FYS3500 Spring 2024 - Problem set 7

Topic: Nuclear Models

Concepts of the week

Explain these concepts: Nuclear Models, Fermi-Gas Model, Shell Model

Problem 1 Shell model

a) What are experimental evidences for the nuclear shell model?

b) What are the magic numbers?

c) Give an overview of the spectroscopic notation we use in the shell model. What decides the number of nucleons that fits in each shell?

d) What is the ground state spin ("total angular momentum") and parity J^P of even-even nuclei?

e) How can we determine the ground state spin of odd-even nuclei? Give the gs spin and parity for ^{15}O, ^{16}O and ^{17}O. You will need figure 8.4 in M&S2019.

f) What is the additional challenge for odd-odd nuclei? What other effects may create deviations from the extreme single particle model?

Problem 2 Deduce shell configuration

a) Assume that in the shell model the nucleon energy levels are ordered as shown in figure 8.4. Write down the ground state shell model configuration of the nucleus ^{7}Li and hence find its ground state spin, parity, and magnetic moment (in nuclear magnetons). Give the two most likely configurations for the first excited state, assuming that only protons are excited. Look up the experimentally determined first excited state of ^{7}Li, e.g at `https://www.nndc.bnl.gov/nudat2/`.

b) The ground-state spin of ^{17}F is $J^P = 5/2^+$, and of the first excited state it is $J^P = 1/2^+$. The second excited state is $J^P = 1/2^-$. Give the configurations for protons and neutron of ^{17}F in the ground-state, and the first and second excited states.

Problem 3 LS-coupling in the shell model

In order to get the correct magic numbers in the nuclear shell model we need to include a spin-orbit coupling. This leads to a Hamiltonian on the form:

$$\hat{H} = \hat{H}_0 - V_{\text{SO}}\hat{\vec{l}} \cdot \hat{\vec{s}} \tag{1}$$

Where $\hat{H}_0$ is a Hamiltonian with eigenstates $\hat{H}_0 |N, l\rangle = \hbar\omega(N + 3/2) |N, l\rangle$ with $l = N, N - 2, N - 4, \ldots 1$ or $0, l \geq 0$ and $V_{\text{SO}}\hat{\vec{L}} \cdot \hat{\vec{S}}$ is the spin-orbit coupling. V_{SO} is the strength of the coupling and can be regarded as a constant in this problem. In this problem we only look at spin-1/2 fermions.

a) The operator for the total spin is $\hat{\vec{j}} = \hat{\vec{l}} + \hat{\vec{s}}$, where $\hat{\vec{l}}$ is the angular momentum operator and $\hat{\vec{s}}$ is the spin operator. Show that:

$$\hat{\vec{l}} \cdot \hat{\vec{s}} = \frac{1}{2}(\hat{\vec{j}}^2 - \hat{\vec{l}}^2 - \hat{\vec{s}}^2)$$

b) What is the expectation value $\langle \hat{\vec{l}} \cdot \hat{\vec{s}} \rangle$?

c) Find an expression for the energy splitting between $j = l + 1/2$ and $j = l - 1/2$ states. Comment on the result. Check in figure 8.4 in M&S2019 which of the states $j = l \pm 1/2$ lies at the lowest energy, and explain this using equation (1).