	Universidade Federal de Campina Grande - UFCG Centro de Ciências e Tecnologia - CCT Unidade Acadêmica de Matemática - UAMat	
DISCIPLINA:	Cálculo Diferencial e Integral II	PERÍODO: 2021.2
CURSO:		TURNO: TARDE
PROFESSOR:		DATA: 15/06/2022
ALUNO(A):		NOTA:

AVALIAÇÃO 1

ATENÇÃO: **Desligue e guarde qualquer aparelho eletrônico. Não remova o grampo da prova! Questões sem os cálculos serão desconsideradas.**

QUESTÃO OBJETIVA (6 pontos) Calcule as integrais e marque a alternativa com a resposta correta.

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1. (1,5 ponto) $\displaystyle\int_0^{\pi/4} 8\cos^3(2x)\sin(2x)dx$

   (a) 0 (b) π (c) 1 (d) 2 (e) -1

2. (1,5 ponto) $\displaystyle\int_0^{\pi/2} \theta^2\sin(2\theta)d\theta$

   (a) 0 (b) π (c) 8π (d) $\dfrac{\pi^2+4}{8}$ (e) $\dfrac{\pi^2-4}{8}$

3. (1,5 ponto) $\displaystyle\int_{\frac{1}{2}}^{\frac{5}{2}} \frac{9}{3x-x^2}dx$

   (a) $\dfrac{\ln 3}{3}$ (b) $3\ln 25$ (c) $\dfrac{\ln 2}{3}$ (d) $3\ln 3$ (e) 0

4. (1,5 ponto) $\displaystyle\int_0^{\frac{1}{2\sqrt{2}}} \frac{2dx}{\sqrt{1-4x^2}}dx$

   (a) $\dfrac{\pi}{4}$ (b) π (c) $\dfrac{\pi}{6}$ (d) 3π (e) 0

QUESTÕES DISCURSIVAS (4 pontos)

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5. (2 pontos) Calcule a integral $\displaystyle\int e^x \sin x dx$.

6. (2 pontos) Analise a convergência das integrais.

   (a) $\displaystyle\int_0^{2\pi} \frac{\sin\left(\frac{x}{2}\right)}{2x^{1/3}}dx$ (b) $\displaystyle\int_2^{\infty} \frac{1}{\sqrt{x-1}}dx$

BOA PROVA!!!

1) Temos

$$\int 8\cos^3(2x)\,\mathrm{sen}(2x)\,dx = \int 8\cos^2(2x)\cos(2x)\,\mathrm{sen}(2x)\,dx$$

$$= 8\int (1-\mathrm{sen}^2(2x))\,\mathrm{sen}(2x)\cos(2x)\,dx$$

$$= 8\int (1-u^2)\,u\,\frac{du}{2}$$

$$= 4\int (u-u^3)\,du$$

$$= 4\cdot\frac{u^2}{2} - 4\cdot\frac{u^4}{4}$$

$$= 2\,\mathrm{sen}^2(2x) - \mathrm{sen}^4(2x) + C$$

Substituisão
$u = \mathrm{sen}(2x)$
$du = 2\cos(2x)\,dx$
$\cos(2x)\,dx = \frac{du}{2}$

Logo,

$$\int_0^{\pi/4} 8\cos^3(2x)\,\mathrm{sen}(2x)\,dx = \Big[2\,\mathrm{sen}^2(2x) - \mathrm{sen}^4(2x)\Big]_0^{\pi/4}$$

$$= 2\,\overset{=1}{\mathrm{sen}^2\tfrac{\pi}{2}} - \overset{=1}{\mathrm{sen}^4\tfrac{\pi}{2}} - 2\,\overset{=0}{\mathrm{sen}^2 0} + \overset{=0}{\mathrm{sen}^4 0}$$

$$= 2 - 1 = 1 \qquad \text{(Resposta (c))}$$

$\boxed{\text{Obs:}}$ No início é mais simples fazer $u = \cos(2x)$ em $\int 8\cos^3(2x)\,\mathrm{sen}(2x)\,dx$!

