

### Rotation Notation/Convention

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = [\Lambda] \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} \hat{x}^T \\ \hat{y}^T \\ \hat{z}^T \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \quad (\text{from global to local})$$

or equivalently:

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = [\Lambda]^T \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

where  $X/Y/Z$  are global coordinates,  $x/y/z$  are local coordinates,  $\Lambda$  is the DCM from global to local, and  $\hat{x} / \hat{y} / \hat{z}$  are the unit vectors of the local coordinate system expressed in the global coordinate system.

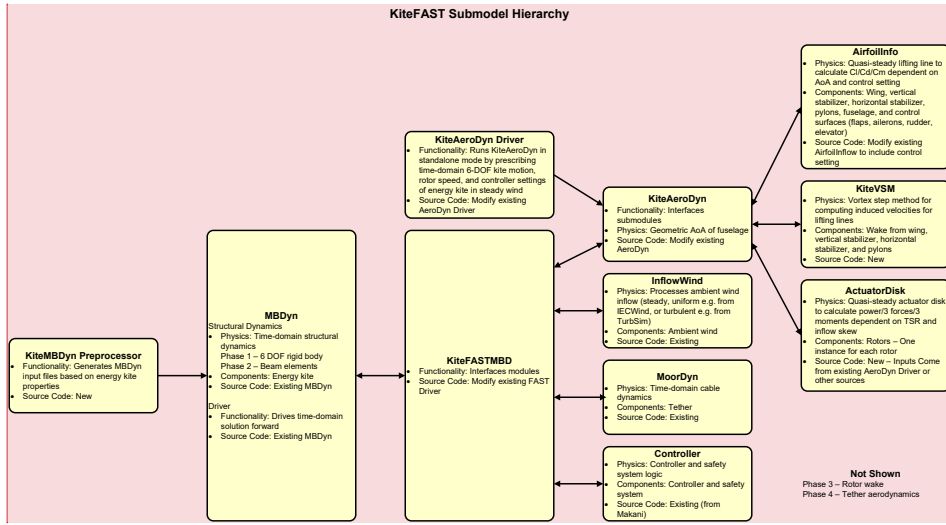
$$\begin{Bmatrix} \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix} = F^{EulerExtract} \left( [\Lambda(\theta_x, \theta_y, \theta_z)] \right)$$

where function  $F^{EulerExtract} ( )$  returns the 3 Euler angles of the x-y-z (1-2-3) rotation sequence used to form  $\Lambda$  (that is, first a rotation  $\theta_x$  about the global  $X$  axis, followed by rotation  $\theta_y$  about the  $Y'$  axis, followed by rotation  $\theta_z$  about the  $Z''$  axis) defined as follows:

$$\begin{aligned} \Lambda(\theta_x, \theta_y, \theta_z) &= \begin{bmatrix} \cos(\theta_z) & \sin(\theta_z) & 0 \\ -\sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_y) & 0 & -\sin(\theta_y) \\ 0 & 1 & 0 \\ \sin(\theta_y) & 0 & \cos(\theta_y) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & \sin(\theta_x) \\ 0 & -\sin(\theta_x) & \cos(\theta_x) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\theta_x)\cos(\theta_z) & \cos(\theta_x)\sin(\theta_z) + \sin(\theta_x)\sin(\theta_y)\cos(\theta_z) & \sin(\theta_x)\sin(\theta_z) - \cos(\theta_x)\sin(\theta_y)\cos(\theta_z) \\ -\cos(\theta_x)\sin(\theta_z) & \cos(\theta_x)\cos(\theta_z) - \sin(\theta_x)\sin(\theta_y)\sin(\theta_z) & \sin(\theta_x)\cos(\theta_z) + \cos(\theta_x)\sin(\theta_y)\sin(\theta_z) \\ \sin(\theta_x) & -\sin(\theta_x)\cos(\theta_y) & \cos(\theta_x)\cos(\theta_y) \end{bmatrix} \end{aligned}$$

Note the following simplifications:

$$\begin{aligned} \Lambda(0, \theta_y, \theta_z) &= \begin{bmatrix} \cos(\theta_y)\cos(\theta_z) & \sin(\theta_z) & -\sin(\theta_y)\cos(\theta_z) \\ -\cos(\theta_y)\sin(\theta_z) & \cos(\theta_z) & \sin(\theta_y)\sin(\theta_z) \\ \sin(\theta_y) & 0 & \cos(\theta_y) \end{bmatrix} \\ \Lambda(\theta_x, 0, \theta_z) &= \begin{bmatrix} \cos(\theta_z) & \cos(\theta_x)\sin(\theta_z) & \sin(\theta_x)\sin(\theta_z) \\ -\sin(\theta_z) & \cos(\theta_x)\cos(\theta_z) & \sin(\theta_x)\cos(\theta_z) \\ 0 & -\sin(\theta_x) & \cos(\theta_x) \end{bmatrix} \\ \Lambda(\theta_x, \theta_y, 0) &= \begin{bmatrix} \cos(\theta_y) & \sin(\theta_x)\sin(\theta_y) & -\cos(\theta_x)\sin(\theta_y) \\ 0 & \cos(\theta_x) & \sin(\theta_x) \\ \sin(\theta_y) & -\sin(\theta_x)\cos(\theta_y) & \cos(\theta_x)\cos(\theta_y) \end{bmatrix} \end{aligned}$$



**Commented [JJ1]:** We've split up KiteFASTMBD into KiteFASTMBD in C and KiteFASTMBD in Fortran. This plan is for KiteFASTMBD in Fortran.

## KiteFASTMBD

Inputs	Outputs	States	Parameters
<ul style="list-style-type: none"> <li><math>^{MBD} \vec{p}^{Wind}</math> – Position of the base station where the fixed wind measurement is taken (m)</li> <li><math>^{MBD} \vec{p}^{FusO}</math> – Position (origin) of the fuselage (m)</li> <li><math>^{MBD} \vec{\Lambda}^{FusO}</math> – Rotation (absolute orientation) of the fuselage origin (-)</li> <li><math>^{MBD} \vec{v}^{FusO}</math> – Translational velocity (absolute) of the fuselage origin (m/s)</li> <li><math>^{MBD} \vec{\omega}^{FusO}</math> – Rotational velocity (absolute) of the fuselage origin (rad/s)</li> <li><math>^{MBD} \vec{a}^{FusO}</math> – Translational acceleration (absolute) of the fuselage origin (m/s<sup>2</sup>)</li> <li><math>^{MBD} \vec{\alpha}^{FusO}</math> – Rotational acceleration (absolute) of the fuselage origin (rad/s<sup>2</sup>)</li> <li><math>^{MBD} \vec{p}_j^{Fus}</math> – Translational position (absolute) of the <math>j^{th}</math> node of the fuselage mesh (m)</li> </ul>	<ul style="list-style-type: none"> <li><math>^{MBD} \vec{F}_j^{Fus}</math> – Aerodynamic applied concentrated forces at the <math>j^{th}</math> node of the fuselage mesh (N)</li> <li><math>^{MBD} \vec{M}_j^{Fus}</math> – Aerodynamic applied concentrated moments at the <math>j^{th}</math> node of the fuselage mesh (N-m)</li> <li><math>^{MBD} \vec{F}_j^{SWn}</math> – Aerodynamic and tether applied concentrated forces at the <math>j^{th}</math> node of the starboard wing mesh (N)</li> <li><math>^{MBD} \vec{M}_j^{SWn}</math> – Aerodynamic applied and tether concentrated moments at the <math>j^{th}</math> node of the starboard wing mesh (N-m)</li> <li><math>^{MBD} \vec{F}_j^{PWn}</math> – Aerodynamic and tether applied concentrated forces at the <math>j^{th}</math> node of the port wing</li> </ul>	<ul style="list-style-type: none"> <li><math>^{KAD} NewTime</math> – Is this a new time step (in order to only call KiteAeroDyn once per step)? (flag) (other state)</li> <li><math>^{Ctrl} NewTime</math> – Is this a new time step (in order to only call the controller once per step)? (flag) (other state)</li> <li><math>^{MBD} OtherStates</math> – Inputs from MBDyn from the previous time step (stored as other states)</li> <li><math>^{MD} OtherStates</math> – Inputs to MoorDyn from the previous time step (stored as other states)</li> <li><math>^{MD} x</math> – MoorDyn</li> </ul>	<ul style="list-style-type: none"> <li><math>\Delta t</math> – MBDyn time step (s)</li> <li><math>^{KAD} \Delta t</math> – KiteAeroDyn time step (s)</li> <li><math>InterpOrder</math> – Interpolation order for input/output time history (-) {1=linear, 2=quadratic}</li> <li><math>N_{KAD/MBD}</math> – Number of KiteAeroDyn time steps per MBDyn time step (-)</li> <li><math>N_{Ctrl/MBD}</math> – Number of controller time steps per MBDyn time step (-)</li> <li><math>N_{Flaps}</math> – Number of flaps per wing (-)</li> <li><math>N_{Pylons}</math> – Number of pylons per wing (-)</li> <li><math>\Lambda^{FAST2Ctrl}</math> – DCM conversion from the FAST ground system (X pointed nominally downwind; Z pointed vertically opposite gravity; Y transverse) to the ground system used by</li> </ul>

**Commented [JJ2]:** These are the data queried from the MBDyn model at t using GetXCurl to be used within KiteFASTMBD. The outputs from MBDyn are inputs to KiteFASTMBD.

**Commented [JJ3]:** These are the data sent to the MBDyn model from KiteFASTMBD. The inputs to MBDyn are outputs from KiteFASTMBD.

