Stereo Vision

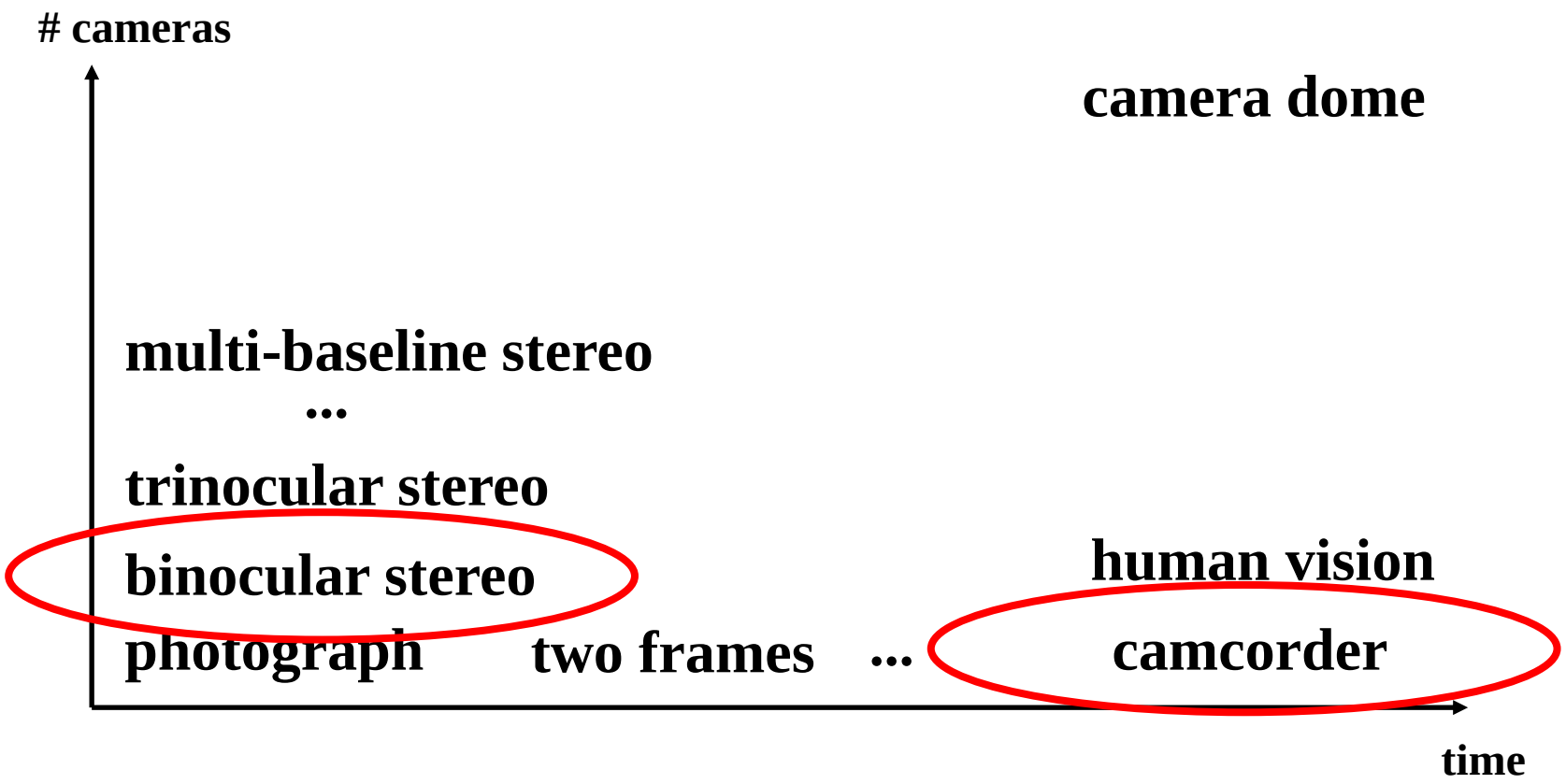
ECE 847:
Digital Image Processing

Stan Birchfield
Clemson University

Outline

- Stereo basics
- Binocular stereo matching
- Advanced stereo techniques

Modeling from multiple views



στερεος – Greek for solid

Stereoscope

Invented by Wheatstone in 1838

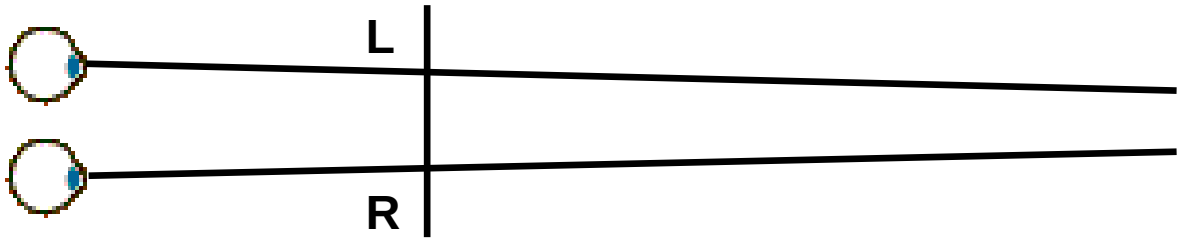
Modern version

Can you fuse these?

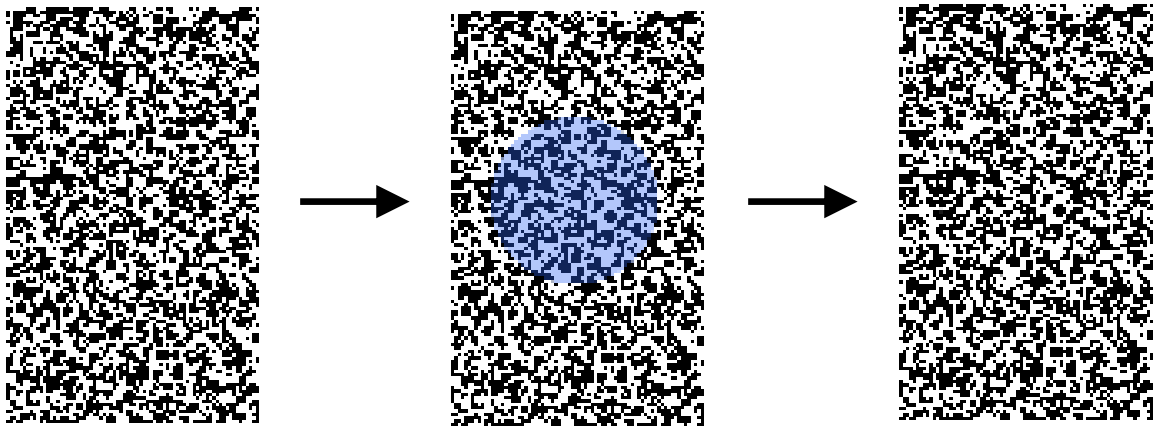
left **right**

No special instrument needed

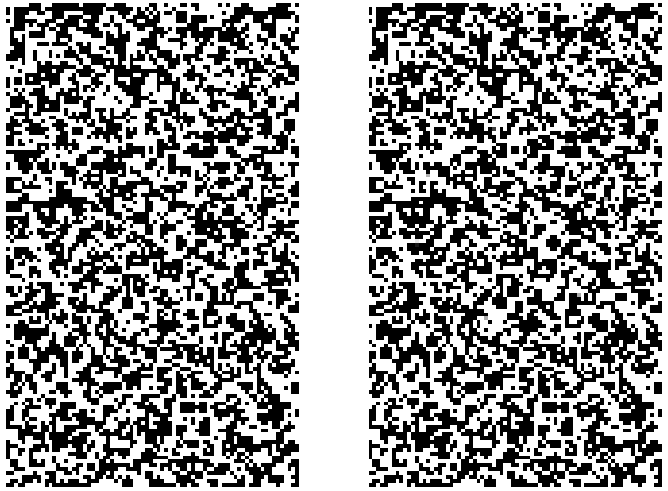
Just relax your eyes



Random dot stereogram

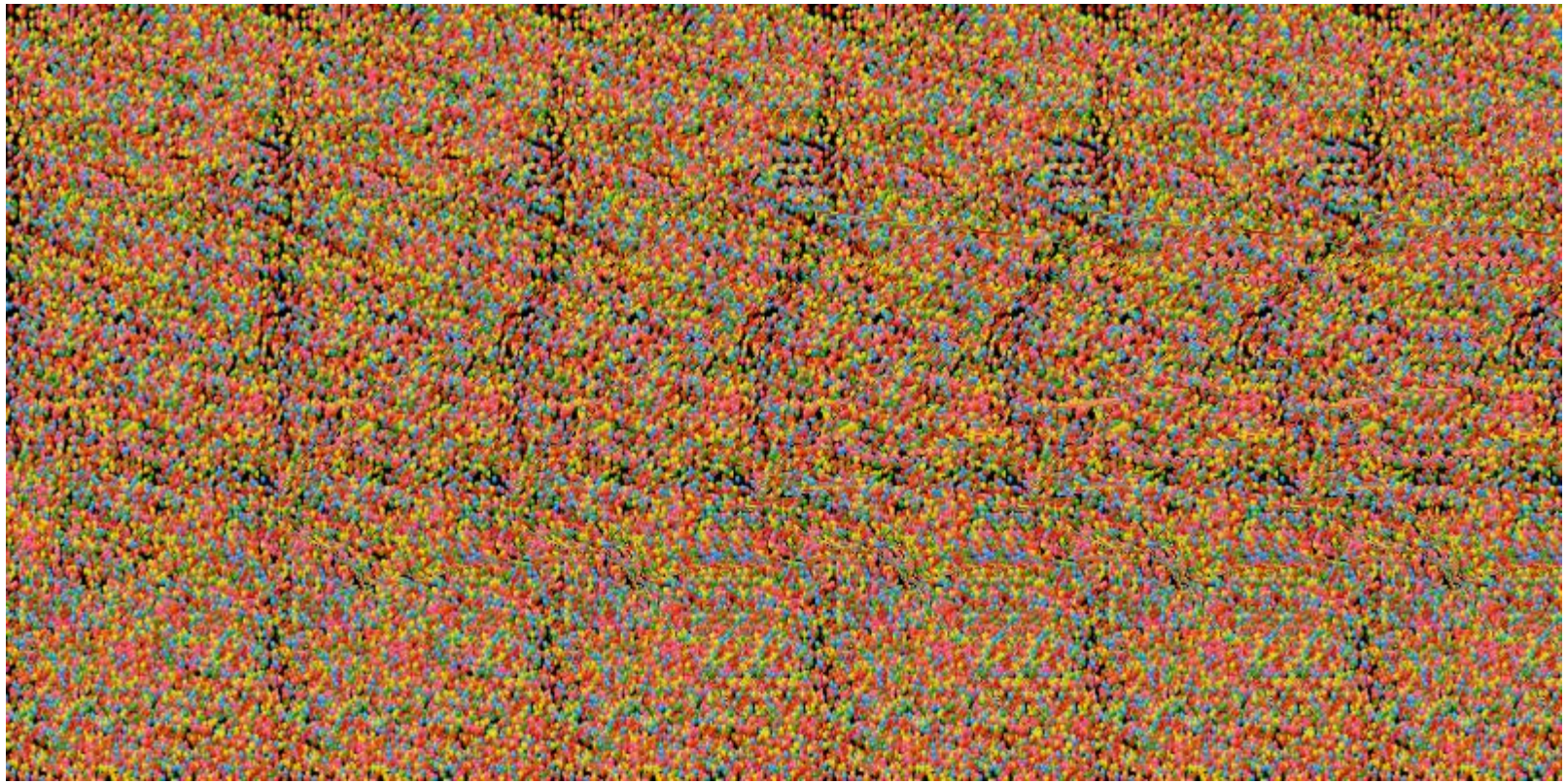


invented by Bela Julesz in 1959



http://www.magiceye.com/faq.htm

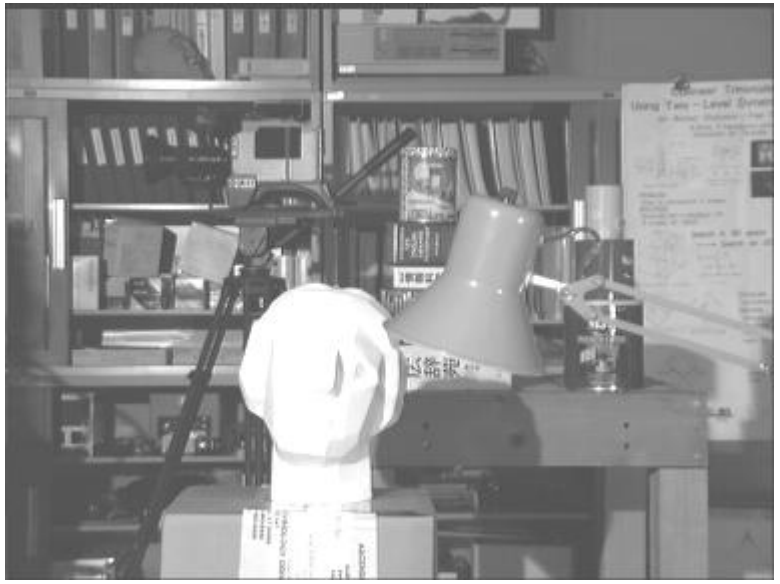
Autostereogram



Do you see the shark?

http://en.wikipedia.org/wiki/Autostereogram

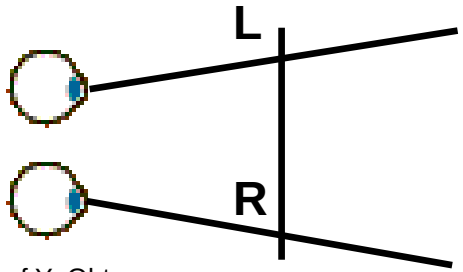
Can you cross-fuse these?



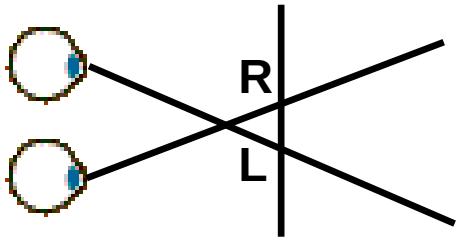
right **left**

Note: **Cross-fusion is necessary if distance between images is greater than inter-ocular distance**

impossible:

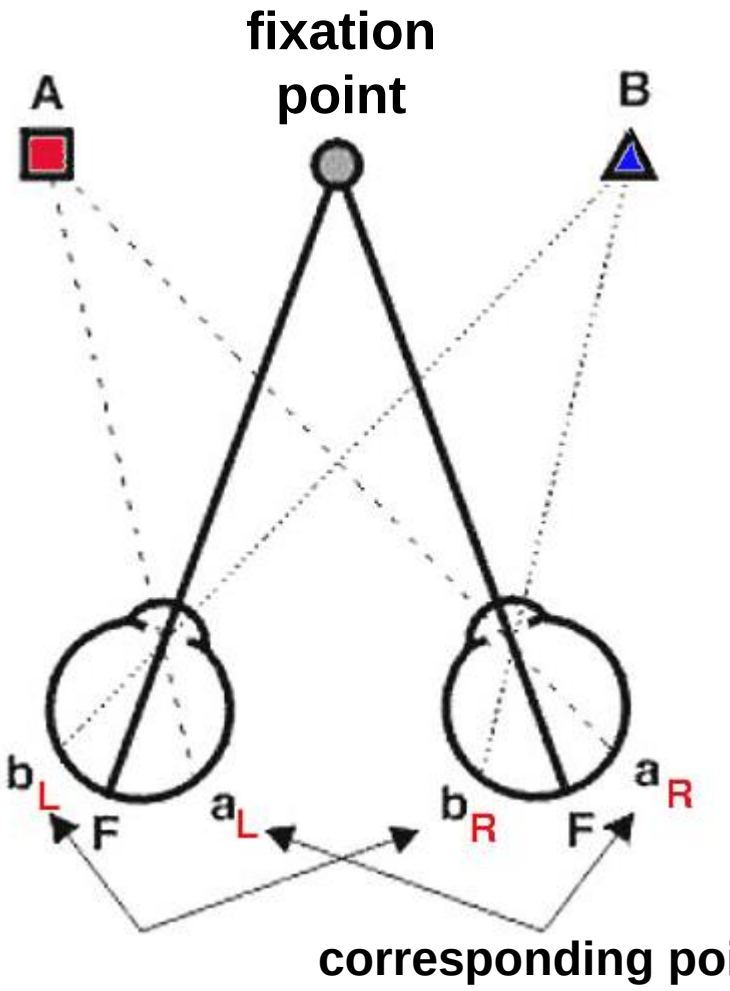


instead, trick the brain:



Tsukuba stereo images courtesy of Y. Ohta and Y. Nakamura at the University of Tsukuba

