FYS3500: Particle Physics
Lecture Notes

Oskar Idland

Contents

1 History

- 1896: Henri Becquerel discovered radioactivity
- 1898: Marie and Pierre Curie discovered radium and polonium
- 1903: Alphas charge to mass ratio
- 1909: Alphas are helium nuclei
- 1911: Rutherford discovers the nucleus
- 1913: Bohr model of the atom
- 1917: Rutherford discovers the proton
- 1930: Neutrinos were postulated
- 1932: Chadwick discovers the neutron by shooting alpha particles at beryllium.
- 1938: Discovery of nuclear fission
- 1956: Neutrinos were detected

1.1 Proton Discovery: The Rutherford Scattering Experiment

Thomson's model of the atom was a positive sphere with electrons embedded in it. Rutherford wanted to test this model by shooting alpha particles at a thin gold foil surrounded by a detector foil. The alpha particles were shot from a radioactive source and when the alpha particles exited, they hit the foil and emitted light.

1.1.1 Conclusion

- Most alpha particles went straight through the foil. This implies the atom is mostly empty space.
- Some alpha particles were deflected by a small angle. This implies the positive charge is concentrated in a small volume.
- Sometimes the particles travel backwards. This implies the positive center has most of the mass of the atom.

1.2 Discovery of the Neutron

- Shooting alpha particles on beryllium which is much lighter than gold. This

2 Nucleus

- Very dense. Carries all the mass. $2.7 \cdot 10^{14}$ times denser than water.
- The atom is mostly empty space. If the nucleus was the size of a coin, the atom would be 2-3 km in radius.

2.1 Notation

- **Notation**: ${}^{A}_{Z}X_{N}$
- Isotope: Same **proton** number Z
- Isotone: Same **neutron** number N
- Isobar: Same **atomic** mass number $A = Z + N$

2.2 Nuclides

- 92 stable elements
- 280 stable isotopes
- 3000 unstable isotopes
- 6000 more predicted to exist

2.2.1 Stable Numbers

$$N = 2, 8, 20, 28, 50, 82, 126 \tag{1}$$

$$Z = 2, 8, 20, 28, 50, 82, \ldots \tag{2}$$

3 Units and Dimensions in Nuclear Physics

3.1 Length

The order of 10^{-15}m = 1fm (fermi/femtometer) meter. This is the distance between nucleons.

3.2 Time Scale

- 10^{-20}s: Unbound, in the case of nuclear reactions and decays.
- $10^{-9}/10^{-12}$s: lifetimes of excited nuclear states through gamma decays.
- Minutes/hours/millions of years: Alpha and beta decays.

3.3 Energy

MeV in nuclear physics.

$$1 \text{ MeV} = 1.6 \times 10^{-13}\text{J}. \tag{3}$$

$$1\text{eV} = 1.6 \times 10^{-19}\text{J} \tag{4}$$

Particle	Mass (kg)	Mass (u)	Mass (Mev/c^2)
1 atomic mass unit	1.660540×10^{-27} kg	1.000 u	931.5 MeV/c^2
neutron	1.674929×10^{-27} kg	1.008664 u	939.57 MeV/c^2
proton	1.672623×10^{-27} kg	1.007276 u	938.28 MeV/c^2
electron	9.109390×10^{-31} kg	0.00054858 u	0.511 MeV/c^2

Figure 1: Table of the masses of the nucleons. $c^2 = 931.5\text{MeV/u}$. In reality, the mass of the proton is slightly less than the mass of the neutron. The proton is 2000 times more massive than the electron.

3.4 Mass

u = unified atomic mass unit. 1 u is defined as 1/12 of the mass of an unbound ^{12}C atom. Mass is equivalent with energy. Therefore:

$$u = 931.5\ \text{MeV}/c^2 = 1.66 \times 10^{-27}\text{kg} \tag{5}$$

4 Nuclear Properties

The parameters which describe the nucleus are. There are two types of nuclear properties: static and dynamic.

