Assignment 4

17.4.1

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Problem 1. Prove Implicit Euler Iteration is of first-order convergence for Dahlquist test.

Proof. According to (2.2.20), $u_n = \frac{1}{(1-a\Delta t)^n}u_0$. For $e_n = u(t_n) - u_n$, we have $e_n = u_0(e^{at_n} - e^{-nln(1-a\Delta t)})$. From Taylor expansion we can know that

$$e_n = -u_0 e^{at_n}(\frac{a^2 t_n}{2}\Delta t + O(\Delta t^2)).$$

In consequence, for a fixed $T = N\Delta t$, $\|u(T) - u(N)\| \leqslant C\Delta t$, which means the implicit iteration is of first-order convergence.

Problem 2. Compute $\frac{du}{dt} = au$ with Euler Iterations, and show the convergence by image.

Result.

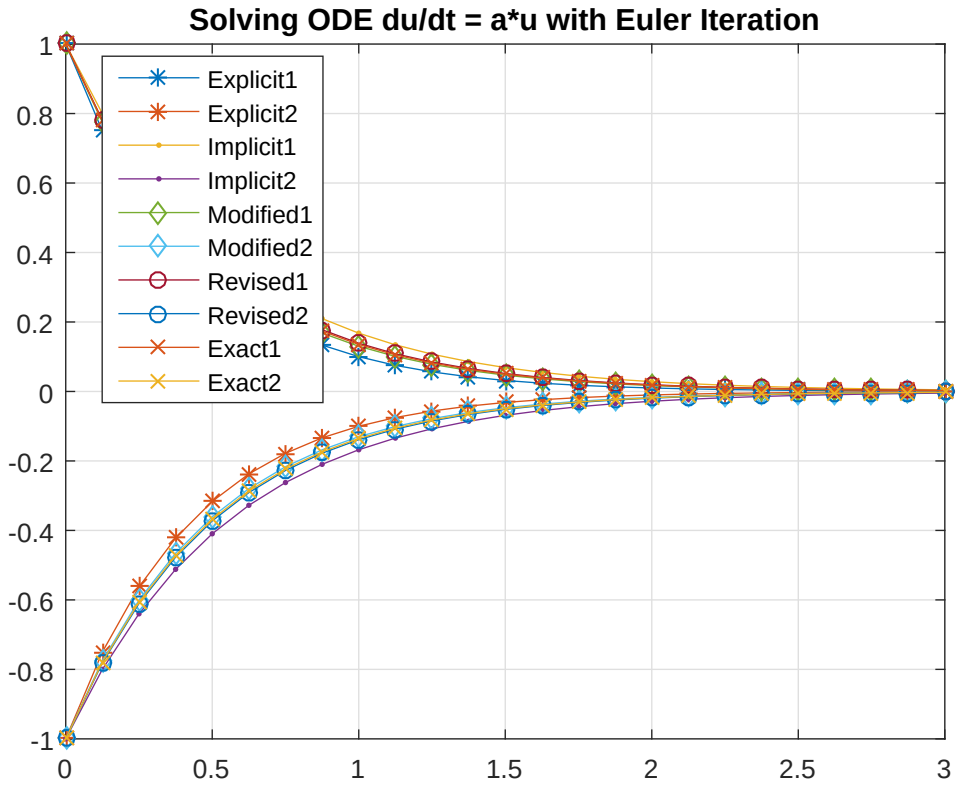

Figure 1: The convergence of Euler Iteration.

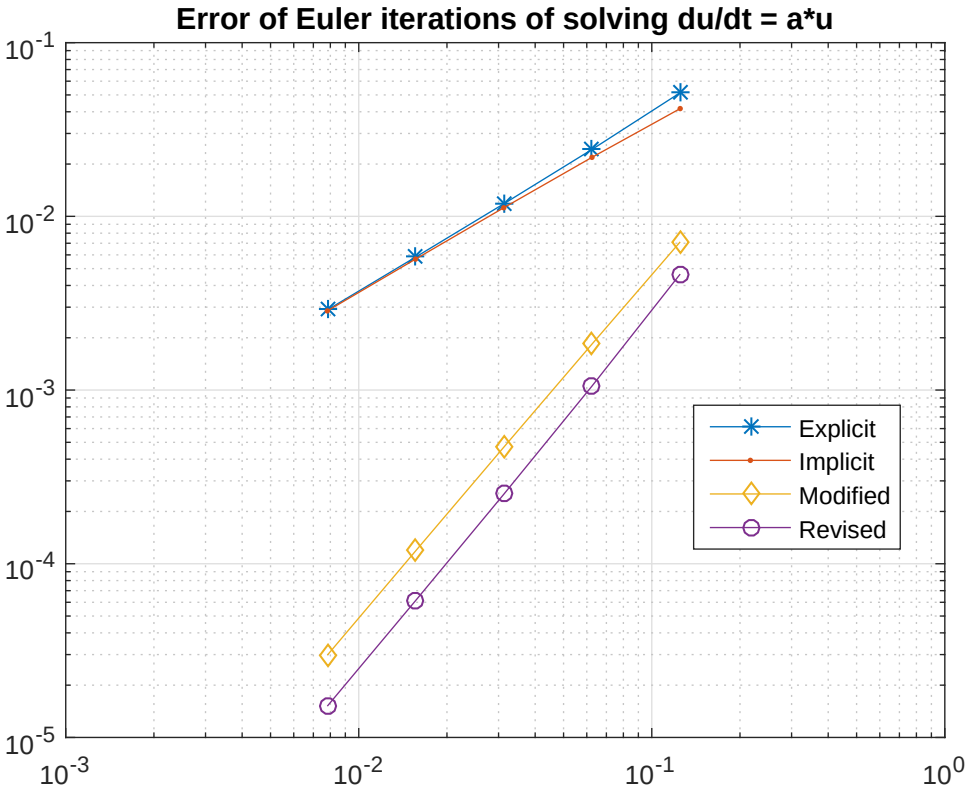

Figure 2: The error of Euler Iteration.