Numerical Analysis
Assignment 8

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Problem 1. Problem 3.42, Page 193

(a). Consider

$$e^{i\frac{2\pi jk}{m}} = \cos\left(\frac{2\pi jk}{m}\right) + i\sin\left(\frac{2\pi jk}{m}\right).$$

Then if k is a multiple of m, we have

$$\sum_{j=0}^{m-1}\cos\left(\frac{2\pi jk}{m}\right) = \sum_{j=0}^{m-1} 1 = m.$$

Otherwise we have

$$\sum_{j=0}^{m-1}\cos\left(\frac{2\pi jk}{m}\right) + i\sum_{j=0}^{m-1}\sin\left(\frac{2\pi jk}{m}\right) = \sum_{j=0}^{m} e^{i\frac{2\pi jk}{m}} = \frac{1-e^{i\frac{2\pi k}{m}m}}{1-e^{i\frac{2\pi k}{m}}} = 0.$$

Thus

$$\sum_{j=0}^{m-1}\cos\left(\frac{2\pi jk}{m}\right) = \sum_{j=0}^{m-1}\sin\left(\frac{2\pi jk}{m}\right) = 0.$$

(b).

$$\sum_{j=0}^{m-1}\cos\left(\frac{2\pi jk}{m}\right)\cos\left(\frac{2\pi jl}{m}\right) = \frac{1}{2}\sum_{j=0}^{m-1}\left(\cos\left(\frac{2\pi j(k+l)}{m}\right)+\cos\left(\frac{2\pi j(k-l)}{m}\right)\right) = \begin{cases} m,\ k=l=\dfrac{m}{2} \\ \dfrac{m}{2},\ k=l\neq\dfrac{m}{2},\ \text{or } k+l=m, k\neq l \\ 0,\ \text{others}\end{cases}$$

$$\sum_{j=0}^{m-1}\sin\left(\frac{2\pi jk}{m}\right)\sin\left(\frac{2\pi jl}{m}\right) = \frac{1}{2}\sum_{j=0}^{m-1}\left(\cos\left(\frac{2\pi j(k-l)}{m}\right)-\cos\left(\frac{2\pi j(k+l)}{m}\right)\right) = \begin{cases} \dfrac{m}{2},\ k=l\neq\dfrac{m}{2} \\ -\dfrac{m}{2},\ k+l=m, k\neq l \\ 0,\ \text{others}\end{cases}$$

$$\sum_{j=0}^{m-1}\cos\left(\frac{2\pi jk}{m}\right)\sin\left(\frac{2\pi jl}{m}\right) = \frac{1}{2}\sum_{j=0}^{m-1}\left(\sin\left(\frac{2\pi j(k+l)}{m}\right)+\sin\left(\frac{2\pi j(l-k)}{m}\right)\right) = 0.$$

Problem 2. Problem 3.43, Page 194

(a).

$$d_k = \frac{1}{m}\sum_{j=0}^{m-1} w_m^{jk}x_j = \frac{1}{m}\sum_{j=0}^{m-1} e^{-i\frac{2\pi jk}{m}}.$$

The same with Problem 1,

$$d_k = \begin{cases} 1,\ k=0 \\ 0,\ k\neq 0\end{cases}$$

(b).

$$d_k = \frac{1}{m}\sum_{j=0}^{m-1} w_m^{jk}x_j = \frac{1}{m}\sum_{j=0}^{m-1}(-1)^k e^{-i\frac{2\pi jk}{m}} = \frac{1}{m}\frac{1-(-1)^m e^{-i2\pi jk}}{1-(-1)e^{-i\frac{2\pi k}{m}}} = \begin{cases} 1,\ k=\dfrac{m}{2},\ m=2n \\ 0,\ \text{other } k,\ m=2n\end{cases}$$

(c).

$$d_k = \frac{1}{m}\sum_{j=0}^{m-1} je^{-i\frac{2\pi jk}{m}} = \frac{1}{e^{-i\frac{2\pi k}{m}}-1}((m-1)e^{-i2\pi k} - \frac{e^{-i\frac{2\pi k}{m}}(1-e^{-i2\pi k})}{1-e^{\frac{-i2\pi k}{m}}}) = \begin{cases} \frac{m-1}{2},\ k=0 \\ \frac{m-1}{e^{-i\frac{2\pi k}{m}}-1},\ k\neq 0 \end{cases}$$