FYS3500 Problem set 2

Problem 1

a) $^{232}_{90}Th_{142}$

$\Rightarrow$ 142 neutrons

$^{235}_{92}U_{143}$

$\Rightarrow$ 143 neutrons

b) Only stable nucleus with $A = 20$:
$^{20}_{10}Ne_{10}$.

c) $^{206}_{82}Pb$, $^{207}_{82}Pb$ and $^{208}_{82}Pb$ are the stable nuclei with $Z = 82$ (^{204}Pb is very long-lived, half-life longer than 10^{17} years).

d) Trick question: there are no stable polonium nuclei ($Z = 84$).

e) The colours tell us the main decay mode of the nucleus:

black: stable
pink: β^--decay
blue: β^+-decay
yellow: α-decay
green: spontaneous fission

(We will learn more about these decay modes later :))

Problem 2

a) ${}^{16}_{8}O_8$, ${}^{15}_{7}N_8$, ${}^{16}_{6}C_{10}$, ${}^{16}_{10}Ne_6$, ${}^{14}_{6}C_8$,

${}^{20}_{7}N_{13}$, ${}^{20}_{10}Ne_{10}$

b) Isotopes : same Z, different N

${}^{15}_{7}N_8$ & ${}^{20}_{7}N_{13}$

${}^{16}_{6}C_{10}$ & ${}^{14}_{6}C_8$

${}^{16}_{10}Ne_6$ & ${}^{20}_{10}Ne_{10}$

c) Isotones : same N, different Z

${}^{16}_{8}O_8$, ${}^{17}_{7}N_8$ & ${}^{14}_{6}C_8$

${}^{16}_{6}C_{10}$ & ${}^{20}_{10}Ne_{10}$

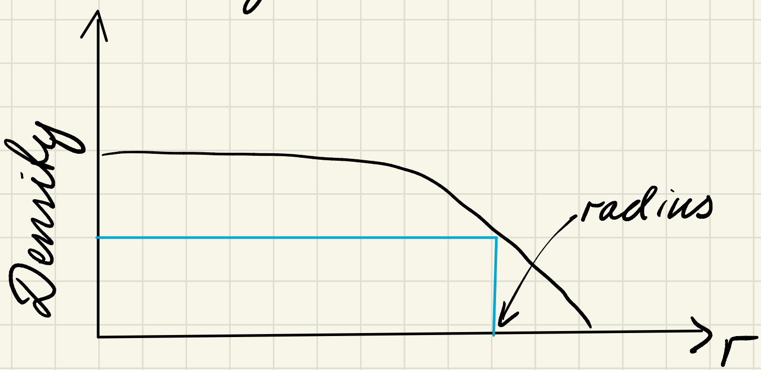
d) Isobars if same A

${}^{16}_{8}O_8$, ${}^{16}_{6}C_{10}$, ${}^{16}_{10}Ne_6$

${}^{20}_{7}N_{13}$ & ${}^{20}_{10}Ne_{10}$

Problem 3

a) The nucleus is not a hard sphere, and thus has no abrupt "end". The density of the nucleus is ≈ constant inside the nucleus, and then drops toward zero:

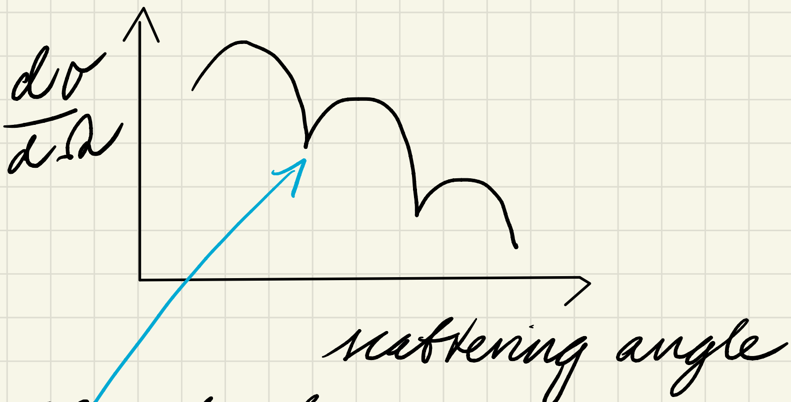


With "nuclear radius" we mean the distance where the density has dropped to half the maximum value.