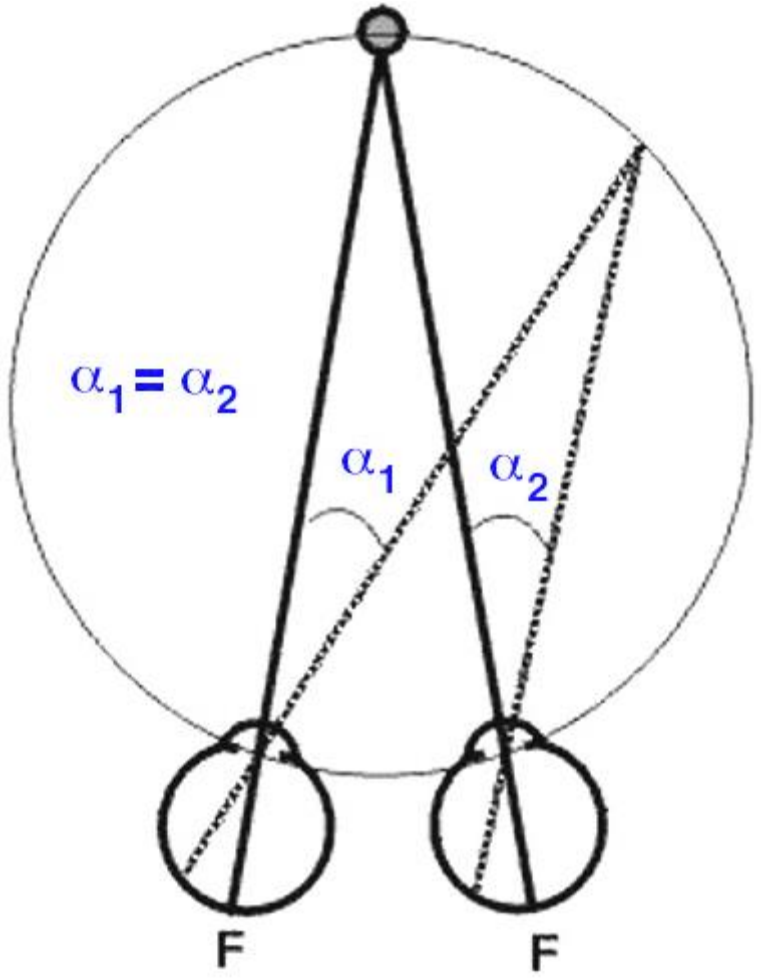
Human stereo geometry



http://webvision.med.utah.edu/space_perception.html

Horopter

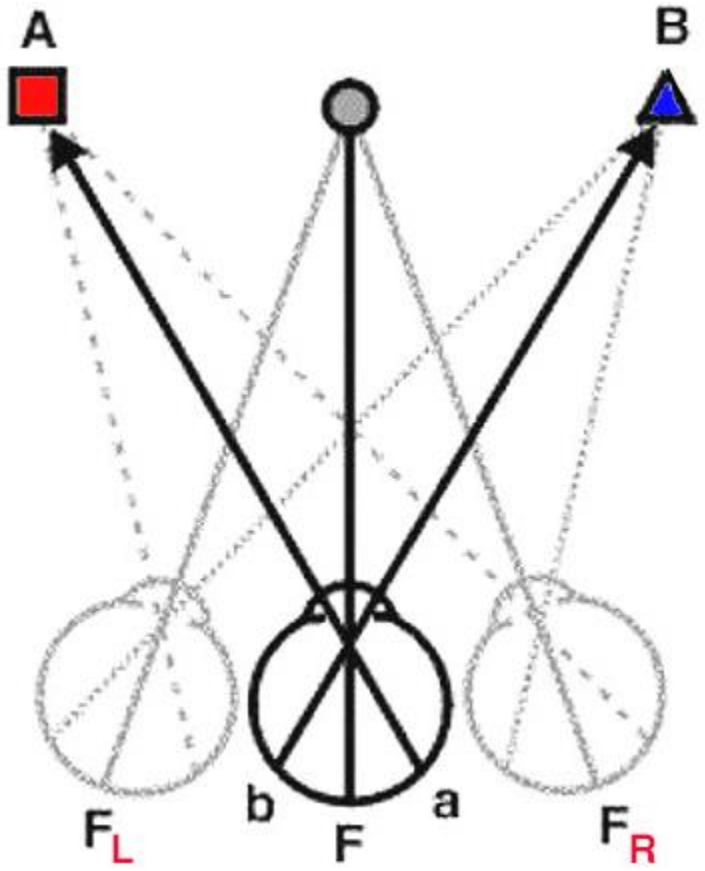
- Horopter: surface where disparity is zero
- For round retina, the theoretical horopter is a circle (Vieth-Muller circle)



http://webvision.med.utah.edu/space_perception.html

Cyclopean image

http://bearah718.tripod.com/sitebuildercontent/sitebuilderpictures/cyclops.jpg



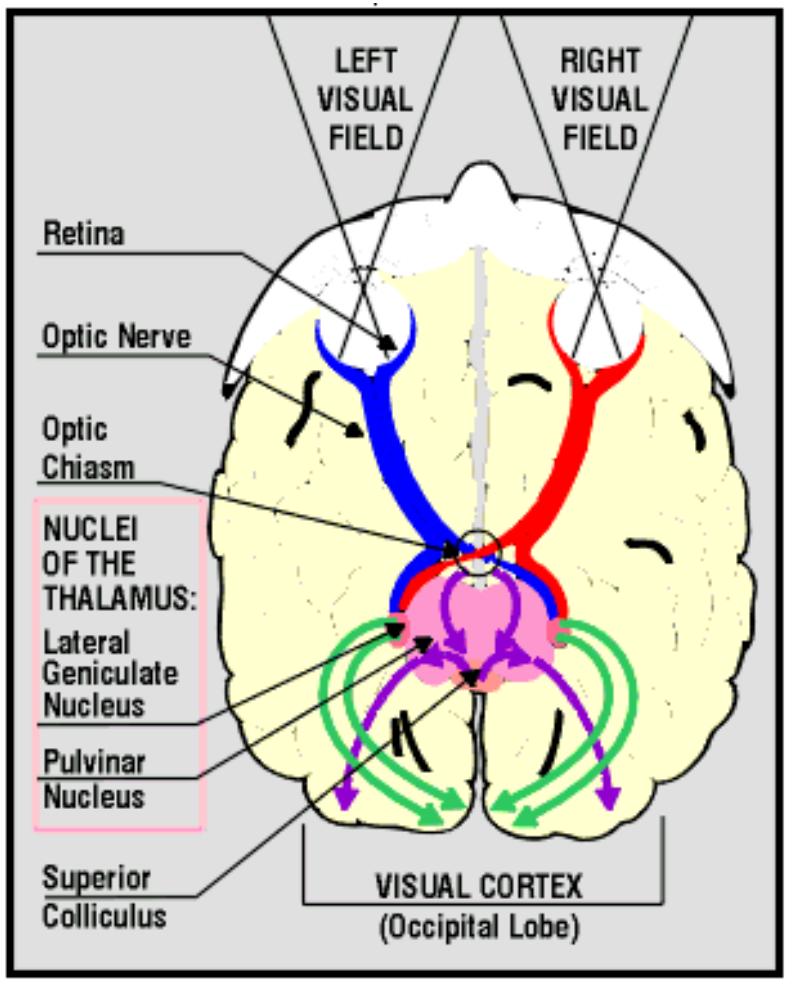
http://webvision.med.utah.edu/space_perception.html

Panum's fusional area (volume)

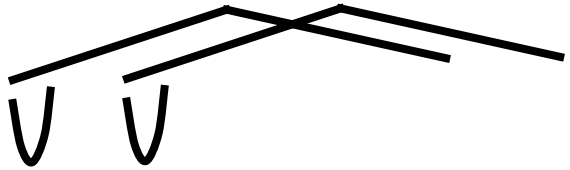
- Human visual system is only capable of fusing the two images with a narrow range of disparities around fixation point
- This area (volume) is Panum's fusional area
- Outside this area we get double-vision (diplopia)

http://www.allaboutvision.com/conditions/double-vision.htm

Human visual pathway

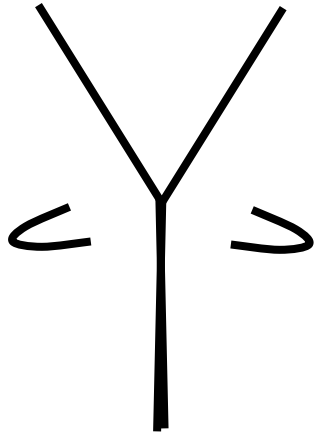


Prey and predator



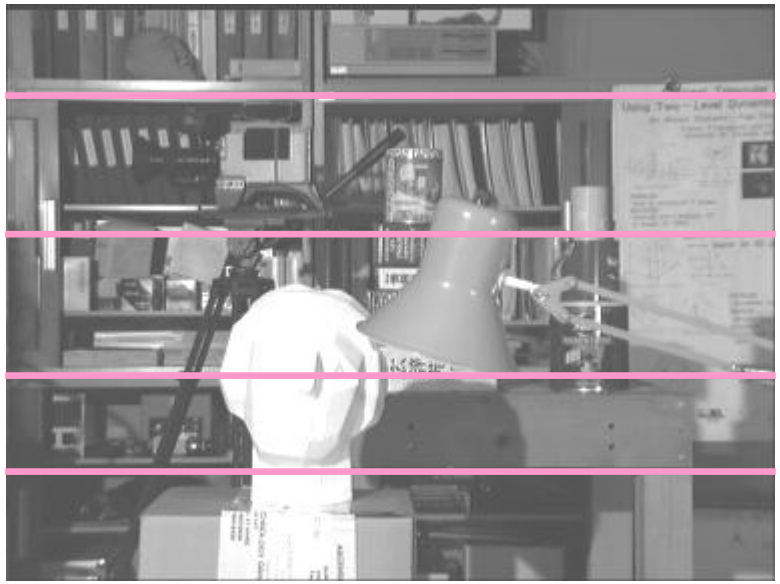
Cheetah: More accurate depth estimation

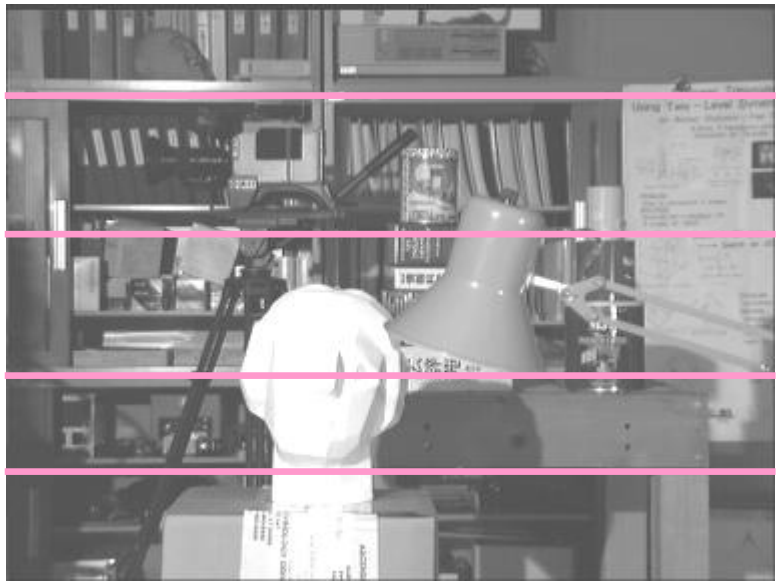
Antelope: larger field of view



photos courtesy California Academy of Science

Example: Motion parallel to image scanlines





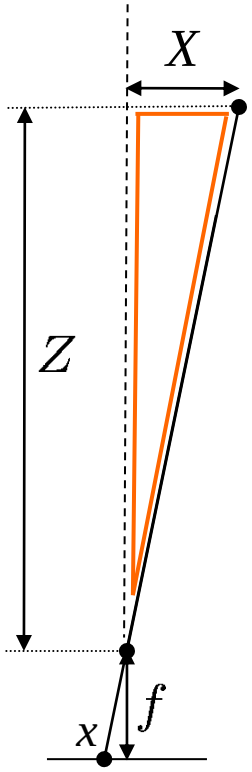
Epipoles are at infinity

Scanlines are the epipolar lines

In this case, the images are said to be “rectified”

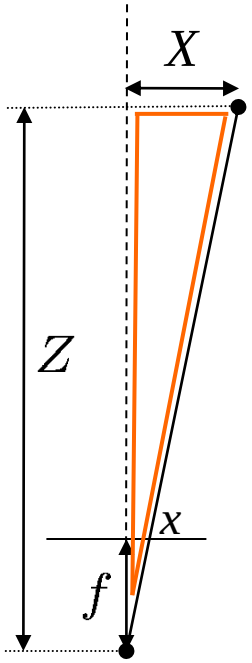
Tsukuba stereo images courtesy of Y. Ohta and Y. Nakamura at the University of Tsukuba

Perspective projection



$$x = -f\frac{X}{Z}$$

$$y = -f\frac{Y}{Z}$$

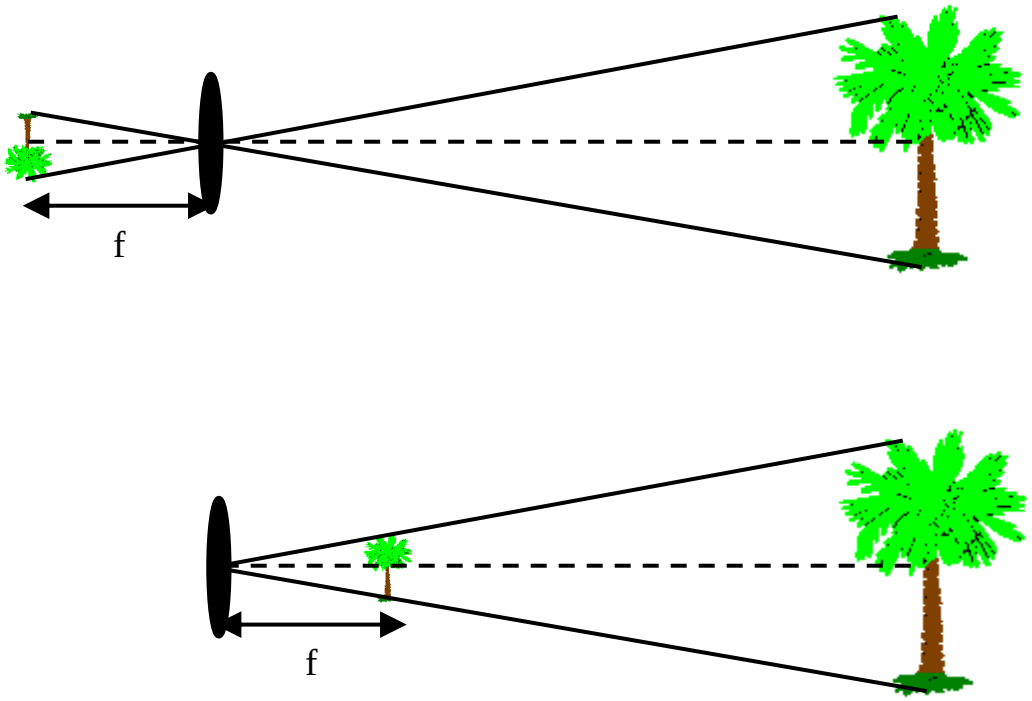


$$x = f\frac{X}{Z}$$

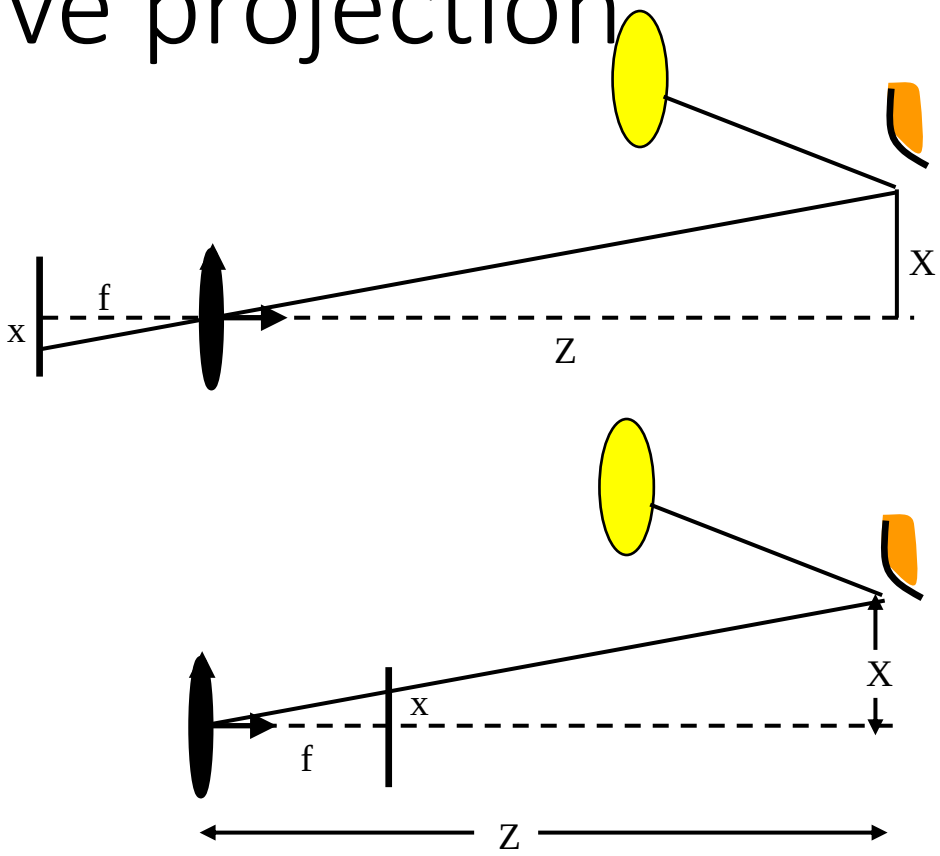
$$y = f\frac{Y}{Z}$$

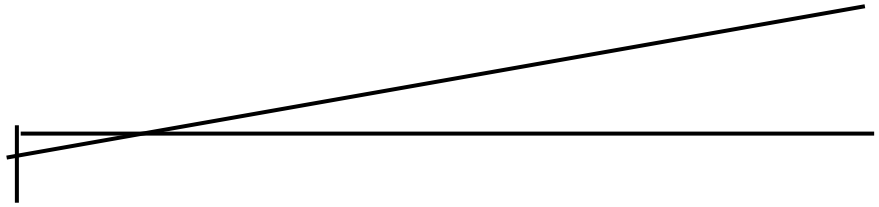
M. Pollefeys, http://www.cs.unc.edu/Research/vision/comp256fall03/

