Machine Learning

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1.

result	Diseased	Not diseased
Positive	0.98	0.03
Negative	0.02	0.97

P → Positive D → disease

N → Negative ND → No disease

$P(D) = 0.008$ $P(ND) = 1 - P(D) = 0.992.$

we want prob. of disease if result is positive

$$P(D/P) = \frac{P(P/D) \cdot P(D)}{P(P/D) \cdot P(D) + P(ND) \cdot P(P/ND)}$$

$$= \frac{0.008 \times 0.98}{0.008 \times 0.98 + 0.992 \times 0.03}$$

$$= \frac{0.00784}{0.00784 + 0.02976}$$

$$= 0.2085$$

So person have less chance of having disease even with positive result.

2) Total vehicles = 1209

vehicles cost > 50000 = 221

vehicles cost < 50000 = 1209 - 221 = 988.

vehicle heights = 1.05 m

> 50000 = 46

< 50000 = 59

$$P(>50000 / \text{height} = 1.05) = \frac{P(\text{height} = 1.05 \mid >50000) \cdot P(>50000)}{P(\text{height} = 1.05)}$$

$$= \frac{\frac{46}{221} \times \frac{221}{1209}}{\frac{46+59}{1209}}$$

$$= \frac{0.208 \times 0.182}{0.087}$$

$$= \underline{\underline{0.438}}$$

So vehicle price probably is not greater than 50000/-.

3) Given

$P(\omega_1/x) = 0.01$

$P(\omega_2/x) = 0.99$

$$R(\alpha_i/x) = \sum_{j=1}^{c} \lambda(\alpha_i/\omega_j) \times P(\omega_j/x)$$

$$R(\alpha_1/x) = \lambda(\alpha_1/\omega_1) \times P(\omega_1/x) + \lambda(\alpha_1/\omega_2) \times P(\omega_2/x)$$

$$= 5 \times 0.01 + 60 \times 0.99$$

$$= 0.05 + 59.4$$

$$= 59.45$$

$$R(\alpha_2/x) = \lambda(\alpha_2/\omega_1) \times P(\omega_1/x) + \lambda(\alpha_2/\omega_2) \times P(\omega_2/x)$$

$$= 50 \times 0.01 + 3 \times 0.99$$

$$= 0.5 + 2.97$$

$$= 3.47$$

$$R(\alpha_3/x) = \lambda(\alpha_3/\omega_1) \times P(\omega_1/x) + \lambda(\alpha_3/\omega_2) \times P(\omega_2/x)$$

$$= 10000 \times 0.01 + 0 \times 0.99$$

$$= 100$$

- Action α_2 (treatement B) takes least loss of 3.47

4) Bayes's rule is optimal if true prior probabilities $P(\omega_i)$ and probability density functions $p(x/\omega_i)$ are known.

5) we know for c classes

$$P\left(\frac{\omega_1}{x}\right) + P\left(\frac{\omega_2}{x}\right) \cdots\cdots + P\left(\frac{\omega_c}{x}\right) = 1 \quad \text{—— (1)}$$

Since total probability is 1.

also

$$P\left(\frac{\omega_{max}}{x}\right) \geqslant P\left(\frac{\omega_i}{x}\right) \qquad i = 1, 2 \cdots c \quad \text{—— (2)}$$

we observe that $P\left(\frac{\omega_{max}}{x}\right)$ is minimum when it is equal to $P\left(\frac{\omega_i}{x}\right)$ (from eqn 2)

So when all $P\left(\frac{\omega_i}{x}\right)$ are equal.

So $c \cdot P\left(\frac{\omega_i}{x}\right) = 1$ —— from eq 1

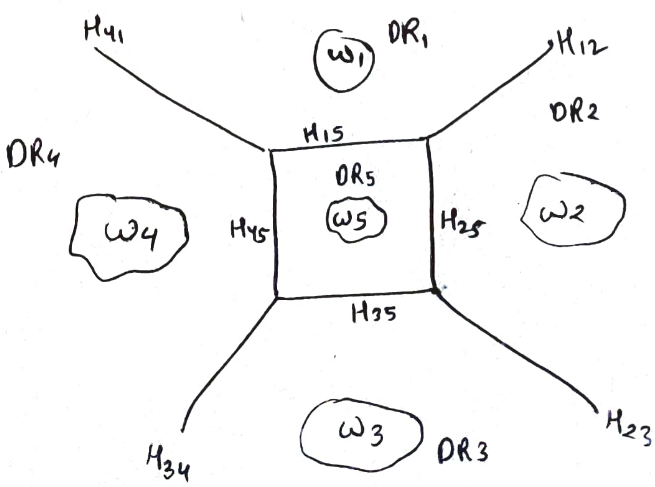
$$P\left(\frac{\omega_i}{x}\right) = \frac{1}{c} \quad \text{—— (3)}$$

Since $P\left(\frac{\omega_{max}}{x}\right) \geqslant P\left(\frac{\omega_i}{x}\right)$

hence $P\left(\frac{\omega_{max}}{x}\right) \geqslant \frac{1}{c}$. from eqn 3.

6) Hyperplane = H (8)

Decision Region = DR (5)



7) Lets consider 2 classes ω_1 & ω_2, with actions α_1 & α_2.

where

α_1 is taken when ω_1 is true class

α_2 is taken when ω_2 is true class

Using conditional risk, we get

$$R(\alpha_1/x) = \lambda_{11} P(\omega_1/x) + \lambda_{12} P(\omega_2/x) \quad \text{— ①}$$

$$R(\alpha_2/x) = \lambda_{21} P(\omega_2/x) + \lambda_{22} P(\omega_2/x) \quad \text{— ②}$$

we decide ω_1 if

$$R(\alpha_1/x) < R(\alpha_2/x)$$

$$\lambda_{11} P(\omega_1/x) + \lambda_{12} P(\omega_2/x) < \lambda_{21} P(\omega_1/x) + \lambda_{22} P(\omega_2/x)$$

$$(\lambda_{11} - \lambda_{21}) P(\omega_1/x) < P(\omega_2/x)(\lambda_{22} - \lambda_{12})$$

$$\frac{P(\omega_1/x)}{P(\omega_2/x)} < \frac{(\lambda_{22} - \lambda_{12})}{(\lambda_{11} - \lambda_{21})}$$

$$\frac{P(x/\omega_1)\, P(\omega_1)}{P(x/\omega_2)\, P(\omega_2)} < \frac{\lambda_{22} - \lambda_{12}}{\lambda_{11} - \lambda_{21}}$$

$$\frac{P(x/\omega_1)}{P(x/\omega_2)} > \frac{\lambda_{12} - \lambda_{22}}{\lambda_{21} - \lambda_{11}} \cdot \frac{P(\omega_2)}{P(\omega_1)}$$

(assuming $\lambda_{21} > \lambda_{11}$)

$\dfrac{P(x/\omega_1)}{P(x/\omega_2)}$ is likelihood ratio

- We will select ω_1 if likelihood ratio exceeds threshold value that will be independent of x. else select class ω_2.