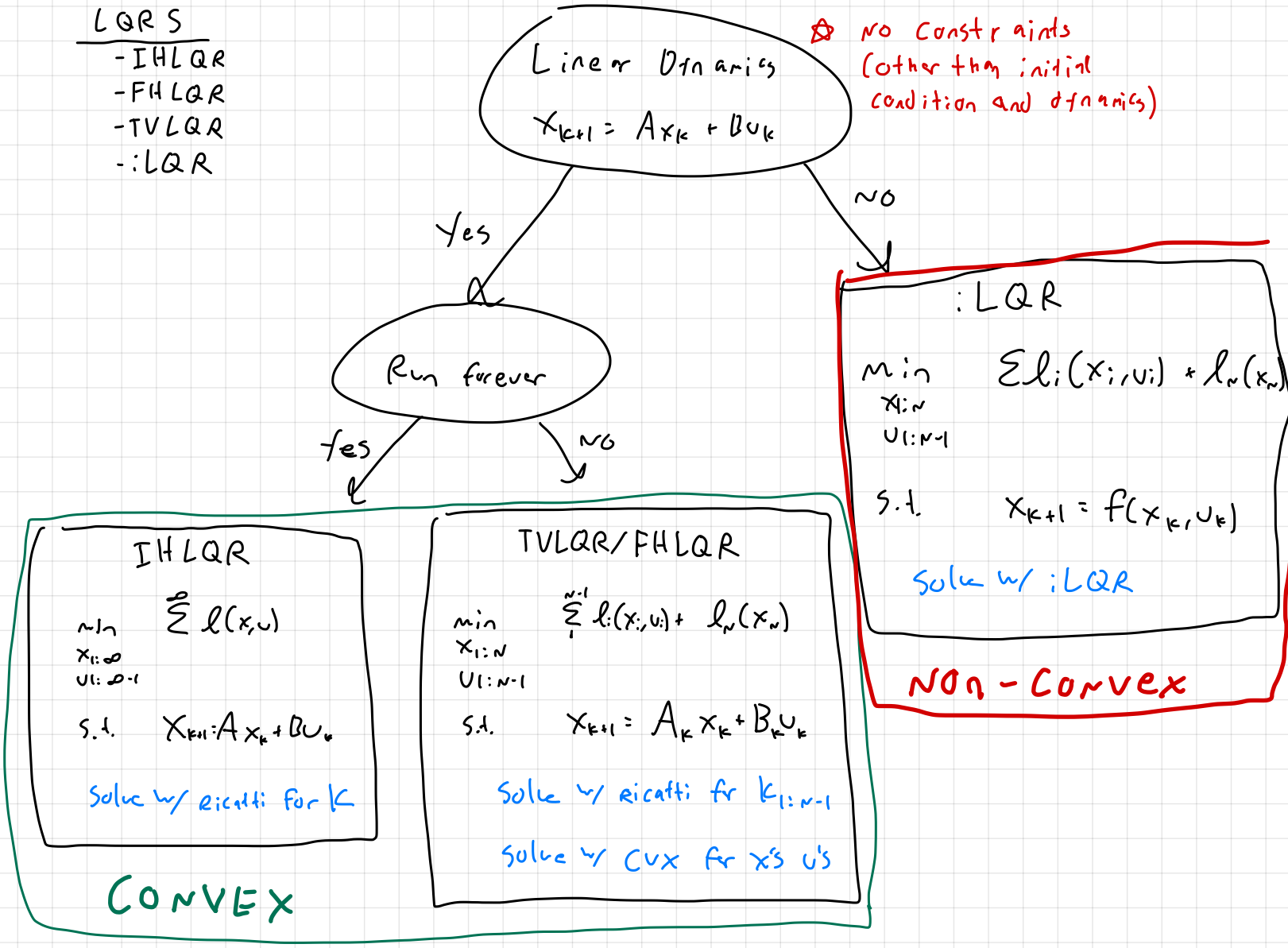
3/22 Recitation

- LQR overview / roadmap
- discretizing
- sparse trajectory optimization

LQRs
- IHLQR
- FHLQR
- TVLQR
- :LQR



- if I add constraints, image to use CVX trajopt
- " " + nonlinear dynamics, non-cvx trajopt
- if I add a term. constraint, $l_N(x_N)$ does nothing for me but I should still keep it

MPC

```
for i = 1 : ...
    U_i = MPC(X_i)   # solve any trajopt problem
    X_{i+1} = f(X_i, U_i)
end
```

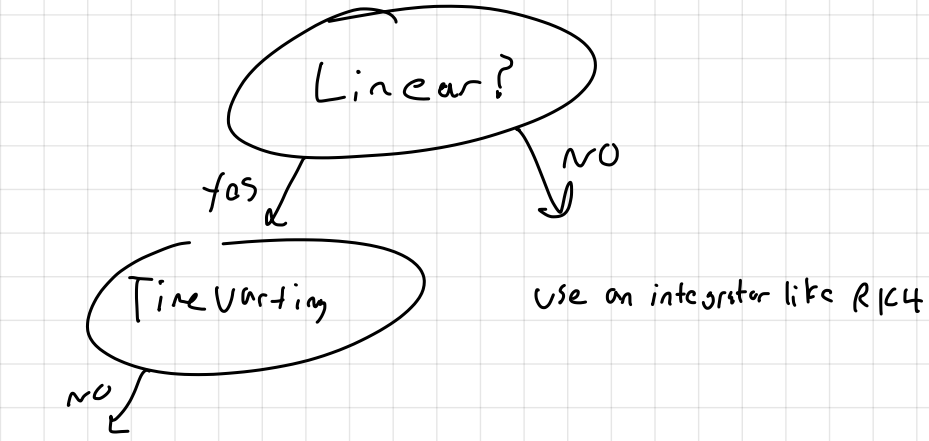
Discretization:

