LNMIIT
The LNM Institute of Information Technology

The LNM Institute of Information Technology, Jaipur

Department of Electronics and Communication Engineering

Engineering Electromagnetics (ECE0332)

End Term Examination

Academic Year: 2017-18 **End Term-2017**

Degree: *B. Tech.* **Program: ECE/CCE**

Time: 180 minutes **Date:04/12/2017** **Maximum Marks: 50**

Instructions: THIS QUESTION PAPER CONTAINS TWO PAGES. EACH OF THE TEN (10) QUESTIONS CARRIES 5 MARKS. ALL QUESTIONS ARE COMPULSORY.
You can bring in FOUR A4 size formula-sheets with you to the Examination Hall. These sheets should be used to write (in your own hand-writing) difficult-to-memorize equations, formulas, vector relationships, values of natural constants, etc.

1. A 50-Ω, 12-cm long lossless transmission line is operating at a frequency of 10 MHz. The line inductance is known to be L= 2.5 mH/m. If the line is terminated with load impedance Z_L= 20 – j30 Ω, how much VSWR will be observed on the line? Also calculate the input impedance of the line.
2. Consider a two-port network with the following ABD parameters: A = 3, B = -j3 ohm, D = 1/3. The network is known to be reciprocal. What is the numerical value of the parameter C of the network? If two such networks are connected in cascade, calculate the scattering matrix of the overall combination. Assume port impedance (for all ports) to be 50 Ω.
3. A 100*MHz* generator with $V_g=10\angle 0°$ (V) and internal resistance 50Ω is connected to a lossless 50Ω air line that is 3.6*m* long and terminated in a 25+*j*25(Ω) load. Find (a) $V(z)$ at a location *z* from the generator, (b) V_i at the input terminals and V_L at the load, (c) the voltage standing-wave radio on the line, and (d) the average power delivered to the load.
4. A). A plane-wave propagating in a lossless dielectric medium has electric field strength given as E_x=10cos(1.51x10^{10}t – 61.6z) mV/m, E_y=0, E_z=0, where t is the time measured in seconds. In which direction is the wave traveling? Also calculate the dielectric constant of the medium, the wavelength, the phase-velocity, and the wave-impedance for this wave.
   B). Consider a discharging capacitor for which the charge is varying with time t according to the following relationship: q (t) = 10 exp (-t) milli-coulombs where t is measured in seconds from the instant the discharge started. What will be value of the displacement current density produced between the plates 1 second after the discharge began? Assume that the medium between plates is air and that the plates are circular in shape with diameter equal to 10 cm.
5. Starting with the four Maxwell's equations, rigorously demonstrate that the intrinsic impedance of a good conductor has phase angle approximately equal to 45 degrees.
6. For a rectangular waveguide operating in its dominant TE_{10} mode, rigorously derive the mathematical expressions for the various field components. Assume that the material inside the waveguide is a lossless dielectric. Make appropriate assumptions if necessary, and clearly state the assumptions made.
7. An X-band air-filled rectangular waveguide (a=22.86mm,b=10.16mm) is operating in its dominant TE_{10} mode at a frequency of 10 GHz. The energy is flowing along the length of the

waveguide (i.e., in z-direction). The z-component of the magnetic field-strength vector is given by H_z=94.00cos (πx/a)exp[j(0.5π-k_zz)] mA/m. Calculate the power being transmitted in the waveguide. Neglect the effects of waveguide losses and the effects of higher-order modes

8. For an air-filled circular waveguide with diameter equal to 10 cm, calculate the cutoff frequencies of the first three TE modes and the first three TM modes. Neatly tabulate your results. Identify 'degenerate' modes if any present. Use Table 8.1 (for zeroes of the derivatives of Bessel functions) and Table 8.2 (for zeroes of Bessel functions).

n	p'_{n1}	p'_{n2}	p'_{n3}	p'_{n4}
0	3.8317	7.0156	10.1735	13.3237
1	1.8412	5.3314	8.5363	11.7060
2	3.0542	6.7061	9.9695	
3	4.2012	8.0152	11.3459	

Table 8.1 Zeroes p'_{nm} of J_n' (x)

n		p_{n1}	p_{n2}	p_{n3}	p_{n4}
0		2.4048	5.5200	8.6537	11.7951
1		3.8317	7.0155	10.1743	
2		5.1356	8.4172	11.6198	
3		6.3801	9.7610		

Table 8.2 Zeroes p_{nm} of J_n (x)

9. A). For the circular waveguide in Q 8, calculate the phase velocity, the group velocity, the wave impedance, and the guided wavelength at an operating frequency 1.25 times the cutoff frequency of the dominant TE mode. Assume that no other modes are propagating.

B). A microstrip line is to be deposited on a 10-mil thick dielectric substrate material whose dielectric constant is equal to 2.22. If the strip width is chosen to be 2 mil, what would be expected value of the characteristic impedance of the line?

10. A). Design an air-filled coaxial line with characteristic impedance of 50 Ω and operating bandwidth of DC to 5.8 GHz. Ignore safety-margin-related concerns regarding the bandwidth of the line. (NOTE: To design means calculating the diameters of the two conductors.)

B). An air-filled coaxial line has the outer conductor diameter = 2.30 cm and the inner conductor diameter = 1.00 cm. The line is L cm long where L is to be determined without using mechanical means. One end of the line (called 'load end') is terminated with a load whose impedance is 50 + j50 Ω at 10 MHz frequency. The other end (called 'input end') is connected to a Vector Network Analyzer (VNA) so that the complex value of the input voltage reflection coefficient can be measured. The port impedances for VNA ports are all equal to 50 Ω. VNA measurements, carried out at 10 MHz frequency, show that the magnitude of the input voltage reflection coefficient is approximately equal to 0.4472 whereas the phase-angle of the same is approximately equal to 26.57 degrees. Determine the numerical value of L.

$Z_L = (50 + j25)\,\Omega$ 184 0.2425

Fundamental quantities

- EF ($\vec{E} \to$ V/m), • MF (H → A/m)
- Flux density ($\vec{D} \to C/m^2$, $\vec{B} \to wb/m^2$)
- ρ (charge density) → C/m^3, C/m^2, C/m
- J (current density) - Amp/m^3, A/m

Maxwell's eqn

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ M1 $\vec{\nabla} \cdot \vec{D} = \rho$ M3

$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$ M2 $\vec{\nabla} \cdot \vec{B} = 0$ M4

$\boxed{EMF = -\frac{\partial \phi}{\partial t}}$ & $\phi = \iint_s \vec{B} \cdot d\vec{s}$

$EMF = \oint_c \vec{E} \cdot d\vec{l}$ Faraday Lenz law

M1 WOP = C, elemental length = dl

$EMF = -\frac{\partial \phi}{\partial t}$, $\vec{E}$ const on dl

$EMF = \boxed{\oint_c \vec{E} \cdot d\vec{l} = -\frac{\partial \phi}{\partial t}}$

$\phi = \iint_s \vec{B} \cdot d\vec{s}$

$\oint_c \vec{E} \cdot dl = -\frac{\partial}{\partial t} \iint \vec{B} \cdot d\vec{s}$

Stokes theorem

$\oint \vec{A} \cdot d\vec{l} = \iint_s (\vec{\nabla} \times \vec{A}) \cdot d\vec{s}$

$\iint_s (\vec{\nabla} \times \vec{E}) d\vec{l} = -\frac{\partial}{\partial t} \iint_s \vec{B} \cdot ds$

$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}}$ (M1)

M2 Ampere law = $\oint_c \vec{H} \cdot d\vec{l} = I$ — (1)

$I = \iint_s \vec{J} \cdot d\vec{s}$ (2)

