FYS3500 Problem set 3

Problem 1

a) We know that

$$M(A,Z) = Z(m_p + m_e) + (A-Z)m_n - \frac{B}{c^2}$$

$$\Rightarrow \frac{B}{c^2} = Z(m_p + m_e) + (A-Z)m_n - M(A,Z)$$

Remember:

$1u = 931.5\ \text{MeV}/c^2$

$m_p = 938.28\ \text{MeV}/c^2$

$m_n = 939.28\ \text{MeV}/c^2$

$m_e = 0.511\ \text{MeV}/c^2$

For ${}^{27}_{13}Al_{14}$:

$$\frac{B}{c^2} = 13(938.28\ \text{MeV}/c^2 + 0.511\ \text{MeV}/c^2)$$
$$+ 14 \cdot 939.28\ \text{MeV}/c^2 -$$
$$26.9815 \cdot 931.5\ \text{MeV}/c^2$$

$$\Rightarrow \underline{\underline{B \approx 225.0\ \text{MeV}}}$$

For $^{235}_{92}U_{143}$: $E^2 = p^2c^2 + m^2c^4$

$$\frac{B}{c^2} = 92(938.28\,MeV/c^2 + 0.511\,MeV/c^2) + 143 \cdot 939.57\,MeV/c^2 - 235.0439 \cdot 931.5\,MeV/c^2$$

$$\Rightarrow B \approx 1783.9\,MeV$$

b)

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym}\frac{(A-2Z)}{A} + \delta$$

$\delta = 0$ since both ^{27}Al & ^{235}U are odd-A

Inserting for $^{27}_{13}Al$ gives

B_semf = 229.0 MeV

while for ^{235}U:

B_semf = 1771.37

We see that there is a deviation between the experimentally determined binding energy and the value given by the semi-empirical binding energy formula of about 5 MeV in the case of Al and 13 MeV in the case of U. The semi-empirical formula assumes that the nucleus is described as a liquid drop (in the so-called liquid drop model), but this is not accurate for all nuclei. Shell effects, where the neutrons and protons are arranged in shells similar to those of the atomic electrons, will affect the resulting binding energy and result in a deviation from the semi-empirical formula.

Problem 2

a) The semi-empirical binding energy formula regards the nucleus as a liquid drop.

The a_v term, called the volume term, says that since all nucleons feel the strong nuclear force, the binding energy of the nucleus increases when we add nucleons. Thus B should increase with A. The coefficient a_v is determined experimentally as are the rest of the coefficients.

$\Rightarrow$ thus the volume term is $a_v A$.

Nucleons on the nuclear surface are not surrounded by other nucleons on all sides, and thus they are less bound than those in the middle of the nucleus. This leads to a decrease in the binding energy, and we must subtract the surface term.

The form of the surface term is determined as follows:

Surface of sphere: $4\pi R^2$

Radius of nucleus: $R \sim R_0 A^{1/3}$

Thus the surface area of a nucleus is

$$4\pi (R_0 A^{1/3})^2 \propto A^{2/3}$$

$\Rightarrow$ surface term is $a_s A^{2/3}$

The Coulomb term describes that the more protons we add to the nucleus, the more the protons will repulse each other, leading to less binding. The magnitude of the Coulomb term increases with the amount of Coulomb repulsion:

Coulomb force: $\propto \frac{q_1 \cdot q_2}{R}$

Z protons in the nucleus, that feel the repulsion of all protons except for itself. $R \propto A^{1/3}$

$\Rightarrow$ the Coulomb term is $a_c \frac{Z(Z-1)}{A^{1/3}}$

The symmetry term accounts for the fact that we observe that nuclei prefer to have N=Z, so that A=2Z. A deviation from this decreases the binding energy. This effect is observed to be less important for heavy nuclei, and thus we divide by A:

