

Bezier Neural Networks

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1 Bezier Curves: Recursive Construction

define the first and last row of this.

We will first define and inspect the core object of study in our paper, the Bezier Curve. To my mind, the Bezier Curve is a function mapping from $R^{m \times n} \rightarrow R^{m \times t}$ parameterized by a real number $t \in [0, 1]$. In effect, we're mapping an m by n matrix $A^{m \times n}$ into a function of a single parameter t . One might write this relation in the function B as

$$B^{n-1}(A) = b(t|A) \quad (1)$$

where B is an operator on a matrix and b is a function $R^{m \times n} \rightarrow R^1 \rightarrow R^m$. With this abstraction, let us now define explicitly. We define an operator $B|R^{m \times m \times t} \rightarrow R^{m \times n-1 \times t}$ on a matrix $A^{m \times n}$ as

$$B(A^1) = A^2(t) \quad (2)$$

Particularly, this operation is a convolution of the column space of A_1 and comes in the form of

$$B(A^1)_i = A_i^1 + t(A_{:,i-1}^1 - A_{:,i}^1) \quad (3a)$$

$$B(A^1)_i = (1-t)A_{:,i}^1 + A_{:,i-1}^1 \quad (3b)$$

As you can see, the resulting matrix upon one operation results in a matrix $A^{n \times m-1|t}$. Suppose we apply this operation l times upon that same matrix where $l < n$. We would then arrive at the matrix

$$B^l(A)_{:,i} = (1-t)B^{l-1}(A)_{:,i} + B^{l-1}(A)_{:,i-1} \quad (4)$$

if $l = n$, we arrive at a vector valued function which is named a bezier curve.

$$b(t|A) = B^n(A)_{:,i} = (1-t)B^{n-1}(A)_{:,i} + B^{n-1}(A)_{:,i-1} \quad (5)$$

We will soon come to see that a treatment of such a curve will enable us to construct a myriad or family of Neural Networks given a set of control points or as given above, a matrix A

2 On the Resolution of Convolution Kernal Parameters

3 On the Synthesis of Bezier and Kernal Parameters