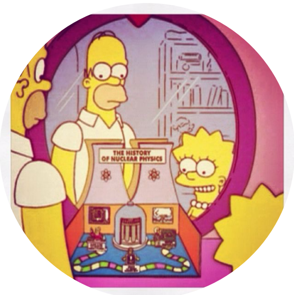
Lecture 1: Introduction to Nuclear Physics

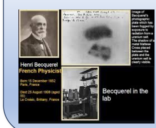
Martin & Shaw chapter 1: 1.1.1.

- **Introduction:**
  - History of Nuclear Physics
  - Nucleus
  - Units and dimensions in Nuclear Physics

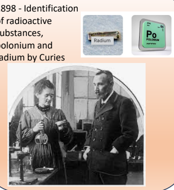


History of nuclear physics

1896 - Discovery of radioactivity by H. Becquerel



1898 - Identification of radioactive substances, polonium and radium by Curies



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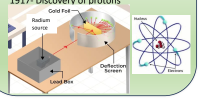
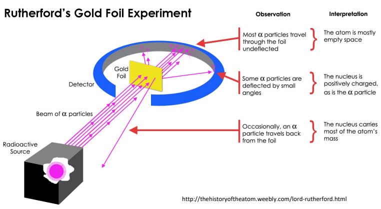
1903 -Alpha's charge-to-mass ration after emitted from radium
1909- alphas are helium nuclei
1911- Discovery of atomic nucleus Rutherford Model
1913- Bohr-Rutherford model
1917- Discovery of protons



1911- Discovery of atomic nucleus



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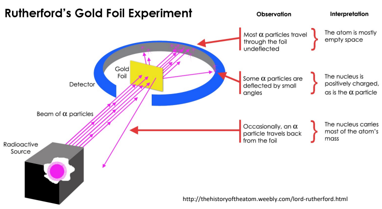

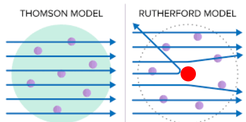

plum pudding model

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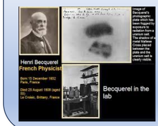

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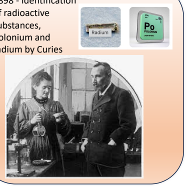

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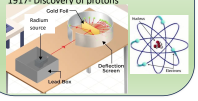
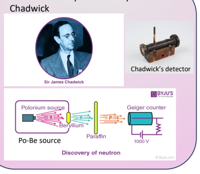


1932 - Discovery neutrons by J. Chadwick



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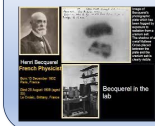

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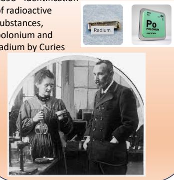

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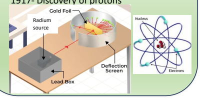

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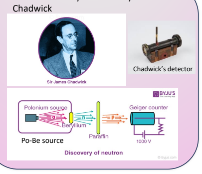


1930 - Existence of neutrino, proposed by Pauli
1956 - Discovery of neutrinos by Reines and Cowan

1930- neutrino by Pauli

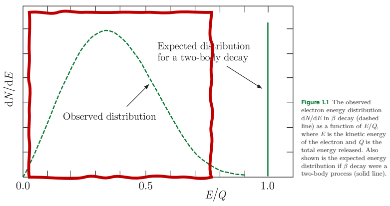


Figure 1.1 The observed electron energy distribution dN/dE in β decay (dashed line) as a function of E/Q, where E is the kinetic energy of the electron and Q is the total energy released. Also shown is the expected energy distribution if β decay were a two-body process (solid line).

Continuous beta-decay energy spectrum was discovered by Chadwick in 1914.

two-body decay was believed for all decays until 1914: α,β,γ decays

$$n \rightarrow p + e^{-}$$

After 1914, β decays had to be excluded!

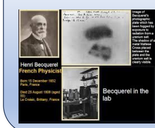
in 1932, Pauli suggested that there must be another neutral particle emitted in addition to electron in the β-decay process.

In that case Beta-decay energy is shared between the two emitted particles: **neutrino** and electron and the energy spectrum is continuous!

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

neutrino → "little neutral one"
very light and weakly interacting

1896 - Discovery of radioactivity by H. Becquerel



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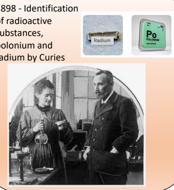

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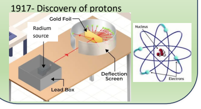

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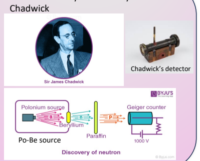

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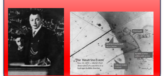
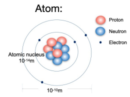


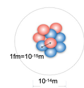
1938 – Discovery of nuclear fission by Lise Meitner and Otto Hahn

n + ^{235}U--> Ba isotopes (1934 E. Fermi)

Nucleus



Atomic nucleus:

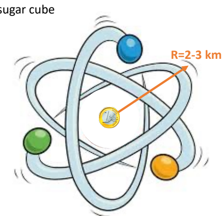


Atom is full about 0.0000000000004% of its total volume.
This means that atom is about 99.9999999999996% empty space

Nucleus is very dense!!

It carries all the mass of the atom in a small volume
density of the nucleus: 2,7x10^{17} kg/m^3
density of water: 1000 kg/m^3

all humans into a sugar cube

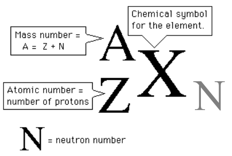


Atom is an empty space!!

If you increase the size of the nucleus to that of a coin the edge of the atom would be at a distance of 2-3 kilometers.

3 km: Blindern - Royal Palace

Nucleus:



$^1_1H_0$ $^{12}_6C_6$ $^{238}_{92}U_{146}$

^1H ^{12}C ^{238}U

Most common to use only ^AX: for example ^{238}U
- Since we know "U" is uranium, i.e., Z=92
- Since N=A-Z= 238-92=146

- ❑ Isoto**p**e (Same **p**roton number, Z)
- ❑ Isoto**n**e (Same **n**eutron number, N)
- ❑ Isob**a**r (Same **a**tomic mass number, A)

Can you write down three isotopes, three isotones, and three isobars using: Carbon($_6$C), Nitrogen($_7$N), Oxygen ($_8$O)?

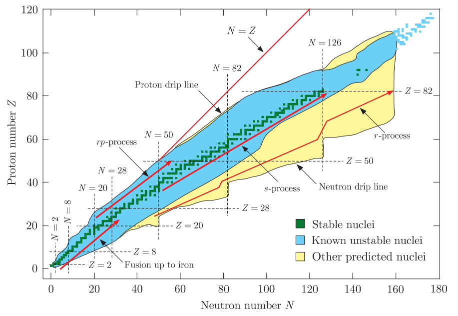
Hint: You can use possible mass numbers A=16,17,18 ☺

Chart of the nuclides

92 stable elements
280 stable isotopes

over 3000 unstable nuclei

over 6000 more are predicted to exist



stability line,
binding energy
neutron drip line
proton drip line
s-,r-,rp-process
Magic numbers:
N=2,8,20,28,50,82,126
Z=2,8,20,28,82,...

Units: length, time, energy, and masses

- **Length** is given in the order of 10^{-15} m (=1 fm, fermi for the name of Enrico Fermi, or femtometer).
- **Time scale** changes enormously in NP.
  - 10^{-20}s : unbound ^{8}Be (→ ^{4}He+^{4}He), ^{5}He (→ ^{4}He+n), ^{5}Li (→^{4}He+p) or nuclear reactions
  - 10^{-9}-10^{-12} s: lifetimes of nuclear excited states of nuclei (through gamma (γ) decay) (What NEP group measures)
  - Minutes-hours or even milions of years: beta (β) and alpha (α) decays (Think about the nuclear waste)
- **Energy unit** is MeV in Nuclear physics

1 eV = 1.602 x 10^{-19} Joule (energy gained by a single electronic charge when accelarated through 1 Volt of potential difference.)

- **Mass unit**: "u" (unified atomic mass unit, a.m.u.)

1u is defined as one twelfth of the mass of an unbound neutral atom of carbon-12 (^{12}C). This means that one nucleon has 1u of mass, thus both protons and neutrons do too.

We usually tell mass unit in terms of energy: 1u = 931.502 MeV/c^2 (easily derived from E=mc^2, where m=>1u=1.66×10^{-27} kg)

Why Carbon-12?
The atomic mass of carbon is 12.01 because natural carbon is a mixture of two stable isotopes, C-12 and C-13. The isotope C-12 is the most abundant, 98.893%, and it has been chosen as the mass reference 12.0000 u.

c^2 = 931.5 MeV/u

Particle	Mass (kg)	Mass (u)	Mass (Mev/c^2)
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proton	1.672623 x 10^{-27} kg	1.007276 u	938.28 MeV/c^2
electron	9.109390 x 10^{-31} kg	0.00054858 u	0.511 MeV/c^2

$m_p \approx m_n \approx 1u$

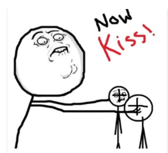
in reality $m_n > m_p$
Is this important?

$m_p \approx 2000\ m_e$

The nucleus holds almost all the mass of the atom

So far ...

- The atomic nucleus is a very dense, positively charged object composed of protons and neutrons
- Nuclei are organized according to their *Z* and *N* values on the Nuclear Chart (or Chart of the Nuclides)
- Nuclei are held together by the strong interaction, and the nuclear force is attractive at short range, but repulsive at very short distances. We will come to this point later.
- Let's continue with nuclear properties



Nuclear properties

Lecture 2: Nuclear properties

Martin & Shaw chapter 2:

Nuclear phenomenology (not 2.1.3 penning trap)

Krane chapter3:
Nuclear properties (see compendium s 66-77)

- Units, length, dimensions (Part we could not finish yesterday)
- **Nuclear Properties**
  - Size
  - Charge
  - Mass
  - …

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m_p≈m_n ≈1u

in reality m_n>m_p
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m_p≈2000 m_e

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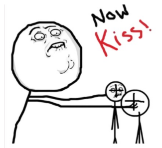


Nuclear properties

Nuclear properties are about the parameters we can describe the nucleus:

What do you think about nucleus? Can you name some of these properties?

Nuclear properties are about the parameters we can describe the nucleus:

- ❑Charge
- ❑Radius
- ❑Mass (Matter)
- ❑Binding energy
- ❑Angular momentum
- ❑Parity
- ❑Magnetuc dipole moment
- ❑Electric quadrupole moments
- ❑Excited states and their energies

Today

These are the "Static properties" of the nucleus.

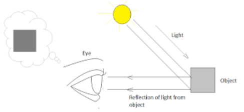
Later we will go through the "dynamic properties" like (nuclear) decay and (nuclear) reactions of nuclei

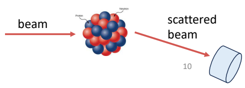
Radius, Charge, and Matter are tighly related to each other

- Charge → charge distribution (protons)
- Matter → matter distribution or nuclear matter (protons and neutrons)
- radius → size of the nucleus (protons and or neutrons)

scattering experiments in 50s were very successful to determine size of the nucleus:

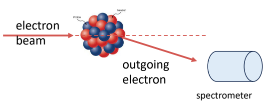
- Electron scattering experiments probed charge distribution (protons) of the nucleus (electromagnetic interaction)
- Alpha (Rutherford), proton, neutron scattering experiments probed matter distribution (strong interaction).





charge distribution

Electron scattering on nuclei

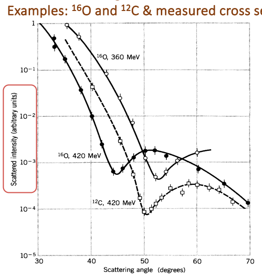


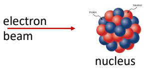
- Main ingredients to meausre charge distribution of the nucleus:
- Charge of the nucleus is given by its protons i.e., "Ze" (where e=1.602x10^{-19} C, Z: atomic number)
- *a charged particle beam is suitable to probe the charge distribution of the nucleus, mainly protons*
- *Its wavelengh should be similar or smaller than the nucleus size (which is about 10 fm, diameter of nucleus)*
- electron was widely used as beam for the scattering experiments in 1950s.
- its energy should be and was accelarated from ***100 MeV up to 1 GeV***.

This energy can be estimated from the de Broglie wave length equation $\lambda = \frac{h}{p}$

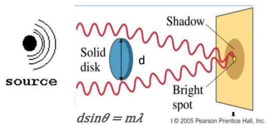
where $\lambda \lesssim 10$ fm, h: planck constant, p: momentum

Electron scattering on nuclei
Examples: ^{16}O and ^{12}C & measured cross sections

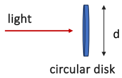


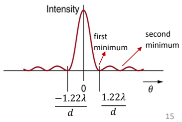


Light scattering on a circular disk

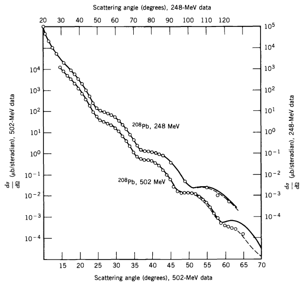


d: diameter of the disk
λ: wavelength of the light emitted from the source

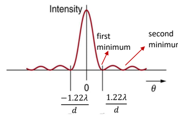




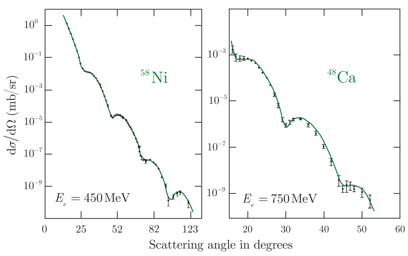
Nuclear Matter



circular disk

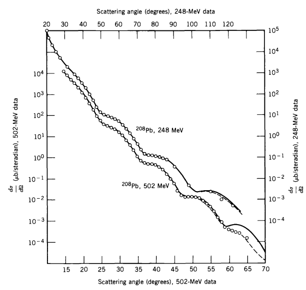


Why do not the minima go all the way to zero like we see in the case of circular disk?

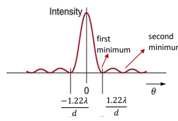


M&S

Nuclear Matter



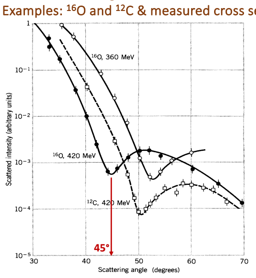
circular disk



Why do not the minima go all the way to zero like we see in the case of circular disk?

A: Because nucleus does not have a sharp boundary. in other words, Nucleus, like an atom, does not have a defined radius. They are not solid spheres like billiard balls.

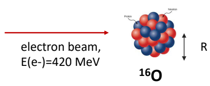
Electron scattering on nuclei
Examples: ^{16}O and ^{12}C & measured cross sections



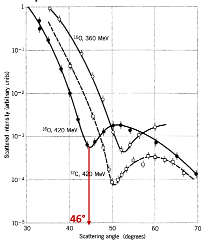
$$\sin\theta = \frac{1.22\lambda}{d}$$

d: diameter of the nucleus (2R)

For E(e-)=420 MeV and measured θ=46°
what is the radius of ^{16}O => R(^{16}O)?



Electron scattering on nuclei
Examples: ^{16}O and ^{12}C

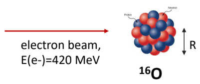


$$\theta = sin^{-1}\left(\frac{1.22\lambda}{d}\right)$$

d: diameter of the nucleus (2R)

Light scattering on a circular disk

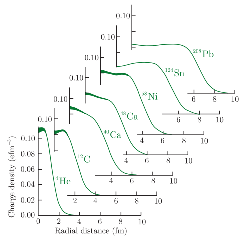
For E(e-)=420 MeV and measured θ=46°
R(^{16}O)=2.5 fm



This value, however, is only rough estimate because potential scattering is a three-dimensional problem while we approximate it to diffraction by a two-dimensional disk.

better determination of the chare distribution can be done using the cross-section distributions from the electron scattering experiments. This is explained in M&S (p 52-55)

charge distribution results from the electron scattering experiments



$\rho(r)$: Charge density corresponds to the charge distribution of the nucleus

What do we interpret from the distributions:

- Radius increases with mass number, A
- The central nuclear charge density is nearly the same for all nuclei. Nucleons do not seem to congregate (concentrate) near the center of the nucleus, but instead have a fairly constant distribution out to the surface.

Thus, the number of nucleons per unit volume is roughly constant and we can calculate the radius from that:

$$\frac{A}{\frac{4}{3}\pi R^3} \sim \text{constant}$$

A: mass of the nucleus
R: mean radius of the nucleus

$$R \propto A^{1/3}$$

$$\boxed{R = R_0 A^{1/3}}$$

R_0: Proportionality constant Obtained as R_0=1.2 fm from the electron scattering measurements.

more quantitavive determination of the relation between nuclear radius and mass number from the electron scattering experiments:

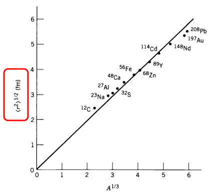


Figure 3.5 The rms nuclear radius determined from electron scattering experiments. The slope of the straight line gives R_0 = 1.23 fm. (The line is not a true fit to the data points, but is forced to go through the origin to satisfy the equation $R = R_0 A^{1/3}$.) The error bars are typically smaller than the size of the points ($\pm$0.01 fm). More complete listings of data and references can be found in the review of C. W. de Jager et al., *Atomic Data and Nuclear Data Tables* **14**, 479 (1974).

rms nuclear radius:
rms stands for root mean square

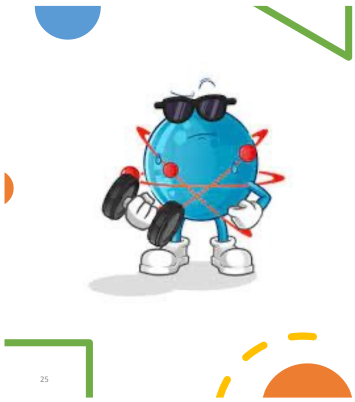
$R=\langle r^2\rangle^{1/2}$

Why square, r^2?

$\frac{d\sigma}{d\Omega}$ ~ $cm^2 \propto r^2$

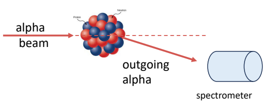
Why mean $\langle r^2 \rangle$

$$\langle r^2 \rangle = \frac{\int r^2 \rho(r) d\tau}{\int \rho(r) d\tau}$$



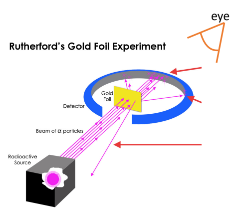
mass distribution

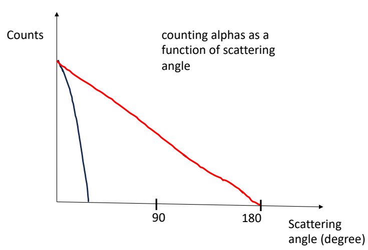
Hadron scattering on nuclei



- electrons only “see” the protons via the coulomb force
- We need a beam particle which interacts with all matter, meaning both protons and neutrons of the nucleus.
- So they are hadrons like protons, neutrons, alpha particles since they interact with both neutrons and protons (via strong nuclear force).
- In this case the radius is characteristic of the *nuclear,* rather than the *Coulomb,* force; these radii therefore reflect the distribution of all nucleons in a nucleus, not only the protons.

Best example is the alpha scattering for mass distribution: Reminder of the Rutherford’s gold exp. (Rutherford scattering)





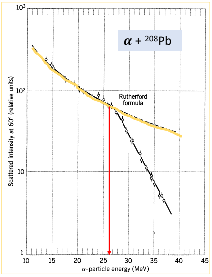
Let’s change the strategy now: Keep the scattering angle fixed at a certain angle and change the incoming energy of the alpha particle

At low energies, alpha and the ^{208}Pb nucleus interact only via Coulomb interaction (Rutherford scattering)

With increasing energy, this drops suddenly:

With increasing energy, the Coulomb repulsion of the nuclei is overcome and they may approach close enough to allow the nuclear force to act. In this case the Rutherford formula no longer holds.

Finally alpha particle is absorbed or nucleus behaves as an absorbing sphere and elastic scattering is only a small part of the total interaction



Let’s change the strategy now: Keep the scattering angle fixed at a certain angle and change the incoming energy of the alpha particle

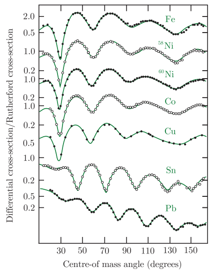


Figure 2.8 Differential cross-sections (normalised to the Rutherford cross-section) for the elastic scattering of 30.3 MeV protons, for a range of nuclei compared with optical model calculations. Source: Satchler (1967). Copyright (1967) Elsevier, reprinted with permission.

Once we know that the alpha incident energy is high enough for the nuclear interaction to happen, diffration-like behaviour again like in the case of electrons!

Conclusion of both type of experiments:

- **the charge and matter radii of nuclei are nearly equal, to within about 0.1fm.**
- Both show the **A$^{1/3}$** dependence with $R_0 \cong 1.2$ fm.

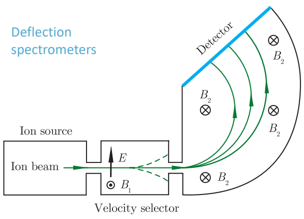
- Because heavy nuclei have about 50% more neutrons than protons, we might have expected the neutron radius to be somewhat larger than the proton radius; however, the proton repulsion tends to push the protons outward and the neutron-proton force tends to pull the neutrons inward, until the neutrons and protons are so completely intermixed that the charge and matter radii are nearly equal.

mass value and mass spectroscopy

We can measure mass of the nucleus, very precisely via mass spectroscopy

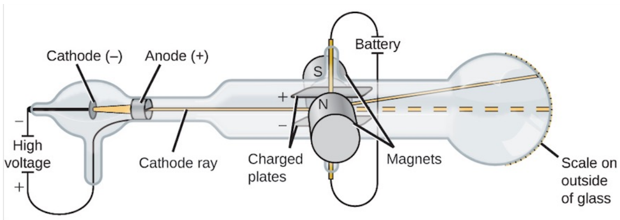
- Different methods exist →Deflection spectrometers & Penning traps. We will briefly see the former.

Deflection spectrometers



The method was applied first time by Thompson in 1912.

Charge-to-mass ratio of electron by Thomspon: 1912

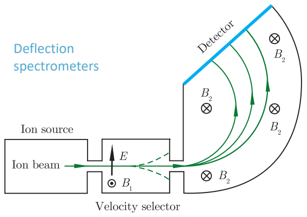


A diagram of J.J. Thomson's cathode ray tube. The ray originates at the cathode and passes through a slit in the anode. The cathode ray is deflected away from the negatively-charged electric plate, and towards the positively-charged electric plate. The amount by which the ray was deflected by a magnetic field helped Thomson determine the mass-to-charge ratio of the particles. Image from Openstax, CC BY 4.0.

We can measure mass of the nucleus, very precisely via mass spectroscopy

- Different methods exist →Deflection spectrometers & Penning traps. We will briefly see the former.

Deflection spectrometers



The method was applied first time by Thompson in 1912.

First: different isotopes are produced. They have different speeds.

Second: By providing E and B_1 balance, only isotopes with a specific speed pass through the velocity selector. The others are deflected (See dashed lines).

Third: Now the remaining isotopes pass through the dipole magnet which has another B value (B_2). They will be deflected according to their m/q value.

- Measured mass values are used in mass models. But they have to be very precise: $\frac{\Delta m}{m}$=10^{-6}
- Such precision cannot be provided from the values of E and B fields.
- Instead there is a technique called "mass doublet technique" which provides a precision of 10^{-8}-10^{-9}

For details: Read M&S page# 44-45

To test nuclear models, $\Delta m/m$ must be determined with a precision of at least 10^{-6}. It is clear from (2.2) that this would require an unrealistically precise knowledge of several physical parameters. However, this problem can be overcome by making a series of measurements comparing pairs of ions with similar masses and combining these to give the mass of the ion under study. This is known as the *mass doublet* method and measurements with precisions as good as 10^{-8} or 10^{-9} can be achieved using it. Consider the example[2] of making an accurate measurement of the mass of the nucleus ^{14}N. Setting the spectrometer for molecules with total mass number 128, the mass difference

$$\Delta_1 = m(C_9H_{20}) - m(C_{10}H_8) = 12m(^1H) - m(^{12}C)$$

can be measured, where the very small difference (of order 10^{-9}u) in the molecular binding energies of the two molecules has been neglected. Then, from the above relation,

$$m(^1H) = \frac{1}{12}[m(^{12}C) + \Delta_1].$$

[2]See p. 61 of Krane (1988).

Lecture 3: Binding Energy

Compendium (Krane):
Nuclear Binding Energy (Chapter 3, Pages: 65-76)
M&S: Chapter 2.3

Reminder for the class: Some or most of my slides have a lot of texts. Do not try to read them during the class. They are for you to read later 🙂

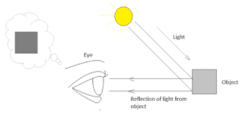
Recap from the last lecture:
Nuclear properties

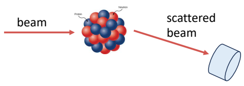
Nuclear properties:

- ❑Charge
- ❑Radius
- ❑Mass (Matter)
- ❑Binding energy
- ❑Angular momentum
- ❑Parity
- ❑Magnetuc dipole moment
- ❑Electric quadrupole moments
- ❑Excited states and their energies

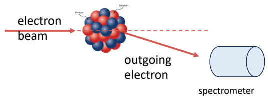
Today

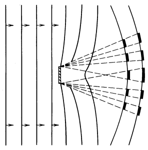
Radius, Charge, and Matter are tighly related to each other

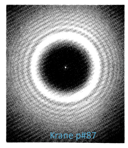


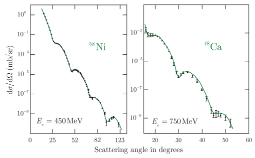


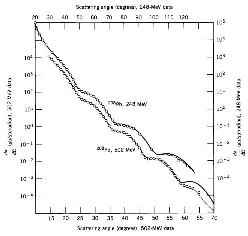
Electron scattering on nuclei



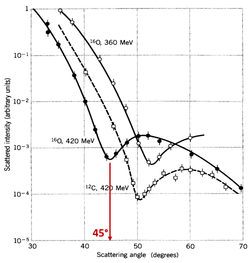






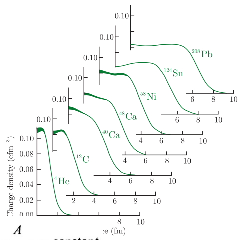


Examples: ^{16}O and ^{12}C & measured cross sections



$sin\,\theta = \frac{1.22\lambda}{d}$ (d=2R)

For E(e-)=420 MeV and measured θ=46°
R(^{16}O)=2.5 fm

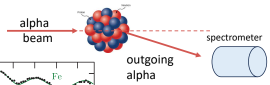


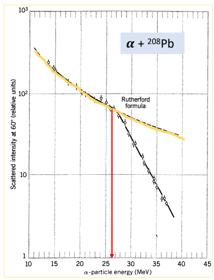
$$\frac{A}{\frac{4}{3}\pi R^3} \sim \text{constant}$$

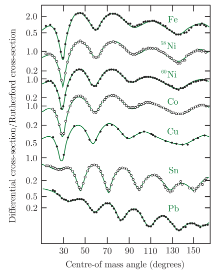
$$R \propto A^{1/3}$$

$$R = R_0 A^{1/3}$$

Hadron scattering on nuclei





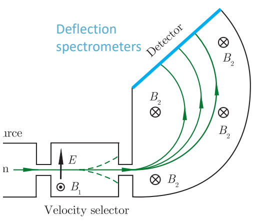


$$R = R_0 A^{1/3}$$

Figure 2.8 Differential cross-sections (normalised to the Rutherford cross-section) for the elastic scattering of 30.3 MeV protons, for a range of nuclei compared with optical model calculations. Source: Satchler (1967). Copyright (1967) Elsevier, reprinted with permission.

mass spectroscopy

- Different methods exist → **Deflection spectrometers** & Penning traps.

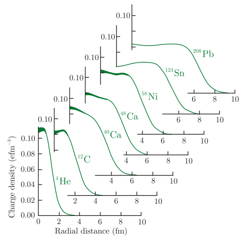


qE=qvB$_1$ => v=$\frac{E}{B_1}$

qvB$_2$ = m $\frac{v^2}{\rho}$=> m=$\frac{q\rho}{E}B_2^2$

q: charge of the isotope
m: mass of the isotope
ρ: curvatire radius of the isotope
v: speed of the isotope
B$_1$: Magnetic strength of the magnet
B$_2$: Magnetic strength of the dipole magnet
Usually B$_1$=B$_2$

Summary of the last lecture:



- Electron scattering experiments probed the charge desntiy/distribution of the nucleus and R=R$_0$A$^{1/3}$
- Hadron scattering experiments probed the matter distribution/nuclear density of the nucleus and R=R$_0$A$^{1/3}$
- Nuclei have no definitive borders.
- Nuclear density is nearly same in all nuclei (changes very little from light to heavy nuclei.)
- Nuclear radius increases with increasing A (R α A$^{1/3}$)
- Nuclear density is roughly constant, i.e. Nucleons have a fairly constant distribution out to the surface.
- Different methods can measure atomic mass of the nucleus very precisely which is important for mass models.

Lecture 3: Binding Energy

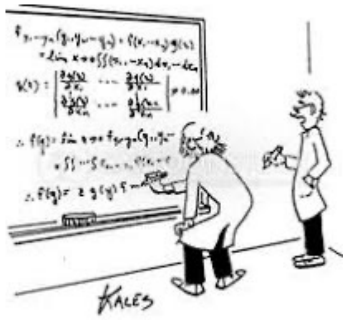
Compendium (Krane):
Nuclear Binding Energy (Chapter 3, Pages: 65-76)
M&S: Chapter 2.3

- ❑Charge
- ❑Radius
- ❑Mass (Matter)
- ❑Binding energy Today
- ❑Angular momentum
- ❑Parity
- ❑Magnetuc dipole moment
- ❑Electric quadrupole moments
- ❑Excited states and their energies

❑Nuclear Binding energy

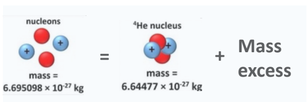
- o**Some formulas, definitions**
- o**Binding energy per nucleon**
- o**Semi-empirical mass formula**
- o**Visiting the nuclear chart again**
- o**Mass parabolas**
- o**...**

Some formulas, definitions



Nuclear binding energy & Mass excess

*The **nuclear binding energy** is the amount of energy required to keep the protons and neutrons together inside the nucleus.*



This definition tells us that the mass of the nucleus is not equal to the sum of the masses of its protons and neutrons but less!

$Zm_p + Nm_n > M_{nucleus}$

This difference in the mass is called **mass excess** given by:

$\Delta m = Zm_p + Nm_n - M_{Nucleus}$

Nuclear binding energy & Mass excess

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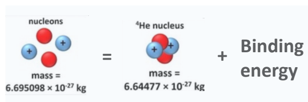

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$Zm_p + Nm_n > M_{nucleus}$

This difference in the mass is called **mass excess** given by:

$\Delta m = Zm_p + Nm_n - M_{Nucleus}$

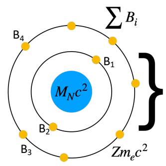
Recalling Einstein's theory of relativity formula $E=mc^2$ (mass and energy is interchangeable!), we can transform the mass excess to Binding energy:

Binding Energy = Δmc^2 (MeV)

The nuclear binding energy is a measure of how tightly nucleons are bonded to each other:

- If the B.E. is large, it takes a lot of energy to separate the nucleons, in other words, the nucleus is very stable
- if the B.E. is small, less amount of energy is needed to separate the nucleons, indicating that the nucleus is less stable.

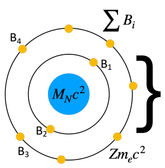
Mass of the nucleus



$M_A c^2 = Mc^2$

c^2 = 931.5 MeV/u
1u = 931.5 MeV/c^2
m_n = 939.57 MeV
m_p = 938.28 MeV
m_e = 0.511 MeV

Mass of the nucleus



$M_A c^2 = Mc^2$

c^2 = 931.5 MeV/u
1u = 931.5 MeV/c^2
m_n = 939.57 MeV
m_p = 938.28 MeV
m_e = 0.511 MeV

$M_{Atom} = M_{Nucleus} + Zm_e - \sum_{i=1}^{Z} B_i/c^2$

$M_{Atom} = M_{Nucleus} + Zm_e$

$M_{Atom}({}^{A}_{Z}X_N) = M_{Nucleus}({}^{A}_{Z}X_N) + Zm_e$

$M({}^{A}_{Z}X_N) = M_{Nucleus}({}^{A}_{Z}X_N) + Zm_e$

$M_{Nucleus}({}^{A}_{Z}X_N) = M({}^{A}_{Z}X_N) - Zm_e$ (recall: => ($E=mc^2$))

$M_{Nucleus}({}^{A}_{Z}X_N)c^2 = \left(M({}^{A}_{Z}X_N) - Zm_e\right)c^2$ (1)

Nuclear binding energy (B.E.)

B.E. = $\{Zm_p + Nm_n - M_N({}^{A}_{Z}X_N)\}c^2$

Nuclear binding energy (B.E.)

B.E. = $\{Zm_p + Nm_n - M_N({}^{A}_{Z}X_N)\}c^2$

Write Eq. (1) for M_N

= $\{Zm_p + Nm_n - (M({}^{A}_{Z}X_N) - Zm_e)\}c^2$

= $\{Z(m_p + m_e) + Nm_n - M({}^{A}_{Z}X_N)\}c^2$

B.E. = $\{Zm({}^{1}H) + Nm_n - M({}^{A}_{Z}X_N)\}c^2$ (2)

Since masses are given in "u" (unified atomic mass unit), we can obtain B.E. unit in MeV:

UNIT: B.E.= $[mc^2]$={ $[uc^2]$={u 931.5 MeV/u]=931.5 MeV

c^2 = 931.5 MeV/u

B.E. = $\{Zm({}^{1}H) + Nm_n - M({}^{A}_{Z}X_N)\}u.\,931.5\ \mathrm{MeV/u}$ (3)

Binding energy is very little compared to the mass energy of the nucleus.

Example of ${}^{4}He$ (Z=2,N=2) → Mc^2 versus B.E.

Binding energy is very little compared to the mass energy of the nucleus.

Example of ^{4}He (Z=2,N=2)

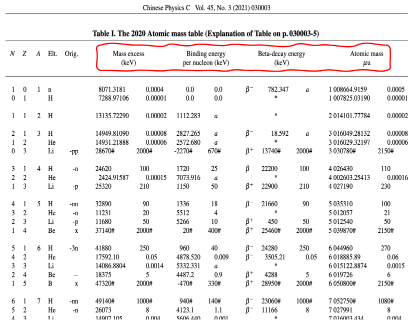
- B.E. = $\{Zm(^{1}H) + Nm_n - M(^{A}_{Z}X_N)\}c^2$

  $= \{2 \times (1.007825\ u) + 2 \times (1.008664\ u) - (4.002603\ u)\} \times 931.5\ MeV/u$

  $= 0.0304 \times 931.5\ MeV =$ **28.3 MeV**

- $M(^{4}He)c^2 = (4.002603\ u) \times 931.5\ MeV/u =$ **3728 MeV**

- $\frac{BE}{Rest\ Mass} = \frac{28.3}{3728} = 0.0075 =$ **0.75%**

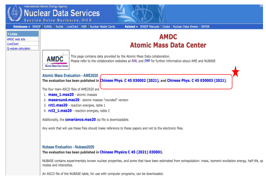


Chinese Physics C Vol. 45, No. 3 (2021) 030003

Table I. The 2020 Atomic mass table (Explanation of Table on p. 030003-5)

N	Z	A	Elt.	Orig.	Mass excess (keV)		Binding energy per nucleon (keV)			Beta-decay energy (keV)		Atomic mass μu	
1	0	1	n		8071.3181	0.0004	0.0	0.0	β^-	782.347	a	1 008664.9159	0.0005
0	1		H		7288.97106	0.00001	0.0	0.0		*		1 007825.03190	0.00001
1	1	2	H		13135.72290	0.00002	1112.283	a		*		2 014101.77784	0.00002
2	1	3	H		14949.81090	0.00008	2827.265	a	β^-	18.592	a	3 016049.28132	0.00008
1	2		He		14931.21888	0.00006	2572.680	a		*		3 016029.32197	0.00006
0	3		Li	-pp	28670#	2000#	-2270#	670#	β^+	13740#	2000#	3 030780#	2150#
3	1	4	H	-n	24620	100	1720	25	β^-	22200	100	4 026430	110
2	2		He		2424.91587	0.00015	7073.916	a		*		4 002603.25413	0.00016
1	3		Li	-p	25320	210	1150	50	β^+	22900	210	4 027190	230
4	1	5	H	-nn	32890	90	1336	18	β^-	21660	90	5 035310	100
3	2		He	-n	11231	20	5512	4		*		5 012057	21
2	3		Li	-p	11680	50	5266	10	β^+	450	50	5 012540	50
1	4		Be	x	37140#	2000#	20#	400#	β^+	25460#	2000#	5 039870#	2150#
5	1	6	H	-3n	41880	250	960	40	β^-	24280	250	6 044960	270
4	2		He		17592.10	0.05	4878.520	0.009	β^-	3505.21	0.05	6 018885.89	0.06
3	3		Li		14086.8804	0.0014	5332.331	a		*		6 015122.8874	0.0015
2	4		Be	–	18375	5	4487.2	0.9	β^+	4288	5	6 019726	6
1	5		B	x	47320#	2000#	-470#	330#	β^+	28950#	2000#	6 050800#	2150#
6	1	7	H	-nn	49140#	1000#	940#	140#	β^-	23060#	1000#	7 052750#	1080#
5	2		He	-n	26073	8	4123.1	1.1	β^-	11166	8	7 027991	8
4	3		Li		14907.105	0.004	5606.440	0.001		*		7 016003.434	0.004

Mass Excess, Binding Energies, and Atomic masses are tabulated and every several year are updated after new mass measurments.



Nucleon separation energies are other interesting properties that are tabulated as well

Nucleon separation energies: S_p and S_n

S_n, one neutron separation energy is the amount of energy that is needed to remove a neutron from a nucleus $^{A}_{Z}X_N$

$^{A}_{Z}X_N \rightarrow {}^{A-1}_{Z}X_{N-1} + n$

$S_n = \{M(^{A-1}_{Z}X_{N-1}) - M(^{A}_{Z}X_N) + m_n\}c^2$

Replacing mass values with Eq (2) in slide#20, we can express S_n in terms of Binding energies as below:

$S_n = B(^{A}_{Z}X_N) - B(^{A-1}_{Z}X_{N-1})$

Comparison of S_n values for Tin (Sn, Z=50) isotopes:

$S_n = B(^{A}_{Z}X_N) - B(^{A-1}_{Z}X_{N-1})$

S_n (^{115}Sn)= B(^{115}Sn) - B (^{114}Sn) = 7.6 MeV

(^{115}Sn) => S_n = 7.6 MeV (N=65) (even-odd)

(^{116}Sn) => S_n = 9.6 MeV (N=66) (even-even)

(^{117}Sn) => S_n = 6.9 MeV (N=67) (even-odd)

(^{118}Sn) => S_n = 9.3 MeV (N=68) (even-even)

(^{119}Sn) => S_n = 6.5 MeV (N=69) (even-odd)

(^{120}Sn) => S_n = 9.1 MeV (N=70) (even-even)

It is easier to remove a neutron from a nucleus with odd number of neutrons. Last unpaired neutron is easy to remove.

evenN- evenZ nucleus has all neutrons in pair. We need more energy to break a pair and remove one neutron. (Pairing makes nucleus more tightly bound and stable!)

Comparison of S_n and S_p values for Antimony (Sb, Z=51) isotopes:

(^{116}Sb) => S_n = 7.9 MeV	S_p = 4.1 MeV	(Z=51, N=65)	(odd-odd)
(^{117}Sb) => S_n = 9.9 MeV	S_p = 4.4 MeV	(Z=51, N=66)	(odd-even)
(^{118}Sb) => S_n = 7.4 MeV	S_p = 4.9 MeV	(Z=51, N=67)	(odd-odd)
(^{119}Sb) => S_n = 9.6 MeV	S_p = 5.1 MeV	(Z=51, N=68)	(odd-even)
(^{120}Sb) => S_n = 7.0 MeV	S_p = 5.6 MeV	(Z=51, N=69)	(odd-odd)
(^{121}Sb) => S_n = 9.3 MeV	S_p = 5.8 MeV	(Z=51, N=70)	(odd-even)

Nucleon separation energies: S_p and S_n

S_p, one proton separation energy is the amount of energy that is needed to remove a proton from a nucleus $^{A}_{Z}X_N$

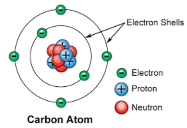
$^{A}_{Z}X_N \rightarrow {}^{A-1}_{Z}Y_{N-1} + p$

$S_p = \{M(^{A-1}_{Z-1}Y_N) - M(^{A}_{Z}X_N) + m(^{1}_{1}H)\}c^2$

Replacing mass values with Eq (2) in slide#20, we can express S_p in terms of Binding energies as below:

$S_p = B(^{A}_{Z}X_N) - B(^{A-1}_{Z-1}Y_N)$

Nucleon separation energies in nuclear physics ⇔ **ionization energies in atomic physics**



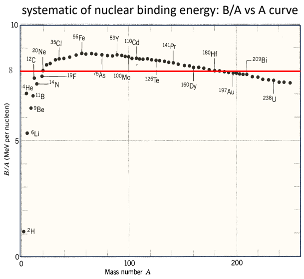
Just like atomic ionization energies, ***Nucleon separation energies*** show evidence for shell structure in nuclei that is similar to the electron shells in the atom. We will come to that later in the discussion of **SHELL MODEL** of Nucleus by seeing the systematics of neutron and proton separation energies.

Average binding energy per nucleon

Average binding energy per nucleon: $\frac{B.E.}{A}$ or $\frac{B}{A}$

Since the binding energy increases approximately linearly with A, it is common to use binding energy per nucleon, **B/A**.

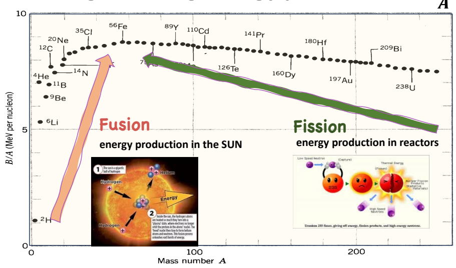
systematic of nuclear binding energy: B/A vs A curve



First visible features of the B/A vs A curve:

- The average binding energy per nucleon is relatively constant and around 8 MeV (except for the very light nuclei)
- Around A=60 the curve reaches its maxiumum. Nuclei in this region are the most tightly bound ones.
- The binding energy per nucleon is the highest for ^{56}Fe (around 8.8 MeV), the most tigthly bound.
- We can climb to ^{56}Fe from left and from right. What happens?

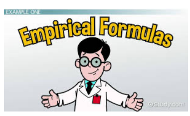
Average binding energy per nucleon: $\frac{B.E.}{A}$



Energy is released when going from a less bound system to a more bound system, because a tighter bound nucleus weighs less!!

$M_N(^A_ZX_N) = Zm_p + Nm_n - B.E.$

$Q=(M_{initial}-M_{final})c^2$ reaction energy. $Q>0$ if $M_{initial} > M_{final}$ (Exothermic, energy released)



Semi-empirical mass formula (SEMF):

Semi-empirical mass formula (SEMF):

- Semi-empirical formula gives us more striking properties of nuclei by explaining the nuclear binding energy as a function of A. It consists of five terms.
- It was proposed in 1935 by German physicist Weizsäcker.
- It is a semi-empirical formula because it contains a set of constants from fitting experimental data but at the same time the formula is based on theory.
- theory: liquid drop model (analogy between a nucleus and a liquid drop)
- To obtain the SEMF, we will first build the Nuclear Binding Energy.
- The nuclear binding energy based on the liquid drop model reproduces the experimental B/A curve successfully. We will see the comparison soon.

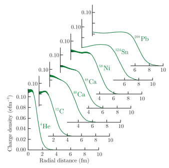
Binding energy formula:

dependence on A and Z

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$

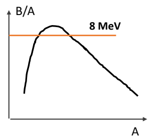
volume, surface, Coulomb, symmetry, pairing

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$



The physical meaning of these five terms can be understood from recalling nuclear radius and B/A:

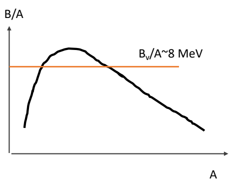
- Nuclear density is nearly same in all nuclei (changes very little from light to heavy nuclei.) → $R=R_0A^{1/3}$
- (Experimental) B/A is almost linear with A (B/A vs A plot)
- → B≈ A. 8 MeV



1- Volume term

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$

Volume term dominates the binding energy.
If we ignore the other terms, the Binding energy should be linear in a first approximation. See the figure here.



$\frac{B_v}{A}\approx$ 8MeV => $B_v \,\alpha\, A$ => B_v=const A

Another way of seeing it:

$B_v \,\alpha\, \frac{4}{3}\pi R^3$ (Volume of a sphere)

$\alpha\, (R_0 A^{1/3})^3$

$\alpha\, A$

$B_v = a_v A$

$a_v \approx 16$ MeV

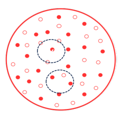
The linear dependence of B on A is surprising and gives us a hint about the nuclear force.

If the nuclear force was "Long range", $B \,\alpha\, A(A-1)$ => $B \,\alpha\, A^2$
(every nucleon would be attracting all the other nucleons.)

Since B varies linearly with A => $B \,\alpha\, A$, this sugests that each nucleon attracts only its closest neighbours and not all of the other nucleons.

From the electron scattering experiment, we know that Nuclear density is roughly constant, and thus each nucleon has about the same number of neighbouring nucleons.

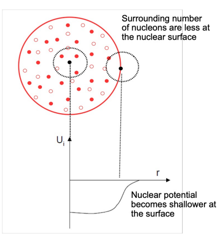
Each nucleon thus contributes roughly the same amount to the binding energy.



2- Surface term

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$

An exception to the argument of the volume term is the fact that a nucleon close to the nuclear surface is surrounded by fewer nucleons and thus experiences less attraction compared to nucleons in the center region. In other words, nucleons in the nuclear surface are less tightly bound. We must therefore subtract this surface effect from the volume term:



$B_s \,\alpha\, \pi R^2$ (surface area)

$B_s \,\alpha\, R^2 \,\alpha\, A^{2/3}$

$B_s = a_s A^{2/3}$

$a_s \approx 16.8$ MeV

3- Coulomb term

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$

The Coulomb repulsion of the protons tends to make the nucleus less tightly bound and this needs to be subtracted.

Unlike the nuclear force, the Coulomb force is a long-range force. Thus, each proton repels all of the others so B_c term will be proportional to Z(Z-1).

If we assume the nucleus as a charge sphere, we can calculate the Coulomb term as:

$$B_c \,\alpha\, \frac{3}{5}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Ze(Z-1)e}{R} = \frac{3}{5}\left(\frac{e^2}{4\pi\varepsilon_0}\right)\frac{Z(Z-1)}{R_0A^{1/3}}$$

$B_c = a_c Z(Z-1)\, A^{-1/3}$

$a_c \approx 0.72$ MeV

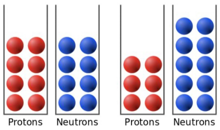
4- Symmetry term (Krane) -> Asymmetry term in M&S

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$

- Light mass stable nuclei tend to have same number of protons and neutrons (N=Z=A/2). Deviations from this symmetry makes the nucleus less bound and less stable.
- This term is very important for light mass nuclei. If we do not include it, the formula would allow stable isotopes of hydrogen with hundreds of neutrons!!!
- This "less stable" effect is seen when we recall the Pauli principle. The Pauli principle allows only two nucleons in the same state. More nucleons should be filling the next state. That is because protons and neutrons are fermions.
- Discuss the case of ^{16}O:
- $a_{asym} \approx 23$ MeV

A=16

STABLE ^{16}O: Z=8, N=8 | **halflife=0.75sec** ^{16}C: Z=6, N=10



N-Z=0 | N-Z=4 (Neutron excess).

