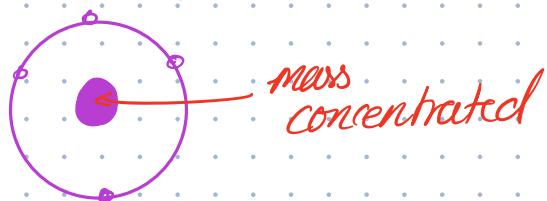


Electron scattering experiments: Electron beam is directed at target containing nuclei. Electrons interact with the nuclei and their trajectory is changed

Radius of atom:  $e^-$ -beam at nucleus and outgoing  $e^-$  angles are measured, these are used to calculate radius of nucleus  $\sin \theta = \frac{r_{atom}}{d}$

Hadron scattering experiment: Hadrons: mesons + baryons, alpha-particles:  
interact with protons and neutrons of the target nucleus  
→ alpha-beam at nucleus and outgoing alphas measured  
→ Rutherford's gold foil experiment:  
• most  $\alpha$  travel through it → atom is mostly empty space  
• some are deflected by small angles → nucleus has positive charge that deflects  $\alpha$  (which is also positive)  
• occasionally an  $\alpha$  travels back → nucleus carries most of atom's mass



At low energies:  $\alpha$  interact w/ nucleus only via Coulomb int.

At high energies: Coulomb force is overcome  $\alpha$ -particles absorbed and we get diffraction pattern again

Conclusion: There exists a nucleus that holds most of the atom's mass

- ## Nuclear charge distribution:
- $e^-$ -scattering experiments probed charge distribution
  - charge of nucleus given by protons
  - From experiments:
    - radius increases with  $A$
    - charge density is nearly same for all nuclei
    - constant distribution of nucleons out to the surface of nucleus
    - number of nucleons per unit volume is roughly constant
  - ↳ we can calculate radius from it:  

$$R = R_0 A^{1/3}$$

## Nuclear matter distribution:

- probed by Hadron scattering
- Radius calculated from diffraction pattern and scattering angles is characteristic of the nuclear force, not Coulomb force.  
 Radius then reflects distribution of nucleons in the nucleus

## Conclusion charge + matter distribution:

- they are nearly equal
- $A^{1/3}$  dependence

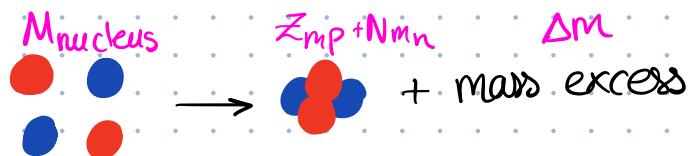
Heavy nuclei: more neutrons than protons, so expect larger neutron radius (= larger matter radius)

→ proton repulsion pushes protons outward and  $n-p$ -force pulls neutrons in  
 so radii are equal

- Nuclear radius:
- average radius calculated by  $R = R_0 A^{1/3}$
  - Radius increases with mass number  $A$
  - Nucleus doesn't have sharp boundary (defined radius)

## Binding energy

energy required to keep protons + neutrons together inside the nucleus



mass of nucleus is not equal to sum of the mass of its nucleons

Mass excess:  $\Delta m = Zm_p + Nm_n - M_{\text{nucleus}}$

Einstein: transform mass excess to binding energy  
 $\Delta m \rightarrow \Delta m c^2$  (MeV)

⇒ measure of how tightly nucleons are bonded to each other

Large B.E. → lot of energy required to separate the nucleons

mass of atom:  $M_{\text{atom}} = M_{\text{nucleus}} + Zm_e - \sum_{i=1}^Z \frac{B_i}{c^2}$

$$M(^A_Z X_N) = M_{\text{nucleus}}(^A_Z X_N) + Zm_e$$

$$M_{\text{nucleus}}(^A_Z X_N) = M(^A_Z X_N) - Zm_e \quad | \cdot c^2$$

$$\underline{M_{\text{nucleus}} \cdot c^2 = (M(^A_Z X_N) - Zm_e) \cdot c^2}$$

Deriving B.E.  $B.E. = (Zm_p + Nm_n - M(^A_Z X_N)) \cdot c^2$

$$= (\underbrace{Z(m_p + m_e)}_{= m(CH)} + Nm_n - M(^A_Z X_N)) c^2$$

$$\qquad \qquad \qquad //$$

$$801,5 \text{ MeV}_4$$

Binding energy is very small compared to rest mass energy of the nucleus

$$\frac{B.E.}{M(^4\text{He})c^2} = 0,75\%$$

Average binding energy:  $\frac{B}{A}$ , increases linearly with  $A$

$\hookrightarrow$  B.E. per nucleon

Energy released,  
because higher  $B/A$   
means less weight  
(lighter bound)

$$Q = (M_i - M_f) c^2 \frac{B/A}{A}$$

$$Q > 0 \text{ if } M_i > M_f \quad 8\text{MeV}$$

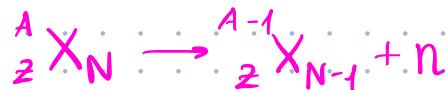
$\hookrightarrow$  exotherm



## Nucleon separation energy

$S_n$  &  $S_p$

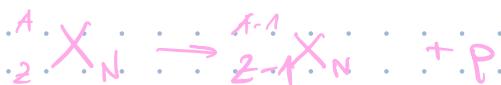
$S_n$ : one separation energy is energy needed to remove a neutron from a nucleus  ${}^A_Z X_N$



$$\begin{aligned} S_n &= (M({}^{A-1}_{Z-1} X_{N-1}) - M({}^A_Z X_N) + m_n) c^2 \\ &= B({}^A_Z X_N) - B({}^{A-1}_{Z-1} X_{N-1}) \end{aligned}$$