**Commented [JJ4]:** Obvious parameters are not listed here.

<ul style="list-style-type: none"> <li>• <math>^{MBD}\vec{A}_j^{Fus}</math> – Displaced rotation (absolute orientation) of the <math>j^{th}</math> node of the fuselage mesh (-)</li> <li>• <math>^{MBD}\vec{V}_j^{Fus}</math> – Translational velocity (absolute) of the <math>j^{th}</math> node of the fuselage mesh (m/s)</li> <li>• <math>^{MBD}\vec{\omega}_j^{Fus}</math> – Rotational velocity (absolute) of the <math>j^{th}</math> node of the fuselage mesh (rad/s)</li> <li>• <math>^{MBD}\vec{a}_j^{Fus}</math> – Translational acceleration (absolute) of the <math>j^{th}</math> node of the fuselage mesh (m/s<sup>2</sup>)</li> <li>• <math>^{MBD}\vec{F}R_j^{Fus}</math> – Reaction force (expressed in the local coordinate system) at the <math>j^{th}</math> Gauss point of the fuselage mesh (N)</li> <li>• <math>^{MBD}\vec{M}R_j^{Fus}</math> – Reaction moment (expressed in the local coordinate system) at the <math>j^{th}</math> Gauss point of the fuselage mesh (N-m)</li> <li>• <math>^{MBD}\vec{p}^{SWnO}</math> – Position (origin) of the starboard wing (m)</li> <li>• <math>^{MBD}\vec{p}_j^{SWn}</math> – Translational position (absolute) of the <math>j^{th}</math> node of the starboard wing (m)</li> <li>• <math>^{MBD}\vec{A}_j^{SWn}</math> – Displaced rotation (absolute orientation) of the <math>j^{th}</math> node of the starboard wing mesh (-)</li> <li>• <math>^{MBD}\vec{V}_j^{SWn}</math> – Translational velocity (absolute) of the <math>j^{th}</math> node of the starboard wing mesh (m/s)</li> <li>• <math>^{MBD}\vec{\omega}_j^{SWn}</math> – Rotational velocity (absolute) of the</li> </ul>	<ul style="list-style-type: none"> <li>mesh (N)</li> <li>• <math>^{MBD}\vec{M}_j^{PWn}</math> – Aerodynamic and tether applied concentrated moments at the <math>j^{th}</math> node of the port wing mesh (N-m)</li> <li>• <math>^{MBD}\vec{F}_j^{VS}</math> – Aerodynamic applied concentrated forces at the <math>j^{th}</math> node of the vertical stabilizer mesh (N)</li> <li>• <math>^{MBD}\vec{M}_j^{VS}</math> – Aerodynamic applied concentrated moments at the <math>j^{th}</math> node of the vertical stabilizer mesh (N-m)</li> <li>• <math>^{MBD}\vec{F}_j^{SHS}</math> – Aerodynamic applied concentrated forces at the <math>j^{th}</math> node of the starboard horizontal stabilizer mesh (N)</li> <li>• <math>^{MBD}\vec{M}_j^{SHS}</math> – Aerodynamic applied concentrated moments at the <math>j^{th}</math> node of the starboard horizontal stabilizer mesh (N-m)</li> <li>• <math>^{MBD}\vec{F}_j^{PHS}</math> – Aerodynamic applied concentrated forces at the <math>j^{th}</math> node of the port horizontal stabilizer mesh (N)</li> <li>• <math>^{MBD}\vec{M}_j^{PHS}</math> – Aerodynamic applied concentrated moments at the <math>j^{th}</math> node of the port horizontal stabilizer mesh (N-m)</li> <li>• <math>^{MBD}\vec{F}_j^{SPy}[n_{Pylons}]</math> – Aerodynamic applied concentrated forces at the <math>j^{th}</math> node of the pylons on the starboard wing mesh (N)</li> <li>• <math>^{MBD}\vec{M}_j^{SPy}[n_{Pylons}]</math> –</li> </ul>	<ul style="list-style-type: none"> <li>continuous states (varied)</li> <li>• <math>^{KAD}\vec{z}</math> – KiteAeroDyn constraint states (varied)</li> <li>• <math>^{KAD}u(\cdot)</math> – Time history of KiteAeroDyn inputs (stored as other states)</li> <li>• <math>^{KAD}y(\cdot)</math> – Time history of KiteAeroDyn outputs (stored as other states)</li> <li>• <math>^{KAD}t(\cdot)</math> – Times associated with history of KiteAeroDyn inputs and outputs (stored as other states)</li> </ul>	<p>the controller (X pointed nominally upwind; Z pointed vertically downward, Y transverse) (-)</p> <ul style="list-style-type: none"> <li>• <math>^{MBD}\vec{g}</math> – Gravity vector expressed in the global inertial-frame coordinate system (m/s<sup>2</sup>)</li> <li>• <math>\rho</math> – Air density (kg/m<sup>3</sup>)</li> <li>• <math>\vec{p}^{Anch}</math> – Position of the base station where the tether attaches (i.e. mooring line anchor) (m)</li> <li>• <math>^{MBD}m^{SPyRtr}[n_{Pylons}, n_2]</math> – Mass of the top and bottom rotors/drivetrains on the pylons on the starboard wing mesh (kg)</li> <li>• <math>^{MBD}I^{SPyRtr}_{Rot}[n_{Pylons}, n_2]</math> – Rotational inertia about the shaft axis of the top and bottom rotors/drivetrains on the pylons on the starboard wing mesh (kg·m<sup>2</sup>)</li> <li>• <math>^{MBD}I^{SPyRtr}_{Tran}[n_{Pylons}, n_2]</math> – Transverse inertia about the rotor reference point of the top and bottom rotors/drivetrains on the pylons on the starboard wing mesh (kg·m<sup>2</sup>)</li> <li>• <math>^{MBD}x_{CM}^{SPyRtr}[n_{Pylons}, n_2]</math> – Distance along the shaft from the rotor reference point of the top and bottom rotors/drivetrains on the pylons on the starboard wing mesh to the center of mass of the rotor/drivetrain (positive along positive x) (m)</li> <li>• <math>^{MBD}m^{PPyRtr}[n_{Pylons}, n_2]</math> – Mass of the top and bottom rotors/drivetrains on the pylons on the port wing mesh (kg)</li> <li>• <math>^{MBD}I^{PPyRtr}_{Rot}[n_{Pylons}, n_2]</math></li> </ul>
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<p><math>j^{\text{th}}</math> node of the starboard wing mesh (rad/s)</p> <ul style="list-style-type: none"> <li>• <math>{}^{MBD}\vec{a}_j^{SWn}</math> – Translational acceleration (absolute) of the <math>j^{\text{th}}</math> node of the starboard wing mesh (m/s<sup>2</sup>)</li> <li>• <math>{}^{MBD}\vec{F}R_j^{SWn}</math> – Reaction force (expressed in the local coordinate system) at the <math>j^{\text{th}}</math> Gauss point of the starboard wing mesh (N)</li> <li>• <math>{}^{MBD}\vec{M}R_j^{SWn}</math> – Reaction moment (expressed in the local coordinate system) at the <math>j^{\text{th}}</math> Gauss point of the starboard wing mesh (N-m)</li> <li>• <math>{}^{MBD}\vec{p}^{PWnO}</math> – Position (origin) of the port wing mesh (m)</li> <li>• <math>{}^{MBD}\vec{p}_j^{PWn}</math> – Translational position (absolute) of the <math>j^{\text{th}}</math> node of the port wing mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{PWn}</math> – Displaced rotation (absolute orientation) of the <math>j^{\text{th}}</math> node of the port wing mesh (-)</li> <li>• <math>{}^{MBD}\vec{v}_j^{PWn}</math> – Translational velocity (absolute) of the <math>j^{\text{th}}</math> node of the port wing mesh (m/s)</li> <li>• <math>{}^{MBD}\vec{\omega}_j^{PWn}</math> – Rotational velocity (absolute) of the <math>j^{\text{th}}</math> node of the port wing mesh (rad/s)</li> <li>• <math>{}^{MBD}\vec{a}_j^{PWn}</math> – Translational acceleration (absolute) of the <math>j^{\text{th}}</math> node of the port wing mesh (m/s<sup>2</sup>)</li> <li>• <math>{}^{MBD}\vec{F}R_j^{PWn}</math> – Reaction force (expressed in the local coordinate system) at the <math>j^{\text{th}}</math> Gauss point of the port wing mesh (N)</li> </ul>	<p>Aerodynamic applied concentrated moments at the <math>j^{\text{th}}</math> node of pylons on the starboard wing mesh (N-m)</p> <ul style="list-style-type: none"> <li>• <math>{}^{MBD}\vec{F}_j^{PPy} [n_{Pylons}]</math> – Aerodynamic applied concentrated forces at the <math>j^{\text{th}}</math> node of the pylons on the port wing mesh (N)</li> <li>• <math>{}^{MBD}\vec{M}_j^{PPy} [n_{Pylons}]</math> – Aerodynamic applied concentrated moments at the <math>j^{\text{th}}</math> node of pylons on the port wing mesh (N-m)</li> <li>• <math>{}^{MBD}\vec{F}^{SPyRtr} [n_{Pylons}, n_2]</math> – Concentrated reaction forces at the top and bottom nacelles on the pylons on the starboard wing mesh at the rotor reference point (N)</li> <li>• <math>{}^{MBD}\vec{M}^{SPyRtr} [n_{Pylons}, n_2]</math> – Concentrated reaction moments at the top and bottom nacelles on the pylons on the starboard wing mesh at the rotor reference point (N-m)</li> <li>• <math>{}^{MBD}\vec{F}^{PPyRtr} [n_{Pylons}, n_2]</math> – Concentrated reaction forces at the top and bottom nacelles on the pylons on the port wing mesh at the rotor reference point (N)</li> <li>• <math>{}^{MBD}\vec{M}^{PPyRtr} [n_{Pylons}, n_2]</math> – Concentrated reaction moments at the top and bottom nacelles on the pylons on the port wing mesh at the rotor reference point (N-m)</li> </ul>	<p>– Rotational inertia about the shaft axis of the top and bottom rotors/drivetrains on the pylons on the port wing mesh (kg·m<sup>2</sup>)</p> <ul style="list-style-type: none"> <li>• <math>{}^{MBD}I_{Tran}^{PPyRtr} [n_{Pylons}, n_2]</math> – Transverse inertia about the rotor reference point of the top and bottom rotors/drivetrains on the pylons on the port wing mesh (kg·m<sup>2</sup>)</li> <li>• <math>{}^{MBD}x_{CM}^{PPyRtr} [n_{Pylons}, n_2]</math> – Distance along the shaft from the rotor reference point of the top and bottom rotors/drivetrains on the pylons on the port wing mesh to the center of mass of the rotor/drivetrain (positive along positive x) (m)</li> <li>• <math>{}^{MBD}\vec{p}_j^{FusR}</math> – Reference position of the <math>j^{\text{th}}</math> node of the fuselage mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{FusR}</math> – Reference orientation of the <math>j^{\text{th}}</math> node of the fuselage mesh (-)</li> <li>• <math>{}^{MBD}\vec{p}_j^{SWnR}</math> – Reference position of the <math>j^{\text{th}}</math> node of the starboard wing mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{SWnR}</math> – Reference orientation of the <math>j^{\text{th}}</math> node of the starboard wing mesh (-)</li> <li>• <math>{}^{MBD}\vec{p}_j^{PWnR}</math> – Reference position of the <math>j^{\text{th}}</math> node of the port wing mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{PWnR}</math> – Reference orientation of the <math>j^{\text{th}}</math> node of the port wing mesh (-)</li> <li>• <math>{}^{MBD}\vec{p}_j^{ISR}</math> – Reference position of the <math>j^{\text{th}}</math> node</li> </ul>
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<ul style="list-style-type: none"> <li>• <math>{}^{MBD}\vec{M}R_j^{PWn}</math> – Reaction moment (expressed in the local coordinate system) at the <math>j^{th}</math> Gauss point of the port wing mesh (N-m)</li> <li>• <math>{}^{MBD}\vec{p}^{VSO}</math> – Position (origin) of the vertical stabilizer (m)</li> <li>• <math>{}^{MBD}\vec{p}_j^{VS}</math> – Translational position (absolute) of the <math>j^{th}</math> node of the vertical stabilizer mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{VS}</math> – Displaced rotation (absolute orientation) of the <math>j^{th}</math> node of the vertical stabilizer mesh (-)</li> <li>• <math>{}^{MBD}\vec{v}_j^{VS}</math> – Translational velocity (absolute) of the <math>j^{th}</math> node of the vertical stabilizer mesh (m/s)</li> <li>• <math>{}^{MBD}\vec{\omega}_j^{VS}</math> – Rotational velocity (absolute) of the <math>j^{th}</math> node of the vertical stabilizer mesh (rad/s)</li> <li>• <math>{}^{MBD}\vec{a}_j^{VS}</math> – Translational acceleration (absolute) of the <math>j^{th}</math> node of the vertical stabilizer mesh (m/s<sup>2</sup>)</li> <li>• <math>{}^{MBD}\vec{F}R_j^{VS}</math> – Reaction force (expressed in the local coordinate system) at the <math>j^{th}</math> Gauss point of the vertical stabilizer mesh (N)</li> <li>• <math>{}^{MBD}\vec{M}R_j^{VS}</math> – Reaction moment (expressed in the local coordinate system) at the <math>j^{th}</math> Gauss point of the vertical stabilizer mesh (N-m)</li> <li>• <math>{}^{MBD}\vec{p}^{SHSO}</math> – Position (origin) of the starboard horizontal stabilizer (m)</li> <li>• <math>{}^{MBD}\vec{p}_j^{SHS}</math> – Translational position (absolute) of the</li> </ul>			<ul style="list-style-type: none"> <li>of the vertical stabilizer mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{VSR}</math> – Reference orientation of the <math>j^{th}</math> node of the vertical stabilizer mesh (-)</li> <li>• <math>{}^{MBD}\vec{p}_j^{SHSR}</math> – Reference position of the <math>j^{th}</math> node of the starboard horizontal stabilizer mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{SHSR}</math> – Reference orientation of the <math>j^{th}</math> node of the starboard horizontal stabilizer mesh (-)</li> <li>• <math>{}^{MBD}\vec{p}_j^{PHSR}</math> – Reference position of the <math>j^{th}</math> node of the port horizontal stabilizer mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{PHSR}</math> – Reference orientation of the <math>j^{th}</math> node of the port horizontal stabilizer mesh (-)</li> <li>• <math>{}^{MBD}\vec{p}_j^{SPyR}[n_{Pylons}]</math> – Reference position of the <math>j^{th}</math> node of the pylons on the starboard wing mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{SPyR}[n_{Pylons}]</math> – Reference orientation of the <math>j^{th}</math> node of the pylons on the starboard wing mesh (-)</li> <li>• <math>{}^{MBD}\vec{p}_j^{PPyR}[n_{Pylons}]</math> – Reference position of the <math>j^{th}</math> node of the pylons on the port wing mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{PPyR}[n_{Pylons}]</math> – Reference orientation of the <math>j^{th}</math> node of the pylons on the port wing mesh (-)</li> <li>• <math>{}^{MBD}\vec{p}^{SPyRtrR}[n_{Pylons}, n_2]</math> – Reference positions (origins) of the top and bottom nacelles on the</li> </ul>
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<p><math>j^{\text{th}}</math> node of the starboard horizontal stabilizer mesh (m)</p> <ul style="list-style-type: none"> <li>• <math>{}^{MBD}\mathcal{A}_j^{SHS}</math> – Displaced rotation (absolute orientation) of the <math>j^{\text{th}}</math> node of the starboard horizontal stabilizer mesh (-)</li> <li>• <math>{}^{MBD}\vec{v}_j^{SHS}</math> – Translational velocity (absolute) of the <math>j^{\text{th}}</math> node of the starboard horizontal stabilizer mesh (m/s)</li> <li>• <math>{}^{MBD}\vec{\omega}_j^{SHS}</math> – Rotational velocity (absolute) of the <math>j^{\text{th}}</math> node of the starboard horizontal stabilizer mesh (rad/s)</li> <li>• <math>{}^{MBD}\vec{a}_j^{SHS}</math> – Translational acceleration (absolute) of the <math>j^{\text{th}}</math> node of the starboard horizontal stabilizer mesh (m/s<sup>2</sup>)</li> <li>• <math>{}^{MBD}\vec{F}R_j^{SHS}</math> – Reaction force (expressed in the local coordinate system) at the <math>j^{\text{th}}</math> Gauss point of the starboard horizontal stabilizer mesh (N)</li> <li>• <math>{}^{MBD}\vec{M}R_j^{SHS}</math> – Reaction moment (expressed in the local coordinate system) at the <math>j^{\text{th}}</math> Gauss point of the starboard horizontal stabilizer mesh (N-m)</li> <li>• <math>{}^{MBD}\vec{p}^{PHSO}</math> – Position (origin) of the port horizontal stabilizer (m)</li> <li>• <math>{}^{MBD}\vec{p}_j^{PHS}</math> – Translational position (absolute) of the <math>j^{\text{th}}</math> node of the port horizontal stabilizer mesh (m)</li> <li>• <math>{}^{MBD}\mathcal{A}_j^{PHS}</math> – Displaced rotation (absolute</li> </ul>			<p>pylons on the starboard wing mesh at the rotor reference point (m)</p> <ul style="list-style-type: none"> <li>• <math>{}^{MBD}\mathcal{A}^{SPyRtrR}[n_{Pylons}, n_2]</math> – Reference orientations of the top and bottom nacelles on the pylons on the starboard wing mesh at the rotor reference point (-)</li> <li>• <math>{}^{MBD}\vec{p}^{PPyRtrR}[n_{Pylons}, n_2]</math> – Reference positions (origins) of the top and bottom nacelles on the pylons on the port wing mesh at the rotor reference point (m)</li> <li>• <math>{}^{MBD}\mathcal{A}^{PPyRtrR}[n_{Pylons}, n_2]</math> – Reference orientations of the top and bottom nacelles on the pylons on the port wing mesh at the rotor reference point (-)</li> </ul>
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<p>orientation) of the <math>j^{\text{th}}</math> node of the port horizontal stabilizer mesh (-)</p> <ul style="list-style-type: none"> <li>• <math>{}^{MBD}\vec{v}_j^{PHS}</math> – Translational velocity (absolute) of the <math>j^{\text{th}}</math> node of the port horizontal stabilizer mesh (m/s)</li> <li>• <math>{}^{MBD}\vec{\omega}_j^{PHS}</math> – Rotational velocity (absolute) of the <math>j^{\text{th}}</math> node of the port horizontal stabilizer mesh (rad/s)</li> <li>• <math>{}^{MBD}\vec{a}_j^{PHS}</math> – Translational acceleration (absolute) of the <math>j^{\text{th}}</math> node of the port horizontal stabilizer mesh (m/s<sup>2</sup>)</li> <li>• <math>{}^{MBD}\vec{F}R_j^{PHS}</math> – Reaction force (expressed in the local coordinate system) at the <math>j^{\text{th}}</math> Gauss point of the port horizontal stabilizer mesh (N)</li> <li>• <math>{}^{MBD}\vec{M}R_j^{PHS}</math> – Reaction moment (expressed in the local coordinate system) at the <math>j^{\text{th}}</math> Gauss point of the port horizontal stabilizer mesh (N-m)</li> <li>• <math>{}^{MBD}\vec{p}^{SPyO} [n_{pylons}]</math> – Positions (origins) of pylons on the starboard wing (m)</li> <li>• <math>{}^{MBD}\vec{p}_j^{SPy} [n_{pylons}]</math> – Translational position (absolute) of the <math>j^{\text{th}}</math> node of the pylons on the starboard wing mesh (m)</li> <li>• <math>{}^{MBD}\Lambda_j^{SPy} [n_{pylons}]</math> – Displaced rotation (absolute orientation) of the <math>j^{\text{th}}</math> node of the pylons on the starboard wing mesh (-)</li> <li>• <math>{}^{MBD}\vec{v}_j^{SPy} [n_{pylons}]</math> – Translational velocity</li> </ul>			
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<p>(absolute) of the <math>j^{\text{th}}</math> node of the pylons on the starboard wing mesh (m/s)</p> <ul style="list-style-type: none"> <li>• <math>{}^{MBD}\vec{\omega}_j^{SPy} [n_{Pylons}] -</math> Rotational velocity (absolute) of the <math>j^{\text{th}}</math> node of the pylons on the starboard wing mesh (rad/s)</li> <li>• <math>{}^{MBD}\vec{a}_j^{SPy} [n_{Pylons}] -</math> Translational acceleration (absolute) of the <math>j^{\text{th}}</math> node of the pylons on the starboard wing mesh (m/s<sup>2</sup>)</li> <li>• <math>{}^{MBD}\vec{F}R_j^{SPy} [n_{Pylons}] -</math> Reaction force (expressed in the local coordinate system) at the <math>j^{\text{th}}</math> Gauss point of the pylons on the starboard wing mesh (N)</li> <li>• <math>{}^{MBD}\vec{M}R_j^{SPy} [n_{Pylons}] -</math> Reaction moment (expressed in the local coordinate system) at the <math>j^{\text{th}}</math> Gauss point of the pylons on the starboard wing mesh (N-m)</li> <li>• <math>{}^{MBD}\vec{p}^{PPyO} [n_{Pylons}] -</math> Positions (origins) of pylons on the port wing (m)</li> <li>• <math>{}^{MBD}\vec{p}_j^{PPy} [n_{Pylons}] -</math> Translational position (absolute) of the <math>j^{\text{th}}</math> node of the pylons on the port wing mesh (m)</li> <li>• <math>{}^{MBD}\mathcal{A}_j^{PPy} [n_{Pylons}] -</math> Displaced rotation (absolute orientation) of the <math>j^{\text{th}}</math> node of the pylons on the port wing mesh (-)</li> <li>• <math>{}^{MBD}\vec{V}_j^{PPy} [n_{Pylons}] -</math> Translational velocity (absolute) of the <math>j^{\text{th}}</math> node of the pylons on the port wing mesh (m/s)</li> </ul>			
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<ul style="list-style-type: none"> <li>• <math>{}^{MBD}\vec{\omega}_j^{PPy} [n_{pylons}] -</math> Rotational velocity (absolute) of the <math>j^{th}</math> node of the pylons on the port wing mesh (rad/s)</li> <li>• <math>{}^{MBD}\vec{a}_j^{PPy} [n_{pylons}] -</math> Translational acceleration (absolute) of the <math>j^{th}</math> node of the pylons on the port wing mesh (m/s<sup>2</sup>)</li> <li>• <math>{}^{MBD}\vec{F}R_j^{PPy} [n_{pylons}] -</math> Reaction force (expressed in the local coordinate system) at the <math>j^{th}</math> Gauss point of the pylons on the port wing mesh (N)</li> <li>• <math>{}^{MBD}\vec{M}R_j^{PPy} [n_{pylons}] -</math> Reaction moment (expressed in the local coordinate system) at the <math>j^{th}</math> Gauss point of the pylons on the port wing mesh (N-m)</li> <li>• <math>{}^{MBD}\vec{p}^{SPyRtr} [n_{pylons}, n_2] -</math> Translational position (absolute) of the top and bottom nacelles on the pylons on the starboard wing mesh at the rotor reference point (m)</li> <li>• <math>{}^{MBD}\vec{A}^{SPyRtr} [n_{pylons}, n_2] -</math> Displaced rotation (absolute orientation) of the top and bottom nacelles on the pylons on the starboard wing mesh at the rotor reference point (-)</li> <li>• <math>{}^{MBD}\vec{v}^{SPyRtr} [n_{pylons}, n_2] -</math> Translational velocity (absolute) of the top and bottom nacelles on the pylons on the starboard wing mesh at the rotor reference point (m/s)</li> <li>• <math>{}^{MBD}\vec{\omega}^{SPyRtr} [n_{pylons}, n_2] -</math> Rotational velocity (absolute) of the top and</li> </ul>			
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<p>bottom nacelles on the pylons on the starboard wing mesh at the rotor reference point (rad/s)</p> <ul style="list-style-type: none"> <li>• <math>{}^{MBD}\vec{a}^{SPyRtr}\left[n_{Pylons},n_2\right] -</math> Translational acceleration (absolute) of the top and bottom nacelles on the pylons on the starboard wing mesh at the rotor reference point (m/s<sup>2</sup>)</li> <li>• <math>{}^{MBD}\vec{\alpha}^{SPyRtr}\left[n_{Pylons},n_2\right] -</math> Rotational acceleration (absolute) of the top and bottom nacelles on the pylons on the starboard wing mesh at the rotor reference point (rad/s<sup>2</sup>)</li> <li>• <math>{}^{MBD}\vec{p}^{PPyRtr}\left[n_{Pylons},n_2\right] -</math> Translational position (absolute) of the top and bottom nacelles on the pylons on the port wing mesh at the rotor reference point (m)</li> <li>• <math>{}^{MBD}\Lambda^{PPyRtr}\left[n_{Pylons},n_2\right] -</math> Displaced rotation (absolute orientation) of the top and bottom nacelles on the pylons on the port wing mesh at the rotor reference point (-)</li> <li>• <math>{}^{MBD}\vec{v}^{PPyRtr}\left[n_{Pylons},n_2\right] -</math> Translational velocity (absolute) of the top and bottom nacelles on the pylons on the port wing mesh at the rotor reference point (m/s)</li> <li>• <math>{}^{MBD}\vec{\omega}^{PPyRtr}\left[n_{Pylons},n_2\right] -</math> Rotational velocity (absolute) of the top and bottom nacelles on the pylons on the port wing mesh at the rotor reference point (rad/s)</li> <li>• <math>{}^{MBD}\vec{a}^{PPyRtr}\left[n_{Pylons},n_2\right] -</math> Translational acceleration</li> </ul>			
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(absolute) of the top and bottom nacelles on the pylons on the port wing mesh at the rotor reference point (m/s <sup>2</sup> )			
<ul style="list-style-type: none"> <li><math>{}^{MBD}\tilde{\alpha}^{PPyRtr} \left[ n_{Pylons}, n_2 \right] -</math></li> </ul> Rotational acceleration (absolute) of the top and bottom nacelles on the pylons on the port wing mesh at the rotor reference point (rad/s <sup>2</sup> )			

MiscVars:  ${}^{Ctrl}y, {}^{MD}y, {}^{IfW}y, {}^{KAD}y, {}^{MBD}u, {}^{KAD}u, {}^{MD}u, {}^{MD}x^{Copy}$

**Commented [JJ5]:** The outputs of each module at time t (as calculated by their respective CalcOutput() routines) are stored as MiscVars in KiteFASTMBD.

**Commented [JJ6]:** The inputs from MBDyn and inputs to MoorDyn at time t and the extrapolated inputs to KiteAeroDyn at t+KAD\*dt are stored as MiscVars in KiteFASTMBD.

