# Answers to Chapter 1

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September 23, 2016

**Problem Page 13, P1.1.1.**

*Proof.* $M = \Pi_{i=1}^{r}(A - x_i I) = A^r - \Sigma_{i=1}^{r} x_i A^{r-1} + \Sigma_{1<x_i<x_j<r} x_i x_j A^{r-2} + ... + (-1)^r \Pi_{i=1}^{r} x_i I$. So the first column of M should be the linear combination of each components in the formula above. Now we give an algorithm to compute the first column of $A^k$.

Algorithm 1.1.1

Input: A n*n matrix A, an integer k.
Output: The first column of $A^1, A^2, ..., A^k$.

```
T = A
for i = 1 to k
    B[:, i] = T[:, 1]
    T = A * T[:, 1]
return B
```

The time complexity of this algorithm is $O(k * n^2)$, and time complexity of calculating the coefficients is $\Sigma_{i=0}^{r} C_r^i = 2^r$. So the total time cost should be $O((r+2) * n^2 + 2^r)$. □

**Problem Page 13, P1.1.3.**

*Proof.* $(xy^T)^k = x((x^T y)^T)^{k-1} y^T = ((x^T y)^T)^{k-1} xy^T$. To calculate $((x^T y)^T)^{k-1}$, time cost is $O(n^2)$; for $xy^T$ time cost is $O(n^2)$. So total time cost is $O(n^2)$. □

**Problem Page 13, P1.1.4.**

*Proof.* □