Exercise 1

1. We just need to know how many remaining barrels of oil the owner of an oil field has in each period t. Let's denote it by x_t where $\sum_{t=0}^{\infty} x_t = B$.

2. The control variables are the amount of barrels of oil the owner of an oil field chooses to sell in each period t. Let's denote it by $y_t \in [0, B]$.

3. The transition equation can be summarized as $x_{t+1} = x_t - y_t$ for each $t \geq 0$ as long as $x_{t+1} \geq 0$.

4. **Sequence problem**:

$$\max_{0 \leq y_t \leq B} \sum_{t=0}^{\infty} (\frac{1}{1+r})^t p_t y_t$$

$$\text{s.t.} \quad x_{t+1} = x_t - y_t$$

$$\sum_{t=0}^{\infty} x_t = B$$

$$x_t \geq 0 \quad \text{for all t} \geq 0$$

Bellman equation:

$$V(x_t) = \max_{0 \leq y_t \leq B} p_t y_t + \frac{1}{1+r} V(x_{t+1})$$

$$\text{s.t.} \quad x_{t+1} = x_t - y_t$$

$$\sum_{t=0}^{\infty} x_t = B$$

$$x_t \geq 0 \quad \text{for all t} \geq 0$$

5. Euler equation looks like a bang-bang situation, i.e.

- for $y_t \in (0, B)$, $p_t = p_{t+1} \frac{1}{1+r}$ for all $t \geq 0$.
- for $y_t = 0$, $p_t < p_{t+1} \frac{1}{1+r}$ for all $t \geq 0$.
- for $y_t = x_t$, $p_t > p_{t+1} \frac{1}{1+r}$ for all $t \geq 0$.

6.

- If $p_{t+1} = p_t$ for all t, then there is no motivation for an owner to wait and hence the optimal solution is selling the all at the initial period, i.e. $y_0 = B$ and $y_t = 0$ for all $t > 0$.

- If $p_{t+1} > p_t(1+r)$ for all t, then the best strategy for an owner is waiting forever. If there is no terminal condition, there would not exist an equilibrium in this case.

  Therefore, for an interior solution, the path of prices should be $p_{t+1}\frac{1}{1+r} = p_t$ for all t.

Exercise 2

1. The state variable is k_t.
2. The control variables are $\{c_t, k_{t+1}\}$.
3. **Bellman equation**

$$
\begin{aligned}
V(k_t) = \max_{c_t \in [0, z_t k_t^\alpha]} & \, u(c_t) + \beta \mathbb{E}_t[V(k_{t+1})] \\
\text{s.t.} \quad & c_t + k_{t+1} = z_t k_t^\alpha + (1-\delta)k_t \\
& k_t \geq 0 \quad \text{for all t} \geq 0 \\
& ln(z_t) \sim N(0, \sigma_z)
\end{aligned}
$$

where $\mathbb{E}_t = \int z_t \phi(z_t) dt$ and $\phi(.)$ is pdf of normal distribution.

4. You can find the answers in my "$EconPS1.ipynb$" file under Exercise 2 section.

Exercise 3

1. **Bellman equation**

$$
\begin{aligned}
V(k_t) = \max_{c_t \in [0, z_t k_t^\alpha]} & \, u(c_t) + \beta \mathbb{E}_t[V(k_{t+1})|z_t] \\
\text{s.t.} \quad & c_t + k_{t+1} = z_t k_t^\alpha + (1-\delta)k_t \\
& k_t \geq 0 \quad \text{for all t} \geq 0 \\
& ln(z_t) = \rho ln(z_{t-1}) + \upsilon_t
\end{aligned}
$$

2. You can find the answers in my "$EconPS1.ipynb$" file under Exercise 3 section.

Exercise 4

1. **Bellman equation**

$$V(w_t) = \max\{\frac{w_t}{1-\beta}, b + \beta \int_0^W V(\tilde{w}) f(\tilde{w}) d\tilde{w}\}$$

where $f(.)$ is density of the wage distribution with cdf F. Here, I assume $F(b) < 1$ (at least some offers are higher than b) and $F(W) = 1$ for some W.

2. **Reservation wage**:

$$V(w_R) = \frac{w_R}{1-\beta} = b + \beta[F(w_R)V(w_R) + \int_{w_R}^W \frac{\tilde{w}}{1-\beta} f(\tilde{w}) d\tilde{w}$$

If we subtract $\frac{\beta}{1-\beta} w_R$, we get

$$w_R - b = \frac{\beta}{1-\beta}[\int_{w_R}^W \frac{\tilde{w}}{1-\beta} f(\tilde{w}) d\tilde{w}]$$

You can find the other answers in my "*EconPS1.ipynb*" file under Exercise 4 section.