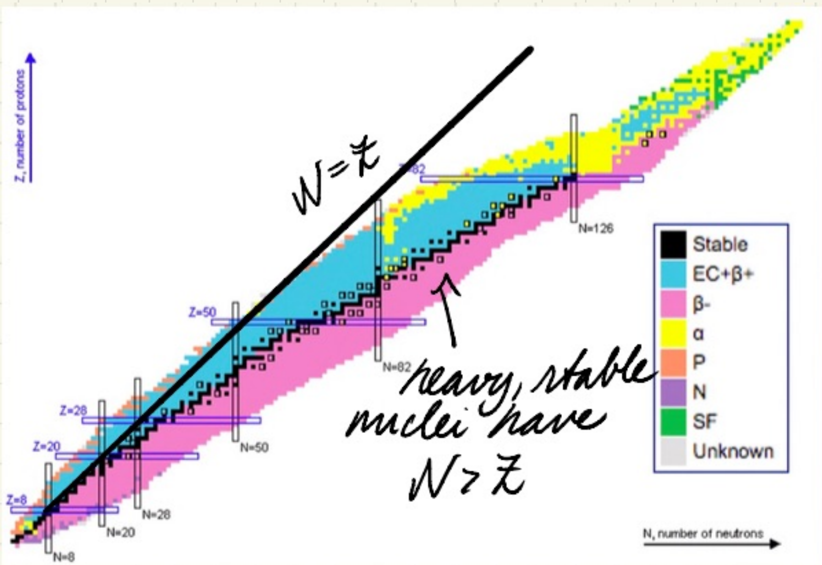
$\Rightarrow$ symmetry term: $a_{sym} \frac{(A-2Z)^2}{A}$

Lastly, the pairing term δ accounts for that nuclei prefer to have an even number of protons and neutrons. Thus we add the paring term (=increase the binding) for even-Z-even-N (even-even) nuclei, subtract it for odd-odd, and have pairing term=0 for odd-A nuclei.

The formula is called semi-empirical because it has basis in physics, while the coefficients are empirically determined.

b) For light nuclei (low-A), the symmetry term dominates. Thus the light nuclei prefer to have N/A approx 0.5, something that is observed when looking at the chart of nuclides (4He, 16O, 12C etc all have N=Z).

As the symmetry term is proportional to 1/A, the importance of this term decreases with A. Instead the Coulomb term (proportional to A**(-1/3)) becomes important. In order for the nucleus to not be ripped apart by the Coulomb repulsion, neutrons must be added to create some distance between the protons. Therefore, heavy nuclei have a ratio N/A > 0.5, and the chart of nuclei "bends off" from the N=Z line



c) Given A, find most stable isobar

→ most stable when binding energy is maximized ⇒ mass is minimized

$$\left.\frac{\partial M(A,Z)}{\partial Z}\right|_{A:\text{const}} = 0$$

$$\begin{cases} M(A,Z) = Z(m_p + m_e) + (A-Z)m_n \\ -\frac{1}{c^2}\Big[a_v A - a_s A^{2/3} - \frac{a_c Z(Z-1)}{A^{1/3}} \\ -a_{sym}\frac{(A-2Z)^2}{A}\Big] \end{cases}$$

$$\begin{cases} = Z(m_p + m_e) + Am_n - Zm_n - \frac{1}{c^2}\Big[a_v A \\ a_s A^{2/3} - \frac{a_c Z^2}{A^{1/3}} + \frac{a_c Z}{A^{1/3}} \\ -a_{sym}\left(\frac{A^2 - 4AZ + 4Z^2}{A}\right)\Big] \end{cases}$$

$$\frac{\partial M}{\partial Z} = m_p + m_e - m_n - \frac{1}{c^2}\left[-\frac{2a_c Z}{A^{1/3}} + \frac{a_c}{A^{1/3}} + a_{sym}\cdot 4 - \frac{a_{sym}\cdot 8 \cdot Z}{A} \right] = 0$$

$$\frac{2a_c Z}{A^{1/3}} + \frac{a_{sym}\cdot 8\cdot Z}{A} = (m_n - m_p - m_e)c^2 + \frac{a_c}{A^{1/3}} + 4\cdot a_{sym}$$

$$Z_{min} = \frac{(m_n - m_p - m_e)c^2 + a_c A^{-1/3} + 4a_{sym}}{2a_c A^{-1/3} + 8a_{sym} A^{-1}}$$