Assignment 2 16.10.2

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Code of this assignment are in the attachment, and they can also be found in GitHub: https://github.com/Orcuslc/Learning/tree/master/Numerical%20Algorithm%20and%20Case%20Studies/Homework2/code

Problem 1 & 2.

Abstract.

Details of the implementation of LU decomposition in MATLAB.

Introduction.

In most cases pivoting is necessary. If we randomly choose a matrix $M \in \mathbb{R}^{n\times n}$, the probability that $M(1,1) = max_i M(i,1)$ is $\frac{1}{n}$. So the efficiency of pivoting is a great concern.

In the experiment below, we generate the matrix M by using MATLAB function $\mathrm{randn(n,\ n)}$; and we use seperately

$$\mathrm{[L,\ U] = lu(A)}$$

$$\mathrm{[L,\ U,\ P] = lu(A)}$$

for the two problems.

Results & Discussion. After trying for several times with different n, the dimension of the random matrix, we found that in most cases, calling $\mathrm{[L,\ U] = lu(A)}$ leads to L not being a lower triangular matrix, but U is always an upper triangular matrix.

When we count the number of non-zero elements of each row in L, we can find out the list of the numbers is a permutation of $\mathrm{[1,\ 2,\ ...,\ n]}$.

Thus I tried to call $\mathrm{[L1,\ U1,\ P] = lu(A)}$, and the P is exactly the permutation matrix:

$$PL1 = L$$

. So it is obvious how MATLAB express the P: it simply multiplied it with the L; In order to represent this, I guess that MATLAB records the number of lines that need interchanging in each step; when the algorithm ends, it conbines the records to gain a permutation matrix.

In order to realize $P*b$, we should use the pernutation record: in each of the n steps, change the elements of two lines, and it will only need $O(n)$ time, and no extra space.

Problem 3.

Abstract & Introduction. As is known to all, using Cholesky decomposition for a symmetric positive definite matrix will need half of the flops comparing LU decomposition; In the mean time, since there is no need of pivoting for a symmetric positive definite matrix, in each step $n-1$ times of comparing and $2n$ times of exchanging will be saved on average.

Result. We generate the symmetric matrix A by

$$\mathrm{A = randn(n,\ n);\ A = A + A';}$$

and we did the experiment with $n = 4, 6, 8, 10$. The result of calling

$$\mathrm{L = chol(A)}$$

is:

$$\text{Error using chol: Matrix must be positive definite.}$$

Analysis. If $A \in \mathbb{R}^{n\times n}$ is not positive definite, then according to the properties of symmetric elementary transformations, $PAP^\top$ is still not positive definite, where P is a permutation matrix.

Thus, we can assume $A(1:2, 1:2)$ is not positive definite. According to the algorithm, we have

$$l_{11} = \sqrt{a_{11}},$$

$$l_{21} = \frac{a_{21}}{l_{11}},$$

$$l_{22} = (a_{22} - l_{21}^2)^{1/2}.$$

That means $l_{22} = a_{22} - \frac{a_{21}^2}{a_{11}} = \frac{a_{22}a_{11} - a_{21}^2}{a_{11}}$. Since A is symmetric and not positive definite, we have $a_{22}a_{11} - a_{21}^2 = a_{22}a_{11} - a21a_{12} < 0$. So l_{22} does not exist.

Problem 4.

Introduction. The $LDL^\top$ decomposition is used for Hermitian indefinite matrices.

Discussion. I found an introduction to $LDL^\top$ decomposition from Golub's *Matrix Computations*, Page 186. It said the method was developed by Bunch and Parlett. I searched their paper in Google Scholar, but it seemed that our school has no access to siam.org, so I cannot read the paper.

Reference

- Gene H. Golub *et al*, *Matrix Computations*, Page 186-187.