Assignment 7 17.4.26

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Problem 1. Derive A-B, A-M and Gear Formula with Newton Interpolation.

Solution. With Newton Interpolation on $t_{n+1}, t_n, \cdots, t_{n+1-k}$, $f(t,u) = f_{n+1-k} + f_{n+1-k,n+2-k}(t - t_{n+1-k}) + \cdots + f_{n+1-k,n+2-k,\cdots,n+1}\Pi_{i=1}^{k}(t - t_{n-k+i}) + f_{n+1-k,n+2-k,\cdots,n+1,t}\Pi_{i=1}^{k+1}(t - t_{n-k+i})$.
Let $p_{n,k}(t) = f(t,u) - f_{n+1-k,n+2-k,\cdots,n+1,t}\Pi_{i=1}^{k+1}(t - t_{n-k+i})$, Integrate within $[t_n, t_{n+1}]$, $u_{n+1} - u_n = \Delta t \Sigma_{i=0}^{k} b_{k,i} f_i$.

Adams-Bashforth Formula

Like Adams-Moulton formula, with Newton interpolation on $t_n, t_{n-1}, \cdots, t_{n-k}$, $f(t,u) = f_{n-k} + f_{n-k,n+1-k}(t - t_{n-k}) + \cdots + f_{n-k,n+1-k,\cdots,n}\Pi_{i=0}^{k-1}(t - t_{n-k+i}) + f_{n-k,n+1-k,\cdots,n,t}\Pi_{i=0}^{k}(t - t_{n-k+i})$.
Let $p_{n,k}(t) = f(t,u) - f_{n-k,n+1-k,\cdots,n,t}\Pi_{i=0}^{k}(t - t_{n-k+i})$, Integrate within $[t_n, t_{n+1}]$, $u_{n+1} - u_n = \Delta t \Sigma_{i=0}^{k} b_{k,i} f_i$.

Gear Formula

Let $I_k u = u_{n-k+1} + u_{n-k+1,n-k+2}(t - t_{n-k+1}) + \cdots + u_{n-k+1,n-k+2,\cdots,n+1}\Pi_{i=n-k+1}^{n}(t - t_i)$.
Use derivative of $I_n u(t)$ to take the place of the derivative of $u(t)$ when $t = t_{n+1}$, there exists

$$f(t,u)|_{t=t_{n+1}} = \frac{du}{dt}|_{t=t_{n+1}} = \frac{dI_k u}{dt}|_{t=t_{n+1}}$$

.
In consequence, $\Delta t f_{n+1} = (\Delta t)^2 (u_{n-k+1} + u_{n-k+1,n-k+2}(t - t_{n-k+1}) + \cdots + u_{n-k+1,n-k+2,\cdots,n+1}\Pi_{i=n-k+1}^{n}(t - t_i))$.

Problem 2. Consider the equation

$$\frac{du}{dt} = \lambda(-u + cos(t))$$

Solution.

The exact solution of the equation.

The exact solution is $u(t) = c_0 e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)} \lambda cos(s) ds$.