2) Vamos utilizar o método tabular no nosso cálculo. Sendo $f(\theta) = \theta^2$ e $g(\theta) = \mathrm{sen}(2\theta)$, temos

$f(\theta)$ e suas derivadas		$g(\theta)$ e suas integrais
θ^2	$(+)$	$\mathrm{sen}(2\theta)$
2θ	$(-)$	$-\frac{1}{2}\cos(2\theta)$
2	$(+)$	$-\frac{1}{4}\mathrm{sen}(2\theta)$
0		$\frac{1}{8}\cos(2\theta)$

Logo,

$$\int_0^{\pi/2} \theta^2\,\mathrm{sen}(2\theta)\,d\theta = \Big[-\frac{\theta^2}{2}\cos(2\theta) + \frac{\theta}{2}\,\mathrm{sen}(2\theta) + \frac{1}{4}\cos(2\theta)\Big]_0^{\pi/2}$$

$$= -\frac{(\pi/2)^2}{2}\overset{=-1}{\cos\pi} + \frac{\pi/2}{2}\overset{=0}{\mathrm{sen}\,\pi} + \frac{1}{4}\overset{=-1}{\cos\pi} + 0 - 0 - \frac{1}{4}\overset{=1}{\cos 0} = \frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4} = \frac{\pi^2-4}{8}$$

(Resposta (e))

②

3) Vamos utilizar frações parciais. Temos

$$\frac{9}{3x-x^2} = \frac{-9}{x^2-3x} = \frac{-9}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$$

Pelo Método de Heaviside, obtemos

$$A = \left.\frac{-9}{x-3}\right|_{x=0} = \frac{-9}{-3} = 3$$

$$B = \left.\frac{-9}{x}\right|_{x=3} = \frac{-9}{3} = -3$$

Logo,

$$\int \frac{9}{3x-x^2}\,dx = \int \frac{3}{x}\,dx + \int \frac{(-3)}{x-3}\,dx$$

$$= 3\ln|x| - 3\ln|x-3| + C$$

$$= 3\left[\ln|x| - \ln|x-3|\right] + C$$

Portanto,

$$\int_{1/2}^{5/2} \frac{9}{3x-x^2}\,dx = 3\Big[\ln|x| - \ln|x-3|\Big]_{1/2}^{5/2}$$

$$= 3\left[\ln\frac{5}{2} - \ln\left|\frac{5}{2}-3\right| - \ln\frac{1}{2} + \ln\left|\frac{1}{2}-3\right|\right]$$

$$= 3\left[\ln\frac{5}{2} - \ln\frac{1}{2} - \ln\frac{1}{2} + \ln\frac{5}{2}\right]$$

$$= 3\left[2\ln\frac{5}{2} - 2\ln\frac{1}{2}\right] = 6\left[\ln\frac{5}{2} - \ln\frac{1}{2}\right]$$

$$= 6\ln\frac{5/2}{1/2} = 6\ln 5 = 3\cdot 2\ln 5 = 3\ln 5^2$$

$$= 3\ln 25 \qquad \text{(Resposta (b))}$$

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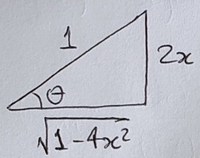
4) Vamos fazer uma substituição trigonométrica.

Temos,



$$\bullet \ \operatorname{sen}\theta = \frac{2x}{1} \Rightarrow x = \frac{\operatorname{sen}\theta}{2}$$

$$\Rightarrow dx = \frac{1}{2}\cos\theta \, d\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\bullet \ \sqrt{1-4x^2} = \sqrt{1 - 4\frac{\operatorname{sen}^2\theta}{4}} = \sqrt{1-\operatorname{sen}^2\theta}$$

$$= \sqrt{\cos^2\theta} = |\cos\theta| = \cos\theta$$

Logo,

$$\int \frac{2dx}{\sqrt{1-4x^2}} = \int \frac{2\cdot\frac{1}{2}\cos\theta\, d\theta}{\cos\theta} = \int d\theta = \theta$$

$$= \operatorname{arc\,sen}(2x) + C$$

e, portanto

$$\int_0^{\frac{1}{2\sqrt{2}}} \frac{2dx}{\sqrt{1-4x^2}} = \operatorname{arc\,sen}\left(2\cdot\frac{1}{2\sqrt{2}}\right) - \operatorname{arc\,sen} 0$$

$$= \operatorname{arc\,sen}\frac{\sqrt{2}}{2} - \operatorname{arc\,sen} 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4} \quad \text{(Resposta (a))}$$

