FYS3500 Solutions 9

Problem 1

a) There is a range of different decays possible for the nucleus, but the most common are:

α - decay

$$^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}Y_{N-2} + \underbrace{^{4}_{2}He}_{\alpha}$$

An α-particle is emitted from the nucleus, common for heavy nuclei

β - decay

$$\beta^{-}: {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+1}Y_{N-1} + e^{-} + \bar{\nu}_e$$

↳ neutron transformed into proton, electron, and anti-ν_e, common for neutron-rich nuclei

$\beta^+ : {}^A_Z X_N \rightarrow {}^{A}_{Z-1} Y_{N+1} + e^+ + \nu_e$

↳ common for proton-rich nuclei

electron capture

${}^A_Z X_N + e^- \rightarrow {}^{A}_{Z-1} Y_{N+1} + \nu_e$

↳ Electron from inner electron shell captured by nucleus

γ decay

${}^A_Z X_N^* \rightarrow {}^A_Z X_N + \gamma$

↳ excited nucleus emits γ ray to rid itself of excitation energy

Other ways: spontaneous fission, internal conversion, internal pair creation, cluster emission

b) The number of nuclei N that decay per time t' is given as: $\frac{dN}{dt'}$

This is equal to $-\lambda N$, where λ is the decay constant $\lambda = 1/\tau$

$$\frac{dN}{dt'} = -\lambda N$$

$$\frac{dN}{N} = -\lambda\, dt'$$

Integrate both sides:

$$\int_{N=N_0}^{N=N(t)} \frac{dN}{N} = -\lambda \int_{t'=0}^{t'=t} dt$$

#nuclei at $t=0$

$$\ln N(t) - \ln N_0 = -\lambda t$$

$$\ln N(t) = \ln N_0 - \lambda t$$

$$e^{\ln N(t)} = e^{\ln N_0 - \lambda t}$$

$$N(t) = e^{\ln N_0} e^{-\lambda t}$$

$$\underline{\underline{N(t) = N_0 e^{-\lambda t}}}$$

Half-life $t_{1/2}$: the time it takes half the nuclei in a sample to decay

$$N(t = t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$\underbrace{\ln 1}_{0} + \ln 2 = +\lambda t_{1/2}$$

$$\lambda = \frac{\ln 2}{t_{1/2}} \Rightarrow \underline{\underline{N(t) = N_0 e^{-\frac{\ln 2}{t_{1/2}} t}}}$$

c) The relation between the width Γ and lifetime τ:

$$\Gamma = \frac{\hbar}{\tau}$$

$\Rightarrow$ longer half-life, shorter width of state

We only have discrete states as long as the widths of different states don't overlap:

-> at high excitation energy, the states are so close that we cannot distinguish between them. Here it doesn't make sense anymore to talk about discrete states (we are in the continum of states)

d) Branching ratios: when a state can decay in different ways such that

$$\Gamma = \sum_i \Gamma_i$$

total width

individual decay widths

ex. $^{73}_{36}Kr$: β^+ 99.3%

p : 0.7%

e)

Because alpha particles, having Z=2, has a very short range in matter and is easily stopped. The alpha particle is stopped by the outermost layer of the skin, beta-particles penetrate to deeper layer of the skin, while gamma radiation might go straight through the hand.

But: to inhale/eat an alpha-emitter can create a lot of damage internally. Which is why Norwegian homes are tested for Radon...

Problem 2

a) $Q = m(^{242}Pu) - m(^{238}U) - m(\alpha)$

$= 242.058743997u - 238.050789466u$
$- 4.002260325413u$
$= 5.694205587 \cdot 10^{-3} u$

$1u = 931.5 \text{ MeV}$

$\Rightarrow \underline{\underline{Q \approx 5.3 \text{ MeV}}}$

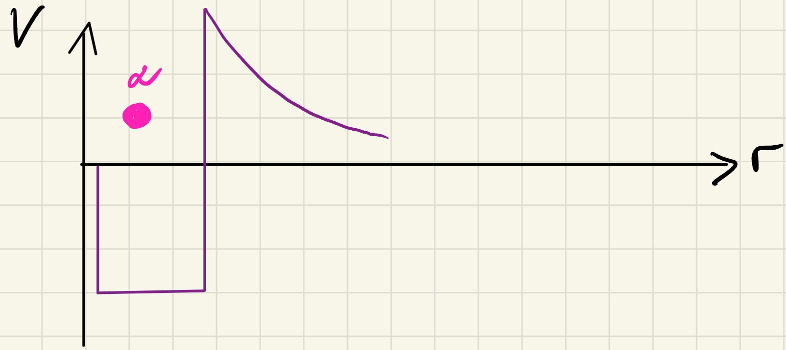
b) Due to recoil, the α cannot get the whole Q-value as kinetic energy. The kinetic energy is given as

$$T_\alpha = \frac{Q}{\left(1 + \frac{m_\alpha}{m_{x'}}\right)} = \frac{5.3 \text{ MeV}}{\left(1 + \frac{4.002260325413u}{238.050789466u}\right)}$$

$$\underline{\underline{\approx 5.2 \text{ MeV}}}$$

Problem 3

a)



The model commonly used for alpha-decay is that the alpha particle is formed in the potential well of the nucleus prior to emission, and gains (most of) the reaction Q-value as kinetic energy. This is not enough to overcome the Coulomb barrier, so the alpha-particle repeatedly "knocks" against the barrier, until it eventually tunnels through.