1,2 $\oint \vec{H} \cdot d\vec{l} = \iint_s \vec{J} \cdot d\vec{s}$ — (3)

ST:

$\iint_s (\vec{\nabla} \times \vec{H}) \cdot d\vec{s} = \iint_s \vec{J} \cdot d\vec{s}$

$\Rightarrow \vec{\nabla} \times \vec{H} = \vec{J}$ Maxwell intuit add extra term

so, $\boxed{\vec{\nabla} \times \vec{H} = \frac{\partial D}{\partial t} + \vec{J}}$ conduction current

displacement density (M2)

M3 Gauss law $\iint_s \vec{D} \cdot d\vec{s} = Q$

$Q = \iiint_v \rho dv$

$\iint_s \vec{D} \cdot d\vec{s} = \iiint_v \rho dv$ — (1)

Divergence theorem:

$\iint_s \vec{A} \cdot d\vec{s} = \iiint_v (\vec{\nabla} \cdot \vec{A}) dv$ (2)

1,2 $\boxed{\vec{\nabla} \cdot \vec{D} = \rho}$ (M3)

M4 Take divergence of M1

$\nabla \cdot (\vec{\nabla} \times \vec{E}) = \nabla \cdot \left(-\frac{\partial B}{\partial t}\right)$

0

$\vec{\nabla} \cdot \frac{dB}{dt} = 0 \Rightarrow \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{B}) = 0$

$\vec{\nabla} \cdot \vec{B} = \text{const}$ without loss of generality

$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$

Take divergence of M2,

$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \left(\frac{\partial \vec{D}}{\partial t} + \vec{J}\right)$

$0 \therefore 0 = \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{D}) + (\vec{\nabla} \cdot \vec{J})$

$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$ using M3 $\vec{\nabla} \cdot \vec{D} = \rho$

Mathematical representation of Eqn of continuity — cons of charge

Integral form of Maxwell's

$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \iint_s \vec{B} \cdot d\vec{s}$ — 1

$\oint \vec{H} \cdot d\vec{l} = \frac{\partial}{\partial t} \iint_s \vec{D} \cdot d\vec{s} + I$ — 2

$\iint_s \vec{D} \cdot d\vec{s} = \iiint_v \rho dv$ — 3

$\iint_s \vec{B} \cdot d\vec{s} = 0$ — 4

Constitutive relationships

Non conducting materials,

$\boxed{\vec{B} = \mu \vec{H}, \ \vec{D} = \varepsilon \vec{E}}$ Dielectric

Magnetic

$\mu_0 = 4\pi \times 10^{-7}$ Henry/m

$\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ Farad/m

Lossless dielectric materials $\varepsilon_r \approx 1$

Lossy dielectric :-

$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$

$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'}$ (loss)

$\varepsilon_r = \varepsilon_r'(1 - j\tan\delta)$

$D = \varepsilon_0 E + P_e$; $\varepsilon_0 \chi_e E$; $D = \varepsilon_0(1+\chi_e)E$

Magnetic material (lossy)

$\mu_r = \mu_r' - j\mu_r''$

$\mu_r = \mu_r'(1 - j\tan\delta_m)$

For conductors,

$\boxed{\vec{J} = \sigma \vec{E}}$ conductivity of material

$\sigma = \frac{A/m^2}{V/m} = \frac{A}{V m} = \frac{1}{\Omega m}$

$\hat{i}_n$ = normal to interface

E_{t_1}, E_{t_2} = tangential comp of EF

H_{t_1}, H_{t_2} = tangential MF

J = current density

D_{n_1}, D_{n_2} = Normal comp of E flux

H_{n_1}, H_{n_2} = Normal comp of M flux

ρ = charge density

Boundary condns

E_{t_1} Med 2, E_{t_2} Δl, h, Med 1

$\oint_c \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \iint_s \vec{B} \cdot ds$ (M1)

$(E_{t_1} - E_{t_2})\Delta l$ + contr due to 2 sides

$= -\frac{\partial}{\partial t}[B \cdot \Delta l \cdot h]$

Allow $h \to 0$,

BC1 $\boxed{E_{t_1} = E_{t_2}}$

$\oint H \cdot d\vec{l} = (H_{t_1}\Delta l - H_{t_2}\Delta l)$ + contribution 2 sides

$= \frac{\partial}{\partial t}(D \cdot \Delta l \cdot h) + I$

Allow $h \to 0 \Rightarrow$

$(H_{t_2} - H_{t_1})\Delta l = I$

$\boxed{H_{t_2} - H_{t_1} = J_s}$ (BC2)

Now same interface, 2 diff med

M3 $\iint \vec{D} \cdot d\vec{s} = \iiint \rho dv$

$\boxed{(-D_{n_1} + D_{n_2})\Delta S = \rho_s \Delta S}$ (BC3)

(M4) $\iint_s \vec{B} \cdot d\vec{s} = 0$

$\boxed{B_{n_2} = B_{n_1}}$ BC4

Cases

- 2 dielectric material (lossless)

$\sigma = 0$ (both) $E_{t_1} = E_{t_2}$

Med 1 | Med 2 $H_{t_1} = H_{t_2}$

$\mu_0, 0, \varepsilon_1$ | $\mu_0, 0, \varepsilon_2$ $B_{N_1} = B_{N_2}$

$D_{N_1} = D_{N_2}$

$\varepsilon_1 E_{n_1} = \varepsilon_2 E_{n_2}$ Normal compon is not equal in the ratio

$\frac{E_{n_2}}{E_{n_1}} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_0 \varepsilon_{r_1}}{\varepsilon_0 \varepsilon_{r_2}}$

- Dielectric conductor surface

ε_r dielectric | conductor med

$E_{t_2} = 0$, $H_{t_2} = 0$ } in conductor

$B_{n_2} = 0 \Rightarrow B_{n_1} = 0 \Rightarrow H_{n_1} = 0$ | $D_{n_2} = 0$, $D_{n_1} = -\rho_s$

$\Rightarrow E_{t_1} = 0$, $\boxed{H_{t_1} = J_s}$

conductor allows some current

If dealing with periodic signals

$e^{j\omega t} \Rightarrow \frac{\partial}{\partial t} \to j\omega$, $\frac{\partial^2}{\partial t^2} = -\omega^2$

so $\vec{\nabla} \times \vec{E} = -j\omega \vec{B}$ ... etc

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Plane waves

Free space:-

$\mu = \mu_0, \varepsilon = \varepsilon_0, \mu_r = 1, \varepsilon_r = 1$

$\sigma = 0, \rho = 0, J = 0$

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -j\omega \vec{B} = -j\omega \mu_0 \vec{H}$ (1)

$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} = j\omega \vec{D} = j\omega \varepsilon_0 \vec{E}$ (2)

$\vec{\nabla} \cdot \vec{D} = \rho = 0 \Rightarrow \vec{\nabla} \cdot (\varepsilon \vec{E}) = 0$, $\vec{\nabla} \cdot \vec{E} = 0$ (3)

$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{\nabla} \cdot \vec{H} = 0$ (4)

Take curl on (1),

$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -j\omega\mu_0(\vec{\nabla} \times \vec{H})$

$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = -\vec{\nabla}^2 \vec{A} + \vec{\nabla}(\vec{\nabla} \cdot \vec{A})$

$-\vec{\nabla}^2 \vec{E} + \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) = -j\omega\mu_0(\vec{\nabla} \times \vec{H})$ — (7)

(3),7 $-\vec{\nabla}^2 \vec{E} = -j\omega\mu_0(\vec{\nabla} \times \vec{H})$

(2),8 $-\vec{\nabla}^2 \vec{E} = -j\omega\mu_0 \times j\omega\varepsilon_0 \vec{E}$

