Problem 3

a)

The nuclear force is charge independent, and it does not distinguish between protons and neutrons. It is, however, *spin dependent.*

It includes a term $\propto \vec{s}_1 \cdot \vec{s}_2$, where $\vec{s}1$ and $\vec{s}2$ are the spins of the nucleons. As we can see from page 101 in Krane, the singlet state decreases the binding, while the triplet state increases it.

As we know from Pauli, identical fermions cannot be in the same state. In a n-n or p-p configuration, the spins cannot be aligned, and they must thus be in the spin-singlet state S=0.

-> Due to the spin dependence of the nuclear force, this decreases the binding, and the system is not bound.

In the p-n case, the particles are not identical, and can thus be found in either the singlet or triplet state. However, due to the same reasons as above, only the triplet state S=1 is bound.

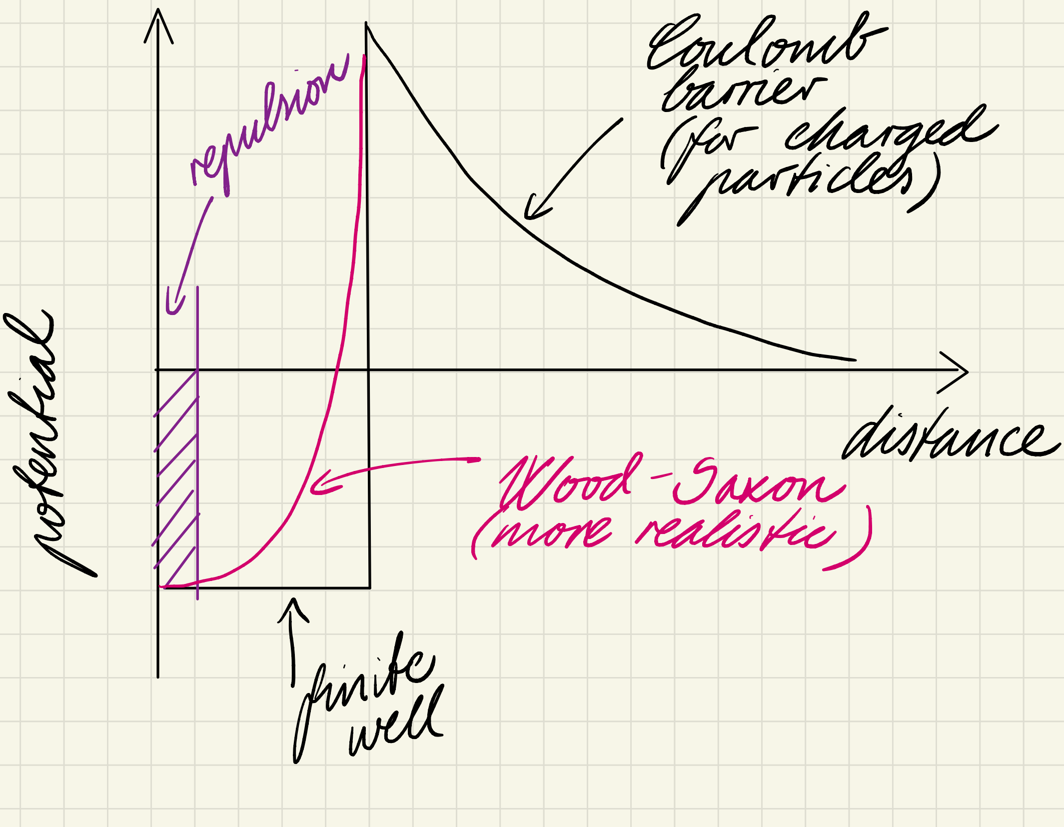
b) The most important contributions to the nuclear potential:

- The finite well created by the strong nuclear force, keeping the nucleus together. Acts at short distances, range about 1.2-1.5 fm. Becomes repulsive at very short distances; nucleons cannot be on top of each other.

- The Coulomb barrier that positive, charged particles must overcome to enter the nucleus.

- The spin-dependent term, giving the triplet state a deeper potential than the singlet state

- An asymmetric term (tensor potential), giving states with $l>0$ noncentral potentials



c) We know that the total ang. mom j is given by

$$\vec{j} = \vec{s} + \vec{l}$$

spin

orbital ang. mom

The p-n can either be in the singlet S=0 state or the triplet S=1 state.

However, due to the spin dependence of the nuclear force, S=1 is the most bound.
-> The ground state has S=1.

As parity is given by (-1)^*l* and we know the ground state has P=+1, only l:even configurations are possible. We would expect mostly *l*=0, but *l*=2 is also possible.

Experimentally: 96% *l*=0, 4% *l=2*

Problem 4

a) The deuteron radial wave function (remember : $\psi \propto \frac{u(r)}{r}$) is, assuming $l=0$, $E<0$ for bound states:

- for $r<R$, inside potential

$$u_{in}(r) = A\sin(k_1 r) + B\cos(k_1 r)$$

$$k_1 = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

- for $r>R$, outside the potential

$$u_{out}(r) = C e^{-k_2 r} + D e^{+k_2 r}$$

$$k_2 = \sqrt{-2mE/\hbar^2}$$

For $r \to \infty$, $u_{out}(r) \to 0$

$\Rightarrow D = 0$

For $r \to 0$, the ratio $\Psi = \frac{u(r)}{r}$

should remain finite. The only way to fix this is to make $u(r) \to 0$ as well.

