Bezier Neural Networks

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1 Bezier Curves: Recursive Construction

We will first define and inspect the core object of study in our paper, the Bezier Curve. To my mind, the Bezier Curve is a function mapping from $R^{mxn} \rightarrow R^{mxt}$ parameterized be a real number $t \in [0, 1]$. In effect, we're mapping an m by n matrix A^{mxn} into a function of a single parameter t. One might write this relation in the function B as

$$B^{n-1}(A) = b(t) \tag{1}$$

where B is an opperator on a matrix and b is a function $R^{mxn} \rightarrow R^1 \rightarrow R^m$. With this abstraction, let us now define explicitly. We define an opperator $B|R^{mxmxt} \rightarrow R^{mxn-1xt}$ on a matrix A^{mxn} as

$$B(A^1) = A^2(t) \tag{2}$$

Particularly, this opperation is a convolution of the column space of A_1 and comes in the form of

$$B(A^1)_i = A^1_i + t(A^1_{:,i-1} - A^1_{:,i}) \tag{3a}$$

$$B(A^1)_i = (1-t)A^1_{:,i} + A^1_{:,i-1} \tag{3b}$$

As you can see, the resulting matrix upon one operation results in a matrix $A^{nxm-1|t}$. Suppose we apply this operation l times upon that same matrix where $l < n$. We would then arrive at the matrix

$$B^l(A)_{:,i} = (1-t)B^{l-1}(A)_{:,i} \tag{4}$$

2 On the Resolution of Convolution Kernal Parameters

3 On the Synthesis of Bezier and Kernal Parameters