$= +\omega^2 \mu_0 \varepsilon_0 \vec{E}$

wave number k_0 of free space

$k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$

$\boxed{(\vec{\nabla}^2 + k_0^2)\vec{E} = 0}$ wave eqn of $\vec{E}$

Take curl on both sides of (2)

we get

$\boxed{(\vec{\nabla}^2 + k_0^2)\vec{H} = 0}$ wave eqn of $\vec{H}$.

wave number, $k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$

$k_0 = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda_0}$ rad/m

Solving wave eqn in free space

Laplacian, $\vec{\nabla}^2$

$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Uniform plane waves (1)

$\vec{E}$ & $\vec{H}$ are ⊥ to dirn of propagation

assume x-dirn

$\frac{\partial^2}{\partial y^2} = 0$, $\frac{\partial^2}{\partial z^2} = 0$, $\frac{\partial}{\partial y} = 0$, $\frac{\partial}{\partial z} = 0$

$(\vec{\nabla}^2 + k_0^2)\vec{E} = 0$

$\frac{d^2 \vec{E}}{dx^2} + k_0^2 \vec{E} = 0$ — (23)

$\frac{d^2 \vec{H}}{dx^2} + k_0^2 \vec{H} = 0$ — (24)

(23) expands to

$\frac{\partial^2}{\partial x^2}[E_x i_x + E_y i_y + E_z i_z]$

$+ k_0^2 [E_x i_x + E_y i_y + E_z i_z] = 0$

$\Rightarrow \frac{\partial^2 E_x}{\partial x^2} + k_0^2 E_x = 0$ (25)

$\Rightarrow \frac{\partial^2 E_y}{\partial y^2} + k_0^2 E_y = 0$ (26)

: (27), — (30)

Use (M3), $\vec{\nabla} \cdot \vec{E} = 0$

$\frac{\partial}{\partial x} E_x + \frac{\partial}{\partial y} E_y + \frac{\partial E_z}{\partial z} = 0$

$\Rightarrow \frac{\partial E_x}{\partial x} = 0$ (31)

Use (M4), $\vec{\nabla} \cdot \vec{H} = 0 \Rightarrow \frac{\partial H_x}{\partial x} = 0$ (32)

(31), (25) $E_x = 0$

(32), (29) $H_x = 0$

TEM mode

$E_y(x) = A e^{-jk_0 x}$ (35)

$E_z(x) = B e^{-jk_0 x}$ (36)

$H_y(x) = C e^{-jk_0 x}$ (37)

$H_z(x) = D e^{-jk_0 x}$ (38)

$e^{j\omega t}$ time dependence

$E_y(x, t) = A e^{j(\omega t - k_0 x)}$ (39)

Similarly for E_z, H_y, H_z (42)

Phase surface (wave front)

$(\omega t - k_0 x)$ (43) ⟷ constant

as ↑t, wave front prop

Phase velo $v_p = \frac{dx}{dt} = \frac{\omega}{k_0}$

Group velo. $v_g = \frac{d\omega}{dk_0}$

$v_p = v_g = \frac{\omega}{k_0} = \frac{\omega}{\omega\sqrt{\mu_0 \varepsilon_0}} = c$

wave travels with speed of light.

Uniform plane wave in free space

$E_x = 0$, $E_y = A e^{-jk_0 x}$, $E_z = B e^{-}$

$H_x = 0$, $H_y = A e^{-jk_0 x}$

$v_p = c$; $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$ Ω

~~characteristic~~ intrinsic impedance

$\vec{E_x}=0\ \ \vec{E_y}=A\cos(\omega t-k_0x);\ \vec{E_z}$

make M1 M2 being satisfied by the solution.

M1 $\nabla\times\vec{E}=-j\omega\mu_0\vec{H}$

$\begin{vmatrix} i_x & i_y & i_z \\ \partial/\partial x & 0 & 0 \\ 0 & E_y & E_z \end{vmatrix} = -j\omega\mu_0(H_x i_x + H_y i_y + H_z i_z)$

compare coeff of $\hat{i}_y$.

$\left(-\frac{\partial}{\partial x}E_z\right) = -j\omega\mu_0 H_y$

$\boxed{H_y = \frac{1}{j\omega\mu_0}\frac{dE_z}{dx}}$ (19)

$Ce^{-jk_0x} = \frac{1}{j\omega\mu_0}\frac{d}{dx}(Be^{-jk_0x})$

$C = \frac{-Bk_0}{\omega\mu_0};\ \frac{B}{C} \overset{Z_w}{=} -\frac{\omega\mu_0}{k_0} = \frac{E_z}{H_y}$ V/A

compare coeff of $\hat{i}_z$ of M1

$\frac{\partial}{\partial x}E_y = -j\omega\mu_0 H_z$

$\frac{\partial E_y}{\partial x} \Rightarrow \frac{A}{D} = \frac{\omega\mu_0}{k_0} = \frac{E_y}{H_z}$

$\boxed{\eta_0 = \frac{E_y}{H_z} = -\frac{E_z}{H_y}}$ (85)

M2 → SS → $Z_w = \left|\frac{E_y}{H_z}\right|$

M3

M4 → 0 = 0 $= \left|\frac{E_z}{H_y}\right|$

$Z_w = \frac{B}{C} = -\frac{\omega\mu_0}{k_0} = \frac{E_z}{H_y}$ V/A

Wave impedance

$\eta_0 = \frac{A}{D} = \frac{E_y}{H_z} = \frac{-E_z}{H_y}$

Uniform plane wave in lossless dielectric medium

$\epsilon_r > 1$ real no, $\epsilon = \epsilon_0\epsilon_r > \epsilon_0$

$\mu = \mu_0,\ \sigma = 0$ (lossless)

Assume x dirn

$\nabla\times\vec{E} = -j\omega\mu_0\vec{H}$ (1)

$\nabla\times\vec{H} = +j\omega\epsilon_0\epsilon_r\vec{E}$ (2)

$\nabla\cdot\vec{E} = 0$ (3)

$\nabla\cdot\vec{H} = 0$ (4)

take curl on (1)

$\nabla\times\nabla\times\vec{E} = -j\omega\mu_0(\nabla\times\vec{H})$

$\Rightarrow -\nabla^2\vec{E} + \nabla(\nabla\cdot\vec{E})$
$= -j\omega\mu_0(j\omega\epsilon_0\epsilon_r)\vec{E}$

$\Rightarrow (\nabla^2 + k^2)\vec{E} = 0$

$\boxed{k = \omega\sqrt{\mu_0\epsilon_r\epsilon_0}}$

similar $(\nabla^2 + k^2)\vec{H} = 0$ $\quad v_p = \frac{c}{\sqrt{\epsilon_r}}$

$Z_w = \frac{\omega\mu_0}{\omega\sqrt{\mu_0\epsilon_0\epsilon_r}}$

$\eta \triangleq \frac{\eta_0}{\sqrt{\epsilon_r}},\ v = \sqrt{\frac{\mu_0}{\epsilon_0}}$

* Uniform lossless magnetic

$K = \omega\sqrt{\mu_0\mu_r\epsilon_0},\ \mu = \mu_0\mu_r$

$v_p = \frac{c}{\sqrt{\mu_r}},\ Z_w = \eta_0\sqrt{\mu_r}$

* Uniform lossy dielectric

Assume +x dirn

(1) $\epsilon_{compl} = \epsilon_r(1 - \tan\delta\, j)$
$= \epsilon_r' - j\epsilon_r''$

(2) $\mu = \mu_0$ M1 $\nabla\times\vec{E} = -j\omega\mu_0\vec{H}$

(3) $\sigma > 0$ M2 $\nabla\times\vec{H} = j\omega\epsilon\vec{E} + \sigma\vec{E}$

