

Homework 2016-03-02

Chuan Lu
13300180056

March 4, 2016

Problem 1.

When $x \rightarrow 0$, will the result of $f(x) = \frac{1-\cos(x)}{x^2}$ be far from 0.5?

Proof. The code is shown as follows.

```
1 % homework1
2 % author: chuanlu
3 % 2016-03-02
4 format long
5 xx = 10 .^ [-1:-1:-16];
6 yy = (1 - cos(xx)) ./ (xx .^ 2);
7 semilogx(xx, yy);
```

The result is shown as follows.

□

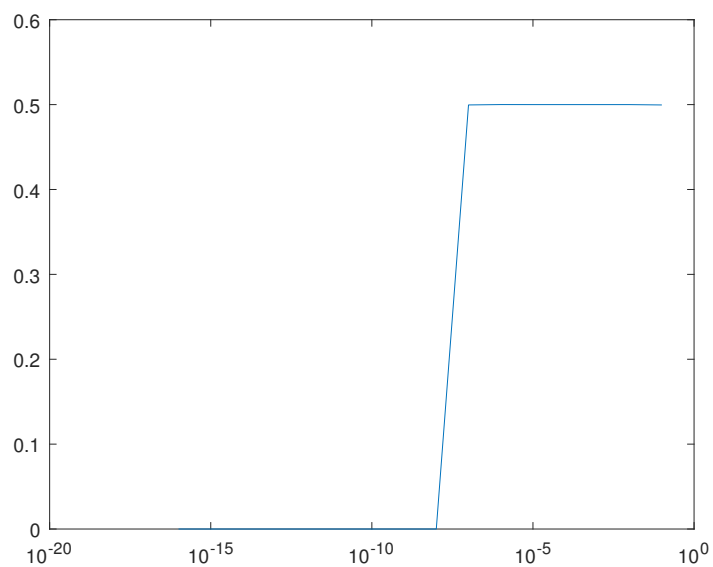


Figure 1: problem1

Problem 2.

Check the relationship of the conds of the two matrixs with their dims.

Proof. The code is shown as follows, which means the statement is proved wrong.

```
1 % make matrix
2 % author: chuanlu
3 % 2016-03-02
4 function [A] = make_matrix(op, n)
5
6     if nargin < 1
7         error('More args needed ---make matrix');
8     elseif nargin == 1
9         op = 'A1';
10    end
11
12    if op == 'A1'
13        c1 = zeros(1, n - 1);
14        c1(2 : n - 1) = -3;
15        c2 = zeros(1, n - 2) + 2;
16        A = eye(n) + diag(c1, -1) + diag(c2, -2);
17    elseif op == 'A2'
18        c1 = zeros(1, n - 1);
19        c1(2 : n - 1) = -3;
20        c2 = zeros(1, n - 2) + 2;
21        A = eye(n) + diag(c1, -1) + diag(c2, -2);
22        A(1, n) = -1;
23    else
24        error('Operation Failed to Match A1 or A2');
25    end
```

```
1 % homework2.m
2 % author: chuanlu
3 % 2016-03-02
4
5 op1 = 'A1';
6 op2 = 'A2';
7 N = 100;
8 cond1 = zeros(1, N);
9 cond2 = zeros(1, N);
10 for n = 1 : N
11     A1 = make_matrix(op1, n);
12     A2 = make_matrix(op2, n);
13     cond1(n) = cond(A1);
14     cond2(n) = cond(A2);
15 end
```

```

16 n = [1:N];
17 figure(1);
18 semilogy(n, cond1, '*-');
19 figure(2);
20 plot(n, cond2, '*-');
21 % disp('cond1: ');
22 % disp(cond1);
23 % disp('cond2: ');
24 % disp(cond2);

```

The result is shown as follows. □

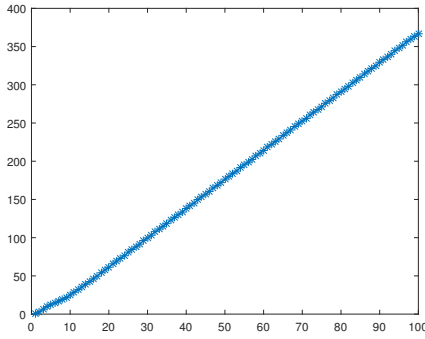


Figure 2: The relationship of $\text{cond}(A_2)$ with $\dim(A_2)$, problem2-1

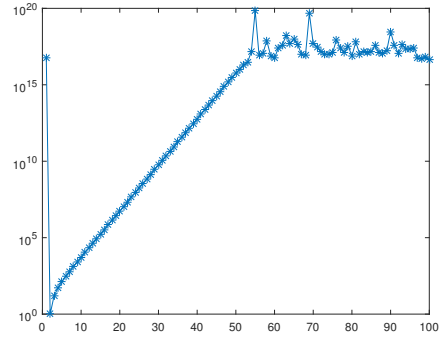


Figure 3: The relationship of $\text{cond}(A_1)$ with $\dim(A_1)$, problem2-2

Problem 3.

Given $\|x_{n+2} - x_{n+1}\| \leq \alpha \|x_{n+1} - x_n\|$, then $\|x^* - x_n\| \leq \frac{\alpha^n}{1-\alpha} \|x_1 - x_0\|$.

Proof. $\|x_n - x_{n-1}\| \leq \alpha \|x_{n-1} - x_{n-2}\| \leq \dots \leq \alpha^{n-1} \|x_1 - x_0\|$

Hence, $\|x^* - x_n\| \leq \|x_n - x_{n+1}\| + \|x_{n+1} - x_{n+2}\| + \dots \leq (\alpha^n + \alpha^{n+1} + \dots) \|x_1 - x_0\|$

$$= \frac{\alpha^n}{1-\alpha} \|x_1 - x_0\|$$

□