- Static: Charge, Radius, mass, Binding energy, Angular momentum, Parity, Magnetic dipole moment, Electric quadrupole moments, Exited states and their energies.
- Dynamic: Shape, Decay

4.1 Connected Terms

- Charge/Charge Distribution: Protons. Found via electron scattering section 4.3 by the Coulomb interaction.
- Matter/Mass Distribution: Nucleons. Found via hadron scattering section 4.5, alpha particles (Rutherford), protons and neutrons by using the strong force.
- Radius: Size of the nucleus (nucleons)

4.2 Charge Distribution

To probe the charge distribution of the nucleus, we use charged particles. We also need the following:

- A beam of charged particles (often protons)
- Wavelength should be similar or smaller than the nucleus (about 10fm in diameter).
- Electrons were popular in the 50's.
- An energy of 100 Mev to 1 GeV is needed.
- Calculating the energy needed is done by using the de Broglie wavelength where $\lambda = h/p$ with $\lambda \leq 10\text{fm}$.

4.3 Nuclear Charge Distribution from Electron Scattering

- Radius increases with mass number A
- The central nuclear charge density is nea4rly the same for all nuclei. Nucleons do not seem to concentrate near the center of the nucleus, but instead have a constant distribution along the surface.
- The number of nucleons per unit volume is roughly constant:

$$\frac{A}{\frac{4}{3}\pi R^3} \approx \text{const} \tag{6}$$

- The radius of the nucleus is proportional to $A^{1/3}$.

$$R = R_0 A^{1/3} \quad , \quad R_0 \approx 1.2 \text{ fm} \tag{7}$$

4.4 Nuclear Size

We can find the radius of a nucleus by using the scattering angle of the local minimum of the Rutherford cross-section, see figure 2. The diffraction pattern is not exactly that of a circular disk, as the nucleus does not have a well-defined surface.

$$\sin\theta = \frac{1.22\lambda}{d} \Rightarrow R = \frac{d}{2} = \frac{1.22\lambda}{2\sin\theta} \tag{8}$$

This is only a rough estimate as the angle is calculated in two dimensions, instead of three.

4.5 Nuclear Mass Distribution from Hadron Scattering

- Electrons only mostly interact with protons. We therefore use hadrons to study the mass distribution of the nucleus.
- The radius is proportional to the nuclear rather than the Coulomb force.
- The Rutherford experiment showed that the nucleus is a point-like object.

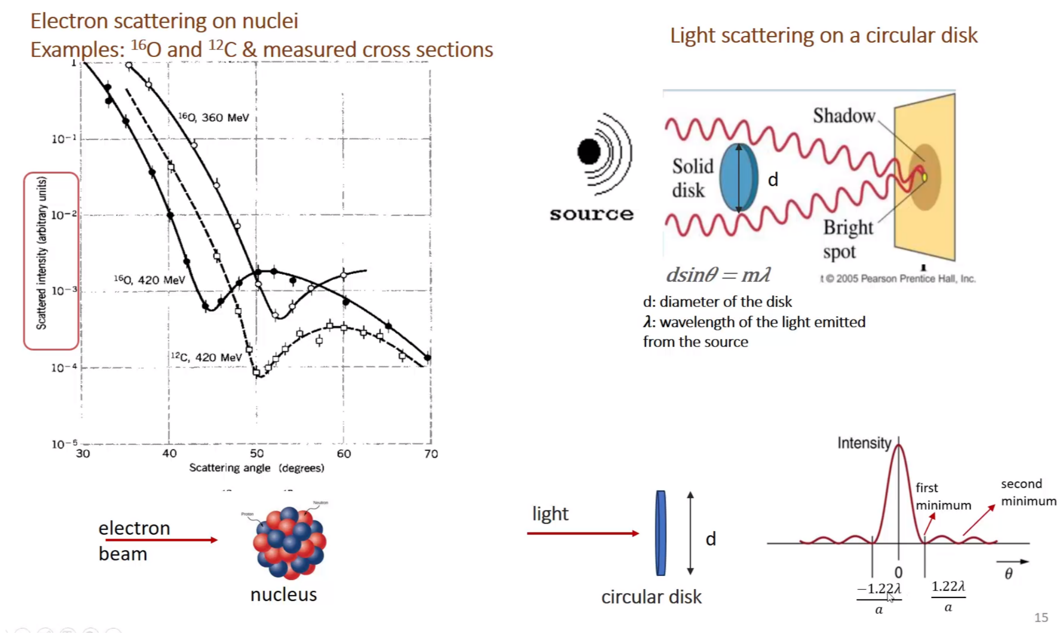

Figure 2: Example of the local minimum of the Rutherford cross-section. The angle is used to calculate the radius of the nucleus.