**Commented [JJ7]:** The temporary states of MoorDyn are stored as MiscVars. In KiteFASTMBD.

#### Mapping of Outputs to Inputs in KiteFASTMBD

Output depends on Input (Y/N)		Inputs				
		MBDyn	KiteAeroDyn	InflowWind	MoorDyn	Controller
Outputs	MBDyn		N	N	N	Y
	KiteAeroDyn	Y				Y
	InflowWind		Y			Y
	MoorDyn	Y				Y
	Controller	N	N			

**Commented [JJ8]:** This may technically not be true, but we can only call the Controller once anyway, so, we'll assume no.

#### Data Flow (stopping when reaching "N")

MBDyn      Controller  
 KiteAeroDyn MBDyn      Controller  
                  Controller  
 InflowWind KiteAeroDyn MBDyn      Controller  
                                  Controller  
                  Controller  
 MoorDyn      MBDyn      Controller  
                  Controller  
 Controller

Thus, no nonlinear solves are required

Order of calls: MBDyn, Controller, MoorDyn, InflowWind, KiteAeroDyn

#### Constructor

This routine initializes KiteFASTMBD at  $t = 0$ :

- Sets parameters
- Initializes states
- Calls module Init routines
- Opens the write output file
- Opens and writes the summary file

**Commented [JJ9]:** t=0 outputs are not set here, except for the Controller

Query the MBDyn model to access the inputs at  $t = 0$ .

Query the MBDyn model to access the names of the KiteAeroDyn, InflowWind, and MoorDyn primary input files

**Commented [JJ10]:** The names of the KiteAeroDyn input file etc., along with switches for enabling/disabling each module, must be queried from the MBDyn model. I haven't specifically included logic below to enable/disable modules, but this should be implemented.

Set the parameters from inputs ( $\Delta t$ ,  $InterpOrder$ ,  $N_{Flaps}$ ,  $N_{Pylons}$ ,  $^{MBD}\tilde{g}$ ,  $^{MBD}\mathbf{m}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}I_{Rot}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}I_{Tran}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\mathbf{x}_{CM}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\mathbf{m}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}I_{Rot}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}I_{Tran}^{PPyRtr}[n_{Pylons}, n_2]$ , and  $^{MBD}\mathbf{x}_{CM}^{PPyRtr}[n_{Pylons}, n_2]$ ). Trigger a fatal error if

$$\begin{aligned} &^{MBD}\mathbf{m}^{SPyRtr}[n_{Pylons}, n_2] < 0, \quad ^{MBD}I_{Rot}^{SPyRtr}[n_{Pylons}, n_2] < 0, \\ &^{MBD}I_{Tran}^{SPyRtr}[n_{Pylons}, n_2] - ^{MBD}\mathbf{m}^{SPyRtr}[n_{Pylons}, n_2] \left( ^{MBD}\mathbf{x}_{CM}^{SPyRtr}[n_{Pylons}, n_2] \right)^2 < 0, \\ &^{MBD}\mathbf{m}^{PPyRtr}[n_{Pylons}, n_2] < 0, \quad ^{MBD}I_{Rot}^{PPyRtr}[n_{Pylons}, n_2] < 0, \quad \text{or} \\ &^{MBD}I_{Tran}^{PPyRtr}[n_{Pylons}, n_2] - ^{MBD}\mathbf{m}^{PPyRtr}[n_{Pylons}, n_2] \left( ^{MBD}\mathbf{x}_{CM}^{PPyRtr}[n_{Pylons}, n_2] \right)^2 < 0. \text{ Note that:} \end{aligned}$$

- The flap indices:  $n_{Flaps} = \{1, 2, \dots, N_{Flaps}\}$
- The pylon indices:  $n_{Pylons} = \{1, 2, \dots, N_{Pylons}\}$
- And:  $n_2 = \{1, 2\}$

Set the DCM conversion parameter from the FAST ground system (X pointed nominally downwind; Z pointed vertically opposite gravity; Y transverse) to the ground system used by the controller (X pointed nominally upwind; Z pointed vertically downward, Y transverse):

$$\mathbf{A}^{FAST2Ctrl} = \begin{bmatrix} -I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & -I \end{bmatrix}$$

Set the reference positions (origins) needed as initialization inputs to KiteAeroDyn:

$$\begin{aligned} ^{KAD}\vec{p}^{SWnOR} &= ^{MBD}\mathbf{A}^{FusO} \left\{ ^{MBD}\vec{p}^{SWnO} - ^{MBD}\vec{p}^{FusO} \right\} \\ ^{KAD}\vec{p}^{PWnOR} &= ^{MBD}\mathbf{A}^{FusO} \left\{ ^{MBD}\vec{p}^{PWnO} - ^{MBD}\vec{p}^{FusO} \right\} \\ ^{KAD}\vec{p}^{VSOR} &= ^{MBD}\mathbf{A}^{FusO} \left\{ ^{MBD}\vec{p}^{VSO} - ^{MBD}\vec{p}^{FusO} \right\} \\ ^{KAD}\vec{p}^{SHSOR} &= ^{MBD}\mathbf{A}^{FusO} \left\{ ^{MBD}\vec{p}^{SHSO} - ^{MBD}\vec{p}^{FusO} \right\} \\ ^{KAD}\vec{p}^{PHSOR} &= ^{MBD}\mathbf{A}^{FusO} \left\{ ^{MBD}\vec{p}^{PHSO} - ^{MBD}\vec{p}^{FusO} \right\} \\ ^{KAD}\vec{p}^{SPyOR}[n_{Pylons}] &= ^{MBD}\mathbf{A}^{FusO} \left\{ ^{MBD}\vec{p}^{SPyO}[n_{Pylons}] - ^{MBD}\vec{p}^{FusO} \right\} \\ ^{KAD}\vec{p}^{PPyOR}[n_{Pylons}] &= ^{MBD}\mathbf{A}^{FusO} \left\{ ^{MBD}\vec{p}^{PPyO}[n_{Pylons}] - ^{MBD}\vec{p}^{FusO} \right\} \\ ^{KAD}\vec{p}^{SPyRtrR}[n_{Pylons}, n_2] &= ^{MBD}\mathbf{A}^{FusO} \left\{ ^{MBD}\vec{p}^{SPyRtr}[n_{Pylons}, n_2] - ^{MBD}\vec{p}^{FusO} \right\} \\ ^{KAD}\vec{p}^{PPyRtrR}[n_{Pylons}, n_2] &= ^{MBD}\mathbf{A}^{FusO} \left\{ ^{MBD}\vec{p}^{PPyRtr}[n_{Pylons}, n_2] - ^{MBD}\vec{p}^{FusO} \right\} \end{aligned}$$

Call KiteAeroDyn\_Init()

Calculate the number of KiteAeroDyn time steps per MBDyn time step:  $N_{KAD/MBD} = NINT\left(\frac{^{KAD}\Delta t}{\Delta t}\right)$

**Commented [JJ11]:** These must be queried from the MBDyn model.

Trigger a fatal error if the KiteAeroDyn time step is not an integer multiple of the MBDyn time step i.e. if

$$N_{KAD/MBD} \Delta t - {}^{KAD} \Delta t \neq 0$$

$${}^{KAD} NewTime = TRUE$$

Set the air density for future reference:  $\rho = {}^{KAD} \rho$

Determine the number of points where wind will be accessed within InflowWind by summing up the nodes on the AeroDyn input meshes, plus one for the fuselage origin and one for the base station:

$$\begin{aligned} {}^{IW} NumWindPoints = & 2 \\ & + {}^{KAD} NumFusNds \\ & + {}^{KAD} NumSWnNds \\ & + {}^{KAD} NumPWnNds \\ & + {}^{KAD} NumVSNds \\ & + {}^{KAD} NumSHSNds \\ & + {}^{KAD} NumPHSNds \\ & + {}^{KAD} NumPylNds (2 N_{Pylons}) \\ & + 4 N_{Pylons} \end{aligned}$$

Call InflowWind\_Init()

Set the initialization inputs to MoorDyn:

$$\begin{aligned} {}^{MD} \mathbf{g} &= \left\| {}^{MBD} \vec{\mathbf{g}} \right\|_2 \\ {}^{MD} rhoW &= \rho \\ {}^{MD} WtrDepth &= 0 \\ {}^{MD} PtfmInit &= \left\{ \begin{array}{l} {}^{MBD} \vec{\mathbf{p}}^{FusO} \\ {}^{MBD} \mathbf{A}^{FusO} \end{array} \right\} \end{aligned}$$

Call MoorDyn\_Init()

Trigger a fatal error if  $({}^{MD} \Delta t \neq \Delta t)$

Trigger a fatal error if there is more than one anchor set in MoorDyn. Also, set the anchor position for future reference based on the initialization output from MoorDyn:

$$\vec{\mathbf{p}}^{Anch} = \text{from MoorDyn initialization output}$$

Call Controller\_Init()

Calculate the number of controller time steps per MBDyn time step:  $N_{Ctrl/MBD} = NINT \left( \frac{{}^{Ctrl} \Delta t}{\Delta t} \right)$

Trigger a fatal error if the controller time step is not an integer multiple of the MBDyn time step i.e. if

$$N_{Ctrl/MBD} \Delta t - {}^{Ctrl} \Delta t \neq 0$$

$${}^{Ctrl} NewTime = FALSE$$

**Commented [JJ12]:** Note: the Controller\_Init() call initializes the controller states and returns the initial controller outputs.

**Commented [JJ13]:** Note: the controller will trigger a fatal error if  $N_{Flaps} \neq 3$  (to match the current controller interface),

$N_{Pylons} \neq 2$  (to match the current controller interface)

**Commented [JJ14]:** If the controller takes larger steps than MBDyn, then we'll need to smooth the controller output to ensure that it is continuous (at least for the rotor velocity and acceleration). That is, the controller would have to be implemented like KiteAeroDyn.

Set the reference positions and orientations of the line2 and point meshes from the inputs:

$$\begin{aligned}
 \vec{p}_j^{FusR} &= MBD \Lambda^{FusO} \left\{ MBD \vec{p}_j^{Fus} - MBD \vec{p}^{FusO} \right\} & (\text{for } j = \{1, 2, \dots, MBD \text{ NumFusNds}\}) \\
 \Lambda_j^{FusR} &= MBD \Lambda_j^{Fus} \left[ MBD \Lambda^{FusO} \right]^T & (\text{for } j = \{1, 2, \dots, MBD \text{ NumFusNds}\}) \\
 \vec{p}_j^{SWnR} &= MBD \Lambda^{FusO} \left\{ MBD \vec{p}_j^{SWn} - MBD \vec{p}^{FusO} \right\} & (\text{for } j = \{1, 2, \dots, MBD \text{ NumSWnNds}\}) \\
 \Lambda_j^{SWnR} &= MBD \Lambda_j^{SWn} \left[ MBD \Lambda^{FusO} \right]^T & (\text{for } j = \{1, 2, \dots, MBD \text{ NumSWnNds}\}) \\
 \vec{p}_j^{PWnR} &= MBD \Lambda^{FusO} \left\{ MBD \vec{p}_j^{PWn} - MBD \vec{p}^{FusO} \right\} & (\text{for } j = \{1, 2, \dots, MBD \text{ NumPWnNds}\}) \\
 \Lambda_j^{PWnR} &= MBD \Lambda_j^{PWn} \left[ MBD \Lambda^{FusO} \right]^T & (\text{for } j = \{1, 2, \dots, MBD \text{ NumPWnNds}\}) \\
 \vec{p}_j^{VSR} &= MBD \Lambda^{FusO} \left\{ MBD \vec{p}_j^{VS} - MBD \vec{p}^{FusO} \right\} & (\text{for } j = \{1, 2, \dots, MBD \text{ NumVSNds}\}) \\
 \Lambda_j^{VSR} &= MBD \Lambda_j^{VS} \left[ MBD \Lambda^{FusO} \right]^T & (\text{for } j = \{1, 2, \dots, MBD \text{ NumVSNds}\}) \\
 \vec{p}_j^{SHSR} &= MBD \Lambda^{FusO} \left\{ MBD \vec{p}_j^{SHS} - MBD \vec{p}^{FusO} \right\} & (\text{for } j = \{1, 2, \dots, MBD \text{ NumSHSNds}\}) \\
 \Lambda_j^{SHSR} &= MBD \Lambda_j^{SHS} \left[ MBD \Lambda^{FusO} \right]^T & (\text{for } j = \{1, 2, \dots, MBD \text{ NumSHSNds}\}) \\
 \vec{p}_j^{PHSR} &= MBD \Lambda^{FusO} \left\{ MBD \vec{p}_j^{PHS} - MBD \vec{p}^{FusO} \right\} & (\text{for } j = \{1, 2, \dots, MBD \text{ NumPHSNds}\}) \\
 \Lambda_j^{PHSR} &= MBD \Lambda_j^{PHS} \left[ MBD \Lambda^{FusO} \right]^T & (\text{for } j = \{1, 2, \dots, MBD \text{ NumPHSNds}\}) \\
 \vec{p}_j^{SPyR} \left[ n_{Pylons} \right] &= MBD \Lambda^{FusO} \left\{ MBD \vec{p}_j^{SPy} \left[ n_{Pylons} \right] - MBD \vec{p}^{FusO} \right\} & (\text{for } j = \{1, 2, \dots, MBD \text{ NumPylNds}\}) \\
 \Lambda_j^{SPyR} \left[ n_{Pylons} \right] &= MBD \Lambda_j^{SPy} \left[ n_{Pylons} \right] \left[ MBD \Lambda^{FusO} \right]^T & (\text{for } j = \{1, 2, \dots, MBD \text{ NumPylNds}\}) \\
 \vec{p}_j^{PPyR} \left[ n_{Pylons} \right] &= MBD \Lambda^{FusO} \left\{ MBD \vec{p}_j^{PPy} \left[ n_{Pylons} \right] - MBD \vec{p}^{FusO} \right\} & (\text{for } j = \{1, 2, \dots, MBD \text{ NumPylNds}\}) \\
 \Lambda_j^{PPyR} \left[ n_{Pylons} \right] &= MBD \Lambda_j^{PPy} \left[ n_{Pylons} \right] \left[ MBD \Lambda^{FusO} \right]^T & (\text{for } j = \{1, 2, \dots, MBD \text{ NumPylNds}\}) \\
 \vec{p}^{SPyRtrR} \left[ n_{Pylons}, n_2 \right] &= MBD \Lambda^{FusO} \left\{ MBD \vec{p}^{SPyRtr} \left[ n_{Pylons}, n_2 \right] - MBD \vec{p}^{FusO} \right\} \\
 \Lambda^{SPyRtrR} \left[ n_{Pylons}, n_2 \right] &= I \\
 \vec{p}^{PPyRtrR} \left[ n_{Pylons}, n_2 \right] &= MBD \Lambda^{FusO} \left\{ MBD \vec{p}^{PPyRtr} \left[ n_{Pylons}, n_2 \right] - MBD \vec{p}^{FusO} \right\} \\
 \Lambda^{PPyRtrR} \left[ n_{Pylons}, n_2 \right] &= I
 \end{aligned}$$

**Commented [JJ15]:** Note: the motion meshes are line2 meshes (except for the rotors, which are point meshes), but the load meshes are point meshes.

Set mesh-mappings between KiteFASTMBD-KiteAeroDyn and KiteFASTMBD-MoorDyn.

Open the write Output File

Open and write a summary file (if SumPrint = TRUE)

KiteFASTMBD Summary File

Predictions were generated on DATE at TIME using KiteFASTMBD (VERSION, DATE)

compiled with

NWTC Subroutine Library (VERSION, DATE)

KiteAeroDyn (VERSION, DATE)

InflowWind (VERSION, DATE) for OpenFAST (VERSION DATE)

MoorDyn (VERSION, DATE)

**Commented [JJ16]:** The mesh-mapping routines can only handle one source and one destination mesh. To do this mapping, the MBDyn meshes for the starboard and port wings (SWn and PWn) have to be copied into a single mesh using a one-to-one transfer of reference positions, reference orientations, and fields (which I label as Wn in the mesh-mappings below).

