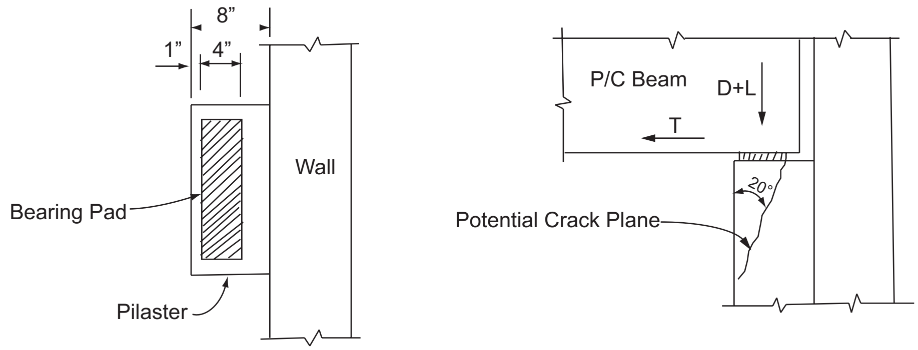
Example 14.2—Shear-Friction Design (Inclined Shear Plane)

For the normalweight reinforced concrete pilaster beam support shown, design for shear transfer across the potential crack plane. Assume a crack at an angle of about 20 degrees to the vertical, as shown below. Beam reactions are D = 25 kips, L = 30 kips. Use T = 20 kips as an estimate of shrinkage and temperature change effects. f'_c = 3500 psi and f_y = 60,000 psi.



Plan Elevation

Calculations and Discussion	Code Reference

1. Factored loads to be considered:

Beam reaction R_u = 1.2D + 1.6L = 1.2 (25) + 1.6 (30) = 30 + 48 = 78 kips — *Eq. (9-2)*

Shrinkage and temperature effects T_u = 1.6 (20) = 32 kips (governs) — *11.8.3.4*
but not less than 0.2 (R_u) = 0.2 (78) = 15.6 kips

Note that the live load factor of 1.6 is used with T, due to the low confidence level in determining shrinkage and temperature effects occurring in service. Also, a minimum value of 20 percent of the beam reaction is considered (see 11.8.3.4 for corbel design).

2. Evaluate force conditions along potential crack plane.

Direct shear transfer force along shear plane:

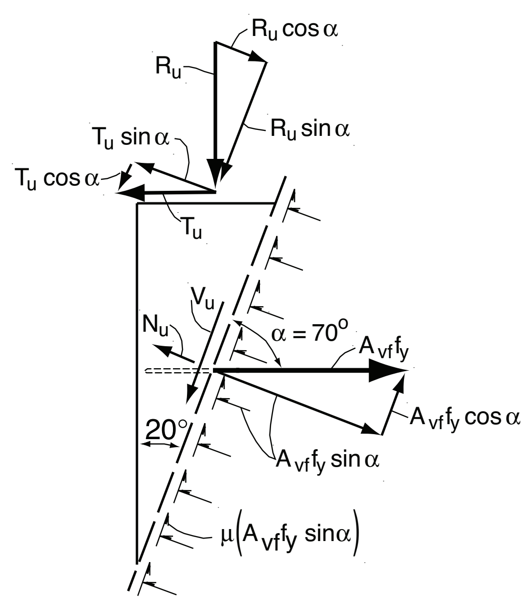
$$V_u = R_u \sin\alpha + T_u \cos\alpha = 78\,(\sin 70°) + 32\,(\cos 70°)$$

$$= 73.3 + 11.0 = 84.3 \text{ kips}$$

Net tension (or compression) across shear plane:

$$N_u = T_u \sin\alpha - R_u \cos\alpha = 32\,(\sin 70°) - 78\,(\cos 70°)$$

$$= 30.1 - 26.7 = 3.4 \text{ kips (net tension)}$$



If the load conditions were such as to result in net compression across the shear plane, it still should not have been used to reduce the required A_{vf}, because of the uncertainty in evaluating the shrinkage and temperature effects. Also, 11.6.7 permits a reduction in A_{vf} only for "permanent" net compression.