- it is easier to remove a neutron from nucleus with odd number of neutrons
  - $\hookrightarrow$  last unpaired neutron easy to remove
- Pairing  $\rightarrow$  more stable nucleus

$S_p$ : — " — to remove a proton from nucleus



- $S_n$  and  $S_p$  show evidence of shell structure in nuclei

Liquid drop model a model that describes the nucleus of an atom as a liquid, held together by the nuclear force

- high, constant density
- classical physics

### SEMF

consists of 5 terms, based on liquid drop model  
based on both theory and experimental data

$$B.E. = \text{Liquid drop model} + \text{shell-model}$$

volume	surface	Coulomb	
$a_v A - a_s A^{2/3}$	$-a_c Z(Z-1) A^{-1/3}$	$-a_{\text{sym}} \frac{(A-2Z)^2}{A} + \delta$	

- dominates B.E.

- linear

-  $a_v = 16 \text{ MeV}$

- energy  $\sim$  volume:

nucleons packed together  
into smallest volume  
in contact w/ each other

- at the surface nucleons

are surrounded by fewer  
nucleons = less attraction/  
tightly bound

- subtract from volume

term

$-a_s = 16, 8 \text{ MeV}$

Coulomb

repulsion of  
protons,  
makes nucleus  
less tightly bound

$\Rightarrow$  long range,  
proportional to  $Z^2$   
 $a_c = 0, 72 \text{ MeV}$

light nuclei have  
equal no. of protons  
+ neutrons, deviation  
from this makes nucleus  
less bound

- Pauli principle:  
 $-a_{\text{sym}} = 23 \text{ MeV}$   
- small for heavy  
nuclei

- linearity of B and A:

if long range nuclear force:  $B \sim A^2$

every nucleon would attract the others

since  $B \sim A$ : means each nucleon attracts  
only its closest neighbors

$$a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3}$$

$$- a_{\text{sym}} \frac{(A-2Z)^2}{A}$$

like nucleons couple pairwise  
to give spin zero  
- closer together than  
when coupled differently  
 $\Rightarrow$  tighter bound



$a_p = 34 \text{ MeV}$   
even-even: more bound  $\delta = +a_p A^{-3/4}$



odd A:  $S = 0$



odd-odd:  $S = -a_p A^{-3/4}$

B.E. can be increased by  
converting one odd nucleon  
to another to form a pair



Volume

Volume + surface

Vol + surf + Coulomb

Vol + surf + Coulomb + sym.

$\hookrightarrow$  results exp. B/A curve

SEMF FOR MASSES: 1. B.E. of nucleus:

$$B.E. = [Zm(H) + Nm_n - M(^A_Z X_N)]c^2$$

2. SEMF included:

$$M(^A_Z X_N) = Zm(H) + Nm_n - B.E(SEMF)/c^2$$

## Chart of nuclides

stability line: stable nuclei that follows the  $N=Z$  line until higher  $N$ , where it curves down

nuclear drip line: boundary at which adding a new proton or neutron to a nucleus will lead to it being unstable or just forming a new nucleus

## Mass parabolas

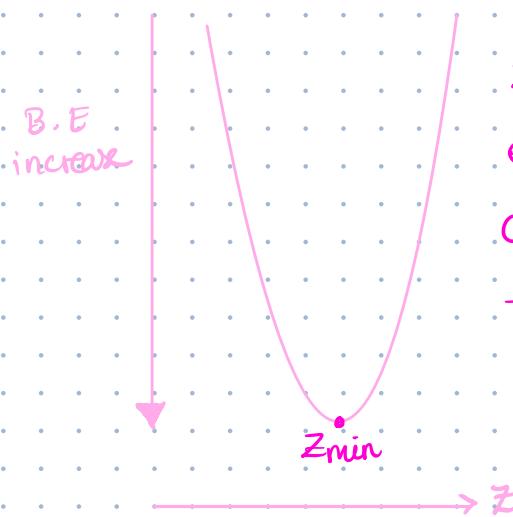
The mass eq.:  $M(A,Z) = Zm(H) + (A-Z)m_n - \frac{B(A,Z)}{c^2}$

also called "Beta Valley"

↳ has the form of a parabola of  $M$  vs.  $Z$ .  
Minimum at  $\frac{\partial M}{\partial Z} = 0$ , obtain  $Z_{min}$

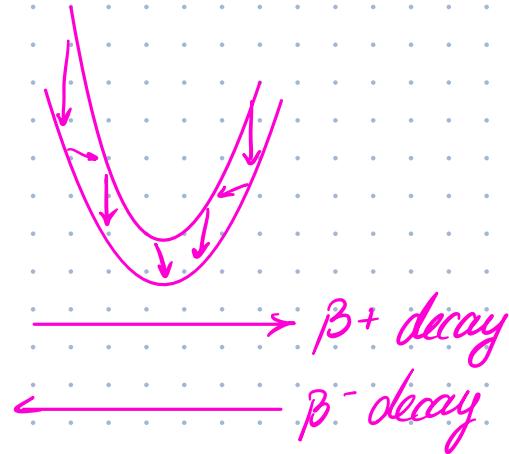
For light nuclei:  $\frac{Z_{min}}{A} = \frac{1}{2} \Rightarrow$  so  $N=Z$ .