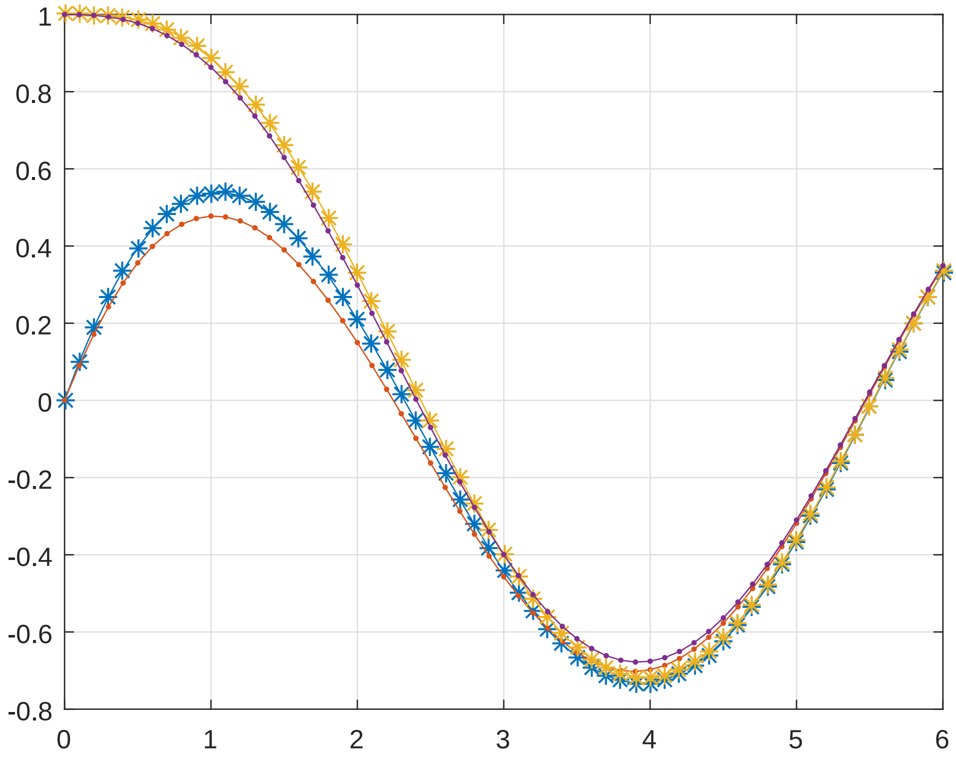

Figure 1: lambda = 1

For $\lambda = 1, 10, 100, 1000,$ **use explicit and implicit Euler iteration to solve.**

Use Adams and Gear iteration to solve the equation when $\lambda = 1000$**.**

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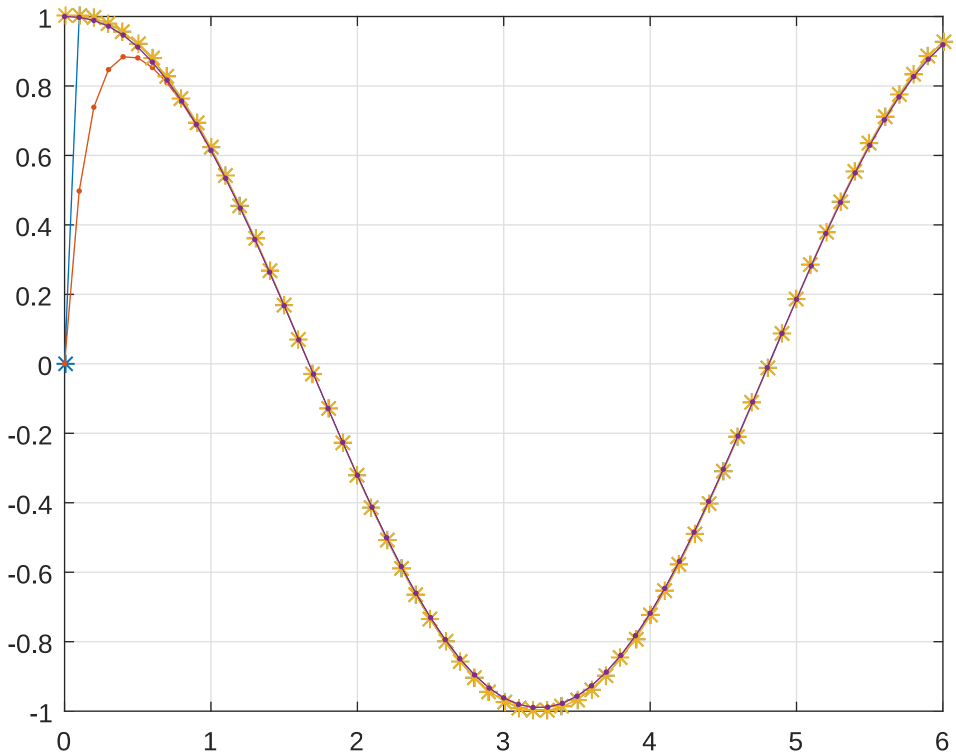