Fermi gas model. We will see it later when discussing about the nuclear models.

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$

- Heavier nuclei instead have more neutrons to compensate the Coulomb repulsion with the nuclear interaction by neutrons. (Volume and Coulomb terms)
- Think about stable ^{208}Pb (Z=82, N=126→ N-Z=44). The effect of this asymmetry terms becomes very small for heavy nuclei.
- Calculate the contribution of the asymmetry term to the B.E. in the case of ^{100}H and ^{208}Pb. Take a_{asym}=23 MeV.

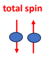
How do you judge it?

5- Pairing term

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$

- The pairing term arises from the tendency of like nucleons in the same spatial state to couple pairwise to give configurations with spin zero. When coupled like this, the wavefunctions of the two nucleons heavily overlap and so on average **they are closer** together than when coupled in other configurations, and hence are more tightly bound. *Even-N even-Z* nuclei (we call them even-even) are therefore more bound.

total spin =zero

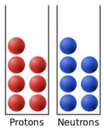


$a_p \approx 34$ MeV

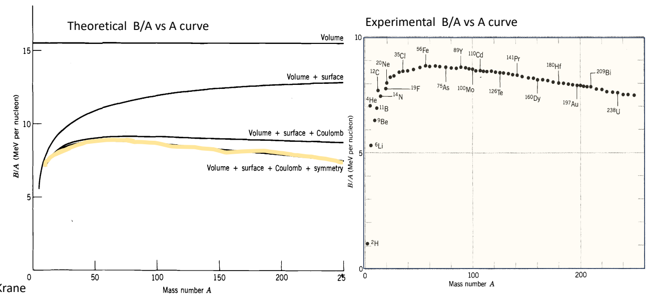
$$\delta = \begin{cases} +a_p A^{-3/4} & evenN \quad evenZ \\ 0 & oddA \\ -a_p A^{-3/4} & oddN \quad oddZ \end{cases}$$

- When there is an odd number of nucleons, this term does not contribute. it is zero.
- Therefore, for *odd-N* and *odd-Z* nuclei, the binding energy may be increased by converting one of the odd protons to a neutron (or vice versa) so that it can form a pair with its formerly odd partner. Such case is very rare and there are only 4 stable nuclei with odd-N and odd-Z.
- Instead, there are 167 stable *even-N even-Z* nuclei!!

odd-Z and odd-N



B/A vs A plot from SEMF.



Krane

Comparison of the experimental B.E. data to the Semi-empirical mass formula results

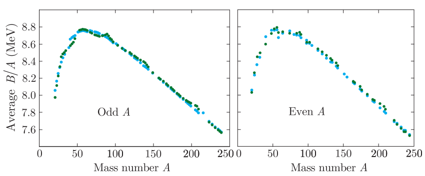


Figure 2.11 Fit to the binding energy data (shown as green circles) for odd-A and even-A nuclei using the SEMF with the coefficients given in the text. The predictions (shown as blue circles) do not lie on smooth curves because Z is not always a smooth function of A.

M&S

SEMF- $M(^{A}_{Z}X_{N})$

1- Earlier we got the binding energy of the nucleus as:
B.E. = $\{Zm(^1H) + Nm_n - M(^{A}_{Z}X_{N})\}c^2$

2- We have the binding energy as:
B.E.(SEMF)= $a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$

3- Now we can write the SEMF for masses:
$M(^{A}_{Z}X_{N}) = Zm(^1H) + Nm_n$ -B.E.(SEMF)/c^2

we will use this formula soon reason ☺

SEMF- Conclusion

SEMF was a first attemt to apply nuclear models to understand the systematic beavhour of a nuclear property which is "Binding energy".

$$\text{B.E.(SEMF)} = a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$

Liquid-drop mass model properties of a droplet of liquid, energy calculations

Shell-model individual nucleons and their interaction to each other

Visiting the nuclear chart again

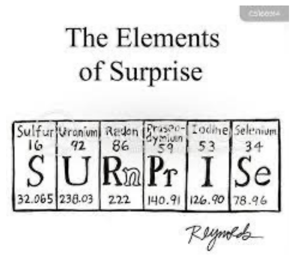
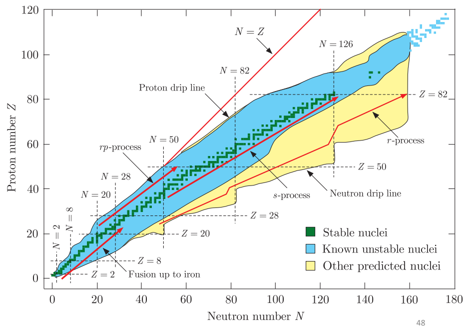


Chart of the nuclides

92 stable elements 280 stable isotopes

over 3000 unstable nuclei

over 6000 more are predicted to exist



stability line
binding energy
neutron drip line
proton drip line

s-,r-,rp-process
Magic numbers:
N=2,8,20,28,50,82,126
Z=2,8,20,28,50,82,...

Mass parabolas

obtaining mass parabolas for Isobars (=isobaric nuclei)

Atomic mass: $M_{Atom}({}^{A}_{Z}X_{N})$

$M({}^{1}H)$

N, neutron number

$$M(A,Z) = Z(\overbrace{m_p + m_e}) + (\overbrace{A - Z})m_n - \frac{1}{c^2}B(A,Z) \qquad \text{(I)}$$

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}}\frac{(A-2Z)^2}{A} (+\delta) \qquad \text{(II)}$$

$$M(A,Z) = Z(m_p + m_e) + (A - Z)m_n - \frac{1}{c^2}B(A,Z) \qquad \text{(I)}$$

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}}\frac{(A-2Z)^2}{A} (+\delta) \qquad \text{(II)}$$

Write Eq. (II) in (I) and Calculate
Eq. (I) represents a parabola of *M vs. Z*

We can find where this parabola reaches its minimum via $\frac{\partial M}{\partial Z} = 0$

If we do that, we get:

$$Z_{\text{min}} = \frac{(m_n - m_p - m_e) + a_c A^{-1/3} + 4a_{sym}}{2a_c A^{-1/3} + 8a_{sym} A^{-1}}$$

$$Z_{min} = \frac{(m_n - m_p - m_e) + a_c A^{-1/3} + 4a_{sym}}{2a_c A^{-1/3} + 8a_{sym} A^{-1}}$$

$a_{asym} \approx 23$ MeV, $a_c \approx 0.72$ MeV

$$\boxed{Z_{min} \simeq \frac{A}{2} \frac{1}{1 + \frac{1}{4} A^{2/3} a_c / a_{sym}}} \quad \text{(III)}$$

For A=10

$Z_{min} \cong 5$ and $\frac{Z_{min}}{A} \sim 1/2$

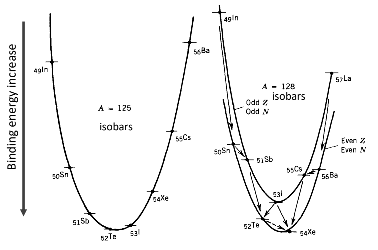
N=Z for light mas stable nuclei

For A=200

$Z_{min} \cong 79$ and $\frac{Z_{min}}{A} \sim 0.4$

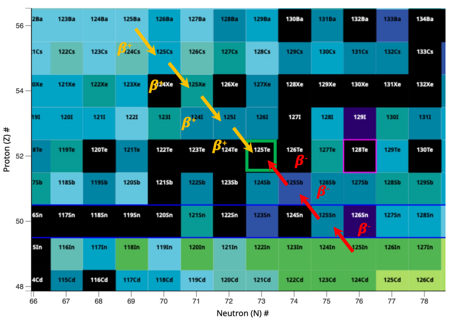
neutron excess for heavy STABLE nuclei

mass parabola→ "Valley of (beta) stability"



For the A=125 isobaric parabola, what Z_{min} value do you expect?

- Z_{min} is the nucleus with the highest binding energy (STABLE) and thus in the bottom of the parabola.
- This parabola is called "Beta Valley" or "Valley of (beta) Stability"
- Anything else in the Beta Valley is unstable and promt to decay towards the bottom of the valley, i.e. ^{125}Te in the left parabola and ^{128}Xe in the right parabola.
- The decay process is through "beta decay".
- Do you see the effect of the pairing term? Double parabola.
- Double-beta decay



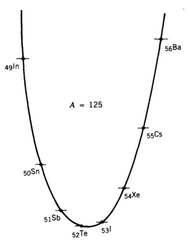
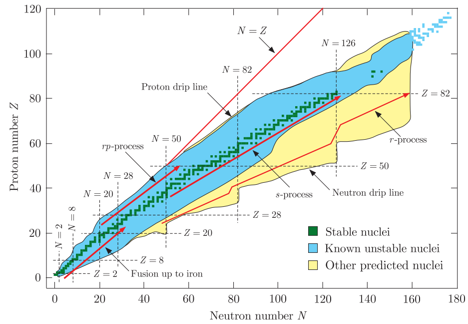


Chart of the nuclides

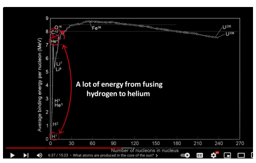
stability line
Valley of beta stability*
binding energy
neutron drip line
proton drip line

s-,r-,rp-process
Magic numbers:
N=2,8,20,28,50,82,126
Z=2,8,20,28,50,82,...



One additional material:
A 15-min Youtube video by Arvin Ash
https://www.youtube.com/watch?v=wgUIB4tD0cM&t=8s

Excellent explanation about the origin of elements in the Universe (so-called nucleosynthesis). It might be interesting to see how he is talking about the (B/A-A) plot and Fusion too!



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Lecture 4: Spin, Parity, EM moments of the nucleus

Compendium (Krane):
Spins, parity, EM moments (Chapter 3, Pages: 70-75)

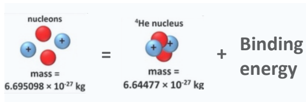
M&S: Chapter 1.5.1

Recap from the last lecture:
Binding energy, SEMF, Beta Valley

- ❑Charge
- ❑Radius
- ❑Mass (Matter)
- ❑Binding energy
- ❑Angular momentum
- ❑Parity
- ❑Magnetuc dipole moment
- ❑Electric quadrupole moments
- ❑Excited states and their energies

Nuclear binding energy & Mass excess

*The **nuclear binding energy** is the amount of energy required to keep the protons and neutrons together inside the nucleus.*



This definition tells us that the mass of the nucleus is not equal to the sum of the masses of its protons and neutrons but less!

$Zm_p + Nm_n > M_{nucleus}$

This difference in the mass is called **mass excess** given by:

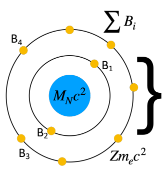
$\Delta m = Zm_p + Nm_n - M_{Nucleus}$

Recalling Einstein's theory of relativity formula $E=mc^2$ (mass and energy is interchangeable!), we can transform the mass excess to Binding energy:

Binding Energy = Δmc^2 (MeV)

The nuclear binding energy is a measure of how tightly nucleons are bonded to each other:

- *If the B.E. is large, it takes a lot of energy to separate the nucleons, in other words, the nucleus is very stable*
- *if the B.E. is small, less amount of energy is needed to separate the nucleons, indicating that the nucleus is less stable.*



$M_A c^2 = Mc^2$

Mass of the nucleus

($E=mc^2$) $M({}^A_Z X_N) = M_{Atom}({}^A_Z X_N)$

$M_{Nucleus}({}^A_Z X_N)c^2 = \left(M({}^A_Z X_N) - Zm_e\right)c^2$ (1)

Nuclear binding energy (B.E.)

$B.E. = \{Zm({}^1H) + Nm_n - M({}^A_Z X_N)\}c^2$ (2)

$B.E. = \{Zm({}^1H) + Nm_n - M({}^A_Z X_N)\}u.\,931.5\ \mathrm{MeV/u}$ (3)

$c^2 = 931.5$ MeV/u
$1u = 931.5$ MeV/c^2
$m_n = 939.57$ MeV
$m_p = 938.28$ MeV
$m_e = 0.511$ MeV

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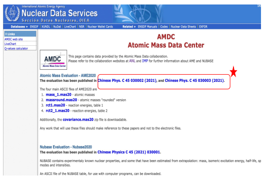
Table I. The 2020 Atomic mass table (Explanation of Table on p. 030003-5)

N	Z	A	Elt.	Orig.	Mass excess (keV)	Errors	Binding energy per nucleon (keV)	Errors		Beta-decay energy (keV)	Errors	Atomic mass μu	Errors
1	0	1	n		8071.3181	0.0004	0.0	0.0	β^-	782.347	*a*	1 008664.9159	0.0005
0	1		H		7288.97106	0.00001	0.0	0.0		*		1 007825.03190	0.00001
1	1	2	H		13135.72290	0.00002	1112.283	*a*		*		2 014101.77784	0.00002
2	1	3	H		14949.81090	0.00008	2827.265	*a*	β^-	18.592	*a*	3 016049.28132	0.00008
1	2		He		14931.21888	0.00006	2572.680	*a*		*		3 016029.32197	0.00006
0	3		Li	-pp	28670#	2000#	-2270#	670#	β^+	13740#	2000#	3 030780#	2150#
3	1	4	H	-n	24620	100	1720	25	β^-	22200	100	4 026430	110
2	2		He		2424.91587	0.00015	7073.916	*a*		*		4 002603.25413	0.00016
1	3		Li	-p	25320	210	1150	50	β^+	22900	210	4 027190	230
4	1	5	H	-nn	32890	90	1336	18	β^-	21660	90	5 035310	100
3	2		He	-n	11231	20	5512	4		*		5 012057	21
2	3		Li	-p	11680	50	5266	10	β^+	450	50	5 012540	50
1	4		Be	x	37140#	2000#	20#	400#	β^+	25460#	2000#	5 039870#	2150#
5	1	6	H	-3n	41880	250	960	40	β^-	24280	250	6 044960	270
4	2		He		17592.10	0.05	4878.520	0.009	β^-	3505.21	0.05	6 018885.89	0.06
3	3		Li		14086.8804	0.0014	5332.331	*a*		*		6 015122.8874	0.0015
2	4		Be	–	18375	5	4487.2	0.9	β^+	4288	5	6 019726	6
1	5		B	x	47320#	2000#	-470#	330#	β^+	28950#	2000#	6 050800#	2150#
6	1	7	H	-nn	49140#	1000#	940#	140#	β^-	23060#	1000#	7 052750#	1080#
5	2		He	-n	26073	8	4123.1	1.1	β^-	11166	8	7 027991	8
4	3		Li		14907.105	0.004	5606.440	0.001		*		7 016003.434	0.004

Mass Excess, Binding Energies, and Atomic masses are tabulated and every several year are updated after new mass measurments.



Nucleon separation energies are other interesting properties that are tabulated as well

Nucleon separation energies: S_p and S_n

S_n, one neutron separation energy is the amount of energy that is needed to remove a neutron from a nucleus $^{A}_{Z}X_N$

S_p, one proton separation energy is the amount of energy that is needed to remove a proton from a nucleus $^{A}_{Z}X_N$

Comparison of S_n values for Tin (Sn, Z=50) isotopes:

$S_n = B(^{A}_{Z}X_N) - B(^{A-1}_{Z}X_{N-1})$

S_n (^{115}Sn)= B(^{115}Sn) – B (^{114}Sn) = 7.6 MeV

(^{115}Sn) => S_n = 7.6 MeV (N=65) (even-odd)
(^{116}Sn) => S_n = 9.6 MeV (N=66) (even-even)
(^{117}Sn) => S_n = 6.9 MeV (N=67) (even-odd)
(^{118}Sn) => S_n = 9.3 MeV (N=68) (even-even)
(^{119}Sn) => S_n = 6.5 MeV (N=69) (even-odd)
(^{120}Sn) => S_n = 9.1 MeV (N=70) (even-even)

It is easier to remove a neutron from a nucleus with odd number of neutrons. Last unpaired neutron is easy to remove.

evenN- evenZ nucleus has all neutrons in pair. We need more energy to break a pair and remove one neutron. (Pairing makes nucleus more tightly bound and stable!)

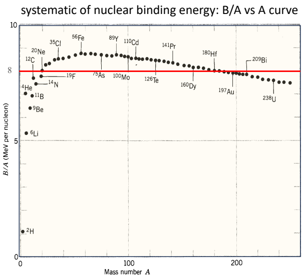
Comparison of $S_n$and S_p values for Antimony (Sb, Z=51) isotopes:

(^{116}Sb) => S_n = 7.9 MeV	S_p = 4.1 MeV	(Z=51, N=65)	(odd-odd)
(^{117}Sb) => S_n = 9.9 MeV	S_p = 4.4 MeV	(Z=51, N=66)	(odd-even)
(^{118}Sb) => S_n = 7.4 MeV	S_p = 4.9 MeV	(Z=51, N=67)	(odd-odd)
(^{119}Sb) => S_n = 9.6 MeV	S_p = 5.1 MeV	(Z=51, N=68)	(odd-even)
(^{120}Sb) => S_n = 7.0 MeV	S_p = 5.6 MeV	(Z=51, N=69)	(odd-odd)
(^{121}Sb) => S_n = 9.3 MeV	S_p = 5.8 MeV	(Z=51, N=70)	(odd-even)

Average binding energy per nucleon: $\frac{B.E.}{A}$ or $\frac{B}{A}$

Since the binding energy increases approximately linearly with A, it is common to use binding energy per nucleon, **B/A**.

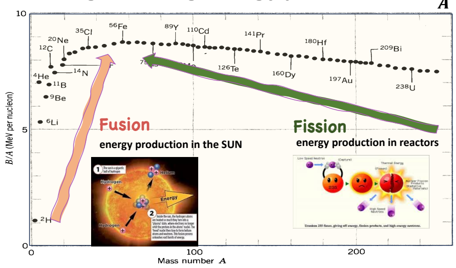
systematic of nuclear binding energy: B/A vs A curve



First visible features of the B/A vs A curve:

- The average binding energy per nucleon is relatively constant and around 8 MeV (except for the very light nuclei)
- Around A=60 the curve reaches its maximum. Nuclei in this region are the most tightly bound ones.
- The binding energy per nucleon is the highest for ^{56}Fe (around 8.8 MeV), the most tigthly bound.
- We can climb to ^{56}Fe from left and from right. What happens?

Average binding energy per nucleon: $\frac{B.E.}{A}$



Energy is released when going from a less bound system to a more bound system, because a tighter bound nucleus weighs less!!

$M_N(^{A}_{Z}X_N)$ = Zm_p+Nmn – B.E.

Q=($M_{initial}$–M_{final})c^2 reaction energy. Q>0 if $M_{initial}$ > M_{final} (Exothermic, energy released)

SEMF: Semi-empirical mass formula

Binding energy formula:

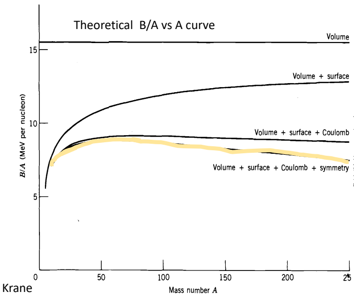
dependence on A and Z

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$

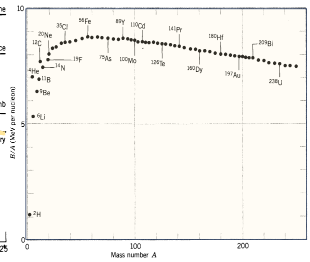
volume | surface | Coulomb | symmetry | pairing

B/A vs A plot from SEMF.

Theoretical B/A vs A curve



Experimental B/A vs A curve



Krane

Mass parabolas

obtaining mass parabolas for Isobars (=isobaric nuclei)

$$M(A,Z) = Z(m_p + m_e) + (A-Z)m_n - \frac{1}{c^2}B(A,Z) \qquad \text{(I)}$$

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}}\frac{(A-2Z)^2}{A}(+\delta) \qquad \text{(II)}$$

Write Eq. (II) in (I) and Calculate
Eq. (I) represents a parabola of *M vs. Z*

We can find where this parabola reaches its minimum via $\frac{\partial M}{\partial Z}=0$

If we do that, we get:

$$Z_{\min} = \frac{(m_n - mp - me) + ac\,A^{-1/3} + 4asym}{2ac\,A^{-1/3} + 8asymA^{-1}}$$

$$Z_{\min} = \frac{(m_n - mp - me) + ac\,A^{-1/3} + 4asym}{2ac\,A^{-1/3} + 8asymA^{-1}}$$

$a_{asym} \approx 23$ MeV, $a_c \approx 0.72$ MeV

$$\boxed{Z_{\min} \simeq \frac{A}{2}\frac{1}{1+\frac{1}{4}A^{2/3}a_c/a_{\text{sym}}}} \qquad \text{(III)}$$

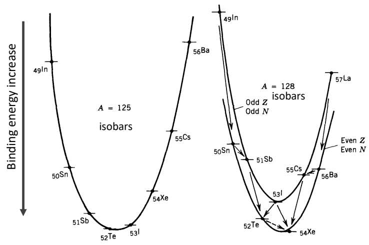
For A=10
$Z_{min} \cong 5$ and $\frac{Z_{min}}{A} \sim 1/2$

N=Z for light mas stable nuclei

For A=200
$Z_{min} \cong 79$ and $\frac{Z_{min}}{A} \sim 0.4$

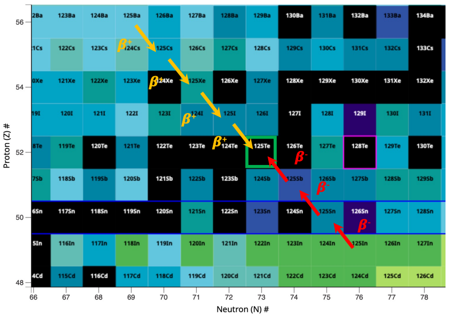
neutron excess for heavy STABLE nuclei

mass parabola→ "Valley of (beta) stability"



For the A=125 isobaric parabola, what Z_{min} value do you expect?

- Z_{min} is the nucleus with the highest binding energy (STABLE) and thus in the bottom of the parabola.
- This parabola is called "Beta Valley" or "Valley of (beta) Stability"
- Anything else in the Beta Valley is unstable and promt to decay towards the bottom of the valley, i.e. ^{125}Te in the left parabola and ^{128}Xe in the right parabola.
- The decay process is through "beta decay".
- Do you see the effect of the pairing term? Double parabola.
- Double-beta decay



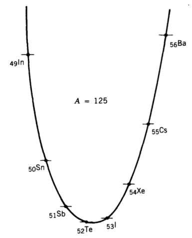
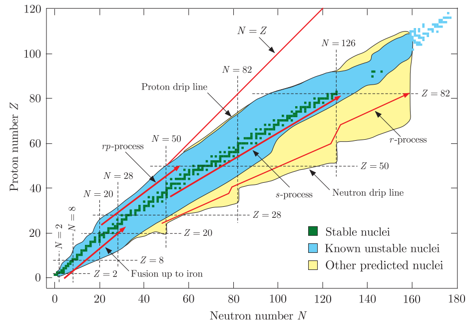


Chart of the nuclides

stability line
Valley of beta stability*
binding energy
neutron drip line
proton drip line

s-,r-,rp-process
Magic numbers:
N=2,8,20,28,50,82,126
Z=2,8,20,28,50,82,...



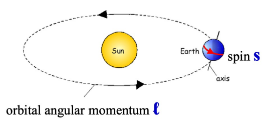
Lecture 4: Spin, Parity, EM moments of the nucleus

Compendium (Krane):
Spins, parity, EM moments (Chapter 3, Pages: 70-75)
M&S: Chapter 1.5.1

- ❑Charge
- ❑Radius
- ❑Mass (Matter)
- ❑Binding energy
- ❑Angular momentum (Spin)
- ❑Parity
- ❑Magnetic dipole moment
- ❑Electric quadrupole moments
- ❑Excited states and their energies

Today

Angular momentum (Spin)



sun ≡ nucleus
earth ≡ electron

Nuclear Angular momentum: The angular momentum of the nucleus

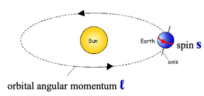
let's start with the total angular momentum of a nucleon (i.e., proton & neutron):

Total angular momentum (=j) of a nucleon is the sum of the orbital angular momentum (=l) and the (intrinsic) spin (=s): **j = l + s**

- *Both l and s are vectors and thus j is a vector too (i.e. they have magnitude and direction).*
- *Both l and s are quantizied, thus j is quantized too.*
- Nucleons are fermions, with spin half integer, s=1/2, like electrons.
- l, orbital angular momentum has classical analog (sun-earth example).
- We can treat "s" as an angular momentum BUT we should keep in mind that nucleons, just like electrons, have structure less and thus cannot rotate.

So the intrinsic spin of nucleons has no classical analog (earth-sun) and it is purely a quantum mechanical concept.

- Let's see "magnitude and direction" of l, s, and j.



sun ≡ nucleus
earth ≡ electron

l, orbital angular momentum

magnitude:

$<l^2>=l(l+1)\hbar^2$

$l=\sqrt{l(l+1)}\hbar$ (1)

$<l_z>=m_l\hbar$ (2)

$m_l=0, \pm1, \pm2, ..., \pm l$

The physical significance of equations (1) and (2) above is that the angular momentum vector $\vec{l}$ can only point in those directions in space, and thus quantised.

l, orbital angular momentum

magnitude:

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The physical significance of equations (1) and (2) above is that the angular momentum vector $\vec{l}$ can only point in l_z directions in space (quantised).

for $\vec{l}=2$

$l=\sqrt{l(l+1)}\hbar$

$=\sqrt{2(3)}\hbar$

$=\sqrt{6}\hbar$ (magnitude)

ℓ	m_ℓ	l	l_z
2	-2	$\sqrt{6}\hbar$	$-2\hbar$
2	-1	$\sqrt{6}\hbar$	$-\hbar$
2	0	$\sqrt{6}\hbar$	0
2	1	$\sqrt{6}\hbar$	$\hbar$
2	-2	$\sqrt{6}\hbar$	$2\hbar$

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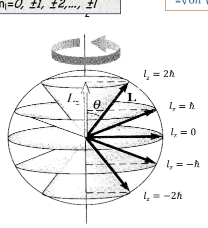
2	2	$\sqrt{6}\hbar$	$2\hbar$

direction:



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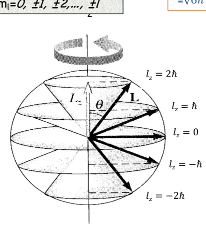
direction:



For a given value of l, similar spectroscopic notation of the atomic states is used in nuclear physics too.

Table 2.6 Spectroscopic Notation

ℓ value	0	1	2	3	4	5	6
Symbol	s	p	d	f	g	h	i

l, orbital angular momentum

magnitude:

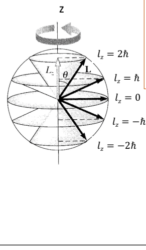
$<l^2>=l(l+1)\hbar^2$

$l=\sqrt{l(l+1)}\hbar$

$<l_z>=m_l\hbar$

$m_l=0, \pm1, \pm2, ..., \pm l$

direction:



for $\vec{l}=2$

$l=\sqrt{l(l+1)}\hbar$

$=\sqrt{2(3)}\hbar$

$=\sqrt{6}\hbar$ (magnitude)

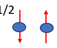
s, (intrinsic)spin

$<s^2>=s(s+1)\hbar^2$

$s=\sqrt{s(s+1)}\hbar$

$<s_z>=m_s\hbar$

$m_s=\pm1/2$ (fermions)



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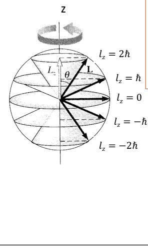
direction:



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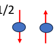


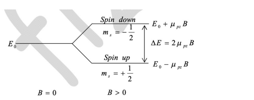


Figure: The energy levels of a proton in a magnetic field are split into spin-up and spin-down sublevels.

l, orbital angular momentum

magnitude:

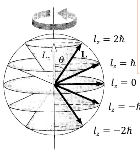
$<l^2>=l(l+1)\hbar^2$

$l=\sqrt{l(l+1)}\hbar$

$<l_z>=m_l\hbar$

$m_l=0, \pm1, \pm2, ..., \pm l$

direction:



for $\vec{l}=2$

$l=\sqrt{l(l+1)}\hbar$

$=\sqrt{2(3)}\hbar$

$=\sqrt{6}\hbar$ (magnitude)

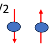
s, (intrinsic)spin

$<s^2>=s(s+1)\hbar^2$

$s=\sqrt{s(s+1)}\hbar$

$<s_z>=m_s\hbar$

$m_s=\pm1/2$ (fermions)



j, total angular momentum

$\vec{j}=\vec{l}+\vec{s}$

$<j^2>=j(j+1)\hbar^2$

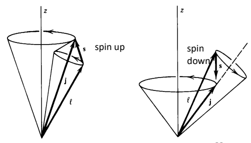
$j=\sqrt{j(j+1)}\hbar$

$<j_z>=m_j\hbar$

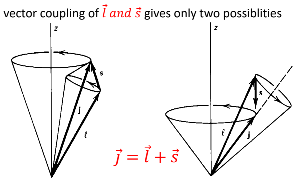
$m_j=-j, -j+1, ..., j-1, j$

$m_j = m_l + m_s = m_l \pm \frac{1}{2}$

vector coupling of $\vec{l}$ *and* $\vec{s}$



vector coupling of $\vec{l}$ *and* $\vec{s}$ gives only two possiblities



$\vec{j}=\vec{l}+\vec{s}$

$j=l+\frac{1}{2}$ $\quad$ $j=l-\frac{1}{2}$

$j=\frac{3}{2}$ $\quad$ $j=\frac{1}{2}$

$P_{3/2}$ $\quad$ $P_{1/2}$

$<j_z>=m_j\hbar$

$m_j=-j, -j+1, ..., j-1, j$

$m_j = m_l + m_s = m_l \pm \frac{1}{2}$

because m_s is always half-integer and m_l is always ingeter, m_j must be always half-integer for one proton or one neutron and thus j must be half-integer.

Table 2.6 Spectroscopic Notation

ℓ value	0	1	2	3	4	5	6
Symbol	s	p	d	f	g	h	i

for l=1 (p states), there are two possible j states.

We indicate j value as a subscript in spectroscopic notation:

Total angular momentum of the nucleus, I:

The total angular momentum of a nucleus containing A nucleons would be the vector sum of the angular momenta of all the nucleons:

nucleus with A nucleons

$\mathbf{I}= \sum_i^A j_i = \sum_i^A(\boldsymbol{l}_i + \boldsymbol{s}_i)$

I is called nuclear spin.

$<I^2>=I(I+1)\hbar^2$

$I=\sqrt{I(I+1)}\hbar$

$<I_z>= m\hbar$

$m=-I,..., +I$

$\vec{j}=\vec{l}+\vec{s}$ one nuclen

Each z component of j is half-integral since ($m_j=m_l+m_s$=integer $\pm\frac{1}{2}$)

($<j_z>=\pm\frac{1}{2}\hbar, \pm\frac{3}{2}\hbar, \pm\frac{5}{2}\hbar,...$) and (j=1/2, 3/2, 5/2,...)

$\boxed{\mathbf{I}= \sum_i^A j_i}$

odd-A nuclei: $\quad I$ = half-integral

even-A nuclei: $\quad I$ = integral

odd-A nuclei: $\quad I$ = half-integral

even-A nuclei: $\quad I$ = integral

The measured values of the nuclear spin can tell us a great deal about the nuclear structure.

Of the hundreds of known (stable and radioactive) even-Z, even-N nuclei, all have spin-0 ground states. (I=0)

This is evidence for the nuclear pairing force the nucleons couple together in spin-0 pairs, giving a total ***I*** of zero.

As a corollary, the ground state spin of an odd-A nucleus must be equal to the *j* of the odd proton or neutron.

We will come back to it in the Shell Model lecture!!

Parity:

Parity

- Parity refers to the behaviour of a state under a spatial reflection, i.e. ***r → -r*** .
- The parity operation causes a reflection of all the coordinates through the origin: ***r → -r*** .
- Parity operation has either of the two effect on a wave function.
- It can take either "+, even" or "-, odd" values.

Cartesian coordinates

$$\hat{P} \begin{cases} x \to -x \\ y \to -y \\ z \to -z \end{cases}$$

Spherical coordinates

$r \to -r$

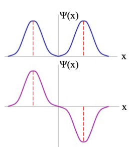
$\theta \to \pi-\theta$

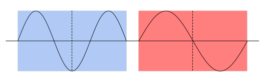
$\phi \to \pi+\phi$

$$\psi(-\boldsymbol{r}) = \pm\psi(\boldsymbol{r})$$

$$\psi(-\boldsymbol{r}) = +\psi(\boldsymbol{r}) \quad \text{"+, even"}$$

$$\psi(-\boldsymbol{r}) = -\psi(\boldsymbol{r}) \quad \text{"-, odd"}$$





Even Wavefunction $\quad$ Odd Wavefunction

Cartesian coordinates

Spherical coordinates

$$\hat{P} \left\{ \begin{array}{l} x \to -x \\ y \to -y \\ z \to -z \end{array} \right. \qquad \begin{array}{l} r \to -r \\ \theta \to \pi - \theta \\ \phi \to \pi + \phi \end{array}$$

$$\Psi_{lmn}(\mathbf{x}) = R_{nl} Y_l^m(\theta, \phi)$$

$$Y_l^m(\theta,\phi) \to Y_l^m(\pi - \theta, \pi + \phi) = (-)^l Y_l^m(\theta,\phi).$$

$\pi = (-1)^l$ parity of a state with orbital angular momentum l

Cartesian coordinates

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$$\pi_{\text{proton}} = +1$$
$$\pi_{\text{neutron}} = +1$$

intrinsic parity: protons and neutrons are fermions. By convention, fermions have +1 parity and anti-fermions have -1 parity

Cartesian coordinates

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parity of a state consisting of particles a and b.

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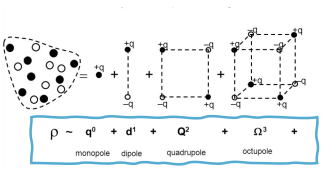
Electric and magnetic moments

- Any distribution of electric charges and currents produces electric and magnetic fields
- Each charge and electric distribution is usually assigned to an electromagnetic multipole moment: **monopole, dipole, quadrupole, octupole, …**
- The simplest distributions of charges and currents give only the **lowest order** of multipole fields.

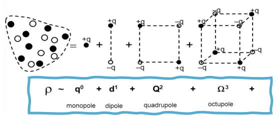
Examples:

- A spherical charge distribution gives only the electric monopole (Coulomb) field and the all higher order fields vanish.
- A circular current loop gives only a magnetic dipole field (magnetic monopole has not been observed so far).
- Nucleus behaves in the similar manner: if a simple, symmetric structure is possible, nuclei tend to acquire that structure.



Parity selection rule applies to the nuclear moments too. There are only certain Electric and Magnetic moments are "allowed".

L being the order of moment,
L=0 Monopole
L=1 Dipole
L=2 Quadrupole
L=3 Octupole etc.



E: L= 0, 1, 2 Parity=$(-1)^L$

E0, Electric Monopole

E2, Electric quadrupole

M: L= 0, 1, 2 Parity=$(-1)^{L+1}$

M1, Magnetic dipole

The **electric dipole moment (E1)** is a measure of the separation of positive and negative electrical charges within a system.

- Should we expect electric dipole moment for nuclei?
- Is there a separation between the neutrons and protons in the nucleus?

The magnetic monopole (M0) has not been observed so far.

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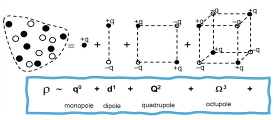

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The **electric dipole moment (E1)** is a measure of the separation of positive and negative electrical charges within a system.

- Should we expect electric dipole moment for nuclei?
- Is there a separation between the neutrons and protons in the nucleus?
- No. Electric dipole moment is zero.

The **magnetic monopole (M0)** has not been observed so far.

Electric monopole moment, E0

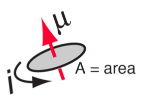
monopole=+q
(Net charge)

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

Electric field arises from the net charge

In atomic nuclei, Electric monopole moment is just the net charge of the nucleus, **Ze**

Magnetic dipole moment, M1



$\mu = NiA$

N: number of loops

Tiny magnet —> “Current loop behaves as magnetic dipole”

Magnetic moment magnitude of an electron, e, moving with speed v in a circle of radius r

$$|\mu| = \frac{e}{(2\pi r/v)}\pi r^2 = \frac{evr}{2} = \frac{e}{2m}|\ell|$$

Period

l: classical angular momentum=mvr

$< l_z >= m_l\hbar$ $\quad m_l = -l, -l+1, \ldots, 0, \ldots, l-1, +l$

$m_l\hbar = +l\hbar$ (The observable magnetic moment corresponds to the maximum component of l)

$$\mu = \frac{e}{2m}|l| = \frac{e\hbar}{2m}l$$

Now l is the orbital angular momentum quantum number

Bohr magneton vs. nuclear magneton

$$\boxed{\mu = \frac{e\hbar}{2m}l}$$

For Atomic motion we use electron mass

$$\mu_B = \frac{e\hbar}{2m_e} = 5.7884x10^{-5}ev/T$$

Bohr magneton

For nucleonic motion, we replace the mass with the proton mass

$$\mu_N = \frac{e\hbar}{2m_p} = 3.1525x10^{-8}ev/T$$

Nuclear magneton

$$\mu_B >> \mu_N$$

Atomic magnetisim has much larger effects than nuclear magnetism

magnetic moment of the nucleus:

$\mu_N = \frac{e\hbar}{2m}$ (m: m_p or m_n)

Magnetic moment of the nucleons because of their orbital motions

Orbital motion

$$\mu = g_l\mu_N.l$$

proton: $g_l = 1$

neutron: $g_l = 0$

g_l: A *g*-factor (also called *g* value) is a dimensionless quantity that characterizes the magnetic moment and angular momentum of an atom, a particle or the **nucleus**.

for example, it is zero for neutrons.

Magnetic dipole moment of the nucleus:

$\mu_N = \frac{e\hbar}{2m}$ (m: m_p or m_n)

Magnetic dipole moment of the nucleons because of their orbital motions Orbital motion

$$\mu = g_l\mu_N.l$$

proton: $g_l = 1$

neutron: $g_l = 0$

Protons and neutrons have also intrinsic spin, s. This there is also spin magnetic dipole moment:
s=1/2 for protons and neutrons

$$\mu = g_s\mu_N.s$$

$g_s = 5.5856912 \pm 0.0000022$

$g_s = -3.8260837 \pm 0.0000018$

- g_s=2 for electron which is a point particle. One could expect the same for proton. But this was not the case.
- Furthermore, gs=0 is expected for neutrons since they are “neutral”.
- **g_s≠0 for neutrons** →(first evident that nucleons are not elementary point particles)

Nuclear structure from magnetic dipole moments

Table 3.2 Sample Values of Nuclear Magnetic Dipole Moments

Nuclide	$\mu(\mu_N)$
n	−1.9130418
p	+2.7928456
^{2}H (D)	+0.8574376
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^{57}Fe	+0.09062293
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In nuclei, the pairing force favors the coupling of nucleons so that their orbital angular momentum and spin angular momentum each add to zero. Thus the paired nucleons do not contribute to the magnetic moment, and we need only consider a few valence nucleons. If this were not so, we might expect on statistical grounds alone to see a few heavy nuclei with very large magnetic moments, perhaps tens of nuclear magnetons. However, no nucleus has been observed with a magnetic dipole moment larger than about $6\mu_N$.

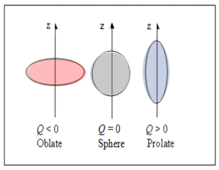
Higher nuclear magnetic dipole moment means that there are more unpaired nucleons (structure information)

Electric quadrupole moment, E2

Electric quadrupole moment of the nucleus reflects Shape of charge distribution and deviation from sphere

Quantum mechanics

$$eQ = e\int \psi^*(3z^2 - r^2)\psi \, dv$$



If $|\Psi|^2$ is spherically symmetric

Q=0

If $|\Psi|^2$ is concentrated on xy-plane (z≅0)

Q ~ - <r²> (negative) Oblate shape

If $|\Psi|^2$ is concentrated along the z-axis (z≅r)

Q ~ +2 <r²> (positive) Prolate shape

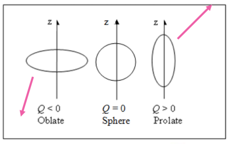
<r²>Mean-square radius of the orbit

Nuclear shape and deformation

The nuclear electric quadrupole moment **gives a measure of the deviation of the nucleus, or more precisely of its charge distribution, from a spherical shape** and hence is an important source of information about nuclear structure such as nuclear shape and deformation.

If $|\Psi|^2$ is concentrated along the z-axis (z≅r)

Q ~ +2 <r²> (positive)



If $|\Psi|^2$ is concentrated on xy-plane (z≅0)

Q ~ - <r²> (negative)

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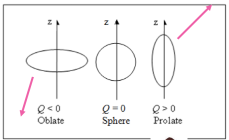
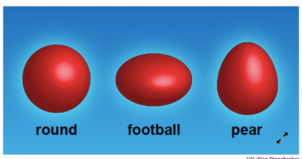


Nucleus is Surprisingly Pear Shaped

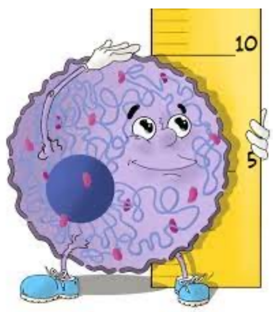
March 17, 2016 • *Physics* 9, s30

Experiments confirm that the barium-144 nucleus is pear shaped and hint that this asymmetry is more pronounced than previously thought.



Octupole
Pear shape

excited states



Level schemes, excited states

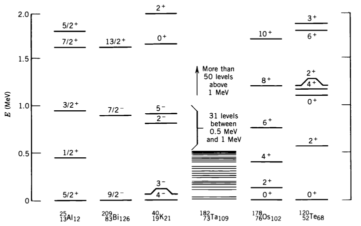


Figure 3.19 Some sample level schemes showing the excited states below 2 MeV. Some nuclei, such as ^{209}Bi, show great simplicity, while others, such as ^{182}Ta, show great complexity. There is a regularity associated with the levels of ^{176}Os that is duplicated in all even-Z, even-N nuclei in the range $150 \le A \le 190$. Structures similar to ^{120}Te are found in many nuclei in the range $50 \le A \le 150$.

- Energy of excitation (excitation energy, level energy)
- lifetimes
- decay modes
- spin and parity
- magnetic dipole moment
- electric quadrupole moment

Lecture 5: Nuclear force

Compendium (Krane):

Nuclear force (Chapter 4, Pages: 80-112)

M&S: Chapter 8.1

Recap from the last lecture:

Spin, parity, EM moment of nuclei

l, orbital angular momentum

magnitude:

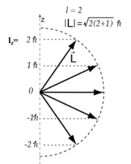
$\langle l^2\rangle = l(l+1)\hbar^2$

$l = \sqrt{l(l+1)}\hbar$

$\langle l_z\rangle = m_l\hbar$

$m_l = 0, \pm 1, \pm 2, \ldots, \pm l$

direction:



for $\vec{l} = 2$

$l = \sqrt{l(l+1)}\hbar$

$= \sqrt{2(3)}\hbar$

$= \sqrt{6}\hbar$ (magnitude)

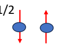
s, (intrinsic)spin

$\langle s^2\rangle = s(s+1)\hbar^2$

$s = \sqrt{s(s+1)}\hbar$

$\langle s_z\rangle = m_s\hbar$

$m_s = \pm 1/2$ (fermions)



j, total angular momentum

$\vec{j} = \vec{l} + \vec{s}$

$\langle j^2\rangle = j(j+1)\hbar^2$

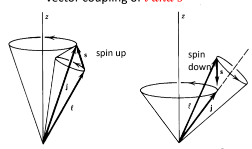
$j = \sqrt{j(j+1)}\hbar$

$\langle j_z\rangle = m_j\hbar$

$m_j = -j, -j+1, \ldots, j-1, j$

$m_j = m_l + m_s = m_l \pm \frac{1}{2}$

vector coupling of $\vec{l}$ and $\vec{s}$



Parity

$\hat{P}$ Cartesian coordinates: $x \to -x$, $y \to -y$, $z \to -z$

Spherical coordinates: $r \to -r$, $\theta \to \pi-\theta$, $\phi \to \pi+\phi$

$$\Psi_{lmn}(\mathbf{x}) = R_{nl}Y_l^m(\theta,\phi)$$

$$Y_l^m(\theta,\phi) \to Y_l^m(\pi-\theta, \pi+\phi) = (-)^l Y_l^m(\theta,\phi).$$

$$\boxed{\pi = (-1)^l}$$ parity of a state with orbital angular momentum l

$\pi_{\text{proton}} = +1$

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intrinsic parity: protons and neutrons are fermions. By convention, fermions have +1 parity and anti-fermions have -1 parity

parity of a state consisting of particles a and b.

$$\pi_{\text{system}} = (-1)^l \pi_a \pi_b$$

Examples:

- Calculate parity of a state consisting of two nucleons in the $p_{3/2}$ orbital.
- Calculate parity of the same state when two nucleons are filling the $g_{9/2}$ orbital.

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Examples:

Table 2.6 Spectroscopic Notation

ℓ value	0	1	2	3	4	5	6
Symbol	s	p	d	f	g	h	i

- Calculate parity of a state consisting of two nucleons in the $2p_{3/2}$ orbital.

$2p_{3/2}$=>nl_j

parity: (-)

p orbital, l=1

- Calculate parity of the same state when two nucleons are filling the $1g_{9/2}$ orbital.

$1g_{9/2}$=>nl_j

parity: (+)

g orbital, l=4

Electric and magnetic moments

Most important moments for nuclei **are magnetic dipole and electric quadrupole** moments

Magnetic dipole moment of the nucleus:

The magnetic moment of a nucleus is the sum of that which is due to the spin of its nucleons (protons and neutrons) and that due to the rotation of its charges.

However, when a nucleon is paired with a nucleon of the same type the spins are oppositely aligned and therefore cancel each other out. Thus the net magnetic moment of a nucleus is due to the presence of unpaired nucleons.

$$\mu = g_l \mu_N . l$$

$$\mu = g_s \mu_N . s$$

$$g_s = \quad 5.5856912 \pm 0.0000022$$
$$g_s = \; -3.8260837 \pm 0.0000018$$

- g_s=2 for electron which is a point particle. One could expect the same for proton. But this was not the case.
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Higher nuclear magnetic dipole moment means that
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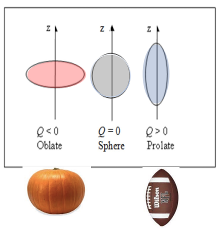
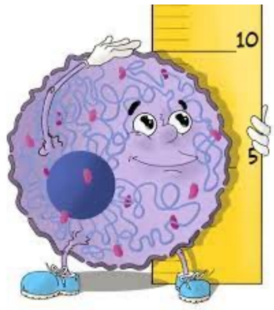
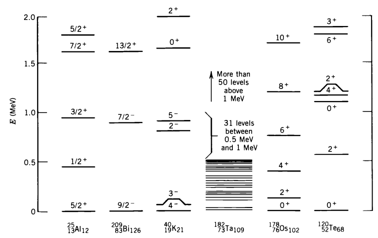


excited states



Level schemes, excited states

$2^{+}=I^{\pi}$



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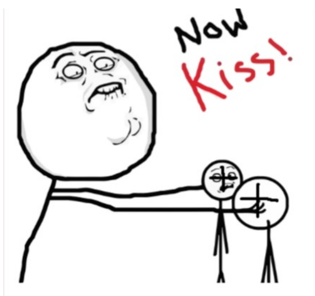
Properties of nuclear force



Nuclear force from what we know so far:

1. Strongly attractive at short distances

At short distances it is stronger than the Coulomb force; the nuclear force can overcome the Coulomb repulsion of protons in the nucleus.