When measuring the nuclear radius from scattering experiments, we either use a neutral incident particle (like the neutron) or a charged one (like the electron). The e^- will only give us information on the distribution of charge in the nucleus ("the charge distribution"), and we find the charge radius. A neutron scattering exp gives only info on the matter distribution ⇒ "mass radius". The two are ~identical as the neutrons and protons are distributed evenly in the nucleus: the Coulomb repulsion pushes the proton away from each other while the strong nuclear force pulls nucleons inward

⇒ n & p completely mixed

b) Light will create an interference pattern when scattering on a sphere. In a similar way, α-particles (that can be regarded as waves according to de Broglie) will create an interference pattern when scattered against a nucleus:



The locations of the minima reflect the size of the nucleus.

OBS: As the nucleus is not a hard sphere, the intensity doesn't drop to 0 in the minima (contrary to the classical picture)

c) The nuclear radius is approximated by the formula:

$$R \simeq R_0 A^{1/3}, \quad R_0 \approx 1.2\ \text{fm}$$

$$R \simeq 1.2\ \text{fm} \cdot (208)^{1/3} \approx \underline{\underline{7.1\ \text{fm}}}$$

d) The Rutherford cross section describes the scattering of a charged particle in an electric field. This formula only represents the scattering cross section as long as the incident particle only feels the Coulomb force; that is, the particle is either not close enough to be affected by the very short-range strong nuclear force, or it does not feel the strong force at all (like for example the electrons). If we increase E_LAB, then the incoming particle (assuming it is one that interacts strongly) eventually comes close enough to feel the strong force. At this point, the measured scattering cross section will start to deviate from the Rutherford cross section.

Problem 4

Figure 3 shows the average binding energy per nucleon for stable nuclei as a function of mass number A. It thus describes how well bound various nuclei are. We see that the average binding energy first increases with mass number until it reaches its maximum for 56Fe, which means 56Fe is the strongest bound nucleus. For heavier nuclei, the average binding energy decreases. Among the light nuclei, we observe a peak for 4He with respect to is neighbors, reflecting that the alpha particle is an exceptionally tightly bound nucleus (due to it being doubly magic - we'll come back to what this means when we discuss the shell model).

Energy is released when going from a system with less binding energy to a system with more binding energy, because a tighter bound nucleus means that it weighs less:

$$M(A,Z) = Z(m_p + m_e) + N m_n - B(A,Z)$$

Higher $B \Rightarrow$ less $M(A,Z)$

Mass and energy are related: $E = mc^2$

<u>Estimate mass of ^{130}Xe:</u>

Can see from Fig 3 that the average binding per A for $A = 130$ is $\sim 8.5\ \text{MeV}/A$

$m_p = 938.3\ MeV/c^2$

$m_n = 939.6\ MeV/c^2$

$m_e = 0.511\ MeV/c^2$

$1u = 931.5\ MeV/c^2$

$$M(A,Z) = 54(m_p + m_e) + 76 m_n - 8.5\ MeV/A \cdot 130 A$$
$$\simeq 1.21 \cdot 10^5\ MeV$$
$$\simeq \underline{129.9 u}$$

⇒ pretty spot on, actually

Fission / fusion:

In stars, energy is released through fusion. As the 1H/2H nuclei are less bound than 4He, energy is released when fusing H to He.

Similarly, energy is released when uranium or other heavy elements fission (splits in two nuclei), because the two resulting nuclei are tighter bound than the original uranium.

-> Binding energy is released as long as we move up in the average binding energy plot.

Nucleosynthesis:

Stars can fuse elements all the way up to 56Fe. Since 56Fe is the most bound nucleus, energy is needed to fuse Fe into heavier elements. How elements heavier than iron were created has long been one of the large unanswered questions: we know something, but not all. We know that 56Fe can capture neutrons several times and then become the radioactive 59Fe, decaying to 59Co in what is called the s-process. This works up to 208Pu. Several of the elements even heavier than this were created in the r-process (Supernovas! Neutron star mergers!)

↳ The nuclear physics group does research on this! Feel free to ask!!