Figure 2: lambda = 10

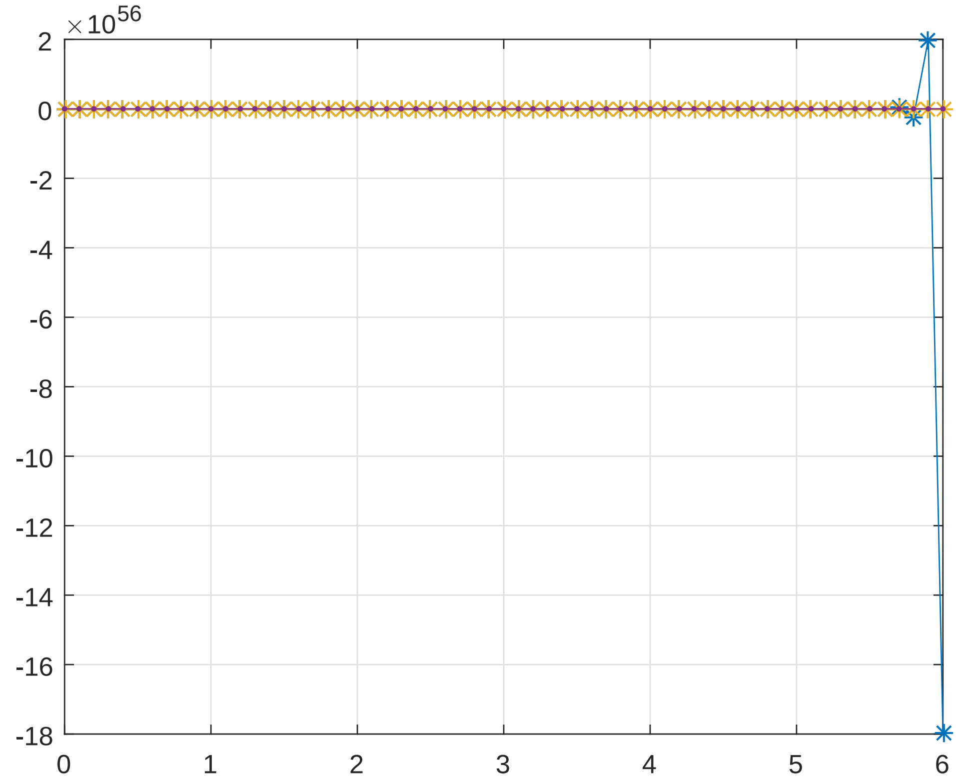

Figure 3: lambda = 100

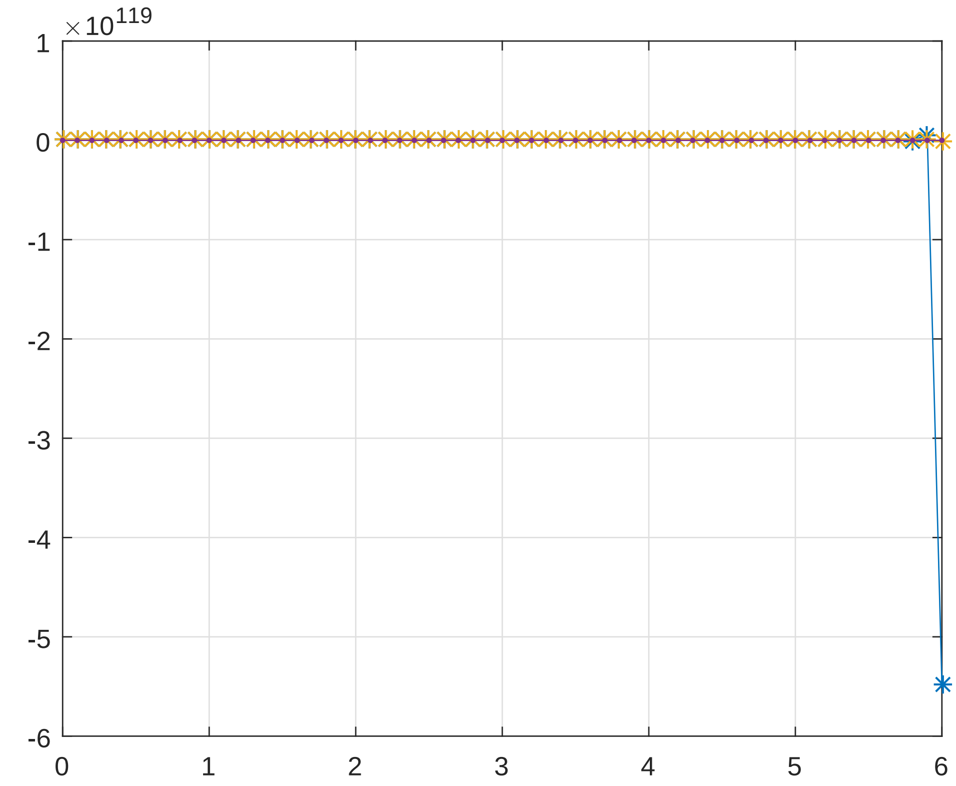

