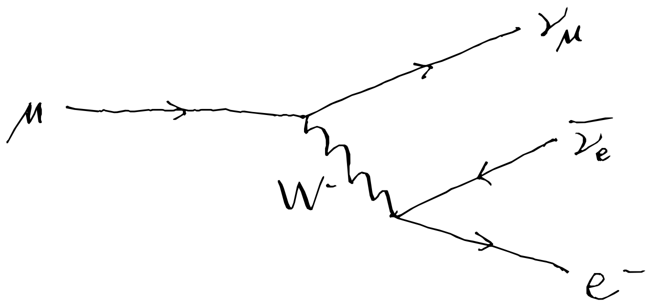
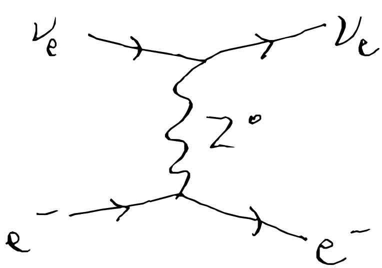
1 - M&S 6.1

- Charged current interactions:
When a $W^\pm$ boson mediates the process.
E.g. μ decay:



- Neutral current: When a Z^0 boson mediates the process. E.g. elastic neutrino scattering:

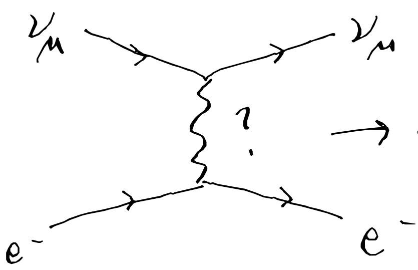


- For neutral current reactions, strangeness (and other flavor quantum numbers) are conserved — there are no flavor changing neutral currents in the SM.

  For charged currents, ΔS can be 0 or ± 1

- $\nu_\mu + e^- \to \nu_\mu + e^-$

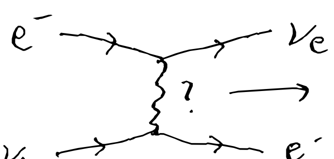
  Lepton numbers and charge must be conserved in each vertex:



$\to$ the mediator must be neutral!

$\nu_e + e^- \to \nu_e + e^-$

The following is possible:



$\longrightarrow$ Mediator is charged!

2 - M&S 6.5

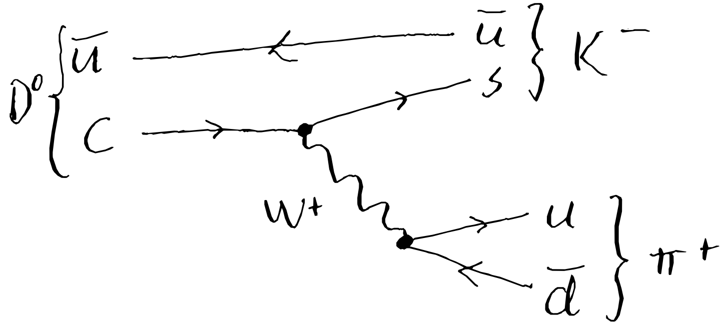
$D^0 \to K^- + \pi^+$ allowed $c\bar{u} \to s\bar{u}u\bar{d}$

$D^+ \to K^0 + \pi^+$ suppressed $c\bar{d} \to d\bar{s}\,u\bar{d}$

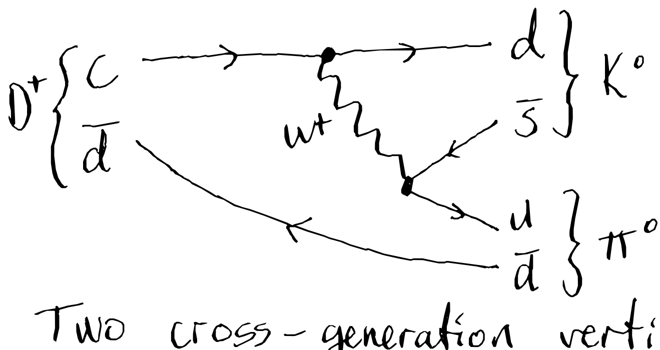
Quark model:

$D^0 = (c\bar{u}), \quad K^- = (s\bar{u}), \quad \pi^+ = (u\bar{d})$

$D^+ = (c\bar{d}), \quad K^0 = (d\bar{s})$



Both weak vertices couple quarks within the <u>same</u> generation



Two cross-generation vertices — strongly Cabibbo-suppressed!

3 - M&S 6.8

Quark generations:

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

c-quark decay can happen by a Wcd or Wcs vertex. $c \to b$ is kinematically forbidden. The Cabibbo-preferred vertex is Wcs, which causes $\Delta C = \pm 1$, $\Delta S = \pm 1$. The transition also causes $\Delta Q_h = \pm 1$, with the excess charge going to the leptons.

$\Rightarrow \Delta C = \Delta S = \Delta Q_h = \pm 1 \Rightarrow$ favored

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The suppressed vertex Wcd has $\Delta C = \pm 1$, $\Delta S = 0$, with $\Delta Q_h = \Delta C$.

$\Rightarrow \Delta C = \Delta Q_h = \pm 1$, $\Delta S = 0$ is suppressed but allowed.