**Commented [JJ17]:** SumPrint must be queried from the MBDyn model

**Commented [JJ18]:** I'm only hand waving here because the implementation should be obvious (similar to other OpenFAST summary files)

**Commented [JJ19]:** (VERSION,DATE) has been replaced with the a git hash

Controller Wrapper (VERSION, DATE)  
 Controller (VERSION, DATE)  
 MBDyn (VERSION, DATE)

Description from the MDyn input file: TITLE

**Commented [JJ20]:** Probably not needed if TITLE is not easily accessible within the MBDyn user element.

Time Step:

Component	Time Step
(-)	(s)
MBDyn	$\Delta t$
KiteAeroDyn	$^{KAD} \Delta t$
MoorDyn	$\Delta t$
Controller	$^{Ctrl} \Delta t$

Reference Points, MBDyn Finite-Element Nodes, and MBDyn Gauss Points

Component	Type	Number	Output Number
x y z	(-)	(-)	(-)
(m) (m) (m)			
Fuselage	Reference point	-	-
0 0 0			
Fuselage	Finite-element node	$j$	$\begin{cases} Fus\langle\beta\rangle & for(FusOutNd[\beta]=j) \\ - & otherwise \end{cases}$
$^{MBD} \vec{p}_j^{FusR}$			
Fuselage	Gauss point	$j$	$\begin{cases} Fus\langle\beta\rangle & for(FusOutNd[\beta]=j) \\ - & otherwise \end{cases}$
$\begin{cases} \left(1 - \frac{\sqrt{3}}{3}\right) ^{MBD} \vec{p}_{j+1}^{FusR} + \left(\frac{\sqrt{3}}{3}\right) ^{MBD} \vec{p}_j^{FusR} & for( Mod(j,2)=1) \\ \left(\frac{\sqrt{3}}{3}\right) ^{MBD} \vec{p}_{j+1}^{FusR} + \left(1 - \frac{\sqrt{3}}{3}\right) ^{MBD} \vec{p}_j^{FusR} & otherwise \end{cases}$			
Starboard wing	Reference point	-	-
$^{KAD} \vec{p}^{SWnOR}$			
Starboard wing	Finite-element node	$j$	$\begin{cases} SWn\langle\beta\rangle & for(SWnOutNd[\beta]=j) \\ - & otherwise \end{cases}$
$^{MBD} \vec{p}_j^{SWnR}$			
Starboard wing	Gauss point	$j$	$\begin{cases} SWn\langle\beta\rangle & for(SWnOutNd[\beta]=j) \\ - & otherwise \end{cases}$
$\begin{cases} \left(1 - \frac{\sqrt{3}}{3}\right) ^{MBD} \vec{p}_{j+1}^{SWnR} + \left(\frac{\sqrt{3}}{3}\right) ^{MBD} \vec{p}_j^{SWnR} & for( Mod(j,2)=1) \\ \left(\frac{\sqrt{3}}{3}\right) ^{MBD} \vec{p}_{j+1}^{SWnR} + \left(1 - \frac{\sqrt{3}}{3}\right) ^{MBD} \vec{p}_j^{SWnR} & otherwise \end{cases}$			

Port wing $KAD \vec{p}^{PWnOR}$	Reference point	-	-
Port wing $MBD \vec{p}_j^{PWnR}$	Finite-element node	$j$	$\begin{cases} PWn\langle\beta\rangle & \text{for } (PWnOutNd[\beta] = j) \\ - & \text{otherwise} \end{cases}$
Port wing	Gauss point	$j$	$\begin{cases} PWn\langle\beta\rangle & \text{for } (PWnOutNd[\beta] = j) \\ - & \text{otherwise} \end{cases}$
$\begin{cases} \left(1 - \frac{\sqrt{3}}{3}\right) MBD \vec{p}_{j+1}^{PWnR} + \left(\frac{\sqrt{3}}{3}\right) MBD \vec{p}_j^{PWnR} & \text{for } (Mod(j, 2) = 1) \\ \left(\frac{\sqrt{3}}{3}\right) MBD \vec{p}_{j+1}^{PWnR} + \left(1 - \frac{\sqrt{3}}{3}\right) MBD \vec{p}_j^{PWnR} & \text{otherwise} \end{cases}$			
Vertical stabilizer $KAD \vec{p}^{VSR}$	Reference point	-	-
Vertical stabilizer $MBD \vec{p}_j^{VSR}$	Finite-element node	$j$	$\begin{cases} VS\langle\beta\rangle & \text{for } (VSOOutNd[\beta] = j) \\ - & \text{otherwise} \end{cases}$
Vertical stabilizer	Gauss point	$j$	$\begin{cases} VS\langle\beta\rangle & \text{for } (VSOOutNd[\beta] = j) \\ - & \text{otherwise} \end{cases}$
$\begin{cases} \left(1 - \frac{\sqrt{3}}{3}\right) MBD \vec{p}_{j+1}^{VSR} + \left(\frac{\sqrt{3}}{3}\right) MBD \vec{p}_j^{VSR} & \text{for } (Mod(j, 2) = 1) \\ \left(\frac{\sqrt{3}}{3}\right) MBD \vec{p}_{j+1}^{VSR} + \left(1 - \frac{\sqrt{3}}{3}\right) MBD \vec{p}_j^{VSR} & \text{otherwise} \end{cases}$			
Starboard horizontal stabilizer $KAD \vec{p}^{SHSR}$	Reference point	-	-
Starboard horizontal stabilizer $MBD \vec{p}_j^{SHSR}$	Finite-element node	$j$	$\begin{cases} SHS\langle\beta\rangle & \text{for } (SHSOutNd[\beta] = j) \\ - & \text{otherwise} \end{cases}$
Starboard horizontal stabilizer	Gauss point	$j$	$\begin{cases} SHS\langle\beta\rangle & \text{for } (SHSOutNd[\beta] = j) \\ - & \text{otherwise} \end{cases}$
$\begin{cases} \left(1 - \frac{\sqrt{3}}{3}\right) MBD \vec{p}_{j+1}^{SHSR} + \left(\frac{\sqrt{3}}{3}\right) MBD \vec{p}_j^{SHSR} & \text{for } (Mod(j, 2) = 1) \\ \left(\frac{\sqrt{3}}{3}\right) MBD \vec{p}_{j+1}^{SHSR} + \left(1 - \frac{\sqrt{3}}{3}\right) MBD \vec{p}_j^{SHSR} & \text{otherwise} \end{cases}$			
Port horizontal stabilizer $KAD \vec{p}^{PHSR}$	Reference point	-	-



Port horizontal stabilizer	Finite-element node	$j$	$\begin{cases} PHS\langle\beta\rangle & \text{for } (PHSOutNd[\beta]=j) \\ - & \text{otherwise} \end{cases}$
$^{MBD}\tilde{\vec{p}}_j^{PHSR}$			

Port horizontal stabilizer	Gauss point	$j$	$\begin{cases} PHS\langle\beta\rangle & \text{for } (PHSOutNd[\beta]=j) \\ - & \text{otherwise} \end{cases}$
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$$\begin{cases} \left(1 - \frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_{j+1}^{PHSR} + \left(\frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_j^{PHSR} & \text{for } (Mod(j,2)=1) \\ \left(\frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_{j+1}^{PHSR} + \left(1 - \frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_j^{PHSR} & \text{otherwise} \end{cases}$$

Starboard pylon $n_{Pylons}$	Reference point	-	-
$^{KAD}\tilde{\vec{p}}^{SPyOR}[n_{Pylons}]$			

Starboard pylon $n_{Pylons}$	Finite-element node	$j$	$\begin{cases} SP\langle n_{Pylons}\rangle\langle\beta\rangle & \text{for } (PylOutNd[\beta]=j) \\ - & \text{otherwise} \end{cases}$
$^{MBD}\tilde{\vec{p}}_j^{SPyR}[n_{Pylons}]$			

Starboard pylon $n_{Pylons}$	Gauss point	$j$	$\begin{cases} SP\langle n_{Pylons}\rangle\langle\beta\rangle & \text{for } (PylOutNd[\beta]=j) \\ - & \text{otherwise} \end{cases}$
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$$\begin{cases} \left(1 - \frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_{j+1}^{SPyR}[n_{Pylons}] + \left(\frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_j^{SPyR}[n_{Pylons}] & \text{for } (Mod(j,2)=1) \\ \left(\frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_{j+1}^{SPyR}[n_{Pylons}] + \left(1 - \frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_j^{SPyR}[n_{Pylons}] & \text{otherwise} \end{cases}$$

Port pylon $n_{Pylons}$	Reference point	-	-
$^{KAD}\tilde{\vec{p}}^{PPyOR}[n_{Pylons}]$			

Port pylon $n_{Pylons}$	Finite-element node	$j$	$\begin{cases} PP\langle n_{Pylons}\rangle\langle\beta\rangle & \text{for } (PylOutNd[\beta]=j) \\ - & \text{otherwise} \end{cases}$
$^{MBD}\tilde{\vec{p}}_j^{PPyR}[n_{Pylons}]$			

Port pylon $n_{Pylons}$	Gauss point	$j$	$\begin{cases} PP\langle n_{Pylons}\rangle\langle\beta\rangle & \text{for } (PylOutNd[\beta]=j) \\ - & \text{otherwise} \end{cases}$
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$$\begin{cases} \left(1 - \frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_{j+1}^{PPyR}[n_{Pylons}] + \left(\frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_j^{PPyR}[n_{Pylons}] & \text{for } (Mod(j,2)=1) \\ \left(\frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_{j+1}^{PPyR}[n_{Pylons}] + \left(1 - \frac{\sqrt{3}}{3}\right)^{MBD}\tilde{\vec{p}}_j^{PPyR}[n_{Pylons}] & \text{otherwise} \end{cases}$$

Top rotor on starboard pylon $n_{Pylons}$	Reference point	-	-
$^{KAD}\tilde{\vec{p}}^{SPyRtrR}[n_{Pylons}, l]$			

Bottom rotor on starboard pylon  $n_{Pylons}$  Reference point - -

$$^{KAD} \vec{p}^{SPyRtrR} [n_{Pylons}, 2]$$

Top rotor on port pylon  $n_{Pylons}$  Reference point - -

$$^{KAD} \vec{p}^{PPyRtrR} [n_{Pylons}, 1]$$

Bottom rotor on port pylon  $n_{Pylons}$  Reference point - -

$$^{KAD} \vec{p}^{PPyRtrR} [n_{Pylons}, 2]$$

Requested Channels in KiteFASTMBD Output Files: NUMBER

Number	Name	Units	Generated by
0	Time	(s)	KiteFASTMBD
NUMBER	NAME	UNITS	(KiteFASTMBD, KiteAeroDyn, InflowWind, MoorDyn, or Controller Wrapper)

#### Deconstructor

This routine ends KiteFASTMBD:

- Calls module End routines
- Deallocates memory
- Closes the write output file

#### AssRes

This routine accesses inputs at  $t$  (from GetXCur) (including  $t = 0$ ) for both the prediction and correction steps of each MBD time step, temporarily updates states from  $t - \Delta t$  to  $t$ , and calculates outputs at  $t$ :

- Calls module UpdateStates and Controller\_Step routines except at  $t = 0$
- Calls module CalcOutput routines

Set the discrete-time counter:

$$n = \frac{t}{\Delta t} - 1$$

Query the MBDyn model to access the inputs at  $t$  (from GetXCur) i.e.  $^{MBD}u$ .

Calculate the translation displacements (relative) of the MBDyn input meshes at  $t$ :

$$^{MBD} \vec{u}_j^{Fus} = ^{MBD} \vec{p}_j^{Fus} - ^{MBD} \vec{p}_j^{FusR} \quad (\text{for } j = \{1, 2, \dots, ^{MBD}NumFusNds\})$$

$$^{MBD} \vec{u}_j^{SWn} = ^{MBD} \vec{p}_j^{SWn} - ^{MBD} \vec{p}_j^{SWnR} \quad (\text{for } j = \{1, 2, \dots, ^{MBD}NumSWnNds\})$$

$$^{MBD} \vec{u}_j^{PWn} = ^{MBD} \vec{p}_j^{PWn} - ^{MBD} \vec{p}_j^{PWnR} \quad (\text{for } j = \{1, 2, \dots, ^{MBD}NumPWnNds\})$$

$$^{MBD} \vec{u}_j^{VS} = ^{MBD} \vec{p}_j^{VS} - ^{MBD} \vec{p}_j^{VSR} \quad (\text{for } j = \{1, 2, \dots, ^{MBD}NumVSNds\})$$

$$^{MBD} \vec{u}_j^{SHS} = ^{MBD} \vec{p}_j^{SHS} - ^{MBD} \vec{p}_j^{SHSR} \quad (\text{for } j = \{1, 2, \dots, ^{MBD}NumSHSNds\})$$

$$^{MBD} \vec{u}_j^{PHS} = ^{MBD} \vec{p}_j^{PHS} - ^{MBD} \vec{p}_j^{PHSR} \quad (\text{for } j = \{1, 2, \dots, ^{MBD}NumPHSNds\})$$

$$^{MBD} \vec{u}_j^{SPy} [n_{Pylons}] = ^{MBD} \vec{p}_j^{SPy} [n_{Pylons}] - ^{MBD} \vec{p}_j^{SPyR} [n_{Pylons}] \quad (\text{for } j = \{1, 2, \dots, ^{MBD}NumPylNds\})$$

$$^{MBD} \vec{u}_j^{PPy} [n_{Pylons}] = ^{MBD} \vec{p}_j^{PPy} [n_{Pylons}] - ^{MBD} \vec{p}_j^{PPyR} [n_{Pylons}] \quad (\text{for } j = \{1, 2, \dots, ^{MBD}NumPylNds\})$$

**Commented [JJ21]:** AssRes could access inputs at t-dt (from GetXPrev), but we save the previous inputs as OtherStates instead.

**Commented [JJ22]:** Note: the module UpdateStates and Controller\_Step routines are not called at t=0 (except for KiteAeroDyn) because the states have already been initialized through the Init calls.

**Commented [JJ23]:** This is necessary because in OpenFAST, UpdateStates shifts from t to t+dt whereas AssRes shifts from t-dt to t.

$$\begin{aligned} {}^{MBD}\vec{u}^{SPyRtr} [n_{Pylons}, n_2] &= {}^{MBD}\vec{p}^{SPyRtr} [n_{Pylons}, n_2] - {}^{MBD}\vec{p}^{SPyRtrR} [n_{Pylons}, n_2] \\ {}^{MBD}\vec{u}^{PPyRtr} [n_{Pylons}, n_2] &= {}^{MBD}\vec{p}^{PPyRtr} [n_{Pylons}, n_2] - {}^{MBD}\vec{p}^{PPyRtrR} [n_{Pylons}, n_2] \end{aligned}$$

Advance the controller only once per controller time step, updating the states to, and obtaining the controller outputs at,  $t$ :

IF ( $Ctrl\_NewTime$ ) THEN

First, calculate the InflowWind outputs at the base station and fuselage using the most converged inputs from MBDyn (as data stored in  ${}^{MBD}OtherStates$  from the previous step):

$$\begin{aligned} {}^{IfW}PositionXYZ(:, 1) &= {}^{MBD}\vec{p}^{Wind} \\ {}^{IfW}PositionXYZ(:, 2) &= {}^{MBD}\vec{p}^{FusO} \end{aligned}$$

Call InflowWind\_CalcOutput()

**Commented [JJ24]:** One can call InflowWind\_CalcOutput() with fewer than  ${}^{IfW}NumWindPoints$ .

Set inputs to Controller using the most converged inputs from MBDyn and the outputs from KiteAeroDyn, InflowWind, and MoorDyn (as data stored in  ${}^{MBD}OtherStates$ ,  ${}^{KAD}y$ , and  ${}^{MD}y$  from the previous step):

$${}^{Ctrl}dcm\_g2b = {}^{MBD}\Lambda^{FusO} [{}^{FAST2Ctrl}]^T$$

$${}^{Ctrl}pqr = {}^{MBD}\Lambda^{FusO} {}^{MBD}\vec{\omega}^{FusO}$$

$${}^{Ctrl}acc\_norm = \left\| {}^{MBD}\vec{a}^{FusO} \right\|_2$$

$${}^{Ctrl}Xg = \Lambda^{FAST2Ctrl} \left\{ {}^{MBD}\vec{p}^{FusO} - \vec{p}^{Anch} \right\}$$

$${}^{Ctrl}Vg = \Lambda^{FAST2Ctrl} {}^{MBD}\vec{v}^{FusO}$$

$${}^{Ctrl}Vb = {}^{MBD}\Lambda^{FusO} {}^{MBD}\vec{v}^{FusO}$$

$${}^{Ctrl}Ag = \Lambda^{FAST2Ctrl} {}^{MBD}\vec{a}^{FusO}$$

$${}^{Ctrl}Ab = {}^{MBD}\Lambda^{FusO} {}^{MBD}\vec{a}^{FusO}$$

$${}^{Ctrl}rho = \rho$$

$${}^{Ctrl}apparent\_wind = \Lambda^{FAST2Ctrl} \left\{ {}^{IfW}VelocityUVW(:, 2) - {}^{MBD}\vec{v}^{FusO} \right\}$$

$${}^{Ctrl}tether\_force\_b = {}^{MBD}\Lambda^{FusO} \left\{ \sum_{i=1}^{NFairs} {}^{MD}PtFairleadLoad\%Force(:, i) \right\}$$

$${}^{Ctrl}wind\_g = \Lambda^{FAST2Ctrl} {}^{IfW}VelocityUVW(:, 1)$$

$${}^{Ctrl}aero\_torque^{SPyRtr} [n_{Pylons}, n_2] = \left\{ {}^{MBD}\hat{x}^{SPyRtr} [n_{Pylons}, n_2] \right\}^T {}^{KAD}\vec{M}^{SPyRtr} [n_{Pylons}, n_2]$$

$${}^{Ctrl}aero\_torque^{PPyRtr} [n_{Pylons}, n_2] = \left\{ {}^{MBD}\hat{x}^{PPyRtr} [n_{Pylons}, n_2] \right\}^T {}^{KAD}\vec{M}^{PPyRtr} [n_{Pylons}, n_2]$$

- Call Controller\_Step()

**Commented [JJ26]:** We are approximating this input to the controller as the vector sum of the fairlead tensions.

**Commented [JJ27]:** These were added to the original controller inputs so that the controller could calculate the rotor/drivetrain acceleration and resulting generator speed and torque.

We should also ensure that the controller is using the same rotor/drivetrain rotational inertia.

Ensure that we only call the controller once per the controller time step:

$${}^{Ctrl}NewTime = FALSE$$

END

Store a copy of the MoorDyn current states at  $t - \Delta t$ :

$$^{MD}\mathbf{x}^{Copy} = ^{MD}\mathbf{x}$$

Set inputs to MoorDyn at  $t$  from MBDyn:

$$^{MD}PtFairleadDisplacement = M_u^{L2P} \left( ^{MBD}\vec{u}_j^{Wn}, ^{MBD}\Lambda_j^{Wn} \right)$$

**Commented [JJ28]:** See earlier comment about mesh mapping with  $Wn$  above.

Advance MoorDyn:

IF ( $t > 0$ ) Call MoorDyn\_UpdateStates()  
Call MoorDyn\_CalcOutput()

**Commented [JJ29]:** Input the time at t-dt in this call.

The input at t-dt comes from  $^{MD}OtherStates$

Advance KiteAeroDyn only once per KiteAeroDyn time step, interpolate the KiteAeroDyn outputs otherwise.

