

# Answers to Chapter 1

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## Problem Page 13, P1.1.1.

*Proof.*  $M = \prod_{i=1}^r (A - x_i I) = A^r - \sum_{i=1}^r x_i A^{r-1} + \sum_{1 < x_i < x_j < r} x_i x_j A^{r-2} + \dots + (-1)^r \prod_{i=1}^r x_i I$ .  
So the first column of  $M$  should be the linear combination of each components in the formula above. Now we give an algorithm to compute the first column of  $A^k$ .

### ALGORITHM 1.1.1

Input: A  $n \times n$  matrix  $A$ , an integer  $k$ .

Output: The first column of  $A^1, A^2, \dots, A^k$ .

```
1  T = A
2  for i = 1 to k
3      B[:, i] = T[:, 1]
4      T = A * T[:, 1]
5  return B
```

The time complexity of this algorithm is  $O(k * n^2)$ , and time complexity of calculating the coefficients is  $\sum_{i=0}^r C_r^i = 2^r$ . So the total time cost should be  $O(r * n^2 + 2^r)$ .  $\square$