Simulation of Electromagnetic Fields: The Finite-Difference Time-Domain (FDTD) Method and Its Applications

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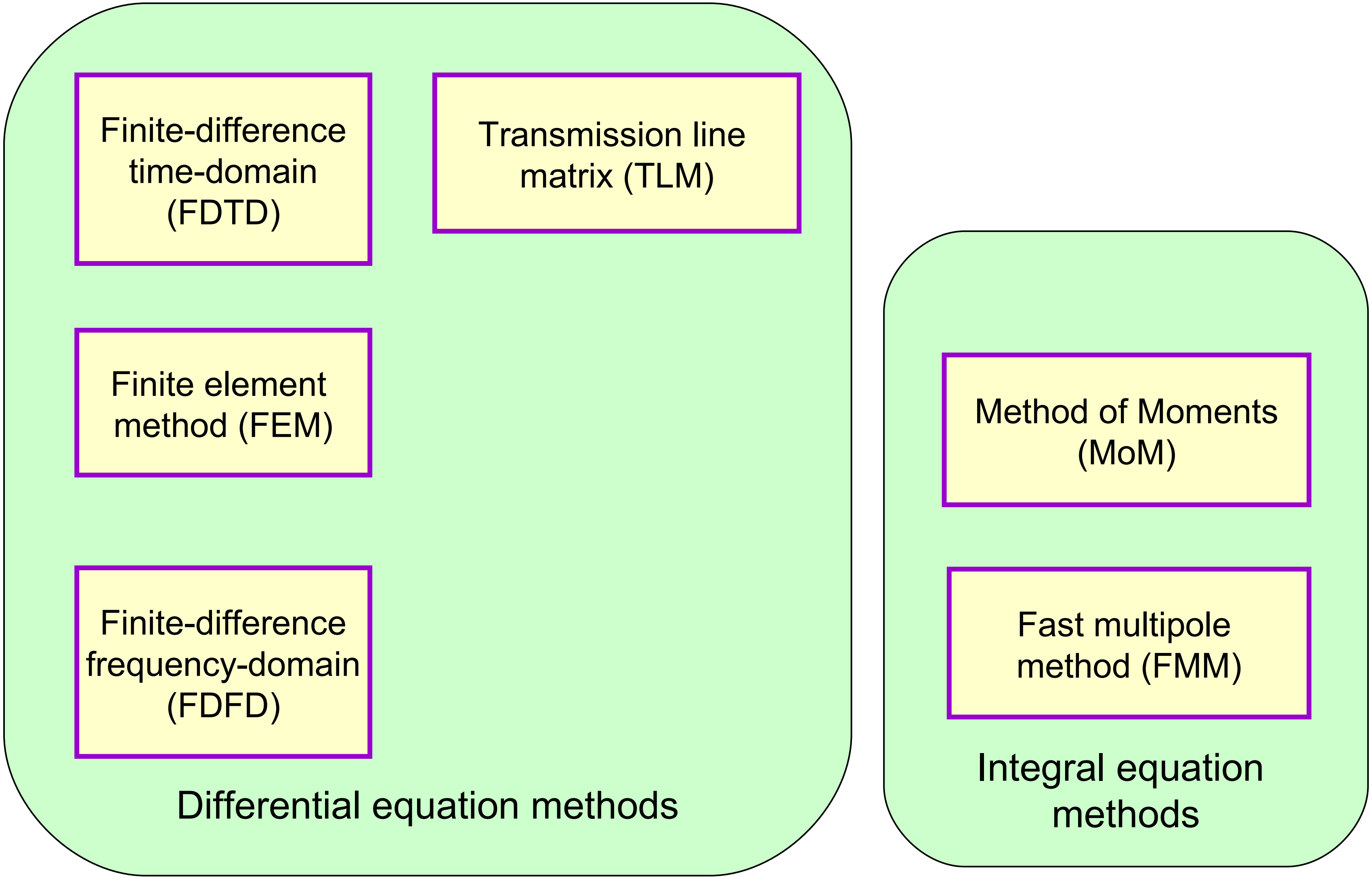
Computational Electromagnetics

- Maxwell's equations can be given in differential or integral form



Computational Electromagnetics

- Maxwell’s equations can be given in time domain or frequency domain

Time-domain methods

- Finite-difference time-domain (FDTD)
- Transmission line matrix (TLM)

- Finite element method (FEM)
- Finite-difference frequency-domain (FDFD)
- Method of Moments (MoM)
- Fast multipole method (FMM)

Frequency domain methods

Commercial software packages

- Commercial software packages

Finite element method (FEM)

ANSOFT

HFSS

Method of Moments (MoM)

IE3D

Zeland Software, Inc.

Agilent Technologies

ADS Momentum

SONNET®

WIPL-D Pro

3D Electromagnetic Solver

Transmission line matrix (TLM)

CST

CST Microstripes

Finite-difference time-domain (FDTD)

CST

XFdtd™

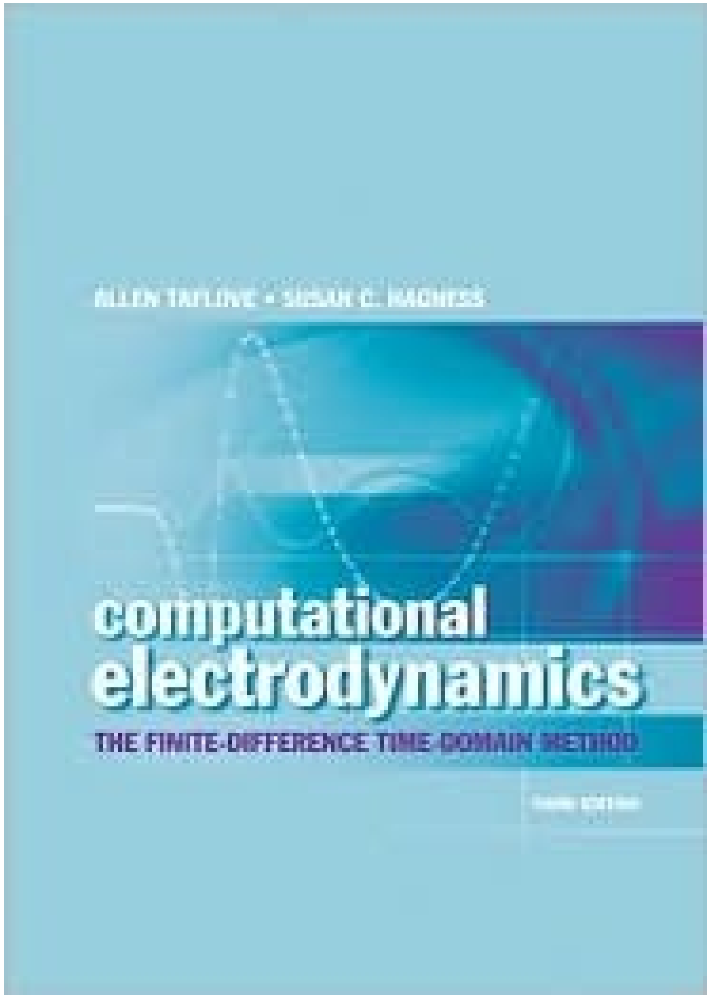
SEMCAD X

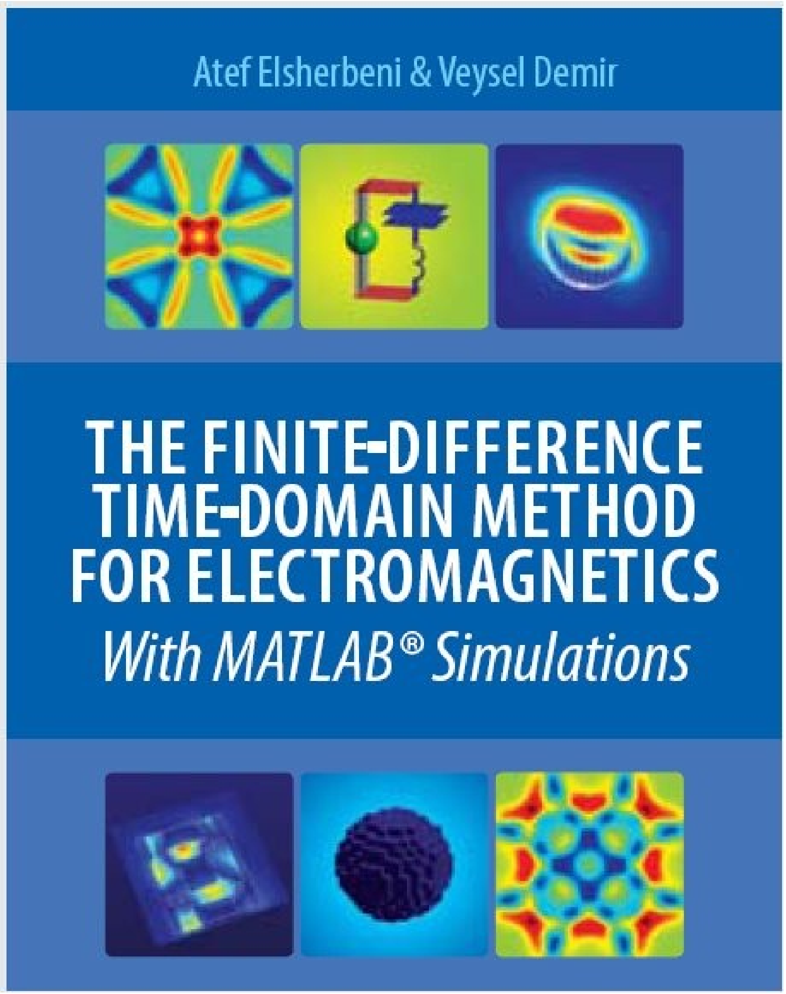
lumerical

illuminating the way

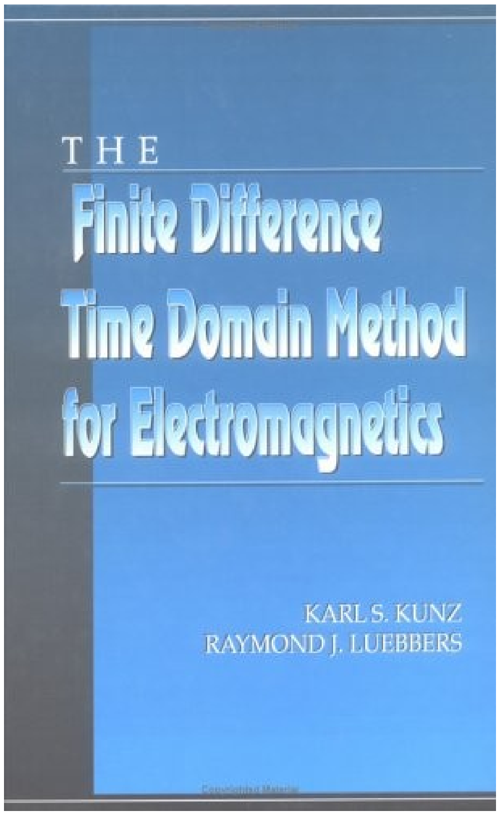
QWED

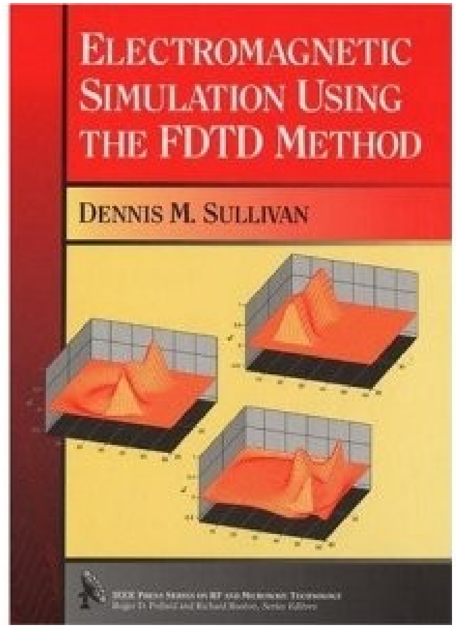
The Finite-Difference Time-Domain Method

