FYS3500 Solutions 14

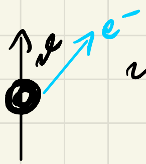
Problem 1 → M&S

We're looking at the decay

$$^{60}Co \rightarrow {}^{60}Ni^{*} + e^{-} + \bar{\nu}_e$$

If ^{60}Co is polarized, the intensity of the emitted electron is:

$$I(v, \vartheta) = 1 + \alpha \frac{v}{c} \cos\vartheta$$



ϑ: angle between e^- & ^{60}Co's spin direction

$J(^{60}Co) = 5$, $J(^{60}Ni) = 4$, $J(e^-) = 1/2$, and $J(\bar{\nu}_e) = 1/2$, and we neglect orbital angular momentum.

⇒ spins of ^{60}Ni, e^- & $\bar{\nu}_e$ must point in same direction to conserve ang. mom.

$$\uparrow {}^{60}Co = \uparrow {}^{60}Ni + \uparrow e^- + \uparrow \bar{\nu}_e$$

The electron is emitted in the ^{60}Co's spin direction

$\Rightarrow \vartheta = 0$

$\Rightarrow$ momentum & spin in same direction

$\Rightarrow$ right-handed electron

We know that in the limit $v \to c$, we only have left-handed fermions and right-handed antifermions

So $I(v,0) = 1 + \alpha \frac{v}{c}$

Since the RH electrons will not exist in the limit $v \to c$, then $I(v \to c, 0) = 0$

$\Rightarrow \underline{\underline{\alpha = -1}}$

Problem 2 → M&S 7.5

$$g_R(f) = -q_f \sin^2\theta_w$$

$$g_L(f) = \pm\frac{1}{2} - q_f \sin^2\theta_w$$

↳ + : ν & u, c, t

↳ − : charget leptons & d, s, b

$$Z^0 \to f\bar{f}; \Gamma_f = \underbrace{\frac{G_F M_Z^2 c^6}{3\pi\sqrt{2}(\hbar c)^3}}_{\Gamma^0} [g_R^2(f) + g_L^2(f)]$$

Assume $\sin^2\theta_w = 1/4$

Can calulate that $\Gamma^0 = 668$ MeV

To neutrinos

ν_L : $g_R(f=\nu) = 0$ & $g_L = 1/2$

$$\Gamma_{\nu_L} = \underbrace{\frac{G_F^2 M_Z^2 c^2}{3\pi\sqrt{2}(\hbar c)^3}}_{\Gamma^0} \left(\frac{1}{2}\right)^2 = \frac{1}{4}\Gamma^0$$

$\Gamma_{\nu e} = \Gamma_{\nu\mu} = \Gamma_{\nu\tau}$, and the total width for decay to neutrinos is

$$\Gamma_\nu = 3 \cdot \frac{\Gamma^0}{4} = \underline{501 \text{ MeV}}$$

To $q\bar{q}$

- For u, c, t

$$g_R = -\frac{2}{3} \cdot \left(\frac{1}{4}\right) = -\frac{1}{6}$$

$$g_L = \frac{1}{2} - \frac{2}{3} \cdot \frac{1}{4} = \frac{6}{12} - \frac{2}{12} = \underline{\frac{1}{3}}$$

$$\Gamma_{u,c,t} = \Gamma^0 \cdot \left[\left(-\frac{1}{6}\right)^2 + \left(\frac{1}{3}\right)^2\right]^2 = \frac{1}{36} + \frac{1}{9}$$

$$= \underline{\frac{5}{36} \cdot \Gamma^0}$$

• For d, s, b

$$g_R = -\left(-\frac{1}{3}\right)\cdot\frac{1}{4} = \frac{1}{12}$$

$$g_L = -\frac{1}{2} - \left(-\frac{1}{3}\right)\cdot\frac{1}{4} = -\frac{5}{12}$$

$$\Gamma_{d,s,b} = \Gamma^0 \, 2\left[\left(\frac{1}{12}\right)^2 + \left(-\frac{5}{12}\right)^2\right] = \frac{13\,\Gamma^0}{72}$$

$$\Gamma_{q\bar{q}} = \Gamma_u + \Gamma_d + \Gamma_c + \Gamma_s + \Gamma_t + \Gamma_b$$

