Homework 4

Instructions: In problems the problems below, references such as III.2.7 refer to Problem 7 in Section 2 of Chapter III in Conway's book.

If you use results from books including Conway's, please be explicit about what results you are using.

Homework 4 is due in class at Midnight March 9.

Do the following problems:

1. IV.7.1

   Sol. (Discussed with a college classmate) In fact, I don't think that this proposition is correct. For example, pick G the unit disk $B(0,1)$, and $\gamma = \gamma(t) : [0,1] \to B$, s.t. $\gamma(t) = t$ for $0 \le t < 1$, and $\gamma(1) = 0$. Then γ is closed, and by simple calculation we know $V(\gamma) = 2$, which shows γ is rectifiable. Let $f = \frac{1}{z-1}$, then f is analytic in $B(0,1)$. But when $t \to 1$, $f \circ \gamma(t) \to \infty$, hence it is not rectifiable.

2. IV.7.2

   (a) Let $f(z) = z$, pick any $z_0 \in \{z \mid d(z, \partial G) < \frac{1}{2}r\}$, then since there is only one point $z = z_0$ satisfies $f(z) = z_0$, by Thm 7.2,
   $$n(\gamma; z_0) = \frac{1}{2\pi i}\int_\gamma \frac{1}{z - z_0}dz$$
   Since $\frac{1}{z-z_0}$ is analytic on $\{z \mid d(z, \partial G) < \frac{1}{2}r\}$, by Prop 2.15, we know the integral is 0. Hence $\{z \mid d(z, \partial G) < \frac{1}{2}r\} \subset H$.

3. V.1.1
4. V.1.4
5. V.1.12
6. V.1.13
7. V.1.17
8. V.2.1
9. V.2.2
10. V.2.3
11. V.2.4
12. V.2.5