

**Problem 1.** Explain the relationship between spectral clustering, normalized spectral clustering and graph cut.

**Solution.** We assume  $k = 2$  in the following explanation.

For spectral clustering, we already know that the eigenvector  $u_2$  corresponding to the second smallest eigenvalue  $\lambda_2$  of the Laplacian matrix of the graph is the solution for the minimization

$$\begin{aligned} \min_{\mathbf{f}} \quad & \mathbf{f}^\top \mathbf{L} \mathbf{f}, \\ \text{s.t.} \quad & \mathbf{f}^\top \mathbf{f} = 1, \mathbf{f}^\top \mathbf{1} = 0 \end{aligned}$$

In fact, if we define

$$G(f) = G(f_1, \dots)$$

For Min Cut, if we define  $\mathbf{f} = (f_1, f_2 \cdots f_n)^\top$