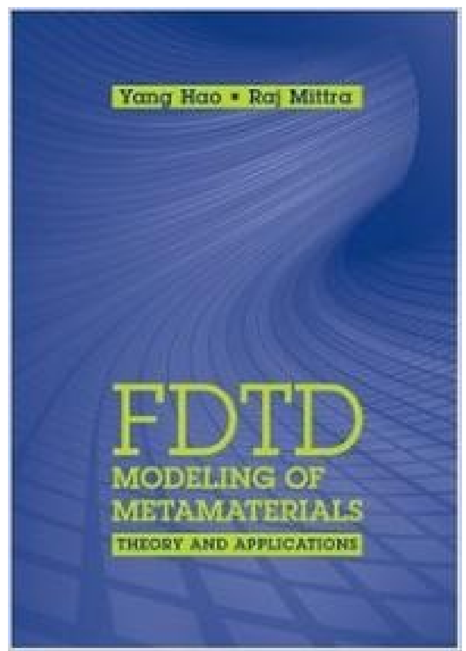


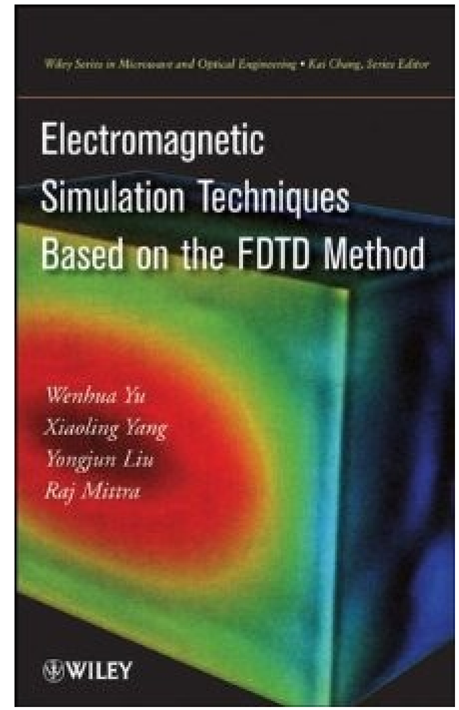


FDTD Books


THE
Finite Difference
Time Domain Method
for Electromagnetics
KARL S. KUNZ
RAYMOND J. LUEBBERS

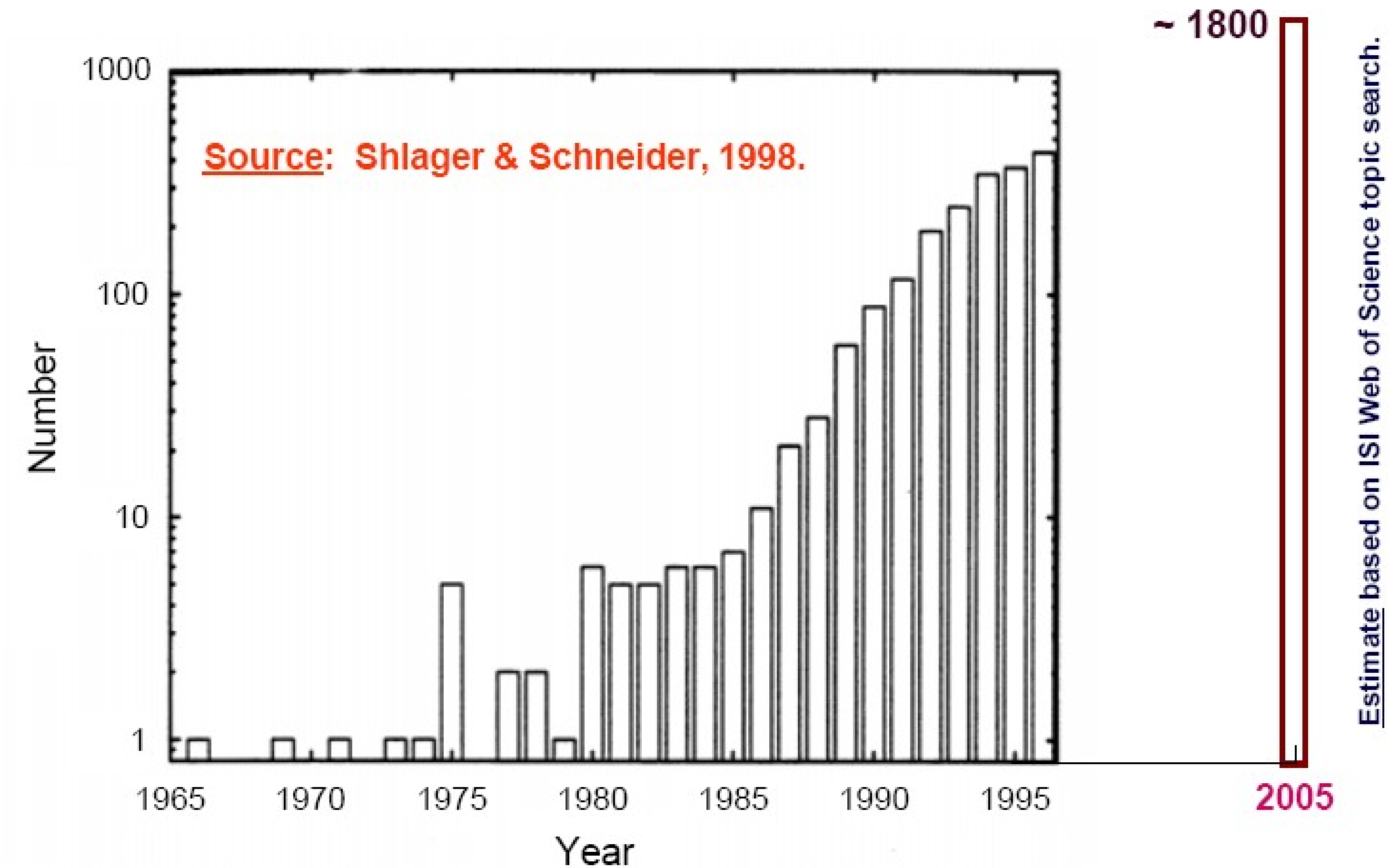

ELECTROMAGNETIC
SIMULATION USING
THE FDTD METHOD
DENNIS M. SULLIVAN


Yang Hao • Raj Mittra
FDTD
MODELING OF
METAMATERIALS
THEORY AND APPLICATIONS


Wiley Series in Microwave and Optical Engineering • Kai Chang, Series Editor
Electromagnetic
Simulation Techniques
Based on the FDTD Method
Wenhua Yu
Xiaoling Yang
Yongjun Liu
Raj Mittra
WILEY

Yearly FDTD Publications

❖ The most popular method in computational electromagnetics



Source: Allen Taflove, "A Perspective on the 40-Year History of FDTD Computational Electrodynamics," **Applied Computational Electromagnetics Society (ACES) Conference, Miami, Florida,** March 15, 2006. Can be found at http://www.ece.northwestern.edu/ecefaculty/Allen1.html

Maxwell's Equations

- The basic set of equations describing the electromagnetic world
- Shows that light is an electromagnetic wave.

Gauss's law
$$\nabla \cdot \bar{D} = \rho_v$$

Gauss's law for magnetism
$$\nabla \cdot \bar{B} = 0$$

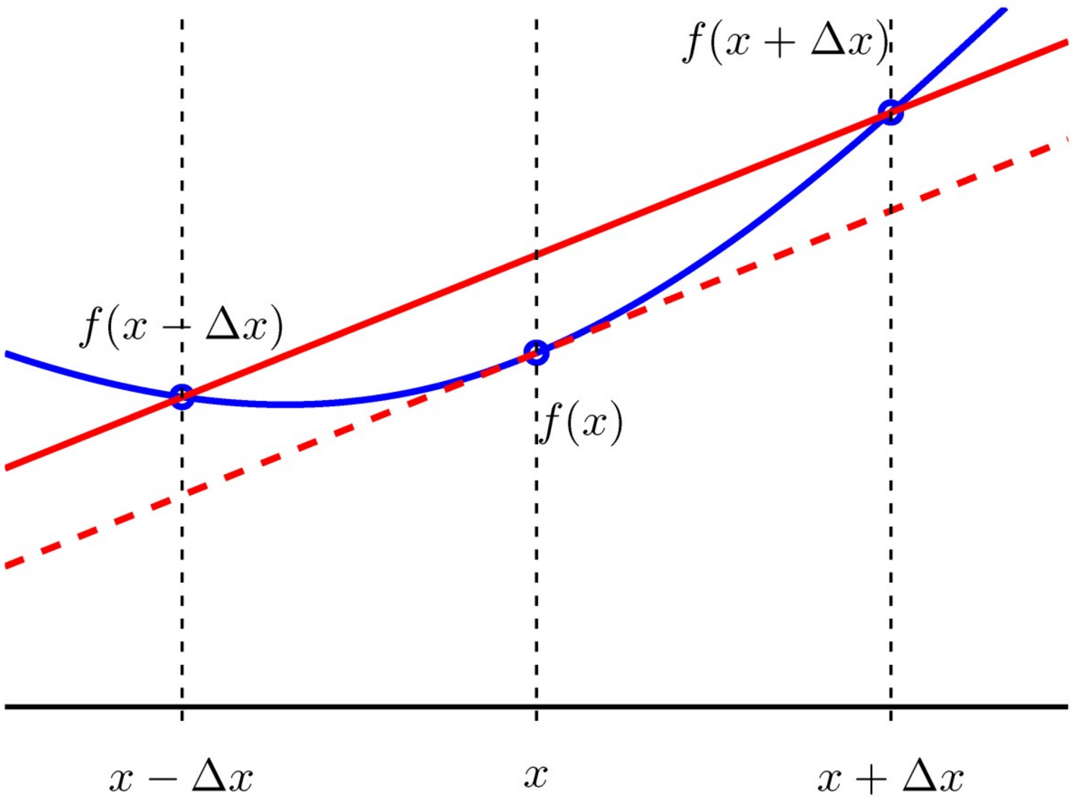
Faraday's law
$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

Ampere's law
$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

Constitutive relations
$$\bar{D} = \varepsilon \bar{E}, \text{and } \bar{B} = \mu \bar{H}$$

FDTD Overview – Finite Differences

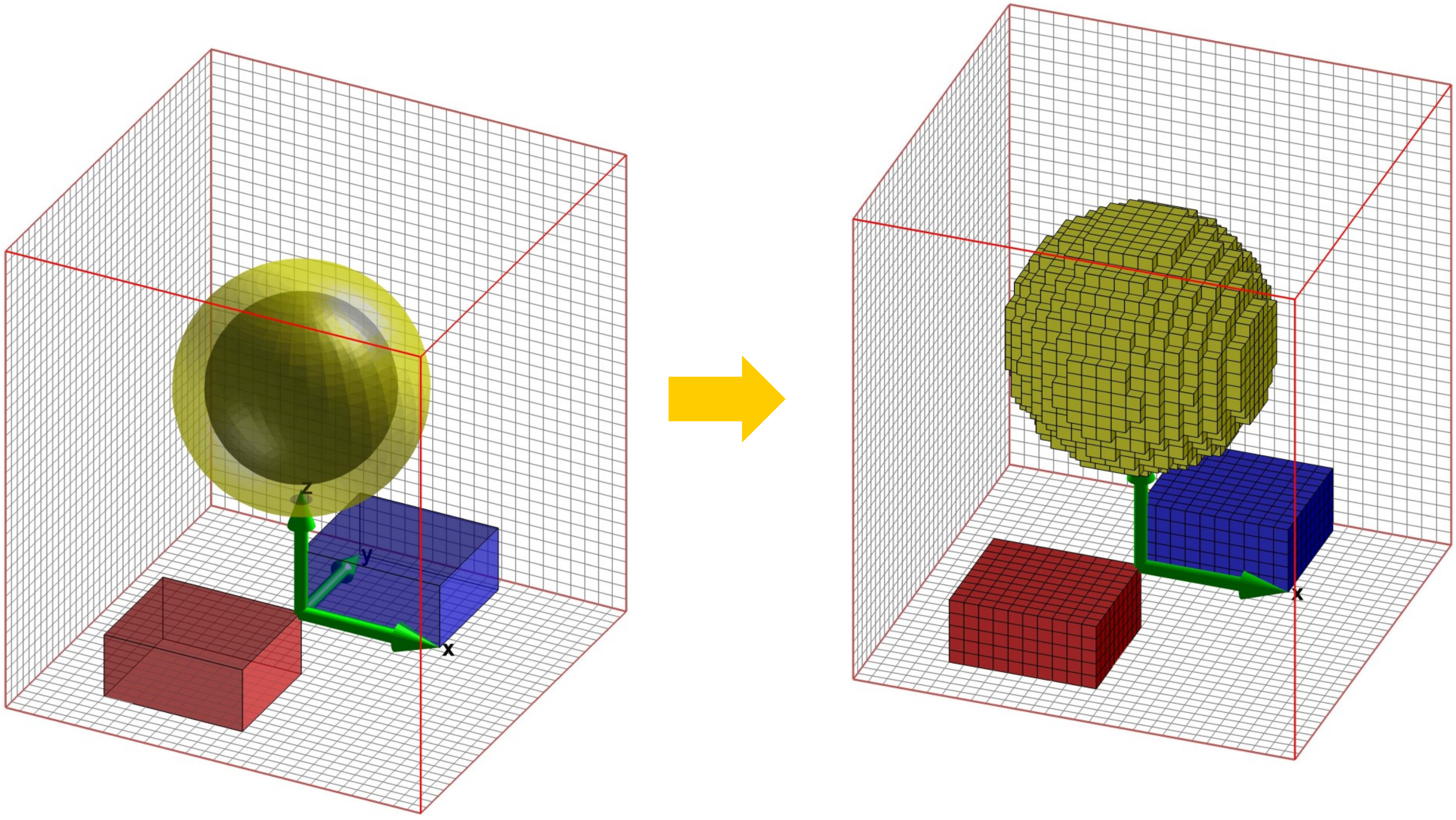
- Represent the derivatives in Maxwell's curl equations by finite differences
- We use the second-order accurate central difference formula



$$\frac{df(x)}{dx} = f'(x) \cong \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

