# Numerical Analysis
# Assignment 8

Chuan Lu

October 17, 2017

**Problem 1.** Problem 3.42, Page 193

***(a).*** Consider

$$e^{i\frac{2\pi jk}{m}} = \cos\left(\frac{2\pi jk}{m}\right) + i\sin\left(\frac{2\pi jk}{m}\right).$$

Then if $k$ is a multiple of $m$, we have

$$\sum_{j=0}^{m-1}\cos\left(\frac{2\pi jk}{m}\right) = \sum_{j=0}^{m-1} 1 = m.$$

Otherwise we have

$$\sum_{j=0}^{m-1}\cos\left(\frac{2\pi jk}{m}\right) + i\sum_{j=0}^{m-1}\sin\left(\frac{2\pi jk}{m}\right) = \sum_{j=0}^{m} e^{i\frac{2\pi jk}{m}} = \frac{1-e^{i\frac{2\pi k}{m}m}}{1-e^{i\frac{2\pi k}{m}}} = 0.$$

Thus

$$\sum_{j=0}^{m-1}\cos\left(\frac{2\pi jk}{m}\right) = \sum_{j=0}^{m-1}\sin\left(\frac{2\pi jk}{m}\right) = 0.$$

***(b).***

$$\sum_{j=0}^{m-1}\cos\left(\frac{2\pi jk}{m}\right)\cos\left(\frac{2\pi jl}{m}\right) = \frac{1}{2}\sum_{j=0}^{m-1}\left(\cos\left(\frac{2\pi j(k+l)}{m}\right) + \cos\left(\frac{2\pi j(k-l)}{m}\right)\right) = \begin{cases} m, & k=l=\frac{m}{2} \\ \frac{m}{2}, & k=l\neq\frac{m}{2}, \text{ or } k+l=m, k\neq l \\ 0, & \text{others}\end{cases}$$

$$\sum_{j=0}^{m-1}\sin\left(\frac{2\pi jk}{m}\right)\sin\left(\frac{2\pi jl}{m}\right) = \frac{1}{2}\sum_{j=0}^{m-1}\left(\cos\left(\frac{2\pi j(k-l)}{m}\right) - \cos\left(\frac{2\pi j(k+l)}{m}\right)\right) = \begin{cases} \frac{m}{2}, & k=l\neq\frac{m}{2} \\ -\frac{m}{2}, & k+l=m, k\neq l \\ 0, & \text{others}\end{cases}$$

$$\sum_{j=0}^{m-1}\cos\left(\frac{2\pi jk}{m}\right)\sin\left(\frac{2\pi jl}{m}\right) = \frac{1}{2}\sum_{j=0}^{m-1}\left(\sin\left(\frac{2\pi j(k+l)}{m}\right) + \sin\left(\frac{2\pi j(l-k)}{m}\right)\right) = 0.$$

**Problem 2.** Problem 3.43, Page 194

***(a).***

$$d_k = \frac{1}{m}\sum_{j=0}^{m-1} w_m^{jk}x_j = \frac{1}{m}\sum_{j=0}^{m-1} e^{-i\frac{2\pi jk}{m}}.$$

The same with Problem 1,

$$d_k = \begin{cases} 1, & k=0 \\ 0, & k\neq 0\end{cases}$$

***(b).***

$$d_k = \frac{1}{m}\sum_{j=0}^{m-1} w_m^{jk}x_j = \frac{1}{m}\sum_{j=0}^{m-1}(-1)^k e^{-i\frac{2\pi jk}{m}} = \frac{1}{m}\frac{1-(-1)^m e^{-i2\pi jk}}{1-(-1)e^{-i\frac{2\pi k}{m}}} = \begin{cases} 1, & k=\frac{m}{2}, \; m=2n \\ 0, & \text{other } k, \; m=2n\end{cases}$$

***(c).***

$$d_k = \frac{1}{m}\sum_{j=0}^{m-1} je^{-i\frac{2\pi jk}{m}} = \frac{1}{e^{-i\frac{2\pi k}{m}}-1}((m-1)e^{-i2\pi k} - \frac{e^{-i\frac{2\pi k}{m}}(1-e^{-i2\pi k})}{1-e^{\frac{-i2\pi k}{m}}}) = \begin{cases} \frac{m-1}{2}, & k=0 \\ \frac{m-1}{e^{-i\frac{2\pi k}{m}}-1}, & k \neq 0 \end{cases}$$

**Problem 3.** Consider approximating $f(x) = e^{\sin(x)}$ on $[0, 2\pi]$, using the triogometric interpolation $P_n(t)$. Create an maximum error table for $n = 1, \cdots, n$.

```
function yy = trigonometric(t, y, xx)
% Trigonometric Interpolation
% t, y: Interpolation points and function values;
% xx: points to evaluate function values;
n = (length(t)-1)/2;
M = zeros(2*n+1, 2*n+1);
for i = 1:(2*n+1)
    M(i, :) = exp(1i*t(i)).^(-n:1:n);
end
C = M\y;

xx = exp(1i*xx);
yy = ones(length(xx), 1)*C(end);
for i = (2*n):-1:1
    yy = yy.*xx+C(i);
end
yy = yy./(xx.^n);
```

```
n = 11;
error = zeros((n-1)/2, 1);
xx = (0:1e-4:2*pi)';
plot(xx, exp(sin(xx)));
hold on;
for i = 1:2:n
    t = (0:(2*pi/(i-1)):2*pi)';
    y = exp(sin(t));
    yy = trigonometric(t, y, xx);
    error((i+1)/2) = max(abs(yy-exp(sin(xx))));
    plot(xx, yy);
    hold on;
end
legend('f', 'n=1', 'n=3', 'n=5', 'n=7', 'n=9', 'n=11', '
   Location', 'Best');
```

```
>> error

error =

    1.7183
    1.7183
    0.3823
    0.4854
```

```
       0.2799
       0.1285
```