Homework 2016-04-08

Chuan Lu
13300180056

April 8, 2016

Problem 1.
Consider the equation

$$\frac{du}{dt} = \lambda(-u + cos(t))$$

Proof. **0.1 The exact solution of the equation.**

The exact solution is $u(t) = c_0 e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)} \lambda cos(s) ds$.

0.2 For $\lambda = 1, 10, 100, 1000$, use explicit and implicit Euler iteration to solve.

The code is shown as follows.

```
lambdalist = [1, 10, 100, 1000];
u0list = [0, 1];
inteval = [0, 6];
delta_t = 0.1;
oplist = {'explicit', 'implicit'};
symbollist = {'*-', '.-'};
for i = 1:4
    figure(i);
    lambda = lambdalist(i);
    func = @(t, u)(lambda .* (-u + cos(t)));
    for j = 1:2
        u0 = u0list(j);
        for k = 1:2
            op = char(oplist(k));
            symbol = cell2mat(symbollist(k));
            [t1, u1] = Euler_iter(func, inteval, u0, delta_t, op);
            plot(t1, u1, symbol);
            hold on
            grid on
        end
    end
end
```

The result is shown as follows.

0.3 Use Adams and Gear iteration to solve the equation when $\lambda = 1000$.

The code is shown as follows.

```
f = @(t, u)(1000 * (-u + cos(t)));
inteval = [0 10];
```

