People strengthen or weaken their relationships depending on whether they agree or disagree, respectively. During the same interaction as in Step 1, the weight of the edge connecting nodes i and j is also changed. The change in weight is

$$\Delta w_{ij} = \beta\, w_{ij}\, (1 - w_{ij})\, (1 - \gamma\, |o_i - o_j|\,) \qquad (2)$$

Here $\beta \in (0,1)$ and $\gamma > 0$ are parameters of the model. If $\gamma \leq 1$ then all weights will converge to 1 over time since differing opinions don't matter enough to decrease edge weights. If $\gamma > 1$, the weight between two nodes will decrease if the opinions of the nodes are different enough — if $|o_i - o_j| > \gamma^{-1}$.

1. Use equation (2) to show that if $\gamma \leq 1$, all edge weights will eventually converge to a value of 1. Do this by showing that the change in weight, Δw_{ij}, is always positive and that w_{ij} will therefore always increase towards 1.
2. (Optional) Determine an upper bound on γ for the model to still make sense. We need to avoid the possibility that edge weights can change too much — drop below 0 or grow above 1 in a step.
3. What happens when two nodes with very different opinions interact? Assume a very large difference in opinion $(o_i - o_j \approx 1)$ and calculate the new difference in opinion and the new weight after an update.
4. What happens when two nodes with very similar opinions interact?
5. What happens when two nodes with somewhat different opinions (differing by 0.4) interact?
6. (Optional) Draw a vector field showing how the edge weight and difference in opinion values change in a single 2-person interaction like the ones you explored above, but for all values of $w_{ij} \in [0,1]$ and $|o_i - o_j| \in [0,1]$. You can use the quiver() plot function in Matplotlib for this.

1.

$$\Delta w_{ij} = \beta\, w_{ij}\, (1 - w_{ij})\, (1 - \gamma\, |o_i - o_j|\,) \qquad (2)$$

Because w is constrained to be positive and smaller than 1, it can never change the sign of w_{ij}. We can thus drop it without effect.
Same logic applies to B and (1-w_{ij}).
Absolute values are also always positive, thus will not change the sign either.
Thus, if y <= 1 -> 1-y >= 0, meaning that weight change can never be negative and weights must thus converge to 1 over time

3.

$$\Delta o_i = \alpha\, w_{ij}\, (o_j - o_i) =$$

Thus if (oi-oj) = 1,
change in oi = αw_{ij}

4.

If oi≈oj, (oi-oj) ≈ 0.
Thus change in oi = αw_{ij}(oi-oj) -> 0

5.

By the same logic,

Thus change in oi = αw_{ij}0.4