Experimental observation: the higher the Q-value, the shorter half-life to alpha decay.

b)

4He is a doubly-magic nucleus, and is thus especially tightly bound. Thus the alpha-particle gets more kinetic energy when it is formed in the potential well, making it easier to tunnel through the Coulomb barrier. Furthermore, it is low-Z, and the barrier increases with Z.

It is found that other, larger clusters can be emitted when heavy nuclei decay, which is referred to as cluster emission.

e.g. $^{233}_{88}Ra \rightarrow ^{209}_{82}Pb + ^{14}_{6}C$

c) α - decay is a two-body decay

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$$

so all the released energy (the Q-value) is shared between the daughter nucleus and the alpha particle. However, the daughter nucleus might be left in an excited state, and since the lowest ex states are discrete, the alpha energy spectrum shows discrete energies:

To conserve momentum, the daughter nucleus recoils and thus some of the Q-value goes to this. Also, the daughter nucleus can be left in an excited state.

Problem 4

$1\mu C$ at $t=0$

$t_{1/2}$: 1.0 s, 1.0 h, 1.0 d

a) From the radioactive decay law that we derived in Problem 1:

$$N(t) = N_0 \, e^{-\left(\frac{\ln 2 \, t}{t_{1/2}}\right)}$$

The activity A is given as

$$A(t) = N(t) \cdot \lambda = N_0 e^{\left(\frac{-\ln 2 \, t}{t_{1/2}}\right)} \cdot \frac{\ln 2}{t_{1/2}}$$

$$A(t=0) = \frac{N_0 \ln 2}{t_{1/2}}$$

$$\Rightarrow N_0 = \frac{A \cdot t_{1/2}}{\ln 2}$$

1 Curie : $3.7 \cdot 10^{10}$ decays per second

- $t_{1/2} = 1.0s$

$$N_0 = \frac{1.0 \cdot 10^{-6} \cdot 3.7 \cdot 10^{10} s^{-1} \cdot 10s}{\ln 2}$$

$$\approx \underline{5.34 \cdot 10^{4}}$$

- $t_{1/2} = 1.0h$

$$N_0 = \frac{1.0 \cdot 10^{-6} \cdot 3.7 \cdot 10^{10} s^{-1} \cdot 3600 s}{\ln 2}$$

$$\approx \underline{1.92 \cdot 10^{8}}$$

- $t_{1/2} = 1.0d$

$$N_0 = \frac{1.0 \cdot 10^{-6} \cdot 3.7 \cdot 10^{10} s^{-1} \cdot 3600 s \cdot 24}{\ln 2}$$

$$= \underline{4.61 \cdot 10^{9}}$$

b)

$$\Delta N = N(t=0) - N(t=1s)$$

$$= N_0 - N_0 e^{-\frac{\ln 2 t}{t_{1/2}}}$$

$$= N_0 \left(1 - e^{-\frac{\ln 2 t}{t_{1/2}}}\right)$$

- $t_{1/2} = 1.0s$

  $\underline{\Delta N \approx 2.67 \cdot 10^4}$

- $t_{1/2} = 1.0h = 3600s$

  $\underline{\Delta N \approx 3.70 \cdot 10^4}$

- $t_{1/2} = 3600s \cdot 24$

  $\underline{\Delta N \approx 3.7 \cdot 10^4}$

Problem 5

a) The timescale must be so that there is a sufficient amount of decays detected, without having all the 14C in the sample having already decayed.

14C has a half-life of approx 5700 years, which determines the time scale of 14C dating.

b) Test of nuclear weapons; more than the usual amount of ^{14}C was produced

c) ^{235}U : $7.04 \cdot 10^8$ yr, 0.72%

^{238}U : $4.4 \cdot 10^9$ yr, 99.28%

equal

$$\frac{N_{238}}{N_{235}} = \frac{\cancel{N_0} e^{\frac{-\ln 2 t}{t_{1/2, 238}}}}{\cancel{N_0} e^{\frac{-\ln 2 t}{t_{1/2, 235}}}} = \frac{\cancel{N} \cdot 0.9928}{\cancel{N} \cdot 0.0072}$$

$$e^{\frac{-\ln 2 t}{t_{1/2, 238}} + \frac{\ln t}{t_{1/2, 235}}} = \frac{0.9928}{0.0072}$$

$$\frac{-\ln 2\, t}{t_{1/2,238}} + \frac{\ln 2\, t}{t_{1/2,235}} = \ln\left(\frac{0.9928}{0.0072}\right)$$

$$8.29448t = \ln\left(\frac{0.9928}{0.0072}\right)$$

$$\Rightarrow \underline{t = 5.9 \cdot 10^9 \text{ years}}$$

(real age: $\sim 4.6 \cdot 10^9$ yr)

This is based on the assumption that 235U and 238U were equally abundant in the beginning. Probably not true. Elements heavier than iron are produced in different processes, like the s-process, r-process, p-process...

d) After $2.5 \cdot 10^9$ yr, $\left(1 - e^{\frac{-\ln 2\, t}{t_{1/2}}}\right)$ of the original material has decayed

$\Rightarrow \sim 32\%$