FYS3500 Solution 6

Problem 1 (Exercise 3.1 in M&S)

In the weak interaction (W.I.), the following quantities should be conserved:

- «the usual» (energy, total angular momentum, linear momentum, charge Q, ...)
- baryon number B
- lepton number L (also the separate lepton numbers L_e, L_μ, L_τ except in neutrino oscillations)
- CPT (product of charge conjugation, parity, and time reversal)

a) Forbidden: $L_\mu(\nu_\mu) = +1$ while $L_\mu(\mu^+) = -1$

$\Rightarrow$ muon number is not conserved, so forbidden reaction

b) Forbidden : Charge before is $Q = +1$ but charge after is $Q = 0$
$\Rightarrow$ charge is not conserved, forbidden reaction

c) $\underset{uds}{\Lambda} \rightarrow \underset{u\bar{d}}{\pi^+} \; e^- \; \bar{\nu_e}$

Forbidden : Baryon nr before is $B = 1$, while after is $B = 0$
$\Rightarrow$ baryon nr not conserved, forbidden reaction

d) $\underset{u\bar{s}}{K^+} \rightarrow \overset{u\bar{u}, d\bar{d}}{\pi^0} \; \mu^+ \; \nu_\mu$

L_μ ✓
B ✓
Q ✓

NB: must also remember to look at the masses when we determine if a decay ($\to A \to B + C$) is allowed: mass of products cannot exceed mass of free, decaying particle! (In collision, $A + B \to C + D$ "gain" center of mass energy from beam)

Energy:

$$\text{mass}(K) > m(\pi^0) + m(\mu^+) + m(\nu_\mu)$$

$\Rightarrow$ ok

Allowed for the weak interaction (strangeness is not necessarily conserved in W.I)

e) $\nu_e + p \rightarrow e^- + \pi^+ + p$

L_e ✓

B ✓

Q ✓

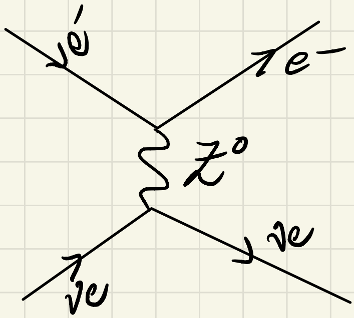
Allowed for the weak interaction

f) $\tau^+ \rightarrow \mu^+ + \bar{\nu}_\mu + \nu_\tau$

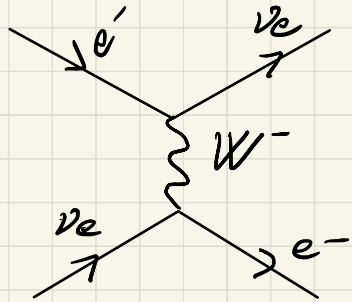
Forbidden: violates both L_μ & L_τ

Problem 2 (Exercise 3.3 in M&S)

The reaction $\nu_e + e^- \rightarrow \nu_e + e^-$ has two possible ways of proceeding:

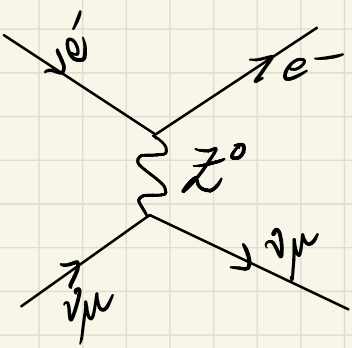


neutral current

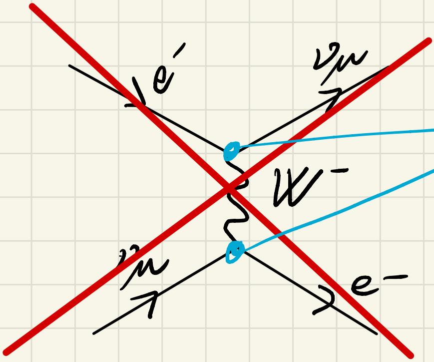


charged current

However, for $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$, only the neutral current is allowed:



The reason is that the charged current reaction would violate lepton no conservation in each vertex:



lepton no not conserved!

Problem 3   Exercise 3.5 in M+S

reactor   200m $\bar{\nu}_e$: $90 \pm 10\,\%$ of the flux if no oscillations

maximal mixing : $\vartheta = 45° = \pi/4$

mean neutrino energy $E(\bar{\nu}_e) = 3\,\mathrm{MeV}$

The probability for $\nu_\alpha \to \nu_\beta$ is given by eq. 3.31a :

$$P(\nu_\alpha \to \nu_\beta) \simeq \sin^2(2\vartheta)\,\sin^2\left[\frac{L\,\overbrace{(m_j^2 - m_i^2)}^{\Delta m_{\alpha\beta}}\,c^4}{4E\,\hbar c}\right]$$

$$= \underbrace{\sin^2(2 \cdot \pi/4)}_{1}\,\sin^2\left[\frac{200\mathrm{m} \cdot \Delta m_{\alpha\beta}^2 \cdot c^4}{4 \cdot 3\,\mathrm{MeV} \cdot \underbrace{\hbar c}_{197.4\,\mathrm{MeV\,fm}}}\right]$$