M3 $\nabla\cdot\vec{E} = 0$

M4 $\nabla\cdot\vec{H} = 0$

$\epsilon = \epsilon_0\epsilon_r(1 - j\tan\delta)$

$\eta_0 = \epsilon' - j\epsilon''$

$\nabla\times\nabla\times\vec{E} = \nabla\times(-j\omega\mu_0\vec{H})$

$\nabla^2\vec{E} + \nabla(\nabla\cdot\vec{E}) = -j\omega\mu_0(\sigma + j\omega\epsilon)\vec{E}$

$(\nabla^2 + k_{eff}^2)\vec{E} = 0$

$k_{eff}^2 = j\omega\mu_0[\sigma + j\epsilon_0\epsilon_r(1 - j\tan\delta)]$

$jk_{eff} = \alpha + j\beta$ ← phase const

attenuation const

$E_x = 0,\ E_y = Ae^{-jk_{eff}x}\ \ E_z = Be^{-jk_{eff}x}$

$H_x = 0,\ H_y = Ce^{-jk_{eff}x},\ H_z = De^{-jk_{eff}x}$

$E_y = Ae^{-\alpha x}e^{-j\beta x},\ H_y = Ce^{-\alpha x}e^{-j\beta x}$

$E_z = Be^{-\alpha x}e^{-j\beta x},\ H_z = De^{-\alpha x}e^{-j\beta x}$

$\boxed{v_p = \frac{\omega}{\beta} = v_g}$ $\quad Z_w = \frac{\omega\mu_0}{k_{eff}} = \frac{j\omega\mu_0}{\alpha + j\beta}$

* Uniform plane wave in good conductor

$\sigma >> 0,\ \mu \approx \mu_0,\ \epsilon \approx \epsilon_0$

$\nabla\times\vec{E} = -j\omega\mu_0\vec{E}$

$\nabla\times\vec{H} = j\omega\epsilon\vec{E} + \sigma\vec{E} \approx \sigma\vec{E}$

$\nabla\cdot\vec{E} = 0,\ \nabla\cdot\vec{H} = 0$

$-\nabla^2\vec{E} = -j\omega\mu_0(\nabla\times\vec{H}) =$
$-j\omega\mu_0\sigma\vec{E}$

$(\nabla^2 + k_{eff}^2)\vec{E} = 0.$

$k_{eff}^2 = -j\omega\mu_0\sigma$

$k_{eff} = \pm j\frac{(1+j)}{\sqrt{2}}\sqrt{\omega\mu_0\sigma}$

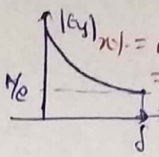
$= \pm\sqrt{\frac{\omega\mu_0\sigma}{2}}(j-1)$

$k_{eff} = \alpha + j\beta = \pm\sqrt{\frac{\omega\mu_0\sigma}{2}}(1-j)$

$\alpha = \sqrt{\frac{\omega\mu_0\sigma}{2}};\ \beta = \sqrt{\frac{\omega\mu_0\sigma}{2}}$

δ_s = skin depth. $|E_y| = A$

$\frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu_0\sigma}}$



$v_p = \omega/\beta$

$Z_w = \sqrt{\frac{\omega\mu_0}{2\sigma}}(1+j)\ \Omega$

POYNTING VECTOR ($\vec{S}$)

Measure of power density (W/m²)

$\vec{S} \triangleq \frac{1}{2}Re[\vec{E}\times\vec{H}^*]$

+x directed uniform plane wave

$E_x = 0 \quad H_x = 0$

$E_y = Ae^{-jk_0x}.\ \ H_y = Ce^{-jk_0x}$

$\frac{B}{C} = \frac{E_z}{H_y} = -Z_w = -\eta_0$ (7)

$\frac{A}{D} = \frac{E_y}{H_z} = Z_w = \eta_0$ (8)

$\vec{E}\times\vec{H}^* = \begin{vmatrix} i_x & i_y & i_z \\ 0 & E_y & E_z \\ 0 & H_y^* & H_z^* \end{vmatrix}$

$\vec{E}\times\vec{H}^* = (E_yH_z^* - E_zH_y^*)\hat{i}_x$

$\vec{S} = S_x\hat{i}_x$ (10)

where, $S_x = \frac{1}{2}Re[E_yH_z^* - E_zH_y^*]$

$\Rightarrow \frac{1}{2}Re\left[E_y\left(\frac{E_y}{\eta_0}\right)^* - E_z\left\{\frac{H_y^*}{\eta_0}\right\}\right]$

$= \frac{|E_y|^2 + |E_z|^2}{2}$ (13)

$= \frac{|E|^2}{2\eta_0}$ (14) $\quad P_{avg} = \frac{E_0^2}{2\eta_0}\cdot Area(?)$

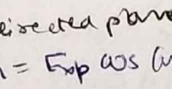
we use rms value then 2 will not be there

Polarization of EM wave

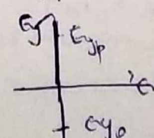
Locus of trace by the tip of $\vec{E}$ vector EM wave

z directed plane

• $E_x = E_{xp}\cos(\omega t - \beta z),\ E_y = 0$, z=0



• $E_y = E_{yp}\sin(\omega t - \beta z),\ E_x = 0$, z=0

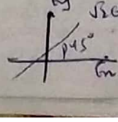


• $E_x = E_0\cos(\omega t - \beta z)$ z=0

$E_y = E_0\cos(\omega t - \beta z)$

$|E| = \sqrt{E_x^2 + E_y^2} = \sqrt{2}E_0\cos\omega t$

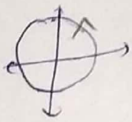
$\theta = \tan^{-1}(E_y/E_x)$



$E_x = E_0\cos(\omega t - \beta z)$
$E_y = E_0\sin(\omega t - \beta z)$ } z=0

$|E| = \sqrt{E_x^2 + E_y^2} = E_0$

$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{\sin\omega t}{\cos\omega t}$
$= \omega t$



Right handed circular polarization.

$E_x = E_0\cos(\omega t - \beta z)$

$E_y = -E_0\sin(\omega t - \beta z)$

⊕ LHCP

elliptical (RHCP/LHCP)

$E_x = E_{x0}\cos(\omega t - \beta z)$

$E_y = E_{y0}\cos(\omega t - \beta z)$

Rectangular waveguide

Used as microwave frequencies (L band to W band)

- low propagation loss (α)
- low dispersion

effect of freq on performance

TEM: $E_z = 0,\ H_z = 0$ NOT possible

TE: $E_z = 0$

TM: no components zero

TE mode analysis

$E_z = 0$ in z dim electric potential

$\vec{E} = \nabla\times(\psi_e\hat{i}_z)$

$E_x\hat{i}_x + E_y\hat{i}_y + E_z\hat{i}_z = \begin{vmatrix} i_x & i_y & i_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & \psi_e \end{vmatrix}$

$E_x = \frac{\partial\psi_e}{\partial y},\ E_y = -\frac{\partial\psi_e}{\partial x}$

$E_z = 0$

M1. $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -j\omega\vec{B}$
$= -j\omega\mu_0\vec{H}$

$-j\omega\mu_0\vec{H} = \begin{vmatrix} i_x & i_y & i_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & 0 \end{vmatrix}$

$H_x = \frac{1}{j\omega\mu_0}\frac{\partial E_y}{\partial z} = \frac{-1}{j\omega\mu_0}\frac{\partial^2\psi_e}{\partial x\partial z}$

$H_y = -\frac{1}{j\omega\mu_0}\frac{\partial^2\psi_e}{\partial z\partial y}$

$H_z = \frac{1}{j\omega\mu_0}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]$

