Introduction

We will start by building a Lagrangian Formulation for Electrodynamics, i.e. a field theory. We will not study the Hamiltonian formulation that is more complicated.

The **Special Relativity** theory, is composed of two concepts:

- the **Relativity Principle**: states the existence of *Inertial Reference Systems* (I.R.S.)
- the existence of the **speed of light** that is the same in every S.D.R.

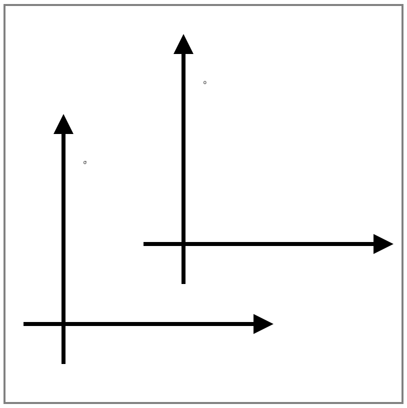


Figure 1: a

The Relativity Principle states the existence of a class of S.D.R.s in which the laws of physics are the same (i.e. the laws af physics are written in a covariant formulation) that are connected by a linear transformation that in the form: “translation + const. speed offset”

The *Galilean Relativity* requires only the Relativity Principle but not the light speed invariance.

Reference system

A **Reference System** is a map that associates eache point of the space-time $\mathcal{M}$ a point in $\mathbb{R}^4$: $\mathcal{M} \longrightarrow \mathbb{R}^4$ Elements of $\mathcal{M}$ are said **Event**s.

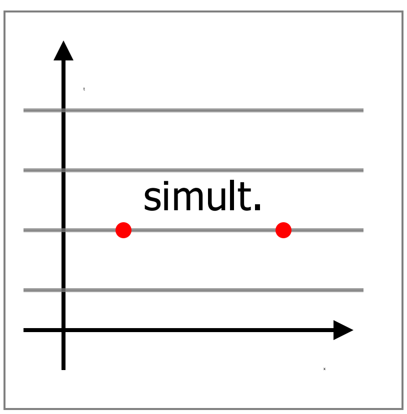


Figure 1: simultaneous events in Galilean Relativity

In Galilean Relativity, it exists a *universal* time, points on a equi-time ??? line are said to be *simultaneous* and events can only be transformed in another point on the same line.

Galilean Transforms are in the form: $\begin{cases} \vec{x'} = \vec{x} + \vec{w}t \;\; \text{(boost)}\\ t' = t \end{cases}$ and $\vec{v'} = \vec{v} - \vec{W}$

We want to find theories that are invariant under these transformations and the Lagrangian formulation is, by construction, galilean covariant because the Lagrangian is a scalar that is transformed in another one that differs only by a total time derivative. $\mathcal{L}$

= \sum_i \frac{1}{2} m_i \dot{\vec{x_i}}^2 - \sum_{i,j} U(|x_i - x_j|) This si not a proper Galilean invariant (it is so under rotations) because: \mathcal{L} \longrightarrow \mathcal{L} + \sum_i \frac{1}{2} m_i ((\dot{\vec{x_i}} - \vec{w})^2 - \dot{\vec{x_i}}^2) that is a constant and therefore, the E.L. equations are the same.

The Newtonian Gravity is a “good” theory as the potential depends only on the distance |x_i - x_j|. Also the potential does not depend on time and this is in accordance to the existence of the concept of simultaneity.

Intro to Special Relativity

The invariance of the speed of light suggested by Electrodynamics breaks the previous concepts: the maxwell equations: \begin{cases} TODO \end{cases} show the c constant that is a characteristic speed!

The oly possible solution is to change the galilean transforms eq. into something more general so that *light intervals* (c|\Delta t| = |\Delta x|) are conserved.

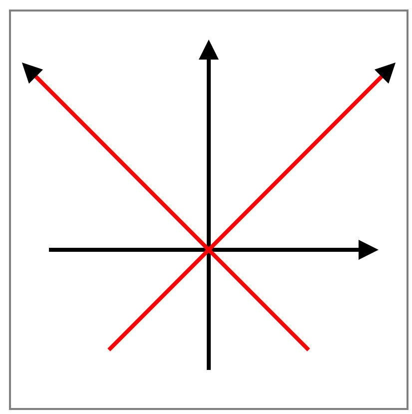
We introduce the map: \begin{split} \mathcal{M} &\longleftrightarrow \begin{pmatrix} x^0 = c t \\ \vec{x} \end{pmatrix} \equiv x^\mu \\ \text{event} &\longleftrightarrow \text{quadrivector} \end{split} x_\mu = (ct,\vec{x})

We introduce the **metric** (Minkowsky metric): \eta_{\mu\nu} = \begin{pmatrix} 1\\ &-1\\ &&-1\\ &&&-1\\ \end{pmatrix} (ct)^2 - \vec{x}^2 = x_\mu \eta_{\mu\nu} x^\nu And we require that \Delta x_\mu \eta_{\mu\nu} \Delta x^\nu = \Delta x'_\mu \eta_{\mu\nu} \Delta x'^\nu We also require:

- linearity
- symmetry
- metric Minkowsky metric (\Delta x_\mu \eta_{\mu\nu} \Delta x^\nu) conservation.

The transform matrix is 4x4 and has 9 degrees of freedom (???): 6 for rotations and 3 for boosts.
We will restrain? on the “direction 1” boost, in the form: \begin{cases} \begin{pmatrix} x'^0 \\ x'^1 \end{pmatrix} = \begin{pmatrix}?&?\\?&?\end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix} \\ x'^2 = x^2\\ x'^3 = x^3 \end{cases}

If we consider a ray starting from the origin: {x'^0}^2 - {x'^1}^2 = {x^0}^2 - {x^1}^2 We will find the Lorentz t… To do that, we



do a change of coordinates: \begin{cases} x^+ = x^0 + x^1\\ x^- = x^0 - x^1 \end{cases}

The eq. becomes: X^+X^- = {X^+}' {X^-}'

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