FYS3500 - Problem set 9

Topic: Nuclear decay, α-decay

Make sure you have read the relevant chapters in the book and compendium before starting the exercises.

Problem 1 Decay

a) What are common ways for a nucleus to decay?

b) Derive the decay law

$$N(t) = N_0 e^{-\lambda t}, \tag{1}$$

where λ is the decay constant. Then use the definition of half-life $t_{1/2}$ to write (1) in terms of $t_{1/2}$ instead of λ.

c) What is the relation between the width Γ of a state and its lifetime? How does Γ effect the notion of discrete states?

d) Explain what branching ratios are.

e) Why can I hold an α source in my hand without necessarily suffering a lot of radiation damage?

Problem 2 Q-value in α decay

The difference in mass before and after a reaction is known as the reaction Q-value, and is for the reaction $x + X \to y + Y$ given as

$$Q = [m(X) + m(X) - m(y) - m(Y)] \tag{2}$$

(see M&S2019 p. 67 or 119).

a) Find the Q-value for the decay $^{242}\text{Pu} \to^{238} \text{U} + \alpha$. Hint: a table of nuclear masses can be found for example here: `https://wwwndc.jaea.go.jp/NuC/index.html`.

b) Calculate the kinetic energy of the α particle.

Problem 3 α decay and cluster emission

a) What is the standard picture for α-decay? Do you know what experimental observations justifies this model?

b) Explain why the α particle is favourable to emit (why e.g. not ^{6}Li instead). In a source of your choice, read about "cluster decay". Do you see the relation to α decays?

c) What does the energy spectrum of the α-particle from an α-decay look like? Why does the energy of the α-particle deviate from the Q-value of the decay?

Problem 4 Radioactive Sources

Three radioactive sources each have activities of 1 μCi at t=0. Their half-lives are, respectively, 1.0 s, 1.0 h, and 1.0 d.

a) How many radioactive nuclei are present at $t = 0$ in each source?

b) How many nuclei of each source decay between $t = 0$ and $t = 1$ s?

Problem 5 Nuclear Archeology and nuclear physics in archeology

^{14}C is used to determine the age of fossils and other organic materials. The idea is that as long an organism is alive, it constantly is exchanging carbon with its environment (eating and excreting) and so the isotopic composition of the organism matches that of the atmosphere. Once the organism dies, this exchange stops, and the ^{14}C trapped in the system start decaying. Thus the ratio $^{14}C/^{12}C$ can tell us something about when the organism died, assuming that the ratio was the same in the past as it is today (which is almost true, but hang on for a surprise . . .).

a) What determines the time-scales that we can use this method on?

b) What gave a sharp rise to the fraction of ^{14}C in the atmosphere from about the 1960s?

If we want to look at events that take much longer than 5000 years, it's useful to look for radioactive decays that have much longer half lives. If you poke around the periodic table, you find that heavy elements often have radioactive isotopes with half lives measured in billions of years. Naturally occurring uranium is a mixture of the 238 U (99.28 %) and 235 U (0.72 %) isotopes.

c) How old must the material of the solar system be if one assumes that at its creation both isotopes were present in equal quantities? Look up their half-lives online, for example at `https://www.nndc.bnl.gov`. How do you interpret this result?

d) How much of the ^{238}U has decayed since the formation of the Earth's crust 2.5×10^9 y ago?