M2: $\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} = j\omega\epsilon_0\vec{E}$

$\begin{vmatrix} i_x & i_y & i_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = j\omega\epsilon_0[E_xi_x + E_yi_y]$

Coeff of $\hat{i}_x$: $\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}$
$= j\omega\epsilon_0 E_x$

$$\frac{\partial}{\partial y}\left[\frac{1}{j\omega\mu_0}\left(\frac{\partial^2\psi_e}{\partial x^2}+\frac{\partial^2\psi_e}{\partial y^2}\right)\right]$$

$$+\frac{\partial}{\partial z}\left[\frac{1}{j\omega\mu_0}\frac{\partial^2\psi_e}{\partial y\,\partial z}\right]=j\omega\varepsilon_0\frac{\partial\psi_e}{\partial y}$$

$$\Rightarrow \frac{1}{j\omega\mu_0}\left[\frac{\partial^3\psi_e}{\partial x^2\,\partial y}+\frac{\partial^3\psi_e}{\partial y^3}\right]$$

$$+\frac{1}{j\omega\mu_0}\frac{\partial^3\psi_e}{\partial y\,\partial z^2}=j\omega\varepsilon_0\frac{\partial\psi_e}{\partial y}$$

$\nabla^2\psi_e$

$$\Rightarrow \frac{1}{j\omega\mu_0}\frac{\partial}{\partial y}\left[\frac{\partial^2\psi_e}{\partial x^2}+\frac{\partial^2\psi_e}{\partial y^2}+\frac{\partial^2\psi_e}{\partial z^2}\right]=j\omega\varepsilon_0\frac{\partial\psi_e}{\partial y}$$

$$=-\omega^2\varepsilon_0\mu_0\psi_e$$

$$\Rightarrow (\nabla^2+k_0^2)\,\psi_e=0$$

(M3) $\vec{\nabla}\cdot\vec{D}=0$

$\Rightarrow \vec{\nabla}\cdot\vec{E}=0$

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}^{0}=0$$

$$\Rightarrow \frac{\partial^2\psi}{\partial y\,\partial x}-\frac{\partial^2\psi}{\partial y\,\partial x}=0$$

$0=0$

(M4) $0=0$

Solution of wave eqn for ψ_e

Assume $\psi_e(x,y,z)$ can be expressed as $X(x)Y(y)Z(z)$; $X(x)\to x$; purely func.

$\Rightarrow \psi_e(x,y,z)=X(x)Y(y)Z(z)$

$\to X''YZ+XY''Z+XYZ''+k_0^2XYZ=0$ method of sepn of variable

$$\frac{X''}{X}+\frac{Y''}{Y}+\frac{Z''}{Z}+k_0^2=0$$

only possible soln is to equal $\frac{X''}{X}, \frac{Y''}{Y}, \frac{Z''}{Z}$ to diff const

$\frac{X''}{X}=-k_x^2$, $\frac{Z''}{Z}=-k_z^2$

$\frac{Y''}{Y}=-k_y^2$

k_x = wave no along x dir

$k_0^2=k_x^2+k_y^2+k_z^2$

(28) $\to \frac{d^2X}{dx^2}+k_x^2X=0$ (29) (30) (31)

soln

$X(x)=A\sin(k_xx)+B\cos(k_xx)$

$Y(y)=C\sin(k_yy)+D\cos(k_yy)$

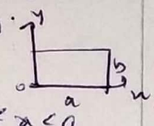
$Z(z)=Ee^{-jk_zz}+Fe^{+jk_zz}$

No propagation in x, y dir - not write in exp form

$\psi_e(x,y,z)$
$=(A\sin k_xx+B\cos k_xx)\cdot$
$(C\sin k_yy+D\cos k_yy)\cdot$
$(Ee^{-jk_zz}+Fe^{-jk_zz})$

Boundary cond due to four walls of waveguide

Tangential component of EF on a perfectly conductor surface $E_{tan}=0$



(1) $E_x|_{y=0}=0$ for $0\le x\le a$, $-\infty<z<\infty$

Top wall :-

(2) $E_x|_{y=b}=0$ for $0\le x\le a$, $-\infty<z<\infty$

(3) $E_z|_{y=b}=0$,,

Left wall

(4) $E_z=0\,|_{x=0}$ $0\le y\le b$, $-\infty<z<\infty$

(5) $E_y=0\,|_{x=0}$,,

Right wall

(6) $E_z=0\,|_{x=a}$ $0\le y\le b$, $-\infty$

(7) $E_y=0\,|_{x=a}$ $-\infty<z<\infty$

Because of TE mode assumption

B.C 1) 3) 5) 7) automatically satisfy at $E_z=0$

BC2 $E_x|_{y=0}=+\frac{\partial\psi_e}{\partial y}\Big|_{y=0}=0$

$C=0$

$\frac{d}{dy}\left[C\sin k_yy+D\cos k_yy\right]_{y=0}=0$

BC4 $\frac{d}{dy}\left[C\sin k_yy+D\cos k_yy\right]_{y=b}=0$

$\sin(k_yb)=0\Rightarrow \boxed{k_y=\frac{n\pi}{b}}$

where $n=0,1,2,3$ —

BC6 $A=0$; $E_y|_{x=0}=0$

$-\frac{\partial}{\partial x}(A\sin k_xx+B\cos k_xx)=0$

BC8 $\boxed{k_x=\frac{m\pi}{a}}$ where $m=0,1,2$.

$\psi_e(x,y,z)=\left(B\cos\frac{m\pi x}{a}\right)\cdot\left[D\cos\frac{n\pi y}{b}\right]\left[Ee^{-jk_zz}+Fe^{jk_zz}\right]$

Forward Reflected wave

$k_0^2=k_x^2+k_y^2+k_z^2$

$$k_z=\sqrt{k_0^2-\left(\frac{m\pi}{a}\right)^2-\left(\frac{n\pi}{a}\right)^2}$$

Assume, waveguide infinitly long or it is terminated with a matched load, the reflected wave can be assumed to be absent. We can write $F=0$

$\psi_{e\,m,n}(x,y,z)=B\cdot D\cdot E$
$\cdot\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$
$\cdot\left[e^{-jk_{z\,m,n}}\right]$

$=\psi^{(0)}_{e\,m,n}\times\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\left(e^{-jk_{z\,m,n}z}\right)$

Peak value

For each (m,n) pair, we get a different soln.

If we choose m=1, n=1, we say waveguide operating in TE_{11} mode

Field expression for $TE_{m,n}$ mode in an air filled rectangular waveguide.

$$\psi_{e\,m,n}=\psi^{(0)}_{e\,m,n}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\cdot e^{-jk_{z\,m,n}}$$