2. Short range and negligible at long distances

At long distances, of the order of atomic sizes, the nuclear force is negligibly feeble; the interactions among nuclei in a molecule can be understood based only on the Coulomb force.

3. Some particles are immune from the nuclear force;

There is no evidence from atomic structure, for example, that electrons feel the nuclear force at all.

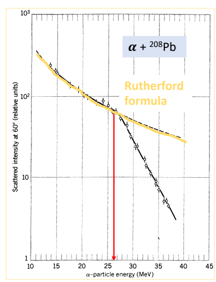
Can you tell any experimental evidence to prove these three features? You might need to use what we have gone through so far these last two weeks ☺

1-Strongly attractive at short distances 2- Short range and negligible at long distances 3. Some particles are immune from the nuclear force;

alpha scattering

At low energies, alpha and the ^{208}Pb nucleus interact only via Coulomb interaction because of the large distance between them (Rutherford scattering)

With increasing energy, the Coulomb repulsion of the nuclei is overcome and the alpha particle may approach close enough to allow the nuclear force to act. In this case the Rutherford formula no longer holds.



1-Strongly attractive at short distances 2- Short range and negligible at long distances 3. Some particles are immune from the nuclear force;

alpha scattering

B/A vs. A plot

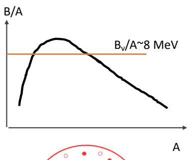
SEMF

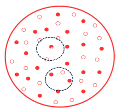
The linear dependence of B on A:

- If the nuclear force was "Long range", B α A(A-1) => B α A²
(every nucleon would be attracting all the other nucleons.)

- Since B varies linearly with A => B α A, this sugests that each nucleon attracts only its closest neighbours and not all of the other nucleons.

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{asym}\frac{(A-2Z)^2}{A} + \delta$$





1-Strongly attractive at short distances 2- Short range and negligible at long distances 3. Some particles are immune from the nuclear force;

alpha scattering

B/A vs. A plot

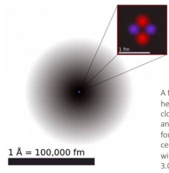
SEMF

Size of atom and nucleus

electron scattering

Nuclear force has short range around 1-2 fm
(=The distance between the nucleons).

Electrons are way far away from the nucleus
size of an atom is 100,000 fm



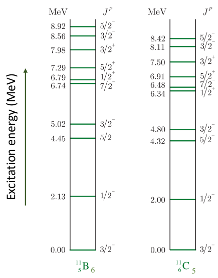
A figurative depiction of the helium-4 atom with the electron cloud in shades of gray. Protons and neutrons are most likely found in the same space, at the central point. Source wikipedia.org License CC BY-SA 3.0

Now we can include experimental results to continue the properties of the nuclear force

4. Nuclear force is charge independent

The nucleon-nucleon force seems to be nearly independent of whether the nucleons are neutrons or protons. This property is called ***"charge independence or charge symmetry".***

We can observe "charge independence " in the excited states of so-called mirror nuclei.



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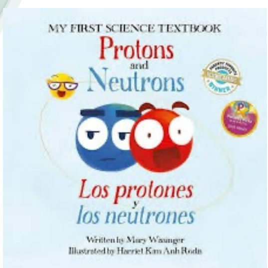
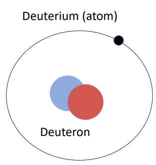
7. The nucleon-nucleon force has a noncentral or ***tensor*** component. This part of the force does not conserve orbital angular momentum, which is a constant of the motion under central forces.



Deuteron ☺

Deuteron the simplest bound state of nucleons and therefore gives us an ideal system for studying the nucleon-nucleon interaction.



- ideally if we could study the electromagnetic transitions between the excited states of deuteron, we would be able to study its structure.
- BUT, deuteron has no excited states!!
- It is such a weakly bound system that any excitation will easily make the system break apart. an that case we will not have a bound system of a proton and a neutron but an unbound system with free proton and neutron.
- Let's start with going through properties of deuteron:
  - Binding energy
  - Spin and parity
  - Magnetic dipole
  - Electric quadrupole

Binding energy

1- mass spectroscopy:

B = [m (^{1}H) + m (n) - m (^{2}H)] c^2 = 2.222463 ± 0.00004 M e V

2- Nuclear reaction (neutron capture reaction via thermal neutron): energy of the gamma-ray (photon) will be equal to the binding energy of the deuteron.

$^1\mathrm{H} + \mathrm{n} \rightarrow {}^2\mathrm{H} + \gamma$ $\quad$ $Q_{reaction}$= ($M_{initial}$ – M_{final})c^2 = [M(^{1}H) + m(n) – M (^{2}H)]=2.22457309 MeV

E(γ)=2.224589 ± 0.000002 MeV –recoil energy of ^{2}H

Chinese Physics C Vol. 45, No. 3 (2021) 030003

Table I. The 2020 Atomic mass table (Explanation of Table on p. 030003-5)

N	Z	A	Elt.	Orig.	Mass excess (keV)		Binding energy per nucleon (keV)		Beta-decay energy (keV)			Atomic mass μu	
1	0	1	n		8071.3181	0.0004	0.0	0.0	β^-	782.347	*a*	1 008664.9159	0.0005
0	1		H		7288.97106	0.00001	0.0	0.0		*		1 007825.03190	0.00001
1	1	2	H		13135.72290	0.00002	1112.283	*a*		*		2 014101.77784	0.00002
2	1	3	H		14949.81090	0.00008	2827.265	*a*	β^-	18.592	*a*	3 016049.28132	0.00008
1	2		He		14931.21888	0.00006	2572.680	*a*		*		3 016029.32197	0.00006
0	3		Li	-pp	28670#	2000#	-2270#	670#	β^+	13740#	2000#	3 030780#	2150#
3	1	4	H	-n	24620	100	1720	25	β^-	22200	100	4 026430	110
2	2		He		2424.91587	0.00015	7073.916	*a*		*		4 002603.25413	0.00016
1	3		Li	-p	25320	210	1150	50	β^+	22900	210	4 027190	230
4	1	5	H	-nn	32890	90	1336	18	β^-	21660	90	5 035310	100
3	2		He	-n	11231	20	5512	4		*		5 012057	21
2	3		Li	-p	11680	50	5266	10	β^+	450	50	5 012540	50
1	4		Be	x	37140#	2000#	20#	400#	β^+	25460#	2000#	5 039870#	2150#
5	1	6	H	-3n	41880	250	960	40	β^-	24280	250	6 044960	270
4	2		He		17592.10	0.05	4878.520	0.009	β^-	3505.21	0.05	6 018885.89	0.06

Binding energy

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E(γ)=2.224589 ± 0.000002 MeV –recoil energy of ^{2}H

3- Inverse reaction of (2): Beam is a γ-ray photon, photodissociation reaction

$\gamma + {}^2\mathrm{H} \rightarrow {}^1\mathrm{H} + \mathrm{n}$

The minimum γ -ray energy for breaking up the deuteron is again equal to the binding energy of the deuteron. Again, one needs to take care of the recoil energy of ^{1}H and neutron.

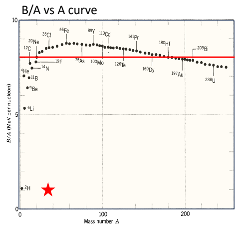
Deuteron is very weakly bound: B/A~1.1 MeV

2.222463 MeV of binding energy corresponds to B/A~1.1 MeV

It is very small compared to the 8 MeV average binding energy per nucleon.

B/A vs A curve

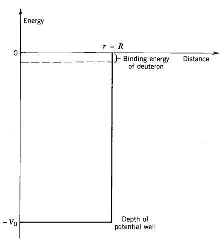
8 MeV



We can do more than seeing this conclusion.

Let's analyse this binding energy to study more about the deuteron.

nucleon-nucleon potential



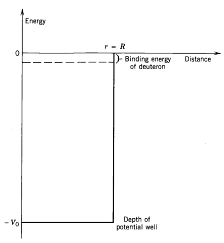
Schrödinger equation, 3D

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + V(r)u(r) = Eu(r)$$

Assume *l=0* $\quad$ ψ(r)→ u(r)/r

m: mass of the nucleon

nucleon-nucleon potential



Schrödinger equation, 3D

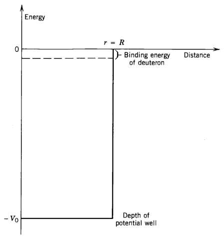
$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + V(r)u(r) = Eu(r)$$

1- choose nuclear potential function V(r). The simplest form is a finite square well

$$V(r) = -V_0 \quad \text{for } r < R$$
$$= 0 \quad \text{for } r > R$$

r: separation between proton and neutror (1fm)
R: diameter of deuteron (2.1 fm)

nucleon-nucleon potential



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$$\begin{aligned} V(r) &= -V_0 \quad \text{for } r < R \\ &= 0 \quad \text{for } r > R \end{aligned}$$

2- solutions for specific E eigenvalues.

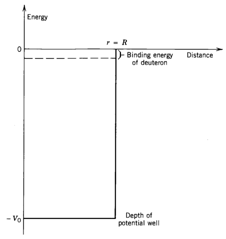
$$u(r) = A\sin k_1 r + B\cos k_1 r$$

with $k_1 = \sqrt{2m(E+V_0)/\hbar^2}$, and for $r > R$,

$$u(r) = Ce^{-k_2 r} + De^{+k_2 r}$$

with $k_2 = \sqrt{-2mE/\hbar^2}$. (Remember, $E < 0$ for bound states.)

nucleon-nucleon potential



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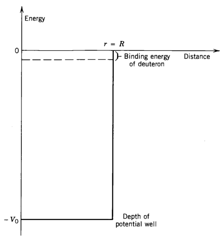
with $k_2 = \sqrt{-2mE/\hbar^2}$. (Remember, $E < 0$ for bound states.)

3- Appply boundary conditions

$$k_1 \cot k_1 R = -k_2$$

nucleon-nucleon potential



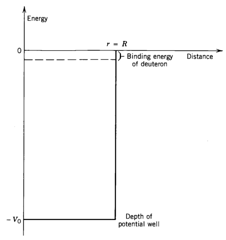
$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + V(r)u(r) = Eu(r)$$

$$\begin{aligned} V(r) &= -V_0 \quad \text{for } r < R \\ &= 0 \quad \text{for } r > R \end{aligned}$$

$$\boxed{k_1 \cot k_1 R = -k_2} \qquad (*)$$

- This condition provides relation between V_0 and R.
- R, radius of the deuteron is known from the electron scattering experiment around 2.1 fm.
- Solving the equation (*) gives a value of V_0=35 MeV.

nucleon-nucleon potential



$$\begin{aligned} V(r) &= -V_0 \quad \text{for } r < R \\ &= 0 \quad \text{for } r > R \end{aligned}$$

- V_0=35 MeV represents the strength of the nucleon-nucleon potential.
- Its sign is negative, corresponding to its being attractive.
- Deuteron is very weakly bound, i.e., its binding energy being close to the top of the potential well.
- If the nucleon-nucleon force were just a bit weaker, the deuteron bound state would not exist! But so nicely this is not the case!!
  - For example, our Sun would not produce energy
  - formation of stable matter from hydrogen wouldn't happen
  - If no stable two-nucleon bound state existed, we would not be here to discuss it!

Wave function of the deuteron:

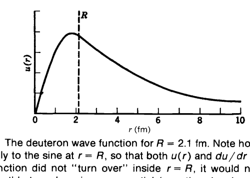


Figure 4.2 The deuteron wave function for R = 2.1 fm. Note how the exponential joins smoothly to the sine at $r = R$, so that both $u(r)$ and du/dr are continuous. If the wave function did not "turn over" inside $r = R$, it would not be possible to connect smoothly to a decaying exponential (negative slope) and there would be no bound state.

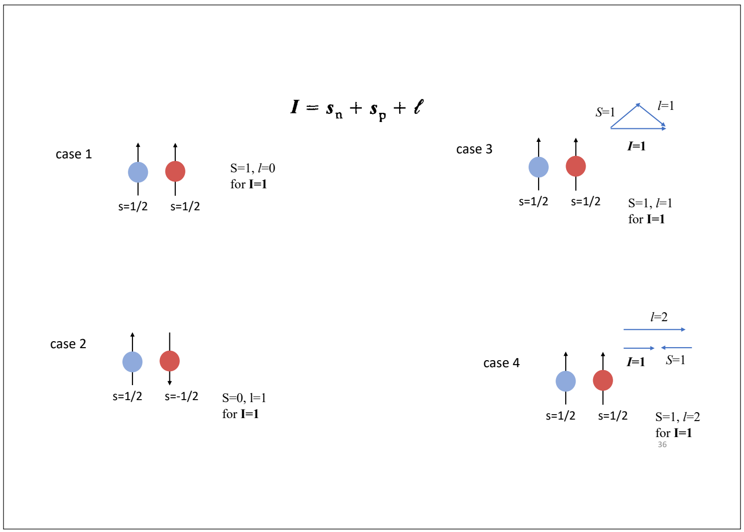
The deuteron wave function is shown in Figure 4.2. The weak binding means that ψ(r) is just barely able to "turn over" in the well so as to connect at $r = R$ with the negative slope of the decaying exponential.

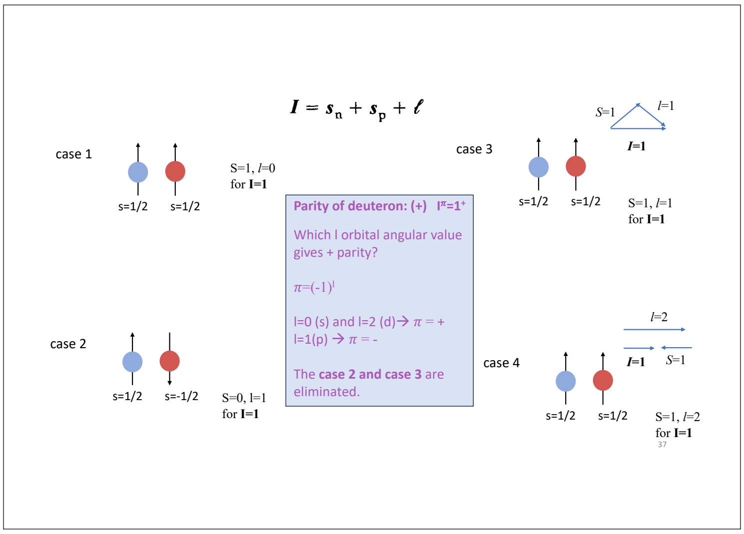
Spin and parity of the deuteron:

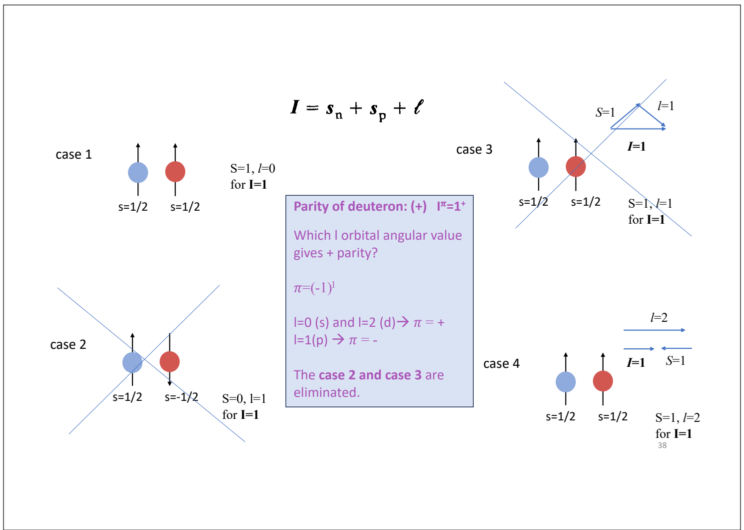
- Total spin of the deuteron is given by

$$\boldsymbol{I} = \boldsymbol{s}_n + \boldsymbol{s}_p + \boldsymbol{\ell}$$

- It is measured to be I=1
- What are the possible spin configurations which give I=1?







l=0 and l=2 are the remaining options/ configurations.

Magnetic dipole moment of the deuteron:

$$\mu_l = g_l \mu_N . l$$
$$\mu_s = g_s \mu_N . s$$

- Let's say l=0 is the correct option.
- In that case μ_l=0 & $\mu_s \neq 0$

$$\mu = \mu_n + \mu_p$$
$$= \frac{g_{sn}\mu_N}{\hbar} s_n + \frac{g_{sp}\mu_N}{\hbar} s_p$$

g_s = 5.5856912 ± 0.0000022
g_s = −3.8260837 ± 0.0000018

$s_n = s_p = +\frac{1}{2}\hbar$

$$\mu = \tfrac{1}{2}\mu_N (g_{sn} + g_{sp})$$
$$= 0.879804\ \mu_N$$

calculated value

- Calculated: 0.879804 μ_N
- Experimental value: 0.8574376 ± 0.0000004 μ_N
- Calculated value seems fine but it is off by more than the error.
- In this scenario, we can assume that the discrepancy is due to a small mixture of the l=2 (d-state) in the deuteron wave functions.

$$\psi = a_s\,\psi(\ell = 0) + a_d\,\psi(\ell = 2)$$

Calculating the magnetic moment from this wave function gives

$$\mu = a_s^2\,\mu(\ell = 0) + a_d^2\,\mu(\ell = 2)$$

$$a_s^2 = 0.96,\ a_d^2 = 0.04;$$

the deuteron is 96% d= 0 and only 4% l= 2. The assumption of the pure l= 0 state, which we made in calculating the well depth, is thus pretty good but not quite exact.

Electric quadrupole moment of the deuteron

Even though it is a small value, deuteron has non-zero Quadrupole moment from the measurements.

$Q = 0.00288 \pm 0.00002$ b

Use the mixed wave function (1) and working with the equation (2)

$\psi = a_s \psi(\ell = 0) + a_d \psi(\ell = 2)$ (1) $\quad eQ = e\int \psi^* (3z^2 - r^2)\psi \, dv$ (2)

Electrick quadrupole moment below relation was found with two components a_s and a_d. Solving it with wave functions from the theory, few % of the (l=2) mixture was obtained, in agreement with 4% from the magnetic dipole discussion.

$$Q = \frac{\sqrt{2}}{10} a_s a_d \langle r^2 \rangle_{sd} - \frac{1}{20} a_d^2 \langle r^2 \rangle_{dd}$$

conclusion 1: tensor interaction

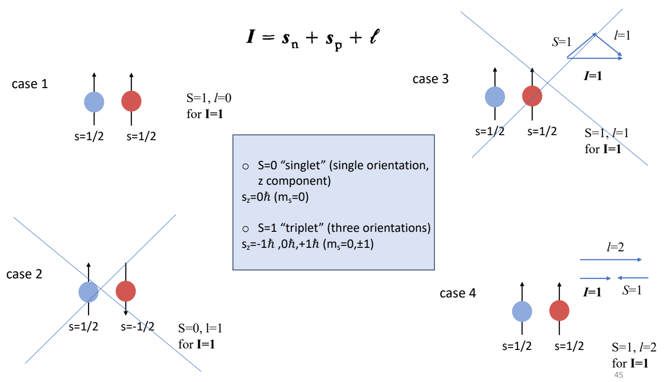
- 4% (l=2) + 96%(l=0) gives the 1^+ ground state of deuteron.
- This conclusion is not purely experimental
  - magnetic dipole was calculated using g_s values for free proton and neutron. Proton and neutron in deuteron are not completely free bound!
  - 4% value of electric quadrupole moment for the l=2 state was obtained without knowing the wave function of the l=2 (d) state.
- precise knowledge of this 4% + 96% combination is important since in this case there will be another component (interaction) added to the nuc-nuc interaction. Unlike V(r) we used, this component should be non-central causing the mixing of two states in deuteron.
- It is called Tensor interaction and we know it should be added to the nuclear force, especially to explain nuclear shell structure of **very exotic nuclei** (far away from stability).

conclusion 2: Nuclear force is strongly spin dependent:

$\vec{S_1} \cdot \vec{S_2}$

Alignment of the spin component of the nucleons makes difference on their bounding and the binding of the nucleus.

$$\boldsymbol{I} = \boldsymbol{s}_n + \boldsymbol{s}_p + \boldsymbol{\ell}$$



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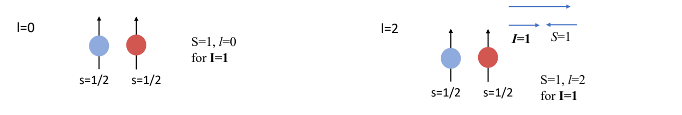
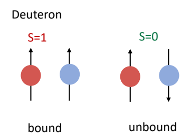
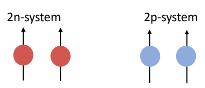




S=1 triplet state increases the binding of deuteron.
S=0 singlet state decreases the binding.

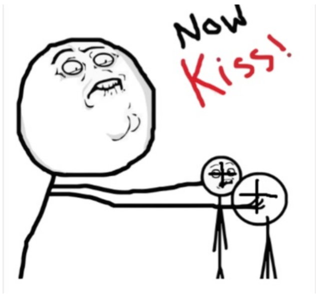
what about two neutron or two proton systems? Can they be bound?



Lecture 6: Nuclear force- II

Compendium (Krane):
Nuclear force (Chapter 4, Pages: 80-112)
M&S: Chapter 8.1

Properties of nuclear force



1. Strongly attractive at short distances
2. Short range and negligible at long distances
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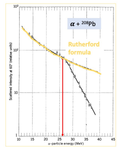
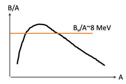
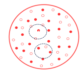
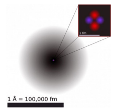
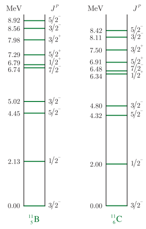
4. Nuclear force is charge independent











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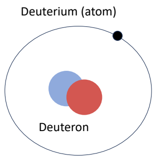

- ideally if we could study the electromagnetic transitions between the excited states of deuteron, we would be able to study its structure.
- BUT, deuteron has no excited states!!
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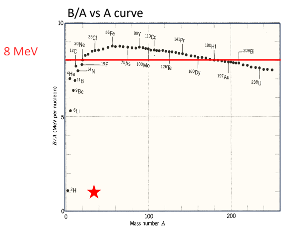
  - Magnetic dipole moment
  - Electric quadrupole moment

Deuteron is very **weakly** bound: B/A~1.1 MeV

2.222463 MeV of binding energy corresponds to B/A~1.1 MeV
It is very small compared to the 8 MeV average binding energy per nucleon.

B/A vs A curve

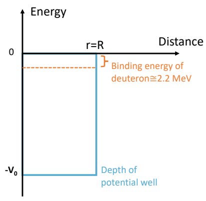


"*weak*" binding indicates "weakness" of the strong nuclear force.

We mean it is "weak", in comparison to the kinetic energy of relative motion of the two nucleons.

In other words, the relative kinetic energy of two nucleons cannot be changed subtsantially by the strong interaction. "Particles are moving independently in a densely packed nuclear medium".

nucleon-nucleon potential



The result: V_0=35 MeV strength of the nuleon-nucleon potential.

Schrödinger equation, 3D

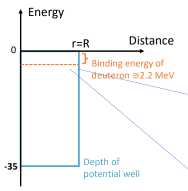
$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + V(r)u(r) = Eu(r)$$

1- Assume *l=0* $\psi(r) \rightarrow u(r)/r$

2- choose nuclear potential function V(r). The simplest form is a finite square well

$$V(r) = -V_0 \quad \text{for } r < R$$
$$= 0 \quad \text{for } r > R$$

r: separation between proton and neutron (1fm)
R: diameter of deuteron (2.1 fm)



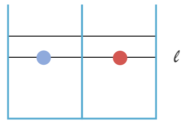
- V_0=35 MeV represents the strength of the nucleon-nucleon potential.
- Its sign is negative, corresponding to its being attractive.
- Deuteron is very weakly bound, i.e., its binding energy being close to the top of the potential well.
- If the nucleon-nucleon force were just a bit weaker, the deuteron bound state would not exist! But so nicely this is not the case!!
- how close the deuteron is to the top of the potential well.

Spin and parity of the deuteron: $J^{\pi} = I^{\pi} = 1^{+}$

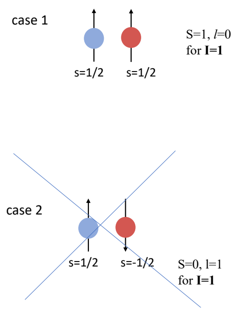
- It is measured to be I=1
- parity of the deuteron is found to be +
- What are the possible spin configurations which give $I=1^{+}$?

- Hint: Total spin of the deuteron is given by

$$\boldsymbol{I} = \boldsymbol{s}_{\rm n} + \boldsymbol{s}_{\rm p} + \boldsymbol{\ell}$$



$$\boxed{\boldsymbol{I} = \boldsymbol{s}_{\rm n} + \boldsymbol{s}_{\rm p} + \boldsymbol{\ell}}$$



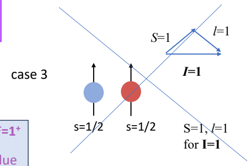
Parity of deuteron: (+) $I^{\pi}=1^{+}$

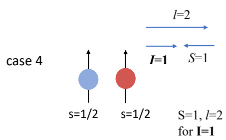
Which l orbital angular value gives + parity?

$\pi=(-1)^{l}$

l=0 (s) and l=2 (d) → π = +
l=1(p) → π = -

The **case 2 and case 3** are eliminated.





ℓ=0 and *ℓ=2* are the remaining options/ configurations.

Magnetic (dipole) moment of the deuteron:

- **Let's say l=0 is the correct option**. i.e. there is one neutron and one proton in the s orbit (l=0) and theyare coupled parallel to each other giving S=1 and thus I=1.
- In this case the magnetic moment is not from the orbital angular momentum of the proton and neutron (since l=0) but only from the intrinsic spin. ($\boldsymbol{\mu}_l$ =0 & $\boldsymbol{\mu}_s \neq 0$).
- Since the spin angular momenta are parallel (S=1), the proton and neutron magnetic moments should add giving:

$$\boldsymbol{\mu} = \boldsymbol{\mu}_{\rm n} + \boldsymbol{\mu}_{\rm p} = \frac{g_{sn}\mu_{\rm N}}{\hbar}\boldsymbol{s}_{\rm n} + \frac{g_{sp}\mu_{\rm N}}{\hbar}\boldsymbol{s}_{\rm p}$$

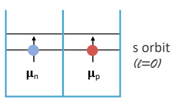
$$\mu = \tfrac{1}{2}\mu_{\rm N}(g_{sn} + g_{sp}) = 0.879804\ \mu_{\rm N}$$

calculated value

$$\mu_{\rm N} = \frac{e\hbar}{2m_{\rm p}}$$

g_{sp}= 5.5856912 ± 0.0000022
g_{sn}= -3.8260837 ± 0.0000018

$s_n=s_p=+\frac{1}{2}\hbar$



- Calculated: 0.879804 $\boldsymbol{\mu}_N$
- Experimental value: 0.8574376 ± 0.0000004 $\boldsymbol{\mu}_N$
- Calculated value seems to be fine compared to the experimental value.
- Conclusion 1: Magnetic moment is additive
- Conclusion2: The S=1 coupling of the proton and neutron spin in the deuteron seems a correct assumption and explains the bound state of the deuteron. (S=0 unbound)
- BUT the calculated value is off by more than the error.
- This discrepancy can be due to a small mixture of the l=2 (d-state) in the deuteron wave functions.

Calculating the magnetic moment from this wave function gives

$$\mu = a_s^2\,\mu(\ell=0) + a_d^2\,\mu(\ell=2)$$

$$a_s^2 = 0.96,\ a_d^2 = 0.04;$$

the deuteron is 96% *l*= 0 and only 4% *l*= 2. The assumption of the pure l= 0 state, which we made in calculating the well depth, is thus pretty good but not quite exact.

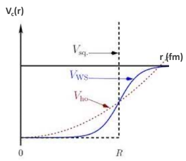
Electric (quadrupole) moment of the deuteron

- The deuteron has non-zero Quadrupole moment from the measurements. Q=0.00288 ± 0.00002 barn.
- If the deuteron were only composed of a proton and a neutron in l=0 orbits (s orbit, S-wave), the Quadrupole moment would vanish.
- Even though it is a small value, non-zero Quadrupole moment indicates a small l=2 component (d orbit, D-wave). d is fine because of the parity conservation. p is not allowed since it gives negative parity. (As we have just seen it ☺)
- Detailed analysis indicated 5 % of the (l=2) mixture. In other words, the quadrupole moment is generated by a D-wave amplitude
- But D-wave is very small compared to the S-wave amplitude
- Conclusion 1: non-zero quadrupole moment of a deuteron shows that the proton-neutron interaction tends to lead to non-spherical nuclear shapes.
- Conclusion2: non-spherical shape of the deuteron cannot be because of the central potential but another component to it which is non-central.

1. Strongly attractive at short distances
2. Short range and negligible at long distances
3. Some particles are immune from the nuclear force
4. Nuclear force is charge independent
5. Interaction between two nucleons (NN interaction) has an attractive central potential.

The Interaction between Two Nucleons Consists to Lowest Order of an Attractive Central Potential

In this chapter we have used for this potential a square-well form, which simplifies the calculations and reproduces the observed data fairly well. Other more realistic forms could just as well have been chosen, but the essential conclusions would not change (in fact, the effective range approximation is virtually independent of the shape assumed for the potential). The common characteristic of these potentials is that they depend only on the internucleon distance r. We therefore represent this central term as $V_c(r)$. The experimental program to study $V_c(r)$ would be to measure the energy dependence of nucleon–nucleon parameters such as scattering phase shifts, and then to try to choose the form for $V_c(r)$ that best reproduces those parameters.



sq: (Finite) Square well
ho: Harmonic oscillator
WS: Woods-Saxon 👍

Kompendium Krane page#100

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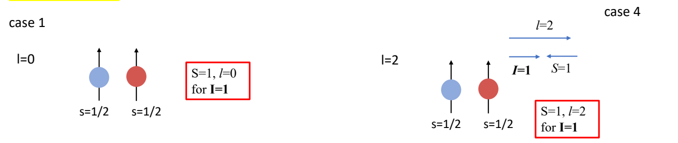
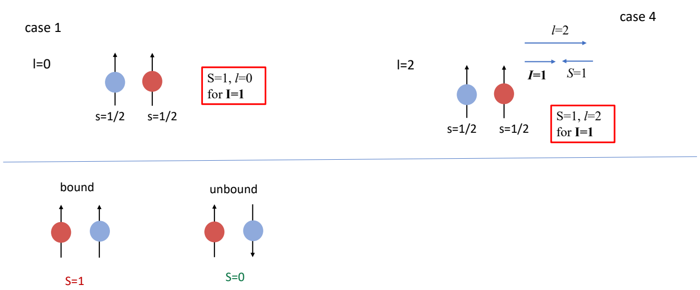
6. The Nucleon - Nucleon Interaction is Strongly Spin Dependent
   - S=1 is more bound than S=0 for p-n interaction
   - S=0 is the only allowed one for p-p and n-n systems/interactions.

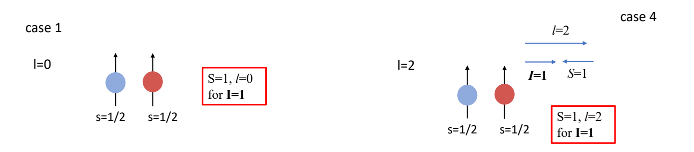
Deuteron (reminder)

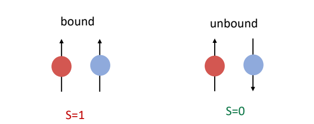




The S = 1 combination has three orientations (corresponding to z components +1,0, -1). The S = 1 combination is called a ***triplet*** state

the S = 0 combination has only a single orientation. The S = 0 combination is called a ***singlet*** state.





The S = 1 combination has three orientations (corresponding to z components +1,0, -1). The S = 1 combination is called a ***triplet*** state

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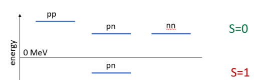
S=1 triplet state increases the binding of deuteron.
S=0 singlet state decreases the binding.



only bound state of deuteron is from S=1

what about two neutron or two proton systems? Can they be bound?

Pauli principle and antisymmetrization

principle: no two identical nucleons can occupy the same place at the same time. More formally, no two nucleons can have identical quantum numbers.

antisymmetrization: a two-particle nuclear wave function, ψ, must be antisymmetric with respect to the interchange of the two partners. For multiparticle states, the antisymmetry must extend to interchanges of any pair of particles.

More general description: The two-fermion wave function must be antisymmetric under fermion exchange

---

let's discuss antisymmetrization for a two like-nucleon system:

Consider a wave function $\psi_{ab}(r_{12})$ of two identical particles occupying a and b orbits and where r_{12} is the distance between the two particles.

For $r_{12}=0$ ($r_1=r_2$), the particles are at the same point in space and thus the Pauli principle requires $\psi_{ab}(r_{12})$ to vanish:

$\psi_a(r_1)\,\psi_b(r_2)=0$ at $r_{12}=0$ $\qquad$ $\psi_{ab}(r_{12})$: Cobined wave function, the product of the two component wave functions

Now consider the wave function:

$$\psi_{ab}(r_{12}) = \psi_a(r_1)\,\psi_b(r_2) - \psi_a(r_2)\,\psi_b(r_1).$$

- for $r_{12}=0$ ($r_1=r_2$), $\psi_{ab}(r_{12}) = 0$ --> thus a two-particle state is not acceptable (minus sign is essential)
- for any r_{12} ($r_1 \neq r_2$)

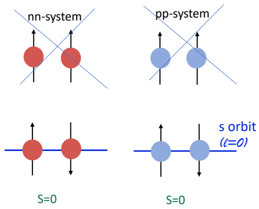
$$\psi_{ab}(r_{12}) = [\psi_a(r_1)\,\psi_b(r_2) - \psi_a(r_2)\,\psi_b(r_1)]$$

$$= -[\psi_a(r_2)\,\psi_b(r_1) - \psi_a(r_1)\,\psi_b(r_2)] = -\psi_{ba}(r_{12}) \Rightarrow \boldsymbol{\psi_{ab}(r_{12}) = -\psi_{ba}(r_{12})}$$

Since the exchange of the particles change the sign, We have an antisymmetric wave function.

---

what about two neutron or two proton systems? Can they be bound?



two protons or two neutrons in the same orbital should couple antiparallely which gives S=0

Pauli principle and antisymmetrization

$$\psi_{ab}(r_{12}) = -\psi_{ba}(r_{12})$$

A two-particle nuclear wave function, ψ, must be antisymmetric with respect to the interchange of the two partners.

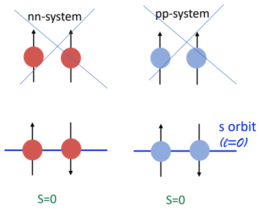
Although we have seen the antisymmetrization in terms of spatial coordinates, it can be extended to other spaces.

For defining a nuclear state, we should include space, spin, and isospin wave functions and the total wave functions should be antisymmetric.

$$\psi = \underset{?}{\psi_{space}} \times \underset{?}{\psi_{spin}} \times \underset{+}{\psi_{isospin}} = \text{antisymmetric}$$

---

what about two neutron or two proton systems? Can they be bound?



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$$\psi = \underset{+}{\psi_{space}} \times \underset{-}{\psi_{spin}} \times \underset{+}{\psi_{isospin}} = \text{antisymmetric}$$

"spin dependence is discussed in Problemset#2→ Problem 3a

Go through Kompendium Krane Page#101 for help.

Problem 3: Nuclear force

a) We say the nuclear force is charge independent. Still, a proton and a neutron can form a bound state but two protons or two neutrons cannot. Why?

terms are also invariant with respect to parity.) The simplest term involving both nucleon spins is $s_1 \cdot s_2$. Let's consider the value of $s_1 \cdot s_2$ for singlet and triplet states. To do this we evaluate the total spin $S = s_1 + s_2$

$$S^2 = S \cdot S = (s_1 + s_2)\cdot(s_1 + s_2) = s_1^2 + s_2^2 + 2 s_1 \cdot s_2$$

Thus

$$s_1 \cdot s_2 = \tfrac{1}{2}\left(S^2 - s_1^2 - s_2^2\right) \tag{4.44}$$

To evaluate this expression, we must remember that in quantum mechanics all squared angular momenta evaluate as $s^2 = \hbar^2 s(s+1)$; see Section 2.5 and Equation 2.69.

$$\langle s_1 \cdot s_2 \rangle = \tfrac{1}{2}\left[S(S+1) - s_1(s_1+1) - s_2(s_2+1)\right]\hbar^2 \tag{4.45}$$

With nucleon spins s_1 and s_2 of $\frac{1}{2}$, the value of $s_1 \cdot s_2$ is, for triplet ($S = 1$) states:

$$\langle s_1 \cdot s_2 \rangle = \tfrac{1}{2}\left[1(1+1) - \tfrac{1}{2}(\tfrac{1}{2}+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\right]\hbar^2 = \tfrac{1}{4}\hbar^2 \tag{4.46}$$

and for singlet ($S = 0$) states:

$$\langle s_1 \cdot s_2 \rangle = \tfrac{1}{2}\left[0(0+1) - \tfrac{1}{2}(\tfrac{1}{2}+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\right]\hbar^2 = -\tfrac{3}{4}\hbar^2 \tag{4.47}$$

---

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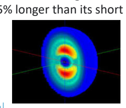
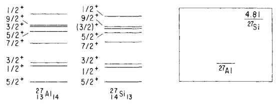
7. The p-n system has tendency to produce non-spherical shapes.
   Non-zero electric quadrupole moment of the deuteron indicated that.
8. The Nucleon-nucleon interaction has a noncentral term, known as a tensor potential
   The wave function of deuteron is mixed and this cannot result from the central potential but a non-central one. This non-central component is called tensor. It has dependence on radial but also the angular coordinates (thus spin) which makes this force a spin-dependent, non-central t force.

Deuteron's long axis is about 5% longer than its short axes.



Better explanation for “4. Nuclear force is charge independent “



Calculate number of pp interactions and
number of nn interactions in the mirror nuclei in the figure above:

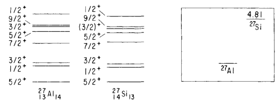
Similar energy structure of mirror nuclei indicated that nuclear force is **charge symmetric**, i.e., **pp** and **nn** interactions are equal.

But their relative binding energies, are different and does not breakdown of the charge symmetry!

It simply reflects the influence of the Coulomb interaction: ^{27}Si has more protons and thus lower binding.

(Repulsive Coulomb interaction lowers the total binding.)

Better explanation for “4. Nuclear force is charge independent “



^{27}Al:
pp: (13x12)/2 = 78
nn: (14x13)/2 = 91
pn: 13x14 = 182

Total: 351

^{27}Si:
pp: (14x13)/2 = 91
nn: (13x12)/2 = 78
pn: 14x13 = 182

Total: 351

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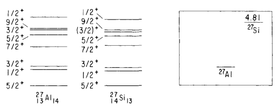

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Total: 351

^{27}Si:
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nn: (13x12)/2 = 78
pn: 14x13 = 182

Total: 351

A more general characteristic of the nuclear force is:
it is **charge independent**, i.e., **pp, nn, and pn** interactions are equal.

Similar energy structure of mirror nuclei indicated that nuclear force is **charge symmetric**, i.e., **pp** and **nn** interactions are equal.

But their relative binding energies, are different and does not breakdown of the charge symmetry!

It simply reflects the influence of the Coulomb interaction: ^{27}Si has more protons and thus lower binding.

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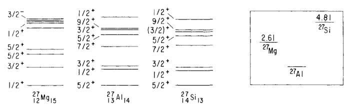
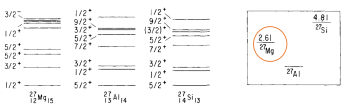


Table 1.1. Nucleon–nucleon interactions in A = 27 Nuclei

	$^{27}_{12}Mg_{15}$	$^{27}_{13}Al_{14}$	$^{27}_{14}Si_{13}$
p–p	66	78	91
n–n	105	91	78

Based on deShalit, 1974.



^{27}Mg is less bound than ^{27}Al even if it has one proton less. It is because of less number of pn interaction with S=1 component.

Table 1.1. Nucleon–nucleon interactions in A = 27 Nuclei

	$^{27}_{12}Mg_{15}$	$^{27}_{13}Al_{14}$	$^{27}_{14}Si_{13}$
p–p	66	78	91
n–n	105	91	78
p–n	180	182	182
Total	351	351	351

Based on deShalit, 1974.

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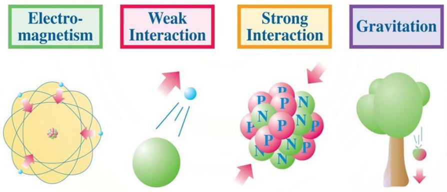
Range of the nuclear force:



Range of the nuclear force

Consider two interacting entities:
One emits an exchange particle and the other one absorbs it. There is non conservation of energy in this process here.

Thus, by the Heisenberg uncertainty principle, there is a finite amount of time during which the exchange can occur : $\Delta t\, \Delta E \geq \hbar$

An important correaltion between the mass (energy) of the virtual particle and the range (time) of the interaction/force: lighter (low E) virtual particles induce smaller violations of energy conservations (smaller ΔE) and can exist for longer period of time, thus allowing longer-range forces:

$r = c\,\Delta t \Rightarrow \Delta t = r/c$ r: range c: speed of light

$r = \frac{\hbar c}{\Delta E} = \frac{\hbar c}{mc^2}$ => for pion, the exchange particle between nucleons (m_{pion}=140 MeV/c^2)

$r = \frac{\hbar c}{mc^2} = \frac{197\ MeV.fm}{140\ MeV} = 1.4$ fm

Range of forces

$R = \frac{\hbar c}{M_x c^2}$ *$M_x c^2$:mass energy of the exchange particle X*
$\hbar c$= 197 MeV.fm

Electromagnetic interaction
- infinite range
- exchange particle: **massless photon**.

Weak interaction
- ranges of approximately $R_{W,Z} \approx 2\times10^{-18}$ m
- exchange particle: **very heavy particles**, the W and Z bosons.

Fundamental strong interaction
- infinite range
- exchange particle: **gluons with zero mass**.

Strong nuclear force
- range of approximately (1 – 2) fm,
- exchange particle: **pions** with the mass predicted by Yukawa.

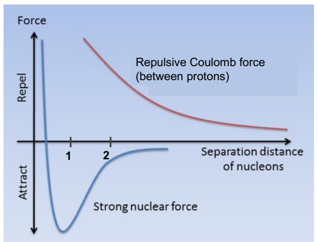
MARTIN & SHAW Page#24-25 x

nuclear force - potential:

Stable nuclei → nuclear force must be **attractive** and much stronger than the Coulomb force. Range is given by the distance between the nucleons ~1 fm

Nuclear force is strongly repulsive for separation distance of nucleons less than 1 fm, around 0.5 fm.

At very short ranges there must be a **repulsive core.** Otherwise, nuclei would collapse in on themselves.



nuclear force - potential:

However, the repulsive core can be ignored in low-energy nuclear structure problems because low-energy particles cannot probe the short-distance behaviour of the potential.

Low-energy particles do not have enough energy to get closer than ~1 fm, which is the range of repulsive core.

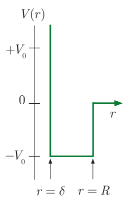


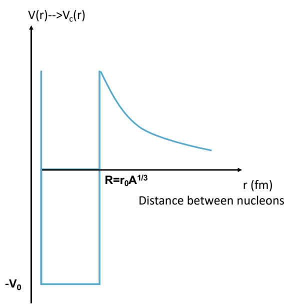
Figure 8.1 Idealised square well representation of the strong interaction nucleon–nucleon potential. The distance R is the range of the nuclear force and $\delta \ll R$ is the distance at which the short-range repulsion becomes important. The depth V_0 is approximately 40 MeV.

nuclear force - potential:

However, the repulsive core can be ignored in low-energy nuclear structure problems because low-energy particles cannot probe the short-distance behaviour of the potential.

Low-energy particles do not have enough energy to get closer than ~1 fm, which is the range of repulsive core.

We should add the Coulomb potential for a complete picture.



Lecture 7: Nuclear Models & Shell Model

Martin & Shaw Ch 8 (8.2 - 8.5)

Focus on Shell model

Recap from the last lecture:
Nuclear force

1. Strongly attractive at short distances
2. Short range and negligible at long distances
3. Some particles are immune from the nuclear force
4. Nuclear force is charge independent and charge symmetric
5. Interaction between two nucleons (Nucleon-nucleon interaction) has an attractive central potential.
6. The Nucleon - Nucleon Interaction is Strongly Spin Dependent
   - S=1 is more bound than S=0 for p-n interaction
   - S=0 is the only allowed one for p-p and n-n systems/interactions.
7. The p-n system has tendency to produce non-spherical shapes.

   Non-zero electric quadrupole moment of the deuteron indicated that.
8. The Nucleon-nucleon interaction has a noncentral term, known as a tensor potential

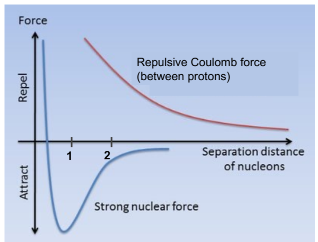
   The wave function of deuteron is mixed and this cannot result from the central potential but a non-central one. This non-central component is called tensor. It has dependence on radial but also the angular coordinates (thus spin) which makes this force a spin-dependent, non-central t force.

nuclear force - potential:

Stable nuclei → nuclear force must be **attractive** and much stronger than the Coulomb force. Range is given by the distance between the nucleons ~1 fm

Nuclear force is strongly repulsive for separation distance of nucleons less than 1 fm, around 0.5 fm.

At very short ranges there must be a **repulsive core.** Otherwise, nuclei would collapse in on themselves.



nuclear force - potential:

However, the repulsive core can be ignored in low-energy nuclear structure problems because low-energy particles cannot probe the short-distance behaviour of the potential.

Low-energy particles do not have enough energy to get closer than ~1 fm, which is the range of repulsive core.

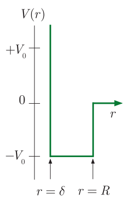


Figure 8.1 Idealised square well representation of the strong interaction nucleon–nucleon potential. The distance R is the range of the nuclear force and $\delta \ll R$ is the distance at which the short-range repulsion becomes important. The depth V_0 is approximately 40 MeV.

nuclear force - potential:

However, the repulsive core can be ignored in low-energy nuclear structure problems because low-energy particles cannot probe the short-distance behaviour of the potential.

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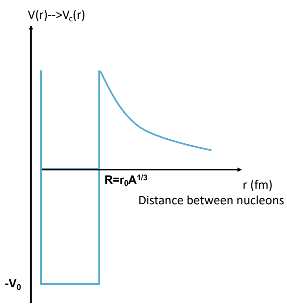


Lecture 7: Nuclear Models & Shell Model

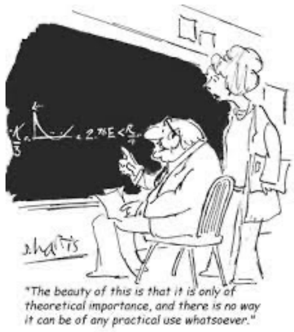
Martin & Shaw Ch 8 (8.2 - 8.5)

Focus on Shell model

Outlook

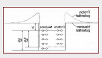
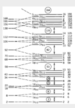
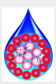
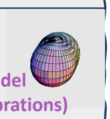
- Nuclear Models
  - Liquid drop model
  - Fermi-gas Model
  - **Shell Model**

Nuclear Models

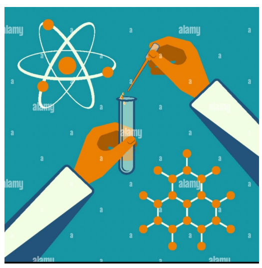


A good model should
- ❑descibe the observed properties
- ❑reproduce the existing data
- ❑predict new properties that can be experimentally tested

we can classify the nuclear models:

Single-particle	Fermi-gas 	Shell Model 
Collective	Liquid-Drop 	Collective Model (rotations, vibrations) 

Liquid drop model



Liquid-drop model

Nucleons in the nucleus behave like molecules in an oscillating liquid droplet:

- All moclecules **attract** each other and are held together by the surface tension
- The droplet is charged and this tends to destabilize the oscillations of the droplet (**less binding**)
- Heavy droplet tends to have a dumbbell shape and then maybe split into two fragments (**Fission**)

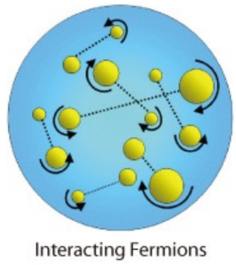
LDM explains:

- Binding energy, thus mass, of nuclei (First three terms of the binding energy in the SEMF - Volume, surface, charge=Coulomb)
- Fission

Cannot explain:
Shell structure, magic numbers in nuclei.