IF ( $^{KAD}NewTime$ ) THEN

Shift the KiteAeroDyn input history:

IF ( $t > 0$ )  
IF ( $InterpOrder == 1$ ) THEN  
 $^{KAD}u(2) = ^{KAD}u(1)$   
ELSEIF ! ( $InterpOrder == 2$ )  
 $^{KAD}u(3) = ^{KAD}u(2)$   
 $^{KAD}u(2) = ^{KAD}u(1)$   
END IF  
END IF

Set inputs to KiteAeroDyn—stored in  $^{KAD}u(1)$ —from Controller at  $t$ :

$$^{KAD}Ctrl^{SFlp} [n_{Flaps}] = \begin{cases} Ctrl kFlapA5 & for (n_{Flaps} = 1) \\ Ctrl kFlapA7 & for (n_{Flaps} = 2) \\ Ctrl kFlapA8 & for (n_{Flaps} = 3) \end{cases}$$

$$^{KAD}Ctrl^{PFlp} [n_{Flaps}] = \begin{cases} Ctrl kFlapA4 & for (n_{Flaps} = 1) \\ Ctrl kFlapA2 & for (n_{Flaps} = 2) \\ Ctrl kFlapA1 & for (n_{Flaps} = 3) \end{cases}$$

$$^{KAD}Ctrl^{Rudr} [n_2] = Ctrl kFlapA10$$

**Commented [JJ30]:** Different controller documentation use kFlapRud in place of kFlapA10

$$^{KAD}Ctrl^{SElv} [n_2] = Ctrl kFlapA9$$

**Commented [JJ31]:** Different controller documentation use kFlapEle in place of kFlapA9

$$^{KAD}Ctrl^{PElv} [n_2] = Ctrl kFlapA9$$

$$^{KAD}\Omega^{SPyRtr} [n_{Pylons}, n_2] = Ctrl \Omega^{SPyRtr} [n_{Pylons}, n_2]$$

$$^{KAD}\Omega^{PPyRtr} [n_{Pylons}, n_2] = Ctrl \Omega^{PPyRtr} [n_{Pylons}, n_2]$$

**Commented [JJ32]:** These were added to the original controller outputs so that the controller could calculate the rotor/drivetrain acceleration and resulting generator speed and torque.

$$^{KAD}\theta^{SPyRtr} [n_{Pylons}, n_2] = 0$$

$$^{KAD}\theta^{PPyRtr} [n_{Pylons}, n_2] = 0$$

**Commented [JJ33]:** The rotor-collective pitch angles are not currently commanded from the controller; assume zero for now.

Set inputs to KiteAeroDyn—stored in  ${}^{KAD}u(I)$ —from MBDyn at  $t$  based on mesh-mapping:

$${}^{KAD}\vec{u}^{FusO} = {}^{MBD}\vec{p}^{FusO}$$

$${}^{KAD}\vec{u}_j^{Fus} = M_u^{L2L} \left( {}^{MBD}\vec{u}_j^{Fus}, {}^{MBD}\Lambda_j^{Fus} \right)$$

$${}^{KAD}\Lambda_j^{Fus} = M_\Lambda^{L2L} \left( {}^{MBD}\Lambda_j^{Fus} \right)$$

$${}^{KAD}\vec{v}_j^{Fus} = M_v^{L2L} \left( {}^{KAD}\vec{u}_j^{Fus}, {}^{MBD}\vec{u}_j^{Fus}, {}^{MBD}\vec{v}_j^{Fus}, {}^{MBD}\vec{\omega}_j^{Fus} \right)$$

$${}^{KAD}\vec{u}_j^{SWn} = M_u^{L2L} \left( {}^{MBD}\vec{u}_j^{SWn}, {}^{MBD}\Lambda_j^{SWn} \right)$$

$${}^{KAD}\Lambda_j^{SWn} = M_\Lambda^{L2L} \left( {}^{MBD}\Lambda_j^{SWn} \right)$$

$${}^{KAD}\vec{v}_j^{SWn} = M_v^{L2L} \left( {}^{KAD}\vec{u}_j^{SWn}, {}^{MBD}\vec{u}_j^{SWn}, {}^{MBD}\vec{v}_j^{SWn}, {}^{MBD}\vec{\omega}_j^{SWn} \right)$$

$${}^{KAD}\vec{u}_j^{PWn} = M_u^{L2L} \left( {}^{MBD}\vec{u}_j^{PWn}, {}^{MBD}\Lambda_j^{PWn} \right)$$

$${}^{KAD}\Lambda_j^{PWn} = M_\Lambda^{L2L} \left( {}^{MBD}\Lambda_j^{PWn} \right)$$

$${}^{KAD}\vec{v}_j^{PWn} = M_v^{L2L} \left( {}^{KAD}\vec{u}_j^{PWn}, {}^{MBD}\vec{u}_j^{PWn}, {}^{MBD}\vec{v}_j^{PWn}, {}^{MBD}\vec{\omega}_j^{PWn} \right)$$

$${}^{KAD}\vec{u}_j^{VS} = M_u^{L2L} \left( {}^{MBD}\vec{u}_j^{VS}, {}^{MBD}\Lambda_j^{VS} \right)$$

$${}^{KAD}\Lambda_j^{VS} = M_\Lambda^{L2L} \left( {}^{MBD}\Lambda_j^{VS} \right)$$

$${}^{KAD}\vec{v}_j^{VS} = M_v^{L2L} \left( {}^{KAD}\vec{u}_j^{VS}, {}^{MBD}\vec{u}_j^{VS}, {}^{MBD}\vec{v}_j^{VS}, {}^{MBD}\vec{\omega}_j^{VS} \right)$$

$${}^{KAD}\vec{u}_j^{SHS} = M_u^{L2L} \left( {}^{MBD}\vec{u}_j^{SHS}, {}^{MBD}\Lambda_j^{SHS} \right)$$

$${}^{KAD}\Lambda_j^{SHS} = M_\Lambda^{L2L} \left( {}^{MBD}\Lambda_j^{SHS} \right)$$

$${}^{KAD}\vec{v}_j^{SHS} = M_v^{L2L} \left( {}^{KAD}\vec{u}_j^{SHS}, {}^{MBD}\vec{u}_j^{SHS}, {}^{MBD}\vec{v}_j^{SHS}, {}^{MBD}\vec{\omega}_j^{SHS} \right)$$

$${}^{KAD}\vec{u}_j^{PHS} = M_u^{L2L} \left( {}^{MBD}\vec{u}_j^{PHS}, {}^{MBD}\Lambda_j^{PHS} \right)$$

$${}^{KAD}\Lambda_j^{PHS} = M_\Lambda^{L2L} \left( {}^{MBD}\Lambda_j^{PHS} \right)$$

$${}^{KAD}\vec{v}_j^{PHS} = M_v^{L2L} \left( {}^{KAD}\vec{u}_j^{PHS}, {}^{MBD}\vec{u}_j^{PHS}, {}^{MBD}\vec{v}_j^{PHS}, {}^{MBD}\vec{\omega}_j^{PHS} \right)$$

$${}^{KAD}\vec{u}_j^{SPy} [n_{Pylons}] = M_u^{L2L} \left( {}^{MBD}\vec{u}_j^{SPy} [n_{Pylons}], {}^{MBD}\Lambda_j^{SPy} [n_{Pylons}] \right)$$

$${}^{KAD}\Lambda_j^{SPy} [n_{Pylons}] = M_\Lambda^{L2L} \left( {}^{MBD}\Lambda_j^{SPy} [n_{Pylons}] \right)$$

$${}^{KAD}\vec{v}_j^{SPy} [n_{Pylons}] = M_v^{L2L} \left( {}^{KAD}\vec{u}_j^{SPy} [n_{Pylons}], {}^{MBD}\vec{u}_j^{SPy} [n_{Pylons}], {}^{MBD}\vec{v}_j^{SPy} [n_{Pylons}], {}^{MBD}\vec{\omega}_j^{SPy} [n_{Pylons}] \right)$$

$${}^{KAD}\vec{u}_j^{PPy} [n_{Pylons}] = M_u^{L2L} \left( {}^{MBD}\vec{u}_j^{PPy} [n_{Pylons}], {}^{MBD}\Lambda_j^{PPy} [n_{Pylons}] \right)$$

$${}^{KAD}\Lambda_j^{PPy} [n_{Pylons}] = M_\Lambda^{L2L} \left( {}^{MBD}\Lambda_j^{PPy} [n_{Pylons}] \right)$$

$${}^{KAD}\vec{v}_j^{PPy} [n_{Pylons}] = M_v^{L2L} \left( {}^{KAD}\vec{u}_j^{PPy} [n_{Pylons}], {}^{MBD}\vec{u}_j^{PPy} [n_{Pylons}], {}^{MBD}\vec{v}_j^{PPy} [n_{Pylons}], {}^{MBD}\vec{\omega}_j^{PPy} [n_{Pylons}] \right)$$

$${}^{KAD}\vec{u}^{SPyRtr} [n_{Pylons}, n_2] = {}^{MBD}\vec{u}^{SPyRtr} [n_{Pylons}, n_2]$$

$${}^{KAD}\Lambda^{SPyRtr} [n_{Pylons}, n_2] = {}^{MBD}\Lambda^{SPyRtr} [n_{Pylons}, n_2]$$

**Commented [JJ34]:** You could use P2P mappings here, but there is no point, because the reference (0,0,0) is the same in both KiteAeroDyn and MBDyn.

$$\begin{aligned}
{}^{KAD}\vec{v}^{SPyRtr} [n_{Pylons}, n_2] &= {}^{MBD}\vec{v}^{SPyRtr} [n_{Pylons}, n_2] \\
{}^{KAD}\vec{u}^{PPyRtr} [n_{Pylons}, n_2] &= {}^{MBD}\vec{u}^{PPyRtr} [n_{Pylons}, n_2] \\
{}^{KAD}\Lambda^{PPyRtr} [n_{Pylons}, n_2] &= {}^{MBD}\Lambda^{PPyRtr} [n_{Pylons}, n_2] \\
{}^{KAD}\vec{v}^{PPyRtr} [n_{Pylons}, n_2] &= {}^{MBD}\vec{v}^{PPyRtr} [n_{Pylons}, n_2]
\end{aligned}$$

**Commented [JJ35]:** You could use P2P mappings here, but there is no point, because the references are the same in both KiteAeroDyn and MBDyn.

**Commented [JJ36]:** You could use P2P mappings here, but there is no point, because the references are the same in both KiteAeroDyn and MBDyn.

Set inputs to InflowWind at  $t$  based on the KiteAeroDyn inputs—stored in  ${}^{KAD}u(I)$ :

$$\begin{aligned}
{}^{IfW}PositionXYZ(:, 1) &= {}^{MBD}\vec{p}^{Wind} \\
{}^{IfW}PositionXYZ(:, 2) &= {}^{MBD}\vec{p}^{FusO} \\
{}^{IfW}PositionXYZ(:, j+2) &= {}^{KAD}In\vec{p}_j^{FusR} + {}^{KAD}\vec{u}_j^{Fus} \quad (\text{for } j = \{1, 2, \dots, {}^{KAD}NumFusNds\}) \\
{}^{IfW}PositionXYZ\left(\begin{array}{c} :, j+2 \\ + {}^{KAD}NumFusNds \end{array}\right) &= {}^{KAD}In\vec{p}_j^{SWnR} + {}^{KAD}\vec{u}_j^{SWn} \quad (\text{for } j = \{1, 2, \dots, {}^{KAD}NumSWnNds\}) \\
{}^{IfW}PositionXYZ\left(\begin{array}{c} :, j+2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \end{array}\right) &= {}^{KAD}In\vec{p}_j^{PWnR} + {}^{KAD}\vec{u}_j^{PWn} \quad (\text{for } j = \{1, 2, \dots, {}^{KAD}NumPWnNds\}) \\
{}^{IfW}PositionXYZ\left(\begin{array}{c} :, j+2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \end{array}\right) &= {}^{KAD}In\vec{p}_j^{VSR} + {}^{KAD}\vec{u}_j^{VS} \quad (\text{for } j = \{1, 2, \dots, {}^{KAD}NumVSNds\}) \\
{}^{IfW}PositionXYZ\left(\begin{array}{c} :, j+2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \end{array}\right) &= {}^{KAD}In\vec{p}_j^{SHSR} + {}^{KAD}\vec{u}_j^{SHS} \quad (\text{for } j = \{1, 2, \dots, {}^{KAD}NumSHSNds\})
\end{aligned}$$

$${}^{IjW}PositionXYZ \begin{pmatrix} \vdots, j+2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \\ + {}^{KAD}NumSHSNds \end{pmatrix} = {}^{KAD}In \vec{p}_j^{PHSR} + {}^{KAD}\vec{u}_j^{PHS} \quad (\text{for } j = \{1, 2, \dots, {}^{KAD}NumPHSNds\})$$

$${}^{IjW}PositionXYZ \begin{pmatrix} \vdots, j+2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \\ + {}^{KAD}NumSHSNds \\ + {}^{KAD}NumPHSNds \\ + {}^{KAD}NumPylNds(n_{Pylons} - 1) \end{pmatrix} = {}^{KAD}In \vec{p}_j^{SPyR} [n_{Pylons}] + {}^{KAD}\vec{u}_j^{SPy} [n_{Pylons}] \quad (\text{for } j = \{1, 2, \dots, {}^{KAD}NumPylNds\})$$

$${}^{IjW}PositionXYZ \begin{pmatrix} \vdots, j+2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \\ + {}^{KAD}NumSHSNds \\ + {}^{KAD}NumPHSNds \\ + {}^{KAD}NumPylNds(N_{Pylons}) \\ + {}^{KAD}NumPylNds(n_{Pylons} - 1) \end{pmatrix} = {}^{KAD}In \vec{p}_j^{PPyR} [n_{Pylons}] + {}^{KAD}\vec{u}_j^{PPy} [n_{Pylons}] \quad (\text{for } j = \{1, 2, \dots, {}^{KAD}NumPylNds\})$$

$${}^{IjW}PositionXYZ \begin{pmatrix} \vdots, n_2 + 2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \\ + {}^{KAD}NumSHSNds \\ + {}^{KAD}NumPHSNds \\ + {}^{KAD}NumPylNds(2N_{Pylons}) \\ + 2(n_{Pylons} - 1) \end{pmatrix} = {}^{KAD}\vec{p}_j^{SPyRtrR} [n_{Pylons}, n_2] + {}^{KAD}\vec{u}^{SPyRtr} [n_{Pylons}, n_2]$$

$$^{IfW}PositionXYZ \begin{pmatrix} \vdots, n_2 + 2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \\ + {}^{KAD}NumSHSNds \\ + {}^{KAD}NumPHSNds \\ + {}^{KAD}NumPylNds(2N_{Pylons}) \\ + 2(N_{Pylons}) \\ + 2(n_{Pylons} - 1) \end{pmatrix} = {}^{KAD}\vec{p}_j^{PPyRtrR} [n_{Pylons}, n_2] + {}^{KAD}\vec{u}^{PPyRtr} [n_{Pylons}, n_2]$$

Call InflowWind\_CalcOutput()

**Commented [JJ37]:** Input the time at t in this call.

Set inputs to KiteAeroDyn—stored in  ${}^{KAD}u(1)$ —from InflowWind at  $t$ :

$${}^{KAD}\vec{V}_j^{Fus} = {}^{IfW}VelocityUVW(\vdots, j + 2) \quad (\text{for}$$

$$j = \{1, 2, \dots, {}^{KAD}NumFusNds\})$$

$${}^{KAD}\vec{V}_j^{SWn} = {}^{IfW}VelocityUVW \begin{pmatrix} \vdots, j + 2 \\ + {}^{KAD}NumFusNds \end{pmatrix} \quad (\text{for}$$

$$j = \{1, 2, \dots, {}^{KAD}NumSWnNds\})$$

$${}^{KAD}\vec{V}_j^{PWn} = {}^{IfW}VelocityUVW \begin{pmatrix} \vdots, j + 2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \end{pmatrix} \quad (\text{for}$$

$$j = \{1, 2, \dots, {}^{KAD}NumPWnNds\})$$

$${}^{KAD}\vec{V}_j^{VS} = {}^{IfW}VelocityUVW \begin{pmatrix} \vdots, j + 2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \end{pmatrix} \quad (\text{for}$$

$$j = \{1, 2, \dots, {}^{KAD}NumVSNds\})$$



$$\begin{aligned}
& {}^{KAD}\vec{V}_j^{SHS} = {}^{I\eta W}VelocityUVW \begin{pmatrix} \vdots, j+2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \end{pmatrix} \quad (\text{for} \\
& j = \{1, 2, \dots, {}^{KAD}NumSHSNds\}) \\
& {}^{KAD}\vec{V}_j^{PHS} = {}^{I\eta W}VelocityUVW \begin{pmatrix} \vdots, j+2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \\ + {}^{KAD}NumSHSNds \end{pmatrix} \quad (\text{for} \\
& j = \{1, 2, \dots, {}^{KAD}NumPHSNds\}) \\
& {}^{KAD}\vec{V}_j^{SPy} [n_{Pylons}] = {}^{I\eta W}VelocityUVW \begin{pmatrix} \vdots, j+2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \\ + {}^{KAD}NumSHSNds \\ + {}^{KAD}NumPHSNds \\ + {}^{KAD}NumPylNds(n_{Pylons} - 1) \end{pmatrix} \quad (\text{for} \\
& j = \{1, 2, \dots, {}^{KAD}NumPylNds\}) \\
& {}^{KAD}\vec{V}_j^{PPy} [n_{Pylons}] = {}^{I\eta W}VelocityUVW \begin{pmatrix} \vdots, j+2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \\ + {}^{KAD}NumSHSNds \\ + {}^{KAD}NumPHSNds \\ + {}^{KAD}NumPylNds(N_{Pylons}) \\ + {}^{KAD}NumPylNds(n_{Pylons} - 1) \end{pmatrix} \quad (\text{for} \\
& j = \{1, 2, \dots, {}^{KAD}NumPylNds\})
\end{aligned}$$

$$\begin{aligned}
{}^{KAD}\vec{V}^{SPyRtr}[n_{Pylons}, n_2] &= {}^{IfW}VelocityUVW \begin{pmatrix} \vdots, n_2 + 2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \\ + {}^{KAD}NumSHSNds \\ + {}^{KAD}NumPHSNds \\ + {}^{KAD}NumPylNds(2N_{Pylons}) \\ + 2(n_{Pylons} - 1) \end{pmatrix} \\
{}^{KAD}\vec{V}^{PPyRtr}[n_{Pylons}, n_2] &= {}^{IfW}VelocityUVW \begin{pmatrix} \vdots, n_2 + 2 \\ + {}^{KAD}NumFusNds \\ + {}^{KAD}NumSWnNds \\ + {}^{KAD}NumPWnNds \\ + {}^{KAD}NumVSNds \\ + {}^{KAD}NumSHSNds \\ + {}^{KAD}NumPHSNds \\ + {}^{KAD}NumPylNds(2N_{Pylons}) \\ + 2(N_{Pylons}) \\ + 2(n_{Pylons} - 1) \end{pmatrix}
\end{aligned}$$

Initialize the KiteAeroDyn input history at  $t = 0$  :

