Answers to Chapter 1

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Problem Page 13, P1.1.1.

Proof. $M = \Pi_{i=1}^{r}(A - x_i I) = A^r - \Sigma_{i=1}^{r} x_i A^{r-1} + \Sigma_{1<x_i<x_j<r} x_i x_j A^{r-2} + ... + (-1)^r \Pi_{i=1}^{r} x_i I$. So the first column of M should be the linear combination of each components in the formula above. Now we give an algorithm to compute the first column of A^k.

Algorithm 1.1.1

Input: A n*n matrix A, an integer k.
Output: The first column of $A^1, A^2, ..., A^k$.

```
1  T = A
2  for i = 1 to k
3        B[:, i] = T[:, 1]
4        T = A * T[:, 1]
5  return B
```

The time complexity of this algorithm is $O(k * n^2)$, and time complexity of calculating the coefficients is $O(C_r^{r/2})$. So the total time cost should be $O(kC_r^{r/2}n^2)$. □