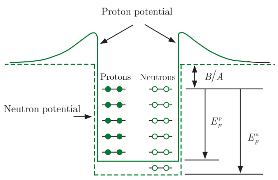
Fermi-gas model

• Thanks to the Uncertainty Relation and the Pauli Principle, whenever a system of identical fermions is compressed, it resists compression with an enormous pressure. No two systems of identical fermions can be merged together; they bounce apart, their probability distributions are not allowed to overlap!



Fermi-gas model

Consider a system of completely non-ineteracting nucleons in a three-dimensional box V.
Gas of protons and neutrons inside a box with volume V.

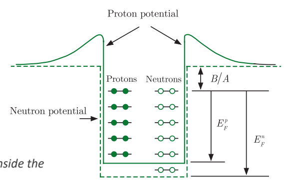


What type of potential well?
How are the nucleons filling the states?
What should be the spin couplings?
Energy spacing between each space?

It is equivalent to putting nucleons in a potential well.

- Protons and neutrons fill the potential well separetely and they all obey the Pauli Principle.
  - S=0 is the right spin cuplings for each state.
- The potential felt by each nucleon is the superposition of the potentials due to all the other nucleons.
- The potential well is in the form of square well like the one in the deuteron.
- The potential well is modified for the proton sector because of the Coulomb force (Coulomb potential). Depth of the neutron potential well is deeper.
  - Since the Fermi levels of the protons and neutrons in a stable nucleus have to be equal (otherwise the nucleus will decay through beta decay to be more), this implies that the depth of the potential well for the neutron gas has to be deeper than for the proton gas.
- For the ground state of the nucleus, the energy levels will fill up from the bottom of the well with equal spacing in between them (separated by the same energy gap, ***Δ***E)

E_F "Fermi energy" : The energy of the highest level that is completely filled.
p_F: Fermi momentum of the moving nucleon at the fermi level.
What is the value of E_F?



We first calculate the number of states (A, number of nucleons) inside the box with volume V

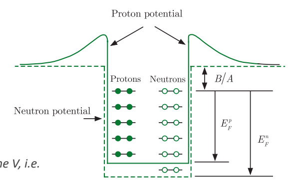
Since nucleons are fermions and obey Fermi statistics, the number of states (nucleons) dA with momentum in a 3-dimensional interval dp is given:

$$dA = 4\times\frac{\text{Phase space volume}}{h^3}$$

$$dA = 4\times\frac{V}{\Delta x^3}$$

Simpler explanation than the book M&S, hopefully...

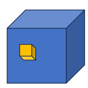
E_F "Fermi energy" : The energy of the highest level that is completely filled.
p_F: Fermi momentum of the moving nucleon at the fermi level.
What is the value of E_F?

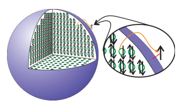


We first calculate the number of states inside the box with volume V, i.e. nucleus as a function of momentum p_F:

Since nucleons are fermions and obey Fermi statistics, the number of states (nucleons) dA with momentum in a 3-dimensional interval dp is given:

$$dA = 4\times\frac{V}{\Delta x^3}$$





$$\Delta p \Delta x \geq \frac{\hbar}{2} = \frac{h}{4\pi} \Rightarrow \Delta x \approx \frac{h}{\Delta p}$$

$$dA = 4\times \frac{V}{\Delta x^3} = 4\times \frac{V \Delta p^3}{h^3} = 4\times \frac{V 4\pi p^2\, dp}{h^3}$$

$$dA = \frac{16V\pi}{h^3} p^2\, dp$$

$$\int dA = \frac{16V\pi}{h^3} \int_0^{p_F} p^2\, dp$$

$$A = \frac{16V\pi}{h^3} \frac{p_F^3}{3}$$

$$p_F^3 = \frac{3h^3 A}{16\pi V}$$

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (r_0 A^{1/3})^3 = \frac{4}{3}\pi (r_0)^3 A$$

where R is the radius of the nucleus and r_0=1.2 fm

$$p_F^3 = \frac{9h^3}{64\pi^2 V (r_0)^3}$$

multiply it with $\frac{8\pi}{8\pi}$ to convert h^3 to $\hbar^3$

$$p_F^3 = \frac{9\hbar^3 \pi}{8 (r_0)^3}$$

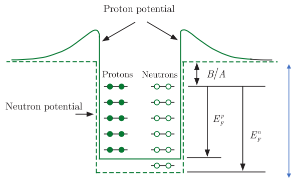
$$p_F = \frac{\hbar}{r_0} \left(\frac{9\pi}{8}\right)^{1/3} \cong 1.5 \frac{\hbar}{r_0}$$

$$p_F = \cong 1.5 \frac{\hbar c}{r_0 c} = 1.5 \frac{197\ MeV.fm}{1.2\ fm\ c} = 246.25\ MeV/c$$

$$p_F \approx 250\ MeV/c$$

$$E_F = \frac{(p_F)^2}{2m} = \frac{(p_F)^2 c^2}{2mc^2} = \frac{250^2\ MeV^2}{2\ (930\ MeV)} \approx \mathbf{33\ MeV}$$

where m is the mass of the nucleon

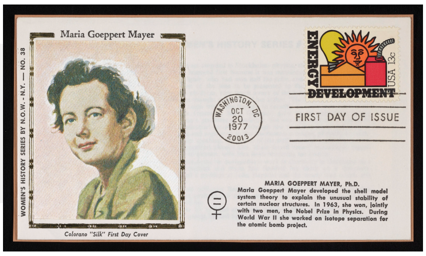


V_0= B/A + E_F= 7 + 33 ~40 MeV (B/A= 7-8 MeV)

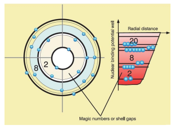
40 MeV is a reasonable estimate for the Nuclear potential. The more detailed calculation (by solving Schrodinger equation for the square well) gives 50 MeV.

Cannot explain:
Shell structure, magic numbers in nuclei, either.

Nuclear Shell Model



Shell Model (1949)



- SM came after large number of experimental evidence.
- Inspired by the shell structure of the atom.
- It explains shell structure and properties for nuclei.
- Nuclei with special numbers ("magic numbers") of protons and/or neutrons are called magic nuclei and
  Magic numbers: Z=2,8,20,28,50,82 N=2,8,20,28,50,82,126
- Nuclei with these magic numbers have **greater stability**
- For example ^{40}Ca (Z=20,N=20), ^{48}Ca (Z=20,N=28), ^{78}Ni (Z=28, N=50), ^{208}Pb (Z=82,N=126)

Nobel Prize in 1963

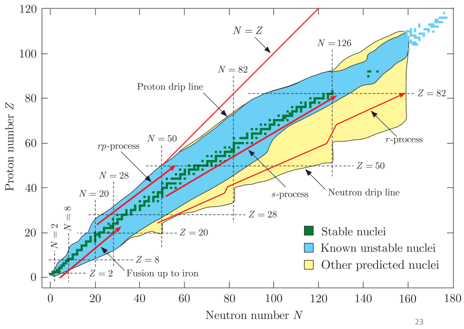
Maria Goeppert Mayer
J. Hans D. Jensen

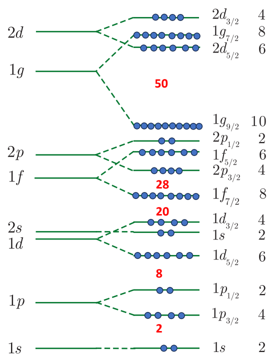
Prize motivation: "for their discoveries concerning nuclear shell structure"

Chart of the nuclides

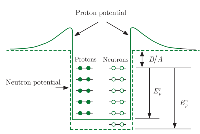
stability line
Valley of beta stability*
binding energy
neutron drip line
proton drip line

s-,r-,rp-process
Magic numbers:
N=2,8,20,28,50,82,126
Z=2,8,20,28,50,82,...





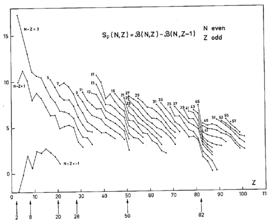
Shell Model

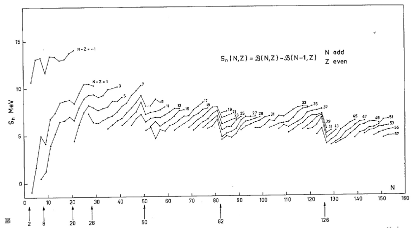


Fermi-gas Model

Expermental evidence for the SM:

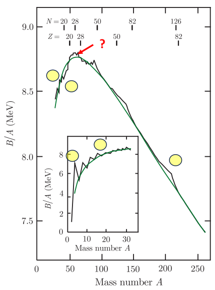
S_p and S_n separation energies:





a sudden drop in the proton (neutron) separation energies takes place after the proton (neutron) number passes the assigned magic number by one neutron.

Nuclei with magic numbers are more tightly bound

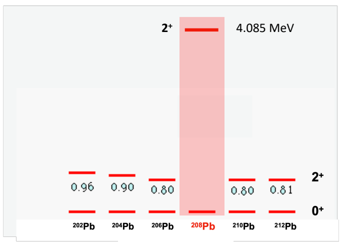


very stable:

$^{4}_{2}He_{2}$
$^{16}_{8}O_{8}$
$^{40}_{20}Ca_{20}$
$^{48}_{20}Ca_{28}$
$^{208}_{82}Pb_{126}$

Figure 8.3 The black line shows the binding energy per nucleon for even values of A. The green curve is the fit from the semi-empirical mass formula. The inset shows the low-A region magnified.

The first excited state, 2^+ lies higher in nuclei with magic numbers.
More energy is required to excite them out of their ground states



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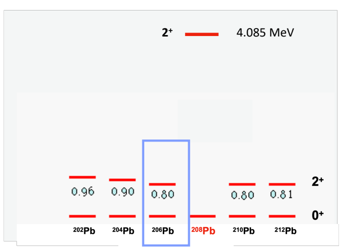
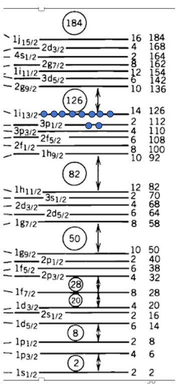


^{206}Pb (Z=82, N=124)



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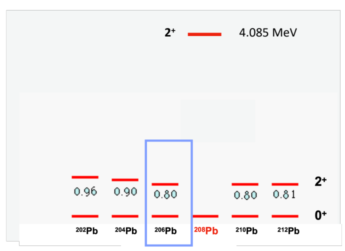

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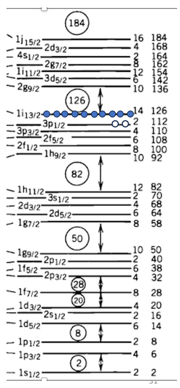

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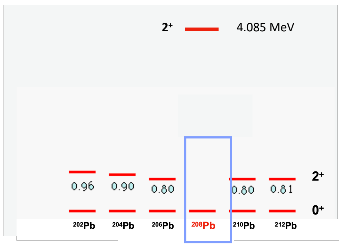
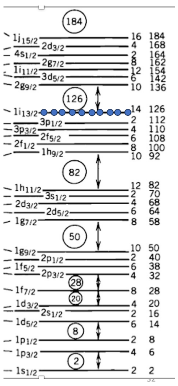


^{208}Pb (Z=82, N=126)

Doubly Magic nucleus:



The first excited state, 2+ lies higher in nuclei with magic numbers.
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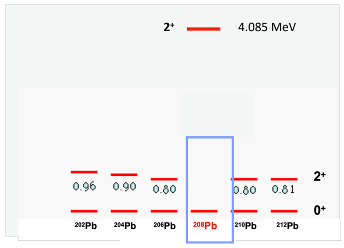
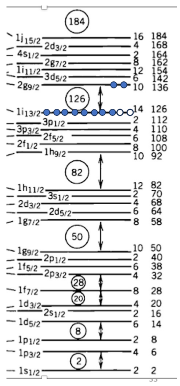


^{208}Pb (Z=82, N=126)

Doubly Magic nucleus:



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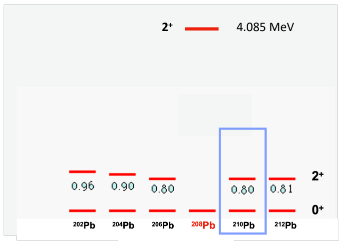
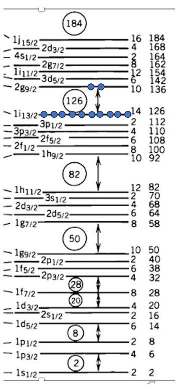


^{210}Pb (Z=82, N=128)



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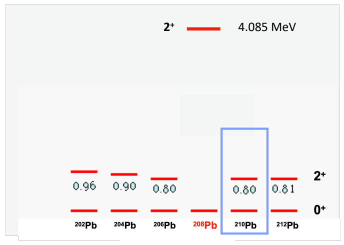
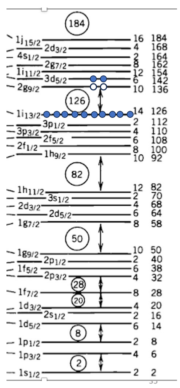


^{210}Pb (Z=82, N=128)



Other phenomena:

Table 19.2 Number of Stable Isotopes with Even and Odd Numbers of Protons and Neutrons

Protons	Neutrons	Number of Stable Isotopes
Odd	Odd	4 ^{2}H: deuteron
Odd	Even	50
Even	Odd	53
Even	Even	164

$N = 2, 8, 20, 28, 50, 82, 126,$

$Z = 2, 8, 20, 28, 50, 82,$

Magic nuclei have more stable isotopes than the other nuclei

They have much smaller Electric quadrupole moments, indicating they are almost spherical, the most tightly bound shape.

Diving into the SM

- Starting point of the shell model is to use “independent particle model” assumption:

IPM describes the nucleus in terms of non-interacting particles in the orbits of spherically symmetric potential U(r) which is itself created by all the nucleons.

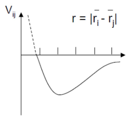
- What is the benefit of this?

A central problem of nuclear structure is to describe the motion of the individual nucleons and to deduce observed facets of nuclear excitations from this basis.

- how the concept of the central potential does arise?

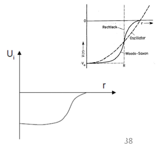
From nucleon-nucleon potential to central potential

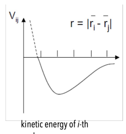
$$H = T + V = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i>k=1}^{A} V_{ik}(\mathbf{r}_i - \mathbf{r}_k)$$



$$H = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] + \sum_{i>k=1}^{A} V_{ik}(\mathbf{r}_i - \mathbf{r}_k) - \sum_{i=1}^{A} U_i(\mathbf{r}) \equiv H_0 + H_{\text{residual}}$$

$$\boxed{H_0 = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] = \sum_{i=1}^{A} H_0^i}$$





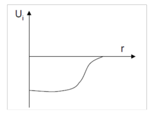
kinetic energy of *i*-th nucleon

2-body nucleon-nucleon interaction between *i*-th and *j*-th nucleons

$$H = \sum_{i=1}^{A} T_i + \sum_{i<j} V_{ij}$$

Complex system

Imagine the complexity of describing a nucleus in which each nucleon is interacting with its nearest neighbours through the nuclear force and at the same time all the protons are pushing on each other with the Coulomb force!



$U(r_i)$: 1-body average potential felt by *i*-th nucleon

$$\begin{aligned} H &= \sum_{i=1}^{A} T_i + \sum_{i<j} V_{ij} \\ &= \sum_{i=1}^{A}(T_i + U(\vec{r}_i)) + \left(\sum_{i<j} V_{ij} - \sum_{i=1}^{A} U(\vec{r}_i)\right) \\ &= H_0 + H_R. \end{aligned}$$

Simple: particle in a box

- Each nucleon moves independently in a potential that represents the average interaction with the other nucleons in a nucleus.
- Existence of a nuclear average potential allows to assume that we can find such a potential, that the residual interaction V_{ij} is small.

x

$$\boxed{H_0 = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] = \sum_{i=1}^{A} H_0^i}$$

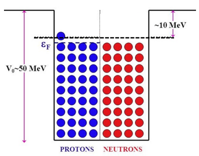
$$H_0^i \psi_i(\mathbf{r}) = E_i \psi_i(\mathbf{r})$$ solutions of the Hamiltonian H_0

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots) = \prod_{i=1}^{A} \psi_i(\mathbf{r})$$

Total nuclear wave function, product of the individual wave functions of each particle orbiting in $U_i(\mathbf{r})$.

wave functions for the individual nucleons in the potential $U_i(\mathbf{r})$ with single-particle energies, E_i.

$$E_o = \sum^{A} E_i$$ Total system energy



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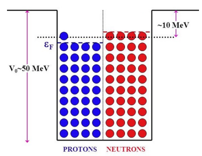
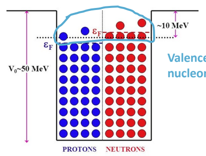




Valence nucleons

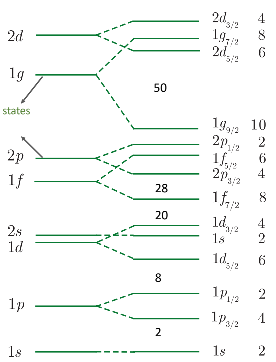
H= H_0+H_{residual}

Next steps are to:

- decide the form of the U(r) potential
- solve this Hamiltonian
- single-particle energies, E_i()
    - i.e., location of the single-particle orbits
- finally reproduce the "magic numbers"

$$\boxed{H_0 = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] = \sum_{i=1}^{A} H_0^i}$$

single-particle states (energies)



Shell Model

Before considering the specific choice of central potential for the nuclear case, it is useful to summarize a few general properties of such potentials. We denote an arbitrary central potential by $U(r)$ and only require that $U(r)$ is attractive and $U(r) \to 0$ as $r \to 0$. The Schrödinger equation for such a potential is

$$H\psi = \left(\frac{\mathbf{P}^2}{2M} + U(\mathbf{r})\right)\psi_{nlm}(\mathbf{r}) = E_{nlm}\psi_{nlm}(\mathbf{r}) \tag{3.2}$$

This equation is separable into radial and angular coordinates and therefore the solutions ψ_{nlm} can be written

$$\psi_{nlm}(\mathbf{r}) = \psi_{nlm}(r\,\theta\,\phi) = \frac{1}{r}R_{nl}(r)\,\psi_{nl}(\theta\,\phi) \tag{3.3}$$

Here, n is the radial quantum number, l the orbital angular momentum and m the eigenvalue of its z-component, l_z. It is conventional in nuclear physics to give names to different l values following the convention:

$$l = 0, 1, 2, 3, 4, 5\ldots \qquad \text{s, p, d, f, g, h}\ldots$$

For a given l, m takes the values $l, l-1, l-2\ldots 0, -1, -2\ldots -(l-1), -l$. Since $U(r)$ is spherically symmetric the $(2l+1)$ energies are independent of m and we will usually delete this index. The radial Schrödinger equation is

$$\frac{\hbar^2}{2M}\frac{d^2R_{nl}(r)}{dr^2} + \left[E_{nl} - U(r) - \frac{\hbar^2}{2M}\frac{l(l+1)}{r^2}\right]R_{nl}(r) = 0 \tag{3.4}$$

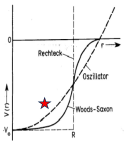
The angular dependence of the wave function is independent of the radial behaviour of the central potential. The angular coordinates give space quantization conditions on **l and m (l_z)**

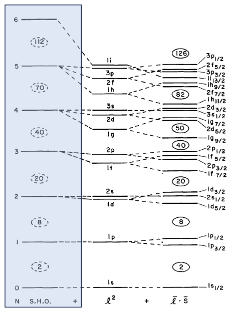
The radial coordinate guves the radial behaviour and energies of the single-particle orbitals (= single particle energies).

Book: Rick Casten, "Nuclear structure from a simple perspective"

$$\frac{\hbar^2}{2M}\frac{d^2 R_{nl}(r)}{dr^2}+\left[E_{nl}-U(r)-\frac{\hbar^2}{2M}\frac{l(l+1)}{r^2}\right]R_{nl}(r)=0$$

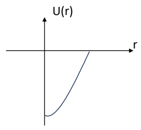
U(r) → $U(r)_{S.H.O.}$
simple harmonic oscillator potential (S.H.O.)

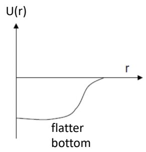




$$\frac{\hbar^2}{2M}\frac{d^2 R_{nl}(r)}{dr^2}+\left[E_{nl}-U(r)-\frac{\hbar^2}{2M}\frac{l(l+1)}{r^2}\right]R_{nl}(r)=0$$

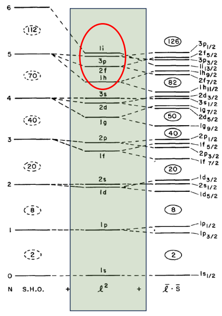
$U(r)_{S.H.O.} + l^2$





- l^2 term flattens the bottom of the SHO potential.
- Larger l orbit will be shifted towards the bottom more!

$l = 0, 1, 2, 3, 4, 5\ldots \quad s, p, d, f, g, h\ldots$

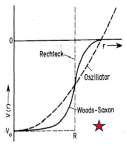


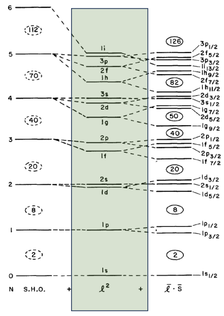
$$\frac{\hbar^2}{2M}\frac{d^2 R_{nl}(r)}{dr^2}+\left[E_{nl}-U(r)-\frac{\hbar^2}{2M}\frac{l(l+1)}{r^2}\right]R_{nl}(r)=0$$

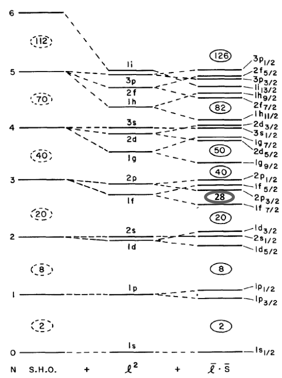
Another similar potential with flatten bottom is the so-called **Woods Saxon potential**, given below:

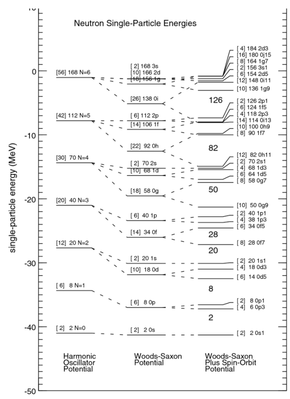
$$V_{\text{central}}(r) = \frac{-V_0}{1+e^{(r-R)/a}}, \qquad (8.20)$$

U(r) is denoted with $V_{central}(r)$ in M&S.
They are the same thing.





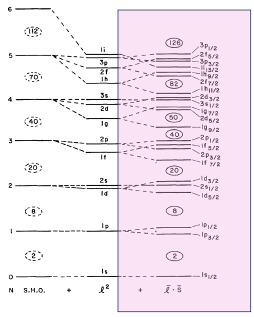




$$\frac{\hbar^2}{2M}\frac{d^2 R_{nl}(r)}{dr^2}+\left[E_{nl}-U(r)-\frac{\hbar^2}{2M}\frac{l(l+1)}{r^2}\right]R_{nl}(r)=0$$

- Still, Woods-Saxon potential alone does not reproduce the Magic numbers.. We need remove degeneracy more.
- The key ingredient is called spin-orbit term (**L.S or l.s**) which should be added to the central potential:

$$V_{\text{total}} = V_{\text{central}}(r) + V_{ls}(r)\,(\mathbf{L}\cdot\mathbf{S}), \qquad (8.21)$$




Maria Goeppert Mayer


J. Hans D. Jensen

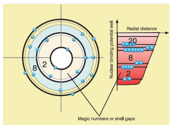
Lecture 8: Shell Model, cont..

Martin & Shaw Ch 8 (8.2 - 8.5)

Recap from the last lecture:
Shell model

Nuclear Shell Model

Shell Model (1949)



- SM came after large number of experimental evidence.
- Inspired by the shell structure of the atom.
- It explains shell structure and properties for nuclei.
- Nuclei with special numbers ("magic numbers") of protons and/or neutrons are called magic nuclei and

  Magic numbers: Z=2,8,20,28,50,82 N=2,8,20,28,50,82,126
- Nuclei with these magic numbers have **greater stability** (compared to those neighbouring ones)
- For example ^{40}Ca (Z=20,N=20), ^{48}Ca (Z=20,N=28), ^{78}Ni (Z=28, N=50), ^{208}Pb (Z=82,N=126)

Nobel Prize in 1963

Maria Goeppert Mayer J. Hans D. Jensen

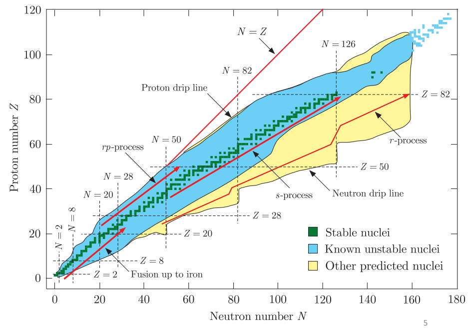
Prize motivation: "for their discoveries concerning nuclear shell structure"

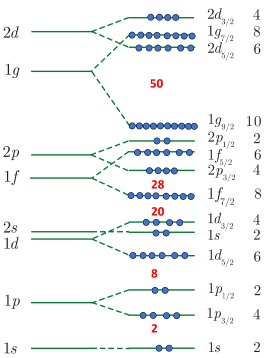
Chart of the nuclides

stability line
Valley of beta stability
binding energy
neutron drip line
proton drip line

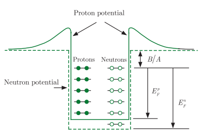
s-,r-,rp-process

Magic numbers*:
N=2,8,20,28,50,82,126
Z=2,8,20,28,50,82,...





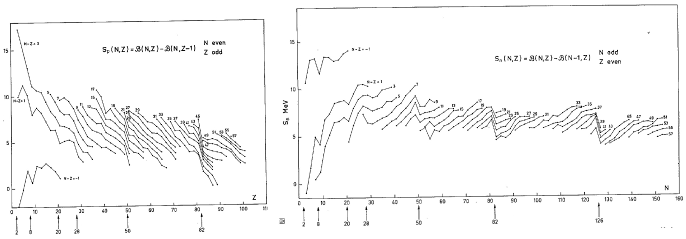
Shell Model



Fermi-gas Model

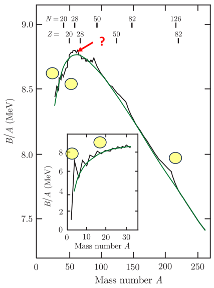
Expermental evidence for the SM:

S_p and S_n separation energies:



a sudden drop in the proton (neutron) separation energies takes place after the proton (neutron) number passes the assigned magic number by one neutron.

Nuclei with magic numbers are more tightly bound

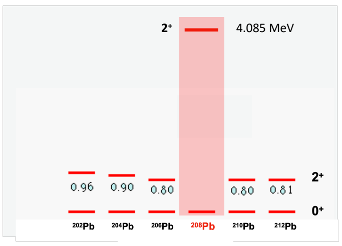


very stable:

$^{4}_{2}He_{2}$
$^{16}_{8}O_{8}$
$^{40}_{20}Ca_{20}$
$^{48}_{20}Ca_{28}$
$^{208}_{82}Pb_{126}$

Figure 8.3 The black line shows the binding energy per nucleon for even values of A. The green curve is the fit from the semi-empirical mass formula. The inset shows the low-A region magnified.

The first excited state, 2^+ lies higher in nuclei with magic numbers.
More energy is required to excite them out of their ground states



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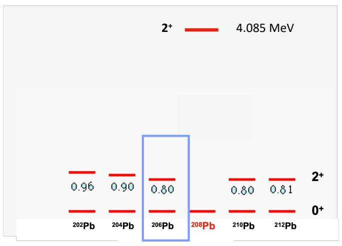
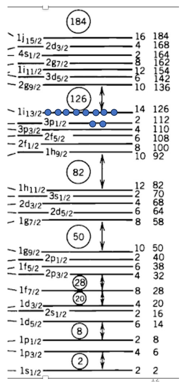


^{206}Pb (Z=82, N=124)



The first excited state, 2^+ lies higher in nuclei with magic numbers.
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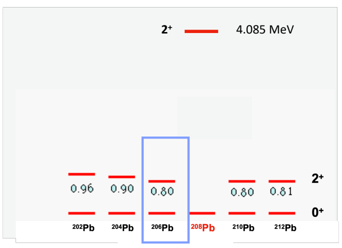
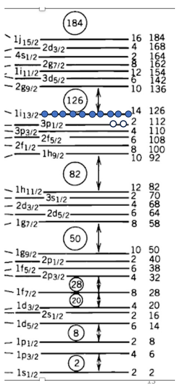


^{206}Pb (Z=82, N=124)



The first excited state, 2^+ lies higher in nuclei with magic numbers.
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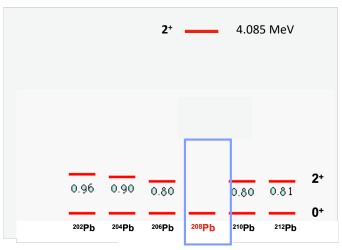
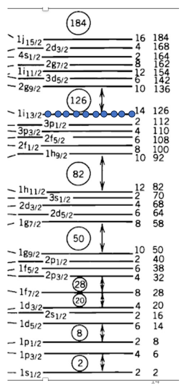
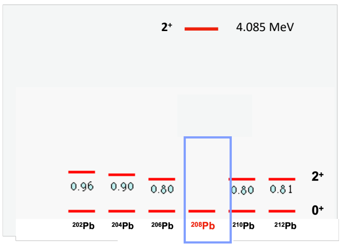


^{208}Pb (Z=82, N=126)

Doubly Magic nucleus:

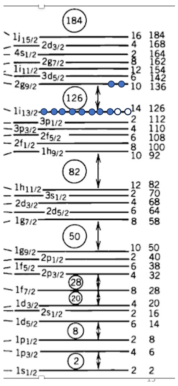


The first excited state, 2^+ lies higher in nuclei with magic numbers.
More energy is required to excite them out of their ground states

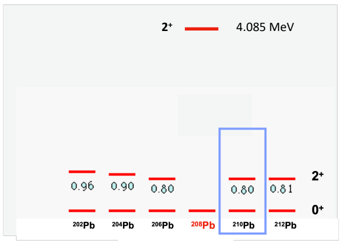


^{208}Pb (Z=82, N=126)

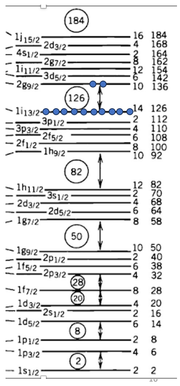
Doubly Magic nucleus:



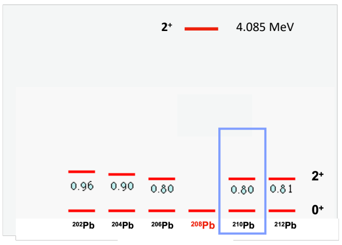
The first excited state, 2^+ lies higher in nuclei with magic numbers.
More energy is required to excite them out of their ground states



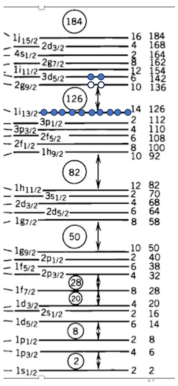
^{210}Pb (Z=82, N=128)



The first excited state, 2^+ lies higher in nuclei with magic numbers.
More energy is required to excite them out of their ground states



^{210}Pb (Z=82, N=128)



Other phenomena:

Table 19.2 Number of Stable Isotopes with Even and Odd Numbers of Protons and Neutrons

Protons	Neutrons	Number of Stable Isotopes
Odd	Odd	4 ^{2}H: deuteron
Odd	Even	50
Even	Odd	53
Even	Even	164

$N = 2, 8, 20, 28, 50, 82, 126,$
$Z = 2, 8, 20, 28, 50, 82,$

Magic nuclei have more stable isotopes than the other nuclei

They have much smaller Electric quadrupole moments, indicating they are almost spherical, the most tightly bound shape.

Magic Numbers

$$N = 2, 8, 20, 28, 50, 82, 126,$$

$$Z = 2, 8, 20, 28, 50, 82,$$

We want SHELL MODEL to predict these magic numbers

Diving into the SM

- Starting point of the shell model is to use "independent particle model" assumption:

  IPM describes the nucleus in terms of non-interacting particles in the orbits of spherically symmetric potential U(r) which is itself created by all the nucleons.

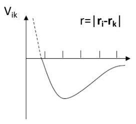
- What is the benefit of this?

  A central problem of nuclear structure is to describe the motion of the individual nucleons and to deduce observed facets of nuclear excitations from this basis.

- how the concept of the central potential does arise?

From nucleon-nucleon potential to central potential

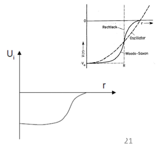
$$H = T + V = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i>k=1}^{A} V_{ik}(\mathbf{r}_i - \mathbf{r}_k)$$

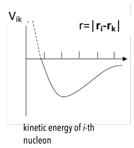


$$H = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] + \sum_{i>k=1}^{A} V_{ik}(\mathbf{r}_i - \mathbf{r}_k) - \sum_{i=1}^{A} U_i(\mathbf{r}) \equiv H_0 + H_{\text{residual}}$$

$$\boxed{H_0 = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] = \sum_{i=1}^{A} H_0^i}$$

The Hamiltonian H_0 for single particle motion

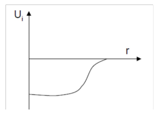




$$H = T + V = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i>k=1}^{A} V_{ik}(\mathbf{r}_i - \mathbf{r}_k)$$

Complex system

Imagine the complexity of describing a nucleus in which each nucleon is interacting with its nearest neighbours through the nuclear force and at the same time all the protons are pushing on each other with the Coulomb force!



$$\boxed{H_0 = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] = \sum_{i=1}^{A} H_0^i}$$

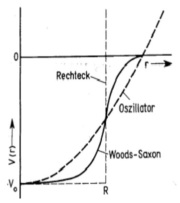
Simple: particle in a box

- Each nucleon moves independently in a potential that represents the average interaction with the other nucleons in a nucleus.
- Existence of a nuclear average potential allows to assume that we can find such a potential, that the residual interaction V_{ij} is small.

- V is a function of the 3 relative positon coordinates of each particle
- Hamiltonian has 3A position coordinates.
- Very difficult to deal with. H has been solved only for a few lightest nuclei.

General properties of U(r):

- U(r) potential is central: depends only on the radial distance from the origin to a given point.
- In other words, we are requring that the potential is sypherically symmetric.
- U(r) is attractive (negative sign)



$$\boxed{H_0 = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] = \sum_{i=1}^{A} H_0^i}$$

The Hamiltonian H_0 for single particle motion

$$H_0^i \psi_i(\mathbf{r}) = E_i \psi_i(\mathbf{r})$$

solutions of the Hamiltonian H_0

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \dots) = \prod_{i=1}^{A} \psi_i(\mathbf{r})$$

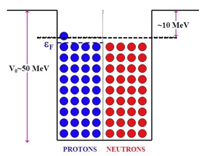
(Anti-symmetrization is ignored here.)

Total nuclear wave function , product of the individual wave functions of each particle orbitting in $U_i(\mathbf{r})$.

wave functions for the individual nucleons in the potential $U_i(\mathbf{r})$ with single-particle energies, E_i.

$$E_o = \sum^{A} E_i$$

Total wave function corresponds to a Total system energy



$$\boxed{H_0 = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] = \sum_{i=1}^{A} H_0^i}$$

The Hamiltonian H_0 for single particle motion

$$H_0^i \psi_i(\mathbf{r}) = E_i \psi_i(\mathbf{r})$$

solutions of the Hamiltonian H_0

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \dots) = \prod_{i=1}^{A} \psi_i(\mathbf{r})$$

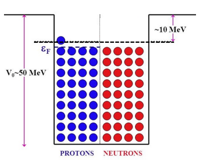
(Anti-symmetrization is ignored here.)

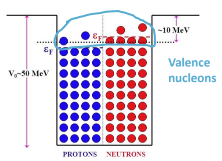
Total nuclear wave function , product of the individual wave functions of each particle orbitting in $U_i(\mathbf{r})$.

wave functions for the individual nucleons in the potential $U_i(\mathbf{r})$ with single-particle energies, E_i.

$$E_o = \sum^{A} E_i$$

Total wave function corresponds to a Total system energy





H= H_0+H_{residual}

Lecture 8: Shell Model, cont..

Martin & Shaw Ch 8 (8.2 - 8.5)

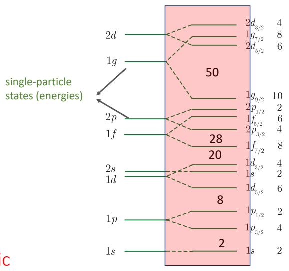
$$N = 2, 8, 20, 28, 50, 82, 126,$$

$$Z = 2, 8, 20, 28, 50, 82,$$

Reminder: We want SHELL MODEL to predict these magic numbers

In order to reproduce the magic numbers we need to

- separate the wave function to radial and angular components
- decide the form of the U(r) potential
- solve this Hamiltonian
- obtain single-particle energies, E_i
  - i.e., location of the single-particle orbits
- the location of the single-particle orbits should be such that finally the correct "magic numbers" (thus shell structures) are reproduced.



Step1:
separate the wave function to radial and angular components

$$\boxed{H_0 = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] = \sum_{i=1}^{A} H_0^i}$$

$$H\,\psi = \left(\frac{\mathbf{p}^2}{2M} + U(\mathbf{r})\right)\psi_{nlm}(\mathbf{r}) = E_{nlm}\,\psi_{nlm}(\mathbf{r})$$

radial angular

$$\psi_{nlm}(\mathbf{r}) = \psi_{nlm}(r\,\theta\,\phi) = \frac{1}{r}R_{nl}(r)\,\psi_{nl}(\theta\,\phi)$$

n: radial quantum number
l: orbital angular momentum quantum number
m: eigenvalues of the z component of the orbital angular momentum quantum number.

$$\psi_{nlm}(\mathbf{r}) = \psi_{nlm}(r\,\theta\,\phi) = \frac{1}{r}R_{nl}(r)\,\psi_{nl}(\theta\,\phi)$$

l=0,1,2,3,4,5, ... s, p, d, f, g, h, ...

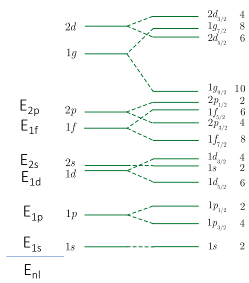
for a given l $\langle l_z \rangle = m_l\hbar$ where $m_l = 0, \pm 1, \pm 2, \ldots, \pm l$

n: radial quantum number
number of nodes (zeros) of the wave function

$\psi_{nl}(\theta\phi)$: The angular dependence of the wave function is independent of the radial behaviour of the central potential.

The angular coordinates give quantization conditions on **l and m (l_z)**

$$\psi_{nlm}(\mathbf{r}) = \psi_{nlm}(r\,\theta\,\phi) = \frac{1}{r}R_{nl}(r)\,\psi_{nl}(\theta\,\phi)$$



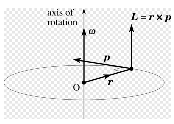
$R_{nl}(r)$: The radial coordinate gives the radial behaviour and energies of the single-particle orbitals (= single particle energies, E_{nl}).

$$\psi_{nlm}(\mathbf{r}) = \psi_{nlm}(r\,\theta\,\phi) = \frac{1}{r}R_{nl}(r)\,\psi_{nl}(\theta\,\phi)$$

We have to write the Schrödinger equation for the radial component. (E=T+V)

For the potential component of the Schrödinger equation:
in addition to the U(r) nuclear potential we have to add also the centrifugal force due to the angular momentum. Thus:

$$U = U(\mathbf{r}) + U_{centrifugal}$$



Centrifugal force is **the apparent outward force on a mass when it is rotated**.

integral of the centrifugal force ($F_{cent} = mw^2r$) gives the centrifugal potential:

$$U_{cent} = \int mw^2 r\,dr = \int \frac{m^2w^2r^4}{mr^3}dr = \int \frac{L^2}{mr^3}dr = \int \frac{l(l+1)\hbar 2}{mr^3}dr = \frac{l(l+1)\hbar 2}{2mr^2}$$

$$where\ L = I\,w = mr^2 w$$

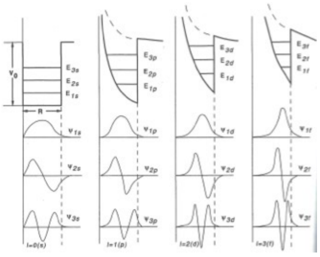
w: angular velocity
I: moment of inertia for a point particle (nucleon)
L: orbital angular momentum

The radial equation becomes:

$$\boxed{\frac{\hbar^2}{2m}\frac{d^2R_{nl}(r)}{dr^2} + \left[E_{nl} - U(r) - \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]R_{nl}(r) = 0}$$

We will not discuss the detailed specifications of the radial wave functions $R_{nl}(r)$ but know that they are proportional to Laguerre polynomial in r^2.

Let's discuss about the solutions of the radial wave functions as a function of n and k. They are given in the figure below. (For now we don't care about the form of U(r).)



- the lowest energy state has n=1
- the states with higher n have higher energy
  - higher n, higher number of nodes, shorter wavelength, higher frequency and thus energy
- for two state with the same n but different l, the state with higher l has larger energy.
  - The centrifugal potential is larger for the particle with larger l and therefore this particle has higher transverse motion (outwards) and is further from the nucleus and therefore less bound.

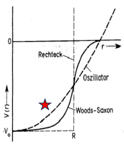
$$U_{cent} = \frac{l(l+1)\hbar 2}{2mr^2}$$

Step2:
decide the form of the U(r) potential

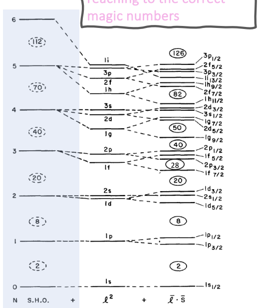
$$\frac{\hbar^2}{2M}\frac{d^2R_{nl}(r)}{dr^2}+\left[E_{nl}-U(r)-\frac{\hbar^2}{2M}\frac{l(l+1)}{r^2}\right]R_{nl}(r)=0$$

Step3:
Solve the Hamiltonian to obtain E_{nl}

U(r) → $U(r)_{S.H.O.}$
simple harmonic oscillator potential (S.H.O.)



Repeat step 2 and 3 until reaching to the correct magic numbers

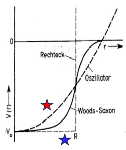


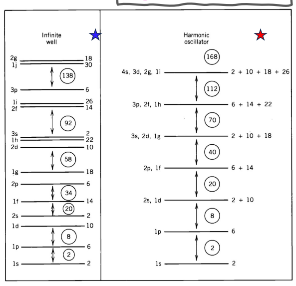
Step2:
Second guess on the form of the U(r) potential

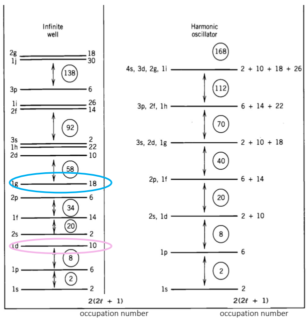
Step3:
Solve again the Hamiltonian to obtain E_{nl}

Repeat step 2 and 3 until reaching to the correct magic numbers

U(r) → $U(r)_{S.H.O.}$
U(r) → $U(r)_{Square}$







Degeneracy: Total number of states with the same energy: $m_s m_l$

$m_l = 2l+1$ $m_s = 2$ → $\boxed{m_{total} = 2(2l+1)}$

$\langle l_z\rangle = m_l\hbar$ $\langle s_z\rangle = m_s\hbar$

$m_l = 0, \pm1, \pm2, ..., \pm l$ $m_s = \pm 1/2$ (fermions)

m_{total} for 1d orbit → $m_{total} = 2(2l+1) = 2\ (2.2+1) = 10$

m_{total} for 1g orbit → $m_{total} = 2(2l+1) = 2\ (2.4+1) = 18$

Degeneracy determines the total number of nucleons each orgbit (energy state) can accept. This is called occupation number. The cumulative occupation number then determines the total number of nucleons for each shell and thus the magic number.

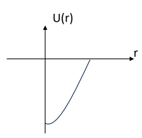
l=0,1,2,3,4,5, ... s, p, d, f, g, h, ...

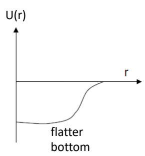
Step2:
Second guess on the form of the U(r) potential

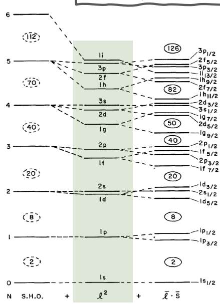
Step3:
Solve again the Hamiltonian to obtain E_{nl}

Getting better but we should improve more!!

U(r) → $U(r)_{S.H.O.} + l^2$



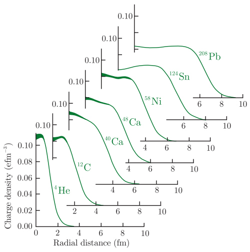




- l^2 term is attractive (negative in sign)
- It flattens the bottom of the SHO potential.
- Larger l orbit will be shifted towards the bottom more!

$l = 0, 1, 2, 3, 4, 5...$ s, p, d, f, g, h...

reminder (lecture 2)
Charge distribution



$\rho(r)$: Charge density corresponds to the charge distribution of the nucleus

Some radial charge distributions for various nuclei obtained by these methods are shown in Figure 2.6. They are well represented by the form

$$\boxed{f(r) = \rho_{\mathrm{ch}}(r) = \frac{\rho^0_{\mathrm{ch}}}{1+e^{(r-a)/b}},} \qquad (2.34)$$

where a and b for medium and heavy nuclei are found to be

$$a \simeq 1.07A^{1/3}\,\mathrm{fm}; \qquad b \simeq 0.54\,\mathrm{fm}. \qquad (2.35)$$

M&S Chapter 2.

40

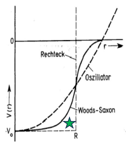
Another similar potential with flatten bottom is the so-called **Woods Saxon potential**, given below:

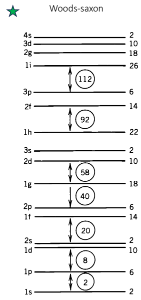
$$V(r) = \frac{-V_0}{1+\exp\left[(r-R)/a\right]}$$

R: nuclear radius
a: surface difuseness

U(r) is denoted with V(r) in Krane. They are the same thing.