4.5.1 Fixed Angle of Observation with Changing Energy

- At low energies the alpha particles and the ^{208}Pb nucleus interact with the Coulomb interacting as with Rutherford scattering.
- With increasing energy, the repulsion from the Coulomb force is overcome, and the strong force becomes the dominant force. The Rutherford formula no longer holds.
- The alpha particles became absorbed by the nucleus and only a small fraction is scattered.
- When energy is high enough, we get the diffraction pattern.

4.6 Conclusion from Charge Radius Experiments

- The charge and mass radii of nuclei is nearly equal to within about 0.1fm.
- Both show the same $A^{1/3}$ dependence with $R_0 = 1.2$fm.
- As heavy nuclei have about 50 % more neutrons than protons, we might expect the neutron distribution to be more extended than the proton distribution. This is not the case as the neutrons pulls inwards, and the protons push outwards, until they are mixed such that the radius is the same.

4.7 Nuclear Mass

4.8 Deflection Spectrometer

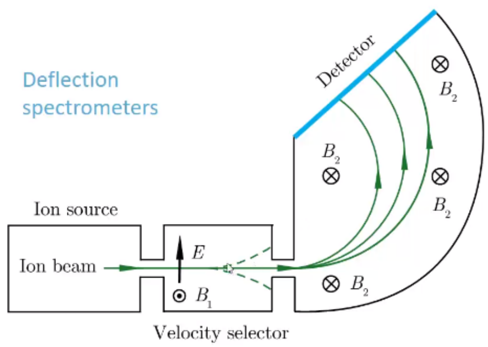

Figure 3: Experimental setup for measuring the mass of a particle.

- Shooting a ray of charge particles affected by a magnetic field and measuring the deflection we can calculate its mass.
- To measure an entire particle they must be ionized. The electrons carry so little mass that they are neglected.
- After ionization, the particles travel through an electric and magnetic field.
- Only the particles with the right velocity will pass through the fields and be subjected to the new magnetic field.
- The new field will deflect the particles according to their m/q value.

4.8.1 Calculating the Mass

$$F_B = q\vec{v} \times \vec{B} \tag{9}$$

The field and velocity are perpendicular.

$$F_B = qvB \tag{10}$$

$$F_E = F_B \Rightarrow qE = qvB \Rightarrow v = \frac{E}{B_1} \tag{11}$$

B_1 is the first magnetic field as seen in figure 3. The force from the magnetic field centripetal force.

$$F_B = \frac{mv^2}{r} = qvB_2 \tag{12}$$

$$\frac{mv}{r} = qB_2 \tag{13}$$

$$\frac{m}{q} = \frac{B_2\rho}{v} \tag{14}$$

The radius of the circle is given by $r = \rho$. Setting $B_1 = B_2$ gives the following for the mass.

$$m = \frac{B_1 B_2 \rho}{E} = \frac{B^2 \rho q}{E} \tag{15}$$

where q is the charge of the particle.

Accuracy

- These measurements are very important for mass models used in other parts of physics.
- The accuracy is about $\Delta m/m = 10^{6-}$, but that is not enough.
- The mass doublet technique gives a precision of 10^{-8} / 10^{-9}

5 Binding Energy

5.1 Formulas and Definitions

- Binding Energy: The energy required to keep the nucleus together. The mass of the nucleus is not equal to the sum of its parts. The mass of the individual nucleons is higher than the mass of the nucleus. The difference is the binding energy.

$$Zm_p + Nm_n - M_{\text{Nucleus}} = \text{Binding Energy} \quad \Rightarrow \quad Zm_p + Nm_n > M_{\text{Nucleus}} \tag{16}$$

5.2 Mass of the Nucleus

The total mass of the atom is the mass of the nucleus and electrons, minus the binding energy of the electrons.

$$M_{\text{Atom}} = M_{\text{Nucleus}} + Zm_e - \underbrace{\sum_{i=1}^{Z} B_i/c^2}_{\text{Often negligible}} \tag{17}$$

$$M_{\text{Atom}} = M_{\text{Nucleus}} + Zm_e \tag{18}$$

M usually refers to the mass of the entire atom, and so the subscript "Atom" is often omitted. We usually write the atom using the following notation:

$$M\left({}^{A}_{Z}X_N\right) = M_{\text{Nucleus}}\left({}^{A}_{Z}X_N\right) + Zm_e \tag{19}$$