- Consult "Linearization Warmup" HW2 Q2
- Matrix exponential

$$\dot{x} = Ax + Bu \xrightarrow{\text{matrix exp.}} x_{k+1} = A_d x_k + B_d u_k$$

Linear systems

NOT LINEARIZED SYS.



WHAT NOT TO DO

$$\dot{x} = f(x,u) \approx f(\bar{x},\bar{u}) + \overset{A_c}{\frac{\partial f}{\partial x}}\Delta x + \overset{A_c}{\frac{\partial f}{\partial u}}\Delta u$$ - Linearize

$\Downarrow$

$A_d, B_d \Rightarrow$ matrix exp. on A_c, B_c - discretize

DO

$x_{k+1} = f_d(x_k, u_k)$ discretize it w/ RK4 or the like - discretize

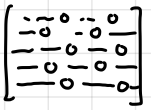
$A_d = \frac{\partial f_d}{\partial x}$, $B_d = \frac{\partial f_d}{\partial u}$ - linearize

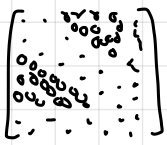
(1) discretize first

(2) then linearize

Sparse Trajopt

Sparse Matrices - don't store the 0's

Store as dense  NNZ => # of non-zeros

store as sparse 

Sparsity pattern in Ipopt

$$\min_{X_{1:N},\, U_{1:N-1}} \quad \ell(X, U)$$

$$\text{s.t.} \quad X_{k+1} - f(x_k, u_k) = 0$$

$$X_1 = x_1$$

$$X_N = x_f$$

$$Z = \begin{bmatrix} X_1 \\ U_1 \\ X_2 \\ U_2 \\ \vdots \\ X_{N-1} \\ U_{N-1} \\ X_N \end{bmatrix} \qquad C(Z) = \begin{bmatrix} X_2 - f(X_1, U_1) \\ X_3 - f(X_2, U_2) \\ X_4 - f(X_3, U_3) \\ \vdots \\ X_N - f(X_{N-1}, U_{N-1}) \end{bmatrix} \begin{matrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_{N-1} \end{matrix}$$

$$\text{Conjac} = \frac{\partial C}{\partial Z} =$$

	X_1	U_1	X_2	U_2	X_3	U_3	$\cdots$	X_{N-1}	U_{N-1}	X_N
C_1	$\frac{\partial C_1}{\partial X_1}$	$\frac{\partial C_1}{\partial U_1}$	$\frac{\partial C}{\partial X_2}$	0	0	0	$\cdots$	0	0	0
C_2	0	0	$\frac{\partial C_2}{\partial X_2}$	$\frac{\partial C_2}{\partial U_2}$	$\frac{\partial C_2}{\partial X_3}$	0	$\cdots$	0	0	0
C_3	0	0	0	0	$\frac{\partial C_3}{\partial X_3}$	$\frac{\partial C_3}{\partial U_3}$	$\cdots$	0	0	0
$\vdots$										
C_{N-1}	0	0	0	0	0	0	0	$\frac{\partial C_{N-1}}{\partial X_{N-1}}$	$\frac{\partial C_{N-1}}{\partial U_{N-1}}$	$\frac{\partial C_{N-1}}{\partial X_N}$

Z

0's

0's

$$z_L \leq C(z) \leq z_U$$

Dense fmincon

$$\min_z \quad f(z)$$

$$\text{s.t.} \quad C_1(z) = 0$$

$$C_L \leq C_2(z) \leq C_U$$

$$z_L \leq z \leq z_U$$

Sparse fmincon

$$\min_z \quad f(z)$$

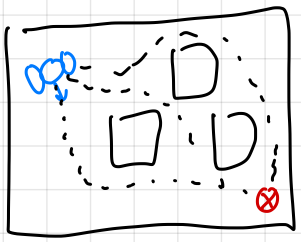
$$\text{s.t.} \quad C_L \leq C(z) \leq C_U$$

$$z_L \leq z \leq z_U$$

$$x = \begin{bmatrix} q \\ v \end{bmatrix} \begin{matrix} \text{- config} \\ \text{- velocity} \end{matrix} \qquad \dot{x} = \begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \text{kinematics}(q, v) \\ \text{dynamics}(q, v) \end{bmatrix}$$

$$x = \begin{bmatrix} r \\ \dot{r} \end{bmatrix} = \quad \dot{x} = \begin{bmatrix} \dot{r} \\ m \cdot a \end{bmatrix}$$

- planning, solves for q's, maybe v's
  - usually just q's, maybe v's to satisfy kinematics
  - A-star, RRT, PRM
  - we'd like to satisfy kinematics



- Trajopt, solves for q's, v's, u's
  - satisfy kinematics and dynamics
  - forces, torques, controls
  - much more expensive than planning