

Numerical Analysis

Assignment 9

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Problem 1. Problem 4.1, Page 239.

Solution. When $n = 4$,

$$\begin{aligned}p_4(x) &= \sum_{k=0}^4 C_4^k f\left(\frac{k}{4}\right) x^k (1-x)^{4-k} = f(0)(1-x)^4 + 4f\left(\frac{1}{4}\right)x(1-x)^3 + 6f\left(\frac{1}{2}\right)x^2(1-x)^2 + 4f\left(\frac{3}{4}\right)x^3(1-x) + f(1)x^4 \\&= (6 - 4\sqrt{2})x^4 + 8\sqrt{2}x^3 - (12 + 6\sqrt{2})x^2 + 2\sqrt{2}x + 6 \\&= (6 - 4\sqrt{2})\left(x - \frac{1}{2}\right)^4 - 3\left(x - \frac{1}{2}\right)^2 + 3 + \frac{\sqrt{2}}{4}\end{aligned}$$

And the fourth degree Taylor polynomial expanded about $\frac{1}{2}$ is

$$\begin{aligned}q_4(x) &= f\left(\frac{1}{2}\right) + f'\left(\frac{1}{2}\right)\left(x - \frac{1}{2}\right) + \frac{1}{2}f''\left(\frac{1}{2}\right)\left(x - \frac{1}{2}\right)^2 + \frac{1}{6}f^{(3)}\left(\frac{1}{2}\right)\left(x - \frac{1}{2}\right)^3 + \frac{1}{24}f^{(4)}\left(\frac{1}{2}\right)\left(x - \frac{1}{2}\right)^4 \\&= 1 - \frac{1}{2}\pi^2\left(x - \frac{1}{2}\right)^2 + \frac{1}{24}\pi^4\left(x - \frac{1}{2}\right)^4\end{aligned}$$

When $x \rightarrow \frac{1}{2}$, $q_4(x) \rightarrow 1 = f\left(\frac{1}{2}\right)$, while $p_4(x) \rightarrow 3 + \frac{\sqrt{2}}{4}$. Thus Bernstein polynomials are poor approximations.

Problem 2. Problem 4.5, Page 240

Problem 3. Problem 4.6, Page 240

Problem 4. Problem 4.10, Page 241

(a). The linear Taylor polynomial to $f = \ln(x)$ expanding about $\frac{3}{2}$ is

$$p_1(x) = \frac{3}{2} + \frac{2}{3}\left(x - \frac{3}{2}\right) = \frac{2}{3}x + \frac{1}{2}.$$

The error of is as follows, the left is error graph for (a) and the right is for (b).

(b). The linear minimax approximation to f is

$$p_2(x) = ax + b.$$

Then there exists x_0 , s.t.

$$\ln(1) - (a + b) = \ln(2) - (2a + b) = -(\ln(x_0) - (ax_0 + b)) = \rho.$$

and

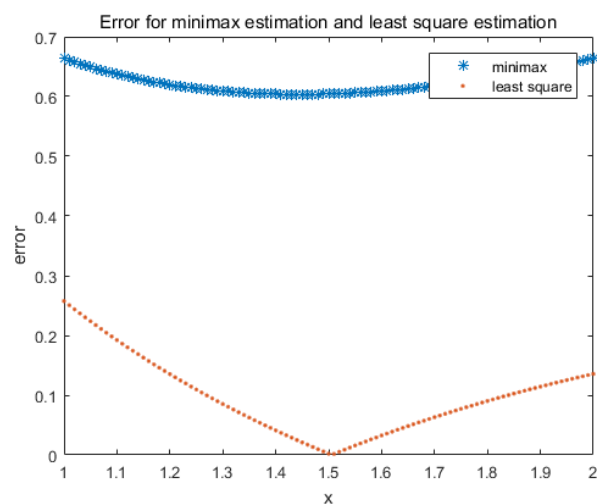
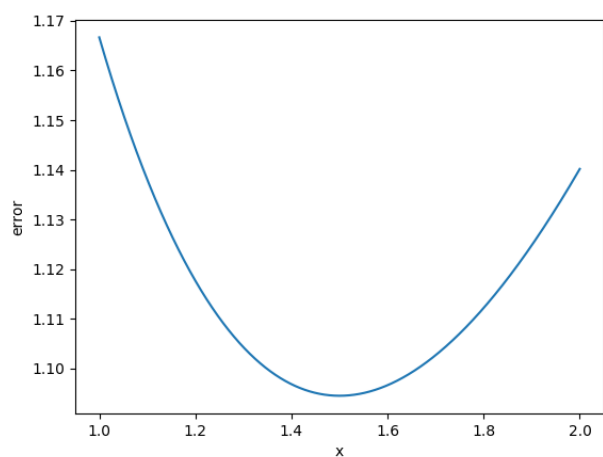
$$(\ln(x) - (ax + b))'|_{x_0} = 0$$

Then we have

$$a = \ln(2), \quad x_0 = \frac{1}{\ln(2)}, \quad b = \frac{1}{2}(\ln\left(\frac{1}{2}\ln(2)\right) + 1),$$

thus

$$p_2(x) = \ln(2)x + \frac{1}{2}(\ln\left(\frac{1}{2}\ln(2)\right) + 1)$$



Problem 5. Problem 4.12, Page 241

Solution. The linear least square approximation to f is

$$q(x) = ax + b.$$

Then

$$E = \int_1^2 (\ln(x) - ax - b)^2 dx,$$

and

$$\frac{\partial E}{\partial a} = \int_1^2 (-2x)(\ln(x) - ax - b) dx = 0,$$

$$\frac{\partial E}{\partial b} = \int_1^2 (-2)(\ln(x) - ax - b) dx = 0.$$

Then

$$\begin{cases} \frac{3}{2}a + \frac{3}{2}b = 2\ln(2) - 1 \\ \frac{14}{3}a + 3b = 4\ln(2) - 2 \end{cases}$$

solving this equations, we get

$$a = 0.3, \quad b = \frac{4}{3}\ln(2) - \frac{29}{30}.$$