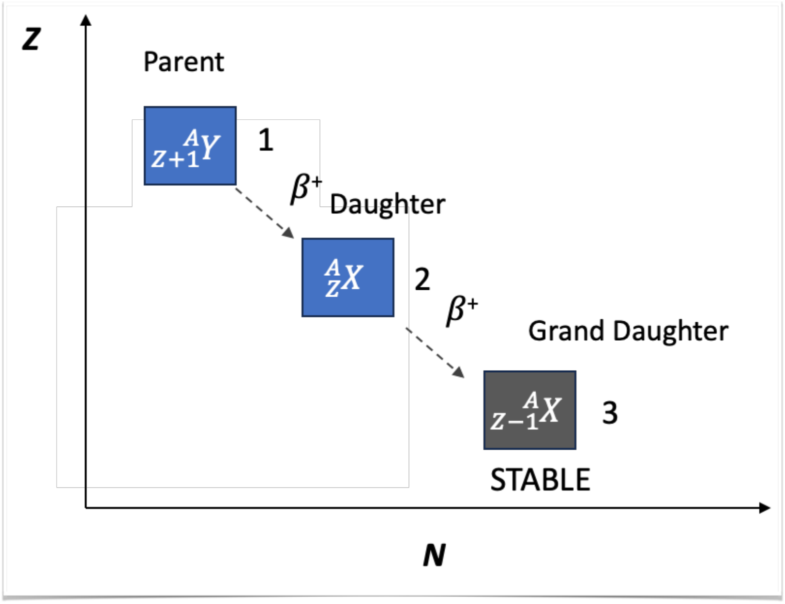
Radioactive decay chain and Bateman equations

Consider a chain of radioactive decays of nucleus 1, 2, and 3:

$1 \rightarrow 2 \rightarrow 3,$

where nucleus 3 is stable and the decays $1 \rightarrow 2$ and $2 \rightarrow 3$ are characterized by the decay constants $\lambda_1$and λ_2 respectively. The amount of nuclei 1, 2, and 3 that we have at time t is described by $N_1(t), N_2(t),$ and $N_3(t)$.

At $t = 0,\ N_1(t = 0) = N_0$ and $N_2(t = 0) = N_3(t = 0) = 0.$



It can be shown that:

$$N_1(t) = N_0 e^{-\lambda_1 t}$$

$$N_2(t) = N_0 \frac{\lambda_1 (e^{-\lambda_1 t} - e^{-\lambda_2 t})}{\lambda_2 - \lambda_1}$$

$$N_3(t) = N_0 \frac{\lambda_1 (1 - e^{-\lambda_2 t}) - \lambda_2 (1 - e^{-\lambda_1 t})}{\lambda_1 - \lambda_2}$$

These equations are called Bateman equations. You don't have to derive them unless you really want to.

We look at the decay chain ^{139}Cs → ^{139}Ba → ^{139}La, observed from an initially pure sample of 1mCi ^{139}Cs.

a) Calculate the amount of ^{139}Cs present at $t = 0$.
b) Write a script that plots the number of ^{139}Cs, ^{139}Ba, and ^{139}La as a function of time. Also plot the activities, and let the time axis run from 0 to 12 hours in both cases. Explain what you see.
c) What is the maximum activity of ^{139}Ba and when does it occur?
d) At what time are the activities of ^{139}Cs and ^{139}Ba equally large? Comment in light of what you found in c).