The maximal velocity of the ground track relative to the ground is 8.4 $\mathrm{km\,s^{-1}}$, obtained for a retrograde satellite moving at ground level. More realistically, the velocity of the ground track at the equator for an operational Sun-synchronous satellite is 6.6 $\mathrm{km\,s^{-1}}$. For a geosynchronous satellite with $i = 0$, hence geostationary, we may check that $w_{\mathrm{E}} = w_{\mathrm{E}}(i = 0) = 0.00$.

The last satellite in the table is not as artificial as it may look. It is in fact a simplified model of the Moon, in circular, Keplerian orbit at a distance of 380 000 km, with period about 27 day. Note that, in $\Re$, the sidereal period is equal to 27.32 day, whilst in $\Re_{\mathrm{T}}$, the synodic period[106] which takes the Earth's motion into account, is equal to 29.53 day, a lunar month.[107] The synodic period T' (in days) is calculated using the relation (2.23), with $T = 27.32$ and $T_1 = N_{\mathrm{sid}}$.

5.6 Appendix: Satellite Visibility Time

The span of time over which a satellite S is visible from a given point P on the Earth is called the visibility time of S from P. This idea arises mainly in the study of satellite constellations.

5.6.1 Satellite in Circular Orbit

Consider an arbitrary point P on the (spherical) surface of the Earth and a satellite S whose orbital plane passes through P at a given time. This is shown schematically in Fig. 5.23 (upper). The satellite S can be seen from P as long as it remains above the local horizon for P, represented by the straight line S_1PS_2, i.e., on the circular arc S_1AS_2. The angle $\alpha = (\boldsymbol{OP}, \boldsymbol{OS}_1)$ can be found immediately (reduced distance η):

$$\cos\alpha = \frac{R}{R+h} = \frac{R}{a} = \frac{1}{\eta}\,. \tag{5.38}$$

The period of the satellite is taken equal to the Keplerian period T_0. The visibility time Δt_{v} is therefore

[106] The noun ἡ σύνοδος, ου, 'synod', is composed from σύν, 'with, together' and ἡ ὁδός, ου, 'way, journey'. In Ancient Greek, it already had the two meanings of 'meeting' and 'conjunction of heavenly bodies', both of which illustrate the idea of arriving at the same time.

[107] In English it is no accident that the word 'month' should be so similar to the word 'Moon'. This similarity can be found in German and other related languages. The Indo-European root **men, *mes* refers to the Moon, to lunation (= month), and to measurement (of time). Many languages in this family have held on to this proximity of meaning, although this has not happened in Greek or Latin. These two languages referred to the Moon as 'the shining one' (ἡ σελήνη, ης and *luna, æ*). See also the note on Chandrasekhar.

$$\Delta t_{\mathrm{v}} = \frac{\alpha}{\pi} T_0 \,. \tag{5.39}$$

Expressing α and T_0 as functions of the semi-major axis a (here the radius) of the orbit, we obtain

$$\Delta t_{\mathrm{v}} = 2\sqrt{\frac{a^3}{\mu}} \arccos \frac{R}{a} \,. \tag{5.40}$$

Using the altitude h and the value of the period $T_{0(h=0)}$ defined by (2.17), we have (for time in minutes and angles in radians)

$$\Delta t_{\mathrm{v}} \text{ (min) } = 84.5 \left(1 + \frac{h}{R}\right)^{3/2} \frac{1}{\pi} \arccos \frac{R}{R+h} \,. \tag{5.41}$$

One may also consider the question by fixing a minimal zenithal angle of sight ζ, with $\zeta = (\boldsymbol{PA}, \boldsymbol{PS}'_1)$. The visibility time then corresponds to the time taken by the satellite to travel along the arc $S'_1 A S'_2$. In this case, the angle $\alpha = (\boldsymbol{OP}, \boldsymbol{OS}'_1)$ depends on η and ζ. Simple trigonometric considerations (projecting S'_1 on PA) show that this angle is the solution of the equation

$$\cos\alpha - \frac{1}{\tan\zeta} \sin\alpha = \frac{1}{\eta} \,.$$

Hence,

$$\alpha = 2 \arctan \frac{\sqrt{1 + Z^2 - H^2} - Z}{1 + H} \,. \tag{5.42}$$

$$\text{with} \quad H = 1/\eta \,, \quad Z = 1/\tan\zeta \,,$$

and we obtain the visibility time from (5.39).

When the satellite does not pass through the vertical at the relevant point, the visibility time is obviously shorter.