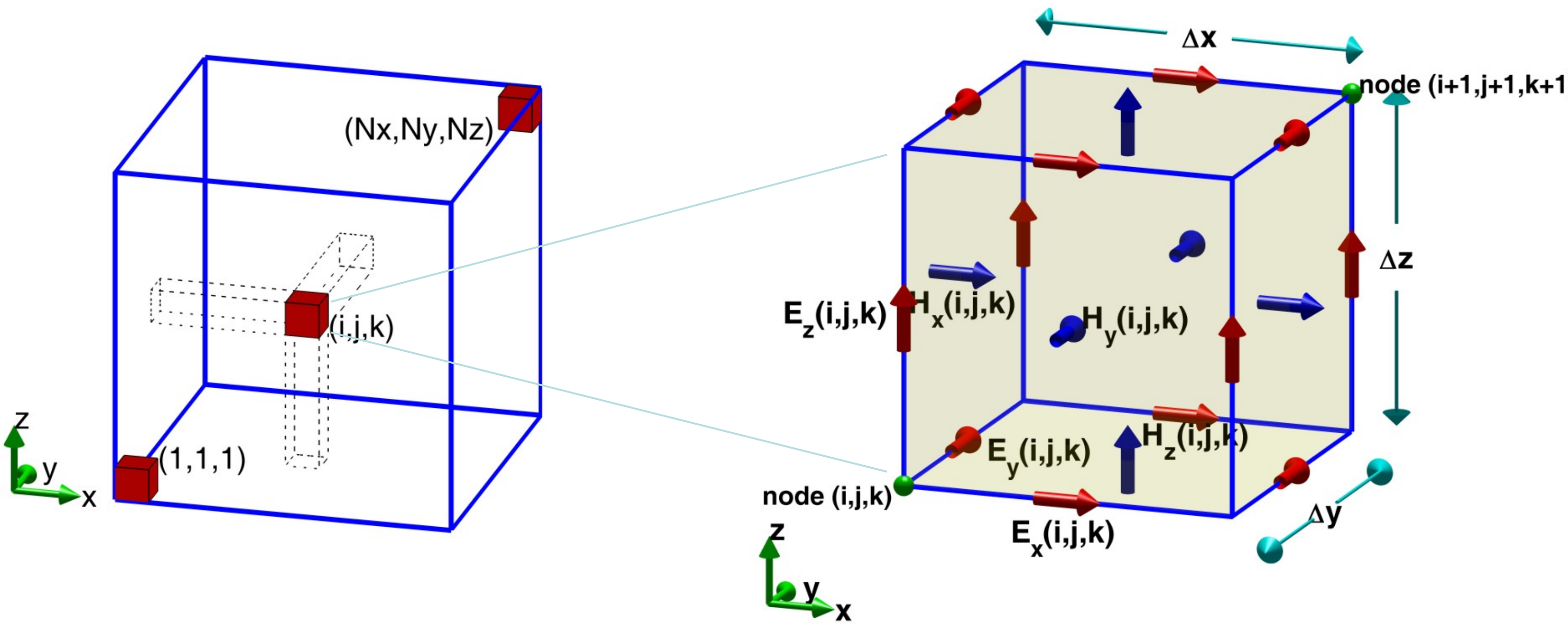
FDTD Overview – Cells

- A three-dimensional problem space is composed of cells



FDTD Overview – The Yee Cell

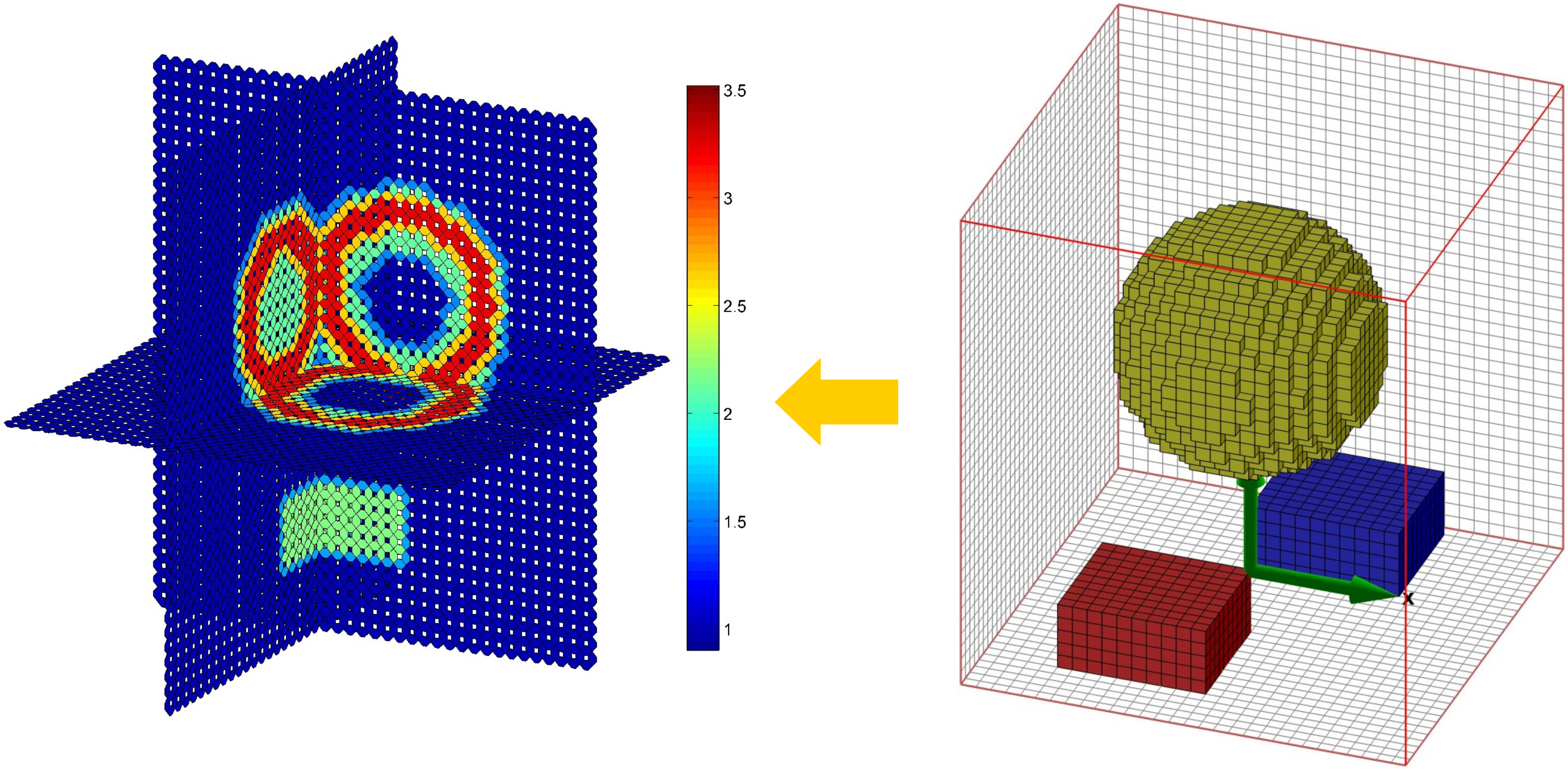
- The FDTD (Finite Difference Time Domain) algorithm was first established by Yee as a three dimensional solution of Maxwell's curl equations.



K. Yee, IEEE Transactions on Antennas and Propagation, May 1966.

FDTD Overview – Material grid

- A three-dimensional problem space is composed of cells



FDTD Overview – Updating Equations

- Three scalar equations can be obtained from one vector curl equation.

$$\varepsilon \frac{\partial \bar{E}}{\partial t} = \nabla \times \bar{H}$$

$$\varepsilon_x \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}$$

$$\varepsilon_y \frac{\partial E_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}$$

$$\varepsilon_z \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$$

$$\mu_x \frac{\partial H_x}{\partial t} = \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}$$

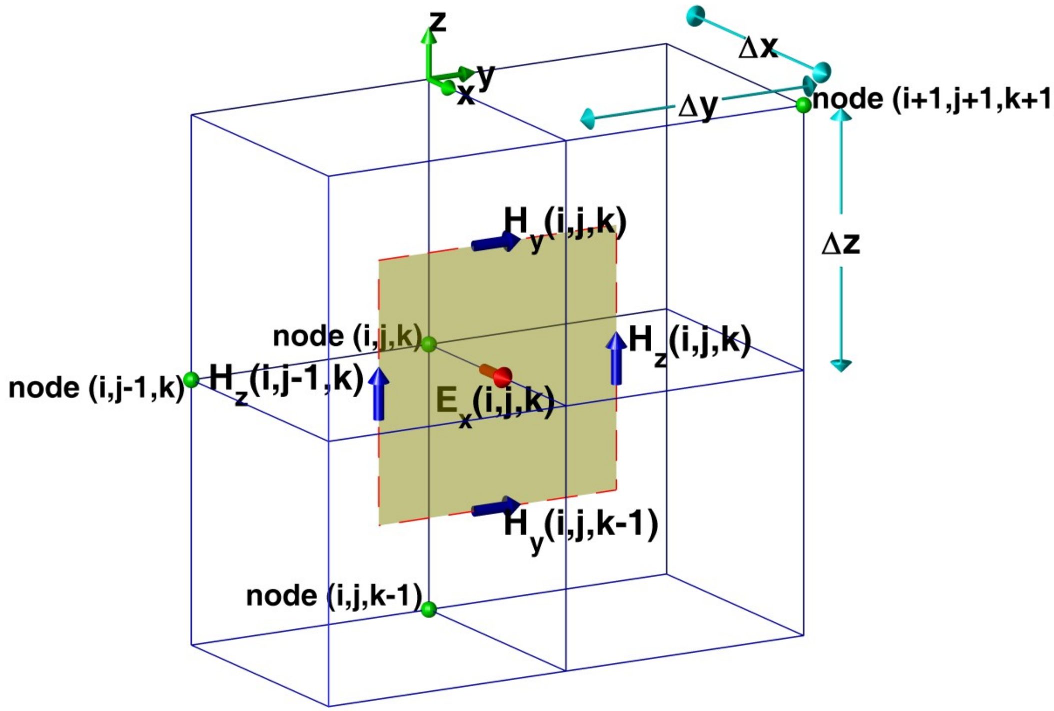
$$\mu_y \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}$$

$$\mu_z \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}$$

$$\mu \frac{\partial \bar{H}}{\partial t} = -\nabla \times \bar{E}$$

FDTD Overview – Updating Equations

❖ Represent derivatives by finite-differences



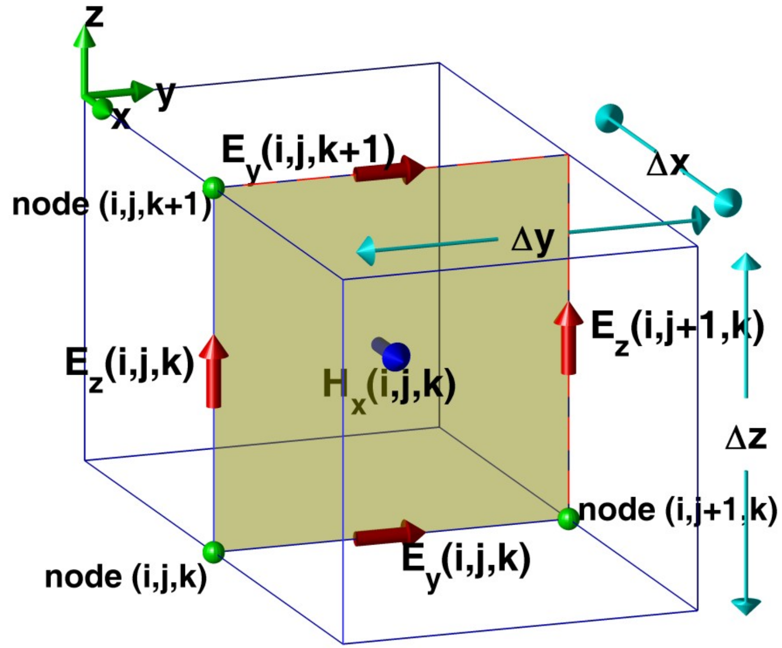
$$\varepsilon_x \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}$$

$$\varepsilon_x(i,j,k)\frac{E_x^{n+1}(i,j,k) - E_x^{n}(i,j,k)}{\Delta t} =$$

$$\frac{H_z^{n+0.5}(i,j,k) - H_z^{n+0.5}(i,j-1,k)}{\Delta y} - \frac{H_y^{n+0.5}(i,j,k) - H_y^{n+0.5}(i,j,k-1)}{\Delta z}$$