$m=0,1,2,\ldots$

$n=0,1,2,\ldots$

$$k_{z\,m,n}=\sqrt{k_0^2-\left(\frac{m\pi}{a}\right)^2-\left(\frac{n\pi}{b}\right)^2}$$

$$k_0=\omega\sqrt{\mu_0\varepsilon_0}=\frac{\omega}{c}=\frac{2\pi}{\lambda}$$

$$E_{x\,m,n}=\frac{\partial\psi_{e\,m,n}}{\partial y}=-\frac{n\pi}{b}\psi_{e\,m,n}\cdot\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\cdot e^{-jk_{z\,m,n}}$$

$$=E^{(0)}_{x\,m,n}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-jk_{z\,m,n}}$$

where $E_{x\,m,n}\triangleq-\frac{n\pi}{b}\psi_e^{\,m,n}$

$$E_{y\,m,n}=-\frac{\partial\psi_{e\,m,n}}{\partial x}=-\frac{m\pi}{a}\psi^{0}_{e\,m,n}*\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-jk_{z\,m,n}}$$

$$\Rightarrow E^{(0)}_{y\,m,n}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\cdot e^{-jk_{z\,m,n}}$$

where $\Rightarrow E^{(0)}_{y\,m,n}\triangleq-\frac{m\pi}{a}\psi_{e\,m,n}$

$\Rightarrow E_{z\,m,n}=0$

$$H_{x\,m,n}=\frac{1}{j\omega\mu_0}\frac{\partial^2\psi_{e\,m,n}}{\partial x\,\partial y}$$

$$=-\frac{jk_{z\,m,n}}{j\omega\mu_0}\left(\frac{m\pi}{a}\right)\sin\frac{m\pi x}{a}*\cos\frac{n\pi y}{b}\times e^{-jk_{z\,m,n}z}$$

$$\Rightarrow H_{y\,m,n}=\frac{-1}{j\omega\mu_0}\frac{\partial^2\psi_{e\,m,n}}{\partial y\,\partial z}$$

$$=-\frac{jk_{z\,m,n}}{j\omega\mu_0}\left(\frac{n\pi}{b}\right)\cos\frac{m\pi x}{a}*\sin\frac{n\pi y}{b}e^{-jk_{z\,m,n}z}$$

$$\Rightarrow H_{z\,m,n}=\frac{1}{j\omega\mu_0}\left[\frac{\partial^2\psi_{e\,m,n}}{\partial x^2}+\frac{\partial^2\psi_{e\,m,n}}{\partial y^2}\right]$$

Cut off mode of operation

$$k_{z\,m,n}=\sqrt{k_0^2-\left(\frac{m\pi}{a}\right)^2-\left(\frac{n\pi}{b}\right)^2}$$

$k_0=\omega/c=2\pi f/c$

f = operating frequency at which signal generator feed signal to the waveguide

If f is low, k_0 would be low and we get

(3)

$$k_0^2<\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2$$

$k_{z\,m,n}=j\alpha_{m,n}$

$$\alpha_{m,n}=\pm\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2-k_0^2}$$

↳ attenuation const

All field component will display $e^{-\alpha_{m,n}z}$ type of z dependance

Nothing will go through if frequency is low

Propagation mode :-

If f is sufficiently high $k_0^2>\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2$

No attenuation of field comp will be seen. It acts as high pass filter. In hypothetical case no attenuation of field component will be seen

Cutoff wave number

$k_{c\,m,n}$:- is defined as that value of k_0 at which waveguide switches from cutoff to propagation mode.

$$k_{c\,m,n}=\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

cutoff $\lambda_{c\,m,n}$:-

$$\lambda_{c\,m,n}=\frac{2\pi}{k_{c\,m,n}}$$

$$\lambda_{c\,m,n}=\frac{2\pi}{\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}}=\frac{2}{\sqrt{(m/a)^2+(n/b)^2}}$$

cutoff $f_{c\,m,n}$

$$f_{c\,m,n}=\frac{c}{\lambda_{c\,m,n}}=\frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

Mode with lowest cutoff freq = dominant mode/ fundamental mode

Degenerate mode = same cutoff freq.

→ 6.56 to 13.11 = official band width

$X_{band}\doteq 8\,GHz-12\,GHz$ practically recom—

Dominant TE_{10} mode by an air filled rectangular waveguide.

for TE_{mn} we know the

$\psi_{e\,mn} = \psi_{e\,mn}\cos\frac{m\pi y}{a}$

$K_{z\,mn} = \sqrt{k_0^2 - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}}$

$k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, $\frac{\omega}{c} = \frac{2\pi}{\lambda}$

TE_{10} ; m=1, n=0

$\psi_e = \psi_e^{(0)}\cos\frac{\pi x}{a}e^{-jk_z z}$

$\psi_e^{(0)} = \psi_{e\,(1,0)}^{(0)}$

and $k_{z0} = k_{z1,0}$

$k_z = \sqrt{k_0^2 - (\frac{\pi}{a})^2}$

phase constant: amount of phase shift introduced in the various fields component/unit length

Field comp for TE_{10},

$E_x = \frac{\partial\psi_e}{\partial y} = 0$

$E_y = -\frac{\partial\psi_e}{\partial x} = \frac{\pi}{a}\psi_e^{(0)}\sin\frac{\pi x}{a}e^{-jk_z z}$

$E_z = 0$

$H_x = \frac{-1}{j\omega\mu_0}\frac{\partial^2\psi_e}{\partial x\partial y} = \frac{-jk_z}{j\omega\mu_0}(\frac{\pi}{a})$

• $\psi_e^{(0)}\sin\frac{\pi x}{a}e^{-jk_z z}$

$H_y = \frac{-1}{j\omega\mu_0}\frac{\partial^2\psi_e}{\partial y\partial z} = 0$

$H_z = 0$

$H_x = \frac{-1}{j\omega\mu_0}\frac{\partial^2\psi_e}{\partial x\partial y} = \frac{-jk_z}{j\omega\mu_0}(\frac{\pi}{a})$

• $\psi_e^{(0)}\sin\frac{\pi x}{a}e^{-jk_z z}$

$H_y = \frac{1}{j\omega\mu_0}\left[\frac{\partial^2\psi_e}{\partial x^2} + \frac{\partial^2\psi_e}{\partial y^2}\right]$

$= -\frac{k_0^2 k_z^2}{j\omega\mu_0}\psi_e^{(0)}\cos\frac{\pi x}{a}e^{-jk_z z}$

Phase constant

$k_z = \sqrt{k_0^2 - \frac{\pi^2}{a^2}}$

$= k_0\sqrt{1-(\frac{\pi}{ak_0})^2}$

$\frac{2\pi}{2ak_0} = \frac{2\pi}{2a\omega\sqrt{\mu_0\varepsilon_0}}$

$= \frac{2\pi c}{2a(2\pi f)} = \frac{\lambda}{a} = \frac{\lambda}{\lambda_c} = \frac{f_c}{f}$

cutoff freq ⇒ $\boxed{f_c = \frac{c}{2a}}$

$\boxed{\lambda = 2a}$ $\boxed{k_z = k_0\sqrt{1-\frac{f_c^2}{f^2}}}$

wave Impedance

$Z_w \triangleq \left|\frac{E_y}{H_x}\right| = \frac{\omega\mu_0}{k_z}$

$= \frac{\omega\mu_0}{k_0\sqrt{1-(\frac{f_c}{f})^2}} = \frac{\eta}{\sqrt{1-(\frac{f_c}{f})^2}}$

Guided wavelength (λ_g)

$\lambda_g \triangleq \frac{2\pi}{k_z} = \frac{2\pi}{k_0\sqrt{1-(\frac{f_c}{f})^2}}$

$= \frac{\lambda}{\sqrt{1-(\frac{f_c}{f})^2}}$

Phase velocity

$\omega t - k_z = const$

wkt $v_p = \frac{\omega}{k_z} = \boxed{\frac{c}{\sqrt{1-(\frac{f_c}{f})^2}}}$

Group velocity

$v_g = \frac{d\omega}{\partial k_z} = c\sqrt{1-(\frac{f_c}{f})^2}$

$\boxed{v_g \cdot v_p = c^2}$

Energy transfer will occur at group velocity

Power carried by TE_{10} mode in an air field rectangular waveguide

$\bar{S} = \frac{1}{2}Re(\vec{E}\times\vec{H}^*)$

$\vec{E}\times\vec{H} = \begin{vmatrix} i_x & i_y & i_z \\ 0 & E_y & 0 \\ H_x^* & 0 & H_z^* \end{vmatrix}$

$\vec{E}\times\vec{H}^* = (E_y H_z^*)\hat{i}_x + (-E_y H_x^*)i_z$

$P_{10} = \frac{1}{2}Re\int_0^a\int_0^b(\vec{E}\times\vec{H}^*)\cdot i_z\,dy\,dx$

$= \frac{ab}{4}\frac{|E_{y0}|^2}{Z_W}$ — (4)

where E_y^0 is peak value of E_y.