Example 5.6. *Calculate the visibility time for LEO and MEO satellites.*

For a (SPOT-type) LEO satellite with $h = 800$ km, we find $\eta = 1.125$ and $T_0 = 101$ min. Using (5.38), we obtain $\alpha = 27°$ and hence, by (5.39),

$$\Delta t_{\mathrm{v}} = (27/180) \times 101 = 15 \text{ min} \,.$$

If the visibility condition consists in requiring that the satellite should be at least 15° above the horizon, i.e., $\zeta = 75°$, the calculation with (5.42) gives $\alpha = 16°$ and we deduce that $\Delta t_{\mathrm{v}} = 9$ min. These visibility times are maximal values, assuming that the satellite passes through the local vertical.
For a (NAVSTAR/GPS-type) MEO satellite with $h = 20\,200$ km, we find $\eta = 4.167$ and $T_0 = 718$ min. It follows that $\alpha = 76°$ and hence,

$$\Delta t_{\mathrm{v}} = (76/180) \times 718 = 304 \text{ min} \approx 5 \text{ hr} \,.$$

With $\zeta = 60°$ (visibility if the satellite is more than 30° above the horizon), we have $\alpha = 48°$ and $\Delta t_{\mathrm{v}} = 192$ min ≈ 3 hr.

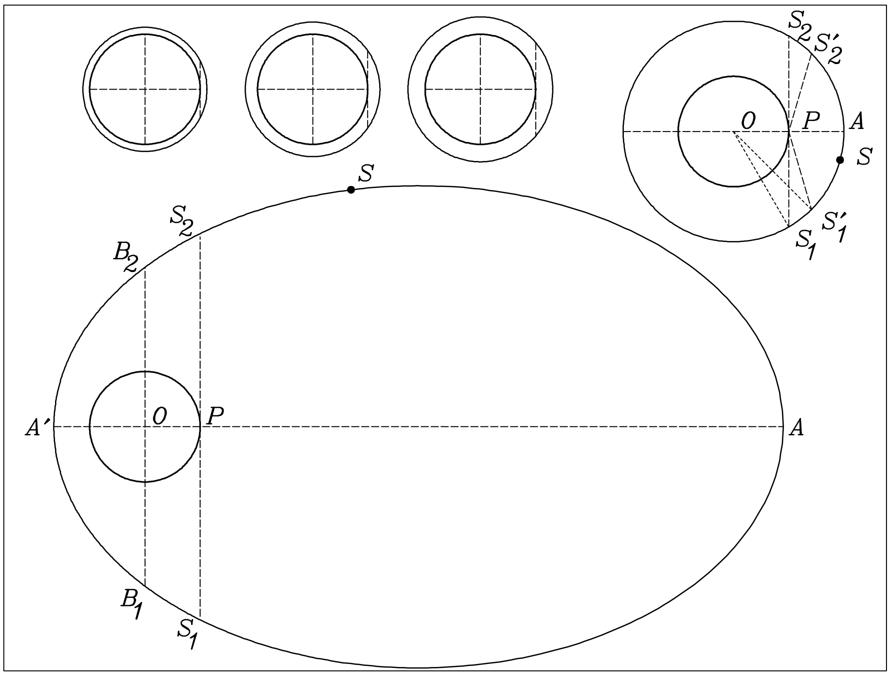

Figure 5.23. Schematic representation of the Earth and satellite trajectories. The Earth and the orbits are drawn on the same scale. *Upper*: circular orbits. LEO orbits are shown with $h = 800$, 1400 and 2000 km, together with the orbit $h = R$, indicating the points mentioned in the text. *Lower*: HEO orbit with period $T \approx$ 24 hr, $e = 0.75$, indicating the points mentioned in the text

5.6.2 Satellite in Highly Eccentric Orbit

For a highly eccentric elliptical orbit, with eccentricity e and type HEO, we consider once again the most favourable situation: the relevant point P is the subsatellite point when the satellite goes through its apogee A, as shown in Fig. 5.23 (lower).

The satellite S can be seen from P as long as it remains on the elliptical arc S_1AS_2. Given the approximations made here, one may replace the local horizon S_1PS_2 by the parallel B_1OB_2 which goes through the centre of the Earth O. We thus find the visibility time as the time taken by the satellite S to travel along the elliptical arc B_1AB_2.

At a given time, the position of S is specified relative to the perigee A' by the true anomaly $v = (\boldsymbol{OA'}, \boldsymbol{OS})$. The mean anomaly M of the point B_1 is calculated from (1.54) with $v = \pi/2$. We obtain

$$M(B_1) = 2\arctan\sqrt{\frac{1-e}{1+e}} - e\sqrt{1-e^2}\,. \tag{5.43}$$