U(r) → U(r)$_{\text{Wood-Saxon}}$





41

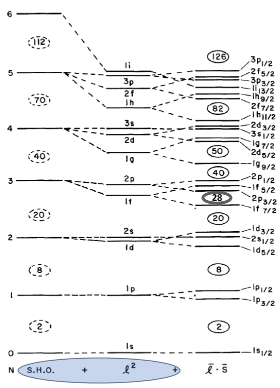
$$\boxed{V_{\mathrm{central}}(r) = \frac{-V_0}{1+e^{(r-R)/a}}, \qquad (8.20)}$$

Woods-Saxon in M&S

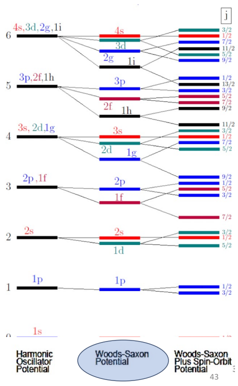
$$\boxed{V(r) = \frac{-V_0}{1+\exp\left[(r-R)/a\right]}}$$

Woods-Saxon in Krane

42

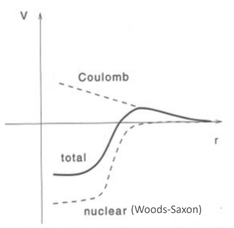


when you search "Shell model orbits" on Google, you will have different versions but they result the same.

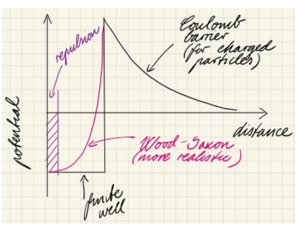


43

now we are left with a more realistic nuclear potential for which is Woods-Saxon type and in between harmonic and square well potentials.

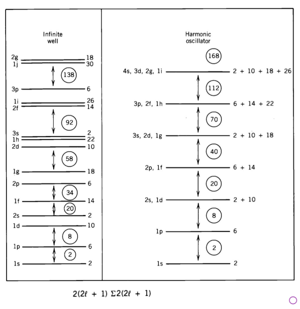


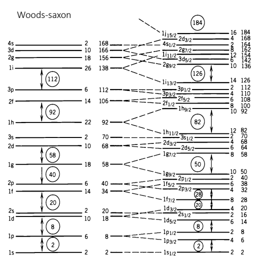
Radial dependence of the nucleon, Coulomb and total 1-body potentials.



Please feel free to ignore the repulsive core from the radial dependence on the nuclear potential. In the nuclear scale, nucleus does not experience the repulsive core.

44





- Still, Woods-Saxon potential alone does not reproduce the Magic numbers. We need to remove degeneracy more.

45

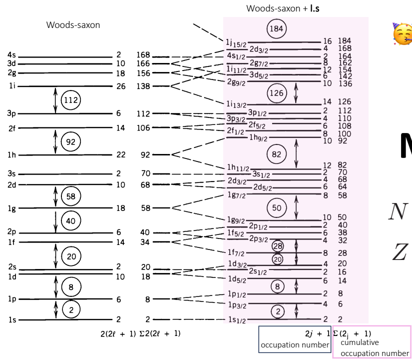
- Since the form of the Woods-Saxon potential is a rather good one and reflects the behaviour of the nuclear density from the scattering experiments, we cannot make any radical change on it.
- Instead we can add another term.
- The term for removing degeneracy is called spin-orbit term (**L.S or l.s**) and it is the key ingredient to obtain the correct magic numbers.

$$\boxed{V_T(r) = \frac{-V_0}{1+\exp[(r-R)/a]} - V_{ls}(r)(\vec{l}.\vec{s})}$$


Maria Goeppert Mayer   J. Hans D. Jensen

$$\boxed{V_{\text{total}} = V_{\text{central}}(r) + V_{ls}(r)\,(\mathbf{L}\cdot\mathbf{S}), \qquad (8.21)}$$

in M&S



Now we can celebrate of the result of adding the spin-orbit term!

Magic Numbers

$$N = 2, 8, 20, 28, 50, 82, 126,$$

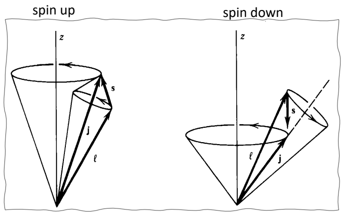
$$Z = 2, 8, 20, 28, 50, 82,$$

$$\boxed{-Vls(r)(\vec{l}.\vec{s})}$$

The form of the $V_{ls}(r)$ is not particularly important but it is the l.s which causes the reordering of the levels and deserves more attention:

- **l.s** coupling causes "l" orbit to split into two components: l+1/2 and l-1/2
- l+1/2 (s and l are parallel aligned) l-1/2 (l and s are anti-parallel aligned)
- In the presence of the spin-orbit interaction, it is appropriate to label the levels with *the total angular momentum j=l+s*

reminder (Lecture 4):
vector coupling of $\vec{l}$ *and* $\vec{s}$



j=l+1/2   **j=l-1/2**

$$\boxed{-Vls(r)(\vec{l}.\vec{s})}$$

The form of the $V_{ls}(r)$ is not particularly important but it is the l.s which causes the re-ordering of the levels and deserves more attention:

- **l.s** coupling causes "l" orbit to split into two components: l+1/2 and l-1/2
- l+1/2 (s and l are parallel aligned) l-1/2 (l and s are anti-parallel aligned)
- In the presence of the spin-orbit interaction, it is appropriate to label the levels with *the total angular momentum j=l+s*

Expectation value of **l.s** can be calculated using a common trick:

$$j^2 = (\ell + s)^2:$$
$$j^2 = \ell^2 + 2\ell\cdot s + s^2$$
$$\ell\cdot s = \tfrac{1}{2}(j^2 - \ell^2 - s^2)$$

$$\langle \ell\cdot s\rangle = \tfrac{1}{2}[j(j+1) - \ell(\ell+1) - s(s+1)]\hbar^2$$

$$\langle \ell\cdot s\rangle = \tfrac{1}{2}[j(j+1) - \ell(\ell+1) - s(s+1)]\hbar^2 = \hbar^2\begin{cases} l/2 & \text{for } j = l+1/2 \\ -(l+1)/2 & \text{for } j = l-1/2 \end{cases}$$

larger j member
smaller j member

Splitting between the two levels

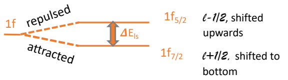
$$\boxed{\Delta E_{ls} = \frac{2l+1}{2}\hbar^2\langle V_{ls}\rangle}$$

- l.s coupling causes the splitting of the "l" orbital angular momentum state
- (l+1/2) is lowered more than (l-1/2) i.e., attracted more towards the bottom of the potential, more bound.
- Larger l, larger splitting (ΔE_{ls})

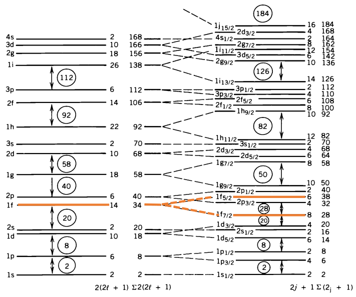
$$\langle \boldsymbol{\ell}\cdot\mathbf{s}\rangle = \tfrac{1}{2}[j(j+1)-\ell(\ell+1)-s(s+1)]\hbar^2 = \hbar^2\begin{cases} l/2 & \text{for } j=l+1/2 \\ -(l+1)/2 & \text{for } j=l-1/2 \end{cases}.$$

Splitting between the two levels

$$\boxed{\Delta E_{ls} = \frac{2l+1}{2}\hbar^2\langle V_{ls}\rangle}$$



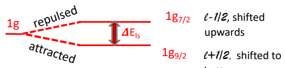
Starting from 1f, the effect of l.s is more powerful so that we can start reaching the magic numbers (for instance 28!!)

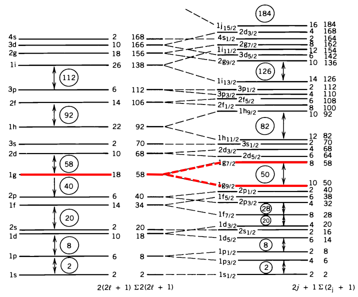


$$\langle \boldsymbol{\ell}\cdot\mathbf{s}\rangle = \tfrac{1}{2}[j(j+1)-\ell(\ell+1)-s(s+1)]\hbar^2 = \hbar^2\begin{cases} l/2 & \text{for } j=l+1/2 \\ -(l+1)/2 & \text{for } j=l-1/2 \end{cases}.$$

Splitting between the two levels

$$\boxed{\Delta E_{ls} = \frac{2l+1}{2}\hbar^2\langle V_{ls}\rangle}$$

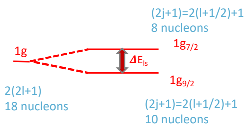




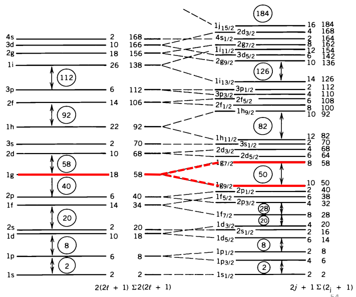
$$\langle \boldsymbol{\ell}\cdot\mathbf{s}\rangle = \tfrac{1}{2}[j(j+1)-\ell(\ell+1)-s(s+1)]\hbar^2 = \hbar^2\begin{cases} l/2 & \text{for } j=l+1/2 \\ -(l+1)/2 & \text{for } j=l-1/2 \end{cases}.$$

Splitting between the two levels

$$\boxed{\Delta E_{ls} = \frac{2l+1}{2}\hbar^2\langle V_{ls}\rangle}$$



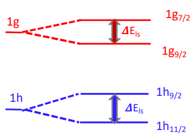
Degeneracy = occupation numbers

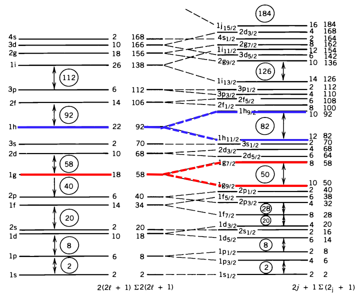


$$\langle \boldsymbol{\ell}\cdot\mathbf{s}\rangle = \tfrac{1}{2}[j(j+1)-\ell(\ell+1)-s(s+1)]\hbar^2 = \hbar^2\begin{cases} l/2 & \text{for } j=l+1/2 \\ -(l+1)/2 & \text{for } j=l-1/2 \end{cases}.$$

Splitting between the two levels

$$\boxed{\Delta E_{ls} = \frac{2l+1}{2}\hbar^2\langle V_{ls}\rangle}$$

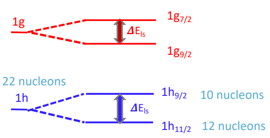


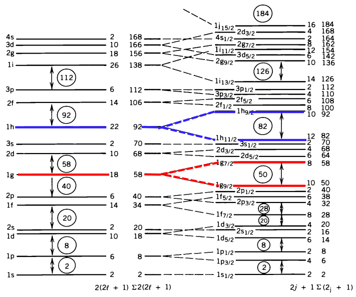


$$\langle \boldsymbol{\ell}\cdot\mathbf{s}\rangle = \tfrac{1}{2}[j(j+1)-\ell(\ell+1)-s(s+1)]\hbar^2 = \hbar^2\begin{cases} l/2 & \text{for } j=l+1/2 \\ -(l+1)/2 & \text{for } j=l-1/2 \end{cases}.$$

Splitting between the two levels

$$\boxed{\Delta E_{ls} = \frac{2l+1}{2}\hbar^2\langle V_{ls}\rangle}$$





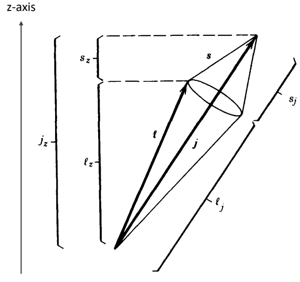


Figure 5.8 As the total angular momentum j precesses about the z axis keeping j_z constant, the vectors ℓ and s precess about j. The components of ℓ and s along j remain constant, but ℓ_z and s_z vary.

- Now we have a coupling between l and s vectors (**l.s**) What happens?
- As the j total angular momentum precceses about the z-axis, l and s precess about j.
- j_z remains constant.
- s_z and l_z vary while l_j and s_j remains constant.
- In other words, m_s and m_l are not good quantum numbers anymore.
- Instead, m_j is a good quantum number.

He did not always fit in well with the group think of the other scientists.

Predictions of Shell Model

- ❑ground state spin-parity
- ❑excite states spin-parity
- ❑magnetic moments
- ❑electric moments

Predictions of Shell Model

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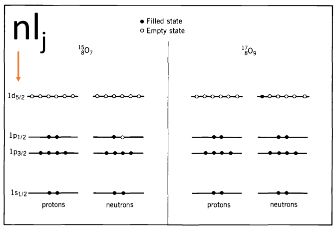
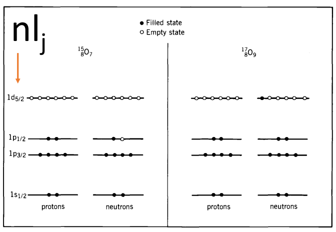
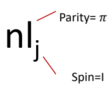
I. First fill your neutrons and protons into the shell model orbits (levels)
II. check the level and quantum numbers of the last unpaired nucleon.

Filling of levels needed to produce a given nucleus. Examples: ^{15}O and ^{17}O



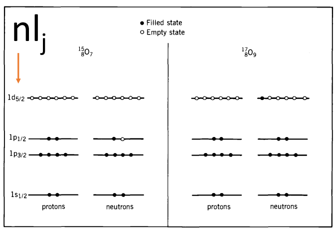
What is the ground state spin-parity (I^{π}) for ^{15}O and ^{17}O?

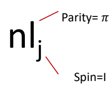




All nucleon pairs coupled to give the spin zero. The remaining last neutron or proton determines the ground state spin and parity:

What is the ground state spin-parity (I^{π}) for ^{15}O and ^{17}O?





All nucleon pairs coupled to give the spin zero. The remaining last neutron or proton determines the ground state spin and parity:

^{15}O: the last neutron is in $1p_{1/2}$ → I =1/2 & $\pi=(-1)^l$= - $I^{\pi}_{g.s.} = 1/2^-$

^{17}O: the last neutron is in $1d_{5/2}$ → I =5/2 & $\pi=(-1)^l$= + $I^{\pi}_{g.s.} = 5/2^+$

We should check if we are correct: NNDC → National Nuclear Data Center (https://www.nndc.bnl.gov/nudat3/chartNuc.jsp)

ADOPTED LEVELS, GAMMAS for ^{15}O

E(level) (keV)	XREF	Jπ(level)	$T_{1/2}$(level)	E(γ) (keV)	I(γ)	M(γ)	Final level	
0.0	ABCDEFG LMNOPQRSTU	1/2-	122.24 s 16 % ε = 100					
5183 1	C E G LM OP RST	1/2+	5.7 fs 7	5182 1	100	[E1]	0.0	1/2-
5240.9 3	BCDE G LMNOP RSTU	5/2+	2.25 ps 21	5239.9 3	100	[M2+E3]	0.0	1/2-
6176.3 17	C E G LMNOPQRSTU	3/2-	< 1.74 fs	6174.9 17	100	[M1+E2]	0.0	1/2-
6793.1 17	C E G LM O T	3/2+	< 20 fs	6791.4 17	100	[E1(+M2)]	0.0	1/2-
6859.4 9	BC E G LM OP TU	5/2+	11.1 fs 17	1618.4 10	100	[M1(+E2)]	5240.9	5/2+
7275.9 6	BCDEF LMNO R TU	7/2+	0.49 ps 11	2034.9 7 7274.0 6	100.0 12 4.0 12	[M1] [E3]	5240.9 0.0	5/2+ 1/2-
7556.5 4	E G LMNO R T	1/2+	0.99 keV 10 % IT = 0.0042 % p = 99.9958	697.1 10 763.4 17 1380.1 17 2373 1 7554.5 4	<11 40.3 11 100.0 7 27.5 11 6.1 9	 [M1] [E1] [M1] [E1]	6859.4 6793.1 6176.3 5183 0.0	5/2+ 3/2+ 3/2- 1/2+ 1/2-

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6176.3 17	C E G LMNOPQRSTU	3/2-	< 1.74 fs	6174.9 17	100	[M1+E2]	0.0	1/2-
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7556.5 4	E G LMNO R T	1/2+	0.99 keV 10 % IT = 0.0042 % p = 99.9958	697.1 10 763.4 17 1380.1 17 2373 1 7554.5 4	<11 40.3 11 100.0 7 27.5 11 6.1 9	 [M1] [E1] [M1] [E1]	6859.4 6793.1 6176.3 5183 0.0	5/2+ 3/2+ 3/2- 1/2+ 1/2-

ADOPTED LEVELS, GAMMAS for ^{17}O

E(level) (keV)	XREF	Jπ(level)	$T_{1/2}$(level)	E(γ) (keV)	I(γ)	M(γ)	Final level	
0	ABCDEFG IJKL PQRS UVWXYZa defghi lmnopqr wxyz01 34 67	5/2+	STABLE					
870.756 20	AB DEFG IJKLM PQRS VWXYZa defghi lmnopqr uvwxyz01 34567	1/2+	179.6 ps 27 % IT = 100	870.732 20	100	E2	0	5/2+
3055.40 6	A EFG JKL PQRS WX Za d f hi lmno r uvw z01 3456	1/2-	110 fs +24-21 % IT = 100	2184.49 5	100	E1	870.756	1/2+
3842.8 4	A EFG IJKL PQRSTU WX Za def lmno s uv z01 3456	5/2-	92×10^{-3} eV 6 % IT = 100	3842.3 4	100		0	5/2+
4143.27 13 S	h	1/2+		1087.89 4 3272.02 8 4142.6 6	100.00 62 20.15 50 4.18 30	E1 M1 E2	3055.40 870.756 0	1/2- 1/2+ 5/2+
4551.8 7	A EFG JKL PQ S X Za de g j lm o tuv 01 3 5	3/2-	38.7 keV 28 % IT = 9.5E-3 % n = 99.9905	3680.6 7 4551.1 7	100 100	E1 E1	870.756 0	1/2+ 5/2+
5086.8 9	A DEF IJKL PQ Za d j lmn q tu 01	3/2+	90 keV 3 % IT = 1.1E-3 % n = 99.9988					

Lecture 8: Shell Model final piece

Martin & Shaw Ch 8 (8.2 - 8.5)

Predictions of Shell model

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with the group think of the other scientists.

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- ❑excite states spin-parity
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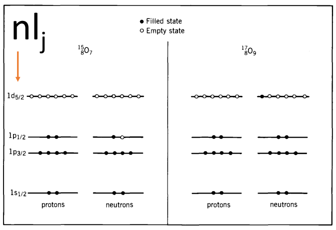
Examples: ^{15}O and ^{17}O



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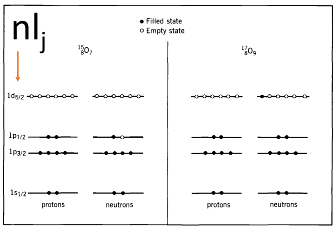
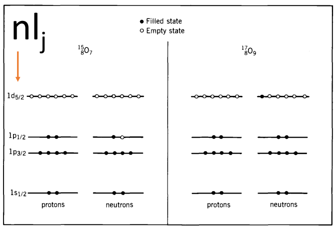


nl_j Parity= π Spin=I

All nucleon pairs coupled to give the spin zero. The remaining last neutron or proton determines the ground state spin and parity:

What is the ground state spin-parity (I^{π}) for ^{15}O and ^{17}O?



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$I^{\pi}_{g.s.} = 5/2^+$

We should check if we are correct: NNDC → National Nuclear Data Center (https://www.nndc.bnl.gov/nudat3/chartNuc.jsp)

ADOPTED LEVELS, GAMMAS for ^{15}O

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7275.9 6	BCDEF LMNO R TU	7/2+	0.49 ps 11	2034.9 7 7274.0 6	100.0 12 4.0 12	[M1] [E3]	5240.9 5/2+ 0.0 1/2-
7556.5 4	E G LMNO R T	1/2+	0.99 keV 10 % IT = 0.0042 % p = 99.9958	697.1 10 763.4 17 1380.1 17 2373 1 7554.5 4	<11 40.3 11 100.0 7 27.5 11 6.1 9	 [M1] [E1] [M1] [E1]	6859.4 5/2+ 6793.1 3/2+ 6176.3 3/2- 5183 1/2+ 0.0 1/2-

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ADOPTED LEVELS, GAMMAS for ^{17}O

E(level) (keV)	XREF	Jπ(level)	$T_{1/2}$(level)	E(γ) (keV)	I(γ)	M(γ)	Final level
0	ABCDEFG IJKL PQRS UVWXYZa defghi lmnopqr wxyz01 34 67	5/2+	STABLE				
870.756 20	AB DEFG IJKLM PQRS VWXYZa defghi lmnopqr uvwxyz01 34567	1/2+	179.6 ps 27 % IT = 100	870.732 20	100	E2	0 5/2+
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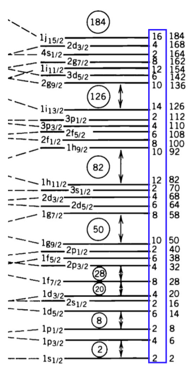
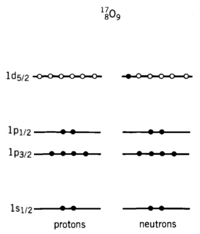
4143.27 13 S	h	1/2+		1087.89 4 3272.02 8 4142.6 6	100.00 62 20.15 50 4.18 30	E1 M1 E2	3055.40 1/2- 870.756 1/2+ 0 5/2+
4551.8 7	A EFG JKL PQ S X Za de g j lm o tuv 01 3 5	3/2-	38.7 keV 28 % IT = 9.5E-3 % n = 99.9905	3680.6 7 4551.1 7	100 100	E1 E1	870.756 1/2+ 0 5/2+
5086.8 9	A DEF IJKL PQ Za d j lmn q tu 01	3/2+	90 keV 3 % IT = 1.1E-3 % n = 99.9988				



- Because of the **l.s** coupling, we now have m_j as a good quantum number to label the energy levels. m_l and m_s are not good quantum numbers anymore.
- The degenracy of each level is then given by the total number of m_j states
–j, -j+1, -j+2, …0, +j → 2j+1
- each state is represented by nl_j quantum number.



When we want to find the ground state spin and parity of a given nucleus we should fill all the protons and neutrons into the respective shell orbits (= energy levels) and the last unpaired nucleon determines the spin and parity of the whole system=nucleus.
Why so? The answer is related to pairing interaction.

$$\boxed{\begin{aligned}\mathbf{I} &= \textstyle\sum_i^A j_i \\ &= 5/2+0+0+0+\ldots+0=5/2\end{aligned}}$$

$$\boxed{\pi = \prod_i^A (-1)^{l_i}}$$

$$\pi = (+)(+)\ (-)(-)\ (-)(-)\ (-)(-)\ (+)(+)\ (-)(-)\ (-)(-)\ (-)(-)\ (+)$$

Let's do few more: This time it is your turn.

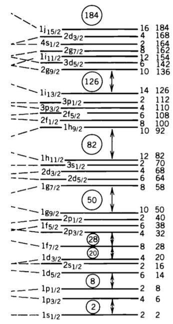
A) ^{49}Ca (Z=20)

B) ^{101}Sn (Z=50)

C) ^{69}Cu (Z=29)

D) ^{209}Bi (Z=83)

E) ^{238}U (Z=92) ^{208}Pb (Z=82), ^{132}Sn, ^{48}Ca



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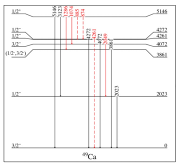
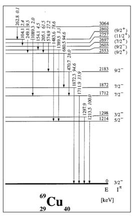
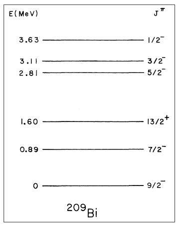






$\boxed{\text{E) } ^{238}U \text{ (Z=92) } ^{208}Pb \text{ (Z=82), } ^{132}Sn, ^{48}Ca}$

ADOPTED LEVELS, GAMMAS for ^{101}Sn

E(level) (keV)	XREF	Jπ(level)
0.0	AB	(5/2+)
171.7 6	B	(7/2+)

Jπ(level): From shell-model interpretation

Total spin and parity of the nucleus:

$$\boxed{\mathbf{I} = \textstyle\sum_i^A j_i}$$

odd-A nuclei: I = half-integral

even-A nuclei: I = integral

$$\boxed{\pi = \prod_i^A (-1)^{l_i}}\ *$$

Does not matter what is the spin of each nucleon in an even-even nucleus, each pair will always be coupled to spin zero giving the total spin "zero"

Does not matter what is the parity of each nucleon in an even-even nucleus: The total parity will always be positive (+):
(-) (-) = (+)
(+) (+) = (+)

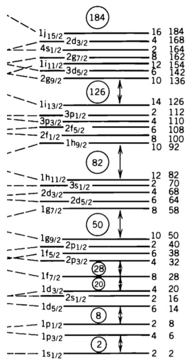
* We ignored the intrinsic parity of each nucleon in the system since nucleons are fermions and their parity is (+1)

$\boxed{\text{E) } ^{238}U \text{ (Z=92) } ^{208}Pb \text{ (Z=82), } ^{132}Sn, ^{48}Ca}$

they are all even-even the ground-state spin and parity is 0^+ for all of them.

GROUND STATE J^π VALUES OF SOME ODD MASS NUCLEI

Z=20	$3/2^+$ ^{37}Ca	$3/2^+$ ^{39}Ca	$7/2^-$ ^{41}Ca	$7/2^-$ ^{43}Ca	$7/2^-$ ^{45}Ca	$7/2^-$ ^{47}Ca	$3/2^-$ ^{49}Ca
Z=50		$9/2^+$ ^{87}Zr	$9/2^+$ ^{89}Zr	$5/2^+$ ^{91}Zr	$5/2^+$ ^{93}Zr	$5/2^+$ ^{95}Zr	
Z=39	$1/2^-$ ^{85}Y	$1/2^-$ ^{87}Y	$1/2^-$ ^{89}Y	$1/2^-$ ^{91}Y	$1/2^-$ ^{93}Y	$1/2^-$ ^{95}Y	$1/2^-$ ^{97}Y
Z=41		$9/2^+$ ^{91}Nb	$9/2^+$ ^{93}Nb	$9/2^+$ ^{95}Nb	$9/2^+$ ^{97}Nb	$9/2^+$ ^{99}Nb	
N=50	$3/2^-$ $^{85}_{35}Br$	$3/2^-$ $^{87}_{37}Rb$	$1/2^-$ $^{89}_{39}Y$	$9/2^+$ $^{91}_{41}Nb$	$9/2^+$ $^{93}_{43}Tc$	$9/2^+$ $^{95}_{45}Rb$	



More challenging case: Ground state of odd-odd nuclei

^{110}In (Z=49, N=61)

- In the case of odd-odd nuclei, there is one unpaired proton and one unpaired neutron. So they will make coupling.
- We cannot then make a precise prediction about the net spin because of the vectorial way that angular momenta combine
- all we can say is that the nuclear spin will lie in the range

$$|j_p - j_n| \le I \le j_p + j_n$$

$$parity = (-1)^{l_p} (-1)^{l_n}$$

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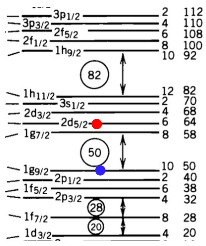

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$$|j_p - j_n| \le I \le j_p + j_n$$

$$|9/2-5/2| \le I \le 9/2 + 5/2$$

$I = 2,3,4,5,6,7$

$parity = (-1)^{l_p} (-1)^{l_n} = (+)(+) = (+)$

$I^\pi = 7^+$ experimental spin-parity for ^{110}In

Predictions of Shell Model

- ❑ground state spin-parity
- ❑excited states spin-parity
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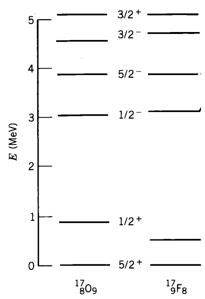
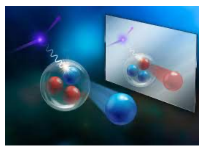
- ❑electric moments

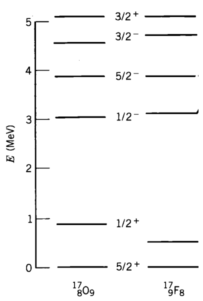
Why do this two nuclei have very similar level schemes?



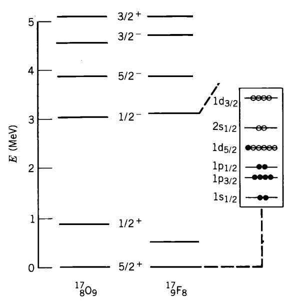
Mirror nuclei



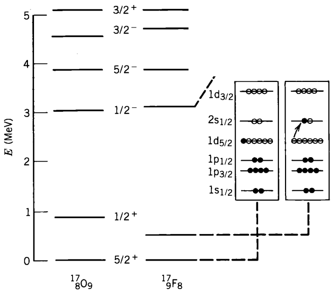
Shell model interpretation of the excited states Step by step:



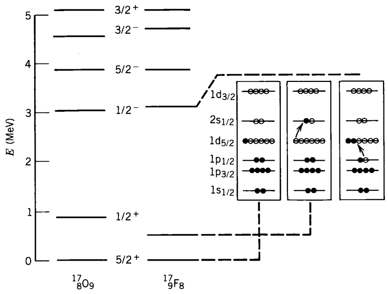
Shell model interpretation of the excited states Step by step:



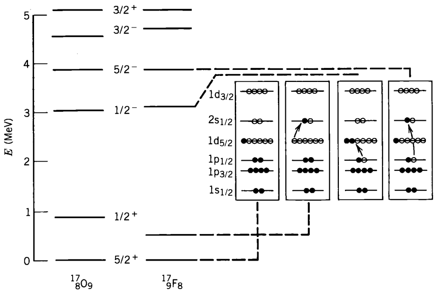
Shell model interpretation of the excited states Step by step:



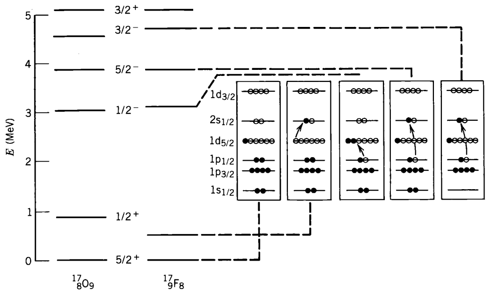
Shell model interpretation of the excited states Step by step:



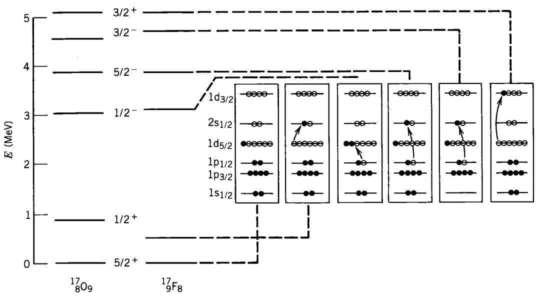
Shell model interpretation of the excited states Step by step:



Shell model interpretation of the excited states Step by step:



Shell model interpretation of the excited states Step by step:



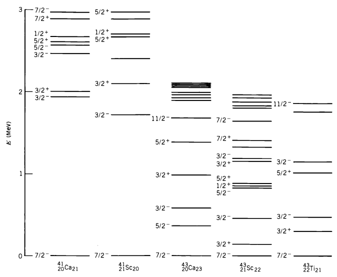
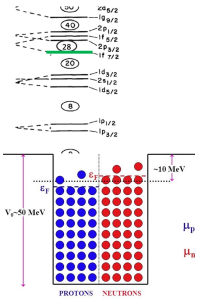
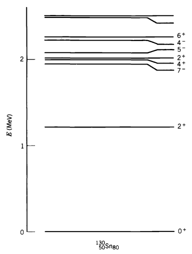


Figure 5.12 Energy levels of nuclei with odd particles in the $1f_{7/2}$ shell.



H= H_0+$H_{residual}$

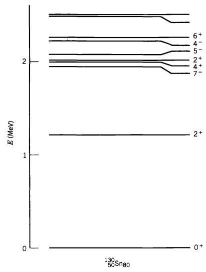
How do we make 2^+ state in even-even nuclei?



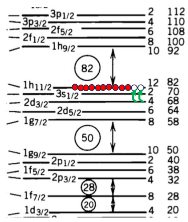
It requires ~ 2MeV to break a pair
But the energy of the 2^+ state is around 1 MeV.

There must be another way which is less energy costly!

How do we make 2^+ state in even-even nuclei?



To excite a pair without breaking (separating) them!!



Predictions of Shell Model

- ❑ground state spin-parity
- ❑excited states spin-parity
- ❑magnetic moments
- ❑electric moments

Reminder (Lecture 4) Magnetic moments

$$\mu = g_l \mu_N . l + g_s \mu_N . s$$
$$= \mu_N (g_l l_z + g_s s_z)/\hbar \qquad \text{(I)}$$

proton: $g_l = 1, g_s = 5.5856912$

neutron: $g_l = 0, g_s = -3.8260837$

Magnetic moment of the nucleus is the combination of its intrinsic magnetic moment due to its s spin and its orbital angular magnetic moment due to its orbital motion.

- g_s=2 for electron which is a point particle. One could expect the same for proton. But this was not the case.
- Furthermore, gs=0 is expected for neutrons since they are "neutral".
- **gs≠0 for neutrons** →(first evident that nucleons are not elementary point particles)

$$\mu = \mu_N \, g_j j_z/\hbar$$

$$\mu = \mu_N \, g_j j$$

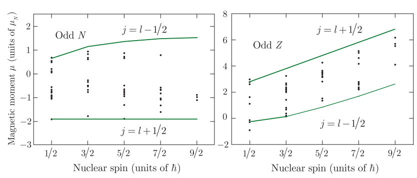
$$g_j = \frac{j(j+1)+l(l+1)-s(s+1)}{2j(j+1)} g_l + \frac{j(j+1)-l(l+1)+s(s+1)}{2j(j+1)} g_s, \qquad (8.29)$$

$$\boxed{\begin{aligned} jg_p &= l + \tfrac{1}{2}\times 5.6 = j + 2.3 \text{ for } j = l+1/2,\\ jg_p &= j\left(1+\frac{1}{2l+1}\right) - 5.6\times j\left(\frac{1}{2l+1}\right) = j - \frac{2.3j}{j+1} \text{ for } j = l-1/2,\\ jg_n &= -\tfrac{1}{2}\times 3.8 = -1.9 \text{ for } j = j = l+1/2,\\ jg_n &= 3.8\times j\left(\frac{1}{2l+1}\right) = \frac{1.9j}{j+1} \text{ for } j = l-1/2. \qquad (8.31)\end{aligned}}$$

Within the Shell model;
We cannot evaluate eq (I) directly since l_z and s_z do not have precise values when we work in a system in which j_z is precisely defined (good quantum number).

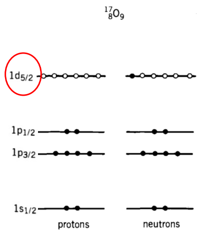
1) We have to rewrite eq (I) using **j=l+s** (vectors)
2) recall the values of g_s and g_l for proton and g_s for neutron (neutrons have zero net charge, thus no magnetic moment from the orbital motion (g_l=0))
3) We also have to remember that j has two components: l+1/2 and l-1/2

$$\boxed{\begin{aligned} jg_p &= l + \tfrac{1}{2}\times 5.6 = j + 2.3 \text{ for } j = l+1/2,\\ jg_p &= j\left(1+\frac{1}{2l+1}\right) - 5.6\times j\left(\frac{1}{2l+1}\right) = j - \frac{2.3j}{j+1} \text{ for } j = l-1/2,\\ jg_n &= -\tfrac{1}{2}\times 3.8 = -1.9 \text{ for } j = j = l+1/2,\\ jg_n &= 3.8\times j\left(\frac{1}{2l+1}\right) = \frac{1.9j}{j+1} \text{ for } j = l-1/2. \qquad (8.31)\end{aligned}}$$



- For simplicity, we compute these equations for odd-A nuclei where there is either one unpaired neutron (Odd N nuclei) or one unpaired proton (Odd Z nuclei)
- Similar to the total nuclear spin, all nucleon pairs result in zero magnetic moment and the only magnetic moment value of the nucleus comes from the unpaired nucleon.

- solid lines are called Schmidt values. They were calculated first time by Schmidt first time in 1937. The points are experimental values measured for some nuclei. We can identify one dot value for ^{17}O as an example



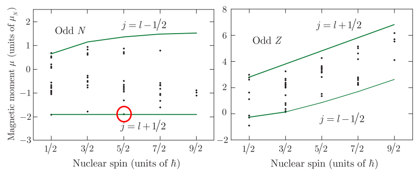
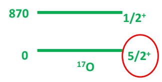
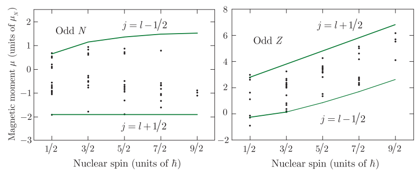


Table 3.2 Sample Values of Nuclear Magnetic Dipole Moments

Nuclide	$\mu(\mu_N)$
n	-1.9130418
p	$+2.7928456$
^{2}H (D)	$+0.8574376$
^{17}O	-1.89379
^{57}Fe	$+0.09062293$
^{57}Co	$+4.733$
^{93}Nb	$+6.1705$



l+1/2
component



Shell model does not predict magnetic moments precisely.

Mismatch between the predictions (solid lines) and experimental data (filled circles).

What are the possible reasons?

from
Krane

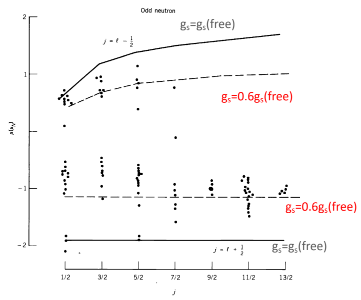


Figure 5.9 Experimental values for the magnetic moments of odd-neutron and odd-proton shell-model nuclei. The Schmidt lines are shown as solid for g_s = g_s(free) and dashed for g_s = 0.6g_s(free).

Re-calculated Shcmidt values for 0.6 times free nucleons match the experimental values well.

from
Krane

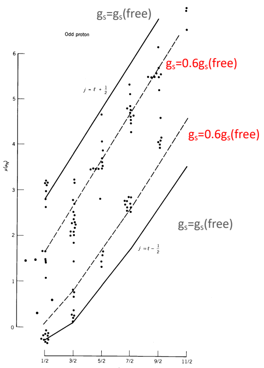


Figure 5.9 Continued.

Re-calculated Shcmidt values for 0.6 times free nucleons match the experimental values.

Perhaps nucleons are not exactly free inside the nucleus

It might arise from the meson cloud surrounding the nucleons. (nucleons are made up quarks and interact via pi mesons→ pions)

"The meson cloud model" considers the nucleon as a system of three valence quarks surrounded by a meson cloud

Predictions of Shell Model

- ❑ground state spin-parity
- ❑excited states spin-parity
- ❑magnetic moments
- ❑electric moments

Electric quadrupole moment, E2

Electric quadrupole moment of the nucleus reflects Shape of charge distribution and deviation from sphere

Quantum mechanics

$$eQ = e\int \psi^*(3z^2 - r^2)\psi \, dv$$

If $|\Psi|^2$ is spherically symmetric

Q=0

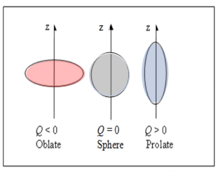
If $|\Psi|^2$ is concentrated on xy-plane (z≅0)

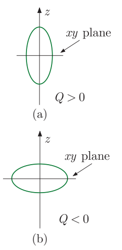
Q ~ - $\langle r^2 \rangle$ (negative) Oblate shape

If $|\Psi|^2$ is concentrated along the z-axis (z≅r)

Q ~ +2 $\langle r^2 \rangle$ (positive) Prolate shape

$\langle r^2 \rangle$Mean-square radius of the orbit





$$eQ = \sum \int \psi^* q_i \,(3z_i^{\,2} - r^2)\psi \mathrm{d}^3\mathbf{r}, \qquad (8.33)$$

$$\boxed{Q \approx -R^2 \frac{(2j-1)}{2(j+1)}.}$$

We again consider odd-A nuclei and predict the Q values using the SM approach, i.e. for the last unpaired neutron or proton.

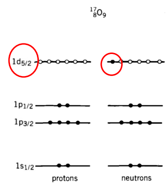
All paired nuclei should contribute zero Quadrupole moment (pairing interaction)

Neutrons contribute zero quadrupole moment due to their being neutral.

Table 5.1 Shell-Model Quadrupole Moments

Shell-Model State	Calculated Q (single proton)	Measured Q Single Particle p	Single Particle n	Single Hole p	Single Hole n
$1p_{3/2}$	−0.013	−0.0366(^{7}Li)		+0.0407(^{11}B)	+0.053(^{9}Be)
$1d_{5/2}$	−0.036	−0.12(^{19}F)	−0.026(^{17}O)	+0.140(^{27}Al)	+0.201(^{25}Mg)
$1d_{3/2}$	−0.037	−0.08249(^{35}Cl)	−0.064(^{33}S)	+0.056(^{39}K)	+0.45(^{35}S)
$1f_{7/2}$	−0.071	−0.26(^{43}Sc)	−0.080(^{41}Ca)	+0.40(^{59}Co)	+0.24(^{49}Ti)
$2p_{3/2}$	−0.055	−0.209(^{63}Cu)	−0.0285(^{53}Cr)	+0.195(^{67}Ga)	+0.20(^{57}Fe)
$1f_{5/2}$	−0.086		−0.20(^{61}Ni)	+0.274(^{85}Rb)	+0.15(^{67}Zn)
$1g_{9/2}$	−0.13	−0.32(^{93}Nb)	−0.17(^{73}Ge)	+0.86(^{115}In)	+0.45(^{85}Kr)
$1g_{7/2}$	−0.14	−0.49(^{123}Sb)		+0.20(^{139}La)	
$2d_{5/2}$	−0.12	−0.36(^{121}Sb)	−0.236(^{91}Zr)		+0.44(^{111}Cd)

from Krane



Experimental Q values

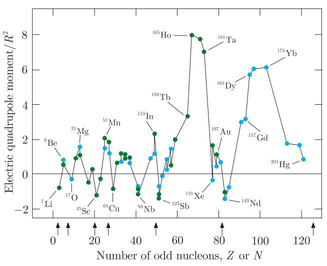
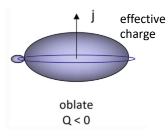


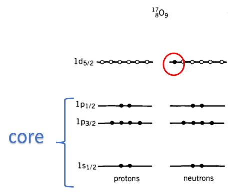
Figure 8.7 Some measured electric quadrupole moments for odd-A nuclei, normalised by dividing by R^2, the squared nuclear radius. Blue circles denote odd-N nuclei and green circles odd-Z nuclei. The solid lines have no theoretical significance. The arrows denote the position of closed shells.

Core polarization - effective charge

Intuitively, due to the attractive proton-neutron interaction, a valence neutron orbiting around the core will cause a core polarization because overlap with the core nucleons minimizes the energy.
So, the ^{17}O nucleus is not spherical but non-spherical (deformed!)

Simplistic shell model does not consider such core polarization.





Collective model (not in the curriculum)

Nuclear vibrations

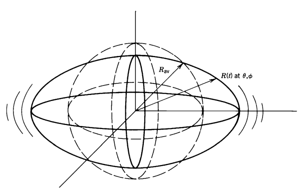


Figure 5.17 A vibrating nucleus with a spherical equilibrium shape. The time-dependent coordinate $R(t)$ locates a point on the surface in the direction θ, ϕ.

Collective model (not in the curriculum)

Nuclear vibrations

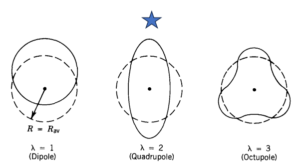
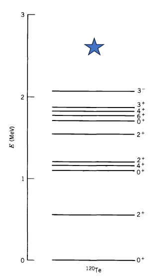


Figure 5.18 The lowest three vibrational modes of a nucleus. The drawings represent a slice through the midplane. The dashed lines show the spherical equilibrium shape and the solid lines show an instantaneous view of the vibrating surface.



Nuclear rotations

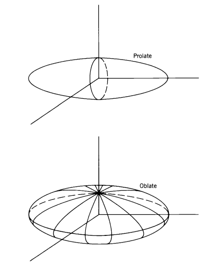
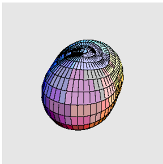


Figure 5.20 Equilibrium shapes of nuclei with permanent deformations. These sketches differ from Figures 5.17 and 5.18 in that these do not represent snapshots of a moving surface at a particular instant of time, but instead show the static shape of the nucleus.

Rotational motion can be observed only in nuclei with non-spherical equilibrium shapes. These nuclei can have substantial distortions from spherical shape and are often called *deformed nuclei.* They are found in the mass ranges **150 < *A* < 190** and ***A* > 220** (rare earths and actinides).



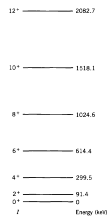
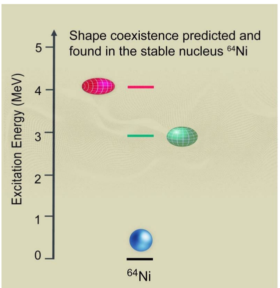


Figure 5.22 The excited states resulting from rotation of the ground state in ^{164}Er.

ple, the rotational levels are given by $E_J = J(J+1)\hbar^2/2I$, where I is the moment of inertia and J is the spin of the nucleus. The predictions of this

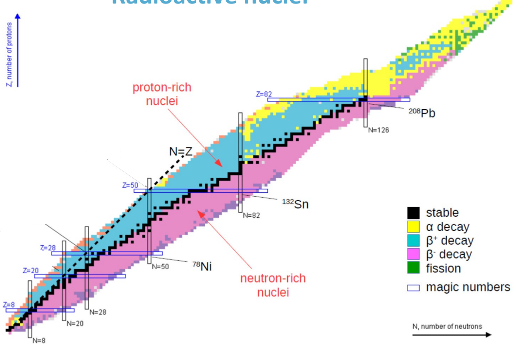


Lecture 9: Radioactive decay

Martin & Shaw Ch 2.4 and 2.5

Krane Ch.6 (Uploaded on Canvas and Timeplan)

Radioactive nuclei

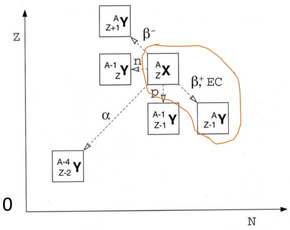


Radioactive decay

- Naturally occurring radioactive nuclei undergo a combination of α, β, E capture, and γ decays. These are spontaneous decays.
- Reminder: Spontaneous decay to occur Q-value should be larger than zero:

$$Q = (M_{initial} - Mf_{inal})c^2 > 0$$

$$Q_{\beta^+} = \{M(^{A}X) - [M(^{A}Y) + \mathrm{m(p)}]\}\, c^2 > 0$$



Q-value: total energy released in a given nuclear decay or a reaction

Decay chain of ^{238}U up to ^{206}Pb

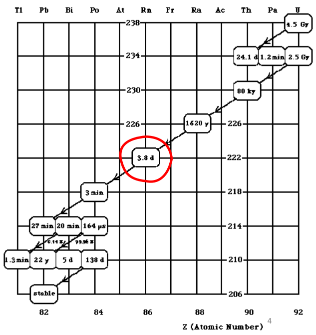
complex "chains" of alpha and beta decays

$^{232}Th \rightarrow \rightarrow$ ^{208}Pb (14.1 billion year half-life),

$^{235}U \rightarrow \rightarrow$ ^{207}Pb (700 million year half-life),

$^{238}U \rightarrow \rightarrow$ ^{206}Pb (4.5 billion year half-life)

^{222}Rn (radon) with a half-life of 3.8 days, is responsible for higher levels of background radiation in many parts of the world. This is primarily because it is a gas and can easily seep out of the earth into unfinished basements and then into the house.