FDTD Overview – Updating Equations

- Represent derivatives by finite-differences



$$\mu_x \frac{\partial H_x}{\partial t} = \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}$$

$$\mu_x(i,j,k)\frac{H_x^{n+0.5}(i,j,k) - H_x^{n-0.5}(i,j,k)}{\Delta t} =$$

$$\frac{E_y^{n}(i,j,k+1) - E_y^{n+0.5}(i,j,k)}{\Delta z} - \frac{E_z^{n}(i,j+1,k) - E_z^{n}(i,j,k)}{\Delta y}$$

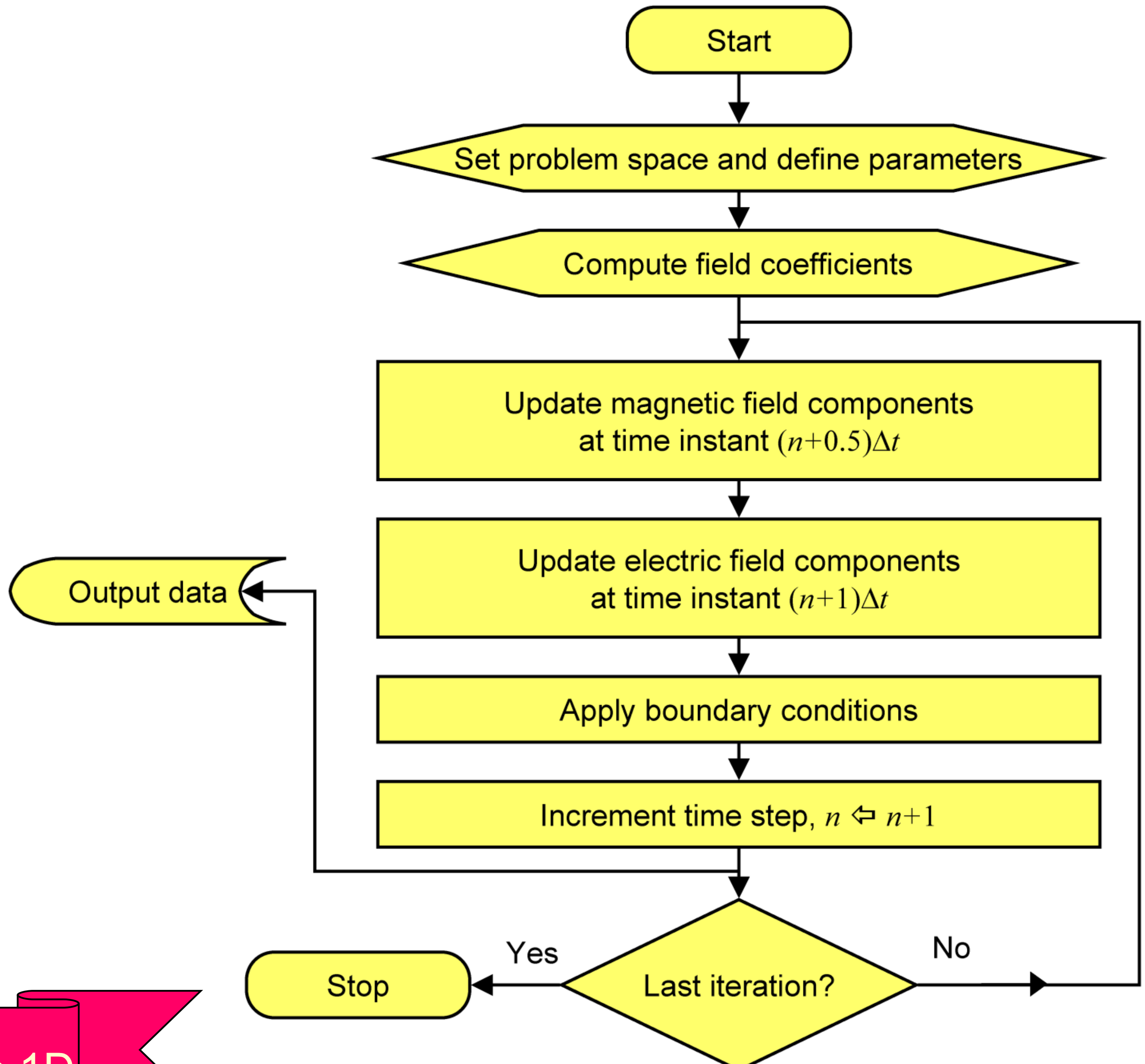
FDTD Overview – Updating Equations

- Express the future components in terms of the past components

$$E_x^{n+1}(i,j,k) = E_x^{n}(i,j,k) - \frac{\Delta t}{\varepsilon_x(i,j,k)}\left(\frac{H_z^{n+0.5}(i,j,k) - H_z^{n+0.5}(i,j-1,k)}{\Delta y} - \frac{H_y^{n+0.5}(i,j,k) - H_y^{n+0.5}(i,j,k-1)}{\Delta z}\right)$$

$$H_x^{n+0.5}(i,j,k) = H_x^{n-0.5}(i,j,k) + \frac{\Delta t}{\mu_x(i,j,k)}\left(\frac{E_y^{n}(i,j,k+1) - E_y^{n+0.5}(i,j,k)}{\Delta z} - \frac{E_z^{n}(i,j+1,k) - E_z^{n}(i,j,k)}{\Delta y}\right)$$

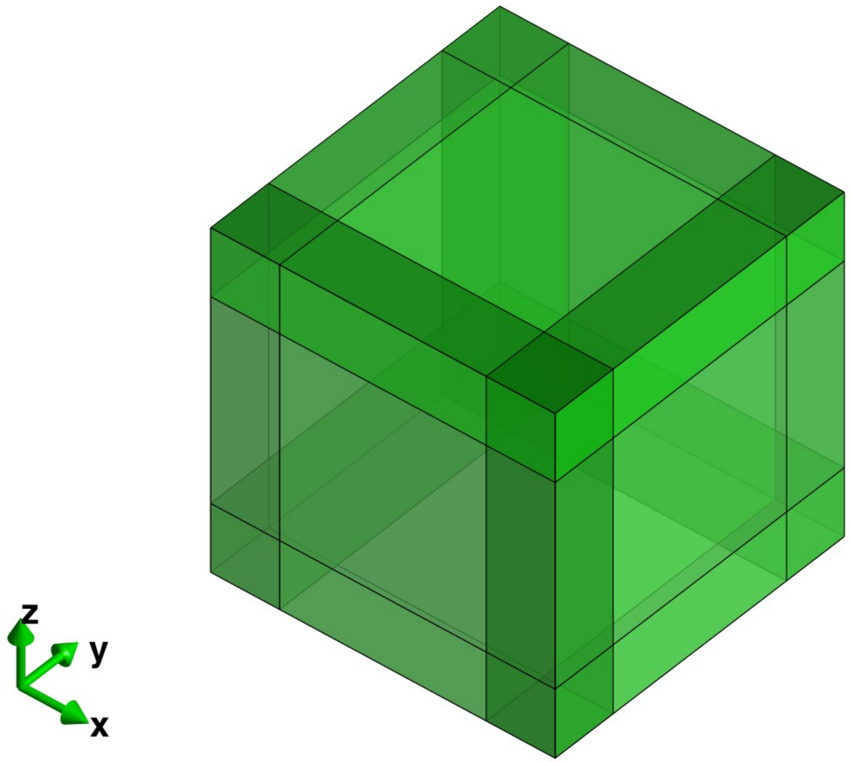
FDTD Overview – Leap-frog Algorithm



Exercise 1D

Absorbing Boundary Conditions

- The three-dimensional problem space is truncated by absorbing boundaries
- Most popular absorbing boundary is Perfectly Matched layers (PML)

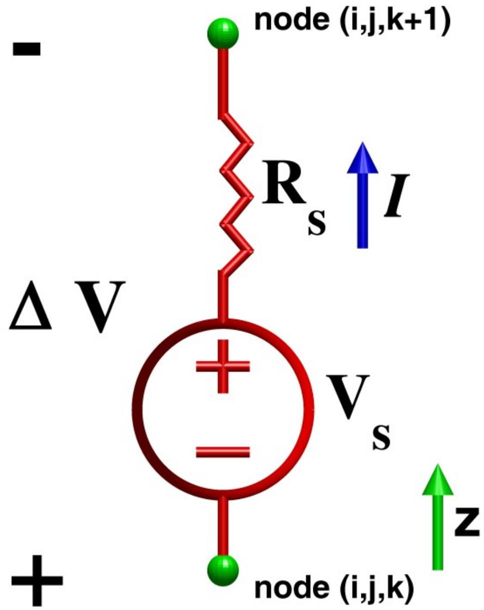


Exercise 2D PML

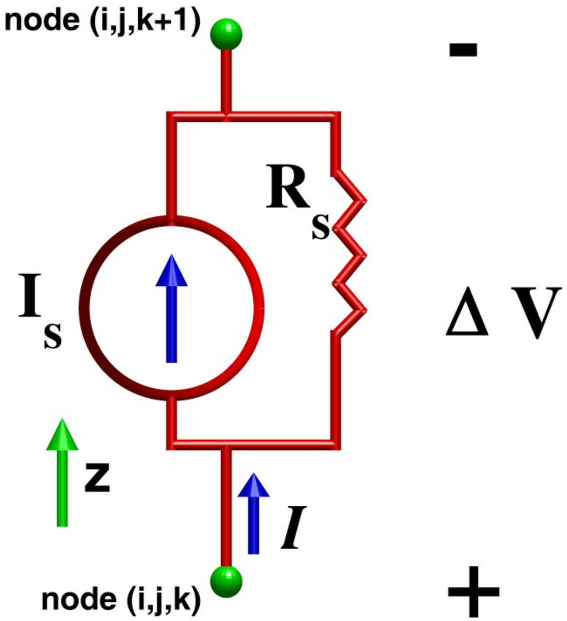
Active and Passive Lumped Elements

- Active and passive lumped elements can be modeled in FDTD

$$\nabla \times \bar{H} = \varepsilon \frac{\partial \bar{E}}{\partial t} + \sigma \bar{E} + \boxed{\bar{J}}$$