$\sin(k_1 r) \to 0$ when $r \to 0$, but

$\cos(k_1 r) \to 1$

$\Rightarrow B = 0$

The wave function must be continous at the border $r = R$

- $u_{in}(r=R) = u_{out}(r=R)$

1) $A \sin(k_1 R) = C e^{-k_2 R}$

- $\frac{du_{in}}{dr}(r=R) = \frac{du_{out}}{dr}(r=R)$

2) $Ak_1 \cos(k_1 R) = -Ck_2 e^{-k_2 R}$

By dividing 2) on 1):

$$k_1 \cot(k_1 R) = -Ck_2$$

The wave function must also be properly normalized:

$$\int dV \, \Psi^* \Psi = 1$$

$\int dV$ in radial coordinates:

$$\iiint r^2 \sin\varphi \, dr \, d\varphi \, d\vartheta = \int_0^\infty 4\pi r^2 dr$$

As $\Psi = \frac{u(r)}{r}$

$$\Rightarrow \int_0^\infty 4\pi u^2(r) \, dr = 1$$

Divide $u(r)$ in $0 \to R$ and $R \to \infty$:

$$\left\{ \int_0^R 4\pi\, u_{in}^2(r)\, dr + \int_R^\infty 4\pi\, u_{out}^2(r)\, dr = 1 \right.$$

$$\left\{ \int_0^R 4\pi A^2 \sin^2(k_2 r)\, dr + \int_R^\infty 4\pi C^2 e^{-2k_2 r}\, dr = 1 \right.$$

$$\left\{ \begin{array}{l} 4\pi A^2 \left[\frac{r}{2} - \frac{\sin(2k_1 r)}{4k_1} \right]_0^R + \\ 4\pi C^2 \left[-\frac{e^{-2k_2 r}}{2k_2} \right]_R^\infty = 1 \end{array} \right.$$

$$\left\{ \begin{array}{l} 4\pi A^2 \left[\frac{R}{2} - \frac{\sin(2k_1 R)}{4k_1} - \frac{0}{2} + \frac{\sin(2k_1 \cdot 0)}{0} \right] \\ + 4\pi C^2 \left[\underbrace{\frac{-e^{-2k_2 r \to \infty}}{2k_2}}_{0} + \frac{e^{-2k_2 R}}{2k_2} \right] = 1 \end{array} \right.$$

$$\left\{ 2\pi A^2 \left(R - \frac{\sin(2k_1 R)}{2k_1} \right) + \frac{2\pi C^2 e^{(-2k_2 R)}}{k_2} = 1 \right.$$

Using that

$$A\sin(k_1R) = Ce^{-k_2R}$$

$$Ak_1\cos(k_1R) = -Ck_2e^{-k_2R}$$

insert

$$2\pi A^2\left(R - \frac{\sin(2k_1R)}{2k_1}\right) + \frac{2\pi C^2 e^{(-2k_2R)}}{k_2} = 1$$

$$2\pi A^2\left(R - \frac{\sin(2k_1R)}{2k_1}\right) + \frac{2\pi A^2\sin^2(k_1R)}{k_2} = 1$$

Use that $\sin(2x) = 2\sin x\cos x$

$$2\pi A^2\left(R - \frac{1}{k_1}\sin(k_1R)\cos(k_1R) + \frac{\sin^2(k_1R)}{k_2}\right) = 1$$

$$A = \left[2\pi\left(R - \frac{\sin(k_1R)\cos(k_1R)}{k_1} + \frac{\sin^2(k_1R)}{k_2}\right)\right]^{-1/2}$$

Using that

$$\frac{k_1 \cos(k_1 R)}{\sin(k_1 R)} = -k_2 \Rightarrow k_1 = -\frac{\sin(k_1 R)}{\cos(k_1 R)} k_2$$

$$\left\{ A = \left[2\pi \left(R - \frac{\sin(k_1 R)\cos(k_1 R)}{k_1} + \frac{\sin^2(k_1 R)}{k_2} \right] ^{-1/2} \right.$$

$$A = \left[2\pi \left(R + \frac{\cos^2(k_1 R)}{k_2} + \frac{\sin^2(k_1 R)}{k_2} \right] ^{-1/2}$$

$$\underline{\underline{A = \left[2\pi \left(R + 1/k_2 \right) \right]^{-1/2}}}$$

Using that

$$A \sin(k_1 R) = C e^{-k_2 R}$$

$$\Rightarrow C = \frac{A \sin(k_1 R)}{e^{-k_2 R}}$$

$$\underline{\underline{= \left[2\pi \left(R + 1/k_2 \right) \right]^{-1/2} \frac{\sin(k_1 R)}{e^{-k_2 R}}}}$$

b)

For a particle to have a magnetic (dipole) moment, it must have spin and charge; as we know, charged particles in motion creates magnetic fields. It has contribution from both the orbital angular momentum *l* and the spin *s* of a state.

The neutron has a non-zero magnetic moment even if it has zero charge, as it consists of quarks with charge. The quarks generate the magnetic moment, and this confirms that the neutron is not an elementary particle.

c) The magnetic moment of the deuteron is measured to be

$(0.85744376 \pm 0.0000004)\mu_N$

We know from exercise 1c) that both $l=0$ and $l=2$ are allowed for the ground state of the deuteron, and thus its wave function will include components of both.

We use the measured value for the magnetic moment to set up the following equation:

$\mu_{exp} = a_s^2 \mu(l=0) + a_d^2 \mu(l=2)$, where normalization requires

$$a_s^2 + a_d^2 = 1$$

$$0.85744376 = a_s^2 \cdot 0.8798 + (1-a_s^2) \cdot 0.3101$$

$$\Rightarrow \underline{\underline{a_s^2 = 0.96 \text{ and } a_d^2 = 0.04}}$$