5) Vamos usar integração por partes. Temos

$$u = e^x \qquad dv = \operatorname{sen} x\, dx$$

$$du = e^x dx \qquad v = -\cos x$$

Daí,

$$\int \underbrace{e^x}_{u} \underbrace{\operatorname{sen} x\, dx}_{dv} = uv - \int v\,du = -e^x\cos x - \int(-\cos x)e^x dx$$

$$= -e^x\cos x + \int e^x \cos x\, dx \quad (\mathrm{I})$$

Utilizamos, novamente, integração por partes em $\int e^x \cos x dx$.

Temos,

$u = e^x \qquad dv = \cos x\, dx$

$du = e^x dx \qquad v = \operatorname{sen} x$

Daí,

$$\int \underbrace{e^x}_{u} \underbrace{\cos x\, dx}_{dv} = uv - \int v\, du = e^x \operatorname{sen} x - \int e^x \operatorname{sen} x\, dx \quad (\text{II})$$

Substituindo (II) em (I), obtemos

$$\int e^x \operatorname{sen} x\, dx = -e^x \cos x + \left(e^x \operatorname{sen} x - \int e^x \operatorname{sen} x\, dx\right)$$

$$2\int e^x \operatorname{sen} x\, dx = e^x \operatorname{sen} x - e^x \cos x$$

$$\int e^x \operatorname{sen} x\, dx = \frac{1}{2}\left(e^x \operatorname{sen} x - e^x \cos x\right) + C.$$

5 (a) Como $-1 \leq \operatorname{sen} x \leq 1$, para todo $x \in [0, 2\pi]$, temos

$$\frac{\operatorname{sen}\left(\frac{x}{2}\right)}{2x^{1/3}} \leq \frac{1}{2x^{1/3}}, \quad \text{para todo } 0 \leq x \leq 2\pi.$$

Mas,

$$\int_0^{2\pi} \frac{1}{2x^{1/3}}\, dx = \frac{1}{2} \lim_{a \to 0^+} \int_a^{2\pi} x^{-1/3}\, dx = \frac{1}{2} \lim_{a \to 0^+} \left[\frac{x^{-1/3+1}}{-1/3+1}\right]_a^{2\pi}$$

$$= \frac{1}{2} \lim_{a \to 0^+} \left[\frac{3}{2} x^{2/3}\right]_a^{2\pi} = \frac{3}{4} \lim_{a \to 0^+} \left[(2\pi)^{2/3} - a^{2/3}\right]$$

$$= \frac{3}{4}(2\pi)^{2/3}$$

Assim, como $\int_0^{2\pi} \frac{1}{2x^{1/3}}\, dx$ converge, segue pelo Teste da Comparação Direta que $\int_0^{2\pi} \frac{\operatorname{sen}\left(\frac{x}{2}\right)}{2x^{1/3}}\, dx$ também converge.

5

6 (b) Sejam $f(x) = \dfrac{1}{\sqrt{x-1}}$ e $g(x) = \dfrac{1}{\sqrt{x}}$. Temos

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{1}{\sqrt{x-1}} \cdot \sqrt{x} = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x-1}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{x}{x-1}} = \lim_{x\to\infty} \sqrt{\frac{1}{1-1/x}} = \sqrt{\lim_{x\to\infty} \frac{1}{1-1/x}}$$

$$= 1 > 0$$

e

$$\int_2^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty} \int_2^b x^{-1/2}\,dx = \lim_{b\to\infty} \left[2x^{1/2}\right]_2^b$$

$$= \lim_{b\to\infty} \left[2\sqrt{b} - 2\sqrt{2}\right] = \infty$$

Como $\int_2^\infty \frac{1}{\sqrt{x}}\,dx$ diverge, segue pelo Teste da Comparação no Limite, que $\int_2^\infty \frac{1}{\sqrt{x-1}}\,dx$ também diverge.