Homework 2016-03-02

Chuan Lu
13300180056

March 2, 2016

Problem 1.
When $x \to 0$, will the result of $f(x) = \frac{1-cos(x)}{x^2}$ be far from 0.5?

Proof. The code is shown as follows.

```
% homework1
% author: chuanlu
% 2016-03-02
format long
xx = 10 .^ [-1:-1:-16];
yy = (1 - cos(xx)) ./ (xx .^ 2);
semilogx(xx, yy);
```

The result is shown as follows. □

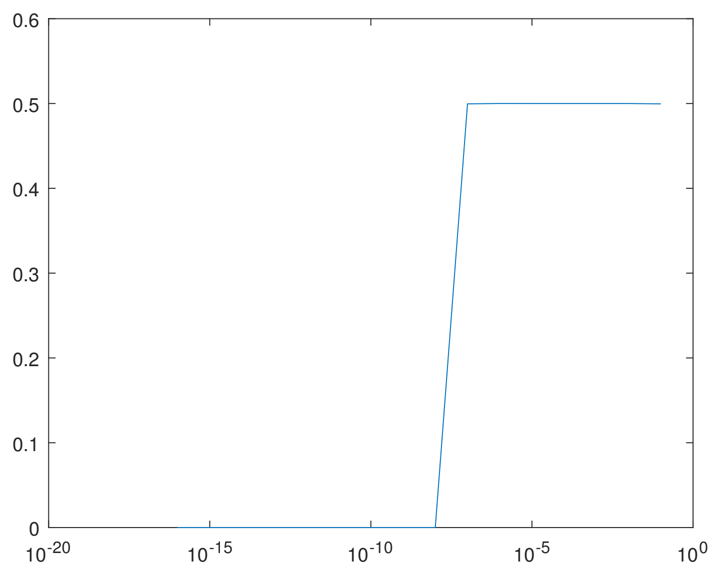

Figure 1: problem1

Problem 2.
Check the relationship of the conds of the two matrixs with their dims.

Proof. The code is shown as follows, which means the statement is proved wrong.

```
% make matrix
% author: chuanlu
% 2016-03-02
function [A] = make_matrix(op, n)

    if nargin < 1
        error('More args needed --make matrix');
    elseif nargin == 1
        op = 'A1';
    end

    if op == 'A1'
        c1 = zeros(1, n - 1);
        c1(2 : n - 1) = -3;
        c2 = zeros(1, n - 2) + 2;
        A = eye(n) + diag(c1, -1) + diag(c2, -2);
    elseif op == 'A2'
        c1 = zeros(1, n - 1);
        c1(2 : n - 1) = -3;
        c2 = zeros(1, n - 2) + 2;
        A = eye(n) + diag(c1, -1) + diag(c2, -2);
        A(1, n) = -1;
    else
        error('Operation Failed to Match A1 or A2');
    end
```

```
% homework2.m
% author: chuanlu
% 2016-03-02

op1 = 'A1';
op2 = 'A2';
N = 100;
cond1 = zeros(1, N);
cond2 = zeros(1, N);
for n = 1 : N
    A1 = make_matrix(op1, n);
    A2 = make_matrix(op2, n);
    cond1(n) = cond(A1);
    cond2(n) = cond(A2);
end
```

```
n = [1:N];
figure(1);
semilogy(n, cond1, '*-');
figure(2);
plot(n, cond2, '*-');
%       disp('cond1:');
%       disp(cond1);
%       disp('cond2:');
%       disp(cond2);
```

The result is shown as follows. $\square$

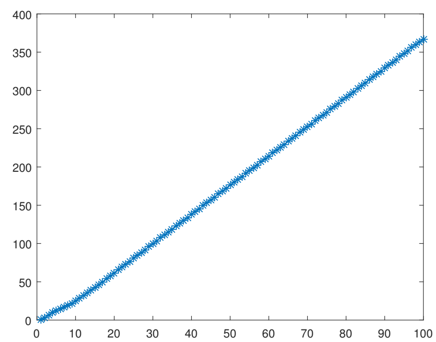

Figure 2: The relationship of cond(A_2) with dim(A_2), problem2-1

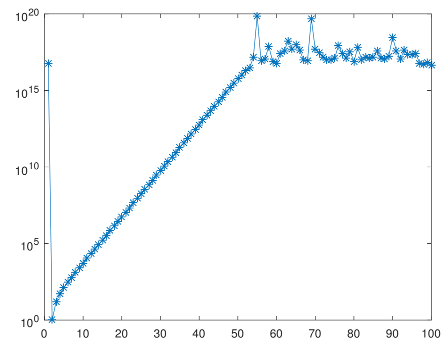

Figure 3: The relationship of cond(A_1) with dim(A_1), problem2-2

Problem 3.
Given $\|x_{n+2} - x_{n+1}\| \leqslant \alpha \|x_{n+1} - x_n\|$, then $\|x^\star - x_n\| \leqslant \frac{\alpha_n}{1-\alpha}\|x_1 - x_0\|$.

Proof. $\|x_n - x_{n-1}\| \leqslant \alpha \|x_{n-1} - x_{n-2}\| \leqslant \cdots \leqslant \alpha^{n-1}\|x_1 - x_0\|$

Hence, $\|x_\star - x_n\| \leqslant \|x_n - x_{n+1}\| + \|x_{n+1} - x_{n+2}\| + \cdots \leqslant (\alpha^n + \alpha^{n+1} + \dots)\|x_1 - x_0\|$

$= \frac{\alpha^n}{1-\alpha}\|x_1 - x_0\|$ $\square$