Assignment 5 17.4.5

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Problem 1. Analysis the stability and absolute stable interval of Modified and Revised Euler Iteration for Dahlquist test.

For Modified Euler Iteration:.

$$u_{n+1} = u_n + \frac{\Delta t}{2}(f_n + f_{n+1})$$

Assume that the disturbed initiate value is $\widetilde{u_0}$ and the exact initiate value is u_0. Mark the following value calculated by the Modified Euler Iteration as $\widetilde{u_i}, i = 1, 2, \cdots$. Then $\widetilde{u_n} = \widetilde{u_{n-1}} + \frac{\Delta t}{2}(a\widetilde{u_{n-1}} + a\widetilde{u_n})$, $\quad \widetilde{u_n} = \frac{1+\frac{a\Delta t}{2}}{1-\frac{a\Delta t}{2}}\widetilde{u_{n-1}}$, $\quad u_n = \frac{1+\frac{a\Delta t}{2}}{1-\frac{a\Delta t}{2}}u_{n-1}$. Hence, $\|\widetilde{u_n} - u_n\| = \frac{1+\frac{a\Delta t}{2}}{1-\frac{a\Delta t}{2}}\|\widetilde{u_{n-1}} - u_{n-1}\| = \cdots = (\frac{1+\frac{a\Delta t}{2}}{1-\frac{a\Delta t}{2}})^n\|\widetilde{u_0} - u_0\|$.
If we expect that to a fixed Δt, when $n \to \infty$, the error can still be of control, then it must be true:

$$\|\frac{1 + \frac{a\Delta t}{2}}{1 - \frac{a\Delta t}{2}}\| \leqslant 1.$$

Hence, because that $\Delta t \geqslant 0$, it can only be true when $a \leqslant 0$, and $0 \leqslant \Delta t < \frac{2}{|a|}$.

On the other hand, take $z = a\Delta t$, we have $\|\frac{2+z}{2-z}\| \leqslant 1, \quad z \in \mathcal{Z}$.

For Revised Euler Iteration:.

$$u_{n+1} = u_n + \Delta t a(u_n + \frac{\Delta t}{2}au_n)$$

Claimed as above, we have $\widetilde{u_{n+1}} = \widetilde{u_n}(1 + \Delta ta + \frac{1}{2}(\Delta ta)^2)$. Hence, $\|\widetilde{u_n} - u_n\| = (1 + \Delta ta + \frac{1}{2}(\Delta ta)^2)^n\|\widetilde{u_0} - u_0\|$. As shown above, $\|1 + \Delta ta + \frac{1}{2}(\Delta ta)^2\| \leqslant 1$. On the other hand, take $z = a\Delta t$, we have $\|z^2 + 2z + 2\| \leqslant 2$.

Problem 2. Draw the convergence order of 1st-4th-order format

Result.

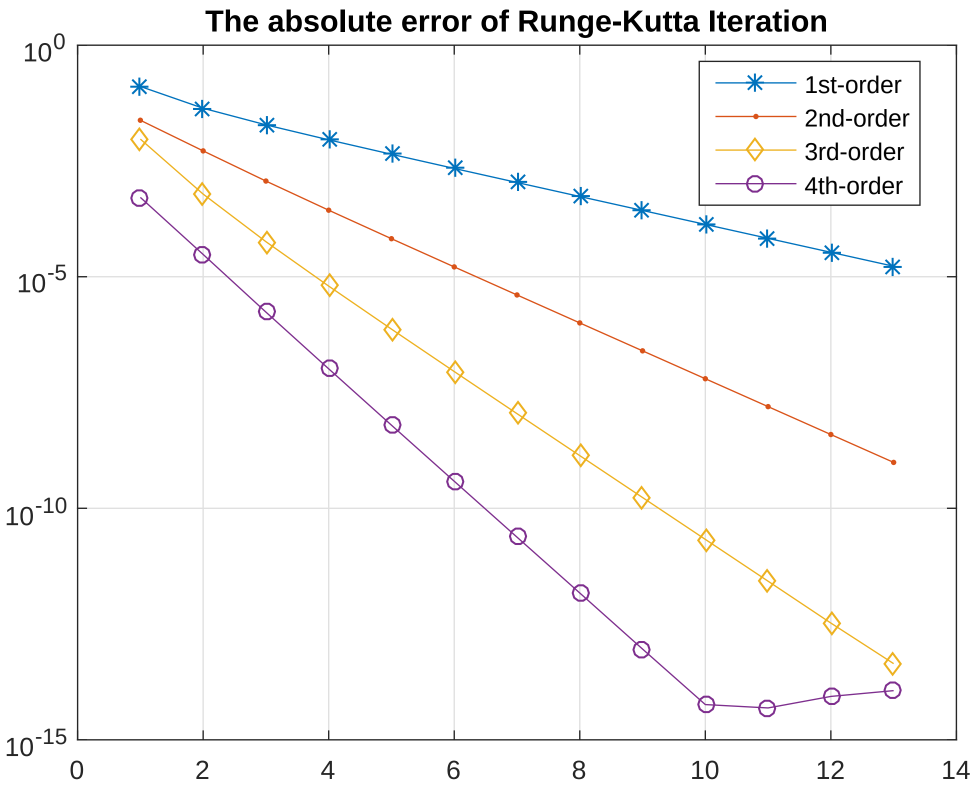

Figure 1: The convergence order of 1-4 order format