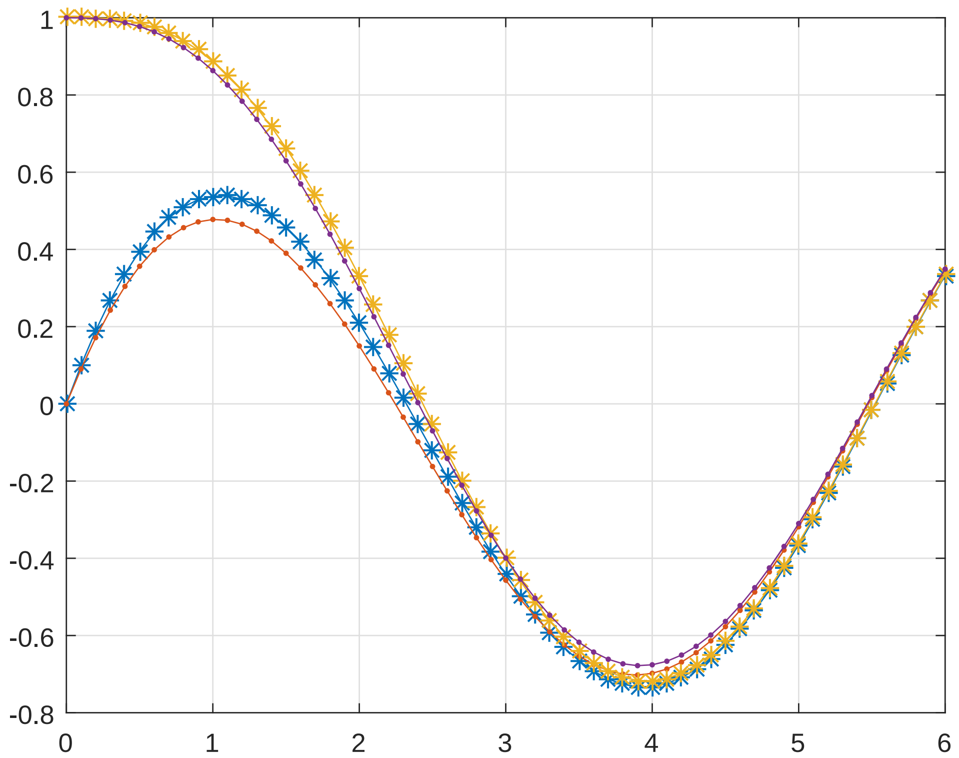

Figure 1: lambda = 1

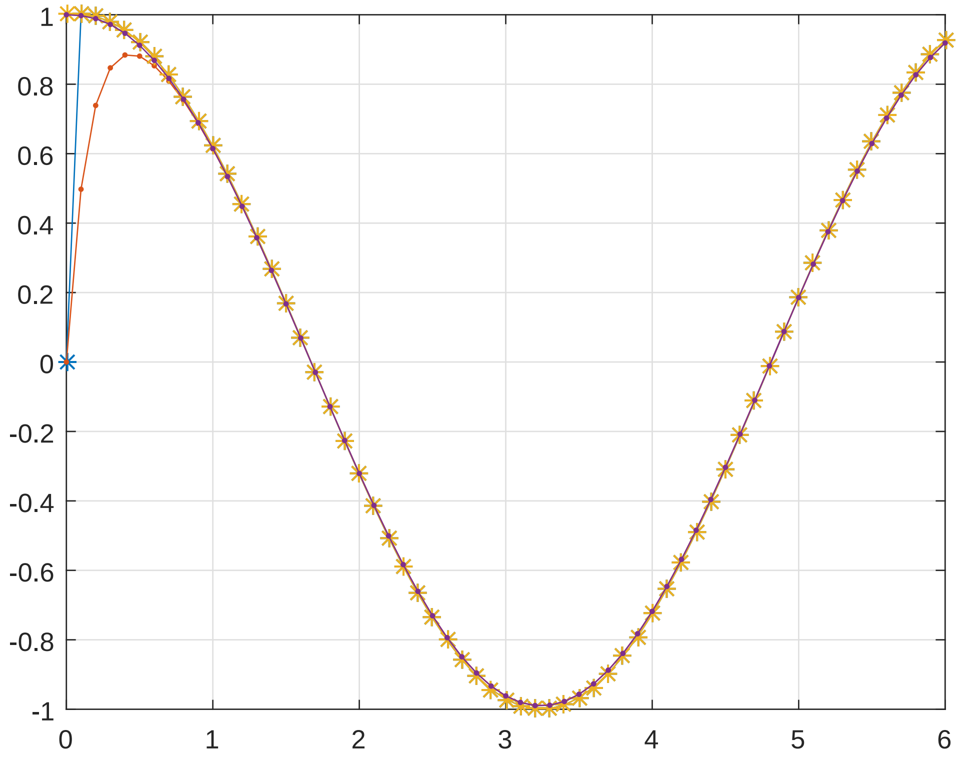

Figure 2: lambda = 10

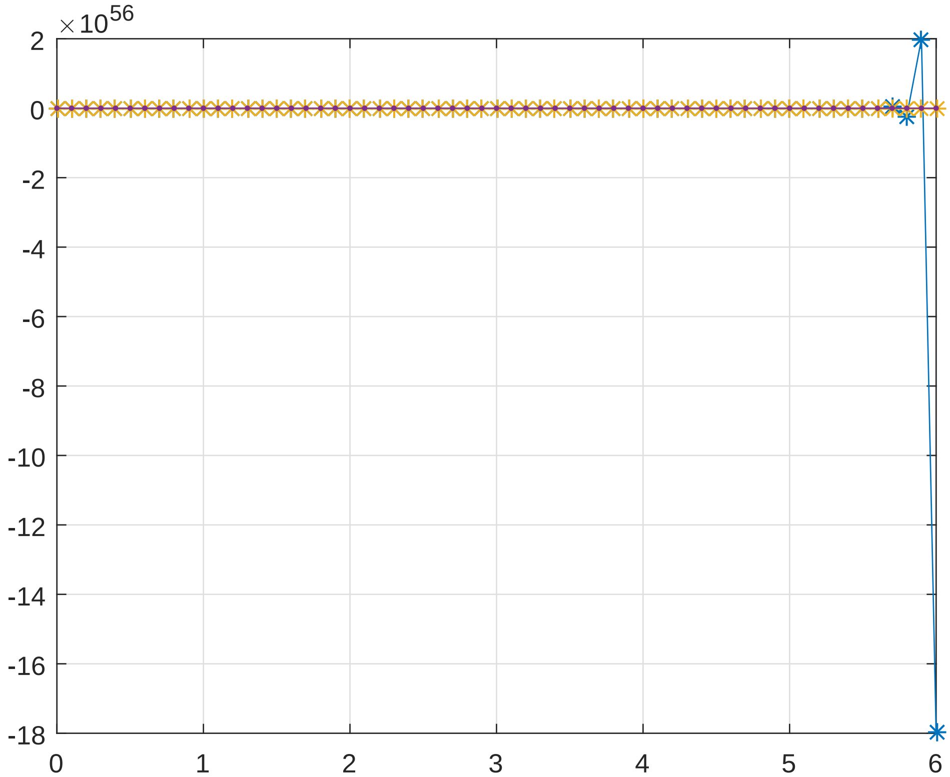

Figure 3: lambda = 100

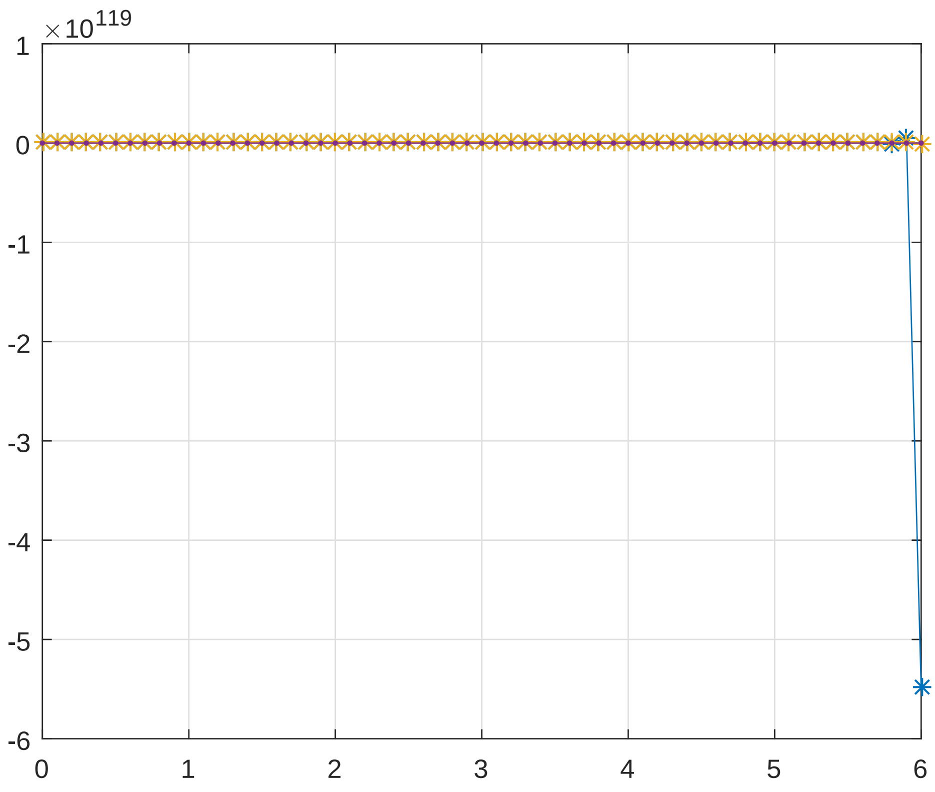

Figure 4: lambda = 1000

```
init = [0  0];