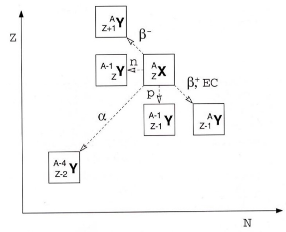
Radioactive decay

- **Naturally occurring** radioactive nuclei undergo a combination of α, β, E capture, and γ decays. These are spontaneous decays.
- Reminder: For a spontaneous decay to occur Q-value should be larger than zero:

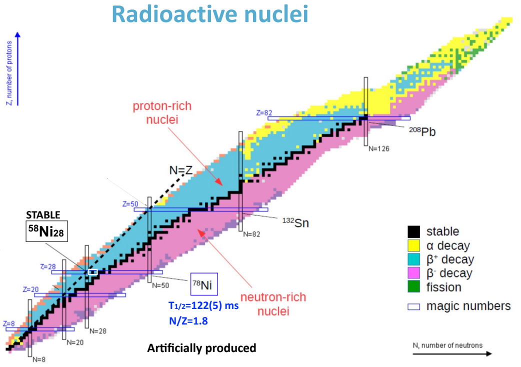
$$Q = (M_{initial} - M f_{inal})c^2 > 0$$

$$Q_{\beta^+} = \{M(^AX) - [M(^AY) + m(e^+)]\}\, c^2 > 0$$

- **Artificially produced** nuclei may also decay by spontaneous fission, neutron emission and even proton and heavy-ion emission in addition to α, β, E capture, and γ decays .



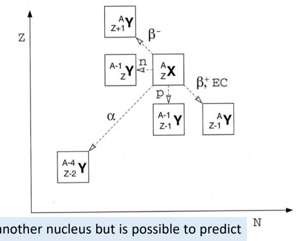
Radioactive nuclei



Radioactive decay

- Radioactivity was discovered in 1896 by Henri Becquerel.
- Decay rate of a pure radioactive substance decreases with time according to an **exponential decay**.
- Decay is statistical in nature. What does this mean?

it is impossible to know when a nucleus will decay into another nucleus but is possible to predict the probability of nuclear decay in a unit of time and consequently the mean number of decaying nuclei per second.



If N radioactive nuclei present at time t and no new nuclei are introduced in the sample, then the number of dN decaying in a time dt is proportional to N, and λ is the proportionality constant.

$$\frac{dN}{dt} \propto N$$

Number of radioactive nuclei which decay in a dt time period (interval)

Decay constant (decay probability)

$$\frac{dN}{dt} = -\lambda N$$

time period

Number of radioactive nuclei present in a sample

Exponential decay law

$$\frac{dN}{N} = -\lambda\, dt$$

$$\int_{N_0}^{N(t)} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln N(t) - \ln N_0 = -\lambda\, t$$

$$\ln \frac{N(t)}{N_0} = -\lambda t$$

N_0: Number of unstable nuclei at t=0
N(t):Number of unstable nuclei remaining at time t
λ: decay constant

$$\boxed{N(t) = N_0\, e^{-\lambda t}}$$

Exponential decay law

Half-life

*The half-life is defined as the time elapsed when the intensity of the radiation is reduced to **one half of its original value. ($t_{1/2}$)***

$$N(t) = N_0\, e^{-\lambda t}$$

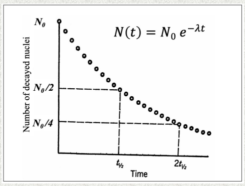
$$N(t_{1/2}) = N_0/2 = N_0\, e^{-\lambda t_{1/2}}$$

$$1/2 = e^{-\lambda t_{1/2}}$$

$$\ln 1 - \ln 2 = -\lambda t_{1/2}$$

$$\ln 2 = \lambda t_{1/2}$$

$$t_{1/2} = \frac{\ln 2}{\lambda}$$



$$\boxed{t_{1/2} = \frac{0.693}{\lambda}}$$

shorter half-life ~ higher decay constant (probability)

10

Mean lifetime

Mean lifetime is the average time that a nucleus is likely to survive before it decays. (Calculated statistically)

$N(t)$: *the number that survive to time t*

$\frac{dN}{dt}dt$: *the number that decay between t and t+dt*

$$\tau = \frac{\int_0^\infty t\,|dN/dt|\,dt}{\int_0^\infty |dN/dt|\,dt}$$

hint1: $\frac{dN}{N} = -\lambda\, dt$

$\frac{dN}{dt} = -\lambda N$

$\boxed{= -\lambda N_0\, e^{-\lambda t}}$

hint2: Use "partial integral" for solving the integaral in the *nominator*

$$\boxed{\tau = \frac{1}{\lambda}}$$

shorter lifetime ~ higher decay constant (probability)

mean lifetime, τ is usually called "lifetime"

11

The number of dN decaying nuclei in a dt time is proportional to the number of nuclei present in a time t. $\boldsymbol{\lambda}$ is proportionality constant.

$$\boldsymbol{\frac{dN}{dt} = -\lambda N(t)}$$

Exponential decay formula

$$N(t) = N_0\, e^{-\lambda t}$$

N_0: Number of unstable nuclei at t=0
N(t):Number of unstable nuclei remaining at time t
λ: decay constant

time formulas

$$t_{1/2} = \frac{0.693}{\lambda} \qquad \boldsymbol{\tau} = \frac{1}{\lambda} \qquad t_{1/2} = 0.693\boldsymbol{\tau}$$

12

Activity

- *Recall the exponential decay law formula: $N(t) = N_0\, e^{-\lambda t}$*
- *N(t) was the number of undecayed nuclei after a time t. This is a difficult quantity to measure.*
- *The exponential decay law formula in this form is then not very useful for us in the laboratory.*
- *Instead of counting the number of undecayed nuclei in a sample, it is easier to count number of decays (by observing the emitted radiations, for example $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ radiations.)*
- *This leads us to **"Activity"**.*
- *Activity is "the number of decays per unit time" i.e. $\frac{dN}{dt}$*

13

$$\frac{dN(t)}{dt} = \frac{d(N_0 e^{-\lambda t})}{dt}$$

$$= -\lambda N_0 e^{-\lambda t}$$

$$\boxed{\boldsymbol{A(t) = A_0 e^{-\lambda t}}}$$

Activity of a radioactive nucleus (or a sample) at a time t.

Where A_0 is the initial activity at t=0 (A_0=$\boldsymbol{\lambda}N_0$)

14

Units of Activity

1 Becquerel (Bq) = 1 decays/s

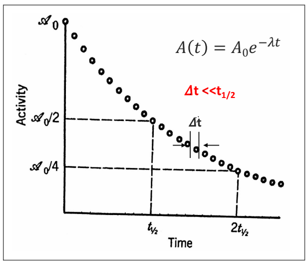
1 Curie (Ci) = 3.7 x 10^{10} decays/s (dps) (activity of 1g ^{226}Ra)

What do we mean when we say a source is highly radioactive?

- Generally, this means **the number of decays per unit time** (i.e. Activity) is very high.
- Knowing activity of a source does not give information about its lifetime or half-life.
- Similarly, by knowing activity of a source we still don't know the kind of radiation emitted nor its energy.

15

Activity vs time plot



We can measure the activity as a function of time by counting the numebr of decays in short Δt time intevals.

The steeper the curve the shorter the life time and the larger the decay constant, λ

The Activity never goes to zero because of (-) exponential

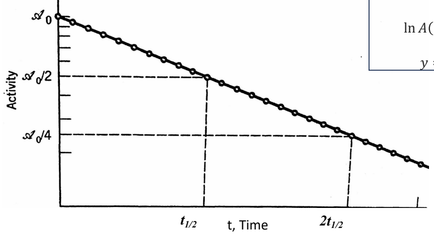
Nevertheless, experimentally we accept the activity to be nearly zero after 10 $t_{1/2}$.

So if $t_{1/2}$=1 min the activity is nearly zero after 10 min.

Example (good case): $t_{1/2}$=10 min **Δt=10 seconds** 😉

too long or too short Δt? What happens?

Semilog of Activity plot (ln A vs t)



$\ln A(t) = \ln(A_0 e^{-\lambda t})$

$\ln A(t) = \ln A_0 + \ln(e^{-\lambda t})$

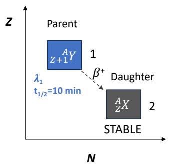
$\ln A(t) = \ln A_0 - \lambda t$

$y = -mx + c$

- The plot of *ln A vs t* gives a straight line with a slope of $-\lambda$, thus $t_{1/2}$ (since $t_{1/2} = \frac{0.693}{\lambda}$)

When does the exponential decay law work and when does not?

When radioactive nucleus 1 decays with λ_1 to stable nucleus 2



Number of nuclei present

$N_1 = N_0\, e^{-\lambda_1 t}$

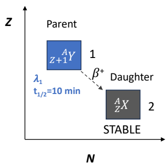
$N_2 = N_0(1 - e^{-\lambda_1 t})$

What is the number of N_1 and N_2 nuclei

a) t=0
b) t=10 min

When does the exponential decay law work and when does not?

When radioactive nucleus 1 decays with λ_1 to stable nucleus 2



Number of nuclei present

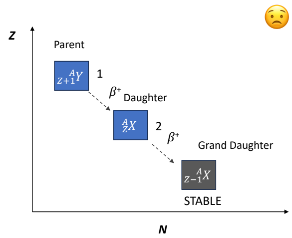
$N_1 = N_0\, e^{-\lambda_1 t}$

$N_2 = N_0(1 - e^{-\lambda_1 t})$

Answer:

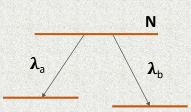
a) $N_1=N_0$ $N_2=0$
b) $N_1=N_0/2$ $N_2=N_0/2$
c) $N_1=0$ $N_2=N_0$ when t → ∞

When radioactive nucleus 1 decays with λ_1 to another radioactive nucleus 2



Many isotopes decay by more than one different ways

$\frac{dN}{dt} = -\lambda N$ *decay rate (activity)*



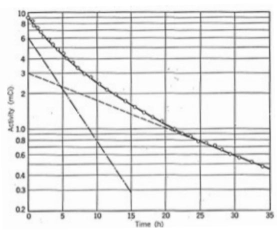
$N\lambda_{tot} = N\lambda_a + N\lambda_b$ total decay rate

$\lambda_{tot} = \lambda_a + \lambda_b$ total decay constant

$\lambda_{tot} = \sum_i \lambda_i$

$$\lambda_{tot} = \frac{1}{\tau_{tot}} = \sum_i \frac{1}{\tau_i}$$

Sample with more than one type of radioactive isotope: example ^{64}Cu and ^{61}Cu



reminder: activity of a given source goes nearly to zero after 10 half-lives

$t_{1/2}$

^{64}Cu (12.7 h)

^{61}Cu (3.4 h)

read the explanation in Krane Chapter 6 Page#165

types of radioactive decays

α decay

$$ {}^{A}_{Z}X_N \rightarrow {}^{A-4}_{Z-2}X'_{N-2} + {}^{4}_{2}He_2 $$

$$ {}^{226}_{88}Ra_{138} \rightarrow {}^{222}_{86}Rn_{136} + \alpha $$ $t_{1/2}$=1600 y

β decay

$n \rightarrow p + e^-$ $\quad \beta^-$ decay

$p \rightarrow n + e^+$ $\quad \beta^+$ decay

$p + e^- \rightarrow n$ $\quad$ electron capture (ε)

${}^{131}_{53}I_{78} \xrightarrow[\beta^-]{} {}^{131}_{54}Xe_{77}$ $\quad t_{1/2} = 8.0$ d

${}^{25}_{13}Al_{12} \xrightarrow[\beta^+]{} {}^{25}_{12}Mg_{13}$ $\quad t_{1/2} = 7.2$ s

${}^{54}_{25}Mn_{29} \xrightarrow[\varepsilon]{} {}^{54}_{24}Cr_{30}$ $\quad t_{1/2} = 312$ d

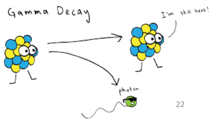
γ decay

${}^{A}{}_{Z}X \quad \rightarrow \quad {}^{A}_{Z}X + {}^{0}_{0}\gamma$ $\quad t_{1/2} = 10^{-12}$ s=1 ps

m for metastable

${}^{Am}_{Z}X \quad \rightarrow \quad {}^{A}_{Z}X + {}^{0}_{0}\gamma$ $\quad t_{1/2}$ from 1ns to 1μs to 1 minutes, hours, years

isomeric states or metastable



Branching ratios and partial half-lives

parent
daughter
grand daughter

More often, a given radioactive nucleus decay via more than one or two decay branches creating a more complex decay schemes.

In such cases, we specify the relative intensities of the competing modes by their "branching ratios".

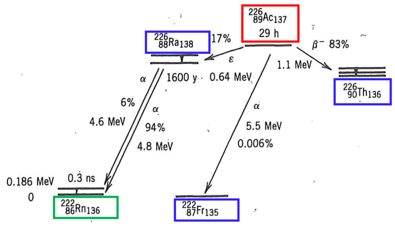
Example in the figure:

${}^{226}Ac \rightarrow {}^{226}Ra$ BR=17% **ε**

${}^{226}Ac \rightarrow {}^{222}Fr$ BR=0.006% **α**

${}^{226}Ac \rightarrow {}^{226}Th$ BR=83% **β⁺**

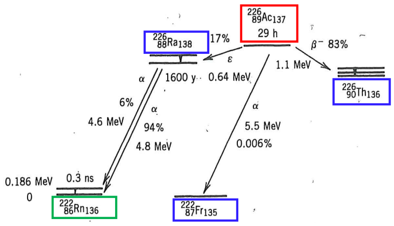
∑BR= 17 + 0.006 + 83 =100



Branching ratios and partial half-lives

parent
daughter
grand daughter

Can you give another example using the same figure?
Example 2:



Branching ratios and partial half-lives

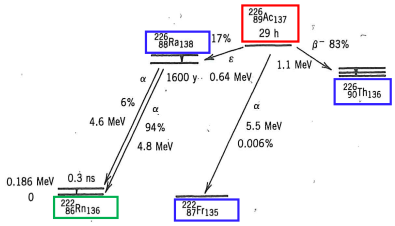
parent
daughter
grand daughter

Can you guve another example using the same figure?
Example 2:

${}^{226}Ra \rightarrow {}^{222}Ra$ + BR=94 % **α**

${}^{226}Ra \rightarrow {}^{222}Ra$ + BR=6 % **α**

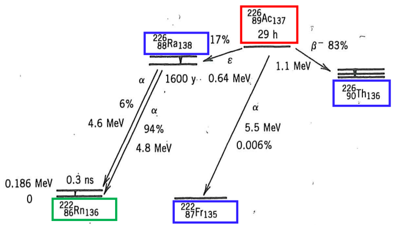
∑BR= 6+ 94 =100



Branching ratios and partial half-lives

parent
daughter
grand daughter

Can you calculate the partial decay constants and half-lives for these three decay branches?



Branching ratios and partial half-lives

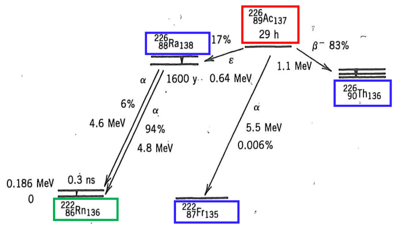
parent
daughter
grand daughter

Can you calculate the partial decay constants and half-lives for these three decay branches?

$$ \lambda_t = \frac{0.693}{t_{1/2}} = 0.024\ h^{-1} = 6.6 \times 10^{-6}\ s^{-1} $$

$$ \lambda_\beta = 0.83\lambda_t = 5.5 \times 10^{-6}\ s^{-1} $$
$$ \lambda_\varepsilon = 0.17\lambda_t = 1.1 \times 10^{-6}\ s^{-1} $$
$$ \lambda_\alpha = 6 \times 10^{-5}\lambda_t = 4 \times 10^{-10}\ s^{-1} $$

$$ t_{1/2,\beta} = \frac{0.693}{\lambda_\beta} = 1.3 \times 10^{5}\ s = 35\ h $$
$$ t_{1/2,\varepsilon} = \frac{0.693}{\lambda_\varepsilon} = 6.1 \times 10^{5}\ s = 170\ h $$
$$ t_{1/2,\alpha} = \frac{0.693}{\lambda_\alpha} = 1.7 \times 10^{9}\ s = 55\ y $$



Relation between the width Γ of a state and its lifetime τ

Stationary state

It does not decay (i.e. it leaves forever)
Energy of the stationary state is exact/precise.
So the uncertainty is zero.

$$\Delta E = \sqrt{<E^2> - <E>^2} = 0$$

$$\Delta E \Delta t \geq \frac{\hbar}{2} \qquad for\ \Delta E = 0\ \Delta t = \infty$$

A state with exact energy lives forever!

Relation between the width Γ of a state and its lifetime τ

Stationary state

It does not decay (i.e. it leaves forever)
Energy of the stationary state is exact/precise.
So the uncertainty is zero.

$$\Delta E = \sqrt{<E^2> - <E>^2} = 0$$

$$\Delta E \Delta t \geq \frac{\hbar}{2} \qquad for\ \Delta E = 0\ \Delta t = \infty$$

A state with exact energy lives forever!

Non-stationary state

It eventually decays
Energy of the non-stationary state is not precisely known.
So the uncertainty is not zero.

$$\Delta E = \sqrt{<E^2> - <E>^2} \neq 0$$

$$\Delta E \Delta t \geq \frac{\hbar}{2} \qquad for \Delta E \neq 0 \Delta t = \tau \quad \text{Mean lifetime}$$

$$\Delta E = \Gamma \quad \text{Width of the state}$$

$$\Gamma \tau \geq \frac{\hbar}{2}$$

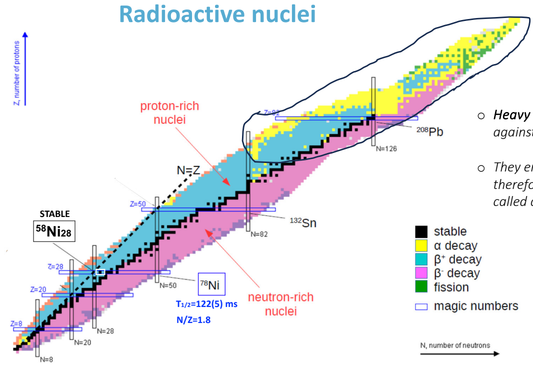
$$\tau \simeq \frac{\hbar}{\Gamma} \qquad \tau \simeq \frac{1}{\lambda}$$

λ: Decay rate = Decay probability

Lecture 10: Alpha decay

Martin & Shaw Ch 8.6
Krane Kompendium p.246 - 269

Radioactive nuclei



- ***Heavy nuclei*** *are unstable against alpha decay.*
- *They emit alpha decay and therefore they are usually called as* ***alpha emitters.***

Why heavy nuclei?

Fill in the blanks using SEMF ☺

------------- force $\propto$ A while ------------- force $\propto Z^2$

Why heavy nuclei?

- *Heavy nuclei experience strong Coulomb repulsion between protons and this causes heavy nuclei to decay by emitting particles to get rid of their Z and mass (A).*
- *Alpha particle seems to be the favorite one apparently. We then ask the second question: Why alpha particle?*

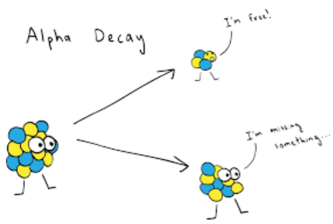
Answer:

Nuclear ------------- force $\propto$ A while Coulomb ------------- force $\propto Z^2$

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3} - a_{asym} \frac{(A-2Z)^2}{A} + \delta$$

$$\delta = \begin{cases} +a_p A^{-3/4} & evenN \quad evenZ \\ 0 & oddA \\ -a_p A^{-3/4} & oddN \quad oddZ \end{cases}$$

Why alpha particle?



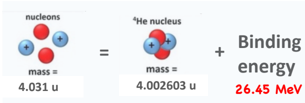
alpha particle: ^{4}He (Z=2, N=2)

1u= 931 (MeV/c^2)

m_α= 4.002603 u
$2m_p+2m_n$= 4.031 u

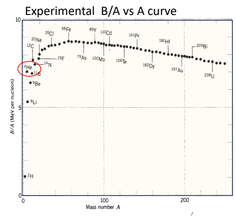
$\Delta m = (Zm_p + Nm_n) - M_{nucleus} > 0$
BE = Δmc^2 (MeV)

BE(^{4}He) = (4.031-4.002603) 931 (MeV/c^2)c^2
= **26.45 MeV**
BE/A = **6.6125 MeV**



- *^{4}He is tightly bound!*
- *So is ^{6}Li. Yet ^{4}He is chosen. Let's continue…*

Experimental B/A vs A curve



Q values

Table 8.1 Energy Release (Q value) for Various Modes of Decay of ^{232}U[a]

Emitted Particle	Energy Release (MeV)	Emitted Particle	Energy Release (MeV)
n	−7.26	^{4}He	+5.41
^{1}H	−6.12	^{5}He	−2.59
^{2}H	−10.70	^{6}He	−6.19
^{3}H	−10.24	^{6}Li	−3.79
^{3}He	−9.92	^{7}Li	−1.94

[a] Computed from known masses.

$Q = (M_{initial} - M_{final})c^2 > 0$ *spontaneous decay*

Let's calculate decay of ^{232}U for two cases:
1. emission of neutron
2. emission of alpha

- *m(n)= 1.0086649159 u*
- *m(^{4}He) = 4.00260325413 u*
- *m(^{232}U) = 232.0371548 u*
- *m(^{231}U) = 231.0362922 u*
- *m(^{228}U) = 228.0287397 u*
- *c^2=931.5 MeV/u*

^{232}U → ^{231}U + ^{1}n

$Q_n = \{M(^{232}U) - [M(^{231}U) + m(n)]\}c^2$

Q_n =- 7.268 MeV

^{232}U → ^{228}Th + **^{4}He**

$Q_\alpha = \{M(^{232}U) - [M(^{228}Th) + m(^4_2He)]\}c^2$

Q_α =+5.41 MeV

So far:

- ✓*Alpha emission occurs as a result of strong Coulomb repulsion in heavy nuclei (heavier than ^{208}Pb).*
- ✓*These nuclei are unstable against alpha decay and called alpha emitters.*
- ✓*Alpha particles are ^{4}He nuclei and ^{4}He is a very tightly bound nucleus compared to other light mass nuclei. We will use this for "theory of alpha emission" later.*

Energetics of alpha decay

$$^A_ZX_N \rightarrow {}^{A-4}_{Z-2}X'_{N-2} + \alpha$$

Energy conservation:

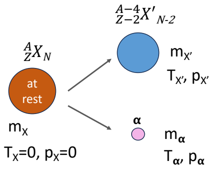
$$m_Xc^2 = m_{X'}c^2 + T_{X'} + m_\alpha c^2 + T_\alpha$$

$$\underbrace{(m_X - m_{X'} - m_\alpha)c^2}_{\text{Q value}} = T_{X'} + T_\alpha$$

$$\boxed{Q = T_{X'} + T_\alpha} \qquad \text{(I)}$$

Momentum conservation:

$$\boxed{p_\alpha = p_{X'}} \qquad \text{(II)}$$



*Alpha decays typically release about **5 MeV** of energy. Thus, for both X' and **α**, kinetic energy is much lower than their rest mass energy **(T<<mc^2)**. So, we may safely use nonrelativistic kinematics.*

Recalling T = p^2/2m and using Equations (I) and (II) gives the kinetic energy of the alpha particle in terms of the Q value:

$$T_\alpha = \frac{Q}{(1 + m_\alpha/m_{X'})}$$

since $m_\alpha << m_{X'}$, we can use Taylor expansion and ignore the terms after the first two terms:

$$\frac{1}{1+\frac{m_\alpha}{m_{X'}}} \Longrightarrow \frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots \text{ (Taylor expansion)}$$

$$T_\alpha = Q\left(1 - \frac{m_\alpha}{m_{X'}}\right)$$

$$T_\alpha = Q(1 - 4/A) \qquad \text{Where } m_\alpha=4 \text{ and } < m_{X'}=A \qquad \text{(III)}$$

$$T_\alpha = Q(1 - 4/A) \quad \text{Eq. (III)}$$

For a typical Q=5 MeV decay energy and A=234 (i.e. alpha decay of ^{238}U to ^{234}Th) we can calculate the kinetic energy of the alpha particle using Eq. (III) as T_α=0.98Q

- *This tells us that in alpha decay, the most of the Q decay energy (energy released in the decay) goes to the alpha particle while only 2% of the Q value goes to the recoiling daughter nucleus.*
- *So when we say Q energy of any given alpha decay process, we also mean the kinetic energy of the alpha particle (Q≈T_α).*
- *This is a measurable quantity since we can detect the emitted alpha.*

alpha decay systematics: (log $t_{1/2}$ vs Q)

Geiger and Nutall gathered the results of many alpha decaying species in 1928.

They study the plot of (log $t_{1/2}$ vs Q) decay energy systematically for many cases given in the Figure 8.1 here.

Their consclusion: The alpha emitters with large decay energies (Q) have shorter half-lives ($t_{1/2}$)

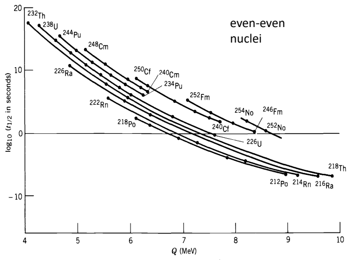


Figure 8.1 The inverse relationship between α-decay half-life and decay energy, called the Geiger-Nuttall rule. Only even-Z, even-N nuclei are shown. The solid lines connect the data points.

alpha decay systematics: (log $t_{1/2}$ vs Q)

Geiger and Nutall gathered the results of many alpha decaying species in 1911.

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Their consclusion: The alpha emitters with large decay energies (Q) have shorter half-lives ($t_{1/2}$)

The variation is so sharp! Compare ^{232}Th and ^{218}Th to see this striking effect of Q energy on the half life:

^{232}Th(**1.4** x10^{10} y; Q = 4.08 MeV)
^{218}Th(1.0 x 10^{-7}s; Q = 9.85 MeV)
A factor of 2 in energy means a factor of 10^{24} in half-life! So the Q energy is very important for decay probability (**λ**).

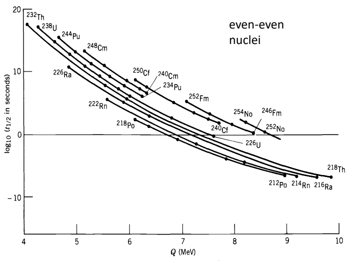


Figure 8.1 The inverse relationship between α-decay half-life and decay energy, called the Geiger-Nuttall rule. Only even-Z, even-N nuclei are shown. The solid lines connect the data points.

Geiger-Nutall formula:

$$\boxed{\log_{10} T_{1/2} = \frac{A(Z)}{\sqrt{E}} + B(Z)} \quad (E \rightarrow Q)$$

where $T_{1/2}$ is the half-life, E the total kinetic energy (of the alpha particle and the daughter nucleus), and A and B are coefficients that depend on the isotope's atomic number Z. The law works best for nuclei with even atomic number and even atomic mass. The trend is still there for even-odd, odd-even, and odd-odd nuclei but is not as pronounced.

source: wikipedia

Nevertheless, for these even-odd, odd-even, and odd-odd alpha emitters, periods are 2-1000 times longer than those for even-even types with the same Z and Q.

Interesting case occurs for ^{235}U (even Z, odd N):

If the half-life ^{235}U were 1000 times shorter, this important nucleus would not occur in nature, and we probably would not have nuclear reactors today!

Q vs A systematics

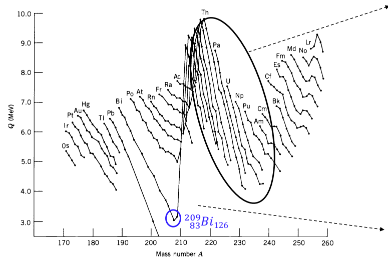


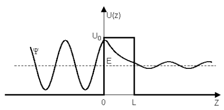
Figure 8.2 Energy released in α decay for various isotopic sequences of heavy nuclei. In contrast to Figure 8.1, both odd-A and even-A isotopes are shown, and a small amount of odd-even staggering can be seen. The effects of the shell closures at N = 126 (large dip in data) and Z = 82 (larger than average spacing between Po, Bi, and Pb sequences) are apparent.

Adding neutron makes the nucleus more stable so the alpha decay energy reduces and according to the Geiger-Nutall rule, the half-lives increases.

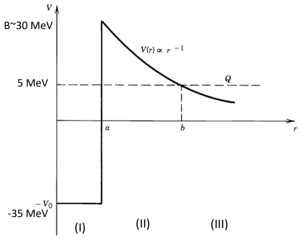
Shell effects are seen near the isotopes around Z=82 and N=126. It is more difficult for alpha particles to be emitted due to large shall gaps near magic numbers.

Theory of alpha emission:

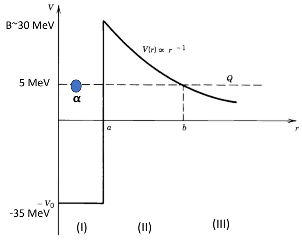
- *The results of the systematics and the Geiger-Nutall rule inspired Gamow to propose the theory of alpha decay.*
- ***He borrowed the quantum tunnelling theory.***
- *Alpha particle exists before the decay.*
- *Alpha particle is treated in non-relativistic manner.*
- *One-body theory.*



Probability=e^{-2k_2L} where $k_2 = \sqrt{\frac{2m(V-E)}{\hbar}}$

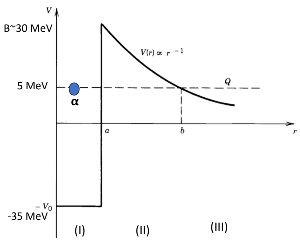


- Total potential energy of an alpha and an heavy daughter nucleus system. Notice the large Coulomb Barrier.
- Potential well is square well for simplicity.
- a is the sum of the radius of the alpha particle and the daughter nuclei
- b is the radial distance where the alpha decay energy, Q starts becoming greater than the Coulomb barrier.
- Three regions appear in the dyagram:



Classic mechanically,
(I) **α** particle can move inside the potential well (with a kinetic energy of Q+V_0) but cannot escape the potential well.
(II) The region of potential well. **α** particle cannot enter this region since the V(r) is gretaer than the alpha kinetic energy (~Q)
(III) This is the only region where **α** particle is classically permitted to be outside of the barrier since Q is larger then V(r).

Quantum mechanically,
- **α** particle will try to penetrate the barrier many times and eventually will escape the nucleus after many trials!
- Here, the barrier delays its emission! That's why alpha emitters have long halflives.
- The decay probablity, **λ**, depends on the frequency (number of trials) and the barrier penetration probability for alpha waves (**λ**=fP)
- Lets find out this frequency:



Consider the case of $^{235}U \rightarrow {}^{231}Th + {}^{4}He$ (Th-> Z=90, U-> Z=92)

Calculate barrier height: B

$$B = \frac{e^2}{4\pi\varepsilon_0}\frac{(Z_1)(Z_2)}{R_1+R_2} = 1.44\ MeV.fm\frac{(2)(90)}{1.2\,(4^{1/3}+231^{1/3})} \approxeq 28 \text{ MeV}$$

Q decay energy for alpha decaying from ^{235}U
Q= 4.6 MeV

Kinetic energy of alpha inside the potential well:
E_α= 5 + 35=40 MeV → v≅ 4.5 10^7 m/sec

Time,t, that the alpha particle takes to reach the barrier: t=R/v
R=a=$r_0\ 235^{1/3}$≅ 7.5 fm
$t = \frac{7.5\ 10^{-15} m}{4.5\ 10^7\ m/sec}$=1.67 10^{-21} sec → $f = \frac{1}{t}$= 6 10^{21} sec^{-1} (number of trials per second)

*since the half life of ^{235}U is $t_{1/2}(^{235}U)$~ 10^9 years, this means the total number of trials of the alpha particle is around 10^{41} times. So, even if the P, probability for penetrating the barrier is low (**λ**=fP) , the high frequency will make it possible for alpha to decay!!*

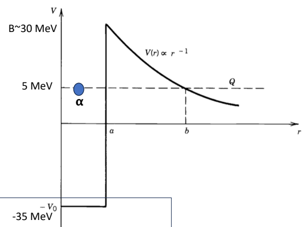


Table 8.2 Calculated α-Decay Half-lives for Th Isotopes

		$t_{1/2}$ (s)	
A	Q (MeV)	Measured	Calculated
220	8.95	10^{-5}	3.3×10^{-7}
222	8.13	2.8×10^{-3}	6.3×10^{-5}
224	7.31	1.04	3.3×10^{-2}
226	6.45	1854	6.0×10^{1}
228	5.52	6.0×10^{7}	2.4×10^{6}
230	4.77	2.5×10^{12}	1.0×10^{11}
232	4.08	4.4×10^{17}	2.6×10^{16}

Structural effects need to be considered. The most important one is the deformation.

Alpha energy spectrum

Two-body—> monoenergetic alpha peaks
98% of Q values is transferred to alpha

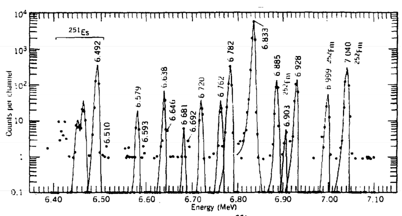


Figure 8.11 α spectrum from the decay of ^{251}Fm. The top portion shows the

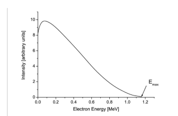
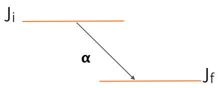
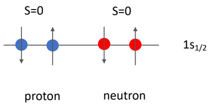


Figure 1: Electron spectra from β-decay of ^{210}Bi.

Spin and parity selection rules in alpha decay

The total angular momentum carried by an α particle in a decay process is purely **orbital** in character→ ℓ_α





SPIN: $|J_i - J_f| \le L \le J_i + J_f$

PARITY: $\pi = (-1)^{l\alpha}$

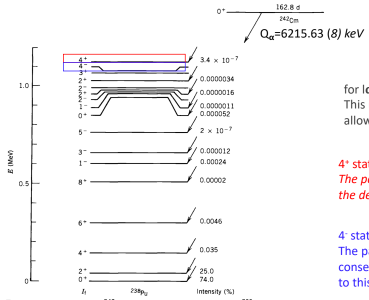


Figure 8.7 α decay of ^{242}Cm to different excited states of ^{238}Pu. The intensity of each α-decay branch is given to the right of the level.

Find out whether alpha decay is allowed for 4^+ and 4^- states in the level scheme?
If the parity is not conserved, the decay is not allowed.

for $l\alpha$=4: $\pi=(-1)^{l\alpha}=(-1)^4=+$
This means that parity of the final state should be **positive** for allowed alpha decay.

4^+ state:
The parity of this state is positive (+) so it is conserved and the decay is allowed. Check the arrow!

4^- state:
The parity of this state is negative (-) so Parity is not conserved. The decay is not allowed. Do you see any arrow to this state?

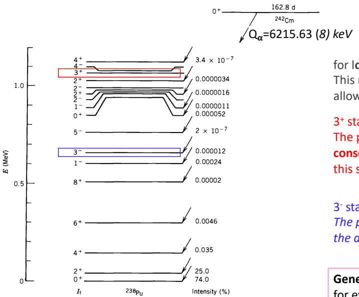


Figure 8.7 α decay of ^{242}Cm to different excited states of ^{238}Pu. The intensity of each α-decay branch is given to the right of the level.

Find out whether alpha decay is allowed for 3^+ and 3^- states in the level scheme?

for $l\alpha$=3: $\pi=(-1)^{l\alpha}=(-1)^3=-$
This means that parity of the final state should be **negative** for allowed alpha decay.

3^+ state:
The parity of this state is positive (+) so Parity is **not conserved**. The decay is not allowed. Do you see any arow to this state?

3^- state:
The parity of this state is negative (-) so it is ***conserved*** *and the decay is allowed. Check the arrow!*

Generalised alpha decay selection rule ($\pi=(-1)^{l\alpha}$)
for even l_α→ (+) parity states are allowed ($0^+,2^+,4^+$…)
for odd l_α→(-) parity states are allowed ($1^-,3^-,5^-$…)

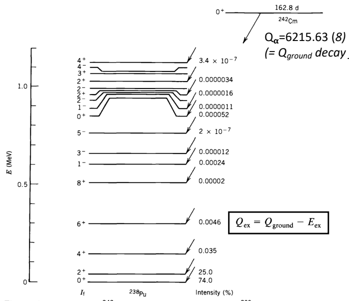


Figure 8.7 α decay of ^{242}Cm to different excited states of ^{238}Pu. The intensity of each α-decay branch is given to the right of the level.

Intensity (decay probability) of each final state depends on:

- Q_{ex} value (lower Q_{ex}, less intensity)
- J total spin (lower spin, higher intensity)

Because of the centrifugal potential.
It increases the potential energy, thus the barrier height and thus the thickness of the barrier in the (II) region.

Reminder from Lecture 8: Shell model

For the potential component of the radial Schrödinger equation:
in addition to the U(**r**) nuclear potential we have to add also the centrifugal force due to the angular momentum.

$$U = U(\mathbf{r}) + U_{centrifugal}$$

The radial equation becomes:

$$\boxed{\frac{\hbar^2}{2m}\frac{d^2R_{nl}(r)}{dr^2} + \left[E_{nl} - U(r) - \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]R_{nl}(r) = 0}$$

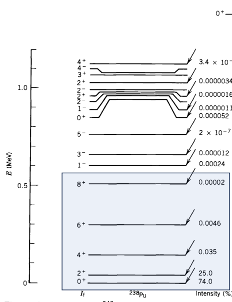


Figure 8.7 α decay of ^{242}Cm to different excited states of ^{238}Pu. The intensity of each α-decay branch is given to the right of the level.

Q_{ex} =5700 keV

8^+ intensity is least compared to the $0^+,2^+,4^+,6^+$:
Because J is the highest and Q_{ex} is the lowest.

Q_{ex} =6175 keV
Q_{ground} =6215.63 keV

2^+ intensity is less than the intensity of the 0^+ ground state: Because J is higher and Q_{ex} is lower.

Lecture 11: Beta decay

Martin & Shaw Ch 8.7

Krane Kompendium p.272 - 292

recap from last lecture: Alpha decay

- Heavy nuclei undergo alpha decay because of the repulsive Coulomb interaction. heavy guys want to get rid of their protons to be stable.
- Alpha particle is preferred instead of protons or other particles. Because: (help me)

Q values

Table 8.1 Energy Release (Q value) for Various Modes of Decay of ^{232}U[a]

Emitted Particle	Energy Release (MeV)	Emitted Particle	Energy Release (MeV)
n	−7.26	^{4}He	+5.41
^{1}H	−6.12	^{5}He	−2.59
^{2}H	−10.70	^{6}He	−6.19
^{3}H	−10.24	^{6}Li	−3.79
^{3}He	−9.92	^{7}Li	−1.94

[a] Computed from known masses.

$Q = (M_{initial} - M_{final})c^2 > 0$ *spontaneous decay*

Let's calculate decay of ^{232}U for two cases:
1. emission of neutron
2. emission of alpha

- *m(n)= 1.0086649159 u*
- *m(^{4}He) = 4.00260325413 u*
- *m(^{232}U) = 232.0371548 u*
- *m(^{231}U) = 231.0362922 u*
- *m(^{228}U) = 228.0287397 u*
- *c²=931.5 MeV/u*

$^{232}U \rightarrow {}^{231}U + {}^1n$

$Q_n = \{M(^{232}U) - [M(^{231}U) + m(n)]\}c^2$

Q_n =- 7.268 MeV

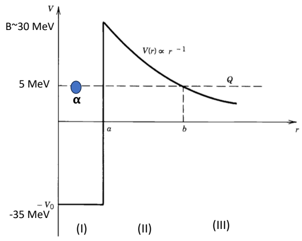
$^{232}U \rightarrow {}^{228}Th + {}^4He$

$Q_\alpha = \{M(^{232}U) - [M(^{228}Th) + m(^4_2He)]\}c^2$

Q_α =+5.41 MeV

Classic mechanically,
- (I) α particle can move inside the potential well (with a kinetic energy of Q+V$_0$) but cannot escape the potential well.
- (II) The region of potential well. α particle cannot enter this region since the V(r) is gretaer than the alpha kinetic energy (~Q)
- (III) This is the only region where α particle is classically permitted to be outside of the barrier since Q is larger then V(r).

theory of alpha decay



Quantum mechanically,
- α particle will try to penetrate the barrier many times and eventually will escape the nucleus after many trials!
- Here, the barrier delays its emission! That's why alpha emitters have long halflives.
- The decay probability, λ, depends on the frequency (number of trials) and the barrier penetration probability for alpha waves (λ=fP)
- Lets find out this frequency:

$$t_{1/2} = 0.693\frac{a}{c}\sqrt{\frac{mc^2}{2(V_0+Q)}}\exp\left\{2\sqrt{\frac{2mc^2}{(\hbar c)^2 Q}}\frac{zZ'e^2}{4\pi\epsilon_0}\left(\frac{\pi}{2} - 2\sqrt{\frac{Q}{B}}\right)\right\} \quad (8.18)$$

$$B = \frac{e^2}{4\pi\epsilon_0}\frac{(Z_1)(Z_2)}{R_1+R_2}$$

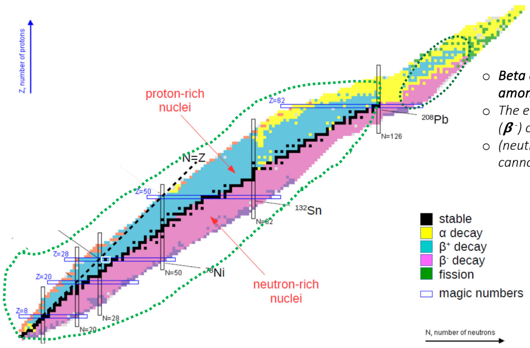
Compare how likely is the decay of ^{220}Th via ^{12}C emission compared to the decay of the same nucleus via an alpha emission:
(a) half life comparison Formula is given above but No need to calculate. (it is given in Krane p.254)
Explain the reason.
(b) (relative) barrier height comparison. Explain the reason

$^{220}Th \rightarrow {}^{12}C + {}^{208}Po$ (Q=32.21 MeV)

$^{220}Th \rightarrow {}^4He + {}^{216}Ra$ Q=8.95 MeV

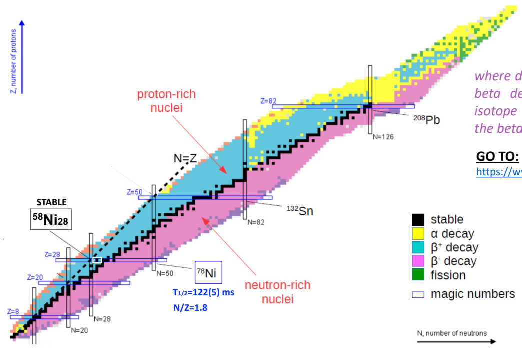
^{220}Th (Z=90)
^{208}Po (Z=84)
^{208}Ra (Z=88)

Radioactive nuclei



- ***Beta decay is very popular among nuclei***
- *The emitted particle is electron (β^-) or positron (β^+).*
- *(neutrinos are emitted to but we cannot detect them.)*

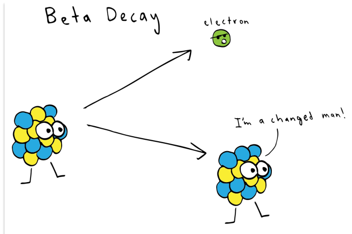
Radioactive nuclei



where does ^{78}Ni end up after beta decays? which stable isotope will be at the end of the beta-decay chain?

GO TO:
https://www.nndc.bnl.gov/nudat3/

What happens when a nucleus undergoes beta decay?



β⁻ decay

n → p + e⁻ + $\bar{\nu}_e$ + Energy (Q_β)

1. neutron turns into proton
2. electron and electron anti neutrino are emitted.

1. down quark turns into up quark
2. electron and electron anti neutrino are emitted.

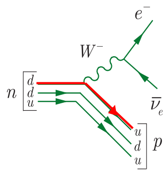


Figure 3.11 Quark Feynman diagram for the decay $n \to pe^-\bar{\nu}_e$ in the spectator model.

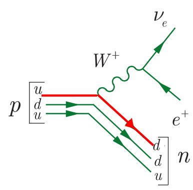
time

β⁺ decay

p → n + e⁺ + ν_e + Energy (Q_β)

1. proton turns into neutron
2. positron and electron neutrino are emitted.

1. up quark turns into down quark
2. positron and electron neutrino are emitted.



time

In alpha decay the alpha particle was initially ready inside the nucleus before the decay. How is it in beta decay?

- *Both electron (positron) and anti-neutrino (neutrino) do not exist before the decay and created at the instant of the decay from the available decay energy (Q_β).*

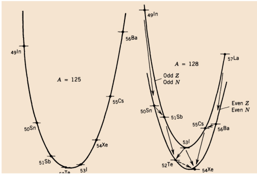
Beta decay does not change A but N and Z!

$${}^{A}_{Z}X_N \rightarrow {}_{Z+1}{}^{A}X'_{N-1} + e^- + \bar{\nu}$$

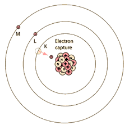
$${}^{A}_{Z}X_N \rightarrow {}_{Z-1}{}^{A}X'_{N+1} + e^+ + \nu$$

Z → Z ± 1 & N → N ∓ 1 ☞ **A =constant**

Beta decay provides a convenient way for an unstable nucleus to "Slide down" the mass parabola!



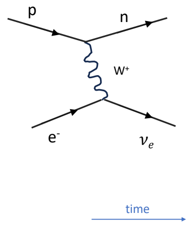
electron capture, ε



p + e⁻ → n + ν_e + Energy (Q_ε)

1. proton captures an orbital electron and turns into neutron
2. electron neutrino is emitted.

1. up quark turns into down quark
2. electron neutrino is emitted.



Electron capture does not occur in the absence of electrons. Fully-ionised atoms cannot decay via electron capture!😎

A free proton does not decay but a bound proton inside a nucleus decays. Why?

- *Proton is stable (or at least much longer than the life of earth)*
- *Proton is the lightest (and therefore least energetic) baryon and will not decay into another baryon of three quark, like neutron, on their own.*
- *Grand Unified Theories propose that It could decay to a neutral meson + positron* $p \rightarrow \pi^0 e^+$
  - *B-L is conserved*
  - *Large energy will be released.*
  - *And such decay results in 10^{34} years of half-life.*

Table 3.3 Some examples of baryons and mesons, with their major decay modes. Masses are in MeV/c^2

Particle	Mass	Lifetime (s)	Major decays
$\pi^+(u\bar{d})$	139.6	2.60×10^{-8}	$\mu^+\nu_\mu$ (~100%)
$\pi^0(u\bar{u}, d\bar{d})$	135.0	8.52×10^{-17}	$\gamma\gamma$ (~100%)
$K^+(u\bar{s})$	493.7	1.24×10^{-8}	$\mu^+\nu_\mu$ (64%) $\pi^+\pi^0$ (21%)
$D^-(d\bar{c})$	1869.7	1.04×10^{-12}	Several seen
$B^-(b\bar{u})$	5279.3	1.64×10^{-12}	Several seen
$p(uud)$	938.3	Stable	None
$n(udd)$	939.6	880.2	$pe^-\bar{\nu}_e$ (100%)
$\Lambda(uds)$	1115.7	2.63×10^{-10}	$p\pi^-$ (64%) $n\pi^0$ (36%)
$\Xi^0(uss)$	1314.9	2.90×10^{-10}	$\Lambda\pi^0$ (99%)
$\Delta^{++}(uuu)$	~1232	$\sim 0.6 \times 10^{-23}$	$p\pi^+$ (100%)
$\Omega^-(sss)$	1672.5	0.82×10^{-10}	ΛK^- (68%) $\Xi^0\pi^-$ (24%)
$\Lambda_c{}^+(udc)$	2286.5	2.00×10^{-13}	Several seen

A free proton does not decay but a bound proton inside a nucleus decays. Why?

