Assignment 1

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Code can be obtained at https://github.com/orcuslc/Learning/

Problem 1. Show the numerical result of $f(x) = \frac{1-cos(x)}{x^2}$ when $x \to 0^+$.

Solution. The result is as follows.

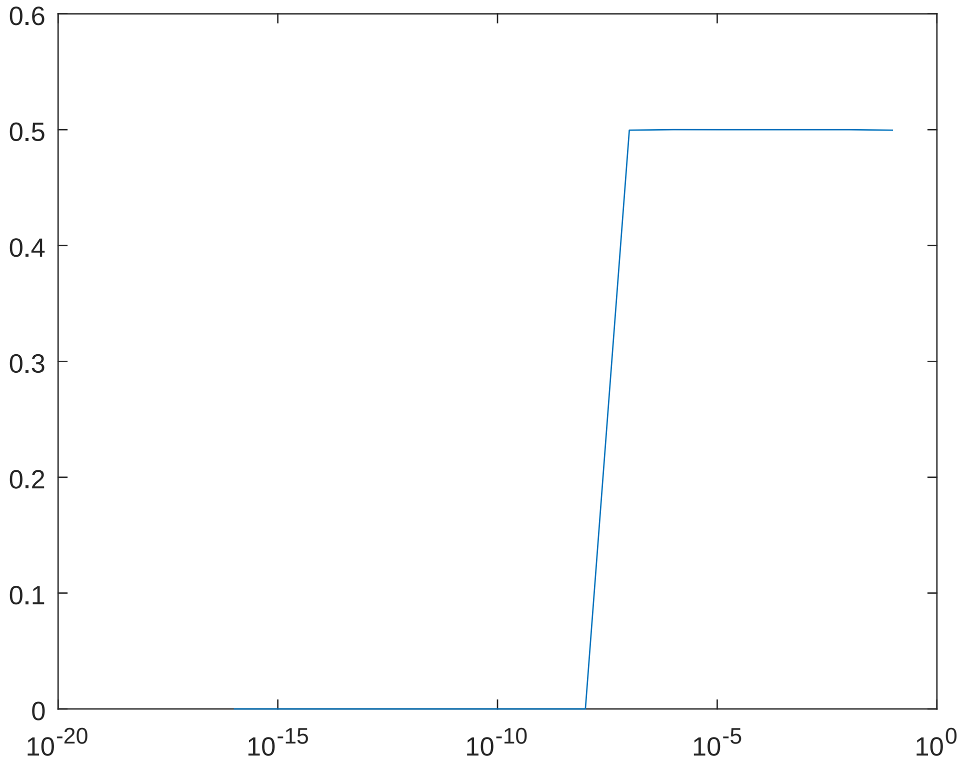

Figure 1: problem1

Problem 2. Analyse the inversed iteration of $u_{n+2} = 3u_{n+1} - 2u_n$

Solution. The inversed iteration is

$$u_n = \frac{3}{2}u_{n+1} - \frac{1}{2}u_{n+2}.$$

And the computed value is

$$\hat{u_n} = \frac{3}{2}\hat{u_{n+1}} - \frac{1}{2}\hat{u_{n+2}} + \epsilon_n.$$

If we let

$$w_n = \begin{pmatrix} u_n \\ u_{n+1} \end{pmatrix}, A = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ 1 & 0 \end{pmatrix}, \hat{w_n} = \begin{pmatrix} \hat{u_n} \\ \hat{u_{n+1}} \end{pmatrix}, \hat{\epsilon_n} = \begin{pmatrix} \epsilon_n \\ 0 \end{pmatrix}$$

Then

$$w_n = Aw_{n+1}, \hat{w_n} = A\hat{w_{n+1}} + \hat{\epsilon_n}$$

Define the error as $e_n = \hat{w_n} - w_n$, then

$$e_n = Ae_{n+1} + \hat{\epsilon_n}$$

Since A has the eigenvalue decomposition

$$P^{-1}AP = \Lambda, P = \begin{pmatrix} 1 & \frac{1}{2} \\ 1 & 1 \end{pmatrix}, \Lambda = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

the iteration can be wrote as

$$P^{-1}e_n = \Lambda P^{-1}e_{n+1} + P^{-1}\hat{\epsilon_n}$$

With the same analysis of Page 13 of the textbook, we can know that e_0 is under control by a constant value of e_n.

Problem 3. Prove (1.2.8) of the textbook

Proof. From (1.2.7) we can know

$$\|x_{n+1} - x_n\| \leqslant \alpha^n \|x_1 - x_0\|$$

So

$$\|x^* - x_n\| \leqslant \sum_{k=n}^{\infty} \|x_{k+1} - x_k\| \leqslant \sum_{k=n}^{\infty} \alpha^k \|x_1 - x_0\| = \frac{\alpha^n}{1-\alpha}\|x_1 - x_0\|$$