$90 \pm 10\,\%$ of flux remains

$\to$ On 1σ confidence ; $P(\bar{\nu}_e \to \bar{\nu}_e) \in [80, 100]\,\%$

$P(\bar{\nu}_e \to \bar{\nu}_x) \in [0, 20]\,\%$

$$P(\overline{\nu_e} \to \overline{\nu_x}) = 0 \rightarrow \Delta m = 0$$

$$P(\overline{\nu_e} \to \overline{\nu_x}) = 0.2$$

$$0.2 = \sin^2\left[\frac{200m \cdot \Delta m_{\alpha\beta}^2 \cdot c^4}{4 \cdot 3 MeV \cdot \hbar c}\right]$$

$$\sqrt{0.2} = \sin\left[\frac{200m \cdot \Delta m_{\alpha\beta}^2 \cdot c^4}{4 \cdot 3 MeV \cdot \hbar c}\right]$$

$$\sin^{-1}(\sqrt{0.2}) = \frac{200m \cdot \Delta m_{\alpha\beta}^2 c^4}{4 \cdot 3 MeV \cdot 197.4 MeV fm}$$

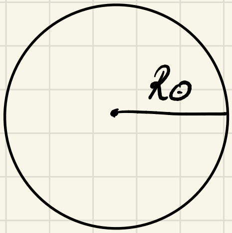
$$\Delta m_{\alpha\beta}^2 = \frac{\sin^{-1}(\sqrt{0.2}) \cdot 4 \cdot 3 MeV \cdot 197.4 MeV fm}{200 m \cdot c^4}$$

$$\Delta m_{\alpha\beta}^2 = 5.49 \cdot 10^{-15} MeV^2/c^4$$

$$= 5.49 \cdot 10^{-3} eV^2/c^4$$

$$\Rightarrow \underline{\underline{0 < \Delta m^2 < 5.49 \cdot 10^{-3} eV^2/c^4}}$$

Problem 4   3.6 in M&S



$R_\odot = 7 \cdot 10^5 \text{ km}$

$M_\odot = 2 \cdot 10^{30} \text{ kg}$

$\sigma = 0.7\, E_L \cdot 10^{-42} \text{ m}^2$

E_L : ν energy in lab [GeV]

Dominant reaction : $p + p \to d + e^+ + \nu_e$

ν_e mean energy : $0.26 \text{ MeV} = 0.26 \cdot 10^{-3} \text{ GeV}$

Calculate the mean free path λ :

$$\lambda = 1/n\sigma$$

nucleons per volume

ν-n cross section

Calculate number of nucleons :

$$N = \frac{M_\odot}{m_{nuc}} = \frac{2 \cdot 10^{30} \text{ kg}}{1.67 \cdot 10^{-27} \text{ kg}} = \underline{1.2 \cdot 10^{57}}$$

Density of nucleons n:

$$n = \frac{N}{V} = \frac{1.2 \cdot 10^{57}}{\frac{4\pi}{3}\left(7 \cdot 10^{5} \cdot 10^{3}\,m\right)^{3}}$$

$$= \underline{8.35 \cdot 10^{29}\ \text{nucleons}/m^{3}}$$

ν - nucleon cross section:

$$\sigma = 0.7\, E_{\nu} \cdot 10^{-42}\, m^{2}$$

$$= 0.7 \cdot 0.26 \cdot 10^{-3} \cdot 10^{-42}\, m^{2}$$

$$= \underline{1.8 \cdot 10^{-46}\, m^{2}}$$

Finally, calculate λ:

$$\lambda = \frac{1}{n\sigma} = \left(8.35 \cdot 10^{29}\, m^{-3} \cdot 1.8 \cdot 10^{-46}\, m^{2}\right)^{-1}$$

$$= \left(1.5 \cdot 10^{16}\, m^{-1}\right)^{-1}$$

$$= \underline{\underline{6.7 \cdot 10^{15}\, m}}$$

$\sim 10^{7}$ times $R_{\odot}$

Problem 5 Exercise 1.3 in M&S

$$p + \bar{p} \to \pi^0 + \pi^0$$

Strong interaction
$\Rightarrow$ both parity and charge conjugation must be conserved

Parity before: $P(p) \cdot P(\bar{p}) \, (-1)^{l_i}$

By convention: $P(p) = 1$, $P(\bar{p}) = -1$
p & $\bar{p}$ are in an s-state $\Rightarrow l_i = 0$

$$P_{before} = 1 \cdot (-1) \cdot (-1)^{l=0} = -1$$

Parity after:
$P(\pi^0) \cdot P(\pi^0) \, (-1)^{l_f}$

$$= \underbrace{P(u)}_{+1} \underbrace{P(\bar{u})}_{-1} \underbrace{P(u)}_{+1} \underbrace{P(\bar{u})}_{-1} (-1)^{l_f}$$

Parity: $(-1)^{l_f}$

So parity conserved if l_f: odd

π^0: spin-0 boson, needs symmetric wave function under interchange of pions

$$\Psi_{\pi_0\pi_0} = \Psi_{spin}\Psi_{space}$$

pions are spin-0 $\Rightarrow$ Ψ_{spin} is symmetric
$\rightarrow$ Ψ_{space} must thus be symmetric
$\rightarrow$ l_f: even
$\Rightarrow$ Controversy! Needed l_f: odd to conserve parity
$\Rightarrow$ cannot be strong interaction