Numerical Analysis
Assignment 1

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Problem 1. Problem 1.1, Page 43

(a). In fact,

$$\frac{1}{n}n\inf_{a\le x\le b}f(x)\le\frac{1}{n}\sum_{i=1}^{n}f(i)\le\frac{1}{n}n\sup_{a\le x\le b}f(x).$$

So according to intermediate theorem, there exists $\zeta\in[a,b]$, $S=f(\zeta)$.

(b). The proposition now becomes as this:

$$S=\sum_{i=1}^{n}w_if(x_i)=f(\zeta)\sum_{i=1}^{n}w_i,$$

for some $\zeta\in[a,b]$. The proof is just the same as (a).

Problem 2. Problem 1.2, Page 43

(a). I believe without the condition that x,z are in the neighbourhood of 0, it can not be derived by only Taylor series.
Fix $z<0$, let $f(t)=|e^(t+z)-e^z|-|t|$. When $t>0$, $f'(t)=e^(t+z)-1$. So $f(t)$ reaches it's maximum when $t=-z$, and $\max f(t)=0$. It is similar when $t<0$. With the arbitrariness of z, we can derive the inequality.

(b). Similar with (a), we choose to use other methods. Fix z, denote $t=x-z$, let $f=|\tan(z+t)-\tan(z)|-|t|$. When $t>0$, we have $f'(t)=\frac{1}{\cos^2(z+t)}-1>=0$, so $\min f=0$. Similarly when $t<0$, we have $\min f=0$. So $|x-z|\le|\tan(x)-\tan(z)|$.

(c). Denote $\xi=x-y$, then

$$x^p-y^p=(y+\xi)^p-y^p=py^{p-1}\xi+R,\ R>0.$$

and we have

$$x^p-y^p=x^p-(x-\xi)^p=px^{p-1}\xi+R,\ R<0.$$

So the inequality stands.

Problem 3. Problem 1.4, Page 44

Proof. According to Integral Mean Value theorem,

$$\int_0^h x^2(h-x)^2g(x)dx=g(\xi)\int_0^h x^2(h-x)^2dx=\frac{1}{30}h^5g(\xi),$$

for some $\xi\in[0,h]$.

Problem 4. Problem 1.5, Page 44

(a).

$$\frac{1}{x}\int_0^x e^{-t^2}dt=$$