FYS3500 Spring 2024 - Problem set 2

Topic: Binding energy, Properties of the nuclear force

Concepts of the week

Explain these concepts: Semi-Empirical Mass Formula (SEMF), Liquid Drop Model (LDM), the Nuclear Force

Problem 1: Binding energy

a) The mass of ^{27}Al and ^{235}U has been measured to be 26.9815 u and 235.0439 u respectively. Use this to estimate the binding energy of the two nuclei from equation (1)

b) Find the binding energies of ^{27}Al and ^{235}U using the semi empirical binding energy formula in equation (2). How well does it agree with the result in a) and what are possible reasons for the deviations?
(Use $a_v = 15.5$ MeV, $a_s = 16.8$ MeV, $a_c = 0.72$ MeV and $a_{\text{sym}} = 23$ MeV).

Problem 2: Semi-empirical mass formula

The relation between the mass and the binding energy is

$$M(A,Z) = Z(m_p + m_e) + (A - Z)m_n - \frac{1}{c^2}B(A,Z) \tag{1}$$

The semi-empirical binding energy formula is

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}} \frac{(A-2Z)^2}{A} (+\delta) \tag{2}$$

where a_v, a_s, a_c and a_{sym} are constants.

a) Explain the origin of the different terms in $B(A,Z)$. Why is the formula called semi-empirical?

b) Explain which terms in B(A,Z) that dominates for light and heavy nuclei respectively. What does this mean for the observed $\frac{N}{A}$ ratio among light and heavy nuclei?

c) Use (1) and 2 to show that for a given A, the Z value that corresponds to the lowest-mass isobar is

$$Z_{\text{min}} = \frac{m_n c^2 - (m_p + m_e)c^2 + a_c A^{-1/3} + 4a_{\text{sym}}}{2a_c A^{-1/3} + 8a_{\text{sym}} A^{-1}}$$

Problem 3: Nuclear force

a) We say the nuclear force is charge independent. Still, a proton and a neutron can form a bound state but two protons or two neutrons cannot. Why?

b) List and briefly explain the most important contributions to the nuclear potential.

c) We know from experiment that the parity P of the ground state of the deuteron is $(+1)$. What can we then learn about the ground state spin and orbital angular momentum configurations? (Hint: you also need to know about the spin-dependence of the nuclear force)

Problem 4 Deuteron and the nuclear force

A simple quantum mechanical model of the deuteron is based on a spherically symmetric potential well given by the following equation:

$$\begin{aligned} V(r) &= -V_0 \qquad \text{for } r < R \\ &= 0 \qquad \text{for } r > R \end{aligned} \tag{3}$$

a) *Extra* Use the continuity and normalization conditions to evaluate the coefficients A and C in equation (4.4) and (4.5) in the deuteron wave function, pg 82 in Krane. Show that

$$A = [2\pi(R + 1/k_2)]^{-\frac{1}{2}} \tag{4}$$

and

$$C = [2\pi(R + 1/k_2)]^{-\frac{1}{2}} \cdot \frac{\sin(k_1 R)}{e^{-k_2 R}} \tag{5}$$

b) What is the origin of the magnetic moment of a nucleon? Why does the neutron have a non-zero magnetic moment?

c) Search online to find the experimentally determined value for the magnetic moment of the deuteron. Using the values $\mu_{d,L=0} = 0.8798\ \mu_N$ and $\mu_{d,L=2} = 0.3101\ \mu_N$, find the percentage admixture of the $(L = 2)$ state in the ground state of the deuteron. Remember, the wave function must be normalized!