add to complement ($set1[i] - set(i)$)

4) diff-fuzz

- 1) for (i ranges from 0 to n)
- 2) if $set1[i] \geq set2_comp[i]$
- 3) add to difference $set2_comp[i]$
- 4) else
- 5) add to difference $set1[i]$.

5) max-min-compo:-

- 1) for (i ranges from 0 to m_1)
- 2) for (j ranges from 0 to n_2)
- 3) for (k ranges from 0 to n_1)
- 4) add to $res[i][j]$ $\max(res[i][j], \min(arr1[i][k], arr2[k][j]))$;
- 5)

* Conclusion:-

I successfully implemented fuzzy sets and their operations. Also implemented max-min composition.

Basis	Fuzzy Set	Crisp Set
Definition	Prescribed by vague or ambiguous properties.	Element is either member of set or not
Property	Elements are allowed to be partially included in set	Element is either member of set or not
Applications	Used in fuzzy controllers	Digital design