Multiplying by c^2 we get the mass in energy units ($E = mc^2$):

$$M_{\text{Nucleus}}\left({}^{A}_{Z}X_N\right) = M\left({}^{A}_{Z}X_N\right) - Zm_e c^2 \tag{20}$$

$$\underline{\underline{M_{\text{Nucleus}}\left({}^{A}_{Z}X_N\right)c^2 = M\left({}^{A}_{Z}X_N\right)c^2 - Zm_e c^2}} \tag{21}$$

5.3 Nuclear Binding Energy (B.E.)

This energy is very small compared to the mass energy of the nucleus. We can derive this from the mass of the nucleus.

$$B.E. = \left(Zm_p + Nm_n - M_N\left({}_Z^AX_N\right)\right)c^2 \tag{22}$$
$$= \left(Zm_p + Nm_n - \left(M\left({}_Z^AX_N\right) - Zm_e\right)\right)c^2 \tag{23}$$
$$= \left(Z(\underbrace{m_p + m_e}_{\text{Hydrogen}}) + Nm_n - M\left({}_Z^AX_N\right)\right)c^2 \tag{24}$$
$$\tag{25}$$
$$B.E. = \underline{\underline{\left(Zm\left({}^1H\right) + Nm_n - M\left({}_Z^AX_N\right)\right)c^2}} \tag{26}$$

As the units so far has been energy (mc^2) we can switch to MeV.

$$B.E. = \left[mc^2\right] = \left[uc^2\right] = u931.5\text{MeV }/u \quad \Rightarrow \quad c^2 = 931.5\text{MeV/u} \tag{27}$$
$$B.E. = \underline{\underline{\left(Zm\left({}^1H\right) + Nm_n - M\left({}_Z^AX_N\right)\right)931.5\text{MeV/u}}} \tag{28}$$

5.3.1 Example: Helium ${}_2^4H_2$

We use the formula for binding energy from equation (28) to calculate the binding energy of the hydrogen atom ${}_2^4He_2$.

$$B.E. = \left(2m_p + 2m_n - M\left({}_2^4He_2\right)\right)931.5\text{MeV/u} \tag{29}$$
$$= (2 \cdot 1.007825u + 2 \cdot 1.008664u - 4.002603u) \cdot 931.5\text{MeV/u} \tag{30}$$
$$= \underline{\underline{0.0304 \cdot 931.5 \text{ MeV} = 28.3 \text{ MeV}}} \tag{31}$$

The ratio between the binding energy and the rest mass of the nucleus is very small. Using the binding energy from equation (31) and the mass of the helium nucleus, we can calculate the ratio:

$$\frac{28.3}{3728} = 0.75\% \tag{32}$$

5.4 Nuclear Separation Energy

The energy required to separate a proton S_p or a neutron S_n from the nucleus.

5.4.1 Neutron Separation Energy

It requires lower energy to remove a neutron from a nucleus with an odd number of neutrons. This is because one is unpaired.

$$S_n = \left(M\left({}_Z^{A-1}X_{N-1}\right) - M({}_Z^AX_N + m_n)\right)c^2 \tag{33}$$

This can also be expressed using binding energies as mass and energy are equivalent through $E = mc^2$:

$$S_n = B\left({}_Z^AX_N\right) - B\left({}_Z^{A-1}X_{N-1}\right) \tag{34}$$

5.4.2 Proton Separation Energy

Using the same logic as for the neutron separation energy section 5.4.1, we can express the proton separation energy through the binding energies. It's important to keep in mind that after loosing a proton, the element changes.

$$S_p = \left(M\left({}^{A-1}_{Z-1}Y_N\right) - M({}^{A}_{Z}X_N + \underbrace{m_p + m_n}_{{}^1_1H}) \right) c^2 \tag{35}$$

$$S_p = B\left({}^{A}_{Z}X_N\right) - B\left({}^{A-1}_{Z-1}Y_N\right) \tag{36}$$

5.5 Average Binding Energy

- Except for very light nuclei, the binding energy per nucleon is linear. It's almost constant at around 8 MeV/nucleon.
- The highest binding energy per nucleon is around $A = 60$ with the highest binding energy per nucleon at ^{56}Fe.
- When going from heavier elements towards iron we get nuclear fission
- When going from lighter elements towards iron we get nuclear fusion