Perspective projection

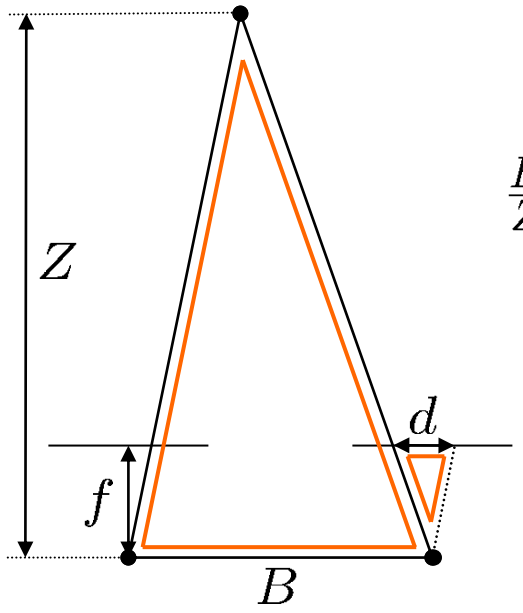


Perspective projection

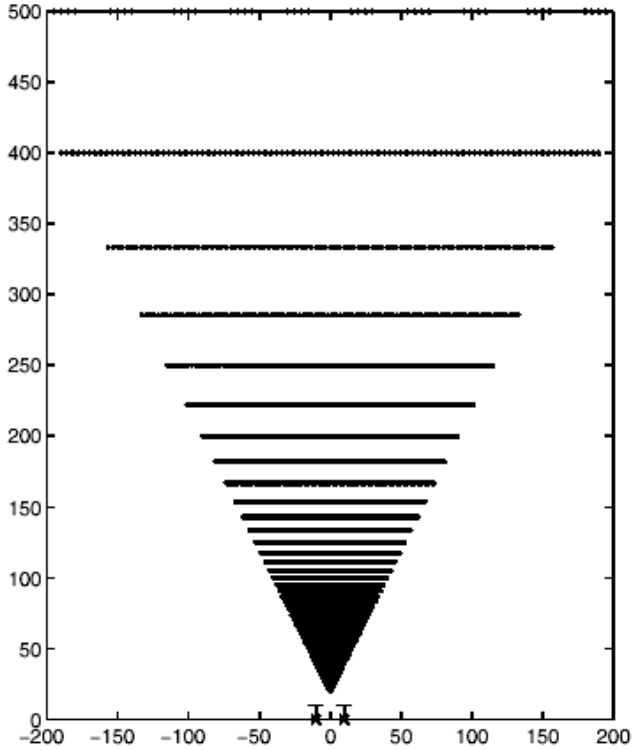




Standard stereo geometry



$$\frac{B}{Z} = \frac{d}{f}$$

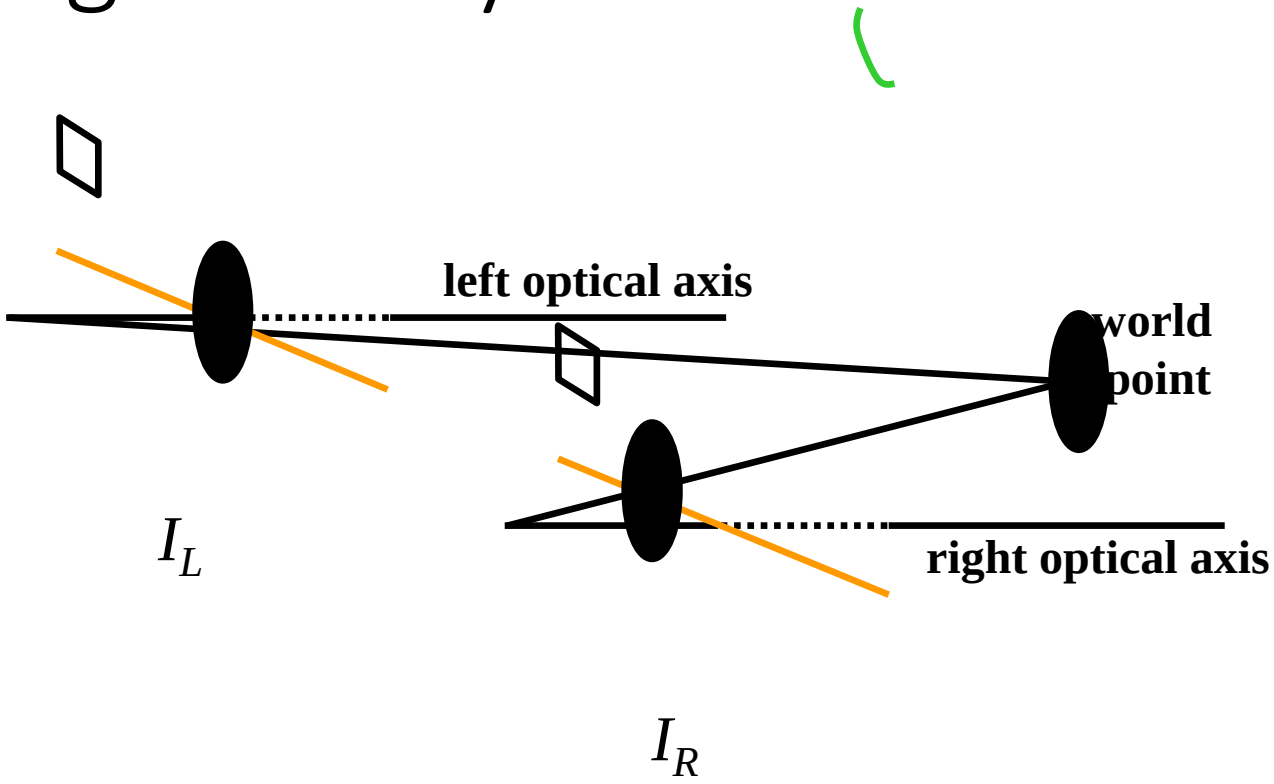


$$d = x_L - x_R = f\frac{X_L}{Z} - f\frac{X_R}{Z} = f\frac{X_L - X_R}{Z} = f\frac{B}{Z}$$

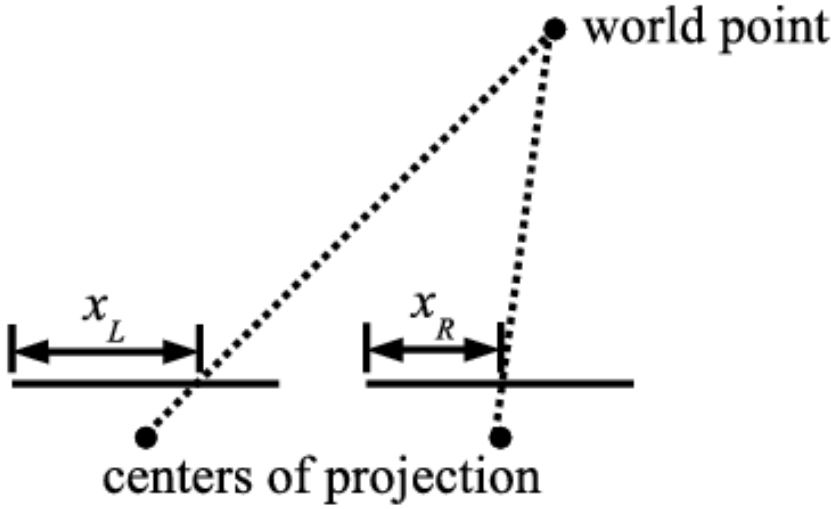
- **disparity is inversely proportional to depth**
- **stereo vision is less useful for distant objects**

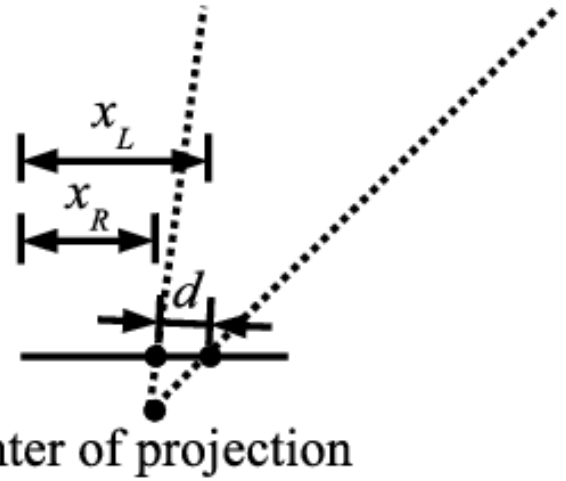
M. Pollefeys, http://www.cs.unc.edu/Research/vision/comp256fall03/

Rectified geometry



Rectified geometry



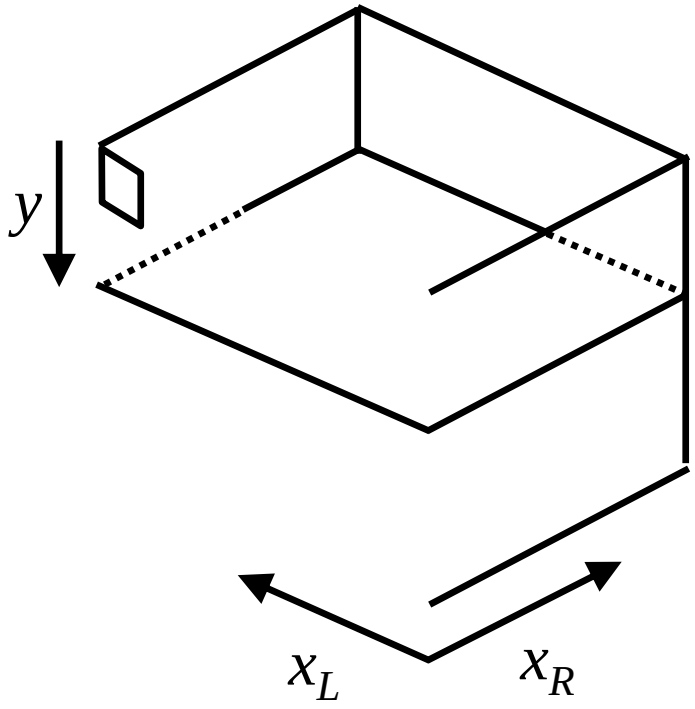


two cameras **overlapped (for display)**

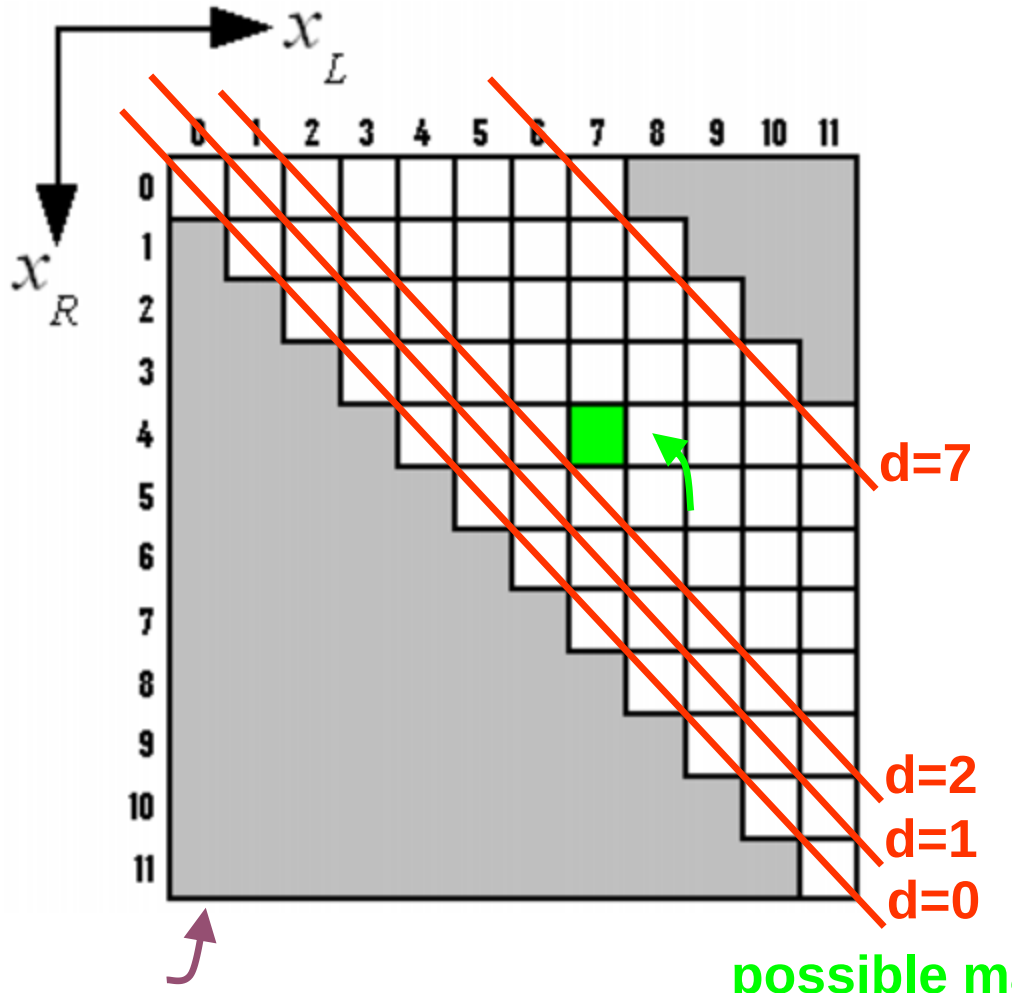
$$d = x_1 - x_2 = f\,(X_1 - X_2) / Z = f\,b / Z$$

disparity **baseline** **depth**

Matching space



Matching space



$$d = x_L - x_R$$

impossible matches

possible match between pixel 7 in left scanline and pixel 4 in right scanline

Stereo Matching

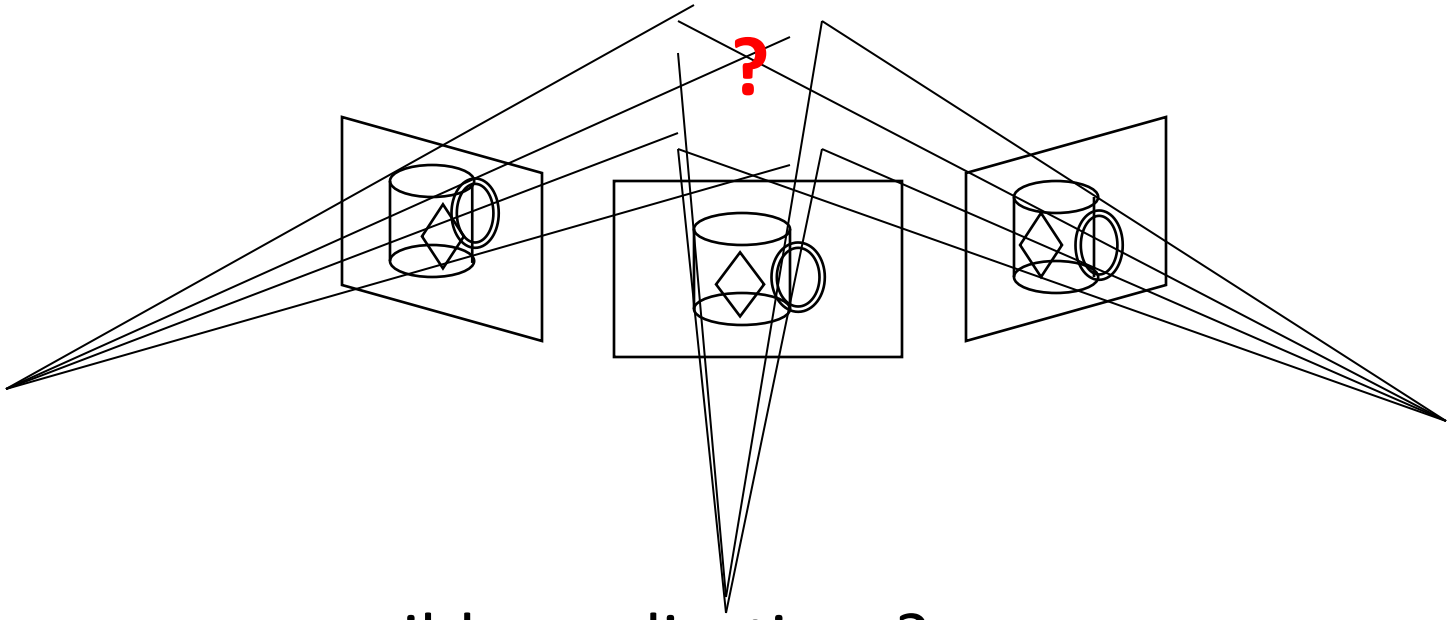
Computer Vision

CSE576, Spring 2005

Richard Szeliski

Stereo Matching

- Given two or more images of the same scene or object, compute a representation of its shape



- What are some possible applications?

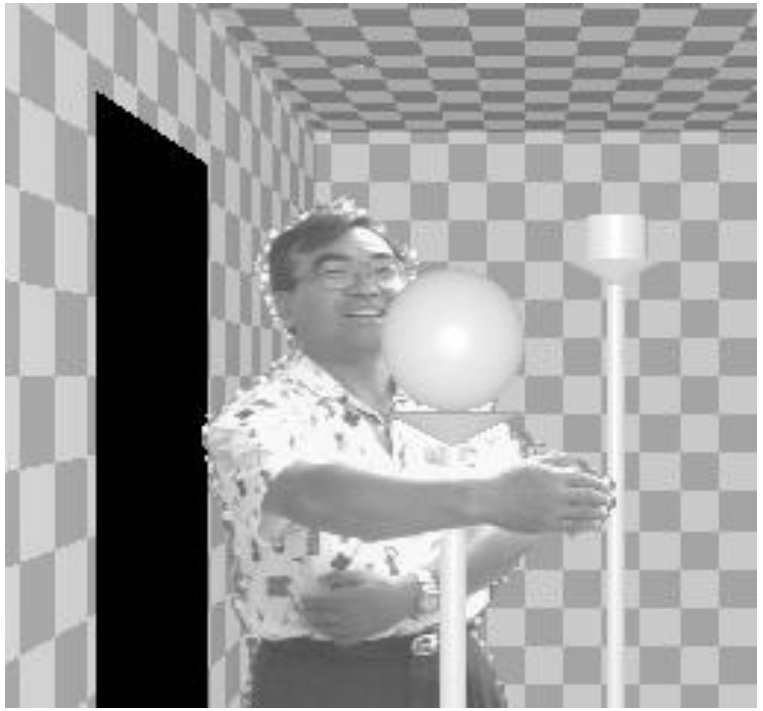
Face modeling

- From one stereo pair to a 3D head model

[Frederic Deverney, INRIA]

Z-keying: mix live and synthetic

- Takeo Kanade, CMU (Stereo Machine)



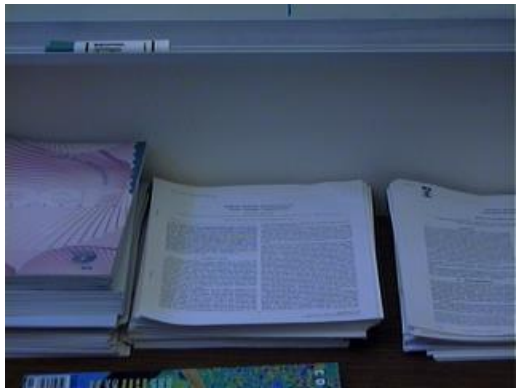
View Interpolation

- Given two images with correspondences, *morph* (warp and cross-dissolve) between them [Chen & Williams, SIGGRAPH'93]

[Matthies,Szeliski,Kanade'88]

More view interpolation

- Spline-based depth map

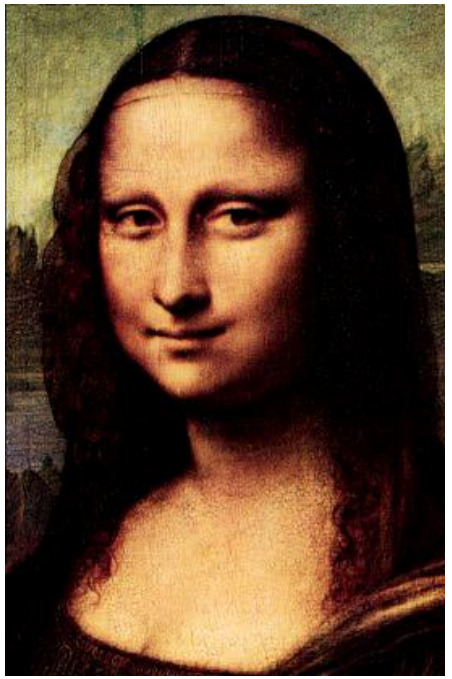


input depth image novel view

- [Szeliski & Kang ‘95]

View Morphing

- Morph between pair of images using epipolar geometry [Seitz & Dyer, SIGGRAPH'96]

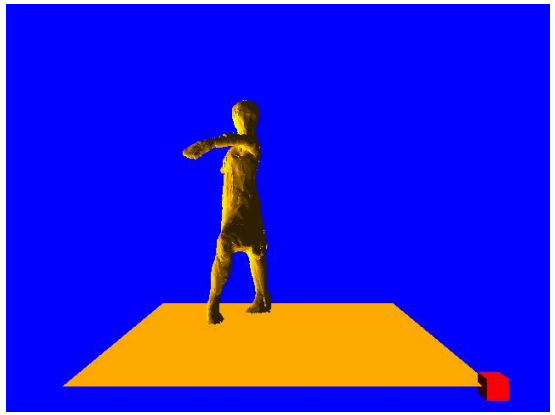


Video view interpolation


Massive Arabesque

Virtualized Reality™

- [Takeo Kanade *et al.*, CMU]
  - collect video from 50+ stream
  - reconstruct 3D model sequences



  - steerable version used for SuperBowl XXV "eye vision"

- http://www.cs.cmu.edu/afs/cs/project/VirtualizedR/www/VirtualizedR.html

Real-time stereo

Nomad robot searches for meteorites in Antartica
http://www.frc.ri.cmu.edu/projects/meteorobot/index.html

- Used for robot navigation (and other tasks)
  - Software-based real-time stereo techniques

Additional applications

- Real-time people tracking (systems from Pt. Gray Research and SRI)
- “Gaze” correction for video conferencing [Ott,Lewis,Cox InterChi’93]
- Other ideas?