for heavy nuclei:  $\frac{Z_{min}}{A} = 0,4 \rightarrow$  neutron excess



$Z_{min}$  = most tightly bound nucleus  
everything else is unstable and decays towards  $Z_{min}$   
→ beta decay

For odd-odd and even-even, we get double parabola  
↳ PAIRING TERM S       $\Rightarrow$  double beta decay



## Nuclear force

the force acting between the protons and neutrons in the atom, binds them together

Properties: 1. strongly attractive at short range  
 $\rightarrow$  stronger than Coulomb at short range

Evidence:  $\alpha$ -scattering

2. negligible at long distances  
 $\rightarrow$  nuclei in molecules don't interact by nuc force

Evidence: linear dependence of  $B$  on  $A$   
long range would mean  $B \sim A^2$

3. electrons don't feel it

Evidence:  $e$  farther away from nucleus than range of force

4. Charge independent

$\rightarrow$  nearly independent whether its protons or neutrons

$\rightarrow$   $p p$ ,  $n n$ ,  $p n$  interactions are equal

Evidence: mirror nuclei have same excited states

ex.  $\underline{\underline{5}}B_6$  &  $\underline{\underline{6}}C_5$

OBS: still have different B.E.

5. dependence on parallel and anti-parallel spin  
 → stronger for aligned pairs



6. includes a repulsive term, that keeps nucleons at certain distance

7. has a noncentral/tensor component that doesn't conserve orbital angular momentum  
 non-spherical shape → spin-dependent

- Deuteron
- bound state of 1 N and 1 Z
  - no excited states
  - low B.E., weakly bound

### Spin & Parity

$$I = S_n + S_p + l = \underline{\underline{1}} \quad \text{and Parity} = \underline{\underline{+1}}$$

two allowed spin alignments that give  $I=1$ :

$$\begin{array}{ccc} l=0 & \begin{array}{c} \uparrow \\ \bullet \\ \uparrow \end{array} & \begin{array}{c} \uparrow \\ \bullet \\ \uparrow \end{array} \\ S=1 & s=\frac{1}{2} & s=\frac{1}{2} \end{array} \quad \begin{array}{ccc} l=2 & \begin{array}{c} \uparrow \\ \bullet \\ \uparrow \\ \bullet \\ \uparrow \end{array} & \begin{array}{c} \uparrow \\ \bullet \\ \uparrow \end{array} \\ S=1 & s=\frac{1}{2} & s=\frac{1}{2} \end{array}$$

$\xrightarrow[l=2]{S=1}$

$$\mathcal{N} = (-1)^l = (-1)^0 = (-1)^2 = \underline{\underline{+1}}$$

### Magnetic moment

$$\mu = \mu_n + \mu_p \quad \text{for } l=0, \mu_e = 0$$

$$\mu_s \neq 0$$

$$\mu_s = g_s \mu_N \cdot s$$

calculated  $\mu_N$  deviates from experimental value,  
 assume that there is a mixture of  $l=2$  state  
 in the deuteron wavefunction

$$\Psi = a_s \Psi(l=0) + a_d \Psi(l=2)$$

$$a_S^2 = 0,86 \quad a_D^2 = 0,04$$

The deuteron is 96%  $d=0$  and 4%  $d=2$

### Electric quadrupole moment

reflects shape of charge distribution  
- deviates from sphere  
-  $l=2$  introduces non-spherical shape

### Conclusions

- the mixture of  $l=0$  and  $l=2$  gives the  $1^+$  ground state (Spin  $I=1$  and Parity  $=+1$ )
- the mix indicates that there is another non-central component causing the mixing of two states
  - ↳ Tensor interaction
- $S=1$  triplet state increases the binding,  
 $S=0$  singlet state decreases binding

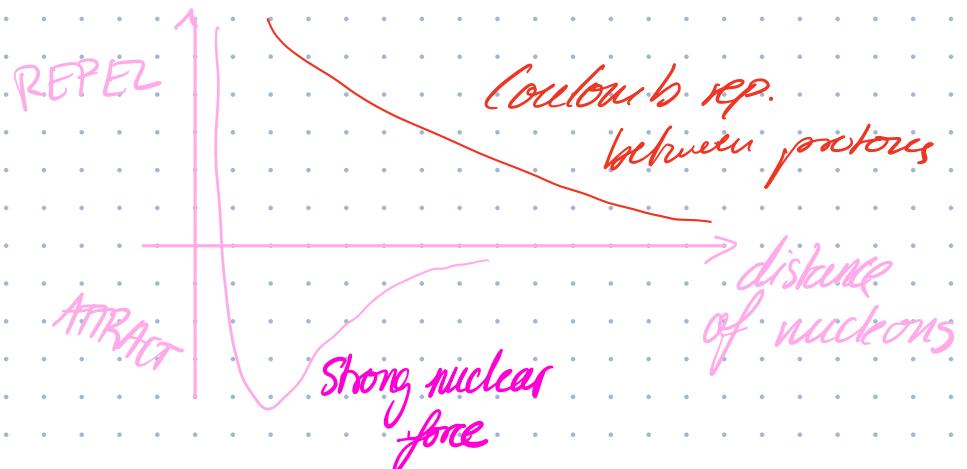


- $S=0$  only allowed for pp or nn interactions  
(antiparallel due to Pauli)