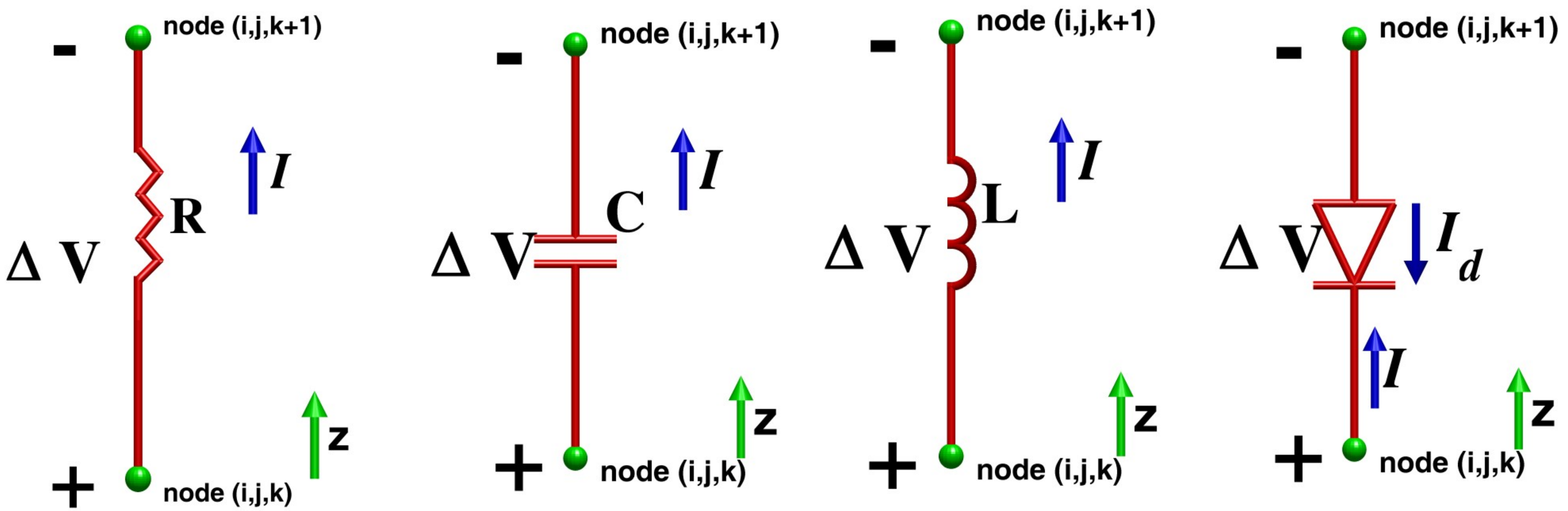


Voltage source



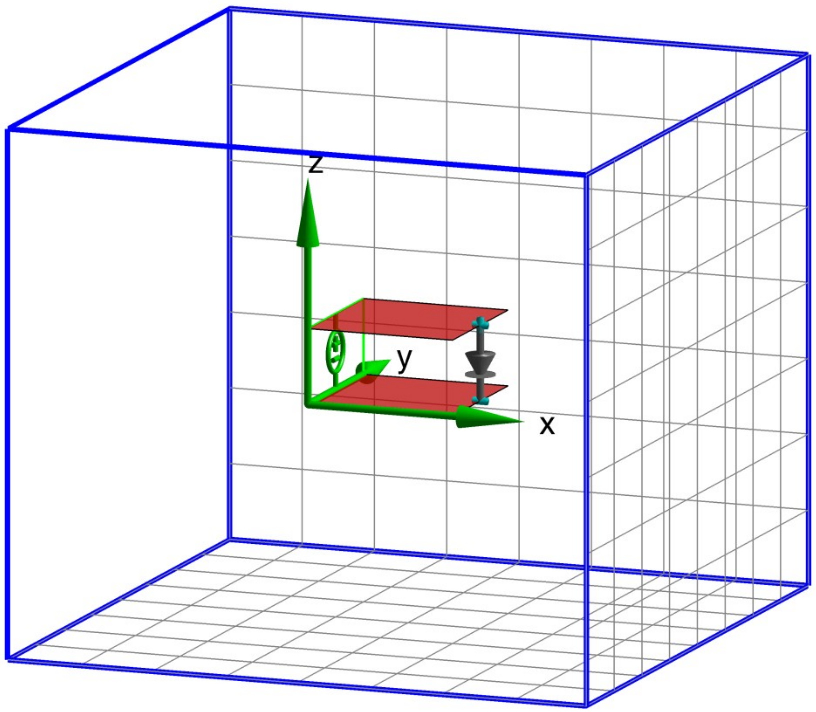
Current source

Active and Passive Lumped Elements

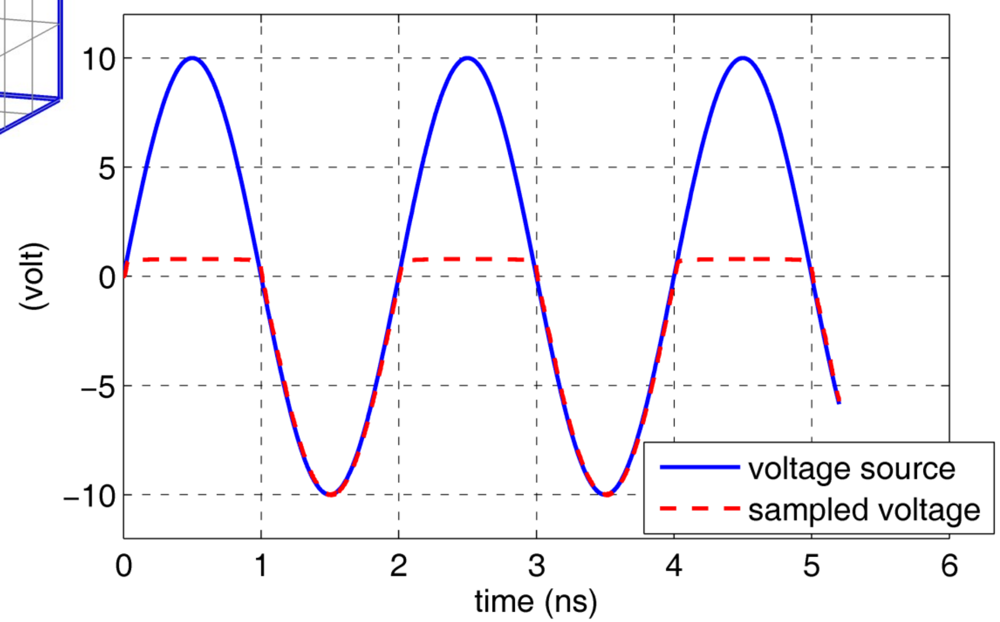


Resistor Capacitor Inductor Diode

Active and Passive Lumped Elements



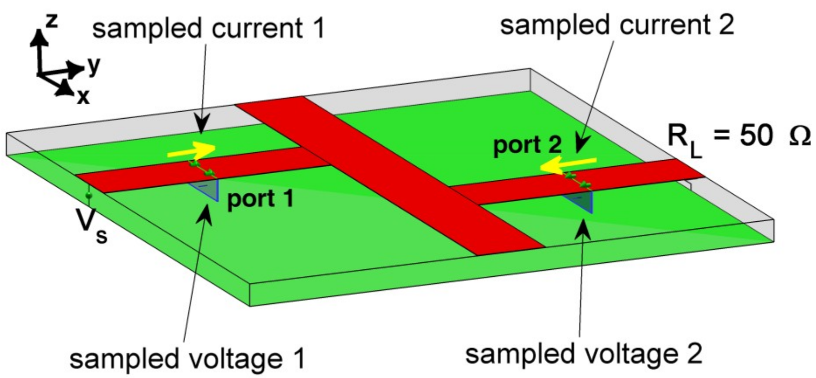
A diode circuit

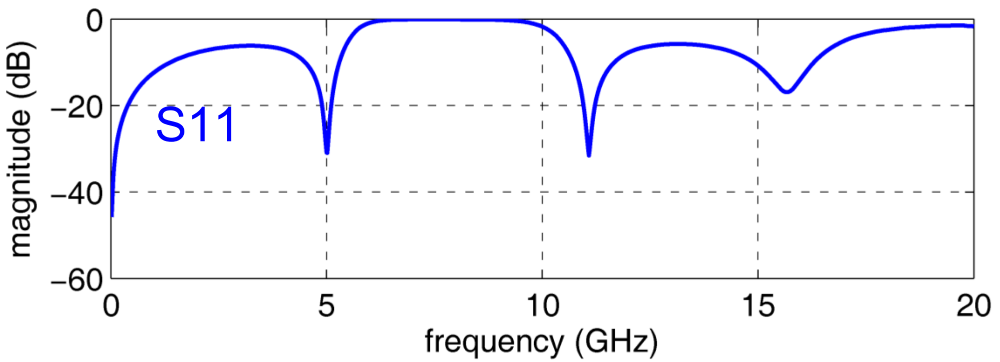


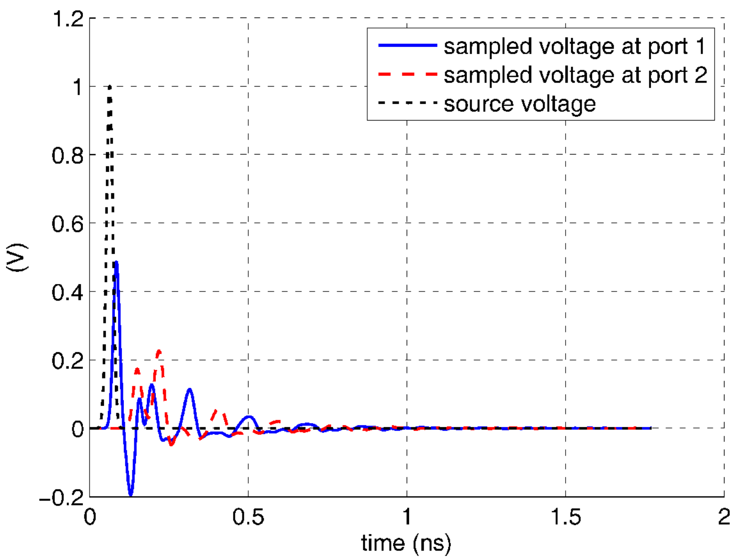
Transformation from Time-Domain to Frequency-Domain

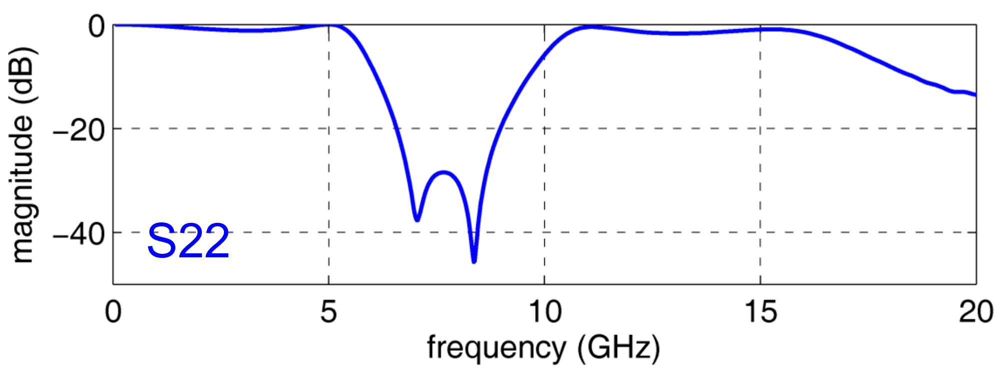
- Results can be obtained for frequency domain using Fourier Transform

A low-pass filter





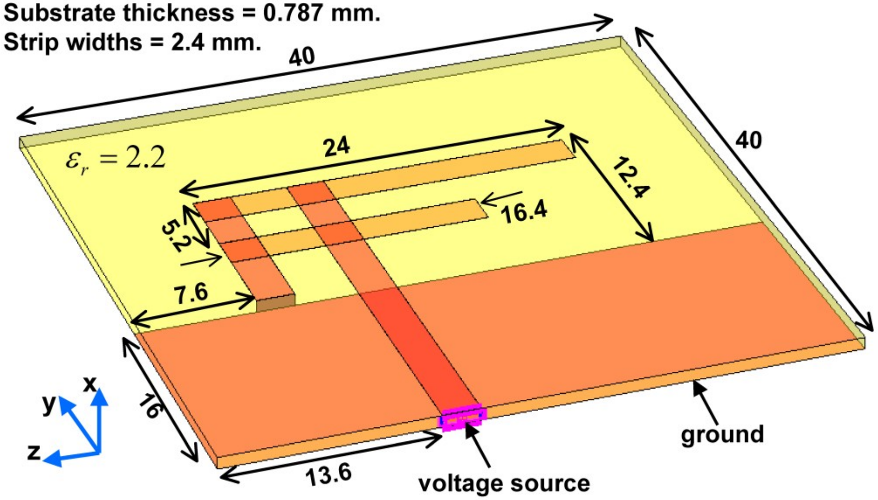




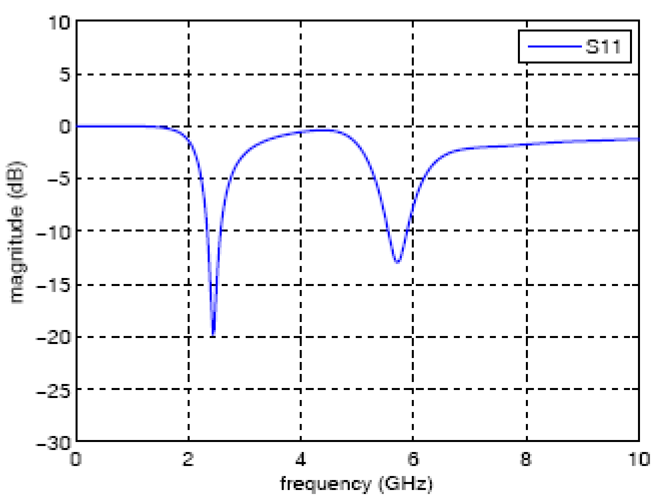
Exercise 2D object

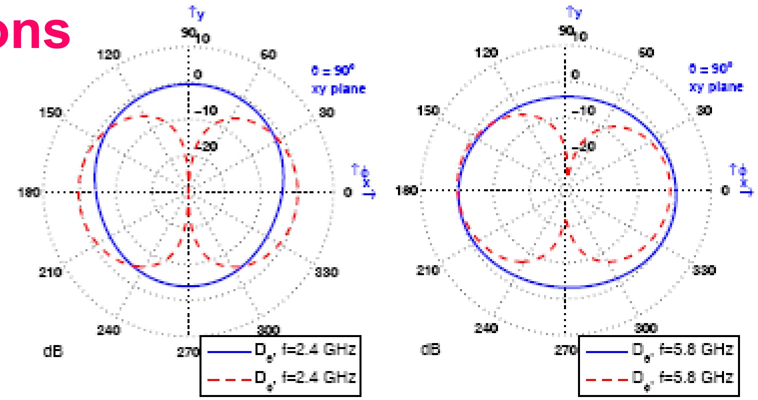
Near-Field to Far-field Transformations

Dimensions are in mm.
Substrate thickness = 0.787 mm.
Strip widths = 2.4 mm.