Stereo Matching

- Given two or more images of the same scene or object, compute a representation of its shape

- What are some possible representations?
  - depth maps
  - volumetric models
  - 3D surface models
  - planar (or offset) layers

Stereo Matching

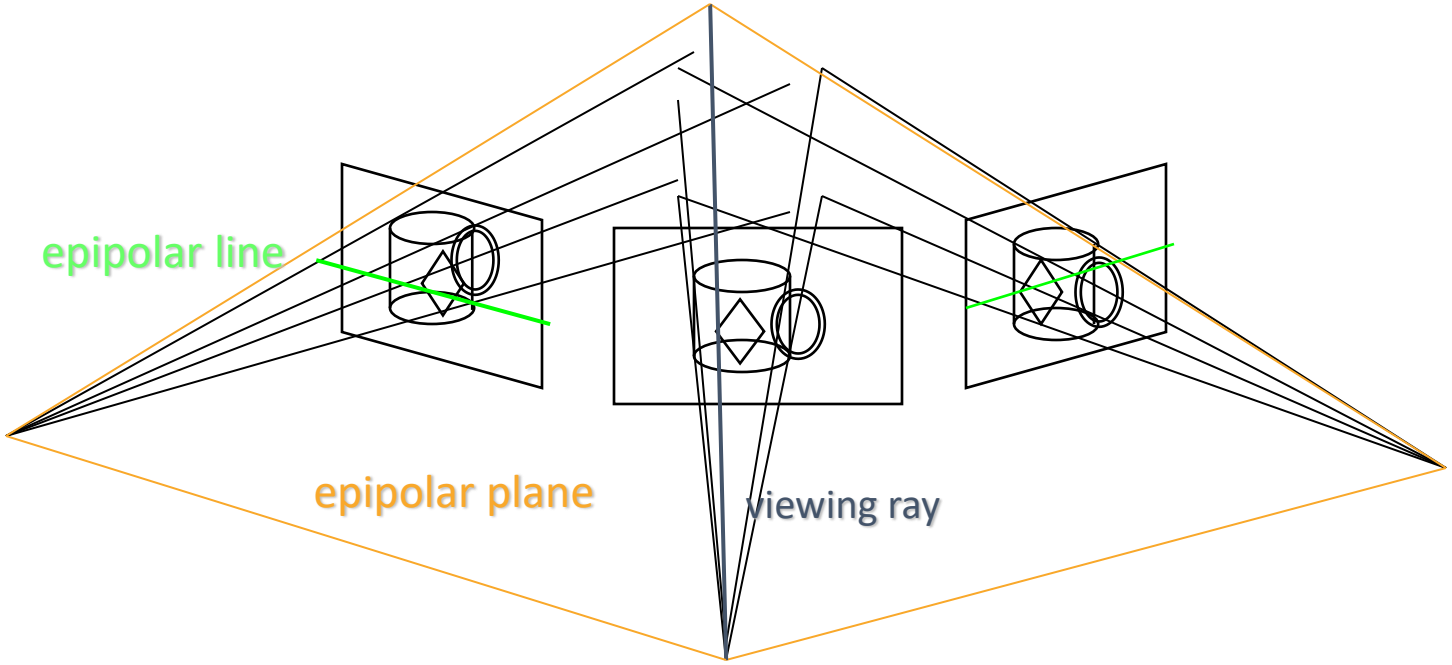
- What are some possible algorithms?
  - match “features” and interpolate
  - match edges and interpolate
  - match all pixels with windows (coarse-fine)
  - use optimization:
    - iterative updating
    - dynamic programming
    - energy minimization (regularization, stochastic)
    - graph algorithms

Outline (remainder of lecture)

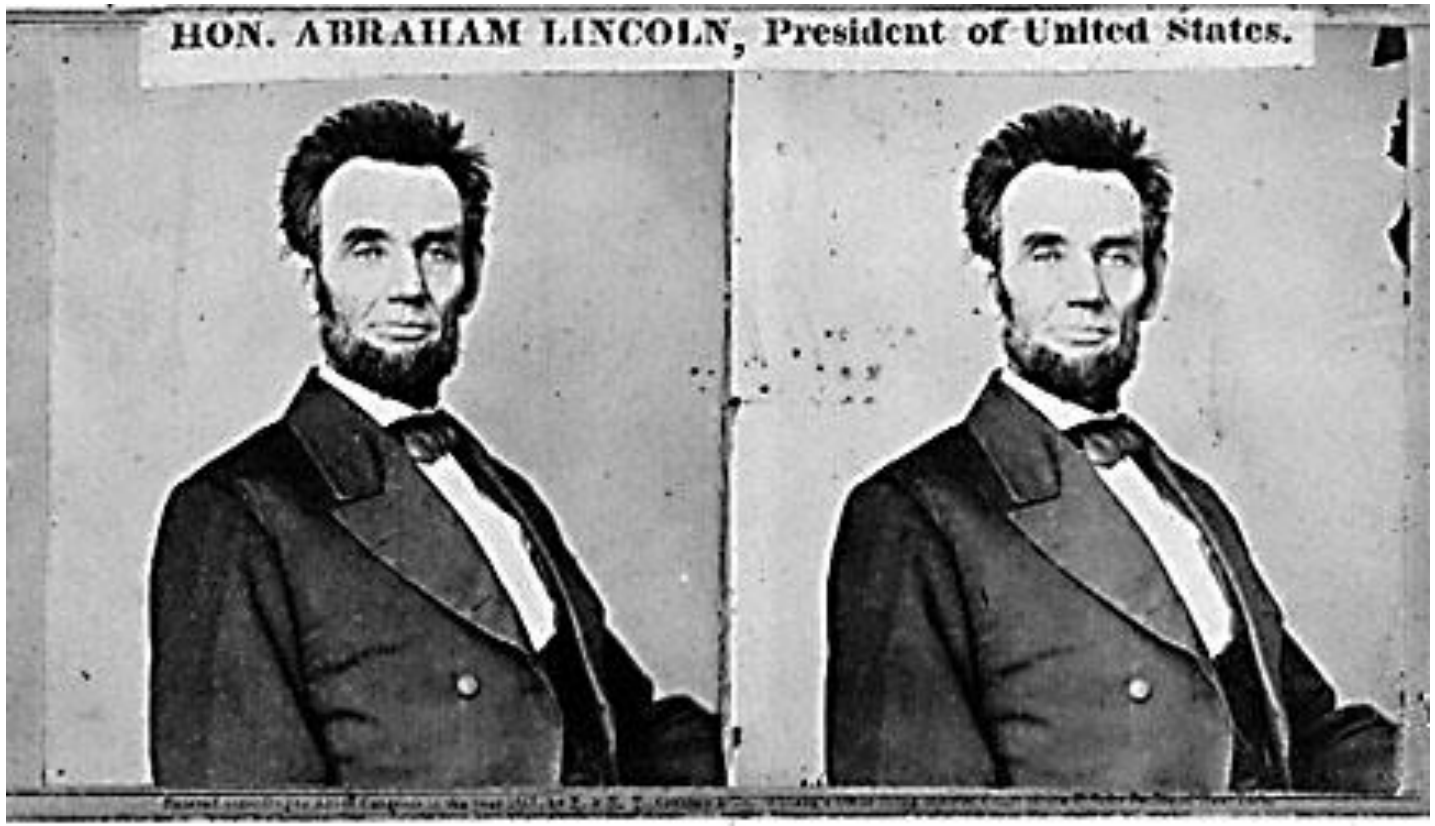
- Image rectification
- Matching criteria
- Local algorithms (aggregation)
  - iterative updating
- Optimization algorithms:
  - energy (cost) formulation & Markov Random Fields
  - mean-field, stochastic, and graph algorithms
- Multi-View stereo & occlusions

Stereo: epipolar geometry

- Match features along epipolar lines



Stereo image pair



Anaglyphs

http://www.rainbowsymphony.com/freestuff.html

(Wikipedia for images)

Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923

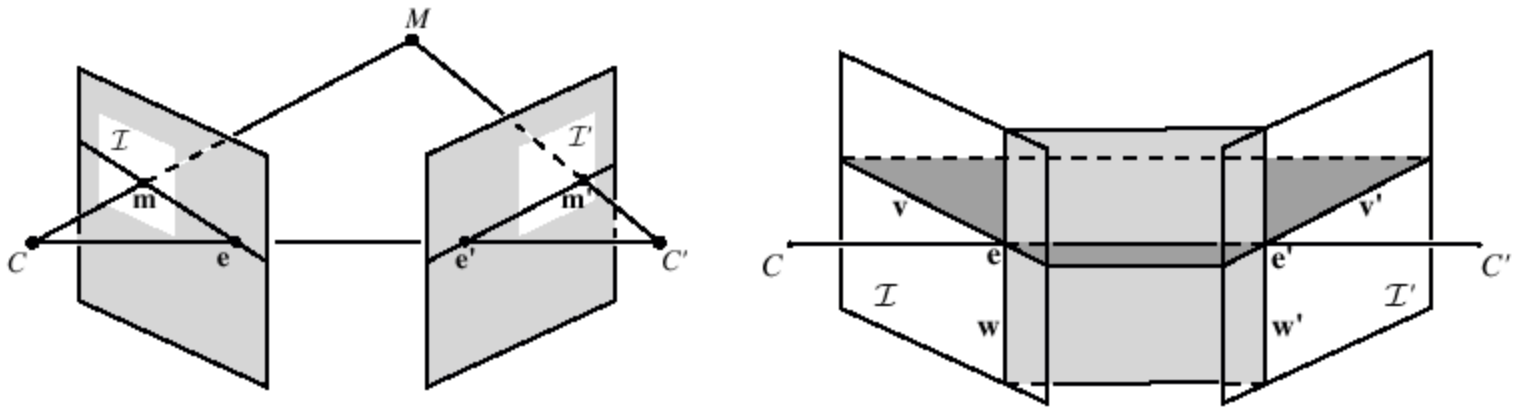
Stereo: epipolar geometry

- for *two* images (or images with collinear camera centers), can find epipolar lines
- epipolar lines are the projection of the *pencil* of planes passing through the centers

- **Rectification:** warping the input images (perspective transformation) so that epipolar lines are horizontal

Rectification

- Project each image onto same plane, which is parallel to the epipole
- Resample lines (and shear/stretch) to place lines in correspondence, and minimize distortion



- [Loop and

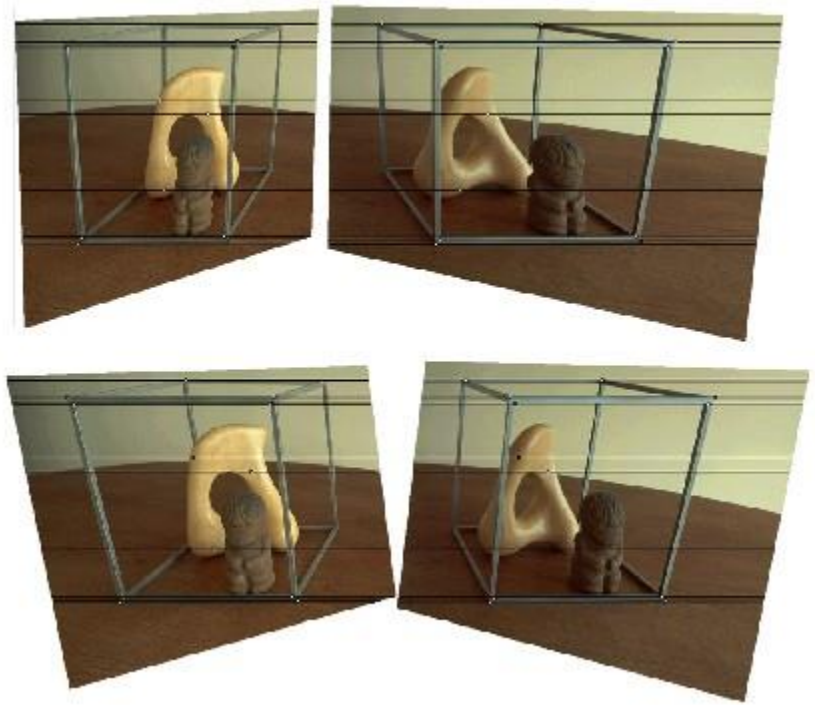
Rectification

(a) Original image pair overlayed with several epipolar lines.

(b) Image pair transformed by the specialized projective mapping $\mathbf{H}_p$ and $\mathbf{H}'_p$. Note that the epipolar lines are now parallel to each other in each image.

BAD!

Rectification



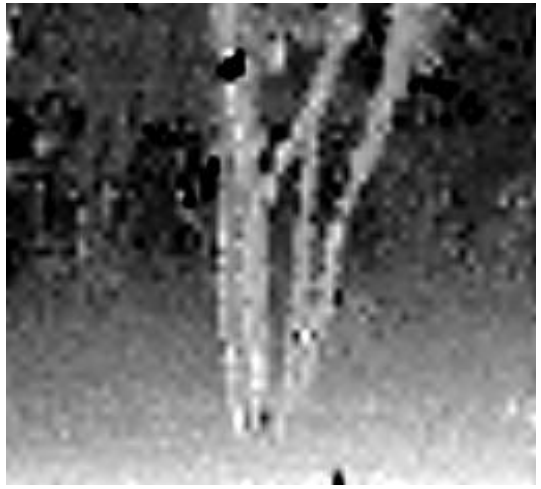
(c) Image pair transformed by the similarity $\mathbf{H}_r$ and $\mathbf{H}'_r$. Note that the image pair is now rectified (the epipolar lines are horizontally aligned).

(d) Final image rectification after shearing transform $\mathbf{H}_a$ and $\mathbf{H}'_a$. Note that the image pair remains rectified, but the horizontal distortion is reduced.

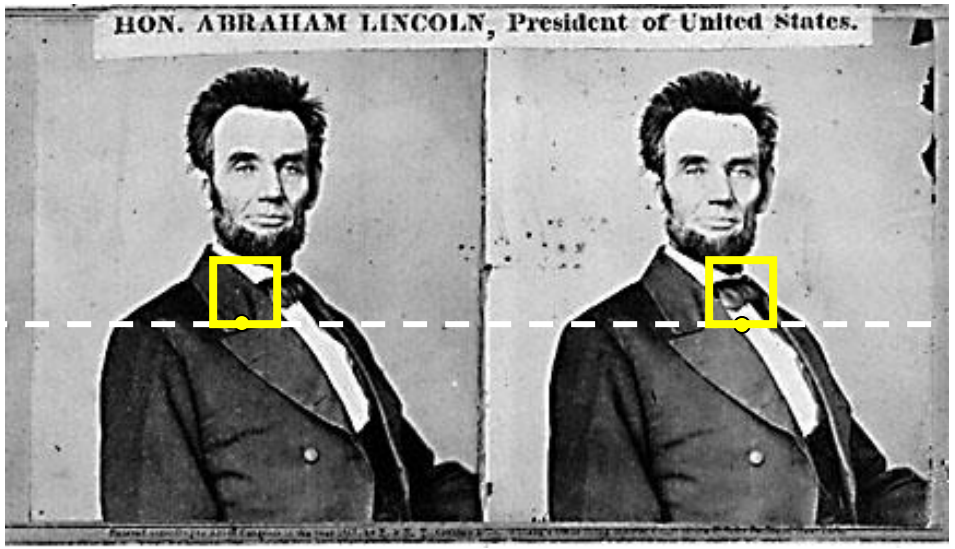
GOOD!

Finding correspondences

- apply feature matching criterion (e.g., correlation or Lucas-Kanade) at *all* pixels simultaneously
- search only over epipolar lines (many fewer candidate positions)



Your basic stereo algorithm



For each epipolar line

For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

Improvement: match ***windows***

- This should look familar...

Image registration (revisited)

- How do we determine correspondences?
  - *block matching* or *SSD* (sum squared differences)

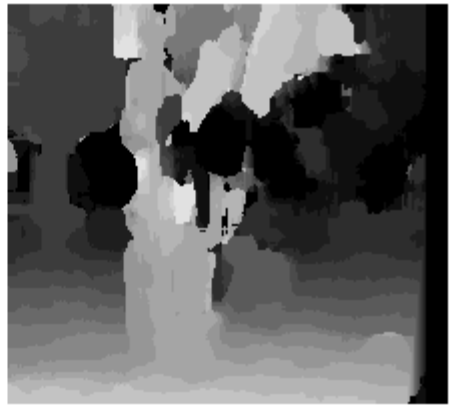
$$E(x,y;d) = \sum_{(x',y')\in N(x,y)} [I_L(x'+d,y') - I_R(x',y')]^2$$

d is the *disparity* (horizontal motion)

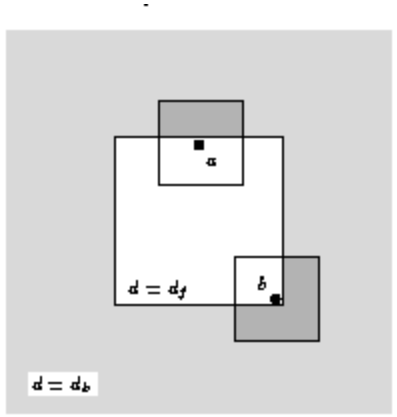
- How big should the neighborhood be?