## Range of the nuclear force

Range (1-2) fm

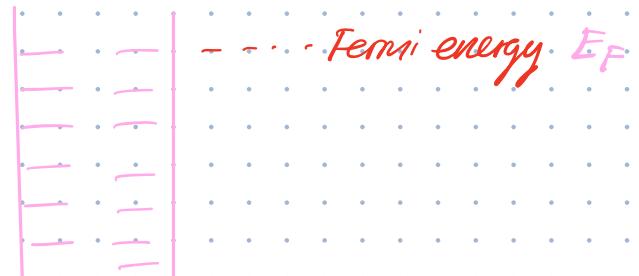
exchange particle: Pions?



## Fermi gas model

model of an ensemble of many non-interacting fermions that obey Fermi-Dirac statistics  $\rightarrow$  identical fermions can't merge (no overlap)

$\rightarrow$  Potential well of  $p + n$ , obey Pauli



## Shell model

-inspo: shell structure of atoms  
nuclei with magic numbers have greater stability

evidence: 1. Sp and  $s_n$  separation energies experience sudden drop after p/n number passes

magic number by one n/p ??

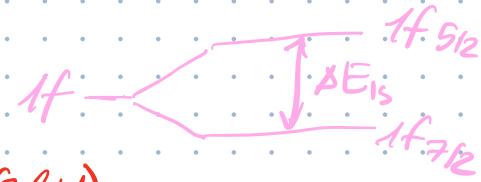
2. higher B.E. for magic nuclei
3. first excited state lies higher (MeV) in nuclei with magic numbers  
→ more E required to excite them
4. smaller electric quadrupole moments  
↳ less spherical = tighter bound

- non-int particles in spherically symmetric potential  $U(r)$
- want shell model to predict magic numbers
- spin-orbit removes degeneracy and obtains the correct magic numbers ??

↳ coupling of  $l+s$ :  $j = l+s$

$\downarrow$   
 $\left\{ \begin{array}{l} l+\frac{1}{2} \text{ and} \\ l-\frac{1}{2} \end{array} \right.$

↳ splitting between them:  $\Delta E_{ls} = \frac{2l+1}{2} \hbar^2 \langle v_{so} \rangle$



$$N = 2(2l+1) = 14$$

$$N = 2j + 1 = 2(l + \frac{1}{2})_j$$

How to use:  
1. fill n and p's into the shell orbits  
2. check level of last unpaired nucleon = 8 and 6

$n l j$

ex.  $1d\ 5/2$  parity  $\pi$   
 $\begin{array}{c} / \\ n \\ \backslash \\ l \\ / \\ j \end{array}$  spin I

I  $\pi$

ex:  $^{15}_8 O_7$ : last unpaired neutron  $N$  is in  $1p_{1/2} \rightarrow n=1, l=p, j=\frac{1}{2}$   
 $l=1$

$^{17}_8 O_9$  → last  $n$  in  $1d_{5/2}$

$n=1, l=d=2, j=\frac{5}{2}$

$$J=L+S \Rightarrow S=J-L=S_{1/2}-2=\frac{1}{2}$$

$$\pi=(-1)^L=(-1)^2=\pm 1$$

$$\pi^L_{gs}=\frac{1}{2}^+$$

$$j=L+S$$

$$S=J-L$$

$$=\frac{1}{2}-1=\underline{-\frac{1}{2}}$$

$$\gamma=(-1)^L=(-1)^1=-1$$

$$I^{\pi}=\frac{1}{2}^-$$

ground state

Degeneracy of each level:  $2j+1$

odd- $A$ :  $I=\text{half-int. ex. } \frac{1}{2}, \dots$

odd-odd: one unpaired  $n$  and one unpaired  $p \rightarrow \text{couple up}$

$$|j_p - j_n| \leq I \leq |j_p + j_n|, \text{ Parity} = (-1)^{j_p} (-1)^{j_n}$$

even- $A$ :  $I=\text{integral, Parity = always positive}$

even-even:  $I=0$  Ground state:  $0^+$

Magnetic moment → all nucleon pairs result in zero magnetic moment

→ magnetic moment value comes from unpaired nucleon

Electric moment → paired nucleons ⇒ zero Quadrupole mom.  
 → neutrons ⇒ 0 quad. mom.

- valence neutron orbiting around core will cause deformation (core polarisation)
- SM does not consider this

## Radioactive decay

process where unstable nucleus loses energy by radiation

main decays:  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $e^-$ -capture

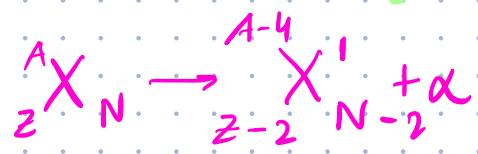
$$\hookrightarrow \text{spontaneous: } Q = (M_i - M_f)c^2 > 0$$