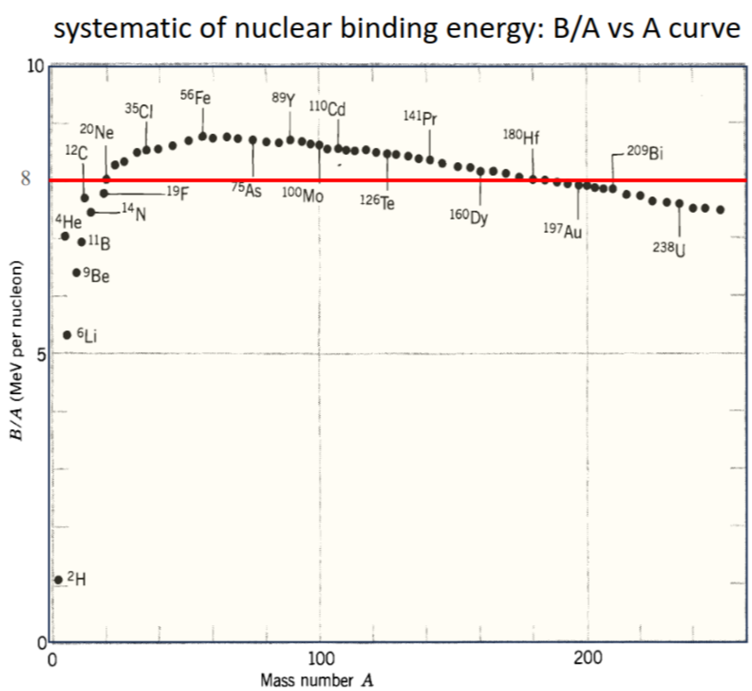

Figure 4

5.6 Semi-Empirical Mass Formula

- Sets out to explain the binding energies of nuclei.
- It is semi-empirical as the five of its constant are found by experiment.
- Tries to recreate the binding energy per nucleus graph in figure 4 by using the *liquid drop model*.

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{\text{asym}} \frac{(A-2Z)^2}{A} + \delta \tag{37}$$

5.6.1 Explanation of the Terms in the Semi-Empirical Mass Formula

- $\mathbf{a_v A}$: **Volume term**. The binding energy is proportional to the volume of the nucleus approximated to a sphere $\left(V = 4\pi R^3/3\right)$. This dominates the binding energy for large nuclei.

  $$a_v \approx 15.8 \text{ MeV} \tag{38}$$

  The linear dependence of the binding energy on the number of nucleons tells us that the strong force is short range as each nucleon only interacts with its nearest neighbors.

- $\mathbf{a_s A^{2/3}}$: **Surface term**. The volume term is not quite accurate as the nucleons on the surface have fewer neighbors. This term corrects for that. The binding energy is proportional to πR^2

  $$a_s \approx 16.8 \text{ MeV} \tag{39}$$

- $\mathbf{a_c Z(Z-1)A^{-1/3}}$: **Coulomb term**. The binding energy is reduced by the repulsion between the protons. It is therefore detracted. The Coulomb force is long range and is therefore proportional to $Z(Z-1)$ as all protons interact.

  $$a_c \approx 0.72 \text{ MeV} \tag{40}$$

- $\mathbf{a_{\text{asym}}(A-2Z)^2 A^{-1}}$: **Asymmetry term**. Stable nuclei have a balance between protons and neutrons. As the ratio of protons to neutrons deviate from 1, the nuclei becomes less stable (lower binding energy). This inhibits Hydrogen or Helium atoms with many neutrons. It is caused by the Pauli exclusion principle as nucleons are fermions and therefore can not occupy the same state at once.

  $$a_{\text{asym}} \approx 23 \text{ MeV} \tag{41}$$

  Heavier nuclei must have more neutrons to fight the Coulomb repulsion. The term gets relatively small as the number of nucleons increases.