- *Inside a nucleus, proton will be converted into a neutron through* $\boldsymbol{\beta}^+$ *decay and/or electron capture, because of the binding energy.*
- *I.e., whether the mass of the entire system is decreased by the decay*
- *whether the binding energy of the entire system is increased by the decay.*
- *We will see the energetics for all three processes soon.*

β^+ decay

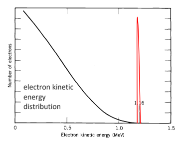
$$M(Z, A) > M(Z-1, A) + 2m_e,$$

electron capture, ε

$$M(Z, A) > M(Z-1, A) + \varepsilon,$$

energetics of beta decay

Curious case of β^- decay of ^{210}Bi in 1920s.



2-body decay **^{210}Bi → ^{210}Po + e⁻**

T_e=Q= 1.16111 MeV

We could observe one single, sharp peak.

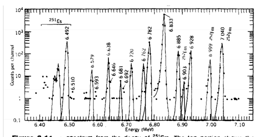
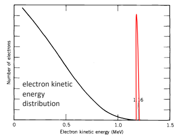
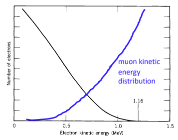


Figure 8.11 α spectrum from the decay of ^{251}Fm. The top portion shows the

Curious case of β^- decay of ^{210}Bi in 1920s.





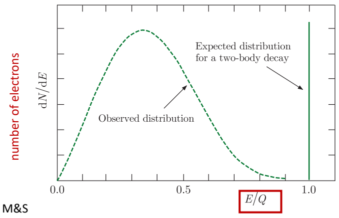
2-body decay $^{210}\mathrm{Bi} \rightarrow {}^{210}\mathrm{Po} + e^-$

$T_e \approx Q$= 1.16111 MeV

We could observe one single, sharp peak.

3-body decay $^{210}\mathrm{Bi} \rightarrow {}^{210}\mathrm{Po} + e^- + \bar{\nu}_e$

Kinetic energy is shared between electron and neutrino. This results in a continuous energy distribution for both particles.

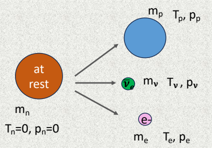


M&S

Figure 1.1 The observed electron energy distribution dN/dE in β decay (dashed line) as a function of E/Q, where E is the kinetic energy of the electron and Q is the total energy released. Also shown is the expected energy distribution if β decay were a two-body process (solid line).

Pauli in 1931:
- "neutrino" particle is the third component in the decay.
- it is very light and a highly penetrating radiation.
- it interacts with matter weakly so it was not stopped in the calorimeter (detector).
- neutral and small=small neutron=neutrino (named by Fermi)

energy conservation in beta decay of neutron: the simplest case



$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}.$$

$$m_n c^2 = m_p c^2 + T_p + m_e c^2 + T_e + m_\nu c^2 + T_\nu$$

$$Q = (m_n - m_p - m_e - m_\nu)c^2 \qquad \text{(I)}$$

$$Q = T_p + T_e + T_\nu$$
$$\approx T_e + T_\nu \quad \text{(since } T_p \text{ is small (~0.3 keV))}$$

Q decay energy is shared between electron and anti-neutrino.
(T_e)max= 0.782 ∓ 0.013 MeV (experimental value)

Using Eq. (I) we can calculate the Q energy:

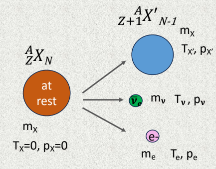
$$\begin{aligned} Q &= m_{\mathrm{n}}c^2 - m_{\mathrm{p}}c^2 - m_{\mathrm{e}}c^2 - m_{\bar{\nu}}c^2 \\ &= 939.573 \text{ MeV} - 938.280 \text{ MeV} - 0.511 \text{ MeV} - m_{\bar{\nu}}c^2 \\ &= 0.782 \text{ MeV} - m_{\bar{\nu}}c^2 \end{aligned}$$

Within the precision of the measured maxium electron energy, we will assume the anti-neutrino as massless and do not include in the calculations from now.

β⁻ decay

$${}^{A}_{Z}\mathrm{X}_N \rightarrow {}_{Z+1}^{\;\;A}\mathrm{X}'_{N-1} + \mathrm{e}^- + \bar{\nu}$$

$$Q_{\beta^-} = \left[m_{\mathrm{N}}({}^{A}_{Z}\mathrm{X}) - m_{\mathrm{N}}({}_{Z+1}^{\;\;A}\mathrm{X}') - m_{\mathrm{e}}\right]c^2$$



Formula to convert from nuclear mass to atomic mass:

$$\underset{\text{atomic}}{m({}^{A}\mathrm{X})c^2} = \underset{\text{nuclear}}{m_{\mathrm{N}}({}^{A}\mathrm{X})c^2} + Zm_{\mathrm{e}}c^2 - \sum_{i=1}^{Z} B_i$$

$$Q_{\beta^-} = \left\{\left[m({}^{A}\mathrm{X}) - Zm_{\mathrm{e}}\right] - \left[m({}^{A}\mathrm{X}') - (Z+1)m_{\mathrm{e}}\right] - m_{\mathrm{e}}\right\}c^2 + \boxed{\left\{\sum_{i=1}^{Z} B_i - \sum_{i=1}^{Z+1} B_i\right\}}$$

Electron binding energies are too small and thus ignored.

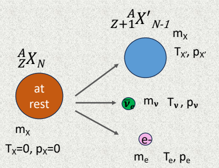
$$\boxed{Q_{\beta^-} = \left[m({}^{A}\mathrm{X}) - m({}^{A}\mathrm{X}')\right]c^2} \qquad \text{(I)}$$

Q decay energy in terms of atomic mass unit (amu or u)

β⁻ decay

$${}^{A}_{Z}\mathrm{X}_N \rightarrow {}_{Z+1}^{\;\;A}\mathrm{X}'_{N-1} + \mathrm{e}^- + \bar{\nu}$$

$$\boxed{Q_{\beta^-} = \left[m({}^{A}\mathrm{X}) - m({}^{A}\mathrm{X}')\right]c^2} \qquad \text{(I)}$$



Energy conservation:

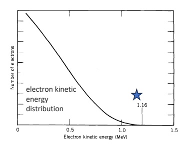
$$\boxed{Q_{\beta^-} = T_e + E_{\bar{\nu}}}$$

$T_{X'}$ is too small, thus ignored ($T_{X'} \approx 0$)

$$\boxed{(T_e)_{\max} = (E_{\bar{\nu}})_{\max} = Q_{\beta^-}} \qquad \text{(II)}$$

Note that both electron and neutrino require relativistic kinematics.
- Since neutrino is massless, its total energy is equal to its kinetic energy $E_\nu = T_\nu$
- For electron, $E_e = T_e + m_e c^2$
- Nuclear recoil (^{A}X') is of very low energy thus can be treated non-relatistically.

Let's go back to the β^- decay of ^{210}Bi in 1920s.



$^{210}\mathrm{Bi} \rightarrow {}^{210}\mathrm{Po} + e^- + \bar{\nu}_e$

$$\begin{aligned} Q_{\beta^-} &= \left[m({}^{210}\mathrm{Bi}) - m({}^{210}\mathrm{Po})\right]c^2 \\ &= (209.984095 \text{ u} - 209.982848 \text{ u})(931.502 \text{ MeV/u}) \\ &= 1.161 \text{ MeV} \end{aligned}$$

$$(T_e)_{\max} = (E_{\bar{\nu}})_{\max} = Q_{\beta^-}$$

$$(T_e)_{\max} = 1.16 \text{ MeV}$$

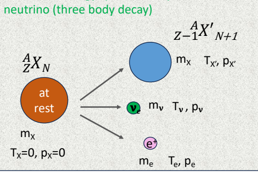
β^- decay

$^A_ZX_N \rightarrow {}^{A}_{Z-1}X'_{N+1} + e^+ + \nu$

$Q_{\beta^+} = \left[m(^AX) - m(^AX') - 2m_e\right]c^2$

Energy conservation: $Q_{\beta^+} = T_e + E_\nu$ ($T_{X'} \approx 0$)

Continous energy spectrum of positron and neutrino (three body decay)



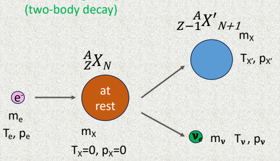
ε capture

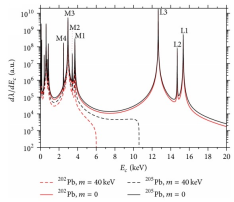
$^A_ZX_N + e^- \rightarrow {}^{A}_{Z-1}X'_{N+1} + \nu$

$Q_\varepsilon = \left[m(^AX) - m(^AX')\right]c^2 - B_n$ n=K,L,M-shell electrons

Energy conservation: $Q_\varepsilon \cong T_\nu$ ($T_{X'} \approx 0$)

Monoenergetic spectrum of neutrino 🤔 (two-body decay)





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Warm Dark Matter Sterile Neutrinos in Electron Capture and Beta Decay Spectra

O. Moreno,[1] E. Moya de Guerra,[1] and M. Ramón Medrano[2]

Show more

Calorimeter spectrum after electron capture in ^{202}Pb (light curves) and in ^{205}Pb (black curves) with emission of a light neutrino, $m_l = 0$ (solid curves) and a heavy neutrino, m_h = 40keV (dashed curves). (a) Full spectrum, showing L and M capture peak labels. (b) Low energy region (E_c = 0-1 keV), showing N and O capture peak labels.

$\boxed{Q_{\beta^+} = \left[m(^AX) - m(^AX') - 2m_e\right]c^2}$ $\boxed{Q_\varepsilon = \left[m(^AX) - m(^AX')\right]c^2 - B_n}$

Looking at Q-values, what interpretation below is correct (B_n<< $2m_ec^2$)?

1- Nucleus for which β⁺ decay is energetically possible can also undergo electron capture.

2- The reverse is also true.

$\boxed{Q_{\beta^+} = \left[m(^AX) - m(^AX') - 2m_e\right]c^2}$ $\boxed{Q_\varepsilon = \left[m(^AX) - m(^AX')\right]c^2 - B_n}$

Looking at Q-values, what interpretation below is correct (B_n<< $2m_ec^2$)?

1- Nucleus for which β⁺ decay is energetically possible can also undergo electron capture. →correct

2- The reverse is also true. → wrong

For a nucleus to spontanously decay via β^+, the Q-value should be equal or gretaer than $2m_ec^2$ ($Q_{\beta^+} \geq 2m_ec^2$).

Like in alpha decay, half-lives in beta decay are very sensitive to Q-values too. The larger the Q value the shorther the half-life (and greater the decay constant/probability)!

Table 9.1 Typical β-Decay Processes

Decay	Type	Q (MeV)	$t_{1/2}$
$^{23}Ne \rightarrow {}^{23}Na + e^- + \bar{\nu}$	β^-	4.38	38 s
$^{99}Tc \rightarrow {}^{99}Ru + e^- + \bar{\nu}$	β^-	0.29	2.1×10^5 y
$^{25}Al \rightarrow {}^{25}Mg + e^+ + \nu$	β^+	3.26	7.2 s
$^{124}I \rightarrow {}^{124}Te + e^+ + \nu$	β^+	2.14	4.2 d
$^{15}O + e^- \rightarrow {}^{15}N + \nu$	ε	2.75	1.22 s
$^{41}Ca + e^- \rightarrow {}^{41}K + \nu$	ε	0.43	1.0×10^5 y

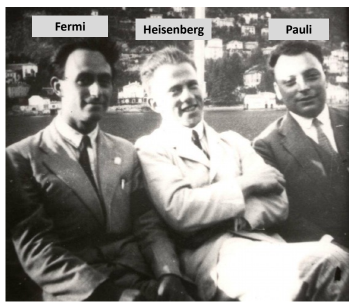
Beta decay

Table 8.2 Calculated α-Decay Half-lives for Th Isotopes

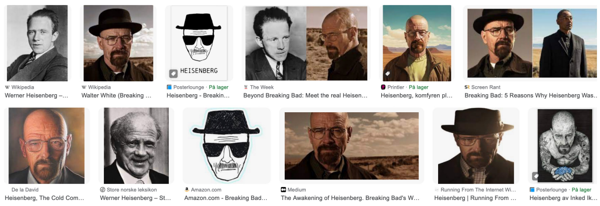
		$t_{1/2}$ (s)	
A	Q (MeV)	Measured	Calculated
220	8.95	10^{-5}	3.3×10^{-7}
222	8.13	2.8×10^{-3}	6.3×10^{-5}
224	7.31	1.04	3.3×10^{-2}
226	6.45	1854	6.0×10^{1}
228	5.52	6.0×10^{7}	2.4×10^{6}
230	4.77	2.5×10^{12}	1.0×10^{11}
232	4.08	4.4×10^{17}	2.6×10^{16}

Alpha decay

Theory of beta decay: Fermi's Golden rule (1934)



“Heisenberg” : on Google images



Fermi’s Golden rule:

$\lambda = \frac{1}{\tau}$

Matrix element

$$\lambda = \frac{2\pi}{\hbar}|V_{fi}|^2\,\rho(Ef)$$

Decay probability

Density of final states

Interaction initial and final states

$$V_{fi} = \int \psi_f^* V \psi_i \, dv$$

Wave function for initial state

Wave function for final state

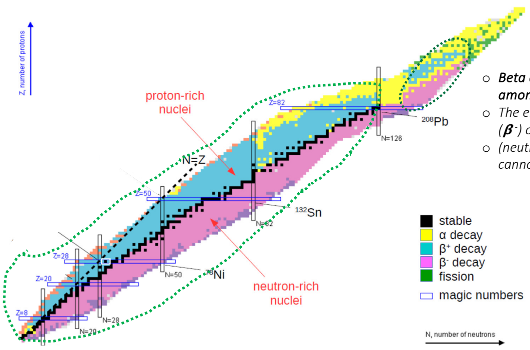
Decay probablity is constant and depends upon on:

- *the density of the final state which decay can proceed. The higher density, the faster the decay (transition).*
- *the matrix element decsribes the strength of the interaction (strong or weak)*
- *it also describes overlap between the wave functions of initial and final states). If the overlap is large, decay (transition) occurs rapidly.*

$$\lambda = \frac{2\pi}{\hbar}|\langle\psi_f|H|\psi_i\rangle|^2\rho(Ef) \quad \text{H=T+V}$$

- *Decay probability (constant) is also called Transition probability (=transition rate)*
- $V_{fi} = M_{fi}$

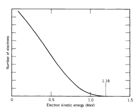
Radioactive nuclei



- *Beta decay is very popular among nuclei*
- *The emitted particle is electron (β^-) or positron (β^+).*
- *(neutrinos are emitted to but we cannot detect them.)*

Fermi’s Golden rule of beta decay

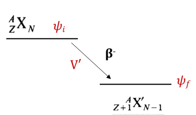
(1)electron and neutrino do not exist before the decay process. The theory must account for their formation.
(2)Electron and neutrino must be treated relativistically
(3)The continuous distribution of electron energies must be reproduced by the theory. (three body)



(1)alpha particle existed before the decay.
(2)Alpha was treated non-relativistically
(3)Alpha energy distributions are monoenergetic. (two-body)

- V’ should be a small perturbation. This is true for beta decay.
- Beta decay is caused by the V’ weak interaction, weak compared to the strong nuclear interaction.
- Characteristics half-lives of beta decay ~ seconds and longer

While Nuclear time scale is much shorter, 10^{-20} s (nuclear reaction=> strong nuclear interaction).



stationary, strong nuclear interaction

no transition
H=T+V

non-stationary, weak interaction

beta transition
H’=T+V+V’

$$V'_{fi} = \int \psi_f^* V' \psi_i \, dv$$

$$\lambda = \frac{2\pi}{\hbar}|V'_{fi}|^2\boldsymbol{\rho}(Ef)$$

Can we calculate **λ** decay rate or **τ** mean lifetime **from the nuclear wave functions**?

Fermi’s Golden rule

$$\lambda = \frac{2\pi}{\hbar}|V'_{fi}|^2\boldsymbol{\rho}(Ef)$$

Where $V'_{fi} = \int \psi_f^* V' \psi_i \, dv$

V_{fi}': Transition matrix element. Its square determines the decay probability, $|V'_{fi}|^2$

V’: Transition operator

ρ(E_f): Density of final states. If the final state E_f is a single isolated state, then the decay probability will be much smaller than it would be in the case that there are many states in a narrow energy band near E_f. If there is a large density of states near Ef, there are more possible final states that can be reached by the transition and thus a larger transition probability.

In order to calculate Decay constant (or mean lifetime) we need to have knowledge on wave functions of initial and final states as well as the V’ interaction which causes the transition between the states.
V’ is like a weak perturbing potential in addition to the original potential V which gives the stationary states.

(i) We first neglect V’ and solve the Schrodinger equation for V potential (H=T+V). The stationary nuclear wave functions will be obtained from that.
(ii) We then use these wave functions to calculate the transition probability between the “stationary” states under the influence of V’. This transition probability is just the decay constant which is given by the Fermi’s Golden rule.

wave function of electron

wave function of initial nuclear state

$$V_{\mathrm{fi}} = \boxed{g} \int [\psi_f^* \varphi_e^* \varphi_\nu^*] \boxed{O_X} \psi_i \, dv$$

wave function of final nuclear state

wave function of neutrino

g: strength constant (determines the strength of the decay)
[…]: entire final system after decay

X determines form of the operator O

V: vector
A: axial vector
S: scalar
P: pseudoscalar
T: tensor

} V-A

(V-A) takes into account for both spin-dependence and parity violation in beta decay:
No spin change (Fermi decay) → V
Spin change (Gamow-Teller decay) → A
Consider (V-A)^2:
V^2 and A^2 conserve parity. But V.A does not.
Parity non-conservation is an interference effect between the two components V and A.

1957 discovery of peity non-conservsation in beta decay.

$$\lambda = \frac{2\pi}{\hbar} \underbrace{|V'_{fi}|^2}_{} \underbrace{\rho(Ef)}_{} \Rightarrow N(p) \propto \underbrace{p^2 (Q - T_e)^2}_{} F(Z', p) \underbrace{|M_{\mathrm{fi}}|^2}_{} S(p, q)$$

Final purpose was to determine momentum and kinetic energy distribution of electrons, N(p)

$$N(p) \propto p^2 (Q - T_e)^2 F(Z', p) |M_{\mathrm{fi}}|^2 S(p, q)$$

p: momentum of electron after decay
q: momentum of neutrino after decay

Statistical factor. It is derived from the number of total final states accesible to the emitted particles. The shape of the distrubution comes from this term!

Fermi function. It accounts for the effect of the nuclear Coulomb field (protons of the daugter nucleus) It is also called Fermi screening factor (in M&S).

The nuclear matrix element. It decsribes the strength of the interaction (strong or weak)

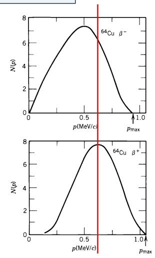
it also describes overlap between the wave functions of initial and final states). If the overlap is large, decay (transition) occurs rapidly.

Shape factor. Additional electron and neutrino momentum dependence term for forbidden transitions.

In the next slides, we will test the Fermi's theory:
- ❑ Shape of the Beta spectrum
- ❑ Kurie plot
- ❑ Total decay rate

Shape of Beta spectrum (either kinetc energy or momentum distributions)

$$N(p) \propto p^2 (Q - T_e)^2$$



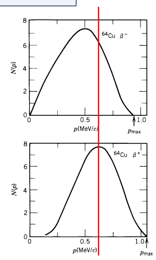
p: momentum of electron after decay
q: momentum of neutrino after decay

$$N(p) = \frac{C}{c^2} p^2 (Q - T_e)^2 = \frac{C}{c^2} p^2 \left[Q - \sqrt{p^2 c^2 + m_e^2 c^4} + m_e c^2\right]^2$$

N(p)=0 for p=0

N(p)=0 for T_e=Q

$$N(p) \propto p^2 (Q - T_e)^2 F(Z', p)$$



p: momentum of electron after decay
q: momentum of neutrino after decay

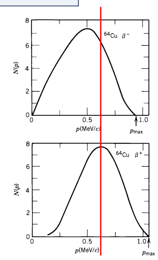
$$N(p) = \frac{C}{c^2} p^2 (Q - T_e)^2 = \frac{C}{c^2} p^2 \left[Q - \sqrt{p^2 c^2 + m_e^2 c^4} + m_e c^2\right]^2$$

N(p)=0 for p=0

N(p)=0 for T_e=Q

positrons are kicked out by the protons of the daugther nucleus so they are faster than beta electrons. Effect is included via F (Z', p) fermi screening factor.

$$N(p) \propto p^2 (Q - T_e)^2 F(Z', p) |M_{\mathrm{fi}}|^2 S(p, q)$$



p: momentum of electron after decay
q: momentum of neutrino after decay

$$N(p) = \frac{C}{c^2} p^2 (Q - T_e)^2 = \frac{C}{c^2} p^2 \left[Q - \sqrt{p^2 c^2 + m_e^2 c^4} + m_e c^2\right]^2$$

N(p)=0 for p=0

N(p)=0 for T_e=Q

positrons are kicked out by the protons of the daugther nucleus so they are faster than beta electrons. Effect is included via F (Z', p) fermi screening factor.

M_{fi} or V_{fi} reflects how allowed or how forbidden is the decay. Forbidden beta decay does not mean" the decay is absolutely forbidden" but "the decay will occur less likely."

$$N(p) \propto p^2(Q - T_e)^2 F(Z', p)|M_{fi}|^2 S(p, q)$$

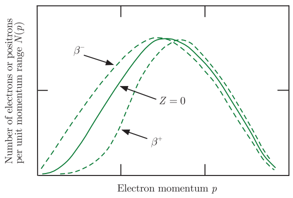


Figure 8.10 Predicted electron spectra: $Z = 0$, without the Fermi screening factor; $\beta^{\pm}$, with the Fermi screening factor.

Fermi-Kurie plot

$$N(p) \propto p^2(Q - T_e)^2 F(Z', p)$$

$$(Q - T_e) \propto \sqrt{\frac{N(p)}{p^2 F(Z', p)}}$$

plot of $\sqrt{\frac{N(p)}{p^2 F(Z', p)}}$ versus T_e is called Fermi-Kurie plot.

- confirms Fermi's theory
- helps determine the decay end point energy, E_{max} and thus Q value.

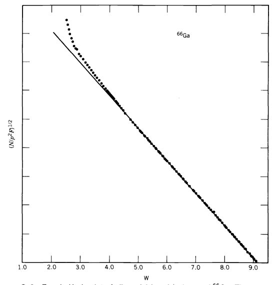


Figure 9.4 Fermi–Kurie plot of allowed $0^+ \rightarrow 0^+$ decay of ^{66}Ga. The horizontal scale is the relativistic *total* energy ($T_e + m_e c^2$) in units of $m_e c^2$. The deviation from the straight line at low energy arises from the scattering of low-energy electrons within the radioactive source. From D. C. Camp and L. M. Langer, *Phys. Rev.* **129**, 1782 (1963).

Fermi-Kurie plot

$$N(p) \propto p^2(Q - T_e)^2 F(Z', p)$$

$$(Q - T_e) \propto \sqrt{\frac{N(p)}{p^2 F(Z', p)}}$$

plot of $\sqrt{\frac{N(p)}{p^2 F(Z', p)}}$ versus T_e is called Fermi-Kurie plot.

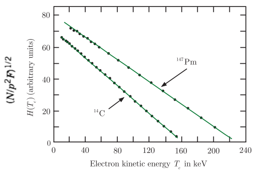


Figure 8.11 Kurie plots for the β decays of ^{14}C and ^{147}Pm. Source: Adapted from Pohm et al. (1955). Copyright (1955) the American Physical Society, reprinted with permission.

- confirms Fermi's theory
- helps determine the decay end point energy, E_{max} and thus Q value.

Fermi-Kurie plot

$$N(p) \propto p^2(Q - T_e)^2 F(Z', p)|M_{fi}|^2 S(p, q)$$

- *Forbidden decays do not follow a straight line.*
- *They need correction → S(p,q) shape factor*
- *(S=p^2+q^2) for first forbidden decays.*

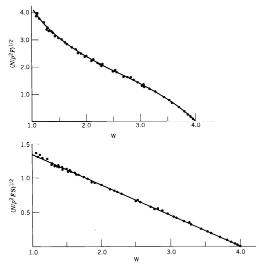


Figure 9.5 Uncorrected Fermi–Kurie plot in the β decay of ^{91}Y (top). The linearity is restored if the shape factor *S(p, q)* is included; for this type of first-forbidden decay, the shape factor $p^2 + q^2$ gives a linear plot (bottom). Data from L. M. Langer and H. C. Price, *Phys. Rev.* **75**, 1109 (1949).

Decay rate: λ and comparable half life $ft_{1/2}$

$$\lambda = \frac{2\pi}{\hbar}|V'_{fi}|^2 \rho(Ef)$$

$$V_{fi} = g\int[\psi_f^* \varphi_e^* \varphi_\nu^*]O_X \psi_i \, dv = g\,|M_{fi}|$$

$$d\lambda = \frac{2\pi}{\hbar} g^2 |M_{fi}|^2 (4\pi)^2 \frac{p^2\, dp\, q^2}{h^6} \frac{dq}{dE_f}$$

partial decay for electrons and neutrinos with momentum p and q, respectively.

$$\lambda = \frac{g^2|M_{fi}|^2}{2\pi^3\hbar^7 c^3}\int_0^{p_{max}} F(Z', p)\, p^2 (Q - T_e)^2\, dp$$

$$\boxed{f(Z', E_0) = \frac{1}{(m_e c)^3 (m_e c^2)^2}\int_0^{p_{max}} F(Z', p)\, p^2 (E_0 - E_e)^2\, dp}$$

fermi-integral.
Dimensioneless. Tabulated for Z' and E_0 values.

$$\lambda = \frac{g^2|M_{fi}|^2}{2\pi^3\hbar^7 c^3} f = \frac{0.693}{t_{1/2}}$$

E_0 : maxiumum electron energy
cp_{max}: maxiumum electron kinetic energy T_e

$$cp_{max} = \sqrt{E_0^2 - m_e^2 c^4})$$

$$\boxed{ft_{1/2} = 0.693 \frac{2\pi^3\hbar^7}{g^2 m_e^5 c^4 |M_{fi}|^2}}$$

$$\boxed{ft_{1/2} = 0.693 \frac{2\pi^3\hbar^7}{g^2 m_e^5 c^4 |M_{fi}|^2}}$$

Comparative half life or ft value
"logft" is more common to use!

Larger M_{fi} smaller logft and thus shorter $t_{1/2}$ → decay is more probable → beta decay is more allowed!

How allowed is the decay???

Most allowed decay is called "superallowed" decay which is a 0^+ → 0^+ decay

Superallowed decays have the smallest logft values thus the shortest halflives ($t_{1/2}$)

M_{fi}=$\sqrt{2}$ $for\ 0+ \rightarrow 0+$ so we expect very similar logft values for all superallowed decays. Is it so?

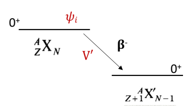


Table 9.2 *ft* Values for $0^+ \rightarrow 0^+$ Superallowed Decays

Decay	*ft* (s)
$^{10}C \rightarrow {}^{10}B$	3100 ± 31
$^{14}O \rightarrow {}^{14}N$	3092 ± 4
$^{18}Ne \rightarrow {}^{18}F$	3084 ± 76
$^{22}Mg \rightarrow {}^{22}Na$	3014 ± 78
$^{26}Al \rightarrow {}^{26}Mg$	3081 ± 4
$^{26}Si \rightarrow {}^{26}Al$	3052 ± 51
$^{30}S \rightarrow {}^{30}P$	3120 ± 82
$^{34}Cl \rightarrow {}^{34}S$	3087 ± 9
$^{34}Ar \rightarrow {}^{34}Cl$	3101 ± 20
$^{38}K \rightarrow {}^{38}Ar$	3102 ± 8
$^{38}Ca \rightarrow {}^{38}K$	3145 ± 138
$^{42}Sc \rightarrow {}^{42}Ca$	3091 ± 7
$^{42}Ti \rightarrow {}^{42}Sc$	3275 ± 1039
$^{46}V \rightarrow {}^{46}Ti$	3082 ± 13
$^{46}Cr \rightarrow {}^{46}V$	2834 ± 657
$^{50}Mn \rightarrow {}^{50}Cr$	3086 ± 8
$^{54}Co \rightarrow {}^{54}Fe$	3091 ± 5
$^{62}Ga \rightarrow {}^{62}Zn$	2549 ± 1280

logft ~3.5

logft values for forbidden decays are much higher. We will see it soon.

what about "g" strength constant of the decay? let's do it for the $0^+ \rightarrow 0^+$ decay

$$\boxed{ft_{1/2} = 0.693\frac{2\pi^3\hbar^7}{g^2 m_e^5 c^4 |M_{fi}|^2}}$$

$M_{fi}=\sqrt{2}$ $for\ 0+ \rightarrow 0+$

$$g = 0.88 \times 10^{-4}\ \mathrm{MeV \cdot fm^3}$$

$$G = \frac{g}{m^{-2}\hbar^3 c^{-1}} = g\frac{m^2 c}{\hbar^3}$$

m: mass of the nucleons

G is strength, dimensionless. This makes the comparison with the other type of interactions easy!

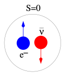
pion-nucleon ("strong")	1
electromagnetic	10^{-2}
β decay ("weak")	10^{-5}
gravitational	10^{-39}

Spin and parity selection rules in beta decay

Allowed decays

electron and neutrino are created at r=0. So, $l=0$. The change of angular momentum is only by s spin (s=1/2 for electron and neutrino)

electron and neutrino spins are anti-parallel: S=0 singlet (L=0)

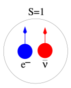


Fermi decay (Superallowed decay)

$$\Delta I = |I_i - I_f| = 0.$$

$$\Delta I = 0 \quad \pi_i \cdot \pi_f = +1\ (\Delta\pi = 0)$$

electron and neutrino spins are parallel: S=1 triplet (L=0)



Gamow-Teller decay

$$\boldsymbol{I_i = I_f + 1}$$

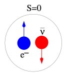
$$\Delta I = 0, \pm 1 \quad I = 0 \not\rightarrow I = 0$$
$$\pi_i \cdot \pi_f = +1\ (\Delta\pi = 0)$$

We therefore have the following *selection rules* for *allowed* β *decay*:

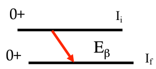
$$\Delta I = 0, 1 \qquad \Delta\pi\ (\text{parity change}) = \text{no}$$

electron and neutrino are created at r=0. So, $l=0$. The change of angular momentum is only by s spin (s=1/2 for electron and neutrino)

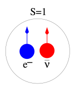
electron and neutrino spins are anti-parallel: S=0 singlet (L=0)



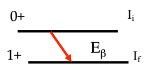
Fermi decay (Superallowed decay)



electron and neutrino spins are parallel: S=1 triplet (L=0)

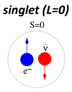


Gamow-Teller decay

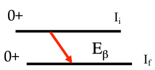


electron and neutrino are created at r=0. So, $l=0$. The change of angular momentum is only by s spin (s=1/2 for electron and neutrino)

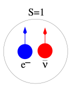
electron and neutrino spins are anti-parallel: S=0 singlet (L=0)



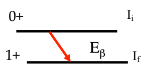
Fermi decay (Superallowed decay)

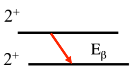


electron and neutrino spins are parallel: S=1 triplet (L=0)



Gamow-Teller decay

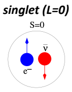




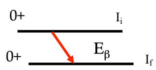
What about this transition? is it a Fermi or Gamow-Teller transition ?

electron and neutrino are created at r=0. So, $l=0$. The change of angular momentum is only by s spin (s=1/2 for electron and neutrino)

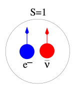
electron and neutrino spins are anti-parallel: S=0 singlet (L=0)



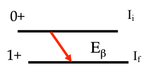
Fermi decay (Superallowed decay)

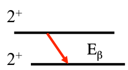


electron and neutrino spins are parallel: S=1 triplet (L=0)



Gamow-Teller decay





Both Fermi and GT transitions are satisfied since $\Delta I=0$ So it is a mixed F + GT transition.

Examples: F or GT transitions?

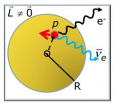
$^{14}O \to {}^{14}N^*$	$0^+ \to 0^+$	
$^{34}Cl \to {}^{34}S$	$0^+ \to 0^+$	
$^{10}C \to {}^{10}B^*$	$0^+ \to 0^+$	
$^{6}He \to {}^{6}Li$	$0^+ \to 1^+$	
$^{13}B \to {}^{13}C$	$(\frac{3}{2}^- \to \frac{1}{2}^-)$	
$^{230}Pa \to {}^{230}Th^*$	$(2^- \to 3^-)$	
$^{111}Sn \to {}^{111}In$	$(\frac{7}{2}^+ \to \frac{9}{2}^+)$	

$n \to p \quad (\frac{1}{2}^+ \to \frac{1}{2}^+)$

$^{14}O \to {}^{14}N^*$	$0^+ \to 0^+$	F
$^{34}Cl \to {}^{34}S$	$0^+ \to 0^+$	F
$^{10}C \to {}^{10}B^*$	$0^+ \to 0^+$	F
$^{6}He \to {}^{6}Li$	$0^+ \to 1^+$	GT
$^{13}B \to {}^{13}C$	$(\frac{3}{2}^- \to \frac{1}{2}^-)$	GT
$^{230}Pa \to {}^{230}Th^*$	$(2^- \to 3^-)$	GT
$^{111}Sn \to {}^{111}In$	$(\frac{7}{2}^+ \to \frac{9}{2}^+)$	GT

$n \to p \quad (\frac{1}{2}^+ \to \frac{1}{2}^+)$ F + GT

Forbidden decays *"Forbidden" → the decay is less probable than an allowed decay*



- Parity of electron and neutrio changes in forbidden decays.
- since Parity=-1^l, L should be odd such as L= 1,3,5. L=3 and L=5 are very unlikely. So L=1
- Consider again S=0 and S=1 cases
- ΔI=L+S coupling
- These decays are called "first-forbidden" decays.

$$\Delta I = 0, 1, 2 \qquad \Delta\pi = \text{yes}$$

$^{17}N \to {}^{17}O \quad (\frac{1}{2}^- \to \frac{5}{2}^+)$

$^{76}Br \to {}^{76}Se \quad (1^- \to 0^+)$

$^{122}Sb \to {}^{122}Sn^* \quad (2^- \to 2^+)$

"second-forbidden" decays

$$\Delta I = 2, 3 \qquad \Delta\pi = \text{no}$$

$^{22}Na \to {}^{22}Ne \quad (3^+ \to 0^+)$

$^{137}Cs \to {}^{137}Ba \quad (\frac{7}{2}^+ \to \frac{3}{2}^+)$

"third-forbidden" decays

$$\Delta I = 3, 4 \qquad \Delta\pi = \text{yes}$$

$^{87}Rb \to {}^{87}Sr \quad (\frac{3}{2}^- \to \frac{9}{2}^+)$

$^{40}K \to {}^{40}Ca \quad (4^- \to 0^+)$

summary of beta selection rules

Type	L	$\Delta\pi$	$\Delta\vec{J}$ $\vec{S}=\vec{0}$ Fermi	$\vec{S}=\vec{1}$ Gam-Tel
super-allowed	0	+	0	0
allowed	0	+	0	0,1
first forbidden	1	-	0,1	0,1,2
second forbidden	2	+	1,2	1,2,3
third forbidden	3	-	2,3	2,3,4

logft values for allowed and forbidden decays

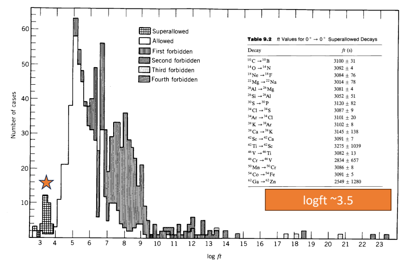
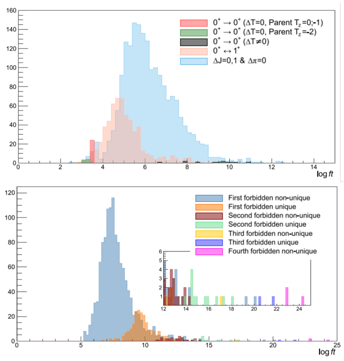


Table 9.2 ft Values for $0^+ \to 0^+$ Superallowed Decays

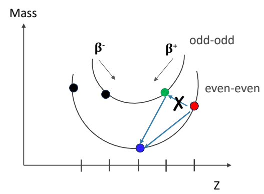
Decay	ft (s)
$^{10}C \to {}^{10}B$	3100 ± 31
$^{14}O \to {}^{14}N$	3092 ± 4
$^{18}Ne \to {}^{18}F$	3084 ± 76
$^{22}Mg \to {}^{22}Na$	3014 ± 78
$^{26}Al \to {}^{26}Mg$	3081 ± 4
$^{26}Si \to {}^{26}Al$	3052 ± 51
$^{30}S \to {}^{30}P$	3120 ± 82
$^{34}Cl \to {}^{34}S$	3087 ± 9
$^{34}Ar \to {}^{34}Cl$	3101 ± 20
$^{38}K \to {}^{38}Ar$	3102 ± 8
$^{38}Ca \to {}^{38}K$	3145 ± 138
$^{42}Sc \to {}^{42}Ca$	3091 ± 7
$^{42}Ti \to {}^{42}Sc$	3275 ± 1039
$^{46}V \to {}^{46}Ti$	3082 ± 13
$^{46}Cr \to {}^{46}V$	2834 ± 657
$^{50}Mn \to {}^{50}Cr$	3086 ± 8
$^{54}Co \to {}^{54}Fe$	3091 ± 5
$^{62}Ga \to {}^{62}Zn$	2549 ± 1280

Figure 9.9 Systematics of experimental log ft values. From W. Meyerhof, *Elements of Nuclear Physics* (New York: McGraw-Hill, 1967).



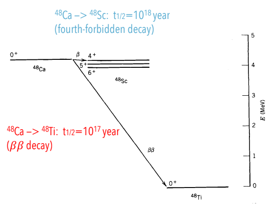
Updated version: 2023

Double beta decay: $\beta\beta$



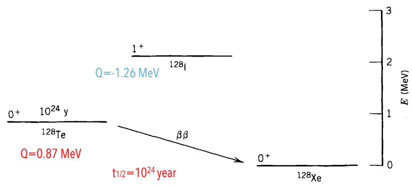
The red nucleus wants to decay back to the stable blue nucleus. It has to do it directly via $\beta\beta$ decay since single β decay to the green one is not energetically allowed.
we shall see some examples:

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{\;\;A}_{Z+2}X''_{N-2} + 2e^{-} + 2\bar{\nu}_{e} $$



- ^{48}Ca can decay to ^{48}Sc via single β decay This is energetically Ok. But still it is very slow. Why?

  See the spin difference? Spin difference is very large. This delays the decay
- But it also decays to ^{48}Ti via $\beta\beta$ decay.
- Both decays are very long. So long that ^{48}Ca is stable for all practical purposes! *(Age of our Earth: 4.4 10^9 year)*



- The double beta decay is the only allowed decay since the intermediate decay is not allowed at all due to the negative Q value (Q=-1.26 MeV for ^{128}Te—>^{128}I) . SO only possible decay is $\beta\beta$ decay!!
- This is the most desired case where the double beta decay can be observed.

Nuclei with double beta decay

Nuclide	Half-life, 10^{21} years	Mode	Transition	Method	Experiment
^{48}Ca	$0.064^{+0.007}_{-0.006} \pm ^{+0.012}_{-0.009}$	$\beta^-\beta^-$		direct	NEMO-3[11]
^{76}Ge	1.926 ± 0.094	$\beta^-\beta^-$		direct	GERDA[10]
^{78}Kr	$9.2^{+5.5}_{-2.6} \pm 1.3$	εε		direct	BAKSAN[10]
^{82}Se	$0.096 \pm 0.003 \pm 0.010$	$\beta^-\beta^-$		direct	NEMO-3[10]
^{96}Zr	$0.0235 \pm 0.0014 \pm 0.0016$	$\beta^-\beta^-$		direct	NEMO-3[10]
^{100}Mo	0.00693 ± 0.00004	$\beta^-\beta^-$		direct	NEMO-3[10]
	$0.69^{+0.10}_{-0.08} \pm 0.07$	$\beta^-\beta^-$	$0^+ \rightarrow 0^+_1$		Ge coincidence[10]
^{116}Cd	$0.028 \pm 0.001 \pm 0.003$ $0.026^{+0.009}_{-0.005}$	$\beta^-\beta^-$		direct	NEMO-3[10] ELEGANT IV[10]
^{128}Te	7200 ± 400 1800 ± 700	$\beta^-\beta^-$		geochemical	[10]
^{130}Te	$0.82 \pm 0.02 \pm 0.06$	$\beta^-\beta^-$		direct	CUORE-0[12]
^{124}Xe	$18 \pm 5 \pm 1$	εε		direct	XENON1T[13]
^{136}Xe	$2.165 \pm 0.016 \pm 0.059$	$\beta^-\beta^-$		direct	EXO-200[10]
^{130}Ba	(0.5 – 2.7)	εε		geochemical	[14][15]
^{150}Nd	$0.00911^{+0.00025}_{-0.00022} \pm 0.00063$	$\beta^-\beta^-$		direct	NEMO-3[10]
	$0.107^{+0.046}_{-0.026}$	$\beta^-\beta^-$	$0^+ \rightarrow 0^+_1$		Ge coincidence[10]
^{238}U	2.0 ± 0.6	$\beta^-\beta^-$		radiochemical	[10]

Nuclei with "neutrinoless" double beta decay

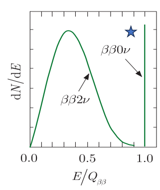
NEMO Highest 0νββ Decay Lower Limits

Isotope	$T_{1/2}$ (yr)	Neutrino mass limit (eV)
^{82}Se	2.1×10^{23}	
^{100}Mo	1.1×10^{24}	0.9
^{116}Cd	1.6×10^{22}	
^{96}Zr	8.6×10^{21}	20.1
^{150}Nd	1.8×10^{22}	6.3
^{48}Ca	1.3×10^{22}	29.7

Neutrino Ettore Majorana Observatory (NEMO experiment)

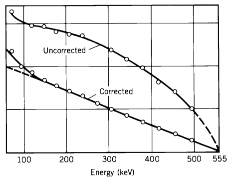
5 years of data taking

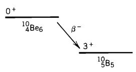
$$ {}^{A}_{Z}X_{N} \rightarrow {}^{\;\;A}_{Z+2}X''_{N-2} + 2e^{-} $$



It has not been observed until now!

Beta decay spectroscopy:





Question: Is it an allowed decay or forbidden? and why?

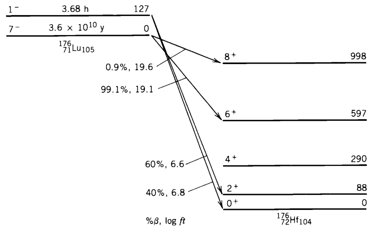
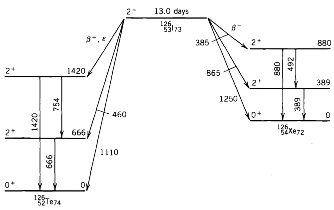


Figure 9.27 The β decay of ^{176}Lu. Level energies are given in keV.

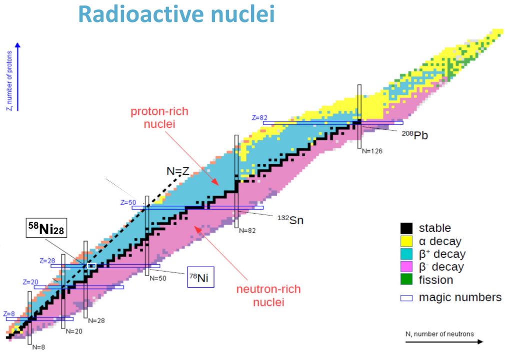


Lecture 12: Gamma decay

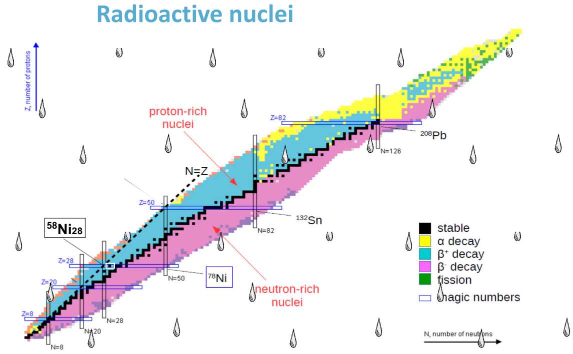
Martin & Shaw Ch 8.8

Additional Compendium: Krane p.327 – 335 & p.341 – 351 & p.359 – 361

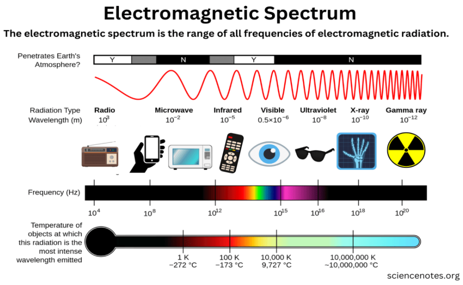
Radioactive nuclei



Radioactive nuclei

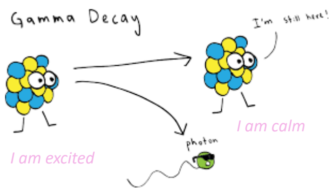


Gamma rays are energetic photons, i.e. light with high frequency.



In Nuclear physics, the range of gamma rays is 0.1-10 MeV (10^4 to 100 fm)

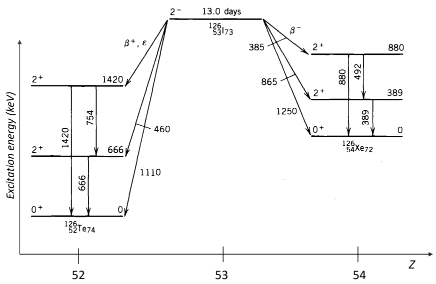
What happens when a nucleus undergoes gamma decay?



- *No charge particles are emitted from the nucleus when it gamma decays.*
- *Only photons are emitted.*
- *Photons are neutral and massless. So they do not carry away mass nor charge.*
- *Because of this, nucleus remains the same but get rids of its excitation energy.*
- *Photons carry away momentum, angular momentum and parity and all these quantities need to be conserved.*

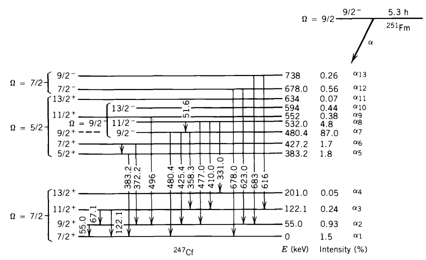
Most of the beta and alpha decays leave the final nucleus in an excited state.

beta decay



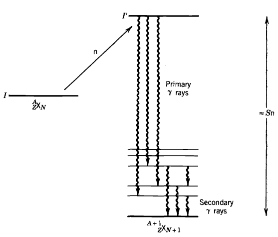
Most of the beta and alpha decays leave the final nucleus in an excited state.

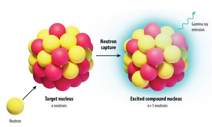
alpha decay



Most of nuclear reactions leave the final nucleus in an excited state.

Example: neutron–capture reactions
A(n,γ)B: The nucleus A captures one neutron and turns into a final nucleus B. The final nucleus is in an excited state and emits γ rays





https://discover.lanl.gov/publications/1663/september-2022/capturing-neutrons-in-the-desert/

Energetics of gamma decay

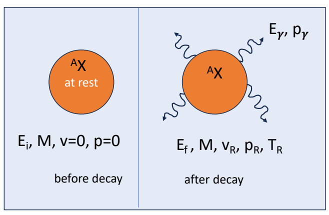
Conservation of energy and momentum:

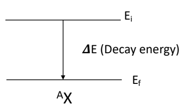
$$E_{\mathrm{i}} = E_{\mathrm{f}} + E_{\gamma} + T_{\mathrm{R}}$$
$$0 = \boldsymbol{p}_{\mathrm{R}} + \boldsymbol{p}_{\gamma}$$

$E_i - E_f = \Delta E = E_\gamma + T_R$

If T_R is very small, we can obtain:

$\boxed{\Delta E \cong E_\gamma}$

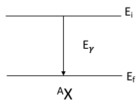




- For low energy gamma rays up to around few MeV, recoil energy of the nucleus is around 1 eV.
- For higher energy gamma rays around 5-10 MeV (for example in neutron-capture experiments) the recoil energy is 100 keV. Radiation damage can be observed as a result of the position change of the recoiling atom in the lattice!