↑  
energy released in decay

Decay: statistical, we can't know when, only how probable

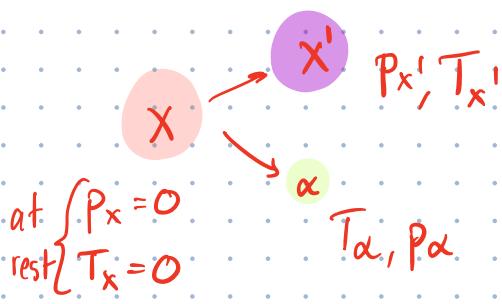
- in time period  $t$ :  $\frac{dN}{dt} = -\lambda N$  decay constant  
no. of product nuclei
  - remaining unstable nuclei at time  $t$ :  $N(t) = N_0 e^{-\lambda t}$
  - time at which intensity of radiation is half of original value:  $t_{1/2} = \frac{\ln 2}{\lambda}$
  - avg. time a nucleus is likely to survive before decay:  $\tau = \frac{1}{\lambda}$
  - number of decays per unit time = activity
- $$A = \frac{dN(t)}{dt} = \underbrace{-\lambda N_0 e^{-\lambda t}}_{A_0} \quad \text{unit: } 1 \text{Bq}$$

## Alpha decay



- occurs in the heaviest nuclei
- Coulomb force  $\sim Z^2$ , so for big  $Z$ , the nucleus decays to get rid of their  $Z$
- $\alpha$ -particle is favorable  
↳  ${}^4\text{He}$  is tightly bound     ${}^4\text{He} \rightarrow (Z=2, N=2)$

Energy conservation:

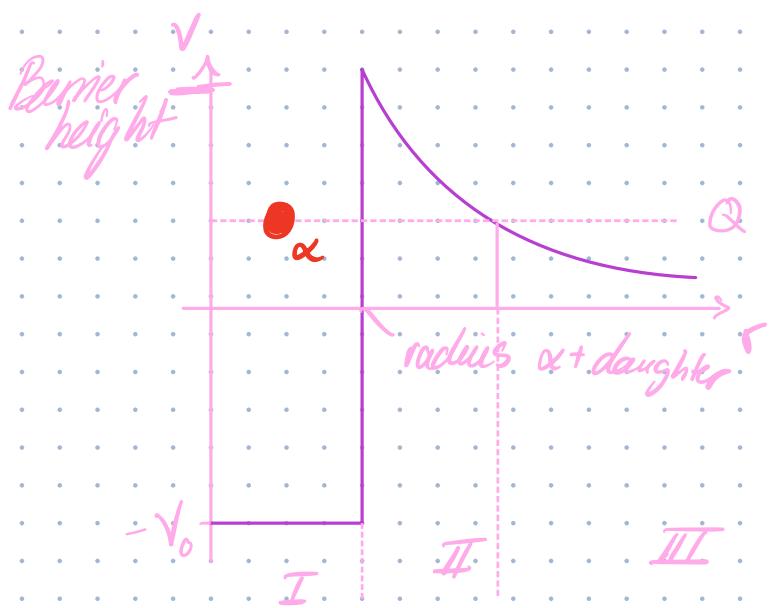


$$m_x c^2 = m_{x'} c^2 + T_{x'} + m_\alpha c^2 + T_\alpha$$

$$Q = T_{x'} + T_\alpha$$

$$\boxed{P_\alpha = P_{x'}}$$

- most of  $Q$  decay energy goes to  $\alpha$ -particle:  $Q \approx T_\alpha$
- higher  $Q \rightarrow$  smaller  $t_{1/2}$  of  $\alpha$
- $t_{1/2}$  shorter for even-even than other combos
- for more stable nuclei → reduced  $Q$ -energy and therefore larger  $t_{1/2}$
- near magic numbers: large shell gaps, less  $\alpha$  emission

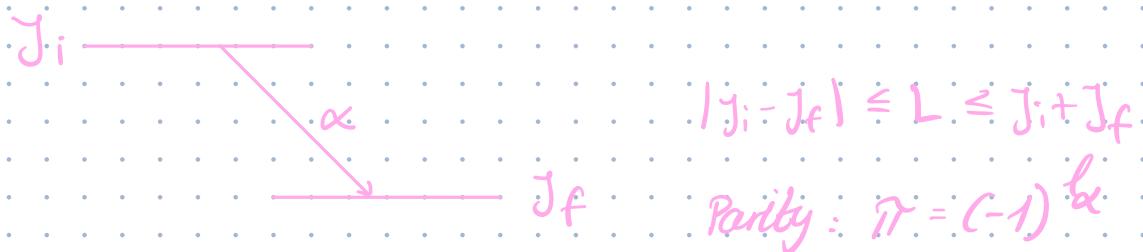


- classical mech. } I.  $\alpha$  inside well with  $T_\alpha = Q + V_0$ , cannot escape  
 classical mech. } II:  $\alpha$  cannot enter here since  $V(r) > Q$   
 classical mech. } III:  $\alpha$  can exist here since  $Q > V(r)$

Quantum mech.:  $\alpha$  tries to penetrate barrier, eventually escapes  
 barrier delays the  $\alpha$ 's emission  $\rightarrow$  long  $t_{1/2}$

