Homework 11

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Problem 1. Problem 1, Page 164.

Properties of $\mathbf{A}+\mathbf{I}$**.** $\mathbf{A}+\mathbf{I}$ is a symmetric and strictly diagonally dominate matrix.

Convergence order of this scheme. On the grid points we have

$$\begin{aligned} -\Delta u(x_l) + u(x_l) &= f(x_l) \\ -\Delta u_{h,l} + u_{h,l} &= f_{h,l}. \end{aligned}$$

Denote error $e_{h,l} = u_{h,l} - u(x_l)$, then $e_{h,l}$ satisfies

$$\Delta_h e_{h,l} + e_{h,l} = R_{h,l},$$

where $R_{h,l}$ is the truncation error. When $u \in C^4(\bar{\Omega})$, using the process in Chapter 2, we have $\|R_h\|_{\Omega_h} \leqslant CMh^2$, where $M = \max(u^{(4)})$.
Notice that $u|_{\partial\Omega} = 1$, with Lemma 3.2.5,

$$\|e_h\|_{\bar{\Omega}_h} \leqslant \hat{C} \|R_h\|_{\Omega_h} \leqslant CMh^2.$$

Thus the scheme is of order-2 convergent.