[t1, u1] = ode113(f, inteval, init);
figure(1);
plot(t1, u1, 'g-');
hold on;
[t2, u2] = ode15s(f, inteval, init);
plot(t2, u2, 'r.-');
legend('Adams', 'Gear');
grid on;
```

The result is shown as follows. □

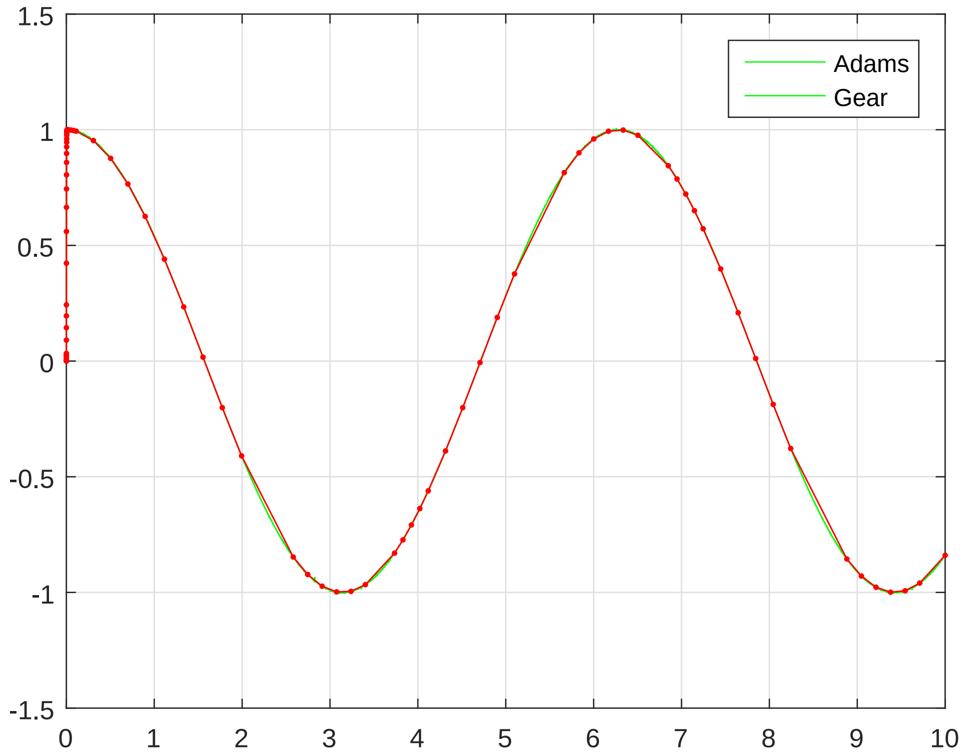

Figure 5: Adams and Gear