### Spin & parity selection rules

total ang. momentum is purely orbital in  $\alpha$  decay



Parity sel. rule:  $\Pi = (-1)^{l_\alpha}$  must be conserved

even  $l_\alpha \rightarrow (+)$  parity

odd  $l_\alpha \rightarrow (-)$  parity

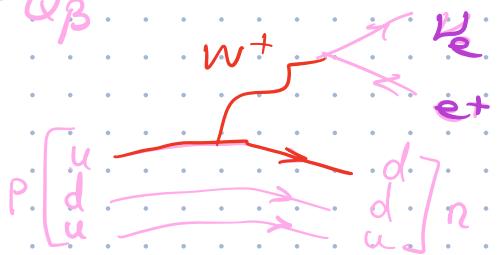
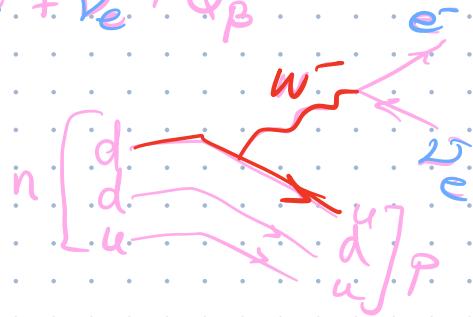
Decay probability of  $\alpha$  decay dependent on:

1. high  $Q_{\text{ex}}$
2. low  $J$  total spin

so low  $l_\alpha$ , high  $Q_{\text{ex}}$  is more probable

## Beta decay

atomic nucleus emits beta particle,  
conversion of protons and neutrons



- ⇒ in  $\beta^-$ -decay the  $e^-/e^+$  and  $\bar{\nu}_e/\nu_e$  do not initially exist, they are created from decay energy  $Q_\beta$
- ⇒  $A$  remains unchanged



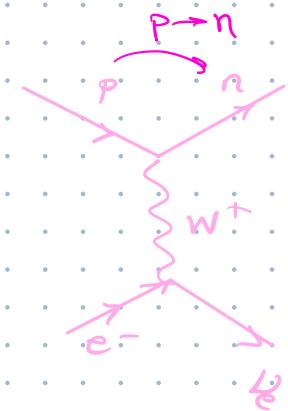
$\Rightarrow A$  is constant



## electron capture E:



proton captures  
an orbital  $e^-$   
and turns into  
a neutron



$$Q_E \approx T_\nu$$

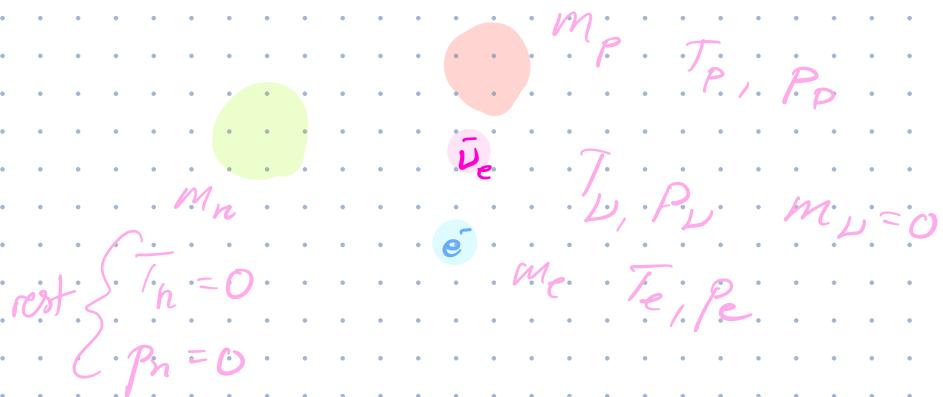
why doesn't a free proton decay?

↳ is the lightest baryon (least energetic)

→ is stable

Energetics:  $Q$  is shared between  $\bar{e}$  and  $\bar{\nu}_e$

$$Q = T_e + T_{\bar{\nu}} + T_p \approx T_e + T_{\bar{\nu}}$$



$$m_{atom}c^2 = m_{nucleus}c^2 + Z \cdot m_e c^2 - \sum_{i=1}^Z B_i$$

$$Q_{\beta^-} = [m_{Nuc}(x) - m_{Nuc}(x') - m_e] c^2$$

$$Q_{\beta^-} = [m(x) - m(x')] c^2$$

$$= T_e + E_{\bar{\nu}}$$

$\bar{e} + \bar{\nu}_e$  require relativistic kinematics!

$$m_{\bar{\nu}} = 0, E_{\bar{\nu}} = T_{\bar{\nu}}$$

$$E_e = T_e + m_e c^2$$

???

3-body: continuous energy spectrum of  $e^+$  and  $\nu_e$

2-body: monoenergetic spectrum of  $\nu$

nucleus can undergo both  $\beta^+$  and  $e^-$ -capture  
if  $\beta^+$ -decay is energetically possible  
for  $\beta^+$  to spontaneously happen:

$$Q_{\beta^+} \geq 2m_e c^2$$

large  $Q \rightarrow$  short  $t_{1/2} \rightarrow$  greater prob.

Theory: Fermi's Golden rule:

decay prob. dependent on: - matrix element (strength of int.)

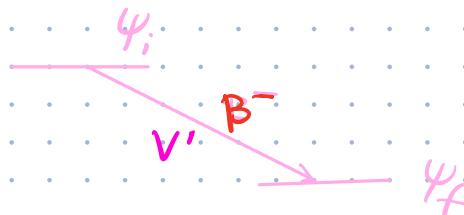
$$\lambda = \frac{2\pi}{\hbar} |V_f|^2 \text{Matrix element} \quad \begin{array}{l} \text{- density of final states} \\ \text{- overlap of initial + final} \\ \text{state wavefunctions} \end{array}$$

$$\lambda = \frac{2\pi}{\hbar} |V_f|^2 g(E_f)$$

density of final states

→ Beta decay caused by weak int.