3. Shear-friction reinforcement to resist direct shear transfer. Use μ for concrete placed monolithically.

$$A_{vf} = \frac{V_u}{\phi f_y\ (\mu \sin\alpha + \cos\alpha)}$$ *Eq. (11-26)*

$$\mu = 1.4\lambda = 1.4 \times 1.0 = 1.4$$ *11.6.4.3*

$$A_{vf} = \frac{84.3}{0.75 \times 60\ (1.4 \sin 70° + \cos 70°)} = 1.13\ \text{in.}^2$$ *[μ from 11.6.4.3]*

4. Reinforcement to resist net tension.

$$A_n = \frac{N_u}{\phi f_y\ (\sin\alpha)} = \frac{3.4}{0.75 \times 60\ (\sin 70°)} = 0.08\ \text{in.}^2$$

Since failure is primarily controlled by shear, use $\phi = 0.75$ (see 11.8.3.1 for corbel design).

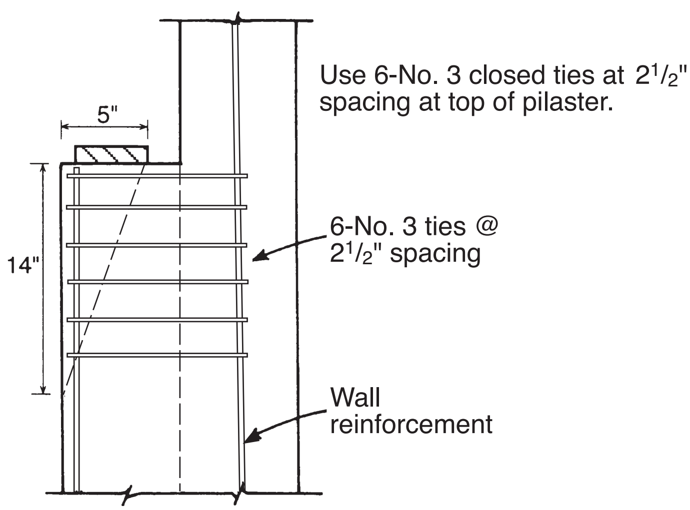
5. Add A_{vf} and A_n for total area of required reinforcement. Distribute reinforcement uniformly along the potential crack plane.

$A_s = 1.13 + 0.08 = 1.21\ \text{in.}^2$

Use No. 3 closed ties (2 legs per tie)

Number required = 1.21 / [2 (0.11)] = 5.5, say 6.0 ties

Ties should be distributed along length of potential crack plane; approximate length = 5/(tan20°) ≈ 14 in.



6. Check reinforcement requirements for dead load only plus shrinkage and temperature effects. Use 0.9 load factor for dead load to maximize net tension across shear plane.

$R_u = 0.9D = 0.9\,(25) = 22.5$ kips, $T_u = 32$ kips

$V_u = 22.5\,(\sin 70°) + 32\,(\cos 70°) = 21.1 + 11.0 = 32.1$ kips

$N_u = 32\,(\sin 70°) - 22.5\,(\cos 70°) = 30.1 - 7.7 = 22.4$ kips (net tension)

$$A_{vf} = \frac{32.1}{0.75 \times 60\,(1.4 \sin 70° + \cos 70°)} = 0.43 \text{ in.}^2$$

$$A_n = \frac{22.4}{0.75 \times 60 \times \sin 70°} = 0.53 \text{ in.}^2$$

$A_s = 0.43 + 0.53 = 0.96 \text{ in.}^2 < 1.21 \text{ in.}^2$

Therefore, original design for full dead load + live load governs.

7. Check maximum shear-transfer strength permitted

$V_{n(max)}$ must not exceed the smallest of:

$\left[0.2f'_c A_c\right]$, $\left[\left(480 + 0.08f'_c\right)A_c\right]$, and $1600A_c$ — 11.6.5

Taking the width of the pilaster to be 16 in.:

$$A_c = \left(\frac{5}{\sin 20°}\right) \times 16 = 234 \text{ in.}^2$$

$V_{n(max)} = 0.2\,(3500)\,(234)/1000 = 164$ kips (governs)

$V_{n(max)} = \left[480 + (0.08)(3500)\right](234)/1000 = 178$ kips

$V_{n(max)} = 1600\,(234)/1000 = 374$ kips

$\phi V_{n(max)} = 0.75\,(164) = 123$ kips

$V_u = 84.3 \text{ kips} \le \phi V_{n(max)} = 123$ kips O.K. — Eq. (11-1)

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