$t\bar{t} > Z^0$ mass, not allowed

$$= \frac{2\cdot 5}{36}\Gamma^0 + \frac{3\cdot 13}{72}\Gamma^0$$

$$= \frac{59}{72}\Gamma^0 = 547\,\text{MeV}$$

• To produce hadrons, we first produce a $q\bar{q}$-pair, which then hadronizes. Each quark is found in three colour states:

$$\Gamma_{\text{hadrons}} = 3\Gamma_q = 1641\,\text{MeV}$$

- If there are N_ν generations of neutrinos with $N_\nu < M_z/2$, then the total decay width of Z° is given by:

$$\Gamma_{Z^\circ, tot} = \Gamma_{hadron} + \Gamma_{lepton} + N_\nu \Gamma_\nu$$

$$\Rightarrow N_\nu = \frac{\Gamma_{Z^\circ, tot} - \Gamma_{hadron} - \Gamma_{lepton}}{\Gamma_\nu}$$

$$= \frac{(2495 \text{ MeV} - 251.9 \text{ MeV} - 1744 \text{ MeV})}{167 \text{ MeV}}$$

$$\underline{\underline{\simeq 3}}$$

$\Rightarrow$ only three generations of neutrinos with $M_\nu < M_{Z^\circ}/2$

Problem 3 ⇒ 7.6 in M&S

The pions have spin-0 and we assume that K^+: spin-0.
Total angular momentum must be conserved, and since the kaon is decaying, it has no orb. ang. mom in its rest frame. So $J_i = 0$, and so $J_f = 0$, giving that the pions have no orbital ang. mom.
The system $\pi^+\pi^-$ has thus parity

$$P_u \cdot P_{\bar{d}} \cdot P_{\bar{u}} \cdot P_d \cdot (-1)^{l=0} = \underline{+1}$$

By the same argumentation, $\pi^+\pi^+\pi^+$ has parity

$$P_u \cdot P_{\bar{d}} \cdot P_u \cdot P_{\bar{d}} \cdot P_u \cdot P_{\bar{d}} = \underline{-1}$$

$\Rightarrow K^+$ decays to both, and thus parity is violated

Problem 4 M&S 7.7

K^0 decays, and in its reference frame it is at rest:

spins

1) $K^0(\vec{p}=0) \rightarrow \pi^-(\vec{p}_1) + e^+(\vec{p}_2, \vec{s}_2) + \nu_e(\vec{p}_3, \vec{s}_3)$,

and momentum conservation gives

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$$

We apply CP:

$$K^0(\vec{p}=0) \xrightarrow{CP} \bar{K}^0(\vec{p}=0)$$

spins left unchanged

$$\pi^-(\vec{p}_1) \xrightarrow{CP} \pi^+(-\vec{p}_1)$$

$$e^+(\vec{p}_2, \vec{s}_2) \xrightarrow{CP} e^-(-\vec{p}_2, \vec{s}_2)$$

$$\nu_e(\vec{p}_3, \vec{s}_3) \xrightarrow{CP} \nu_e(-\vec{p}_3, \vec{s}_3)$$

2) $\bar{K}^0(\vec{p}=0) = \pi^+(-\vec{p}_1) + e^-(-\vec{p}_2, \vec{s}_2) + \nu_e(-\vec{p}_3, \vec{s}_3)$

Since 2) is the CP-transformed reaction of 1) and CP-invariance means equivalent reaction rates, the two should be equal if CP is conserved.

Problem 5 M&S 7.10, $B^0 : \bar{b}d$

7.10 (a) The semi-leptonic decay $\bar{D}^0\pi^-\mu^+\nu_\mu$ can only proceed in lowest-order interaction via $\bar{b} \to \bar{c} + W^+$ with $W^+ \to \ell^+ + \nu_\ell$, together with $q\bar{q}$ pair production. Thus it can only give a positive charged lepton, whereas a $\bar{B}^0$ semi-leptonic decay gives a negative charged lepton. Therefore (a) is a B^0 decay.