IF ( $t == 0$ ) THEN

IF ( $InterpOrder == 1$ ) THEN

$${}^{KAD}u(2) = {}^{KAD}u(1)$$

$${}^{KAD}t(2) = -{}^{KAD}\Delta t$$

$${}^{KAD}t(1) = 0$$

ELSEIF ! ( $InterpOrder == 2$ )

$${}^{KAD}u(3) = {}^{KAD}u(1)$$

$${}^{KAD}u(2) = {}^{KAD}u(1)$$

$${}^{KAD}t(3) = -2{}^{KAD}\Delta t$$

$${}^{KAD}t(2) = {}^{KAD}\Delta t$$

$${}^{KAD}t(1) = 0$$

END IF  
END IF

Advance KiteAeroDyn to  $t + {}^{KAD}\Delta t$  :

Call KiteAeroDyn\_Input\_ExtrapInterp(  ${}^{KAD}u(:)$ ,  ${}^{KAD}t(:)$ ,  ${}^{KAD}u$ ,  $t + {}^{KAD}\Delta t$  )

Call KiteAeroDyn\_UpdateStates()

Call KiteAeroDyn\_CalcOutput()

**Commented [JJ38]:** Input the time at t in this call

**Commented [JJ39]:** Input the time at t+KAD\*dt in this call.

Shift the KiteAeroDyn output history:

IF ( $t > 0$ ) THEN

IF ( $InterpOrder == 1$ ) THEN

${}^{KAD}y(2) = {}^{KAD}y(1)$

${}^{KAD}y(1) = {}^{KAD}y$

${}^{KAD}t(2) = {}^{KAD}t(1)$

${}^{KAD}t(1) = t + {}^{KAD}\Delta t$

ELSEIF ! ( $InterpOrder == 2$ )

${}^{KAD}y(3) = {}^{KAD}y(2)$

${}^{KAD}y(2) = {}^{KAD}y(1)$

${}^{KAD}y(1) = {}^{KAD}y$

${}^{KAD}t(3) = {}^{KAD}t(2)$

${}^{KAD}t(2) = {}^{KAD}t(1)$

${}^{KAD}t(1) = t + {}^{KAD}\Delta t$

END IF

ELSE ! ( $t == 0$ )

IF ( $InterpOrder == 1$ ) THEN

${}^{KAD}y(2) = {}^{KAD}y$

${}^{KAD}y(1) = {}^{KAD}y$

ELSEIF ! ( $InterpOrder == 2$ )

${}^{KAD}y(3) = {}^{KAD}y$

${}^{KAD}y(2) = {}^{KAD}y$

${}^{KAD}y(1) = {}^{KAD}y$

END IF

END IF

Ensure that we only call KiteAeroDyn once per KiteAeroDyn time step:

${}^{KAD}NewTime = FALSE$

END

Call KiteAeroDyn\_Output\_ExtrapInterp(  $^{KAD}y(:,)$ ,  $^{KAD}t(:,)$ ,  $^{KAD}y$ ,  $t$  )

Model the rotor/drivetrain dynamics, including the effects from the Controller and KiteAeroDyn, and calculate the reaction loads on the pylons for transfer to MBDyn at  $t$  :

Call Rotor(  $^{MBD}A^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{\omega}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{a}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{\alpha}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{Ctrl}\Omega^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{Ctrl}T^{GenSPyRtr}[n_{Pylons}, n_2]$ ,  $^{KAD}\vec{F}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{KAD}\vec{M}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{g}$ ,  $^{MBD}m^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}I_{Rot}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}I_{Tran}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\chi_{CM}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{F}^{SPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{M}^{SPyRtr}[n_{Pylons}, n_2]$  )

Call Rotor(  $^{MBD}A^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{\omega}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{a}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{\alpha}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{Ctrl}\Omega^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{Ctrl}T^{GenPPyRtr}[n_{Pylons}, n_2]$ ,  $^{KAD}\vec{F}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{KAD}\vec{M}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{g}$ ,  $^{MBD}m^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}I_{Rot}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}I_{Tran}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\chi_{CM}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{F}^{PPyRtr}[n_{Pylons}, n_2]$ ,  $^{MBD}\vec{M}^{PPyRtr}[n_{Pylons}, n_2]$  )

where:

$$^{Ctrl}T^{GenSPyRtr}[n_{Pylons}, n_2] = \begin{cases} ^{Ctrl}Motor 7 & \text{for}((n_{Pylons} = 1).AND.(n_2 = 1)) \\ ^{Ctrl}Motor 2 & \text{for}((n_{Pylons} = 1).AND.(n_2 = 2)) \\ ^{Ctrl}Motor 8 & \text{for}((n_{Pylons} = 2).AND.(n_2 = 1)) \\ ^{Ctrl}Motor 1 & \text{for}((n_{Pylons} = 2).AND.(n_2 = 2)) \end{cases}$$

$$^{Ctrl}T^{GenPPyRtr}[n_{Pylons}, n_2] = \begin{cases} ^{Ctrl}Motor 6 & \text{for}((n_{Pylons} = 1).AND.(n_2 = 1)) \\ ^{Ctrl}Motor 3 & \text{for}((n_{Pylons} = 1).AND.(n_2 = 2)) \\ ^{Ctrl}Motor 5 & \text{for}((n_{Pylons} = 2).AND.(n_2 = 1)) \\ ^{Ctrl}Motor 4 & \text{for}((n_{Pylons} = 2).AND.(n_2 = 2)) \end{cases}$$

**Commented [JJ40]:** This math assumes the top node of the pylon is node 1 and that the pylons are numbered from inboard to outboard.

**Commented [JJ41]:** This math is now done in the C controller.

Transfer outputs from KiteAeroDyn to MBDyn at  $t$  :

$$^{MBD}\vec{F}_j^{Fus} = M_F^{P2P} \left( ^{KAD}\vec{F}_j^{Fus} \right)$$

$$^{MBD}\vec{M}_j^{Fus} = M_M^{P2P} \left( ^{MBD}\vec{u}_j^{Fus}, ^{KAD}Out\vec{u}_j^{Fus}, ^{KAD}\vec{F}_j^{Fus}, ^{KAD}\vec{M}_j^{Fus} \right)$$

$$^{MBD}\vec{F}_j^{SWn} = M_F^{P2P} \left( ^{KAD}\vec{F}_j^{SWn} \right)$$

$$^{MBD}\vec{M}_j^{SWn} = M_M^{P2P} \left( ^{MBD}\vec{u}_j^{SWn}, ^{KAD}Out\vec{u}_j^{SWn}, ^{KAD}\vec{F}_j^{SWn}, ^{KAD}\vec{M}_j^{SWn} \right)$$

$$^{MBD}\vec{F}_j^{PWn} = M_F^{P2P} \left( ^{KAD}\vec{F}_j^{PWn} \right)$$

$$^{MBD}\vec{M}_j^{PWn} = M_M^{P2P} \left( ^{MBD}\vec{u}_j^{PWn}, ^{KAD}Out\vec{u}_j^{PWn}, ^{KAD}\vec{F}_j^{PWn}, ^{KAD}\vec{M}_j^{PWn} \right)$$

$$^{MBD}\vec{F}_j^{VS} = M_F^{P2P} \left( ^{KAD}\vec{F}_j^{VS} \right)$$

$$\begin{aligned}
{}^{MBD}\vec{M}_j^{VS} &= M_M^{P2P} \left( {}^{MBD}\vec{u}_j^{VS}, {}^{KAD}Out\vec{u}_j^{VS}, {}^{KAD}\vec{F}_j^{VS}, {}^{KAD}\vec{M}_j^{VS} \right) \\
{}^{MBD}\vec{F}_j^{SHS} &= M_F^{P2P} \left( {}^{KAD}\vec{F}_j^{SHS} \right) \\
{}^{MBD}\vec{M}_j^{SHS} &= M_M^{P2P} \left( {}^{MBD}\vec{u}_j^{SHS}, {}^{KAD}Out\vec{u}_j^{SHS}, {}^{KAD}\vec{F}_j^{SHS}, {}^{KAD}\vec{M}_j^{SHS} \right) \\
{}^{MBD}\vec{F}_j^{PHS} &= M_F^{P2P} \left( {}^{KAD}\vec{F}_j^{PHS} \right) \\
{}^{MBD}\vec{M}_j^{PHS} &= M_M^{P2P} \left( {}^{MBD}\vec{u}_j^{PHS}, {}^{KAD}Out\vec{u}_j^{PHS}, {}^{KAD}\vec{F}_j^{PHS}, {}^{KAD}\vec{M}_j^{PHS} \right) \\
{}^{MBD}\vec{F}_j^{SPy} \left[ n_{Pylons} \right] &= M_F^{P2P} \left( {}^{KAD}\vec{F}_j^{SPy} \left[ n_{Pylons} \right] \right) \\
{}^{MBD}\vec{M}_j^{SPy} \left[ n_{Pylons} \right] &= M_M^{P2P} \left( {}^{MBD}\vec{u}_j^{SPy} \left[ n_{Pylons} \right], {}^{KAD}Out\vec{u}_j^{SPy} \left[ n_{Pylons} \right], {}^{KAD}\vec{F}_j^{SPy} \left[ n_{Pylons} \right], {}^{KAD}\vec{M}_j^{SPy} \left[ n_{Pylons} \right] \right) \\
{}^{MBD}\vec{F}_j^{PPy} \left[ n_{Pylons} \right] &= M_F^{P2P} \left( {}^{KAD}\vec{F}_j^{PPy} \left[ n_{Pylons} \right] \right) \\
{}^{MBD}\vec{M}_j^{PPy} \left[ n_{Pylons} \right] &= M_M^{P2P} \left( {}^{MBD}\vec{u}_j^{PPy} \left[ n_{Pylons} \right], {}^{KAD}Out\vec{u}_j^{PPy} \left[ n_{Pylons} \right], {}^{KAD}\vec{F}_j^{PPy} \left[ n_{Pylons} \right], {}^{KAD}\vec{M}_j^{PPy} \left[ n_{Pylons} \right] \right)
\end{aligned}$$

Transfer outputs from MoorDyn to MBDyn at  $t$ :

$$\begin{aligned}
{}^{MBD}\vec{F}_j^{SWn} &= {}^{MBD}\vec{F}_j^{SWn} + M_F^{P2P} \left( {}^{MD}PtFairleadLoad \right) \\
{}^{MBD}\vec{M}_j^{SWn} &= {}^{MBD}\vec{M}_j^{SWn} + M_M^{P2P} \left( {}^{MBD}\vec{u}_j^{SWn}, {}^{MD}PtFairleadDisplacement, {}^{MD}PtFairleadLoad, \vec{0} \right) \\
{}^{MBD}\vec{F}_j^{PWn} &= {}^{MBD}\vec{F}_j^{PWn} + M_F^{P2P} \left( {}^{MD}PtFairleadLoad \right) \\
{}^{MBD}\vec{M}_j^{PWn} &= {}^{MBD}\vec{M}_j^{PWn} + M_M^{P2P} \left( {}^{MBD}\vec{u}_j^{PWn}, {}^{MD}PtFairleadDisplacement, {}^{MD}PtFairleadLoad, \vec{0} \right)
\end{aligned}$$

**Commented [JJ42]:** See earlier comment about mesh mapping with Wn above.

#### Private SUBROUTINES

##### Rotor (SUBROUTINE Rotor)

Implements the structural dynamics of a rotor/drivetrain analytically to calculate the reaction loads (forces and moments) applied on the nacelle, including the applied aerodynamic loads, rotor inertial loads, rotor gyroscopic loads, etc. The analytical formulation assumes that the rotor/drivetrain is a rigid body rotating about the local x-axis of the nacelle coordinate system and that the structure is axisymmetric about this axis (with no imbalances) such that the calculations do not depend on the azimuth angle of the rotor. That is, for a body-fixed (x,y,z) coordinate system in the rotor/drivetrain, it is assumed that:

$${}^{CM}y = {}^{CM}z = 0$$

$$I_{xy} = I_{yz} = I_{xz} = 0$$

$$I_{xx} = I^{Rot}$$

$$I_{yy} = I_{zz} = I^{Tran}$$

Inputs	Outputs	States	Parameters
<ul style="list-style-type: none"> <li><math>\mathcal{A}^{Nac}</math> – Displaced rotation (absolute orientation) of the nacelle (-)</li> <li><math>\vec{\omega}^{Nac}</math> – Rotational velocity (absolute) of</li> </ul>	<ul style="list-style-type: none"> <li><math>\vec{F}^{React}</math> – reaction forces applied on the nacelle at the rotor reference point expressed in the global inertial-frame</li> </ul>		

<p>the nacelle (rad/s)</p> <ul style="list-style-type: none"> <li>• <math>\vec{a}^{Nac}</math> – Translational acceleration (absolute) of the nacelle at the rotor reference point (m/s<sup>2</sup>)</li> <li>• <math>\vec{\alpha}^{Nac}</math> – Rotational acceleration (absolute) of the nacelle (rad/s<sup>2</sup>)</li> <li>• <math>\Omega^{Rtr}</math> – Rotor speed about the shaft axis (relative to the nacelle) (rad/s)</li> <li>• <math>T^{Gen}</math> – electrical generator torque applied to the rotor/drivetrain about the shaft axis (N·m)</li> <li>• <math>\vec{F}^{Aero}</math> – aerodynamic forces applied on the rotor at the rotor reference point expressed in the global inertial-frame coordinate system (N)</li> <li>• <math>\vec{M}^{Aero}</math> – aerodynamic moments applied on the rotor about the rotor reference point expressed in the global inertial-frame coordinate system (N·m)</li> <li>• <math>\vec{g}</math> – gravity vector expressed in the global inertial-frame coordinate system (m/s<sup>2</sup>)</li> <li>• <math>m</math> – rotor/drivetrain mass (kg)</li> <li>• <math>I^{Rot}</math> – rotor/drivetrain rotational inertia about the shaft axis (kg·m<sup>2</sup>)</li> <li>• <math>I^{Tran}</math> – rotor/drivetrain transverse inertia about the rotor reference point (kg·m<sup>2</sup>)</li> <li>• <math>^{CM}x</math> – distance along the shaft from the rotor reference point to the center of mass of the rotor/drivetrain</li> </ul>	<p>coordinate system (N)</p> <ul style="list-style-type: none"> <li>• <math>\vec{M}^{React}</math> – reaction moments applied on the nacelle about the rotor reference point expressed in the global inertial-frame coordinate system (N·m)</li> </ul>		
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**Commented [JJ43]:** This is input in place of:

$\dot{\Omega}^{Rtr}$  – Rotor acceleration about the shaft axis (relative to the nacelle) (rad/s<sup>2</sup>)

(positive along positive x) (m)			
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Compute the inputs relative to the rotor/drivetrain CM and expressed in the local nacelle coordinate system:

$${}^{CM}\vec{r} = {}^{CM}\hat{x}\hat{x}^{Nac}$$

$${}^{CM}I^{Tran} = I^{Tran} - m^{CM}x^2$$

$$\begin{Bmatrix} {}^{CM}F_x^{Aero} \\ {}^{CM}F_y^{Aero} \\ {}^{CM}F_z^{Aero} \end{Bmatrix} = A^{Nac} \vec{F}^{Aero}$$

$$\begin{Bmatrix} {}^{CM}M_x^{Aero} \\ {}^{CM}M_y^{Aero} \\ {}^{CM}M_z^{Aero} \end{Bmatrix} = A^{Nac} \left\{ \vec{M}^{Aero} - {}^{CM}\vec{r} \times \vec{F}^{Aero} \right\}$$

$$\begin{Bmatrix} g_x \\ g_y \\ g_z \end{Bmatrix} = A^{Nac} \vec{g}$$

$$\vec{\omega}^{Rtr} = \vec{\omega}^{Nac} + \Omega^{Rtr} \hat{x}^{Nac}$$

$$\vec{\alpha}^{Rtr} = \vec{\alpha}^{Nac}$$

$$\begin{Bmatrix} \omega_x^{Rtr} \\ \omega_y^{Rtr} \\ \omega_z^{Rtr} \end{Bmatrix} = A^{Nac} \vec{\omega}^{Rtr}$$

$$\begin{Bmatrix} {}^{CM}a_x^{Rtr} \\ {}^{CM}a_y^{Rtr} \\ {}^{CM}a_z^{Rtr} \end{Bmatrix} = A^{Nac} \left\{ \vec{a}^{Nac} + \vec{\alpha}^{Rtr} \times {}^{CM}\vec{r} + \vec{\omega}^{Rtr} \times \left\{ \vec{\omega}^{Rtr} \times {}^{CM}\vec{r} \right\} \right\}$$

$$\begin{Bmatrix} \alpha_x^{Rtr} \\ \alpha_y^{Rtr} \\ \alpha_z^{Rtr} \end{Bmatrix} = A^{Nac} \vec{\alpha}^{Rtr}$$

Compute the reaction loads applied to the rotor/drivetrain at the rotor/drivetrain CM and expressed in the local nacelle coordinate system:

$$\begin{Bmatrix} {}^{CM}F_x^{React} \\ {}^{CM}F_y^{React} \\ {}^{CM}F_z^{React} \end{Bmatrix} = \begin{Bmatrix} -{}^{CM}F_x^{Aero} - mg_x + m^{CM}a_x^{Rtr} \\ -{}^{CM}F_y^{Aero} - mg_y + m^{CM}a_y^{Rtr} \\ -{}^{CM}F_z^{Aero} - mg_z + m^{CM}a_z^{Rtr} \end{Bmatrix}$$

$$\begin{Bmatrix} {}^{CM}M_x^{React} \\ {}^{CM}M_y^{React} \\ {}^{CM}M_z^{React} \end{Bmatrix} = \begin{Bmatrix} T^{Gen} \\ -{}^{CM}M_y^{Aero} + I^{Rot}\alpha_y^{Rtr} + (I^{Rot} - {}^{CM}I^{Tran})\omega_z^{Rtr}\omega_x^{Rtr} \\ -{}^{CM}M_z^{Aero} + I^{Rot}\alpha_z^{Rtr} - (I^{Rot} - {}^{CM}I^{Tran})\omega_y^{Rtr}\omega_x^{Rtr} \end{Bmatrix}$$

**Commented [JJ44]:** The equation implemented neglects the rotor acceleration about the shaft axis. The correct equation should be:

$$\vec{\alpha}^{Rtr} = \vec{\alpha}^{Nac} + \dot{\Omega}^{Rtr} \hat{x}^{Nac}$$

, but the rotor acceleration about the shaft axis is not needed because the generator torque is input instead.