- δ: **Pairing term**. This term is not included in the original formula, but is added to account for the fact that nuclei with an even number of protons and neutrons are more stable. This is because the nucleons in the same space-state can be coupled to have a total spin of 0. They are therefore closer together and therefore more tightly bound with a higher binding energy. This is called even-even nuclei.

  $$\delta = \begin{cases} +a_p S^{-3/4}, & \text{if even(N)-even(Z)} \\ 0, & \text{if odd(A)} \\ -a_p S^{-3/4}, & \text{if odd(N)-odd(Z)} \end{cases} \tag{42}$$

  $$a_p \approx 34 \text{ MeV} \tag{43}$$

5.6.2 SEMF Conclusion

- The semi-empirical mass formula was a first attempt at understanding how binding energy works.
- It is semi-empirical as the constants are found by experiment.
- A negative binding energy means the nucleus is not bound and is therefore not stable.

$$B = \underbrace{a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3}}_{\text{Liquid-drop model for energy calculations}} - \underbrace{a_{\text{asym}}(A-2Z)^2 A^{-1} + \delta}_{\text{Interactions between nucleons}} \tag{44}$$

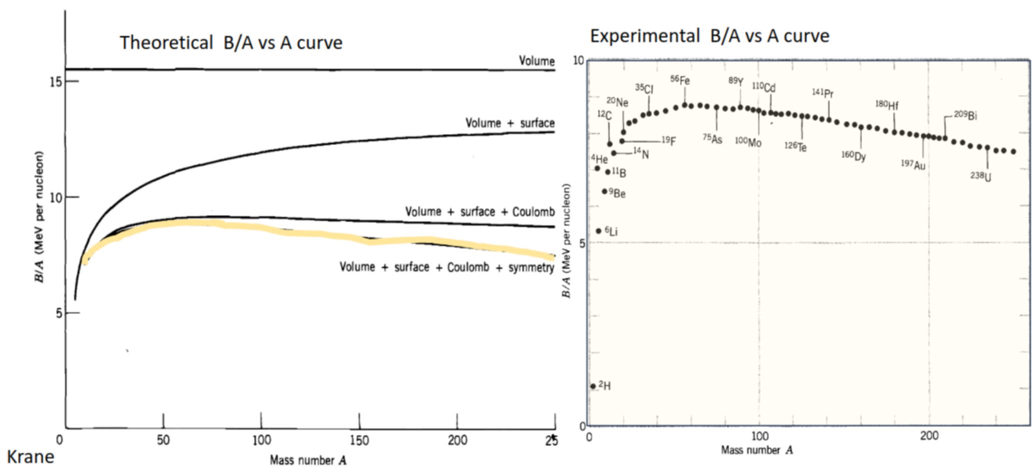

Figure 5: Plot of how the different terms in the semi-empirical mass formula equation (37) gets us closer to the experimental values

5.7 Mass Parabolas of Isobars

Isobars have the same number of nucleons (A).

$$M(A,Z) = Z(\overbrace{m_p + m_e}^{M(^1H)}) + (\underbrace{A-Z}_{\text{neut. num.}})m_n - B(A,Z)/c^2 \tag{45}$$

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + \delta(A,Z) \tag{46}$$

5.7.1 Finding the Minimum of the Mass Parabola

As the parabola is mass M as a function of Z, we can find the minimum by taking the derivative with respect to Z and setting it equal to zero.

$$\frac{\partial M}{\partial Z} = 0 \tag{47}$$

$$Z_{\text{min}} = \frac{(m_n - m_p - m_e) + a_c A^{-1/3} + 4a_{\text{sym}}}{2a_c A^{-1/3} + 8a_{\text{sym}} A^{-1}} \tag{48}$$

We can approximate this as the following:

$$Z_{\text{min}} \approx \frac{A}{2} - \frac{1}{1+\frac{1}{4}A^{2/3}a_c/a_{\text{sym}}} \quad , \quad a_{\text{sym}} \approx 23 \text{ MeV} \; , \; a_c \approx 0.72 \text{ MeV} \tag{49}$$

Example: A = 10 This is stable for smaller nuclei.

$$Z_{\text{min}} \approx 5 \quad \text{and} \quad \frac{Z_{\text{min}}}{A} \approx 0.5 \tag{50}$$

Example: A = 200 A lower ratio is stable for larger nuclei.

$$Z_{\text{min}} \approx 79 \quad \text{and} \quad \frac{Z_{\text{min}}}{A} \approx 0.4 \tag{51}$$

5.7.2 Valley of (beta) stability

- As can be seen in figure 6, we have two parabolas for $A = 128$ as it can be odd-odd or even-even. Higher binding energy is more stable.
- The even-even isobar is more stable as explained in section 5.6.1, because the nucleons can pair up in the same space-state with opposite spins and therefore be closer to each other and thus more stable.
- Only the atom in the bottom of the valley is stable. The others are prompt to beta decay downwards.
- Double beta decay can happen with even numbers of nucleons as can be seen for $A = 128$ with $Z = 52$, as $Z = 53$ has higher energy and it is therefore forced to decay all the way up to $Z = 54$.