$\Delta E \approxeq E_\gamma$

From now, we will always assume that the energy of the emitted gamma ray (E_γ) is equal to the energy difference between the initial and final states (E_i-E_f)



10.1 ENERGETICS OF γ DECAY

Let's consider the decay of a nucleus of mass M at rest, from an initial excited state E_i to a final state E_f. To conserve linear momentum, the final nucleus will not be at rest but must have a recoil momentum p_R and corresponding recoil kinetic energy T_R, which we assume to be nonrelativistic ($T_R = p_R^2/2M$). Conservation of total energy and momentum give

$$E_i = E_f + E_\gamma + T_R \qquad 0 = \boldsymbol{p}_R + \boldsymbol{p}_\gamma \tag{10.1}$$

It follows that $p_R = p_\gamma$; the nucleus recoils with a momentum equal and opposite to that of the γ ray. Defining $\Delta E = E_i - E_f$ and using the relativistic relationship $E_\gamma = cp_\gamma$,

$$\Delta E = E_\gamma + \frac{E_\gamma^2}{2Mc^2} \tag{10.2}$$

which has the solution

$$E_\gamma = Mc^2\left[-1 \pm \left(1 + 2\frac{\Delta E}{Mc^2}\right)^{1/2}\right] \tag{10.3}$$

The energy differences ΔE are typically of the order of MeV, while the rest energies Mc^2 are of order $A \times 10^3$ MeV, where A is the mass number. Thus $\Delta E \ll Mc^2$ and to a precision of the order of 10^{-4} to 10^{-5} we keep only the first three terms in the expansion of the square root:

$$E_\gamma \cong \Delta E - \frac{(\Delta E)^2}{2Mc^2} \tag{10.4}$$

Hint: Use Taylor expension of $(1\text{-}2x)^{1/2}$ in Eq. (10.3)

$$(1-2x)^{1/2} = 1 + x - \frac{x^2}{2} + \cdots \quad \text{(I)}$$

$$(1-2\frac{\Delta E}{Mc^2})^{1/2} = 1 + \frac{\Delta E}{Mc^2} - \frac{\Delta E^2}{2(Mc^2)^2} + \cdots \quad \text{(II)}$$

since $\Delta E << Mc^2$, after the first three terms in the Taylor expension are ignored.

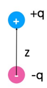
Using the Eq. (II) above, the Eq. (10.3) is written as Eq. (10.4).

some overview about em radiation

- The nucleus is a distribution of charges, which come from the protons, and a distribution of currents, which result from the motion of protons inside the nucleus.
- The oscillation of these charges is what causes the emission of electromagnetic radiation from the nucleus, i.e. Gamma radiation
- The most basic example of a charge distribution that emits electromagnetic radiation is the **electric dipole**. Think of two point charges oscillating in the z-axis with some angular frequency ω for example.
- The most basic example of a magnetic moment is the **magnetic dipole** caused by a current in circular motion in the xy plane.

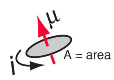
static

electric dipole



d=qz

magnetic dipole



μ=iA

dynamic

- *dynamic charge and current distributions create "a radiation field" (emits gamma rays). Charge distribution vary in time and oscillates.*

d(t)=qzcos(wt) *electric dipole radiation field*

μ(t)=iAcos(wt) *magnetic dipole radiation field*

Important characteristics of **dipole** radiations

*1. Both electric and magnetic dipole radiations have characteristics **angular distributions** which which are **identical** for both E and M. It has a $\sin^2\boldsymbol{\theta}$ dependency. ($\boldsymbol{\theta}$ is the angle with respect to the z- axis (axis of oscillation))*

Similarly, higher order multipoles, such as quadrupole radiation, have their characteristic angular distributions.

*2. Electric and magnetic dipole fields have **opposite parity** (even if they give same angular distributions). Electric dipole has odd parity, magnetic dipole even.*

3. Average radiated power (Energy per unit time) sent out by the electric and magnetic dipole :

$$P = \frac{1}{12\pi\epsilon_0}\frac{\omega^4}{c^3}d^2 \qquad\qquad P = \frac{1}{12\pi\epsilon_0}\frac{\omega^4}{c^5}\mu^2$$

d and $\boldsymbol{\mu}$ represent the *amplitudes* of the time-varying dipole moments.

One can extend these properties of **dipole radiation to multipole radiations** in general. This is done via electromagnetic theory.

General characteristics of **multipole** radiations

1. *Same multipolarity of different types (E2 & M2) have the identical angular distribution.*
2. *Same multipolarity of different types (E2 & M2) have the opposite parity.*

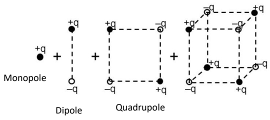
3. The radiated power (energy per unit time) for Electric (σ=E) and for Magnetic (σ=M) for a given L multipolarity.

$$P(\sigma L) = \frac{2(L+1)c}{\epsilon_0 L[(2L+1)!!]^2}\left(\frac{\omega}{c}\right)^{2L+2}[m(\sigma L)]^2$$

Amplitude of the time-varying E or M moment

The $m(\sigma L)$ will be slightly different for E and M.

L:0,1,2,3,4,..(index multipolarity)
2^L: multipole order
Monopole (L=0) → 2^0=1
Dipole (L=1) → 2^1=2
Quadrupole (L=2) → 2^2=4
Octupole (L=3) → 2^3=8



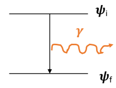
Going from classical to quantum mechanics:

energy radiated per second

$$P(\sigma L) = \frac{2(L+1)c}{\epsilon_0 L[(2L+1)!!]^2}\left(\frac{\omega}{c}\right)^{2L+2}[m(\sigma L)]^2$$

Decay probability is governed by the matrix element which contains information about the nuclear structure.

$$\lambda(\sigma L) = \frac{P(\sigma L)}{\hbar\omega} = \frac{2(L+1)}{\epsilon_0 \hbar L[(2L+1)!!]^2}\left(\frac{\omega}{c}\right)^{2L+1}[m_{fi}(\sigma L)]^2$$



energy of the photon.

$$m_{fi}(\sigma L) = \int \psi_f^* m(\sigma L)\psi_i \, dv$$

$m(\sigma L)$ multipole opeartor has the job of changing the nuclear state from ψ_i to ψ_f while simultaneously creating a photon of the proper energy, parity, and multipole order.

Fermi's golden rule

$$\boxed{\lambda = \frac{2\pi}{\hbar}|V'_{fi}|^2 \rho(Ef)}$$

$\alpha_X = g^2/4\pi\hbar c$

$\alpha \equiv e^2/4\pi\varepsilon_0\hbar c$

beta decay

$$\lambda = \frac{g^2|M_{fi}|^2}{2\pi^3\hbar^7c^3}\int_0^{p_{max}} F(Z',p)p^2(Q-T_e)^2\,dp$$

$$V_{fi} = g\int[\psi_f^*\varphi_e^*\varphi_\nu^*]O_X\psi_i\,dv$$

gamma decay

$$\lambda(\sigma L) = \frac{P(\sigma L)}{\hbar\omega} = \frac{2(L+1)}{\epsilon_0 \hbar L[(2L+1)!!]^2}\left(\frac{\omega}{c}\right)^{2L+1}[m_{fi}(\sigma L)]^2$$

$$m_{fi}(\sigma L) = \int \psi_f^* m(\sigma L)\psi_i\,dv$$

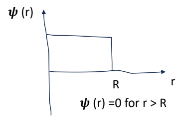
$$\lambda(\sigma L) = \frac{P(\sigma L)}{\hbar\omega} = \frac{2(L+1)}{\epsilon_0 \hbar L[(2L+1)!!]^2}\left(\frac{\omega}{c}\right)^{2L+1}[m_{fi}(\sigma L)]^2$$

integral over nuclear volume

$$\boxed{m_{fi}(\sigma L) = \int \psi_f^* m(\sigma L)\psi_i\,dv}$$

$m(\sigma L)$ → r^L

$\int \psi_f^* \psi_i$ dv ψ (r) → r



ψ (r) =0 for r > R

ψ (r) ~ constant for r < R

Simple assumption: one single proton jumps to another shell-model orbit and thus the transition occurs & a gamma ray is emitted.

Note that in order to calculate decay probability, we need to evaluate the matrix element $m_{fi}(\sigma L)$. This is explained in Krane page 331-332.

Electric transitions

$\alpha_X = g^2/4\pi\hbar c$

$\alpha \equiv e^2/4\pi\varepsilon_0\hbar c$

Radius of the nucleus
$R=R_0A^{1/3}$

$$\boxed{\lambda(EL) \cong \frac{8\pi(L+1)}{L[(2L+1)!!]^2}\frac{e^2}{4\pi\epsilon_0\hbar c}\left(\frac{E}{\hbar c}\right)^{2L+1}\left(\frac{3}{L+3}\right)^2 cR^{2L}}$$

level density

where λ is in s^{-1} and E is in MeV.

E: Energy of the gamma ray

Weisskopf estimates: (single-particle estimates since one single proton causes the gamma decay!)

They give reasonable relative comparisons of the transition rates.

Electric transitions

Radius of the nucleus
$R=R_0A^{1/3}$

$$\boxed{\lambda(EL) \cong \frac{8\pi(L+1)}{L[(2L+1)!!]^2}\frac{e^2}{4\pi\epsilon_0\hbar c}\left(\frac{E}{\hbar c}\right)^{2L+1}\left(\frac{3}{L+3}\right)^2 cR^{2L}}$$

$$\lambda(E1) = 1.0\times10^{14}A^{2/3}E^3$$
$$\lambda(E2) = 7.3\times10^7A^{4/3}E^5$$
$$\lambda(E3) = 34A^2E^7$$
$$\lambda(E4) = 1.1\times10^{-5}A^{8/3}E^9$$

E: Energy of the gamma ray
A: Mass of the nucleus emitting gamma ray.

- Lower multipolarities are dominant.
- λ(E1) wins among other decay constants with higher multipolarities.
- increasing multipolarity by one unit reduces the decay rate by 5 order of magnitude (10^{-5})

factor from gamma ray energy:
$E_\gamma =$ 0.1   0.2 0.5 1   2 MeV

$\frac{dipole}{quadrupole} = \frac{E_\gamma^3}{E_\gamma^5} =$ 100   25   4   1   0,25

Magnetic transitions

$$\lambda(ML) \cong \frac{8\pi(L+1)}{L[(2L+1)!!]^2}\left(\mu_p - \frac{1}{L+1}\right)^2\left(\frac{\hbar}{m_p c}\right)^2\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right) \times \left(\frac{E}{\hbar c}\right)^{2L+1}\left(\frac{3}{L+2}\right)^2 cR^{2L-2}$$

$$\lambda(M1) = 5.6 \times 10^{13} E^3$$
$$\lambda(M2) = 3.5 \times 10^7 A^{2/3} E^5$$
$$\lambda(M3) = 16 A^{4/3} E^7$$
$$\lambda(M4) = 4.5 \times 10^{-6} A^2 E^9$$

where λ is in s^{-1} and E is in MeV.

- Again, lower multipolarities are dominant

Electric versus Magnetic

$$\lambda(E1) = 1.0 \times 10^{14} A^{2/3} E^3$$
$$\lambda(E2) = 7.3 \times 10^7 A^{4/3} E^5$$
$$\lambda(E3) = 34 A^2 E^7$$
$$\lambda(E4) = 1.1 \times 10^{-5} A^{8/3} E^9$$

$$\lambda(M1) = 5.6 \times 10^{13} E^3$$
$$\lambda(M2) = 3.5 \times 10^7 A^{2/3} E^5$$
$$\lambda(M3) = 16 A^{4/3} E^7$$
$$\lambda(M4) = 4.5 \times 10^{-6} A^2 E^9$$

For a given multipole order, electric radiation is more likely than than magnetic radiation. How likely?

For a medium heavy nucleus (A=100)
Assume E_γ=1 MeV

λ(M1)=5.6 x 10^{13} sec^{-1}
λ(M2)=7.5 x 10^{8} sec^{-1}
λ(M3)=7.4 x 10^{3} sec^{-1}

$$\frac{\lambda(M1)}{\lambda(M2)} = 0.75\ x\ 105$$

$$\frac{\lambda(E1)}{\lambda(M1)} = \sim 10^2$$

λ(E1)=2.2 x 10^{15} sec^{-1}
λ(E2)=3.4 x 10^{9} sec^{-1}
λ(E3)=3.4 x 10^{5} sec^{-1}

$$\frac{\lambda(E1)}{\lambda(E2)} = 0.65\ x\ 106$$

1. *The lowest allowed multipole is dominant between magnetic multipoles or electric multipoles*
2. *Electric multipoles are more probable than the same magnetic multipole by a factor ~ 10^2 (however, which one is going to happen depends on the parity)*

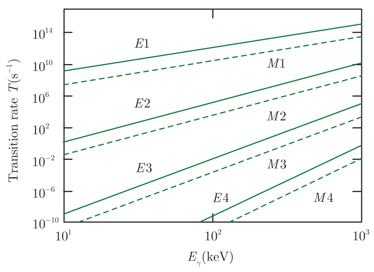


Figure 8.14 Transition rates using the single-particle shell model formulas of Weisskopf as a function of photon energy for a nucleus of mass number $A = 60$.

How to interprete our exoerimental decay rates to Weisskopf estimates (single-particle estimates)

1) if the measured decay rate, λ is much smaller than the Weisskopf estimate, we then say that the there is a poor overlap between the initial and final state wave functions which slows the decay!
2) if the measured value is much greater than the Weisskopf estimate, we can expect that more than one single nucleon is responsible for the transition--> Collective excitations, deformations.
3) if the measured value is similar to the Weisskopf estimate, then the transition is mostly caused by a single nucleon excitation.

reminder for Eda: lifetimes vs decay rates

Angular momentum and parity selection rules:

- Emitted gamma ray carries away parity and angular momentum ($L\hbar$), in addition to energy.
- Since it is an electromagnetic process, total angular momentum and parity should be conserved.
- Consider a gamma transition from an initial state, I_i to a final state, I_f. Assume $\mathbf{I_i \neq I_f}$

Conservation of angular momentum:

$$\boxed{\boldsymbol{I}_i = \boldsymbol{L} + \boldsymbol{I}_f}$$

$$|I_i - I_f| \leq L \leq I_i + I_f$$

Possible L values for the emitted gamma ray.

$$\begin{array}{l} \mathbf{I_i}, \pi_i \\ \downarrow \ \mathbf{L_\gamma}, E_\gamma \\ \mathbf{I_f}, \pi_f \end{array}$$

^{A}X

Parity of Electric and Magnetic radiation

$$\pi(ML) = (-1)^{L+1}$$
$$\pi(EL) = (-1)^{L}$$

Electric and Magnetic multipoles of the same order have the opposite parity.

parity should be conserved :

$\Delta\pi = \pi_i . \pi_f$ = + (no parity change)
$\Delta\pi = \pi_i . \pi_f$ = - (yes parity change)

Angular momentum and parity selection rules:

$$|I_i - I_f| \le L \le I_i + I_f \qquad (\text{no } L = 0)$$

$$\Delta\pi = \text{no}: \quad \text{even electric, odd magnetic}$$

$$\Delta\pi = \text{yes}: \quad \text{odd electric, even magnetic}$$

$$\pi(ML) = (-1)^{L+1}$$
$$\pi(EL) = (-1)^{L}$$

there are no monopole (L = 0) transitions in which a single photon is emitted.

Angular momentum and parity selection rules:

Table 8.1 Selection rules for γ emission

Multipolarity	Dipole		Quadrupole		Octupole	
Type of radiation	E1	M1	E2	M2	E3	M3
L	1	1	2	2	3	3
ΔP	Yes	No	No	Yes	Yes	No

$$\pi(ML) = (-1)^{L+1}$$
$$\pi(EL) = (-1)^{L}$$

Examples:

Table 8.2 Examples of nuclear electromagnetic transitions

$J_i^{P_i}$	$J_f^{P_f}$	ΔP	L	Allowed transitions
0^+	0^+			
$1/2^+$	$1/2^-$			
1^+	0^+			
2^+	0^+			
$3/2^-$	$1/2^+$			
2^+	1^+			
$3/2^-$	$5/2^+$			

Examples:

Table 8.2 Examples of nuclear electromagnetic transitions

$J_i^{P_i}$	$J_f^{P_f}$	ΔP	L	Allowed transitions
0^+	0^+	No	–	None
$1/2^+$	$1/2^-$	Yes	1	E1
1^+	0^+	No	1	M1
2^+	0^+	No	2	E2
$3/2^-$	$1/2^+$	Yes	1, 2	E1, M2
2^+	1^+	No	1, 2, 3	M1, E2, M3
$3/2^-$	$5/2^+$	Yes	1, 2, 3, 4	E1, M2, E3, M4

Dominant multipoles for allowed transitions

$\lambda(M1)=5.6 \times 10^{13}\ \text{sec}^{-1}$
$\lambda(M2)=7.5 \times 10^{8}\ \text{sec}^{-1}$
$\lambda(M3)=7.4 \times 10^{3}\ \text{sec}^{-1}$

$$\frac{\lambda(M1)}{\lambda(M2)} = \sim 10^5$$

$\lambda(E1)=2.2 \times 10^{15}\ \text{sec}^{-1}$
$\lambda(E2)=3.4 \times 10^{9}\ \text{sec}^{-1}$
$\lambda(E3)=3.4 \times 10^{5}\ \text{sec}^{-1}$

$$\frac{\lambda(E1)}{\lambda(E2)} = \sim 10^6$$

$$\frac{\lambda(E1)}{\lambda(M1)} = \sim 10^2$$

$$\frac{\lambda(E1)}{\lambda(M2)} = \sim 10^7$$ L=E1, M2→ E1 is dominant

$$\frac{\lambda(M1)}{\lambda(E2)} = \sim 10^7$$ L=M1, E2→ M1 is dominant

The lowest allowed multipole is dominant among all allowed radiations

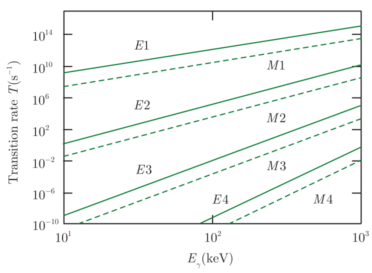


Figure 8.14 Transition rates using the single-particle shell model formulas of Weisskopf as a function of photon energy for a nucleus of mass number $A = 60$.

Examples:

Table 8.2 Examples of nuclear electromagnetic transitions

$J_i^{P_i}$	$J_f^{P_f}$	ΔP	L	Allowed transitions
0^+	0^+	No	–	None
$1/2^+$	$1/2^-$	Yes	1	E1
1^+	0^+	No	1	M1
2^+	0^+	No	2	E2
$3/2^-$	$1/2^+$	Yes	1, 2	E1, M2
2^+	1^+	No	1, 2, 3	M1, E2, M3
$3/2^-$	$5/2^+$	Yes	1, 2, 3, 4	E1, M2, E3, M4

Most dominant multipoles

what about $0^+ \rightarrow 0^+$ transition?

Since photons have non-zero angular momentum, L cannot be zero.
So what carries parity and angular momentum then?

- ***Orbitting electrons near the nucleus*** *carries, energy, parity and angular momentum.*
- *The nucleus gives its energy to the nearest orbitting electron. This process is called* ***Internal conversion****.*

What is internal conversion?

- The excitation energy is transferred directly to electrons near the nucleus and these electrons fly away from the atom.
- The process is called “internal conversion” and the emitted electrons are called (internal) “conversion electrons”.
- It is an electromagnetic process and compete with the emission of gamma rays.

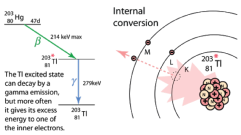
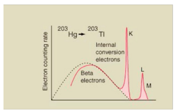
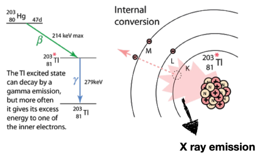


Figure: http://hyperphysics.phy-astr.gsu.edu/hbase/Nuclear/radact2.html

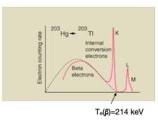
Example of ^{203}Hg decay:

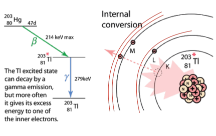


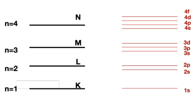


Internal conversion
(a) different than beta decay (In IC, electron already exists in an atomic orbit and is therefore not created)
(b) Not a two-step process (i.e., first a photon is emitted by the nucleus and then interacts with an orbiting electron via PA)

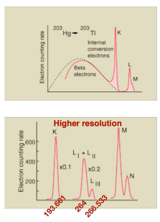
Example of ^{203}Hg decay:

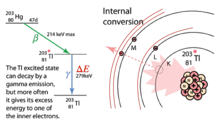






Example of ^{203}Hg decay:



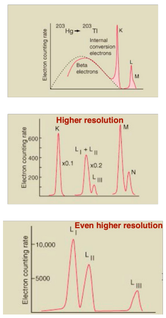


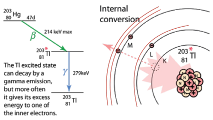
$$B(\mathrm{K}) = 85.529 \text{ keV}$$
$$B(\mathrm{L_I}) = 15.347 \text{ keV}$$
$$B(\mathrm{L_{II}}) = 14.698 \text{ keV}$$
$$B(\mathrm{L_{III}}) = 12.657 \text{ keV}$$
$$B(\mathrm{M_I}) = 3.704 \text{ keV}$$

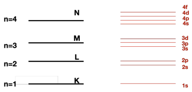
$$T_e = \Delta E - B$$

$$T_e(\mathrm{K}) = 279.190 - 85.529 = 193.661 \text{ keV}$$
$$T_e(\mathrm{L_I}) = 279.190 - 15.347 = 263.843 \text{ keV}$$
$$T_e(\mathrm{L_{II}}) = 279.190 - 14.698 = 264.492 \text{ keV}$$
$$T_e(\mathrm{L_{III}}) = 279.190 - 12.657 = 266.533 \text{ keV}$$
$$T_e(\mathrm{M_I}) = 279.190 - 3.704 = 275.486 \text{ keV}$$

Example of ^{203}Hg decay:







α internal conversion coefficient

Since internal conversion electrons and gamma rays compete against each other in de-exciting nuclei, α **internal conversion coefficient** is defined as the **ratio** of the CE transition probability and the γ-ray transition probability.

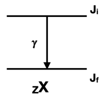
$$\alpha = \frac{\lambda_c}{\lambda_\gamma}$$

$$\lambda_t = \lambda_c + \lambda_\gamma$$

$$\lambda_t = \lambda_\gamma(1+\alpha)$$

$$\lambda_t = \lambda_\gamma(1 + \alpha_K + \alpha_L + \alpha_M + \ldots)$$

$$\alpha_L = \alpha_{L_I} + \alpha_{L_{II}} + \alpha_{L_{III}}$$

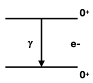


$0^+ \rightarrow 0^+$ Transition

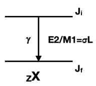
The internal conversion is the only mode for the $0^+ \rightarrow 0^+$ transition in nuclei (E0)

Higher-energy internal conversion can be observed

Very important for understanding structure of nuclei such as sudden deformation and shape changes so-called shape coexistence



α internal conversion coefficient depends very much on *atomic number Z, transition multipolarity EL/ML and transition energy E.*



$$\alpha(EL) \cong \frac{Z^3}{n^3}\left(\frac{L}{L+1}\right)\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)^4\left(\frac{2m_ec^2}{E}\right)^{L+5/2}$$

$$\alpha(ML) \cong \frac{Z^3}{n^3}\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)^4\left(\frac{2m_ec^2}{E}\right)^{L+3/2}$$

$$\alpha = \frac{\lambda_c}{\lambda_\gamma}$$

Gamma decays of **medium and heavy nuclei (high Z)** with **high multipolarity** and **low transition energy** will have high probability of Electron Conversion process, i.e. large α internal conversion ratio or coefficient.

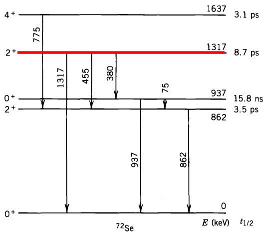


Figure 10.11 Energy levels in ^{72}Se.

We can calculate the total decay constant from the meausured half-life

$$\lambda_t = \frac{0.693}{t_{1/2}} = \frac{0.693}{8.7\times10^{-12}\ \mathrm{s}} = 8.0\times10^{10}\ \mathrm{s}^{-1}$$

$$\boxed{\lambda_t = \lambda_{\gamma,1317} + \lambda_{\gamma,455} + \lambda_{\gamma,380}}$$

$$= \lambda_{\gamma,1317}(1+\alpha_{1317}) + \lambda_{\gamma,455}(1+\alpha_{455}) + \lambda_{\gamma,380}(1+\alpha_{380})$$

We ignore the contribution of the internal conversion . Internal conversion coeeficients are are very small for these gamma rays. Why?

$$\boxed{\lambda_{\gamma,1317} : \lambda_{\gamma,455} : \lambda_{\gamma,380} = 51:39:10}$$ branching ratios

$$\lambda_{\gamma,1317} = 0.51(8.0\times10^{10}\ \mathrm{s}^{-1}) = 4.1\times10^{10}\ \mathrm{s}^{-1}$$

$$\lambda_{\gamma,455} = 0.39(8.0\times10^{10}\ \mathrm{s}^{-1}) = 3.1\times10^{10}\ \mathrm{s}^{-1}$$

$$\lambda_{\gamma,380} = 0.10(8.0\times10^{10}\ \mathrm{s}^{-1}) = 0.80\times10^{10}\ \mathrm{s}^{-1}$$

TOTAL=8.0 x 10^{10} s^{-1}

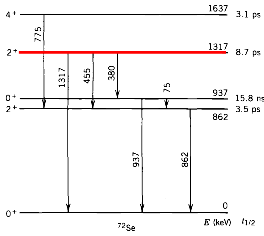


Figure 10.11 Energy levels in ^{72}Se.

$$\lambda_t = \frac{0.693}{t_{1/2}} = \frac{0.693}{8.7\times10^{-12}\ \mathrm{s}} = 8.0\times10^{10}\ \mathrm{s}^{-1}$$

$$\lambda_{\gamma,1317} = 0.51(8.0\times10^{10}\ \mathrm{s}^{-1}) = 4.1\times10^{10}\ \mathrm{s}^{-1}$$

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$$\lambda_{\gamma,380} = 0.10(8.0\times10^{10}\ \mathrm{s}^{-1}) = 0.80\times10^{10}\ \mathrm{s}^{-1}$$

We can calculate the Weisskopf estimates. Shall we calculate it for the total or for three different branches separately?

$$\lambda_{E2,1317} = 8.7\times10^{10}\ \mathrm{s}^{-1}$$

$$\lambda_{E2,455} = 4.3\times10^{8}\ \mathrm{s}^{-1}$$

$$\lambda_{E2,380} = 1.7\times10^{8}\ \mathrm{s}^{-1}$$

Answer: separately since Weisskopf estimates are calculated for a given gamma ray energy.

What branches do the state at 937 keV have?

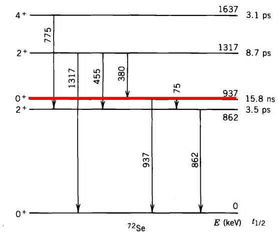


Figure 10.11 Energy levels in ^{72}Se.

$$\lambda_t = \frac{0.693}{15.8\ \text{ns}} = 4.39 \times 10^7\ \text{s}^{-1}$$

$$\lambda_t = \lambda_{t,937} + \lambda_{t,75}$$

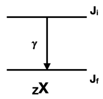
$$= \lambda_{e,937} + \lambda_{\gamma,75}(1 + \alpha_{75})$$

α internal conversion coefficient

Since internal conversion electrons and gamma rays compete against each other in de-exciting nuclei, **α internal conversion coefficient** is defined as the **ratio** of the CE transition probability and the γ-ray transition probability.

$$\alpha = \frac{\lambda_c}{\lambda_\gamma} \qquad \lambda_t = \lambda_c + \lambda_\gamma$$

$$\lambda_t = \lambda_\gamma(1 + \alpha)$$



$$\lambda_t = \lambda_\gamma(1 + \alpha_K + \alpha_L + \alpha_M + \ldots)$$

$$\alpha_L = \alpha_{L_I} + \alpha_{L_{II}} + \alpha_{L_{III}}$$

What branches do the state at 937 keV have?

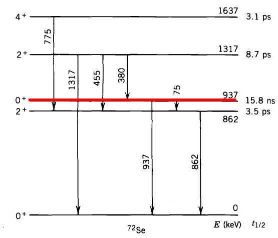


Figure 10.11 Energy levels in ^{72}Se.

$$\lambda_t = \frac{0.693}{15.8\ \text{ns}} = 4.39 \times 10^7\ \text{s}^{-1}$$

$$\lambda_t = \lambda_{t,937} + \lambda_{t,75}$$

$$= \underbrace{\lambda_{e,937}}_{\text{pure internal conversion}} + \underbrace{\lambda_{\gamma,75}(1 + \alpha_{75})}_{\text{gamma decay and inetrnal conversion competes here}}$$

One feature that generally emerges from these calculations is that the measured transition rates are frequently at least an order of magnitude larger than the Weisskopf estimates for $E2$ transitions. This is strong evidence for the *collective* aspects of nuclear structure discussed in Chapter 5—the Weisskopf estimates are based on the assumption that the transition arises from the motion of a *single nucleon*, and the fact that these are too small indicates that *many* nucleons must be taking part in the transition. Figure 10.12 summarizes similar results for many $E2$ transitions, and you can see that this enhancement or acceleration of the single-particle $E2$ rate is quite a common feature. No such effect occurs for $E1$ transitions, which are generally slower than the single-particle rates. On the other hand, consider Figure 10.13 which shows the systematic behavior of $M4$ transitions. Here the agreement between theory and experiment is excellent.

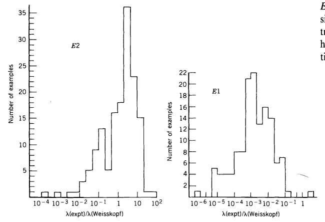


Figure 10.12 Systematics of $E2$ and $E1$ transitions. The figures show the number of cases versus the ratio between the observed decay rate and the value calculated from the Weisskopf formulas. From S. J. Skorka et al., *Nucl. Data* **2**, 347 (1966).

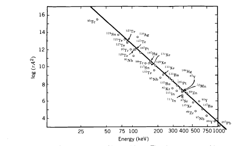


Figure 10.13 Systematics of $M4$ transitions. The data are plotted in terms of the mean life (the reciprocal of the decay constant λ). The straight line is determined from Equation 10.15. Note especially the good agreement between the data points and the expected E^{-9} dependence. From M. Goldhaber and A. W. Sunyar, *Phys. Rev.* **83**, 906 (1951).

Angular distributions:

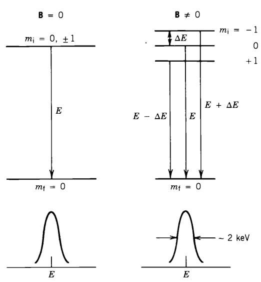
L:0,1,2,3,4,..(index multipolarity)	
2^L: multipole order	
Monopole (L=0)	→ 2^0=1
Dipole (L=1)	→ 2^1=2
Quadrupole (L=2)	→ 2^2=4
Octupole (L=3)	→ 2^3=8

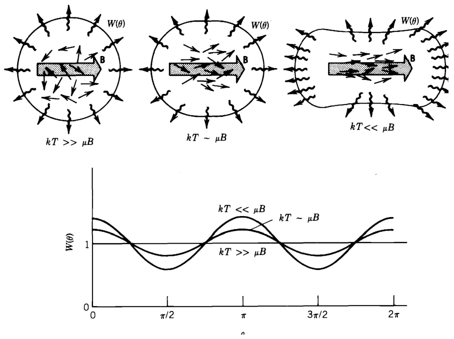
- The angular distribution of 2^L-pole radiation, **relative to a properly chosen direction**, is governed by the *Legendre polynomial* $P_{2L}(\cos\boldsymbol{\theta})$.
- The most common cases are for dipole and quadrupole gamma transitions.

$$P_2 = \tfrac{1}{2}(3\cos^2\theta - 1) \quad \text{For dipole (L=1)}$$

$$P_4 = \tfrac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3) \quad \text{For quadrupole (L=2)}$$

Angular correlation measurements





Angular correlation measurements

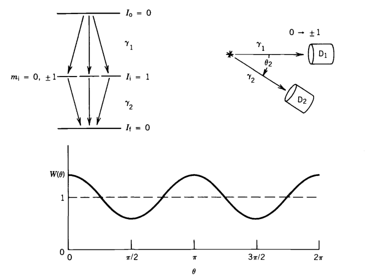
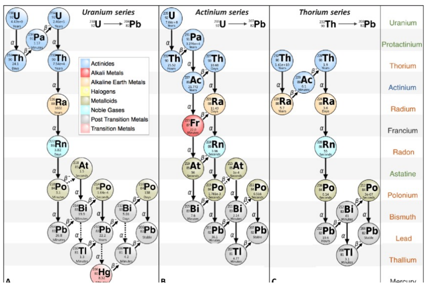
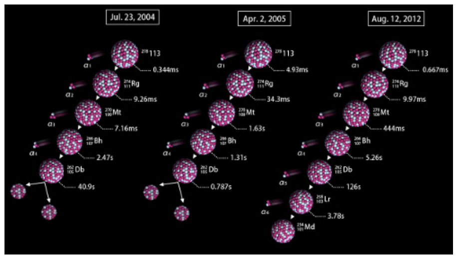


Figure 10.4 Angular correlation measurements. In a cascade of two radiations, here assumed to be $0 \to 1 \to 0$, the angular distribution of γ_2 is measured relative to the direction of γ_1. A typical result for two dipole transitions is shown at the bottom.

Alpha decay in nautrally occuring radionuclides, premordial.



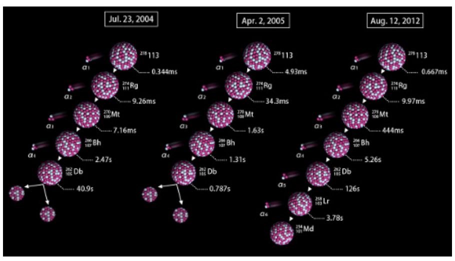
Alpha decay in the discovery of super-heavy elements:



*Though these were good demonstrations, they were not considered conclusive evidence for the existence of element 113, **because the decay chain did not demonstrate "firm connections to known nuclides"** (according to the Joint Working Party's 2011 report).*

https://www.riken.jp/en/news_pubs/research_news/pr/2015/20151231_1/index.html/

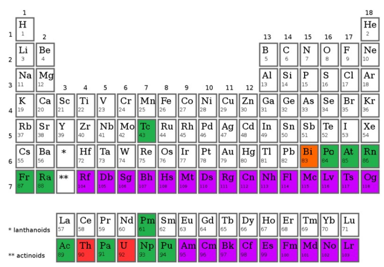
Alpha decay in the discovery of super-heavy elements:



*Then, on August 12, 2012, the group observed the crucial third event. **This time, following the four initial decays, the dubnium-262 continued to undergo alpha decays rather than spontaneous fission**, transforming into lawrencium-258 (element 103) and then finally mendelevium-254 (element 101). As the chain had been clearly characterized, it demonstrated clearly that element 113 was the source of the decay chain.*

https://www.riken.jp/en/news_pubs/research_news/pr/2015/20151231_1/index.html/

Synthetic elements (man-made)



Synthetic elements are elements that do not occur naturally in the universe and have to be created in the laboratory.

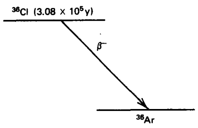
Synthetic elements | Rare radioactive natural elements; often produced artificially | Common radioactive natural elements

Few radionuclides decay only to the ground state : pure beta emitters (beta+ and beta- decays)

Table 1.1 Some "Pure" Beta-Minus Sources

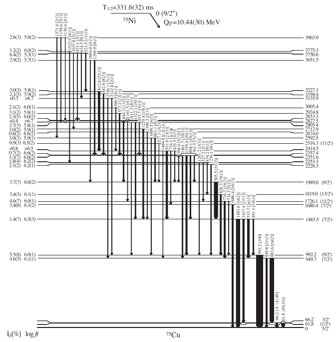
Nuclide	Half-Life	Endpoint Energy (MeV)
^{3}H	12.26 y	0.0186
^{14}C	5730 y	0.156
^{32}P	14.28 d	1.710
^{33}P	24.4 d	0.248
^{35}S	87.9 d	0.167
^{36}Cl	3.08×10^5 y	0.714
^{45}Ca	165 d	0.252
^{63}Ni	92 y	0.067
$^{90}Sr/^{90}Y$	27.7 y/64 h	0.546/2.27
^{99}Tc	2.12×10^5 y	0.292
^{147}Pm	2.62 y	0.224
^{204}Tl	3.81 y	0.766

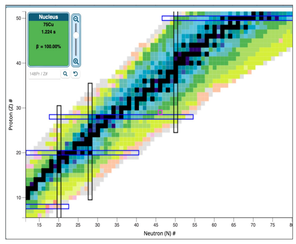
Data from Lederer and Shirley.[1]



Book Leo: Nuclear instrumentation (FYS4505/9505)

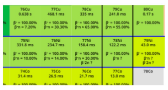
Beta decay

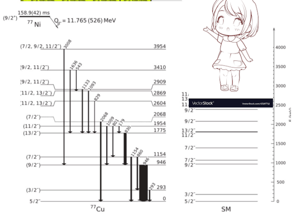


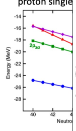


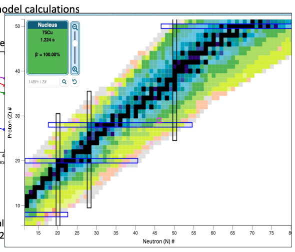
shell evolution near ^{78}Ni

- information about single-particle structure
- crossing of $\pi f_{5/2}$ and $\pi p_{3/2}$ orbitals
- still large gap for $Z = 28$
- benchmark and refine shell model calculations



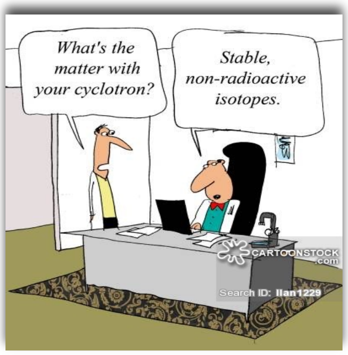




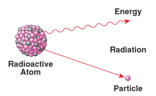


E. Sahin et al
PRL 118, 242

From the guest lecture of Andreas: Exotic nuclei



We need to go to a radioactive beam facility!!

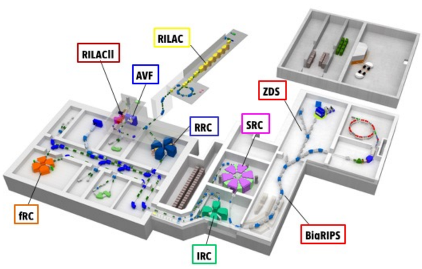


RIKEN Nishina Center, Japan

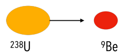
Most intense uranium (^{238}U) beam in the world!

SRC: World Largest (Heaviest) Cyclotron

The SRC is the world's first superconducting ring cyclotron with the ever largest K-value of 2600 MeV, which expresses the maximum bending power of extracted beam from the cyclotron

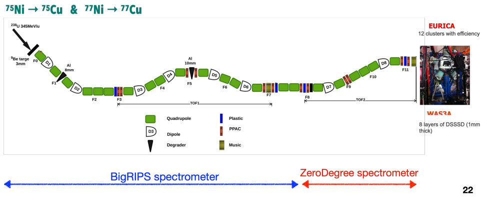


What happens if a very energetic uranium hits a target of beryllium?



Observation of gamma-ray energies and level scheme for the first time

$^{75}Ni \rightarrow {}^{75}Cu$ & $^{77}Ni \rightarrow {}^{77}Cu$

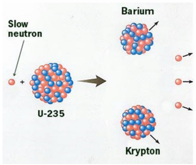


22

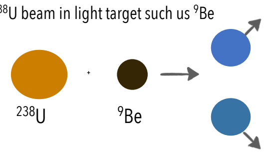
Flight time ~ 200 ns

half-lives of 77Ni are 158 ms

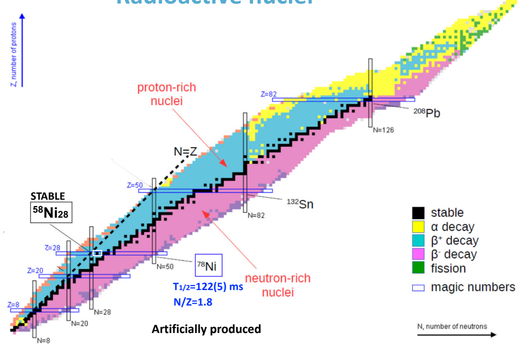
Nuclear Fission



Fission in inverse kinematics:
^{238}U beam in light target such us ^{9}Be



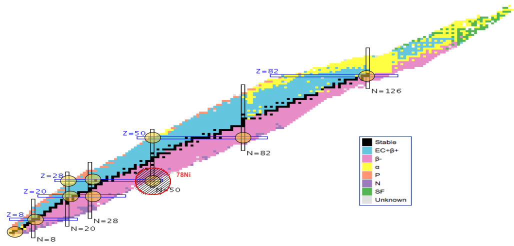
Radioactive nuclei



68

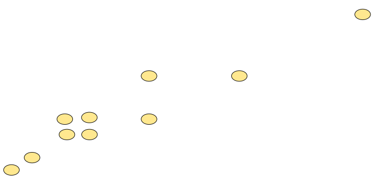
Is ^{78}Ni doubly-magic?
Z=28 N=50

Segre Chart



Segre Chart

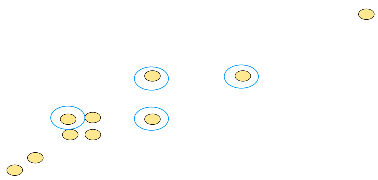
10 doubly-magic nuclei
4He,16O,40Ca,48Ca,48Ni,56Ni,78Ni,100Sn,132Sn,208Pb



Segre Chart

10 doubly-magic nuclei
4He,16O,40Ca,48Ca,48Ni,56Ni,78Ni,100Sn,132Sn,208Pb

4 of them are far from stability
48Ni,78Ni,100Sn,132Sn



Segre Chart

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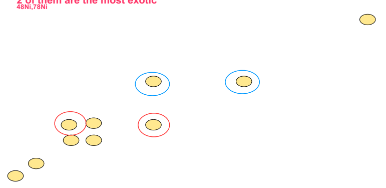
4He,16O,40Ca,48Ca,48Ni,56Ni,78Ni,100Sn,132Sn,208Pb

4 of them are far from stability
48Ni,78Ni,100Sn,132Sn

2 of them are the most exotic
48Ni,78Ni



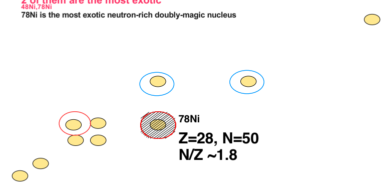
Segre Chart

10 doubly-magic nuclei
4He,16O,40Ca,48Ca,48Ni,56Ni,78Ni,100Sn,132Sn,208Pb

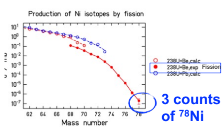
4 of them are far from stability
48Ni,78Ni,100Sn,132Sn

2 of them are the most exotic
48Ni,78Ni

78Ni is the most exotic neutron-rich doubly-magic nucleus

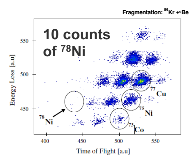


1995, GSI, Germany



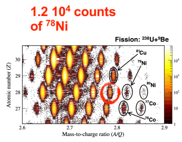
Existence

2005, MSU/NSCL, USA



Half-life 110^{+100}_{-60} ms

2012, RIKEN, Japan

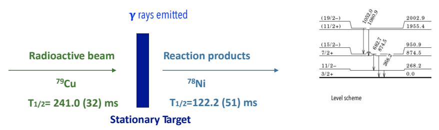


Half-life 122.2 ∓5.1 ms

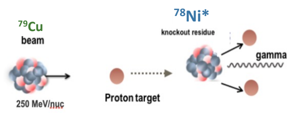
But we want to go further and use these radioactive nuclei as secondary beams and hit another target

In-beam gamma-ray spectroscopy

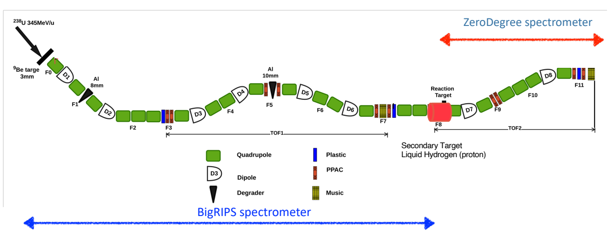
Product
Detect
Identify



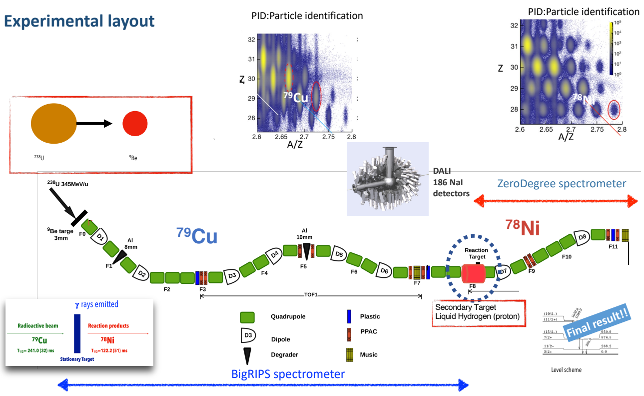
single nucleon knok-out reaction



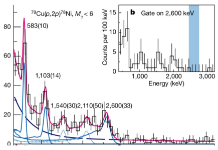
Experimental layout

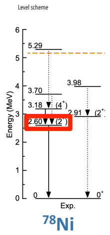


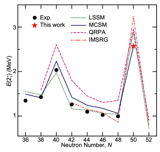
Experimental layout



2+ Energy



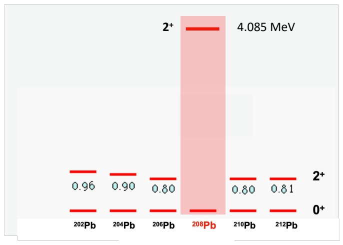




^{78}Ni is doubly-magic
N=50 is still a big gap!!!

R. Taniuchi et al., Nature 2019

The first excited state, 2+ lies higher in nuclei with magic numbers.
More energy is required to excite them out of their ground states



Concepts of NP subjects

- Electron scattering experiments
- Nuclear charge distribution
- Hadron scattering experiments
- Nuclear matter distribution
- Nuclear radius

- Binding energy
- Neutron separation energy
- Liquid drop model
- SEMF (its five components)
- Mass parabolas
- Beta stability valley
- Nuclear force
- Deuteron
  - Spin-parity
  - Magnetic moment
  - Electric moment

- Fermi-gas model
- Shell model
- Independent particle assumption
- Nuclear (central) potentials
- Magic numbers
- Experimental evidences for Magic numbers
- Nuclear chart

- Radioactive decay
- Q-value
- decay constant
- lifetime, half-life
- decay width
- alpha decay
  - theory
  - energetics
  - alpha spectrum
- beta decay
  - theory
  - energetics
  - beta spectrum
  - logfts
  - double-beta decay
- gamma decay
  - theory
  - energetics
  - weisskopf estimates
  - internal conversion
- alpha, beta, gamma-ray spectroscopy
- nuclear fission