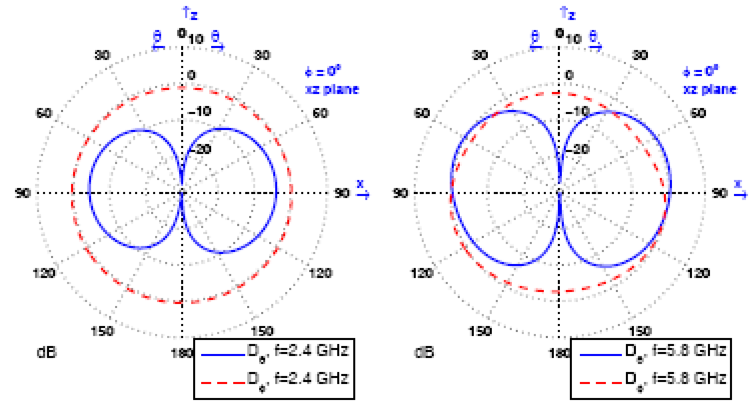


An inverted-F antenna

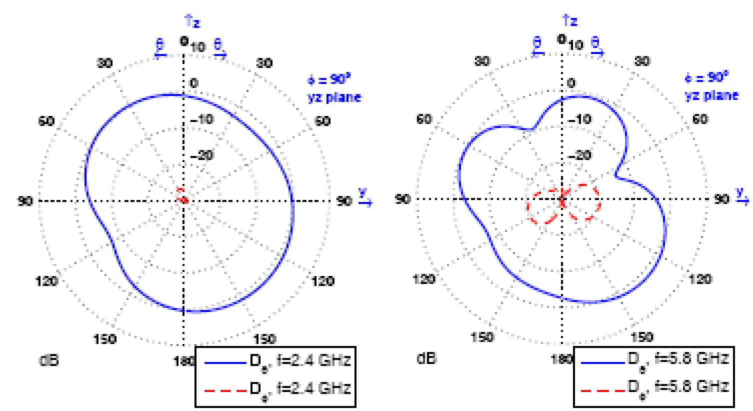




(a) 2.4 GHz (b) 5.8 GHz



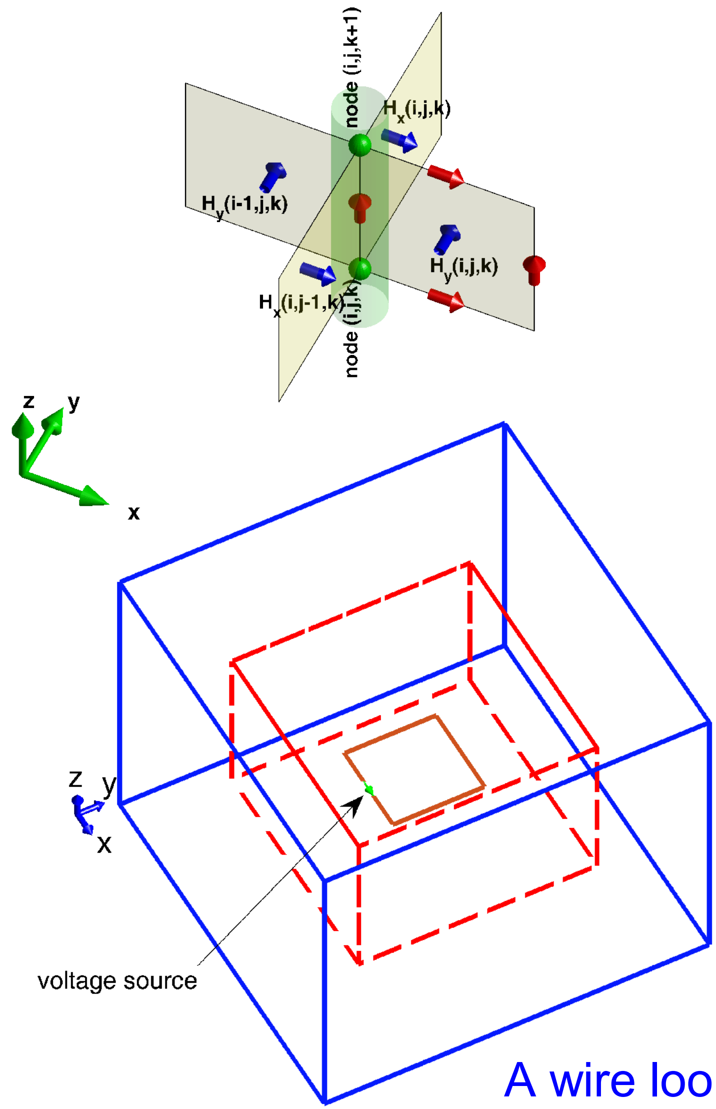
(a) 2.4 GHz (b) 5.8 GHz

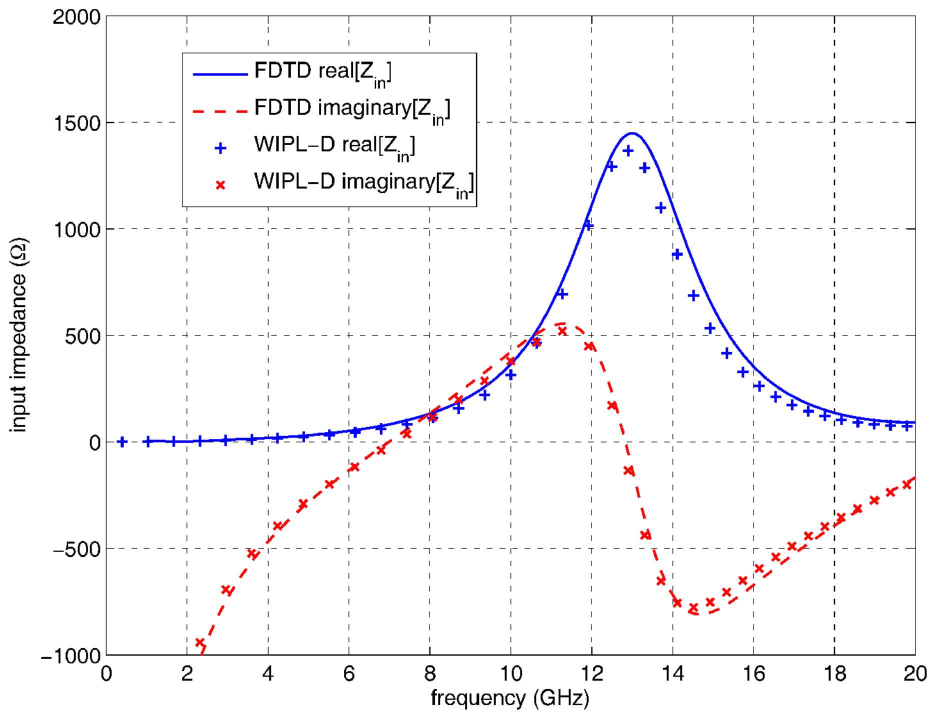


(a) 2.4 GHz (b) 5.8 GHz

Modeling fine geometries

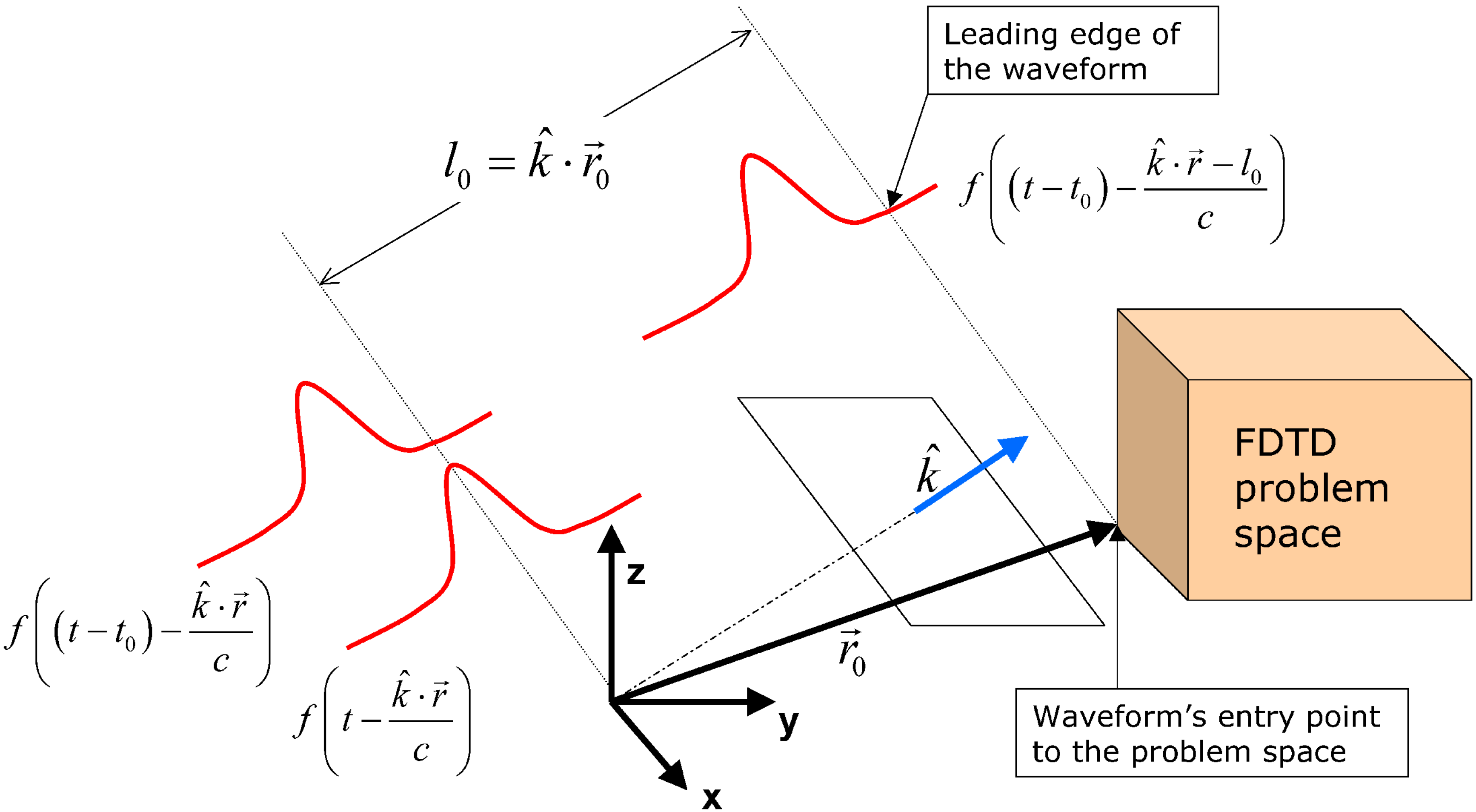
❖ It is possible to model fine structures using appropriate formulations