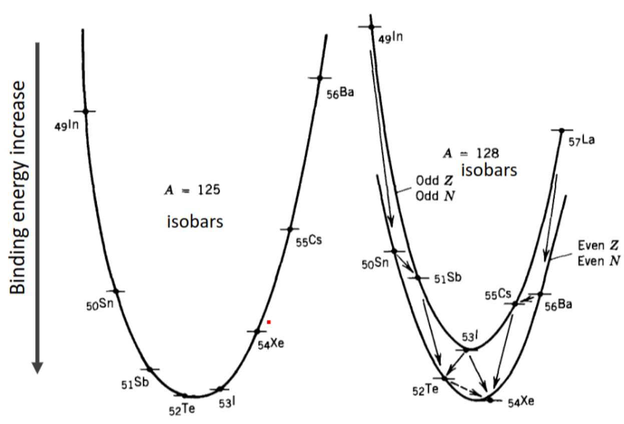

Figure 6: Valley of (beta) stability for different isobars with $A = 125$ and $A = 128$. The higher the binding energy, the more stable the isobar.

Beta Decay

- $\beta+$: Proton rich nuclei decay by converting a proton into a neutron, a positron and a neutrino.
- $\beta-$: Neutron rich nuclei decay by converting a neutron into a proton, an electron and an antineutrino.

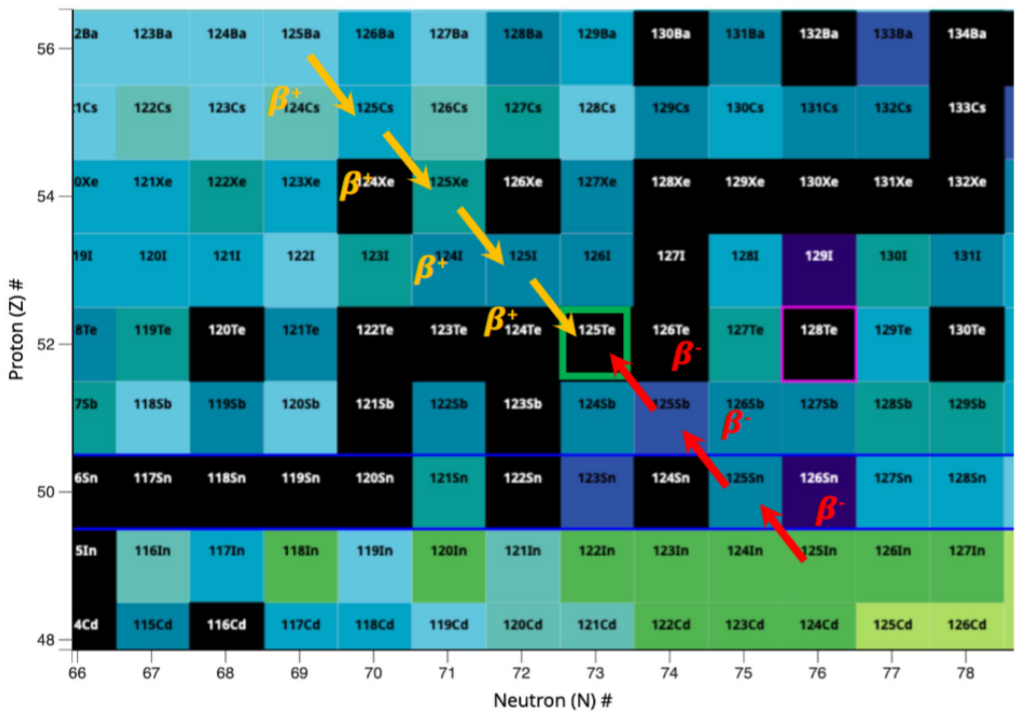

Figure 7: Chart showing different elements and their decays.