# Answers to Chapter 1

## Chuan Lu

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**Problem Page 13, P1.1.1.**

*Proof.* $M = \Pi_{i=1}^{r}(A - x_i I) = A^r - \Sigma_{i=1}^{r} x_i A^{r-1} + \Sigma_{1 < x_i < x_j < r} x_i x_j A^{r-2} + ... + (-1)^r \Pi_{i=1}^{r} x_i I$.
So the first column of M should be the linear combination of each components in the formula above. Now we give an algorithm to compute the first column of $A^k$.

ALGORITHM 1.1.1

   Input: A n*n matrix A, an integer k.
   Output: The first column of $A^1, A^2, ..., A^k$.
1   T = A
2   **for** $i = 1$ **to** $k$
3       $B[:, i] = T[:, 1]$
4       $T = A * T[:, 1]$
5   **return** $B$

The time complexity of this algorithm is $O(k * n^2)$, and time complexity of calculating the coefficients is $\Sigma_{i=0}^{r} C_r^i = 2^r$. So the total time cost should be $O((r + 2) * n^2 + 2^r)$. □

**Problem Page 13, P1.1.3.**

*Proof.* $(xy^T)^k = x((x^T y)^T)^{k-1} y^T = ((x^T y)^T)^{k-1} xy^T$. To calculate $((x^T y)^T)^{k-1}$, time cost is $O(n^2)$; for $xy^T$ time cost is $O(n^2)$. So total time cost is $O(n^2)$. □