A wire loop antenna

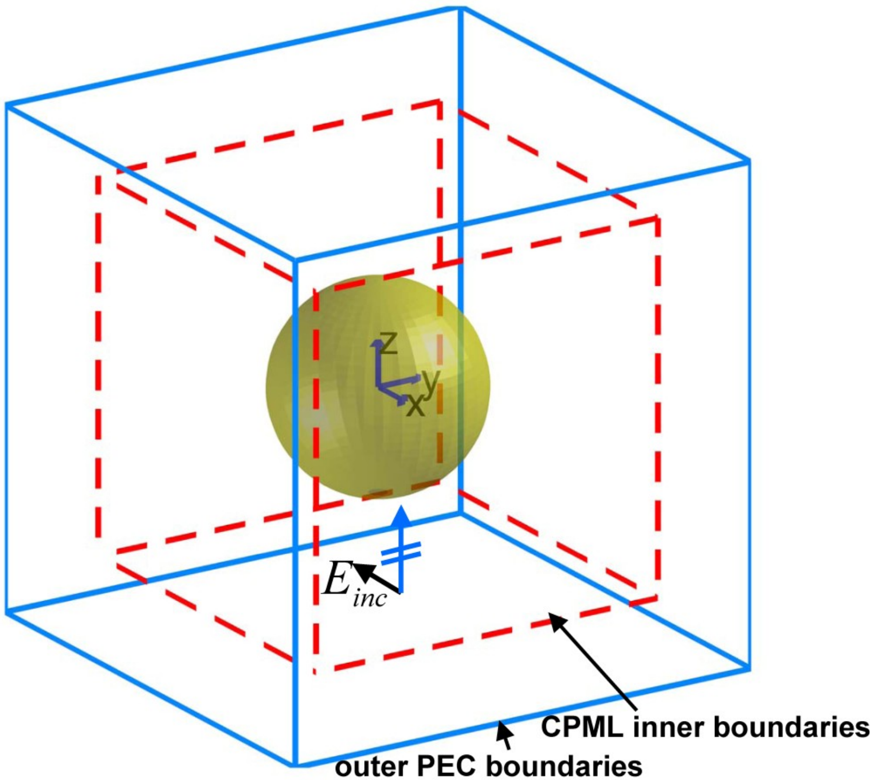
Incident plane wave



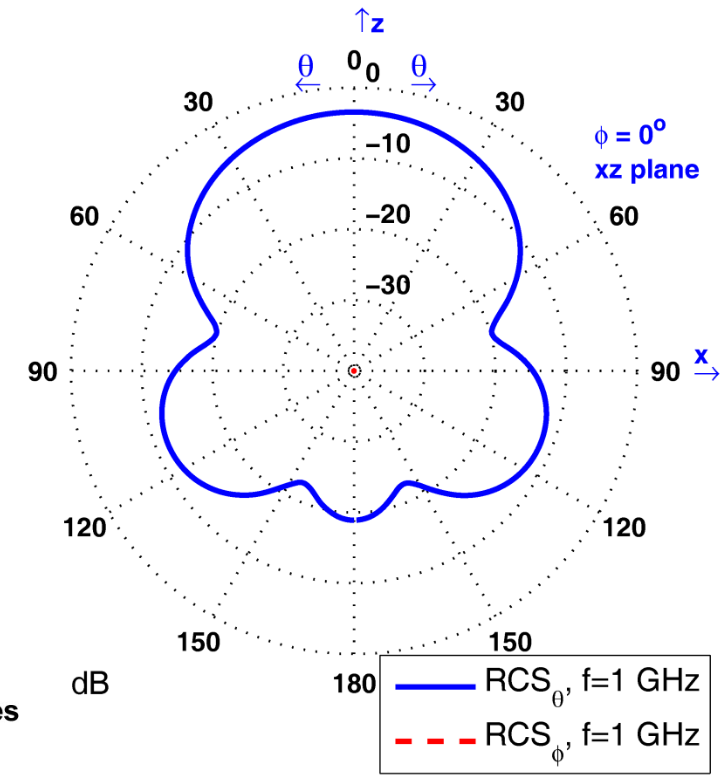
Scattering Problems

$$\nabla \times \left(\bar{H}_{inc} + \bar{H}_{scat} \right) = \varepsilon \frac{\partial}{\partial t} \left(\bar{E}_{inc} + \bar{E}_{scat} \right)$$

$$\nabla \times \bar{H}_{inc} = \varepsilon_0 \frac{\partial}{\partial t} \bar{E}_{inc}$$

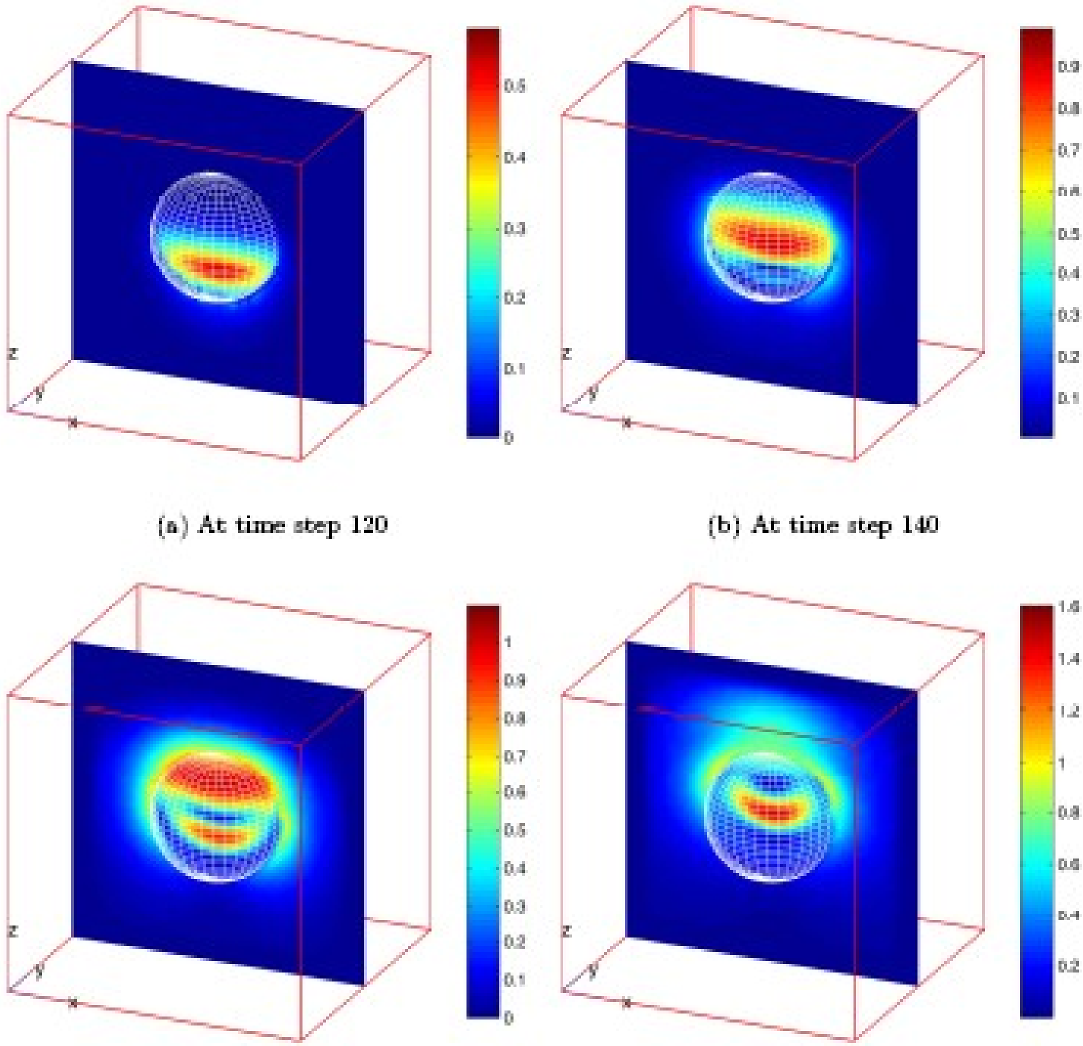


A dielectric sphere



Exercise 3D PML

Scattering from a Dielectric Sphere



(a) At time step 120

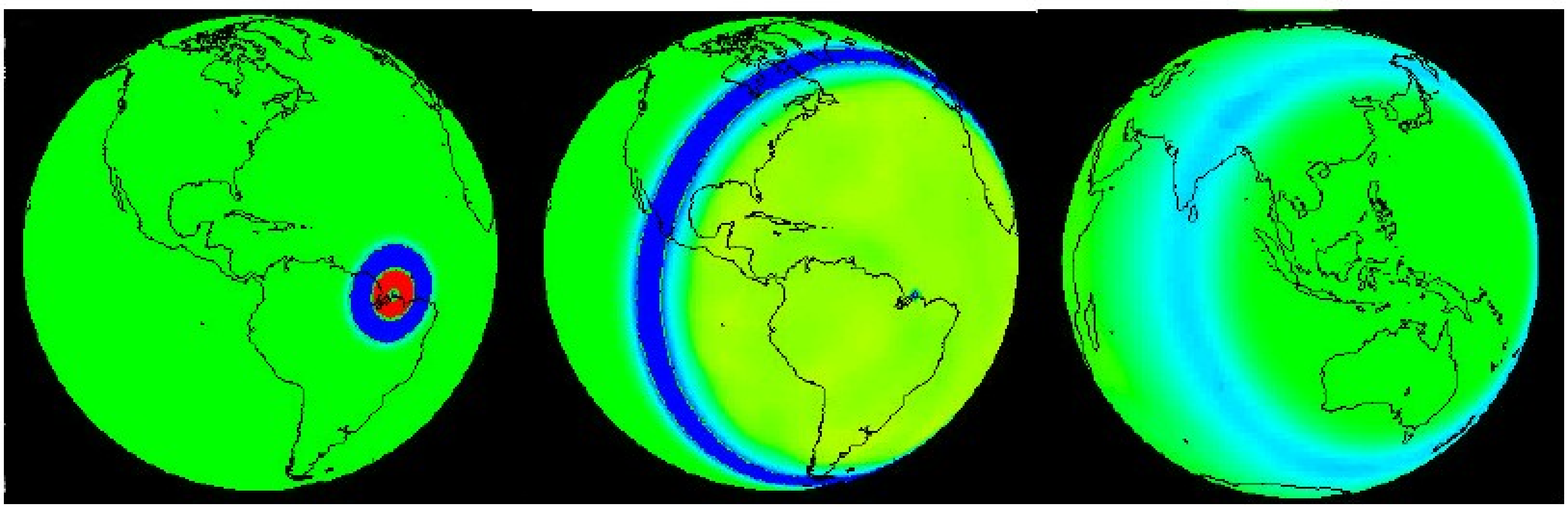
(b) At time step 140

(c) At time step 160

(d) At time step 180

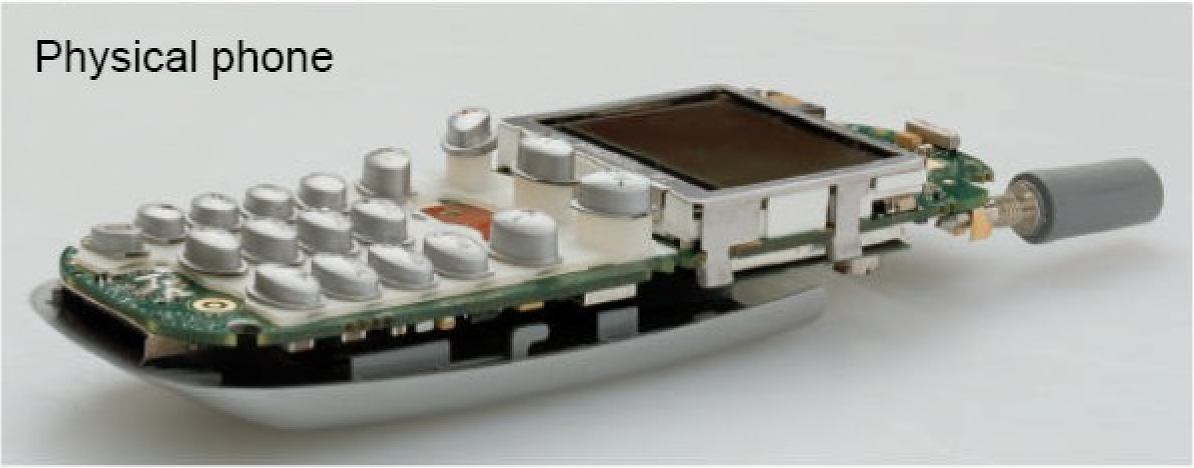
Earth / Ionosphere Models in Geophysics

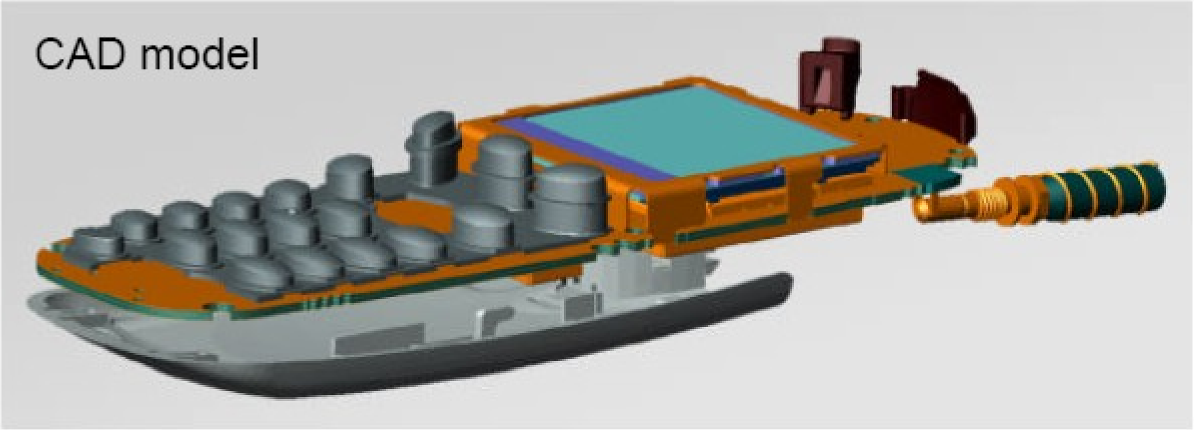
- Snapshots of FDTD-Computed Global Propagation of ELF Electromagnetic Pulse Generated by Vertical Lightning Strike off South America Coast