TM mode in air field rectangular waveguide

$H_z = 0$

$\vec{H} = \vec{\nabla}\times(\psi_m i_z)$

magnetic scalar potential

Expanding (2), we get

$H_x\hat{i}_x + H_y i_y + H_z i_z$

$= \begin{vmatrix} i_x & i_y & i_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & \psi_m \end{vmatrix}$

$H = \frac{\partial\psi_m}{\partial y}$, $H_y = -\frac{\partial\psi_m}{\partial x}$

$H_z = 0$.

M1 → $\vec{\nabla}\times\vec{E} = -j\omega\mu_0\vec{H}$

M2 → $\vec{\nabla}\times\vec{H} = j\omega\varepsilon_0\vec{E}$

$= j\omega\varepsilon_0(E_x i_x + E_y i_y + E_z i_z)$

$\Rightarrow \begin{vmatrix} i_x & i_y & i_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & 0 \end{vmatrix}$

$= j\omega\varepsilon_0(E_x i_x + E_y i_y + E_z i_z)$

$= i_x(-\frac{\partial H_y}{\partial z}) - i_y(\frac{-\partial H_x}{\partial z})$

$+ i_z(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) = j\omega\varepsilon_0$

$\cdot(E_x i_x + E_y i_y + E_z i_z)$

$E_x = \frac{-1}{j\omega\varepsilon_0}\frac{\partial H_y}{\partial z} = \frac{1}{j\omega\varepsilon_0}\frac{\partial^2\psi_m}{\partial x\partial z}$

$\frac{\partial H_x}{\partial z} = j\omega\varepsilon_0 E_y$

$E_y = \frac{1}{j\omega\varepsilon_0}\frac{\partial H_y}{\partial z} = \frac{1}{j\omega\varepsilon_0}\frac{\partial^2\psi_m}{\partial y\partial z}$

$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon_0 E_z$

$\vec{E}_z = \frac{1}{j\omega\varepsilon}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]$

$= \frac{-1}{j\omega\varepsilon_0}\left[\frac{\partial^2\psi_m}{\partial x^2} + \frac{\partial^2\psi_m}{\partial y^2}\right]$

Application of M1, M3, M4 will lead to 0=0 OR to wave eqn $(\nabla^2 + k_0^2)\psi_m = 0$

Using method of separation of variables

$\psi_m(x,y,z) = X(x)Y(y)Z(z)$

Apply B.C. by 4 walls of waveguide

$\psi_{m\,mn}(x,y,z) = \psi_{m\,mn}^{(0)}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-jk_{z\,m,n}z}$

where

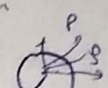
$k_{z\,m,n} = \sqrt{k_0^2 - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2}$

$\boxed{f_{c\,min} = \frac{c}{2}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}}$

CIRCULAR WAVEGUIDE

hollow metal sphere with circular cross section

(ρ, ϕ, z)



$\vec{\nabla}_f = \frac{\partial f}{\partial\rho}i_\rho + \frac{1}{\rho}\frac{\partial f}{\partial\phi}i_\phi + \frac{\partial f}{\partial z}i_z$

For arbitrary vector

$\vec{\nabla}\cdot\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$

$\vec{\nabla}\times\vec{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]i_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right]i_\phi + \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right]i_z$

Bessel eqn

$\frac{1}{\rho}\left(\frac{\partial}{\partial\rho}\left(\rho\frac{df}{d\rho}\right)\right) + \left(k^2 - \frac{n^2}{\rho^2}\right)f = 0$

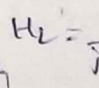
$f(\rho) = AJ_n(k\rho) + BY_n(k\rho)$

Bessel fn of 1st kind; 2nd kind

$J_n(x) = \sum_{m=0}^{\infty}\frac{(-1)^m(\frac{x}{2})^{n+2m}}{m!(n+m)!}$

$Y_n(x) = \frac{2}{\pi}\left[\gamma + \ln\frac{x}{2}\right]J_n(x)$

TE mode for air filled circular waveguide



$E_z = 0$

$\vec{E} = \vec{\nabla}\times(\psi_e i_z)$

M1 $H_\rho = \frac{-1}{j\omega\mu_0}\frac{\partial^2\psi_e}{\partial\rho\partial z}$

$H_\phi = \frac{-1}{j\omega\mu_0}\frac{\partial^2\psi_e}{\partial\phi\partial z}$

$H_z = \frac{1}{j\omega\mu_0}\left[\frac{\partial^2\psi_e}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\psi_e}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\psi_e}{\partial\phi^2}\right]$

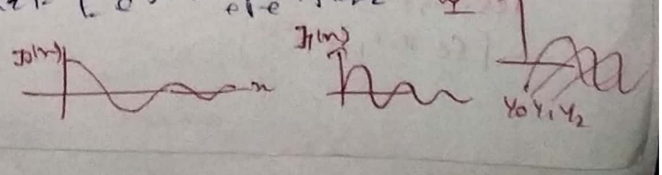
Remaining M2, M3 gives 0=0 or $(\nabla^2 + k_0^2)\psi_e = 0$

$\nabla^2\psi_e = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi_e}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi_e}{\partial\phi^2} + \frac{\partial^2\psi_e}{\partial z^2}$ —(4)

$\psi_e(\rho,\phi,z) = R(\rho)\Phi(\phi)T(z)$

from $T(z) = Ce^{-jk_z z} + De^{+jk_z z}$

providing exponentials



TEM mode in air filled circular waveguide

$\Psi_e(\rho,\phi,z) = R(\rho)S(\phi)T(z)$

$T(z) = F e^{-jk_z z} + F e^{jk_z z}$

forward travel wave — reflected wave

$S(\phi) = C\sin n\phi + D\cos n\phi$

$$\frac{\partial^2 R}{\partial \rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} + \left(k_c^2 - \frac{n^2}{\rho^2}\right)R = 0$$

$\underbrace{}_{\text{STD Bessel eqn}}$

$R(\rho) = A J_n(k_c\rho) + B Y_n(k_c\rho)$

$F = 0$ (∞ long / load trans)

$\Psi_e(\rho,\phi,z) = [A J_n(k_c\rho) + B Y_n(k_c\rho)] \cdot [C\sin n\phi + D\cos n\phi] \cdot e^{-jk_z z}$

$\Rightarrow \Psi_e = J_n(k_c\rho)(C\sin n\phi + D\cos n\phi)$

A is with CD absorbed

Boundary condn

1) $E_z|_{\rho=a} = 0$

2) $E_\phi|_{\rho=a} = 0$

BC1 $-\frac{\partial \Psi_e}{\partial \rho}\Big|_{\rho=a} = 0$

$J_n'(k_c a) = 0 \rightarrow \infty$ no of zeros

If $p'(n,m)$ is m^{th} zero of $J_n'(k_c a)$

$\boxed{k_{c\,nm}\, a = p'_{nm}}$

$k_{c\,n,m} = \frac{p'_{nm}}{a}$

$\lambda_{c\,n,m} = \frac{2\pi}{k_{c\,nm}}$

$f_{c\,n,m} = \frac{c}{\lambda_{c\,nm}}$

dominant mode = 2nd higher mode

10 not possible

Performance parameters of dominant mode TE_{11}

$$Z_w = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$k_z = k_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$v_p v_g = c^2$

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$\lambda_g = \frac{2\pi}{k_z}$