Neighborhood size

- Smaller neighborhood: more details
- Larger neighborhood: fewer isolated mistakes
- 
- w = 3



- w = 20

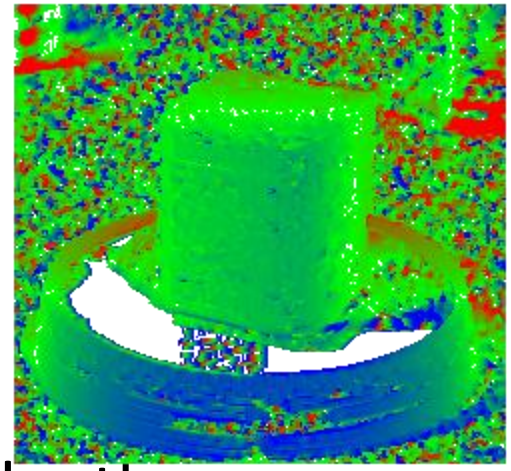
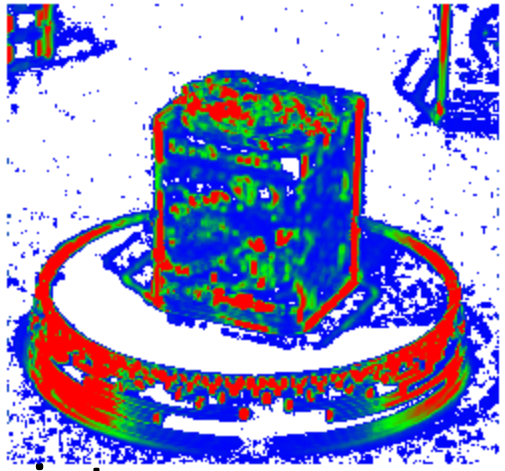


Matching criteria

- Raw pixel values (correlation)
- Band-pass filtered images [Jones & Malik 92]
- “Corner” like features [Zhang, …]
- Edges [many people…]
- Gradients [Seitz 89; Scharstein 94]
- Rank statistics [Zabih & Woodfill 94]

Stereo: certainty modeling

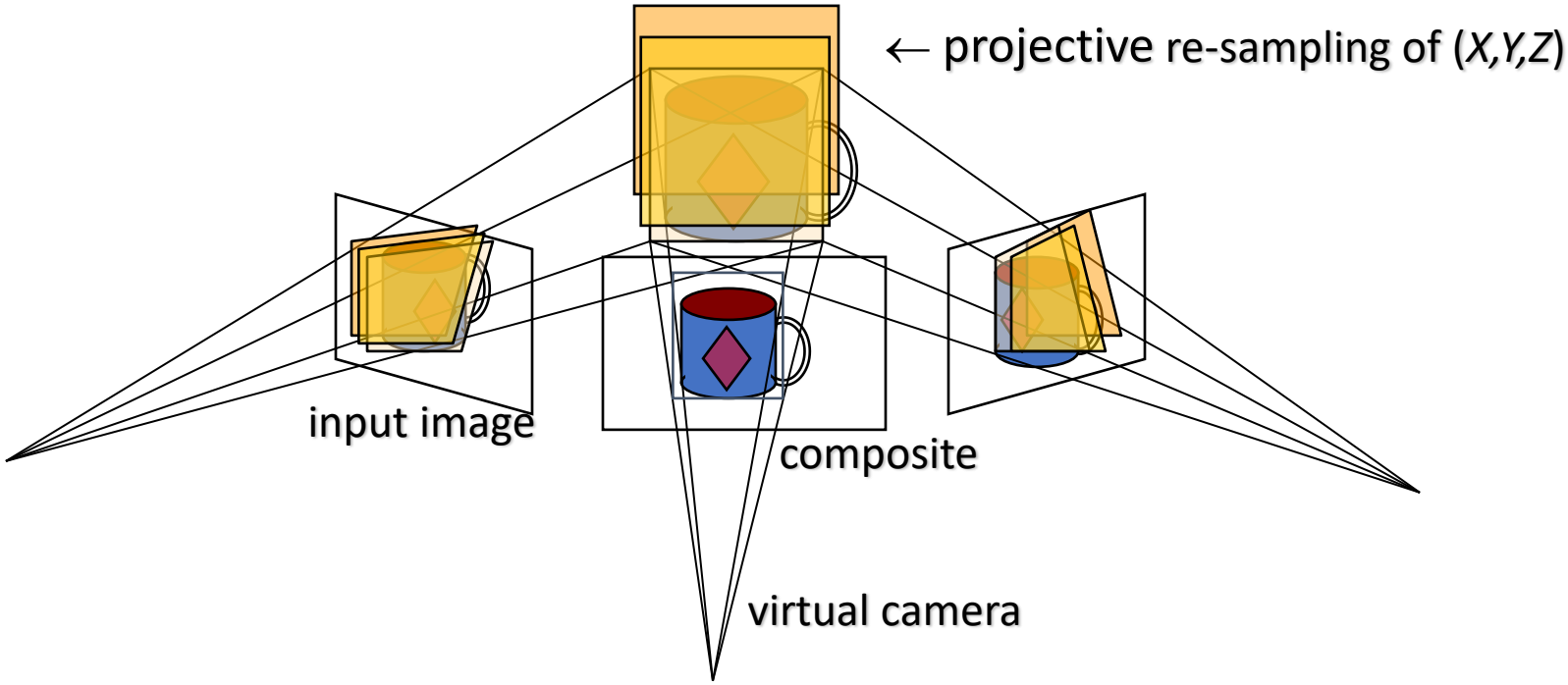
- Compute certainty map from correlations

- input depth map certainty map

Plane Sweep Stereo

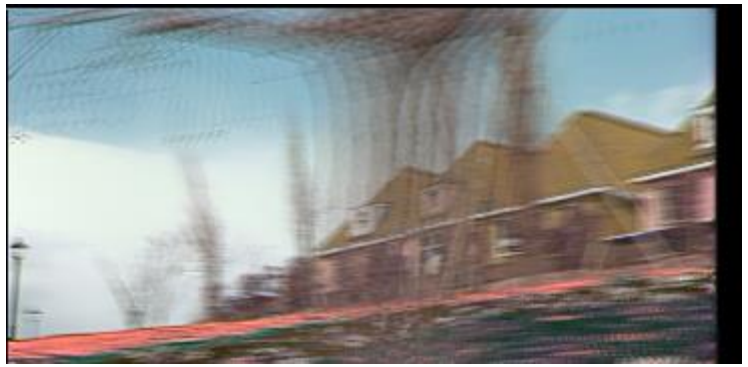
- Sweep family of planes through volume



- each plane defines an image ⇒ composite homography

Plane Sweep Stereo

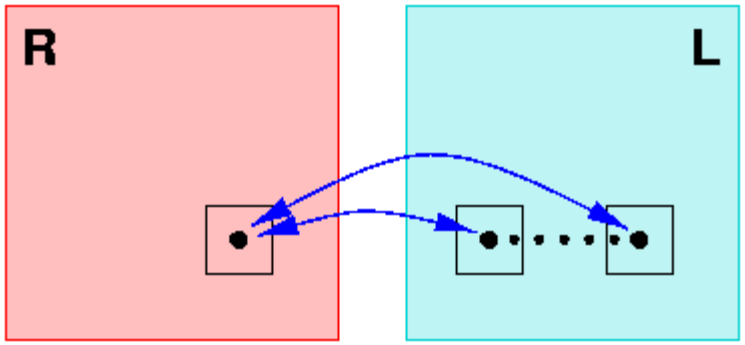
- For each depth plane
  - compute composite (mosaic) image — *mean*

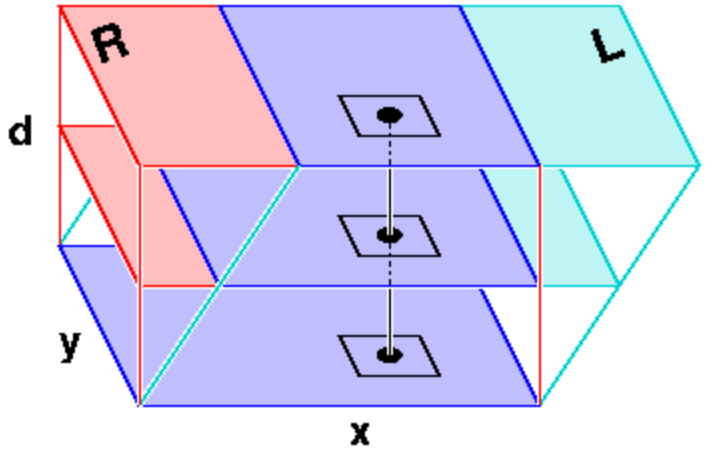


  - compute error image — *variance*
  - convert to confidence and aggregate spatially
- Select winning depth at each pixel

Plane sweep stereo

- Re-order (pixel / disparity) evaluation loops





for every pixel,
for every disparity
compute cost

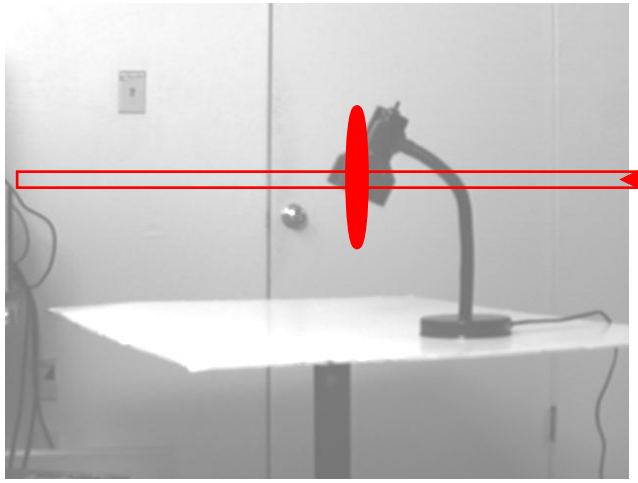
for every disparity
for every pixel
compute cost

Outline

- Stereo basics
- Binocular stereo matching
- Advanced stereo techniques

Binocular rectified stereo

left

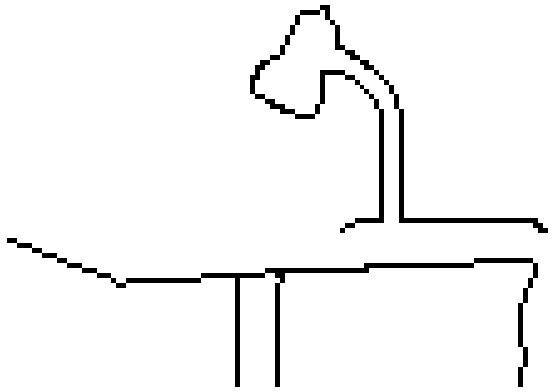


right

epipolar constraint

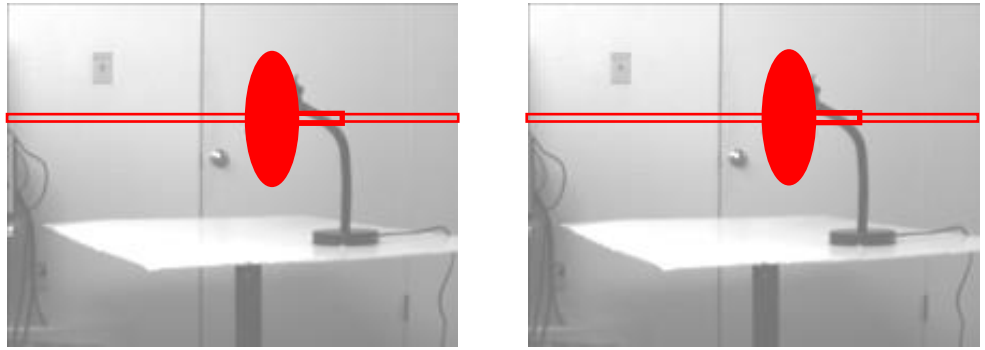
1D search: look for similar pixel in other image

disparity map

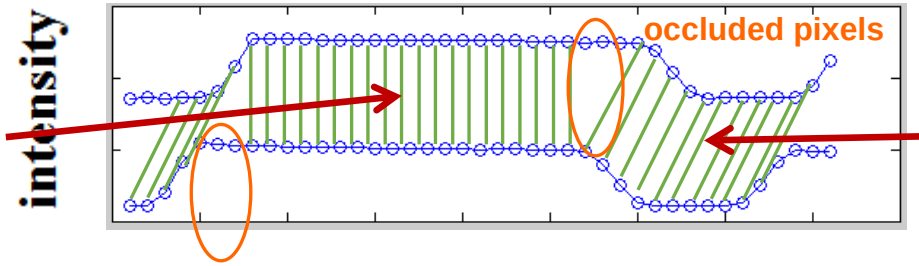


depth discontinuities

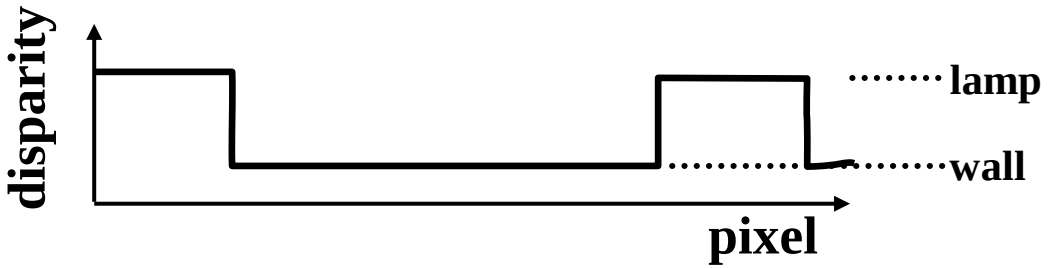
Disparity function



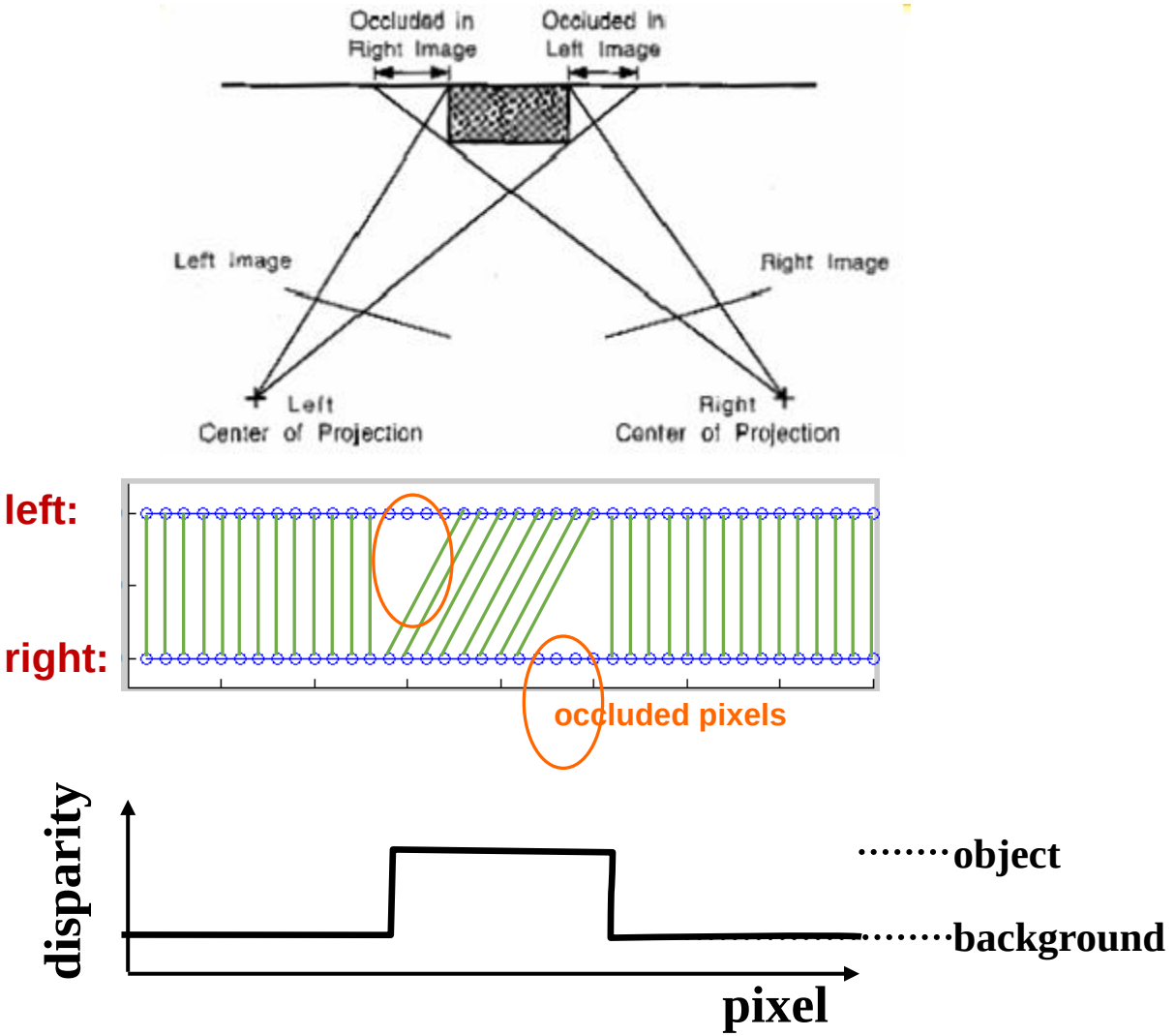
smaller slope =
smaller disparity =
farther from camera



higher slope =
larger disparity =
closer to camera

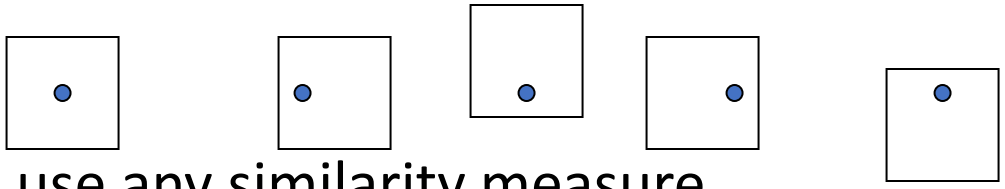


Occlusions


Occluded in
Right Image
Occluded in
Left Image
Left Image
Right Image
Left
Center of Projection
Right
Center of Projection
left:
right:
occluded pixels
disparity
object
background
pixel

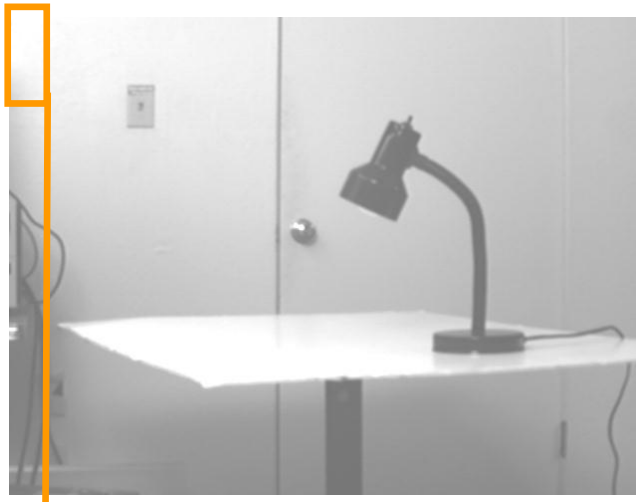
Matching a pixel

- Pixel's value is not unique
  - Only 256 values but ~100,000 pixels!
  - Also, noise affects value
- Solution: use more than one pixel
- Assume neighbors have similar disparity
  - Correlation window around pixel