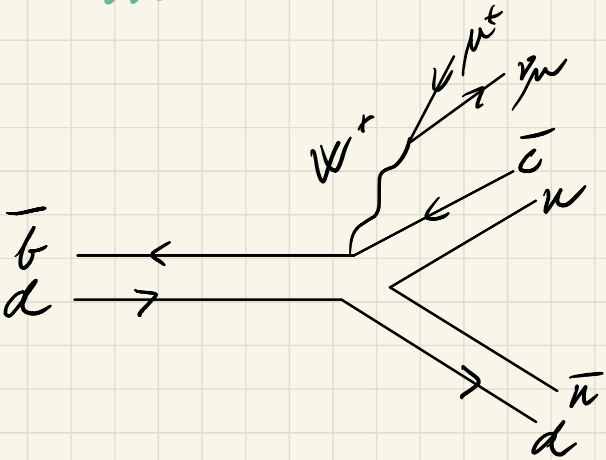
(b) The decay $\bar{B}^0 \to \rho^+ + K^-$ at quark level is $b\bar{d} \to u\bar{d} + s\bar{u}$, and so involves the transition $b \to us\bar{u}$. This can occur via $b \to u + W^-$ with $W^- \to s + \bar{u}$. In contrast, $B^0 \to \rho^+ + K^-$ is $\bar{b}d \to u\bar{d} + s\bar{u}$ at the quark level. However, while $\bar{b} \to \bar{u} + W^+$, $W^+d \not\to u\bar{d}s$ in lowest order. Hence (b) is a $\bar{B}^0$ decay.

(c) The decay $B^0 \to \rho^+ + \pi^-$ at quark level is $d\bar{b} \to u\bar{d} + d\bar{u}$, and so involves the transition $\bar{b} \to u\bar{d}u$. This is allowed via $\bar{b} \to \bar{u}W^+$ with $W^+ \to d\bar{u}$. $\bar{B}^0 \to \rho^-\pi^+$ at the quark level is $b\bar{d} \to u\bar{d} + d\bar{u}$, and so involves the transition $b \to ud\bar{u}$. This can occur via $b \to uW^-$ with $W^- \to d\bar{u}$. Hence (c) could be either B^0 or $\bar{B}^0$.

(d) The decay $B^0 \to D^-D_s^+$ at quark level is $\bar{b}d \to d\bar{c} + c\bar{s}$, i.e. it involves the transition $\bar{b} \to \bar{c}c\bar{s}$, which can proceed via $\bar{b} \to \bar{c}W^+$ with $W^+ \to c\bar{s}$. $\bar{B}^0 \to D^-D_s^+$ at quark level is $b\bar{d} \to d\bar{c} + c\bar{s}$. The b quark can decay via $b \to cW^-$, but $W^-\bar{d} \to d\bar{c}\bar{s}$ is not possible in lowest order. Hence (d) is a B^0 meson.

a) $B^0 \to \bar{D}^0 \pi^- \mu^+ \nu_\mu$

$\bar{b}d \quad \bar{c}u \quad d\bar{u}$



$\Rightarrow$ ok

If $\bar{B}^0$: $\bar{B}^0 \to \bar{D}^0 \pi^- \mu^+ \nu_\mu$

$b\bar{d} \quad \bar{c}u \quad d\bar{u}$

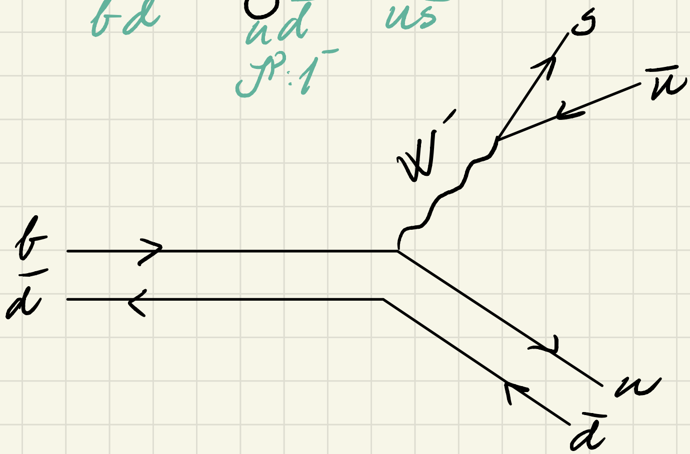
can change $b \to u + W^-$, but W^- would give neg. lepton