Other combinations of ΔC, ΔS, ΔQ_h forbidden in semi-leptonic decays!

a) $D^+ \to K^- + \pi^+ + e^+ + \nu_e$

$(c\bar{d})$ $(s\bar{u})$

$\Delta C = -1, \quad \Delta S = -1, \quad \Delta Q_h = -1$

$\Rightarrow$ Allowed

b) $D^+ \to K^+ + \pi^- + e^+ + \nu_e$

$(c\bar{d})$ $(\bar{s}u)$

$\Delta C = -1, \quad \Delta S = +1$

$\Rightarrow$ Forbidden!

c) $D^+ \to \pi^+ + \pi^+ + e^- + \bar{\nu}_e$

$(c\bar{d})$ $(u\bar{d})$

$\Delta C = -1, \quad \Delta S = 0, \quad \Delta Q_h = +1$

$\Rightarrow$ Forbidden!

d) $D^+ \to \pi^+ + \pi^- + e^+ + \nu_e$

$(c\bar{d})$ $(u\bar{d})$ $(d\bar{u})$

$\Delta C = -1, \quad \Delta S = 0, \quad \Delta Q_h = -1$

$\Rightarrow$ Suppressed!

Problem 4 - M&S 6.9

Decays of strange hadrons:

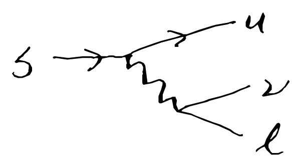
$K^+ = (u\bar{s}) \Rightarrow S = 1$

$\left.\begin{array}{l}\Xi^0 = (uss) \\ \Xi^- = (dss)\end{array}\right\} \Rightarrow S = -2$

$\Omega^- = (sss) \Rightarrow S = -3$

$\Sigma^- = (dds) \Rightarrow S = -1$

For semileptonic decays,

$s \to uW \to u + \text{leptons}$ 

or $\bar{s} \to \bar{u}W \to \bar{u} + \text{leptons}$

so, $\Delta S = \pm 1$ with $\boxed{\Delta Q_h = \Delta S = \pm 1}$

For hadronic weak decays no leptons can carry away excess charge, so $\Delta Q_h = 0$. In general we have 

, where all fermions are quarks. If no vertices involve an s quark we have $\Delta S = 0$. If one vertex involves an s quark and the other not, $\Delta S = \pm 1$. If both involve s quarks, we must have $\Delta S = 0$ since $\Delta Q = 0$. So for hadronic decays,

$$\boxed{\Delta S = \pm 1, 0 \quad \text{and} \quad \Delta Q_h = 0}$$

a) $K^+ \to \pi^+ + \pi^+ + e^- + \overline{\nu_e}$

Semileptonic. $\Delta S = -1$, $\Delta Q_h = +1$.
Forbidden

b) $K^- \to \pi^+ + \pi^- + e^- + \overline{\nu_e}$

Semileptonic. $\Delta S = +1$, $\Delta Q_h = +1$.
Allowed.

c) $\Xi^0 \to \Sigma^- + e^+ + \nu_e$

Semileptonic. $\Delta S = +1$, $\Delta Q_h = -1$
Forbidden

d) $\Omega^- \to \Xi^0 + e^- + \overline{\nu_e}$

Semileptonic. $\Delta S = +1$, $\Delta Q_h = +1$
Allowed

e) $\Xi^0 \to p + \pi^- + \pi^0$

Hadronic. $\Delta S = +2 \Rightarrow$ Forbidden

f) $\Omega^- \to \Xi^- + \pi^+ + \pi^-$

Hadronic. $\Delta S = +1$. Allowed

Problem 5 — M&S 6.10

Estimate BR for

a) $b \to c + e^- + \bar{\nu}_e$

b) $\tau^- \to e^- + \bar{\nu}_e + \nu_\tau$

Ignore Cabibbo-suppressed modes.

Quark-lepton symmetry:
Weak interactions are identical for

$$\begin{pmatrix} u \\ d \end{pmatrix} \leftrightarrow \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix} \leftrightarrow \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix} \leftrightarrow \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

So there is nothing in the interactions themselves that make a difference, but we must account for the possible final states.

a) $b \to c + e^- + \bar{\nu}_e$

Lepton universality tells us that

$b \to c + \mu^- + \bar{\nu}_\mu$, $b \to c + \tau^- + \nu_\tau$

are equally likely, given the mass does not hinder the decay. Here, it does not: $m_b \approx 4.2\,GeV$, $m_c \approx 1.3\,GeV$, $m_\tau \approx 1.8\,GeV$.

There are also the hadronic decays

$b \to c + \bar{u} + d$, $b \to c + \bar{c} + s$

The $\bar{u}+d$ or $\bar{c}+s$ state must be color neutral, but this can happen in 3 ways, so there are 3 such decay paths, per flavor combo.

$$\Rightarrow Br = \frac{1}{3 + (2 \cdot 3)} = \frac{1}{9}$$

leptonic　hadronic flavor　hadronic color

b) $\tau^- \to e^- + \bar{\nu}_e + \nu_\tau$

One also has $\tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau$ from lepton universality.
There is also the path

$$\tau^- \to \nu_\tau + d + \bar{u}$$

in 3 possible color states.
We can in principle have a decay into 2nd generation quarks, as

$$m_c + m_s = 1.27 \text{ GeV} + 93 \text{ MeV}$$
$$= 1.27 \text{ GeV} + 0.09 \text{ GeV}$$
$$= 1.36 \text{ GeV}$$
$$m_\tau = 1.78 \text{ GeV}.$$

But one must also supply enough energy that the quarks can hadronize. The lightest charmed states are D mesons, with $m \simeq 1.9$ GeV.
So we have

$$Br = \frac{1}{2 + (1 \cdot 3)} = \frac{1}{5} = 20\%$$

Measured value: 17.8 %