Source: Allen Taflove, "A Perspective on the 40-Year History of FDTD Computational Electrodynamics," **Applied Computational Electromagnetics Society (ACES) Conference, Miami, Florida,** March 15, 2006. Can be found at http://www.ece.northwestern.edu/ecefaculty/Allen1.html

Wireless Personal Communications Devices



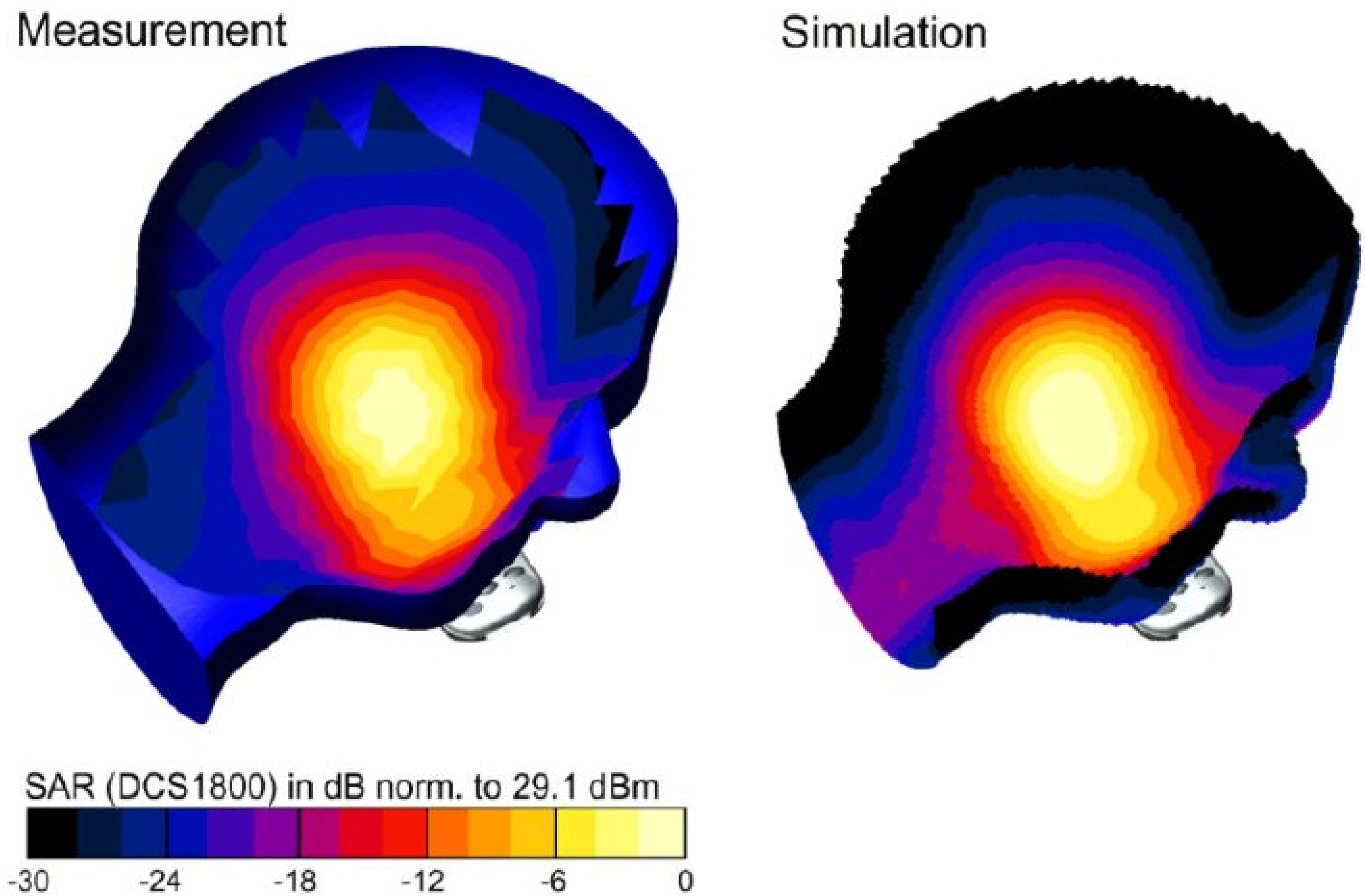


High-resolution FDTD model. The lattice-cell size is as fine as 0.1 mm to resolve individual circuit board layers and the helical antenna.

Source: Chavannes et al., *IEEE Antennas and Propagation Magazine*, Dec. 2003, pp. 52–66.

Source: Allen Taflove, “A Perspective on the 40-Year History of FDTD Computational Electrodynamics,” **Applied Computational Electromagnetics Society (ACES) Conference, Miami, Florida,** March 15, 2006. Can be found at http://www.ece.northwestern.edu/ecefaculty/Allen1.html

Phantom Head Validation at 1.8 GHz



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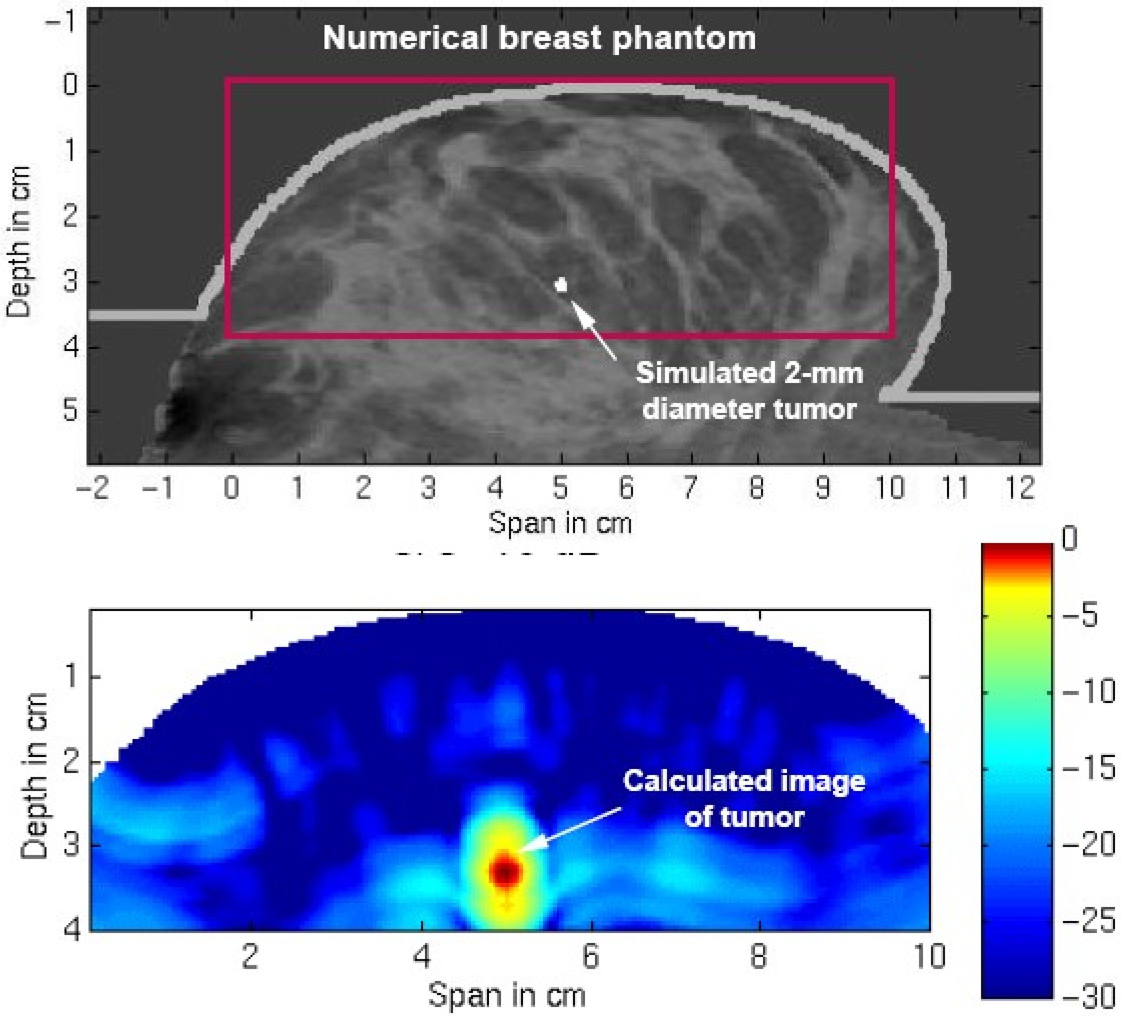
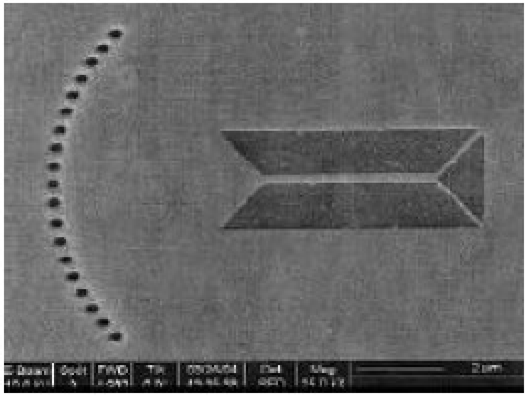
Ultrawideband Microwave Detection of Early-Stage Breast Cancer



FDTD simulation of UWB microwave detection of a 2-mm-diameter malignant tumor embedded 3 cm within an MRI-derived numerical breast model. The cancer's signature is 15 to 30 dB stronger than the clutter due to the surrounding normal tissues. <u>Source</u>: Bond et al., *IEEE Trans. Antennas and Propagation*, 2003, pp. 1690–1705.

Source: Allen Taflove, "A Perspective on the 40-Year History of FDTD Computational Electrodynamics," **Applied Computational Electromagnetics Society (ACES) Conference, Miami, Florida,** March 15, 2006. Can be found at http://www.ece.northwestern.edu/ecefaculty/Allen1.html

Focusing Plasmonic Lens

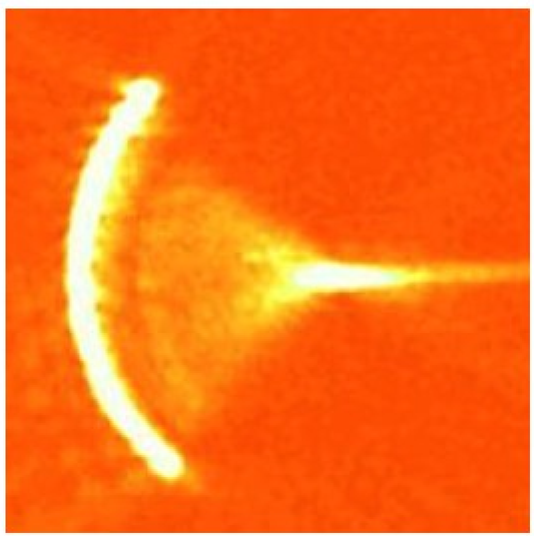


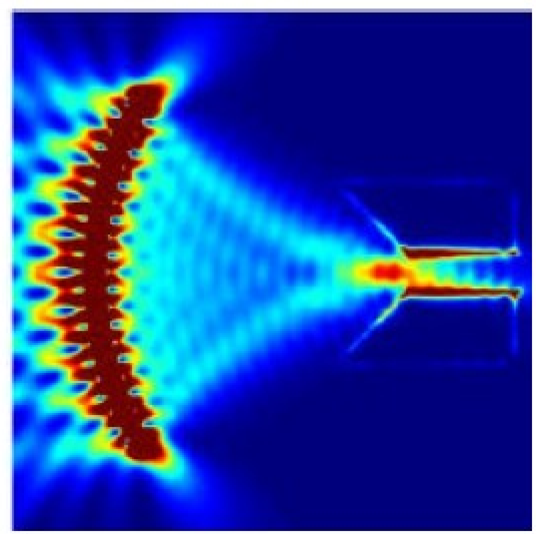
SEM photo

<u>Source</u> (left and bottom left images): L. Yin et al., *Nano Letters* 5, 1399 (2005).

<u>Source</u> (bottom right image): S-H. Chang

Experiment





FDTD

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Thank You