$V'$  is like a weak perturbation that causes the transition between states



Fermi-Kline plot ???

$$\lambda = f = \frac{\ln 2}{t_{1/2}}$$

comparable half-life

$$f t_{1/2} = \ln 2 \frac{2\pi^3 h^7}{g^2 m_e^5 c^4 \Gamma M G F^2}$$

$$\log f t_{1/2} \approx 222$$

most allowed decay:  $0^+ \rightarrow 0^+$  decay

$\hookrightarrow$  has smallest log f t<sub>1/2</sub> value  
shorter t<sub>1/2</sub>  $\Rightarrow$  more allowed

Spin + parity selection rules  $\beta$ -decay

l=0 for e<sup>-</sup> and  $\nu_e$ , only change in ang. mom. is by spin

s=1/2, singlet state so that S=0

Fermi decay  
(superallowed decay)  
 $0^+ \rightarrow 0^+$

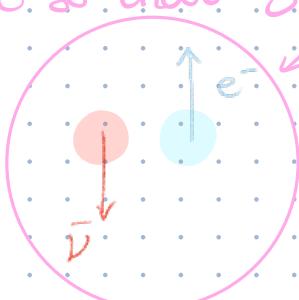
S=1

triplet state

222

Gamow-Teller decay

$0^+ \rightarrow 1^+$



must be 0 since L=0 and no change in S

$$\begin{aligned} I_f &= I_i, -I_f = 0 \\ \sigma J &= I_i, -J_f = 0 \\ \vec{\Pi}_i \cdot \vec{\Pi}_f &= +1 \\ \Delta \vec{\Pi} &= 0 \end{aligned}$$

I<sub>i</sub> = I<sub>f</sub> + 1 (total charge of 1 in the ang. mom)

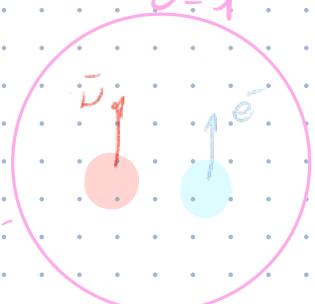
$$\Delta I = 0, \pm 1 \quad I = 0 \not\rightarrow I = 0$$

$$\vec{\Pi}_i \cdot \vec{\Pi}_f = +1 (\Delta \vec{\Pi} = 0)$$

$$\Delta J = 0 \text{ or } 1 \xrightarrow{IJ=1-J=1} \xrightarrow{IJ=1-J=0}$$

$$J = L + S$$

$$\begin{aligned} J &= L + 1 = 1 \\ &= L + 0 = 0 \end{aligned}$$



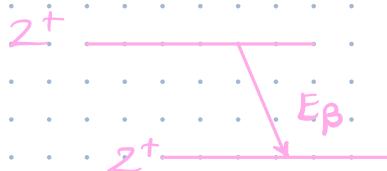
selection rules:

??

$$\Delta I = 0, 1$$

$$\Delta \pi = \text{no}$$

what about  $2^+ \rightarrow 2^+$ ?



$$\Delta L = 0$$

$$J = L + S$$

$$J = S$$

Forbidden decays = (less probable)

↳ when parity  $\Delta \pi \neq 0$  (parity changes)

$$\pi = (-1)^L$$

→  $e^-$  and  $\nu_e$  must be emitted with odd  $L$ -value

$$L = 1, 3, 5 \dots$$

L first - forbidden

for  $L=1$  we have both F and GT decay:

$$F: S=0$$

$$GT: S=1$$

$$J = L+S$$

$$J = L+S = 1+1=2$$

$$= 1+0=1$$

$$\Delta I = 0, 1 \text{ or } 2$$

$$\Delta I = 0 \text{ or } 1$$

selection rules for forbidden decays:

$$\Delta I = 0, 1, 2 \quad \Delta \pi = \text{yes}$$

for  $\Delta I \geq 2$  and  $\Delta \pi = \text{no}$ : second - forbidden

for  $\ell = 2$  we have  $\Delta I = \underline{0, 1, 2, 3}$   
 allowed      2nd forbidden

selection rules second-forbidden decays:

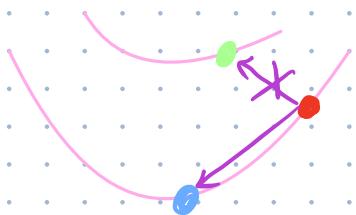
$$\Delta I = 2, 3 \quad \Delta \pi = \text{no}$$

selection rules third-forbidden decays:

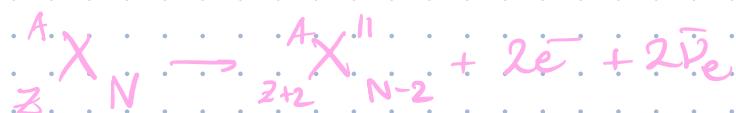
$$\Delta I = 3, 4 \quad \Delta \pi = \text{yes}$$

$$\begin{aligned} \ell &= 3 \quad J = 3+0 \text{ or } 1 \\ &= \underline{0, 1, 2, 3, 4} \\ &\text{first-forbidden} \quad \text{3rd-forbidden} \end{aligned}$$