  - Can use any similarity measure

Block matching

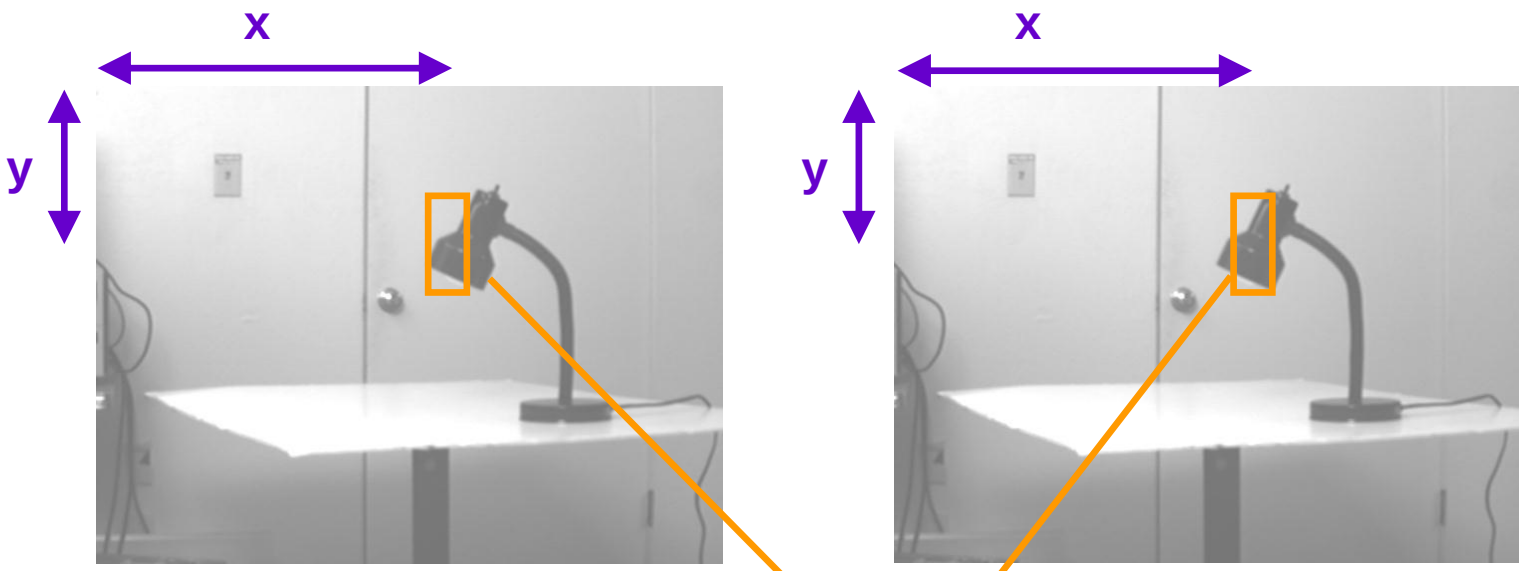


left

disparity map

- **compute best disparity for each pixel**
- **store result in disparity map**

Block matching (cont.)



Note: Window only moves left. Why?

Block matching

$$d_L(x,y) = \arg \min_{0 \le d \le d_{\max}} dissim\left(I_L(x,y), I_R(x-d,y)\right)$$

disparity

dissimilarity

```
Function disparity_map = BlockMatch1(img_left, img_right; min_disp, max_disp)
  for y = 0 to height-1
    for x = 0 to width-1
      ghat = infinity
      for d = min_disp to max_disp
        g = 0
        for j = -w to w
          for i = -w to w
            g = g + dissimilarity(img_left(x+i, y+j), img_right(x+i-d, y+j))
        if g < ghat,
          ghat = g
          dhat = d
      disparity_map(x, y) = dhat
```

5 nested for loops!!!!!

Block matching

$$d_L(x,y) = \arg \min_{0 \le d \le d_{\max}} dissim\left(I_L(x,y), I_R(x-d,y)\right)$$

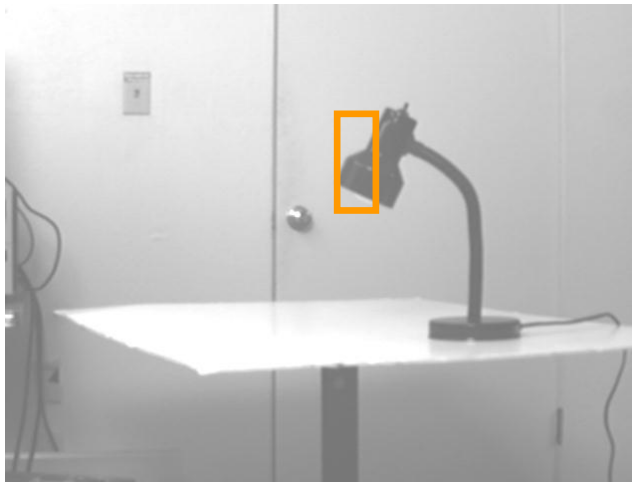
disparity

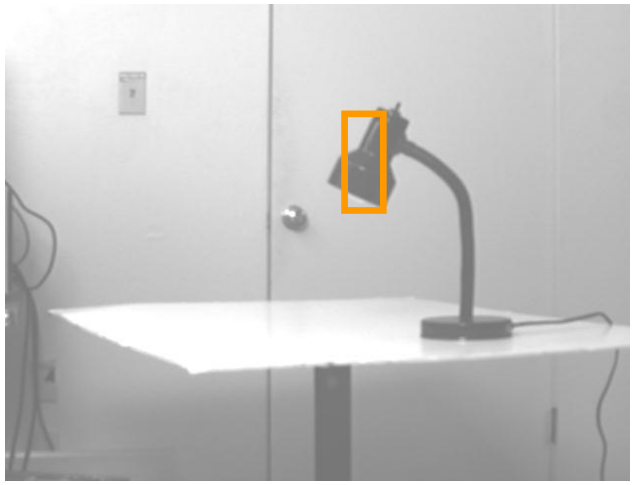
dissimilarity

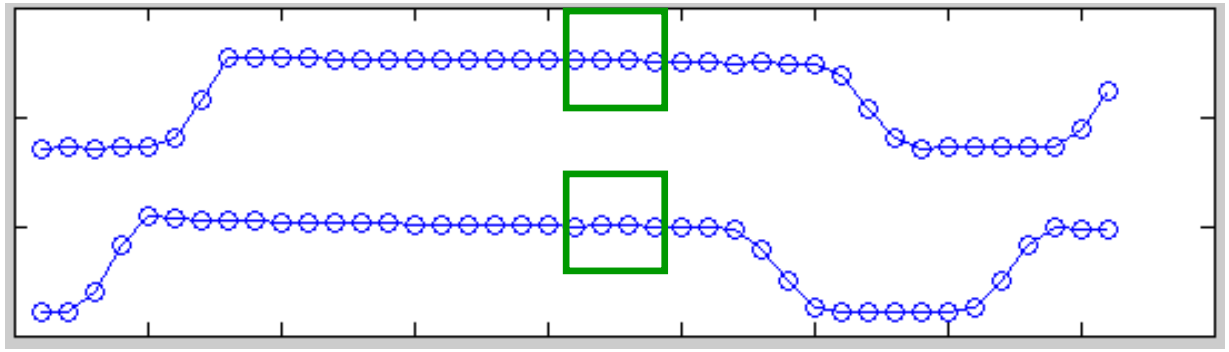
```
BlockMatch1(I_L, I_R, d_min, d_max)
 1  for (x, y) ∈ I_L do
 2        ĝ ← ∞
 3        for d ← d_min to d_max do
 4              g ← 0
 5              for (x̃, ỹ) ∈ W do
 6                    g ← g + dissim(I_L(x + x̃, y + ỹ), I_R(x + x̃ − d, y + ỹ))
 7              if g < ĝ then
 8                    ĝ ← g
 9                    d̂ ← d
10        d_L(x, y) ← d̂
11  return d_L
```

5 nested for loops!!!!!

Eliminating redundant computations



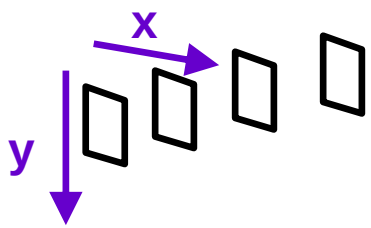




for same disparity, overlapping windows recompute the same dissimilarities for many pixels

Block matching: another view

- Alternatively,
  - precompute
    $\Delta(x,y,d) = dissim(I_L(x,y), I_R(x-d,y))$
    for all *x, y, d*
  - then for each *(x,y)* select the best *d*



More efficient block matching

```
Function dbar = ComputeDbar(img_left, img_right; min_disp, max_disp)
  for d=min_disp:max_disp,
    // compare pixels
    for y=0:height-1,
     for x=0:width-1,
       dbar(x, y, d) = dissimilarity(img_left(x, y), img_right(x-d, y)
    // convolve with 2D box filter to sum over window
    tmp = convolve dbar(:, :, d) with 1D kernel [1 ... 1]
    dbar(:, :, d) = convolve tmp with 1D kernel [1 ... 1]^T
```

} separable

```
Function disparity_map = BlockMatch2(img_left, img_right; min_disp, max_disp)
dbar = ComputeDbar(img_left, img_right; min_disp, max_disp)
  for y=0:height-1,
   for x=0:width-1,
     disparity_map(x, y) = arg min of dbar(x, y, :)
```

Key idea: **Summation over window is convolution with box filter, which is separable**

(only 3 nested for loops!!!)

Running sum improves efficiency even more

More efficient block matching

```
BlockMatch2(I_L, I_R, d_min, d_max)
1  Δ ← ComputeSummedDissimilarities(I_L, I_R, d_min, d_max)
2  for (x, y) ∈ I_L do
3        d_L(x, y) ← arg min_d Δ(x, y, d)
4  return d_L

ComputeSummedDissimilarities(I_L, I_R, d_min, d_max)
1  for d ← d_min to d_max do
2        for (x, y) ∈ I_L do
3              Δ(x, y, d) ← dissim(I_L(x, y), I_R(x − d, y))
4        Δ(:, :, d) ← Convolve(Δ(:, :, d), 1_{w×w})
5  return Δ
```

separable

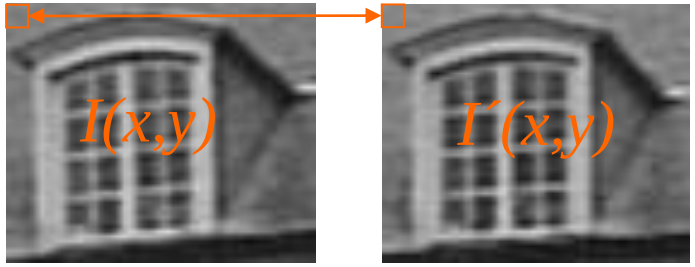
Key idea: **Summation over window is convolution with box filter, which is separable**

(only 3 nested for loops!!!)

Running sum improves efficiency even more

Comparing image regions

Compare intensities pixel-by-pixel



Dissimilarity measures

Sum of Square Differences

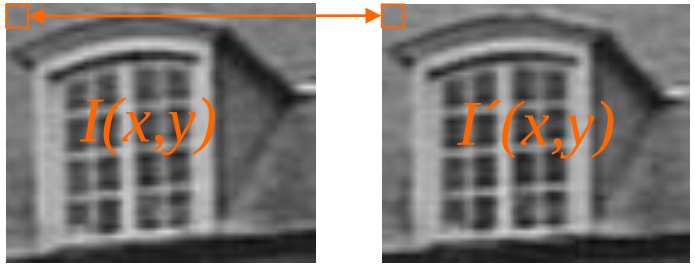
$$SSD = \iint_W \left[I'(x,y) - I(x,y) \right]^2 dxdy$$

Note: **SAD is fast approximation (replace square with absolute value)**

M. Pollefeys, http://www.cs.unc.edu/Research/vision/comp256fall03/

Comparing image regions

Compare intensities pixel-by-pixel



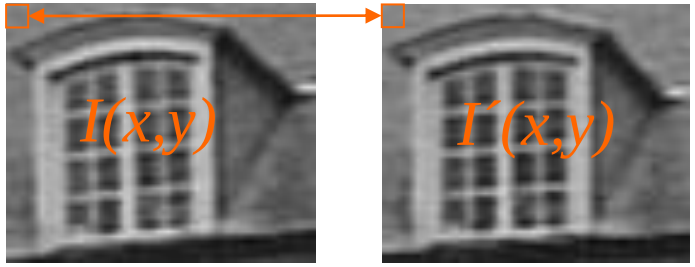
Dissimilarity measures

If energy does not change much, then minimizing SSD equals maximizing cross-correlation

M. Pollefeys, http://www.cs.unc.edu/Research/vision/comp256fall03/

Comparing image regions

Compare intensities pixel-by-pixel



Similarity measures

Zero-mean Normalized Cross Correlation

$$NCC = \frac{N(I', I)}{\sqrt{N(I', I')N(I, I)}}$$

$$N(A, B) = \iint_W \left(A(x, y) - \overline{A}\right)\left(B(x, y) - \overline{B}\right)dxdy$$

M. Pollefeys, http://www.cs.unc.edu/Research/vision/comp256fall03/

Dissimilarity measures

Most common:

$$D(\mathbf{x}_L, \mathbf{x}_R) = [I_L(x_L, y_L) - I_R(x_R, y_R)]^2 \qquad \text{SSD}$$

$$D(\mathbf{x}_L, \mathbf{x}_R) = |I_L(x_L, y_L) - I_R(x_R, y_R)| \qquad \text{SAD}$$

$$D(\mathbf{x}_L, \mathbf{x}_R) = -I_L(x_L, y_L) I_R(x_R, y_R) \qquad \text{cross correlation}$$

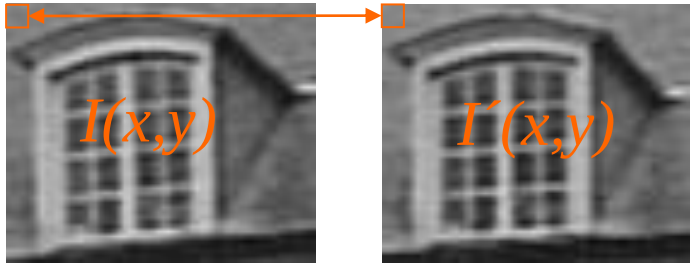
Connection between SSD and cross correlation:

$$\begin{aligned} D(\mathbf{x}_L, \mathbf{x}_R) &= [I_L(x_L, y_L) - I_R(x_R, y_R)]^2 \\ &= [I_L(x_L, y_L)]^2 + [I_R(x_R, y_R)]^2 - 2 I_L(x_L, y_L) I_R(x_R, y_R) \\ &\propto -I_L(x_L, y_L) I_R(x_R, y_R) \end{aligned}$$

Also normalized correlation, rank, census, sampling-insensitive ...

Comparing image regions

Compare intensities pixel-by-pixel



Similarity measures

Census

$$C_I(i,j) = (I(x+i, y+j) > I(x,y))$$

125	126	125
127	128	130
129	132	135

→

0	0	0
0		1
1	1	1

→ [00001111]

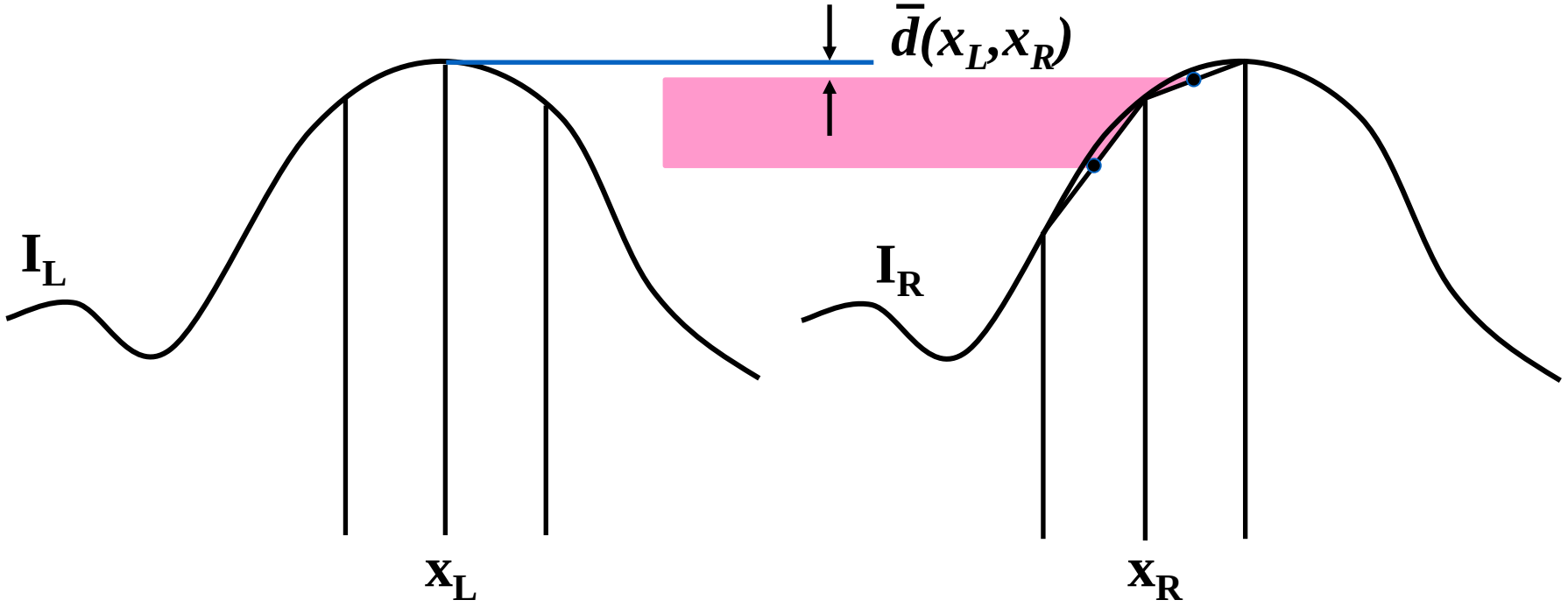
only compare bit signature

using XOR, SAD, or Hamming distance (all equivalent)

(Real-time chip from TZYX based on Census)

M. Pollefeys, http://www.cs.unc.edu/Research/vision/comp256fall03/

Sampling-Insensitive Pixel Dissimilarity



Our dissimilarity measure: $d(x_L,x_R) = \min\{\bar{d}(x_L,x_R)\ ,\bar{d}(x_R,x_L)\}$

[Birchfield & Tomasi 1998]

Dissimilarity Measure Theorems

Given: **An interval A such that**
$[x_L - ½ , x_L + ½] \subseteq A$, and
$[x_R - ½ , x_R + ½] \subseteq A$

Theorem 1:

If $| x_L - x_R | \leq ½$, then $d(x_L,x_R) = 0$

(when A is convex or concave)