$\Rightarrow$ to lowest order, this could not be $\bar{B}^0$

b) $\bar{B}^0 \to \rho^0 K^-$

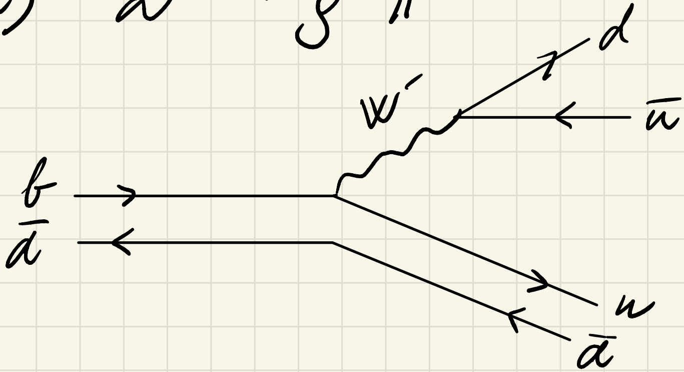
$b\bar{d}$ $\quad u\bar{d}$ $\quad u\bar{s}$

$\rho: 1^-$

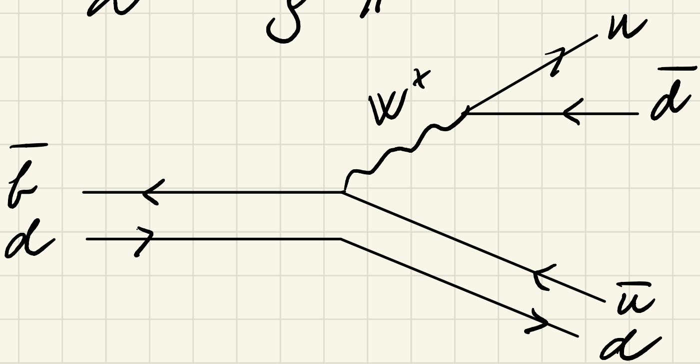


$B^0 \to \rho^0 K^-$ would need multiple $W^\pm$ exchanges, so would not go in lowest order

c) $\bar{B}^0 \to \rho^+ \pi^-$



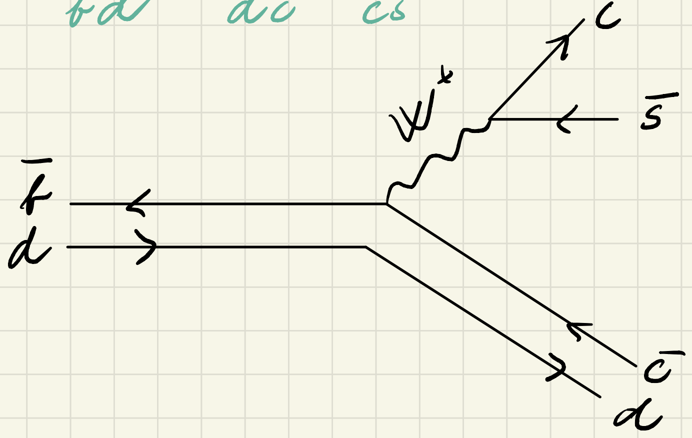
$B^0 \to \rho^+ \pi^-$



$\Rightarrow$ can proceed through both

d) $B^0 \to D^- D_s^+$

$\bar{b}d \quad d\bar{c} \quad c\bar{s}$



$B^0 \to D^- D_s^+$

$b\bar{d} \quad d\bar{c} \quad c\bar{s}$

$\Rightarrow$ cannot go through lowest order