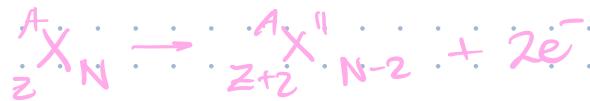
Double  $\beta$ -decay



red nucleus wants to decay  
 to blue, but single  $\beta$ -decay (green)  
 energetically forbidden



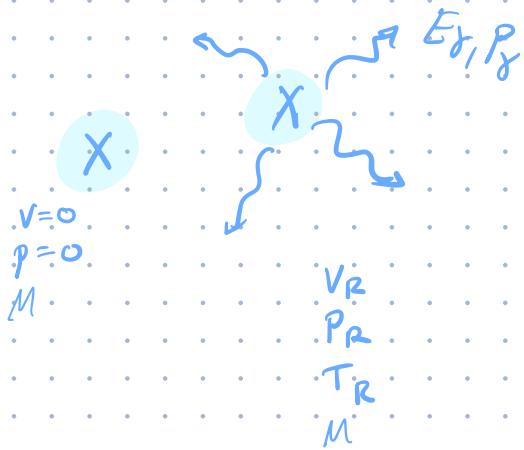
Neutrino-less  $\beta$ -decay



- Gamma decay
- unstable atomic nuclei radiates e.m. energy (ex. photon) and nucleus remains unchanged
  - emitted particle has no charge
  - massless  $\rightarrow$  do not carry away mass
  - nucleus gets rid of its excitation energy

## Energetics

$$E_i = E_f + E_\gamma + T_R$$



P<sub>tot</sub> = P<sub>tot</sub>

$$0 = p_R + p_\gamma$$

$$\begin{aligned} E_i - E_f &= \Delta E = E_\gamma + T_R \\ &\approx E_\gamma \end{aligned}$$

low energy  $\gamma$ -rays:  $T_R \approx 0$

high energy  $\gamma$ -rays: bigger  $T_R$ , radiation damage in lattice

## Cause of $\gamma$ -rays



nucleus is distribution of oscillating charges

- create a radiation field  $\rightarrow$  el. dipole field  $\rightarrow$  magnetic dipole field

### Dipole radiations:

- identical ang. distr. ( $E+M$ ),  $\sin^2\theta$  dependency
- $E$  and  $M$  have opposite parity

$E: \pi = (-)$

$M: \pi = (+)$

- $P$  = power (energy per time) radiated for el.  
+ magn. multipole

$$- \lambda = \frac{P}{\hbar \omega} \sim |m_{el}|^2$$

energy  
of photon

multipole op. responsible  
for changing nuc. state from  
 $\psi_i$  to  $\psi_f$  + creating photon

## Electric transitions

Weisskopf estimate  $\rightarrow$  e.m. transition is due to  
a single proton

- lower multipolarity dominate

## Magnetic transitions

- lower multipolarity dominate

## Compared

- medium heavy nucleus ( $A=100$ )  
 ↳ el. multipoles more probable

## Experimental data

- measured  $\lambda$  much smaller than Weisskopf estimate
  - ↳ poor overlap of init and final wavefunctions
  - ↳ slows decay
- measured  $\lambda \gg$  Weisskopf: more than one single nucleon responsible for the transition  $\Rightarrow$  collective excitations
- measured  $\lambda \approx$  Weisskopf: transition caused by single nucleon excitation

## Selection rules

→  $\gamma$ -ray carries away  $\pi$  and  $L$  + energy  $E\gamma$

→ e.m. process:  $\pi$  and  $L$  conserved

$$\Delta \pi = (-1)^3 (-1)^{-} =$$

$$\Delta L = (-1)^{+1} (-1)^{+1} = (+)$$

$$\pi: \quad \pi_{(ML)} = (-1)^{L+1}$$

$$\pi_{(EL)} = (-1)^L$$

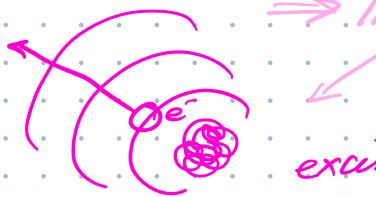
$$\Delta \pi = \pi_i \cdot \pi_f$$

$$J_{\text{init.}} = L + J_{\text{final}} \rightarrow |J_i - J_f| \leq L \leq J_i + J_f \quad (L \neq 0)$$

- lowest  $M$  and  $E$  ( $M_1, E_1$ ) are dominant
- photons have non-zero  $l$ , so  $L \neq 0$ .

what carries  $\pi$  and  $L$  then?  $(0^+ \rightarrow 0^+)$

↳ orbiting nearby  $e^-$ , nucleus gives energy to this  $e^-$   
 $\Rightarrow$  internal conversion



excitation energy transferred to  $e^-$  in nearby shell

- competes with  $\gamma$ -ray
- $e^-$  already exists, not created like photon

$\propto$  int. conversion coefficient

↳ ratio between gamma ray transition prob. and conversion electron prob.

$$\alpha = \frac{\lambda_c}{\lambda_\gamma} \quad \lambda_{\text{tot}} = \lambda_\gamma (1 + \alpha)$$

$0^+ \rightarrow 0^+$  transition

internal conversion is only mode for this transition



high  $\lambda_c$ : medium-to-heavy nuclei, high L and low  $E_{\text{transition}}$

Angular distributions

→ governed by Legendre polynomial