Theorem 2: $| x_L - x_R | \leq ½$ iff $d(x_L,x_R) = 0$

(when A is linear)

[Birchfield & Tomasi 1998]

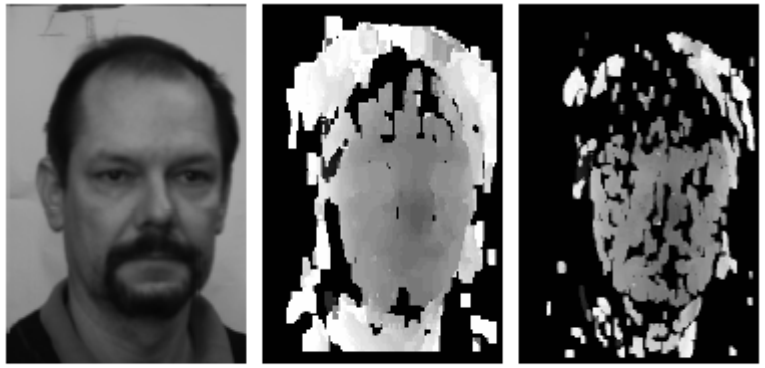
Aggregation window sizes

Small windows

- disparities similar
- more ambiguities
- accurate when correct

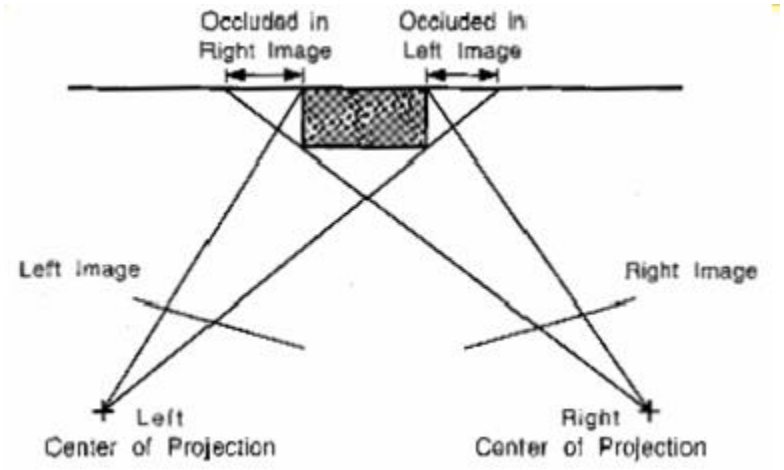
Large windows

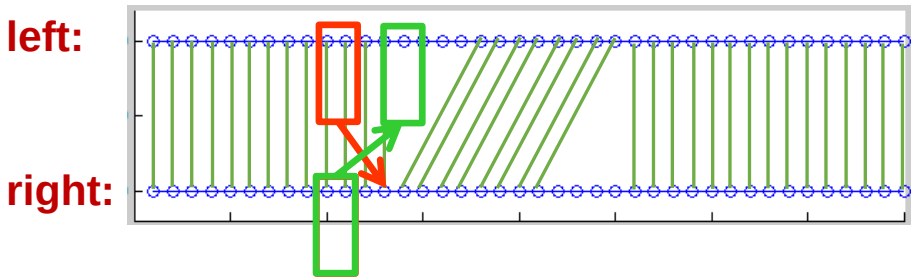
- larger disp. variation
- more discriminant
- often more robust
- use shiftable windows to deal with discontinuities



(Illustration from Pascal Fua)

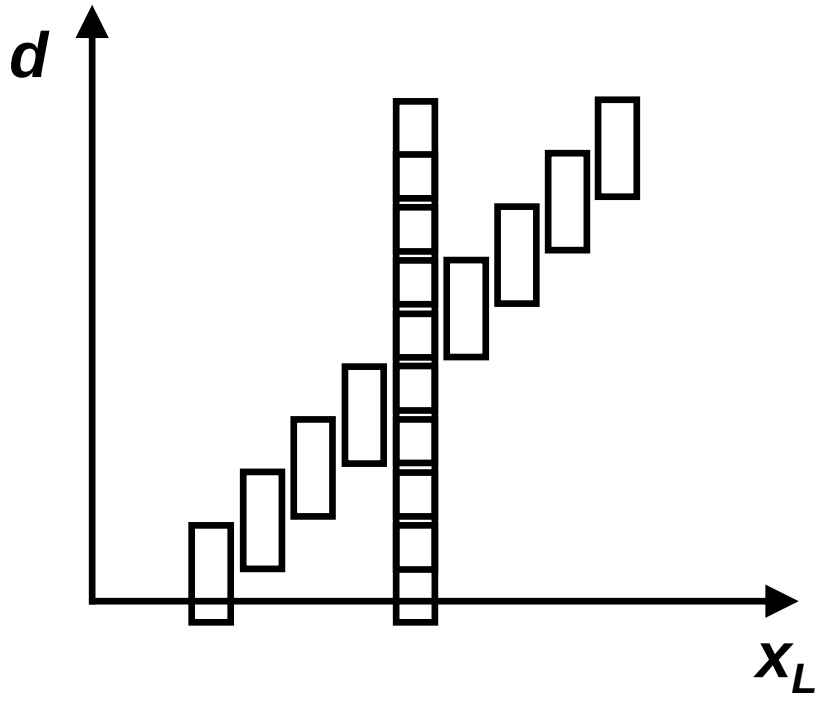
Occlusions





If pixel matches do not agree in both directions, then unreliable

Left-right consistency check



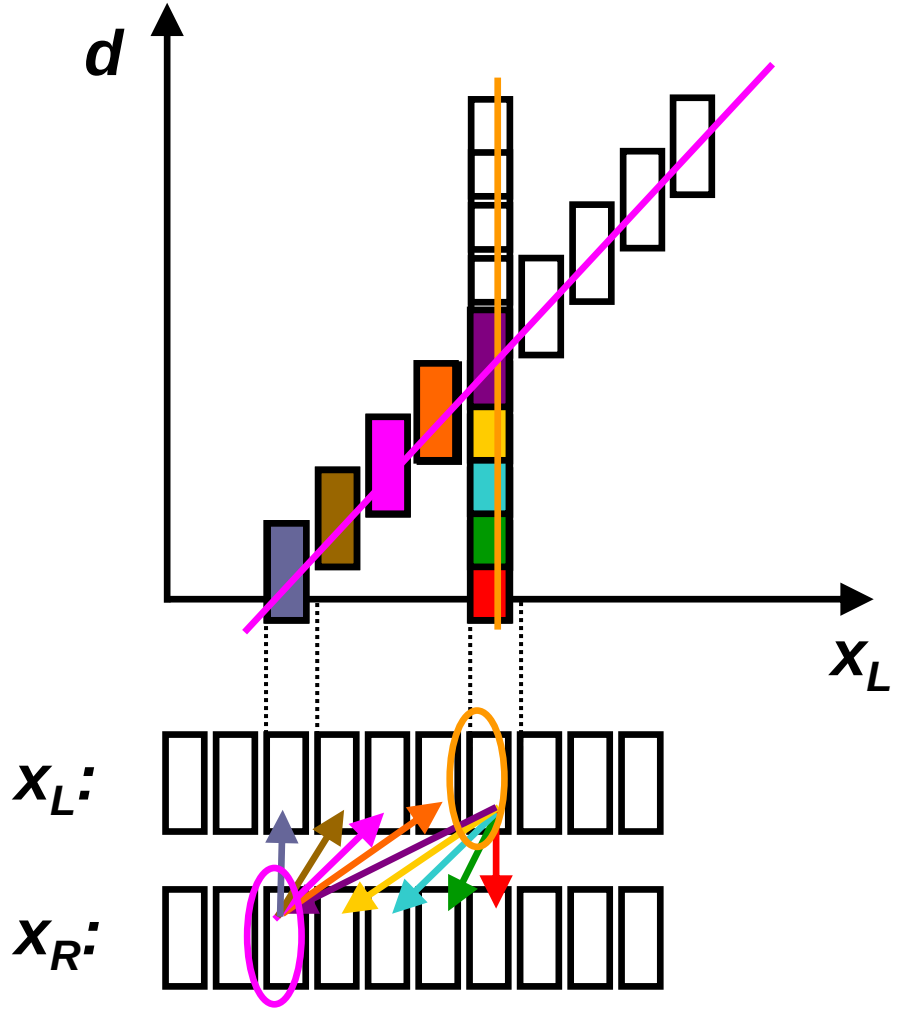
- **Search left-to-right, then right-to-left**
- **Retain disparity only if they agree**

Do minima coincide?

Conceptually,

```
dm_L = BlockMatch(img_left, img_right; 0, max_disp)
dm_R = BlockMatch(img_right, img_left; -max_disp, 0)
for y=0:height-1,
for x=0:width-1,
  if dm_L(x, y) != - dm_R(x - dm_L(x, y), y)
    dm_L(x, y) = NOT_MATCHED
```

Left-right consistency check



for pixel (x,y) in left image, choices are

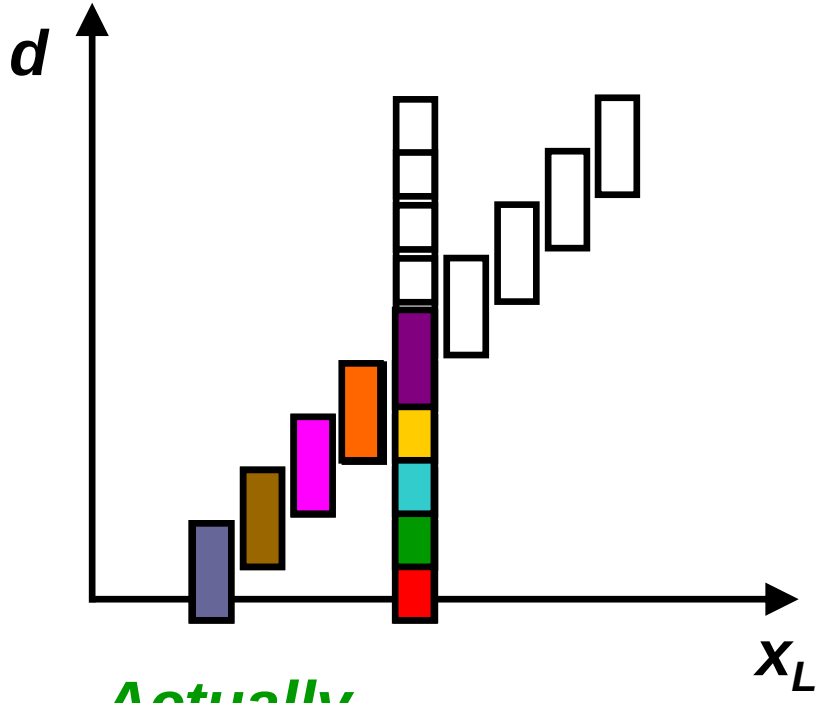
Δ(x,y,0),
Δ(x,y,1),
Δ(x,y,2),
…,
Δ(x,y,max_disp)

for pixel (x,y) in right image, choices are

Δ(x,y,0),
Δ(x+1,y,1),
Δ(x+2,y,2),
…,
Δ(x+max_disp,y,max_disp)

because $x_L = x_R$ + disparity

Left-right consistency check



Actually,

```
Function disparity_map = BlockMatchWithRightLeftCheck(img_left, img_right; max_disp)
△= ComputeDbar(img_left, img_right; 0, max_disp)
for y=0:height-1,
for x=0:width-1,
  // find left answer
  d_left = arg min( △(x,y,0), △(x,y,1), ..., △(x,y,max_disp) )
  d_right = arg min( △(x-d_left,y,0), △(x-d_left+1,y,1), ..., △(x-d_left+max_disp,y,max_disp)
  disp_map(x,y) = (d_left == d_right) ? d_left : NOT_MATCHED
```

With left-right check

inefficient:

BlockMatchWithLeftRightCheck1$(I_L, I_R, d_{\max})$

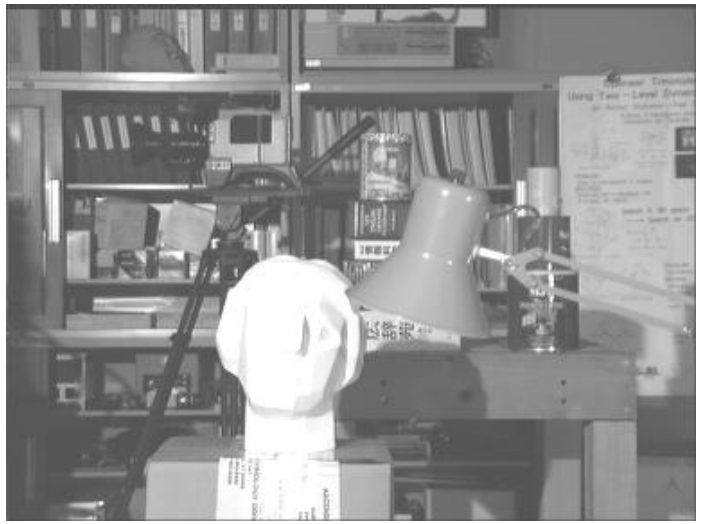
```
1  d_L ← BlockMatch2(I_L, I_R, 0, d_max)
2  d_R ← BlockMatch2(I_R, I_L, −d_max, 0)
3  for (x, y) ∈ I_L do
4        if d_L(x, y) ≠ −d_R(x − d_L(x, y), y) then
5              d_L(x, y) ← NOT-MATCHED
6  return d_L
```

more efficient:

BlockMatchWithLeftRightCheck2$(I_L, I_R, d_{\max})$

```
1  Δ ← ComputeSummedDissimilarities(I_L, I_R, 0, d_max)
2  for (x, y) ∈ I_L do
3        δ_L ← arg min{Δ(x, y, 0), Δ(x, y, 1), ..., Δ(x, y, d_max)}
4        δ_R ← arg min{Δ(x − δ_L, y, 0), Δ(x − δ_L + 1, y, 1), ..., Δ(x − δ_L + d_max, y, d_max)}
5        if δ_L == δ_R then
6              d_L(x, y) ← δ_L
7        else
8              d_L(x, y) ← NOT-MATCHED
9  return d_L
```

Results: correlation



left

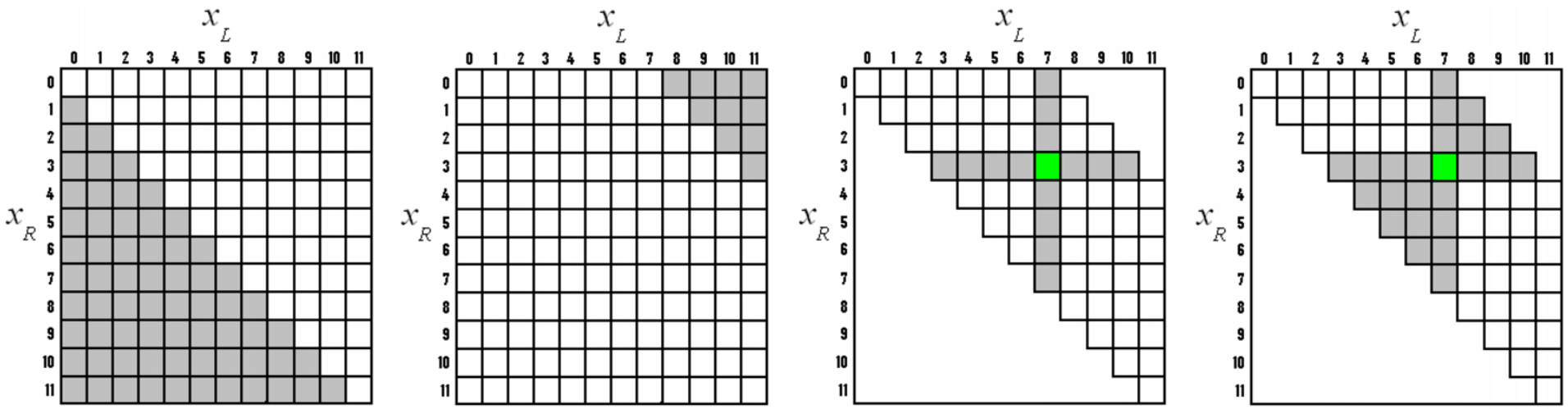
disparity map

with left-right consistency check

Constraints

- Epipolar – match must lie on epipolar line
- Piecewise constancy – neighboring pixels should usually have same disparity
- Piecewise continuity – neighboring pixels should usually have similar disparity
- Disparity – impose allowable range of disparities
  (Panum's fusional area)
- Disparity gradient – restricts slope of disparity
- Figural continuity – disparity of edges across scanlines
- Uniqueness – each pixel has no more than one match (violated by windows and mirrors)
- Ordering – disparity function is monotonic
  (precludes thin poles)

Stereo constraints



cheirality

$$d = x_L - x_R \geq 0$$

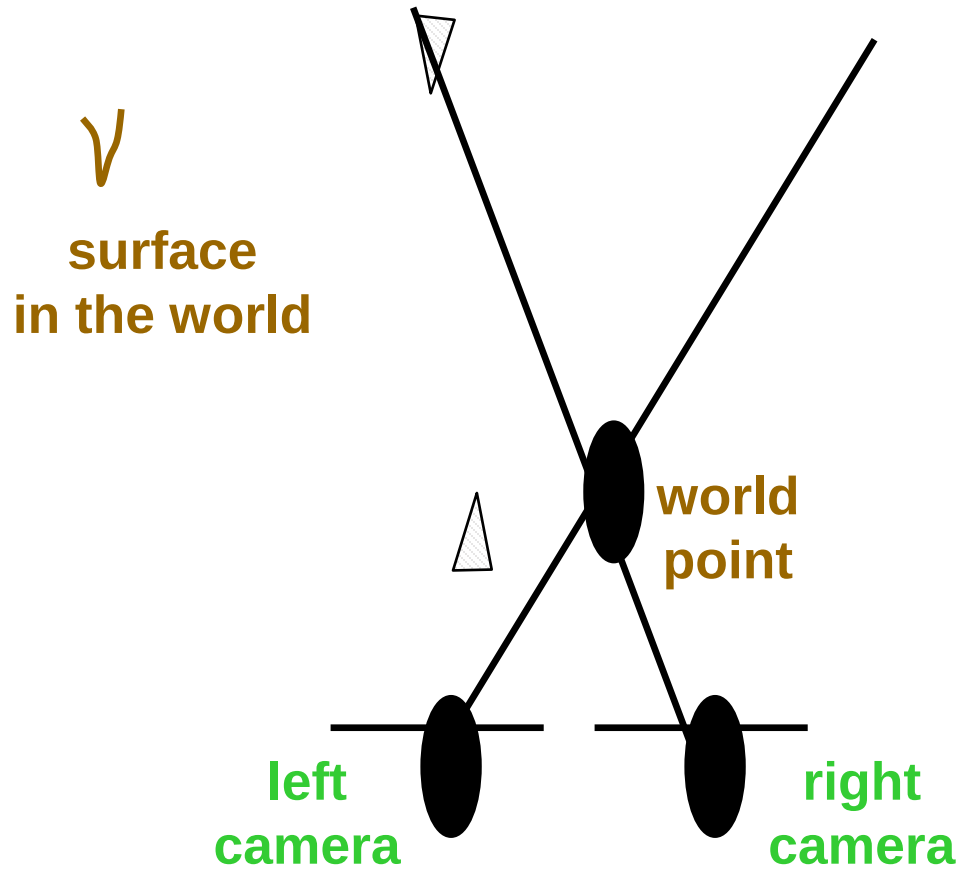
maximum disparity

uniqueness

ordering (monotonicity)

When are these violated?

