Assignment 2

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Code can be obtained at https://github.com/orcuslc/Learning/

Problem 1. Given $G(x) = e^{-x}$ and $F(x) = x - e^{-x}$, compare Fixed-point iteration and Newton-Raphson iteration.

Solution. The result is as follows.

Fixed-point iter	Newton-Raphson iter
1.000000000000000	1.000000000000000
0.367879441171442	0.537882842739990
0.692200627555346	0.566986991405413
0.500473500563637	0.567143285989123
0.606243535085597	
0.545395785975027	
0.579612335503379	
0.560115461361089	
0.571143115080177	
0.564879347391050	
0.568428725029061	
0.566414733146883	
0.567556637328283	
0.566908911921495	
0.567276232175570	
0.567067898390788	
0.567186050099357	
0.567119040057215	
0.567157044001298	
0.567135490206278	
0.567147714260119	
0.567140781458298	
0.567144713346570	
0.567142483401307	
Root	Root
0.567143748099411	0.567143290409784

Analysis. When using Newton-Raphson iteration to extract the root of an equation, according to (1.2.22), the error is of second order, while the error of fixed point iteration is of first order. Hence, the speed of convergence is much quicker for Newton-Raphson iteration.

Problem 2. For a n-dimension vector $\mathbf{x}$, when $1 \leqslant p \leqslant q$, prove $\|\mathbf{x}\|_q \leqslant \|\mathbf{x}\|_p \leqslant n^{\frac{1}{p}-\frac{1}{q}}\|\mathbf{x}\|_q$.

Proof. On one hand, assume $\|\mathbf{x}\|_q = 1$, then

$$\|x_i\| \leqslant 1, \quad for \quad 1 \leqslant i \leqslant n$$

Because $1 \leqslant p \leqslant q$, then $\|x_i\|^q \leqslant \|x_i\|^p$. Hence, $(\sum_{i=1}^n \|x_i\|^p)^{\frac{1}{p}} \geqslant (\sum_{i=1}^n \|x_i\|^q)^{\frac{1}{q}}$, which means $\|x\|_q \leqslant \|x\|_p$.
On the other hand, according to $H\ddot{o}lder$ inequation,

$$\frac{(\sum_{i=1}^n \|x_i\|^p)^{\frac{1}{p}}}{n^{\frac{1}{p}}} \leqslant \frac{(\sum_{i=1}^n \|x_i\|^q)^{\frac{1}{q}}}{n^{\frac{1}{q}}},$$

hence

$$\|\mathbf{x}\|_p \leqslant n^{\frac{1}{p}-\frac{1}{q}} \|\mathbf{x}\|_q$$