$f_c = 17.6$

TM mode in air filled circular waveguide

$\vec{H_z} = 0$

$\vec{H} = (\vec{\nabla} \times \Psi_m i_z)$

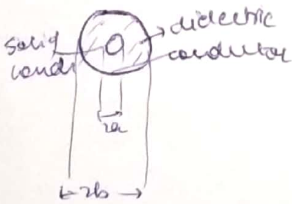
$(\vec{\nabla}^2 + \vec{k_0}^2)\Psi_m = 0$

final eqn for calc for $J_n(k_c a) = 0$

$f_{c\,n,m}(TM) = \frac{c}{2\pi a} p_{n,m}$

Transmission line

coaxial line



a pair of 2 concentric conductors

1) suppose TEM mode
2) cutoff freq = 0 even DC can go through
3) wide band operation possible (DC − 18 GHz)
4) 1st higher mode coaxial line is TE_{11}. ∵ circ waveguide

TEM mode analysis (z dirn)

$E_z = 0;\ H_z = 0$

$E_\rho = \frac{A}{\rho} e^{-jkz}$ (1)

$E_\phi = \frac{B}{\rho} e^{-jkz}$ (2)

$H_\rho = \frac{C}{\rho} e^{-jkz}$ (3)

$H_\phi = \frac{D}{\rho} e^{-jkz}$

$k = k_0\sqrt{\varepsilon_r} = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r}$

$Z_w = \frac{E_\rho}{H_\phi} = -\frac{E_\phi}{H_\rho} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \eta$

By taking field component does not depends on ϕ

for any ϕ, E ⟷ ∴ $\frac{d}{d\phi} = 0$

TEM mode in coaxial transmission line.

Now time and dist depend. $E_\rho = \frac{A}{\rho} e^{j(\omega t - kz)}$

$k = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r} = k\sqrt{\varepsilon_r} = \frac{2\pi}{\lambda}\sqrt{\varepsilon_r}$

$Z_w = \frac{E_\phi}{H_\rho} = \frac{E_\rho}{H_\phi} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \eta$

$v_p = \frac{\omega}{k} = \frac{c}{\sqrt{\varepsilon_r}}$

$v_g = \frac{\partial\omega}{\partial k} = \frac{c}{\sqrt{\varepsilon_r}}$

∴ $\boxed{v_p = v_g}$

Boundary condn

$-\infty < z < \infty$

$E_z|_{\rho=a} = 0$

$E_\phi|_{\rho=a} = 0$

$E_z|_{\rho=b} = 0$

$E_\phi|_{\rho=b} = 0$

B has to 0 ∴ $E_\phi = 0$

Iff $B = 0 \Rightarrow E_\phi = 0$

$H_\rho = \frac{E_\phi}{Z_w} \Rightarrow H_\rho = 0$

$\frac{A}{D} = \frac{E_\rho}{H_\phi} = Z_w$ — E_ρ only H_ϕ non zero compon

voltage $V(z)$ b/n 2 conductors

$V(z) = \int_a^b E_\rho\, d\rho$

$= \int_a^b \frac{A}{\rho} e^{-jkz} d\rho = A e^{-jkz} \ln(b/a)$

current I_z in inner conductor

$I(z) = \int_0^{2\pi} [H_\phi|_{\rho=a}]\, a\, d\phi$

$= \int_0^{2\pi} \frac{D}{a} e^{-jkz} a\, d\phi$

$I(z) = 2\pi D e^{-jkz}$

characteristic impedance

$Z_0 \triangleq \frac{V(z)}{I(z)}$

$= [A \ln b/a]\left[\frac{1}{2\pi D}\right]$

$\eta = 120\pi$

$Z_0 \triangleq \frac{\eta}{2\pi} \ln b/a$

$$\boxed{Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln(b/a)}$$

Higher order modes in coaxial.

1st HOM = TE_{11} mode

∴ $\boxed{k_c = \frac{2}{a+b}}$

$\lambda_c = \frac{2\pi}{k_c} = \frac{2\pi}{2}(a+b)$

$= \pi(a+b)$

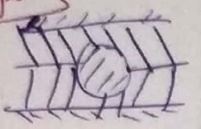
$f_{c\,TE_{11}} = \frac{v}{\lambda_c} = \frac{c}{\pi\sqrt{\varepsilon_r}}$

Microwave Integrated Cht (MIC)

→ based on Tx lines.
→ very compact, attractive,
→ Alternative to bulky waveguide & coaxial lines
→ Thin-film technology used to fabricate MIC

MIC
- Stripline (SL)
- Microstrip (MS)
- Slotline (SL)
- Coplanar waveguide (CPW)
- Coplanar strip (CPS)

Stripline



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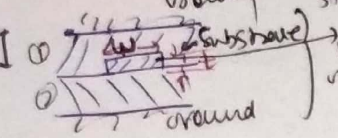
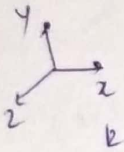


Stripline piece of metal in middle

Space b/n metal is filled by dielectric

$v_p = \frac{c}{\sqrt{\varepsilon_r}} = v_g$



$e^{j(\omega t - kz)}$

$k = \omega\sqrt{\mu_0 \varepsilon_r \varepsilon_0}$

$$\boxed{Z_0 = \frac{30\pi}{\sqrt{\varepsilon_r}} \times \frac{b}{w_e + 0.441b}} \quad \boxed{b = 2h}$$

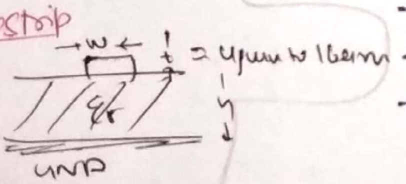
For line,

$\frac{w_e}{b} = \frac{w}{b}$ for $\frac{w}{b} > 0.35$

$\frac{w_e}{b} = \frac{w}{b} - \left(0.35 - \frac{w}{b}\right)^2$ $\frac{w}{b} < 0.35$

1 mil = 0.001"

Microstrip



$v_p = \frac{c}{\sqrt{\varepsilon_{reff}}}$

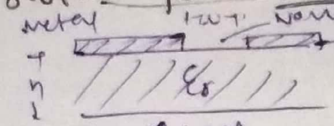
$$\varepsilon_{reff} = \left[\frac{\varepsilon_r + 1}{2}\right] + \left[\frac{\varepsilon_r - 1}{2}\left(\frac{1}{\sqrt{1 + 12\frac{h}{w}}}\right)\right]$$

$\frac{w}{h}$ = Aspect ratio of microstrip

$$Z_0 \simeq \frac{60}{\sqrt{\varepsilon_{reff}}} \ln\left(\frac{8h}{w} + \frac{w}{4h}\right) \quad \boxed{\text{for } \frac{w}{h} \leq 1}$$

$$\simeq \frac{120\pi}{\sqrt{\varepsilon_{reff}}\left[\frac{w}{h} + 1.393 + 0.667\ln\left(\frac{w}{h} + 1.444\right)\right]} \quad \boxed{\text{for } w/h > 1}$$

Slotline for NonTEM mode



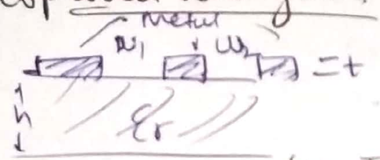
Cross section of slot line

t = Metal thickness

h = substrate thickness

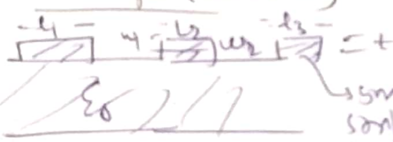
w: slot width

Coplanar waveguide



(Non TEM)

Coplanar strips (NON TEM)



small strips without copper

MMIC's

Monolithic Microwave ICs

→ soldering not required

→ Reliability is higher

→ cheaper also