Figure 4: lambda = 1000

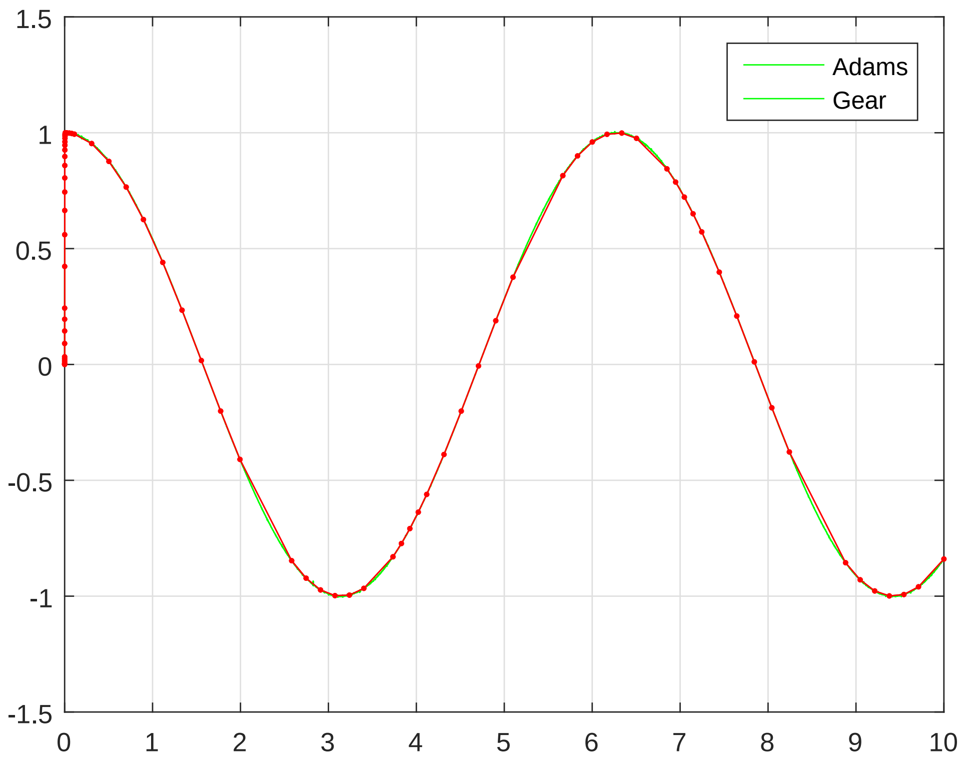

Figure 5: Adams and Gear