Homework 2016-04-01

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April 6, 2016

Problem 1.
Give an example of expotional and polynomial increase of error.

Proof. **0.1 Expotional increase**

Let the iteration formula be as Page 98, Example 2.5.7, in which $u_{n+2} - 3u_{n+1} + 2u_n = \frac{\Delta t}{12}(7f_{n+2} - 8f_{n+1} - 11f_n)$. Use this formula to solve the function shown in the same example, $u' = 0$, we can get the iteration

$$u_{n+2} - 3u_{n+1} + 2u_n = 0.$$

As $u_{n+2} - u_{n+1} = 2(u_{n+1} - u_n)$, assume the initiate error is δ, then $\delta_n = O(2^n\delta)$.

0.2 Polynomial increase

Let the iteration formula be like $u_{n+4} - 2u_{n+3} + 3u_{n+2} - 2u_{n+1} + u_n = \Delta t \Sigma \beta_j f_{n+j}$, then according to Page 99, (2.5.20), the error should be polynomial. □