**Commented [JJ45]:** The first equation should be:

$${}^{CM}M_x^{React} = -{}^{CM}M_x^{Aero} + I^{Rot}\alpha_x^{Rtr}$$

But this equals the equation implemented because the generator torque is input instead of the rotor acceleration about the shaft axis.

Compute the reaction loads applied to the nacelle (this is equal, but opposite to the reaction loads applied to the rotor/drivetrain) at the rotor/drivetrain reference point and expressed in the global inertial frame coordinate system:

$$\vec{F}^{React} = -[A^{Nac}]^T \begin{Bmatrix} {}^{CM}F_x^{React} \\ {}^{CM}F_y^{React} \\ {}^{CM}F_z^{React} \end{Bmatrix}$$

$$\vec{M}^{React} = -[A^{Nac}]^T \begin{Bmatrix} {}^{CM}M_x^{React} \\ {}^{CM}M_y^{React} \\ {}^{CM}M_z^{React} \end{Bmatrix} + {}^{CM}\vec{r} \times \vec{F}^{React}$$

#### AfterPredict

This routine updates the actual states based on the temporary states at the successful completion of time step  $t$  (including  $t=0$ ). That said, time has already been updated to  $t=t+\Delta t$  before this routine is called, so technically, this routine is first called at  $t=\Delta t$ .

```
IF  $\left( MOD \left( NINT \left( \frac{t}{\Delta t} \right), N_{KAD/MBD} \right) = 0 \right)$  THEN
     ${}^{KAD}NewTime = TRUE$ 
END
IF  $\left( MOD \left( NINT \left( \frac{t}{\Delta t} \right), N_{Ctrl/MBD} \right) = 0 \right)$  THEN
     ${}^{Ctrl}NewTime = TRUE$ 
     ${}^{MBD}OtherStates = {}^{MBD}u$ 
END
 ${}^{MD}OtherStates = {}^{MD}u$ 
 ${}^{MD}x = {}^{MD}x^{Copy}$ 
```

#### Output

This routine is called at the successful completion of time step  $t$  (including  $t=0$ ) to write output data to a file.

Calculate the KiteFASTMBD write outputs and write them to the output file, together with the module-level write output data currently stored in MiscVars.

This is a list of all possible output parameters available within the KiteFASTMBD (not including the module-level outputs available from KiteAeroDyn, InflowWind, MoorDyn, and the Controller). The names are grouped by meaning, but can be ordered in the **OUTPUTS** section of the KiteMBDyn Preprocessor input file as you see fit.

Fus $\beta$  refers to output  $\beta$  on the fuselage, where  $\beta$  is a one-digit number in the range [1,9] corresponding to the finite-element node for motions or Gauss point for loads identified by entry  $\beta$  in the **FusOutNd** list. Setting  $\beta > NFusOuts$  yields invalid output.

SWn $\beta$  and PWn $\beta$  refer to output  $\beta$  on the starboard and port wings, respectively, where  $\beta$  is a one-digit number in the range [1,9] corresponding to the finite-element node for motions or Gauss point for loads identified by entry  $\beta$  in the **SWnOutNd** and **PWnOutNd** lists, respectively. Setting  $\beta > NSWnOuts$  and **NPWnOuts**, respectively, yields invalid output.

**Commented [JJ46]:** The new OUTPUT section of the KiteMBDyn Preprocessor input file should look something like this:

```
--- OUTPUT ---
True      SumPrint      Print summary data to
<RootName>.sum? (flag)
"ES10.3E2" OutFmt      Format used for text tabular
output, excluding the time channel; resulting field should be 10
characters (string)
4         NFusOuts      Number of fuselage outputs (-) [0 to 9]
2, 4, 6, 8 FusOutNd      List of fuselage nodes/points whose
values will be output (-) [1 to NFusOuts] [unused for NFusOuts=0]
4         NSWnOuts      Number of starboard wing outputs (-)
[0 to 9]
2, 4, 6, 8 SWnOutNd      List of starboard wing nodes/points
whose values will be output (-) [1 to NSWnOuts] [unused for
NSWnOuts=0]
4         NPWnOuts      Number of port wing outputs (-) [0 to
9]
2, 4, 6, 8 PWnOutNd      List of port wing nodes/points
whose values will be output (-) [1 to NPWnOuts] [unused for
NPWnOuts=0]
2         NVSOuts      Number of vertical stabilizer outputs (-
) [0 to 9]
2, 4         VSOOutNd      List of vertical stabilizer nodes/points
whose values will be output (-) [1 to NVSOuts] [unused for
NVSOuts=0]
1         NSHSOuts      Number of starboard horizontal
stabilizer outputs (-) [0 to 9]
2         SHSOOutNd      List of starboard horizontal stabilizer
nodes/points whose values will be output (-) [1 to NSHSOuts]
[unused for NSHSOuts=0]
1         NPHSOOuts      Number of port horizontal stabilizer
outputs (-) [0 to 9]
2         PHSOOutNd      List of port horizontal stabilizer
nodes/points whose values will be output (-) [1 to NPHSOOuts]
[unused for NPHSOOuts=0]
2         NPyIOuts      Number of pylon outputs (-) [0 to 9]
2, 4         PyIOOutNd      List of pylon nodes/points whose
values will be output (-) [1 to NPyIOuts] [unused for NPyIOuts=0]
OutList      The next line(s) contains a list of output
parameters. See OutListParameters.xlsx for a listing of available
output channels (quoted string)
END of input file (the word "END" must appear in the first 3
columns of this last OutList line)
```



VS $\beta$  refers to output  $\beta$  on the vertical stabilizer, where  $\beta$  is a one-digit number in the range [1,9] corresponding to the finite-element node for motions or Gauss point for loads identified by entry  $\beta$  in the *VSOutNd* list. Setting  $\beta > NVSOuts$  yields invalid output.

SHS $\beta$  and PHS $\beta$  refer to output  $\beta$  on the starboard and port horizontal stabilizers, respectively, where  $\beta$  is a one-digit number in the range [1,9] corresponding to the finite-element node for motions or Gauss point for loads identified by entry  $\beta$  in the *SHSOutNd* and *PHSOutNd* lists, respectively. Setting  $\beta > NSHSOuts$  and *NPHSOuts*, respectively, yields invalid output.

SP $\alpha$  and PP $\alpha$  refer to pylon  $\alpha$  on the starboard and port wings, respectively, where  $\alpha$  is a one-digit number in the range [1,9]. SP $\alpha\beta$  and PP $\alpha\beta$  refer to output  $\beta$  on pylon  $\alpha$  on the starboard and port wings, respectively, where  $\alpha$  is a one-digit number in the range [1,9] and  $\beta$  is a one-digit number in the range [1,9] corresponding to the finite-element node for motions or Gauss point for loads identified by entry  $\beta$  in the *PylOutNd* list. Setting  $\alpha > NumPylons$  or setting  $\beta > NPylOuts$  yields invalid output. If *NumPylons* > 9, only the first 9 pylons can be output.

For the fuselage, wings, vertical stabilizer, horizontal stabilizers, and pylons, the local structural coordinate system is used for output, where  $n$  is normal to the chord pointed toward the suction surface,  $c$  is along the chord pointed toward the trailing edge, and the spanwise ( $s$ ) axis is directed into the airfoil following the right-hand rule i.e.  $s = n \times c$ .

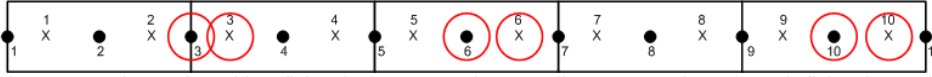


Figure: Example member with 5 finite elements, 11 nodes ( $\bullet$ ), and 10 Gauss points (x) (each finite element in MBDyn has 2 end nodes, 1 middle node, and 2 Gauss points). The red circles identify the finite-element nodes where motions are output and Gauss points where loads are output when *NOuts* = 3 and *OutNd* = 3, 6, 10.

Channel Name(s)	Unit(s)	Description
<i>Fuselage</i>		
Fus $\beta$ TDx, Fus $\beta$ TDy, Fus $\beta$ TDz, Fus $\beta$ RDx, Fus $\beta$ RDy, Fus $\beta$ RDz	(m), (m), (m), (deg), (deg), (deg)	Translational and rotational (angular) deflections at Fus $\beta$ relative to the undeflected rigid-body position/orientation in the kite coordinate system; the rotations are output as Euler angles in a x-y'-z'' (roll-pitch-yaw) rotation sequence
Fus $\beta$ RVn, Fus $\beta$ RVc, Fus $\beta$ RVs	(deg/s), (deg/s), (deg/s)	Absolute rotational (angular) velocity at Fus $\beta$ expressed in the local structural coordinate system
Fus $\beta$ TAn, Fus $\beta$ TAc, Fus $\beta$ TAs	(m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> )	Absolute translational acceleration at Fus $\beta$ expressed in the local structural coordinate system (does not include gravity)
Fus $\beta$ FRn, Fus $\beta$ FRc, Fus $\beta$ FRs, Fus $\beta$ MRn, Fus $\beta$ MRC, Fus $\beta$ MRs	(N), (N), (N), (N·m), (N·m), (N·m)	Shear force and bending moment reaction loads at Fus $\beta$ expressed in the local structural coordinate system
<i>Starboard (Right) Wing</i>		
SWn $\beta$ TDx, SWn $\beta$ TDy, SWn $\beta$ TDz, SWn $\beta$ RDx, SWn $\beta$ RDy, SWn $\beta$ RDz	(m), (m), (m), (deg), (deg), (deg)	Translational and rotational (angular) deflections at SWn $\beta$ relative to the undeflected rigid-body position/orientation in the kite coordinate system; the rotations are output as Euler angles in a x-y'-z'' (roll-pitch-yaw) rotation sequence
SWn $\beta$ RVn, SWn $\beta$ RVc, SWn $\beta$ RVs	(deg/s), (deg/s), (deg/s)	Absolute rotational (angular) velocity at SWn $\beta$ expressed in the local structural coordinate system

SWnβTAn, SWnβTAc, SWnβTAs	(m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> )	Absolute translational acceleration at SWnβ expressed in the local structural coordinate system (does not include gravity)
SWnβFRn, SWnβFRc, SWnβFRs, SWnβMRn, SWnβMRc, SWnβMRs	(N), (N), (N), (N·m), (N·m), (N·m)	Shear force and bending moment reaction loads at SWnβ expressed in the local structural coordinate system
<i>Port (Left) Wing</i>		
PWnβTDx, PWnβTDy, PWnβTDz, PWnβRDx, PWnβRDy, PWnβRDz	(m), (m), (m), (deg), (deg), (deg)	Translational and rotational (angular) deflections at PWnβ relative to the undeflected rigid-body position/orientation in the kite coordinate system; the rotations are output as Euler angles in a x-y'-z'' (roll-pitch-yaw) rotation sequence
PWnβRVn, PWnβRVc, PWnβRVs	(deg/s), (deg/s), (deg/s)	Absolute rotational (angular) velocity at PWnβ expressed in the local structural coordinate system
PWnβTAn, PWnβTAc, PWnβTAs	(m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> )	Absolute translational acceleration at PWnβ expressed in the local structural coordinate system (does not include gravity)
PWnβFRn, PWnβFRc, PWnβFRs, PWnβMRn, PWnβMRc, PWnβMRs	(N), (N), (N), (N·m), (N·m), (N·m)	Shear force and bending moment reaction loads at PWnβ expressed in the local structural coordinate system
<i>Vertical Stabilizer</i>		
VSβTDx, VSβTDy, VSβTDz, VSβRDx, VSβRDy, VSβRDz	(m), (m), (m), (deg), (deg), (deg)	Translational and rotational (angular) deflections at VSβ relative to the undeflected rigid-body position/orientation in the kite coordinate system; the rotations are output as Euler angles in a x-y'-z'' (roll-pitch-yaw) rotation sequence
VSβRVn, VSβRVc, VSβRVs	(deg/s), (deg/s), (deg/s)	Absolute rotational (angular) velocity at VSβ expressed in the local structural coordinate system
VSβTAn, VSβTAc, VSβTAs	(m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> )	Absolute translational acceleration at VSβ expressed in the local structural coordinate system (does not include gravity)
VSβFRn, VSβFRc, VSβFRs, VSβMRn, VSβMRc, VSβMRs	(N), (N), (N), (N·m), (N·m), (N·m)	Shear force and bending moment reaction loads at VSβ expressed in the local structural coordinate system
<i>Starboard (Right) Horizontal Stabilizer</i>		
SHSβTDx, SHSβTDy, SHSβTDz, SHSβRDx, SHSβRDy, SHSβRDz	(m), (m), (m), (deg), (deg), (deg)	Translational and rotational (angular) deflections at SHSβ relative to the undeflected rigid-body position/orientation in the kite coordinate system; the rotations are output as Euler angles in a x-y'-z'' (roll-pitch-yaw) rotation sequence
SHSβRVn, SHSβRVc, SHSβRVs	(deg/s), (deg/s), (deg/s)	Absolute rotational (angular) velocity at SHSβ expressed in the local structural coordinate system
SHSβTAn, SHSβTAc, SHSβTAs	(m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> )	Absolute translational acceleration at SHSβ expressed in the local structural coordinate system (does not include gravity)
SHSβFRn, SHSβFRc, SHSβFRs, SHSβMRn, SHSβMRc, SHSβMRs	(N), (N), (N), (N·m), (N·m), (N·m)	Shear force and bending moment reaction loads at SHSβ expressed in the local structural coordinate system
<i>Port (Left) Horizontal Stabilizer</i>		
PHSβTDx, PHSβTDy, PHSβTDz, PHSβRDx, PHSβRDy, PHSβRDz	(m), (m), (m), (deg), (deg), (deg)	Translational and rotational (angular) deflections at PHSβ relative to the undeflected rigid-body position/orientation in the kite coordinate system;

		the rotations are output as Euler angles in a x-y'-z'' (roll-pitch-yaw) rotation sequence
PHS $\beta$ RVn, PHS $\beta$ RVc, PHS $\beta$ RVs	(deg/s), (deg/s), (deg/s)	Absolute rotational (angular) velocity at PHS $\beta$ expressed in the local structural coordinate system
PHS $\beta$ TAn, PHS $\beta$ TAc, PHS $\beta$ TAs	(m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> )	Absolute translational acceleration at PHS $\beta$ expressed in the local structural coordinate system (does not include gravity)
PHS $\beta$ FRn, PHS $\beta$ FRc, PHS $\beta$ FRs, PHS $\beta$ MRn, PHS $\beta$ MRc, PHS $\beta$ MRs	(N), (N), (N), (N·m), (N·m), (N·m)	Shear force and bending moment reaction loads at PHS $\beta$ expressed in the local structural coordinate system
<i>Pylons</i>		
SP $\alpha\beta$ TDx, SP $\alpha\beta$ TDy, SP $\alpha\beta$ TDz, SP $\alpha\beta$ RDx, SP $\alpha\beta$ RDy, SP $\alpha\beta$ RDz, PP $\alpha\beta$ TDx, PP $\alpha\beta$ TDy, PP $\alpha\beta$ TDz, PP $\alpha\beta$ RDx, PP $\alpha\beta$ RDy, PP $\alpha\beta$ RDz	(m), (m), (m), (deg), (deg), (deg), (m), (m), (m), (deg), (deg), (deg)	Translational and rotational (angular) deflections at SP $\alpha\beta$ and PP $\alpha\beta$ relative to the undeflected rigid-body position/orientation in the kite coordinate system; the rotations are output as Euler angles in a x-y'-z'' (roll-pitch-yaw) rotation sequence
SP $\alpha\beta$ RVn, SP $\alpha\beta$ RVc, SP $\alpha\beta$ RVs, PP $\alpha\beta$ RVn, PP $\alpha\beta$ RVc, PP $\alpha\beta$ RVs	(deg/s), (deg/s), (deg/s), (deg/s), (deg/s), (deg/s)	Absolute rotational (angular) velocity at SP $\alpha\beta$ and PP $\alpha\beta$ expressed in the local structural coordinate system
SP $\alpha\beta$ TAn, SP $\alpha\beta$ TAc, SP $\alpha\beta$ TAs, PP $\alpha\beta$ TAn, PP $\alpha\beta$ TAc, PP $\alpha\beta$ TAs	(m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> )	Absolute translational acceleration at SP $\alpha\beta$ and PP $\alpha\beta$ expressed in the local structural coordinate system (does not include gravity)
SP $\alpha\beta$ FRn, SP $\alpha\beta$ FRc, SP $\alpha\beta$ FRs, SP $\alpha\beta$ MRn, SP $\alpha\beta$ MRc, SP $\alpha\beta$ MRs, PP $\alpha\beta$ FRn, PP $\alpha\beta$ FRc, PP $\alpha\beta$ FRs, PP $\alpha\beta$ MRn, PP $\alpha\beta$ MRc, PP $\alpha\beta$ MRs	(N), (N), (N), (N·m), (N·m), (N·m), (N), (N), (N), (N·m), (N·m), (N·m)	Shear force and bending moment reaction loads at SP $\alpha\beta$ and PP $\alpha\beta$ expressed in the local structural coordinate system
<i>Rotors</i>		
SP $\alpha$ TRtSpd, SP $\alpha$ BRtSpd, PP $\alpha$ TRtSpd, PP $\alpha$ BRtSpd	(rad/s), (rad/s), (rad/s), (rad/s)	Rotor speed of the top (T) and bottom (B) rotor on SP $\alpha$ and PP $\alpha$ (relative to the nacelle)
SP $\alpha$ TRtAcc, SP $\alpha$ BRtAcc, PP $\alpha$ TRtAcc, PP $\alpha$ BRtAcc	(rad/s <sup>2</sup> ), (rad/s <sup>2</sup> ), (rad/s <sup>2</sup> ), (rad/s <sup>2</sup> )	Rotor acceleration of the top (T) and bottom (B) rotor on SP $\alpha$ and PP $\alpha$ (relative to the nacelle)
<i>Energy Kite</i>		
KitePxi, KitePyi, KitePzi, KiteRoll, KitePitch, KiteYaw	(m), (m), (m), (deg), (deg), (deg)	Translational position and rotational (angular) orientation of the energy kite fuselage reference point in the global inertial-frame coordinate system; the rotations are output as Euler angles in a X-Y'-Z'' (roll-pitch-yaw) rotation sequence
KiteTVx, KiteTVy, KiteTVz, KiteRVx, KiteRVy, KiteRVz	(m/s), (m/s), (m/s), (deg/s), (deg/s), (deg/s)	Absolute translational and rotational (angular) velocity of the energy kite fuselage reference point expressed in the kite coordinate system
KiteTAX, KiteTAY, KiteTAZ, KiteRAX, KiteRAY, KiteRAZ	(m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (m/s <sup>2</sup> ), (deg/s <sup>2</sup> ), (deg/s <sup>2</sup> ), (deg/s <sup>2</sup> )	Absolute translational and rotational (angular) acceleration of the energy kite fuselage reference point expressed in the kite coordinate system

These are calculated within KiteFASTMBD as follows:

*Fuselage:*

$$\begin{aligned}
& \begin{Bmatrix} Fus\beta TDx \\ Fus\beta TDy \\ Fus\beta TDz \\ Fus\beta RDx \\ Fus\beta RDy \\ Fus\beta RDz \end{Bmatrix} = \left\{ \frac{180}{\pi} F^{EulerExtract} \left( \left[ \begin{matrix} MBD \Lambda^{FusO} \end{matrix} \right]^T \left[ \begin{matrix} MBD \Lambda^{FusR} \\ MBD \Lambda^{FusOutNd[\beta]} \end{matrix} \right]^T MBD \Lambda^{Fus} \right) \right\} \\
& \begin{Bmatrix} Fus\beta RVn \\ Fus\beta RVc \\ Fus\beta RVs \end{Bmatrix} = \frac{180}{\pi} MBD \Lambda^{Fus} MBD \vec{\omega}_{FusOutNd[\beta]}^{Fus} \\
& \begin{Bmatrix} Fus\beta TAn \\ Fus\beta TAc \\ Fus\beta TAs \end{Bmatrix} = MBD \Lambda^{Fus} MBD \vec{a}_{FusOutNd[\beta]}^{Fus} \\
& \begin{Bmatrix} Fus\beta FRn \\ Fus\beta FRc \\ Fus\beta FRs \\ Fus\beta MRn \\ Fus\beta MRc \\ Fus\beta MRs \end{Bmatrix} = \left\{ \begin{matrix} MBD \vec{R}_{FusOutNd[\beta]}^{Fus} \\ MBD \vec{M}_{FusOutNd[\beta]}^{Fus} \end{matrix} \right\}
\end{aligned}$$

Starboard (Right) Wing:

$$\begin{aligned}
& \begin{Bmatrix} SWn\beta TDx \\ SWn\beta TDy \\ SWn\beta TDz \\ SWn\beta RDx \\ SWn\beta RDy \\ SWn\beta RDz \end{Bmatrix} = \left\{ \frac{180}{\pi} F^{EulerExtract} \left( \left[ \begin{matrix} MBD \Lambda^{FusO} \end{matrix} \right]^T \left[ \begin{matrix} MBD \Lambda^{SWnR} \\ MBD \Lambda^{SWnOutNd[\beta]} \end{matrix} \right]^T MBD \Lambda^{SWn} \right) \right\} \\
& \begin{Bmatrix} SWn\beta RVn \\ SWn\beta RVc \\ SWn\beta RVs \end{Bmatrix} = \frac{180}{\pi} MBD \Lambda^{SWn} MBD \vec{\omega}_{SWnOutNd[\beta]}^{SWn} \\
& \begin{Bmatrix} SWn\beta TAn \\ SWn\beta TAc \\ SWn\beta TAs \end{Bmatrix} = MBD \Lambda^{SWn} MBD \vec{a}_{SWnOutNd[\beta]}^{SWn}
\end{aligned}$$

$$\begin{Bmatrix} SWn\beta FRn \\ SWn\beta FRc \\ SWn\beta FRs \\ SWn\beta MRn \\ SWn\beta MRc \\ SWn\beta MRs \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\bar{F}R_{SWnOutNd[\beta]}^{SWn} \\ {}^{MBD}\bar{M}R_{SWnOutNd[\beta]}^{SWn} \end{Bmatrix}$$

Port (Left) Wing:

$$\begin{Bmatrix} PWn\beta TDx \\ PWn\beta TDy \\ PWn\beta TDz \\ PWn\beta RDx \\ PWn\beta RDy \\ PWn\beta RDz \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\Lambda^{FusO} \left\{ {}^{MBD}\vec{p}_{PWnOutNd[\beta]}^{PWn} - {}^{MBD}\vec{p}^{FusO} \right\} - {}^{MBD}\vec{p}_{PWnOutNd[\beta]}^{PWnR} \\ \frac{180}{\pi} F^{EulerExtract} \left( \left[ {}^{MBD}\Lambda^{FusO} \right]^T \left[ {}^{MBD}\Lambda_{PWnOutNd[\beta]}^{PWnR} \right]^T {}^{MBD}\Lambda_{PWnOutNd[\beta]}^{PWn} \right) \end{Bmatrix}$$

$$\begin{Bmatrix} PWn\beta RVn \\ PWn\beta RVc \\ PWn\beta RVs \end{Bmatrix} = \frac{180}{\pi} {}^{MBD}\Lambda_{PWnOutNd[\beta]}^{PWn} {}^{MBD}\vec{\omega}_{PWnOutNd[\beta]}^{PWn}$$

$$\begin{Bmatrix} PWn\beta TAn \\ PWn\beta TAc \\ PWn\beta TAs \end{Bmatrix} = {}^{MBD}\Lambda_{PWnOutNd[\beta]}^{PWn} {}^{MBD}\vec{a}_{PWnOutNd[\beta]}^{PWn}$$

$$\begin{Bmatrix} PWn\beta FRn \\ PWn\beta FRc \\ PWn\beta FRs \\ PWn\beta MRn \\ PWn\beta MRc \\ PWn\beta MRs \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\bar{F}R_{PWnOutNd[\beta]}^{PWn} \\ {}^{MBD}\bar{M}R_{PWnOutNd[\beta]}^{PWn} \end{Bmatrix}$$

Vertical Stabilizer:

$$\begin{Bmatrix} VS\beta TDx \\ VS\beta TDy \\ VS\beta TDz \\ VS\beta RDx \\ VS\beta RDy \\ VS\beta RDz \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\Lambda^{FusO} \left\{ {}^{MBD}\vec{p}_{VSOutNd[\beta]}^{VS} - {}^{MBD}\vec{p}^{FusO} \right\} - {}^{MBD}\vec{p}_{VSOutNd[\beta]}^{VSR} \\ \frac{180}{\pi} F^{EulerExtract} \left( \left[ {}^{MBD}\Lambda^{FusO} \right]^T \left[ {}^{MBD}\Lambda_{VSOutNd[\beta]}^{VSR} \right]^T {}^{MBD}\Lambda_{VSOutNd[\beta]}^{VS} \right) \end{Bmatrix}$$

$$\begin{Bmatrix} VS\beta RVn \\ VS\beta RVc \\ VS\beta RVs \end{Bmatrix} = \frac{180}{\pi} {}^{MBD}\Lambda_{VSOutNd[\beta]}^{VS} {}^{MBD}\vec{\omega}_{VSOutNd[\beta]}^{VS}$$

$$\begin{Bmatrix} VS\beta TAn \\ VS\beta TAc \\ VS\beta TAs \end{Bmatrix} = {}^{MBD}\Lambda_{VSOutNd[\beta]}^{VS} {}^{MBD}\vec{a}_{VSOutNd[\beta]}^{VS}$$

$$\begin{Bmatrix} VS\beta FRn \\ VS\beta FRc \\ VS\beta FRs \\ VS\beta MRn \\ VS\beta MRc \\ VS\beta MRs \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\vec{F}R_{VSOutNd[\beta]}^{VS} \\ {}^{MBD}\vec{M}R_{VSOutNd[\beta]}^{VS} \end{Bmatrix}$$

Starboard (Right) Horizontal Stabilizer:

$$\begin{Bmatrix} SHS\beta TDx \\ SHS\beta TDy \\ SHS\beta TDz \\ SHS\beta RDx \\ SHS\beta RDy \\ SHS\beta RDz \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\Lambda^{FusO} \left\{ {}^{MBD}\vec{p}_{SHSOutNd[\beta]}^{SHS} - {}^{MBD}\vec{p}^{FusO} \right\} - {}^{MBD}\vec{p}_{SHSOutNd[\beta]}^{SHSR} \\ \frac{180}{\pi} F^{EulerExtract} \left( \left[ {}^{MBD}\Lambda^{FusO} \right]^T \left[ {}^{MBD}\Lambda_{SHSOutNd[\beta]}^{SHSR} \right]^T {}^{MBD}\Lambda_{SHSOutNd[\beta]}^{SHS} \right) \end{Bmatrix}$$

$$\begin{Bmatrix} SHS\beta RVn \\ SHS\beta RVc \\ SHS\beta RVs \end{Bmatrix} = \frac{180}{\pi} {}^{MBD}\Lambda_{SHSOutNd[\beta]}^{SHS} {}^{MBD}\vec{\omega}_{SHSOutNd[\beta]}^{SHS}$$

$$\begin{Bmatrix} SHS\beta TAn \\ SHS\beta TAc \\ SHS\beta TAs \end{Bmatrix} = {}^{MBD}\Lambda_{SHSOutNd[\beta]}^{SHS} {}^{MBD}\vec{a}_{SHSOutNd[\beta]}^{SHS}$$

$$\begin{Bmatrix} SHS\beta FRn \\ SHS\beta FRc \\ SHS\beta FRs \\ SHS\beta MRn \\ SHS\beta MRc \\ SHS\beta MRs \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\vec{F}R_{SHSOutNd[\beta]}^{SHS} \\ {}^{MBD}\vec{M}R_{SHSOutNd[\beta]}^{SHS} \end{Bmatrix}$$

Port (Left) Horizontal Stabilizer:

$$\begin{Bmatrix} PHS\beta TDx \\ PHS\beta TDy \\ PHS\beta TDz \\ PHS\beta RDx \\ PHS\beta RDy \\ PHS\beta RDz \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\Lambda^{FusO} \left\{ {}^{MBD}\vec{p}_{PHSOutNd[\beta]}^{PHS} - {}^{MBD}\vec{p}^{FusO} \right\} - {}^{MBD}\vec{p}_{PHSOutNd[\beta]}^{PHSR} \\ \frac{180}{\pi} F^{EulerExtract} \left( \left[ {}^{MBD}\Lambda^{FusO} \right]^T \left[ {}^{MBD}\Lambda_{PHSOutNd[\beta]}^{PHSR} \right]^T {}^{MBD}\Lambda_{PHSOutNd[\beta]}^{PHS} \right) \end{Bmatrix}$$

$$\begin{Bmatrix} PHS\beta RVn \\ PHS\beta RVc \\ PHS\beta RVs \end{Bmatrix} = \frac{180}{\pi} {}^{MBD}\Lambda_{PHSOutNd[\beta]}^{PHS} {}^{MBD}\vec{\omega}_{PHSOutNd[\beta]}^{PHS}$$

$$\begin{Bmatrix} PHS\beta TAn \\ PHS\beta TAc \\ PHS\beta TAs \end{Bmatrix} = {}^{MBD}\Lambda_{PHSOutNd[\beta]}^{PHS} {}^{MBD}\vec{a}_{PHSOutNd[\beta]}^{PHS}$$

$$\begin{Bmatrix} PHS\beta FRn \\ PHS\beta FRc \\ PHS\beta FRs \\ PHS\beta MRn \\ PHS\beta MRc \\ PHS\beta MRs \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\vec{F}R_{PHSOutNd[\beta]}^{PHS} \\ {}^{MBD}\vec{M}R_{PHSOutNd[\beta]}^{PHS} \end{Bmatrix}$$

Pylons:

$$\begin{Bmatrix} SP\alpha\beta TDx \\ SP\alpha\beta TDy \\ SP\alpha\beta TDz \\ SP\alpha\beta RDx \\ SP\alpha\beta RDy \\ SP\alpha\beta RDz \\ PP\alpha\beta TDx \\ PP\alpha\beta TDy \\ PP\alpha\beta TDz \\ PP\alpha\beta RDx \\ PP\alpha\beta RDy \\ PP\alpha\beta RDz \\ SP\alpha\beta RVn \\ SP\alpha\beta RVc \\ SP\alpha\beta RVs \\ PP\alpha\beta RVn \\ PP\alpha\beta RVc \\ PP\alpha\beta RVs \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\Lambda^{FusO} \left\{ {}^{MBD}\vec{p}_{PylOutNd[\beta]}^{SPy} [\alpha] - {}^{MBD}\vec{p}^{FusO} \right\} - {}^{MBD}\vec{p}_{PylOutNd[\beta]}^{SPyR} [\alpha] \\ \frac{180}{\pi} F^{EulerExtract} \left( \left[ {}^{MBD}\Lambda^{FusO} \right]^T \left[ {}^{MBD}\Lambda_{PylOutNd[\beta]}^{SPyR} [\alpha] \right]^T {}^{MBD}\Lambda_{PylOutNd[\beta]}^{SPy} [\alpha] \right) \\ {}^{MBD}\Lambda^{FusO} \left\{ {}^{MBD}\vec{p}_{PylOutNd[\beta]}^{PPy} [\alpha] - {}^{MBD}\vec{p}^{FusO} \right\} - {}^{MBD}\vec{p}_{PylOutNd[\beta]}^{PPyR} [\alpha] \\ \frac{180}{\pi} F^{EulerExtract} \left( \left[ {}^{MBD}\Lambda^{FusO} \right]^T \left[ {}^{MBD}\Lambda_{PylOutNd[\beta]}^{PPyR} [\alpha] \right]^T {}^{MBD}\Lambda_{PylOutNd[\beta]}^{PPy} [\alpha] \right) \\ {}^{MBD}\Lambda_{PylOutNd[\beta]}^{SPy} [\alpha] \\ {}^{MBD}\Lambda_{PylOutNd[\beta]}^{PPy} [\alpha] \end{Bmatrix}$$

$$\begin{Bmatrix} SP\alpha\beta TAn \\ SP\alpha\beta TAc \\ SP\alpha\beta TAs \\ PP\alpha\beta TAn \\ PP\alpha\beta TAc \\ PP\alpha\beta TAs \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}A_{PyiOutNd}^{SPy}[\beta][\alpha] & {}^{MBD}\vec{a}_{PyiOutNd}^{SPy}[\beta][\alpha] \\ {}^{MBD}A_{PyiOutNd}^{PPy}[\beta][\alpha] & {}^{MBD}\vec{a}_{PyiOutNd}^{PPy}[\beta][\alpha] \end{Bmatrix}$$

$$\begin{Bmatrix} SP\alpha\beta FRn \\ SP\alpha\beta FRc \\ SP\alpha\beta FRs \\ SP\alpha\beta MRn \\ SP\alpha\beta MRc \\ SP\alpha\beta MRs \\ PP\alpha\beta FRn \\ PP\alpha\beta FRc \\ PP\alpha\beta FRs \\ PP\alpha\beta MRn \\ PP\alpha\beta MRc \\ PP\alpha\beta MRs \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\vec{F}R_{PyiOutNd}^{SPy}[\alpha] \\ {}^{MBD}\vec{M}R_{PyiOutNd}^{SPy}[\alpha] \\ {}^{MBD}\vec{F}R_{PyiOutNd}^{PPy}[\alpha] \\ {}^{MBD}\vec{M}R_{PyiOutNd}^{PPy}[\alpha] \end{Bmatrix}$$

*Rotors*

$$\begin{Bmatrix} SP\alpha TRtSpd \\ SP\alpha BRtSpd \\ PP\alpha TRtSpd \\ PP\alpha BRtSpd \end{Bmatrix} = \begin{Bmatrix} {}^{Ctrl}\Omega^{SPyRtr}[\alpha, 1] \\ {}^{Ctrl}\Omega^{SPyRtr}[\alpha, 2] \\ {}^{Ctrl}\Omega^{PPyRtr}[\alpha, 1] \\ {}^{Ctrl}\Omega^{PPyRtr}[\alpha, 2] \end{Bmatrix}$$

$$\begin{Bmatrix} SP\alpha TRtAcc \\ SP\alpha BRtAcc \\ PP\alpha TRtAcc \\ PP\alpha BRtAcc \end{Bmatrix} = \begin{Bmatrix} {}^{Ctrl}\alpha^{SPyRtr}[\alpha, 1] \\ {}^{Ctrl}\alpha^{SPyRtr}[\alpha, 2] \\ {}^{Ctrl}\alpha^{PPyRtr}[\alpha, 1] \\ {}^{Ctrl}\alpha^{PPyRtr}[\alpha, 2] \end{Bmatrix}$$

*Energy Kite*

$$\begin{Bmatrix} KitePxi \\ KitePyi \\ KitePzi \\ KiteRoll \\ KitePitch \\ KiteYaw \end{Bmatrix} = \begin{Bmatrix} {}^{MBD}\vec{p}^{FusO} \\ \frac{180}{\pi} F^{EulerExtract} \left( {}^{MBD}A^{FusO} \right) \end{Bmatrix}$$



$$\begin{Bmatrix} KiteTV_x \\ KiteTV_y \\ KiteTV_z \\ KiteRV_x \\ KiteRV_y \\ KiteRV_z \end{Bmatrix} = \left\{ \begin{array}{c} {}^{MBD}\Lambda^{FusOMB D} \vec{\mathbf{v}}^{FusO} \\ \frac{180}{\pi} {}^{MBD}\Lambda^{FusOMB D} \vec{\omega}^{FusO} \end{array} \right\}$$

$$\begin{Bmatrix} KiteTA_x \\ KiteTA_y \\ KiteTA_z \\ KiteRA_x \\ KiteRA_y \\ KiteRA_z \end{Bmatrix} = \left\{ \begin{array}{c} {}^{MBD}\Lambda^{FusOMB D} \vec{\mathbf{a}}^{FusO} \\ \frac{180}{\pi} {}^{MBD}\Lambda^{FusOMB D} \vec{\alpha}^{FusO} \end{array} \right\}$$