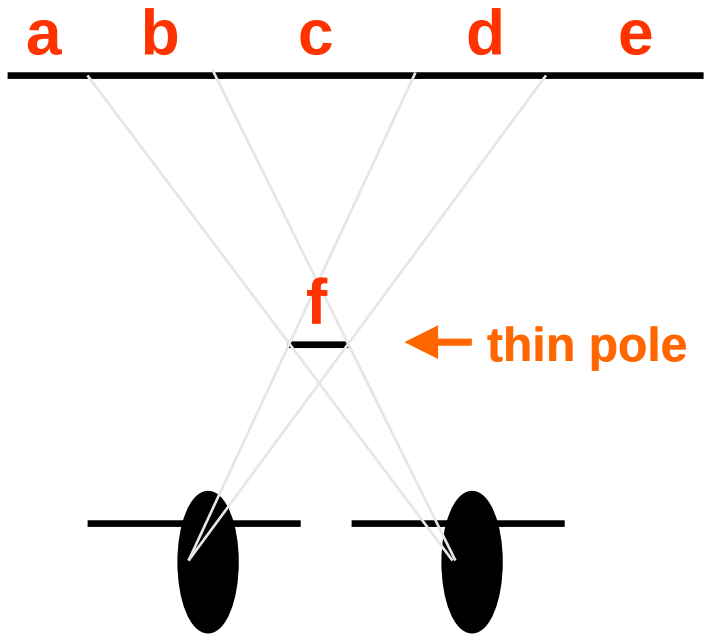
Forbidden zone

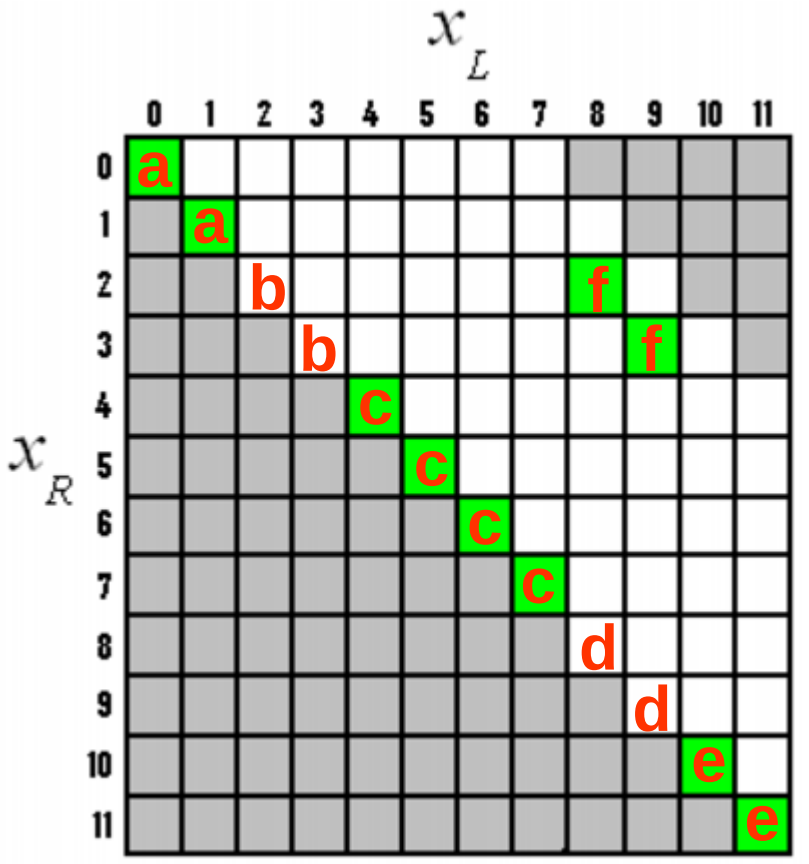


**no matches
are possible in the
hourglass-shaped region
(if surface is continuous)**

(Related to ordering constraint)

Violation of ordering constraint





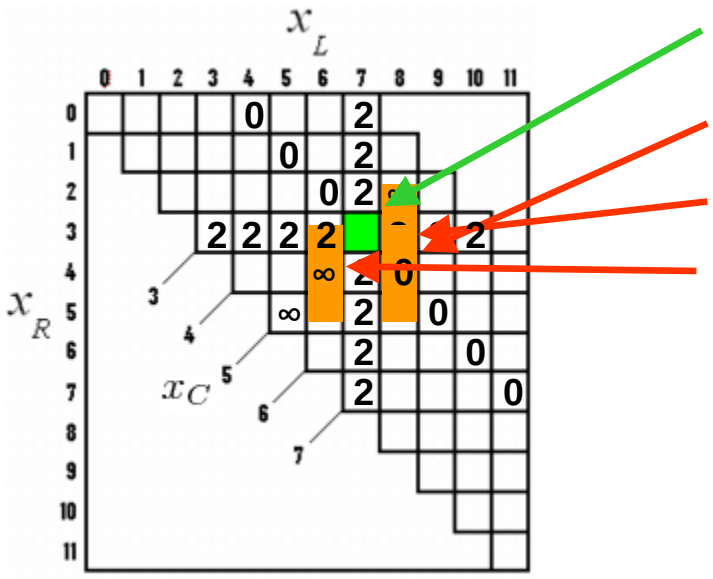
Disparity gradient

$$x_C = \frac{1}{2}(x_L + x_R)$$ ← Cyclopean coordinate

x_1 in I_L matches x'_1 in I_R: $d_1 = x_1 - x'_1$

x_2 in I_L matches x'_2 in I_R: $d_2 = x_2 - x'_2$

disparity gradient: $$\left|\frac{\partial d}{\partial x_c}\right| = \frac{d_2 - d_1}{\frac{1}{2}(x_2 + x'_2) - \frac{1}{2}(x_1 + x'_1)} = \frac{2(d_2 - d_1)}{x_2 + x'_2 - x_1 - x'_1}$$



$d_1 = 7-3 = 4$ $x_c = (7+3)/2 = 5$

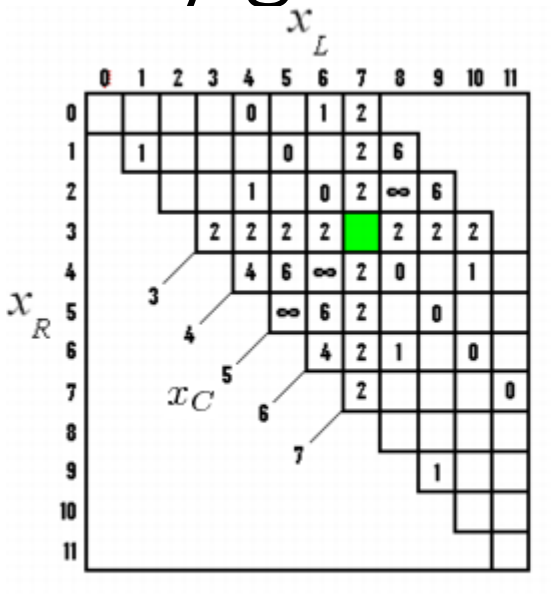
$d_2 = 8-4 = 4$ $x_c = (8+4)/2 = 6$ d.g. = 0

$d_2 = 8-3 = 5$ $x_c = (8+3)/2 = 5.5$ d.g. = 2

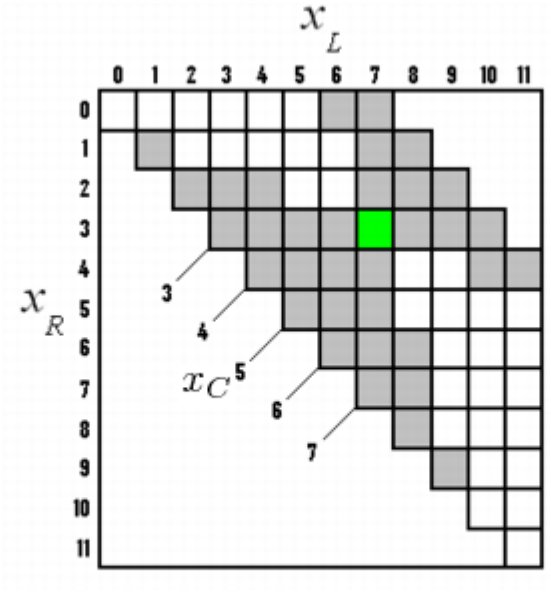
$d_2 = 6-4 = 2$ $x_c = (6+4)/2 = 5$ d.g. = ∞

…

Disparity gradient constraint

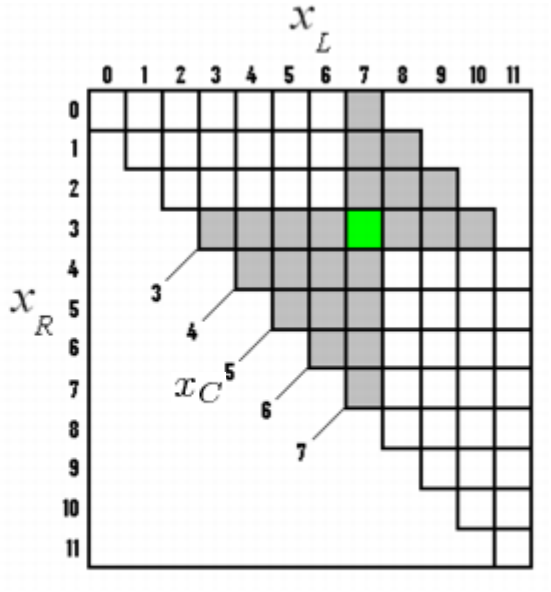


$$\left|\frac{\partial d}{\partial x_c}\right|$$



$$\left|\frac{\partial d}{\partial x_c}\right| \leq 1$$

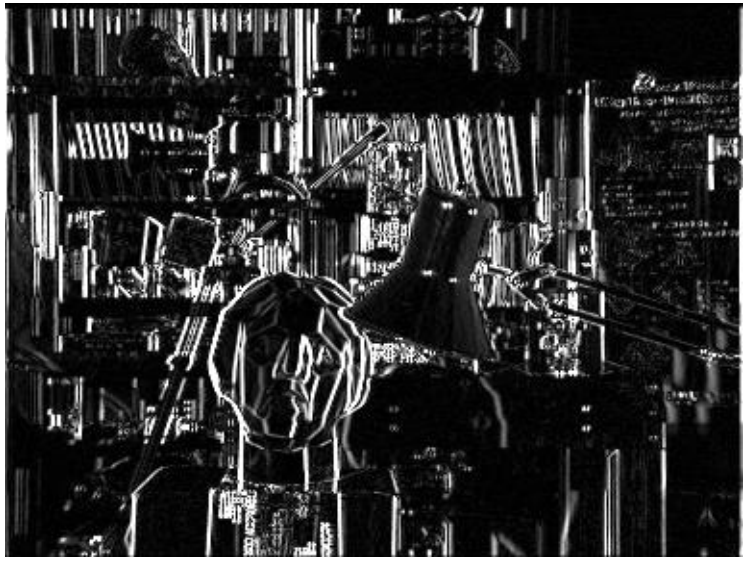
(human visual system imposes this)



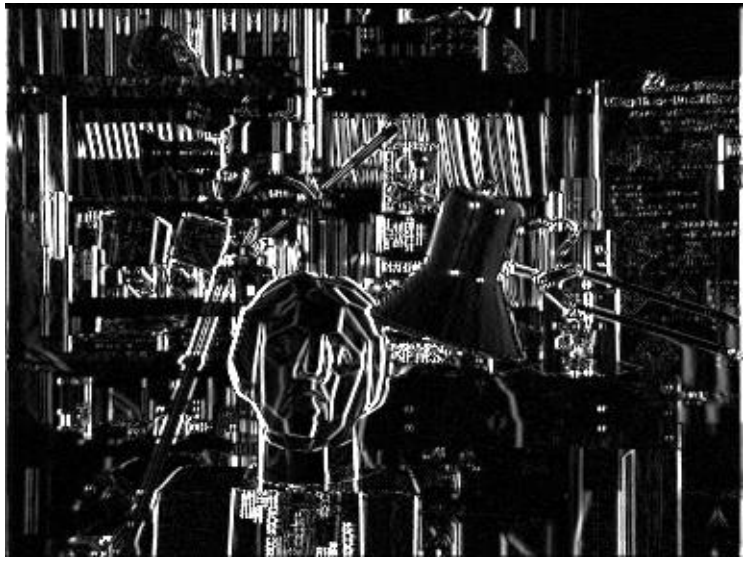
$$\left|\frac{\partial d}{\partial x_c}\right| \leq 2$$

(same as ordering constraint)

Figural continuity constraint



right



left

[University of Tsukuba]

Outline

- Stereo basics
- Binocular stereo matching
- Advanced stereo techniques

Dynamic Programming: 1D Search

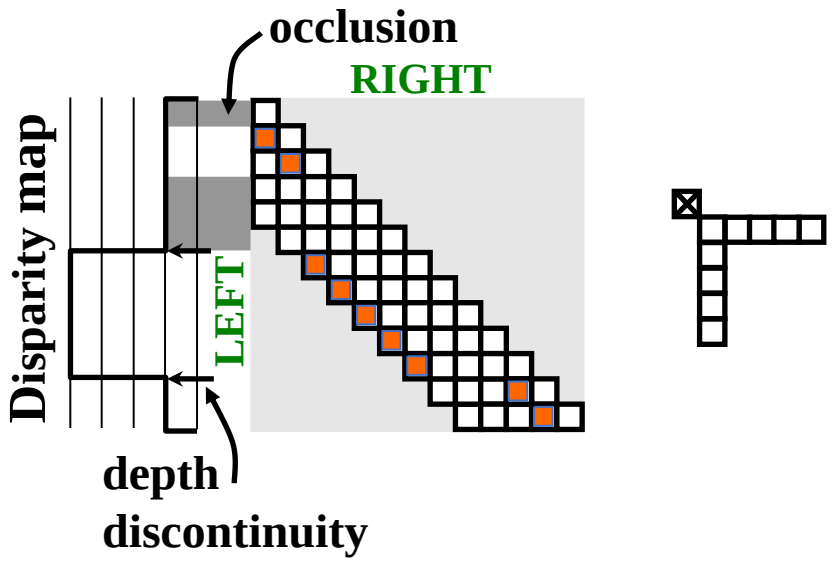
string editing:

		c	a	r	t
	0	1	2	3	4
c	1	0	1	2	3
a	2	1	0	1	2
t	3	2	1	1	1

c a t
| | \
c a r t

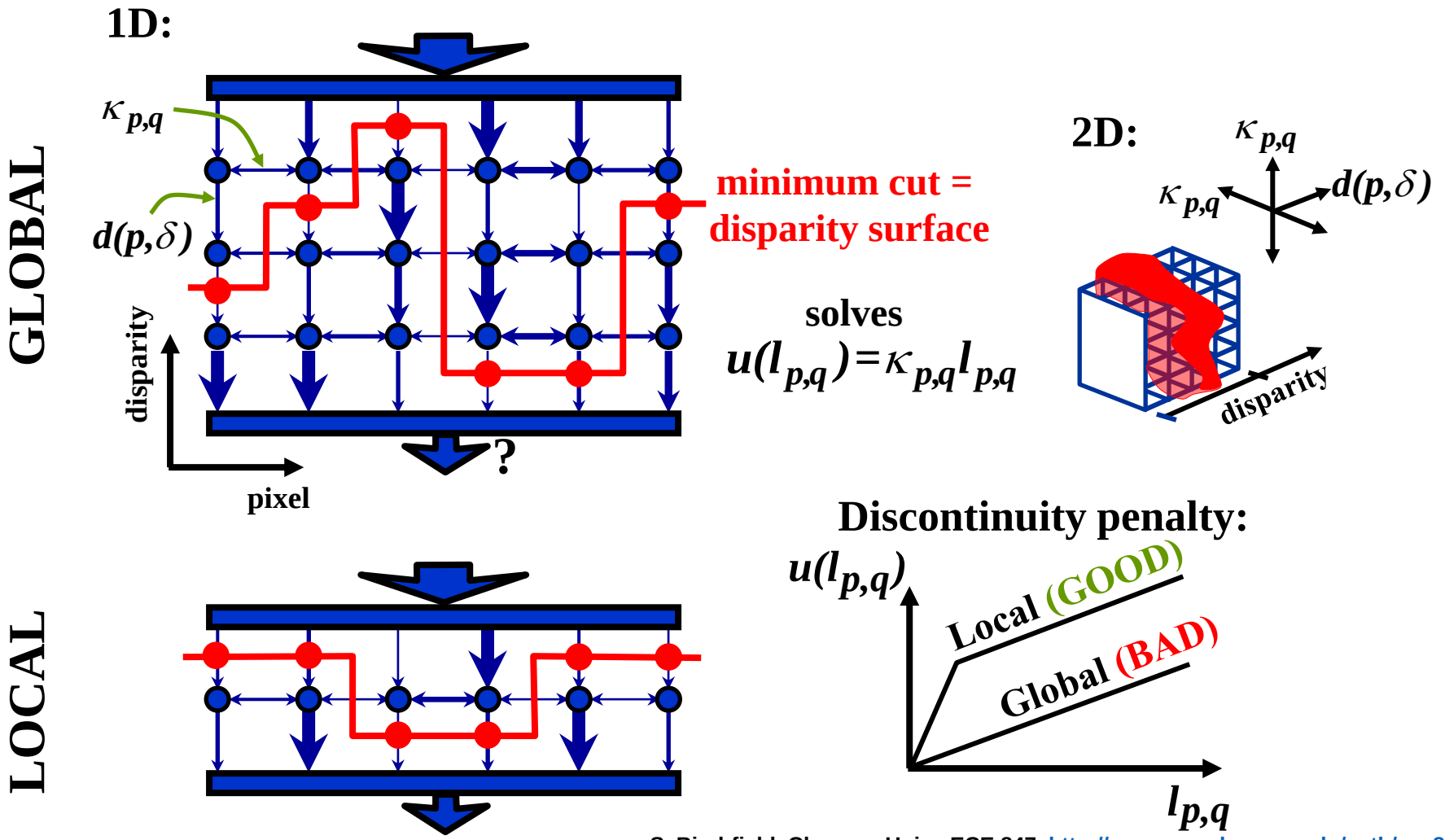
penalties:
- mismatch = 1
- insertion = 1
- deletion = 1

stereo matching:

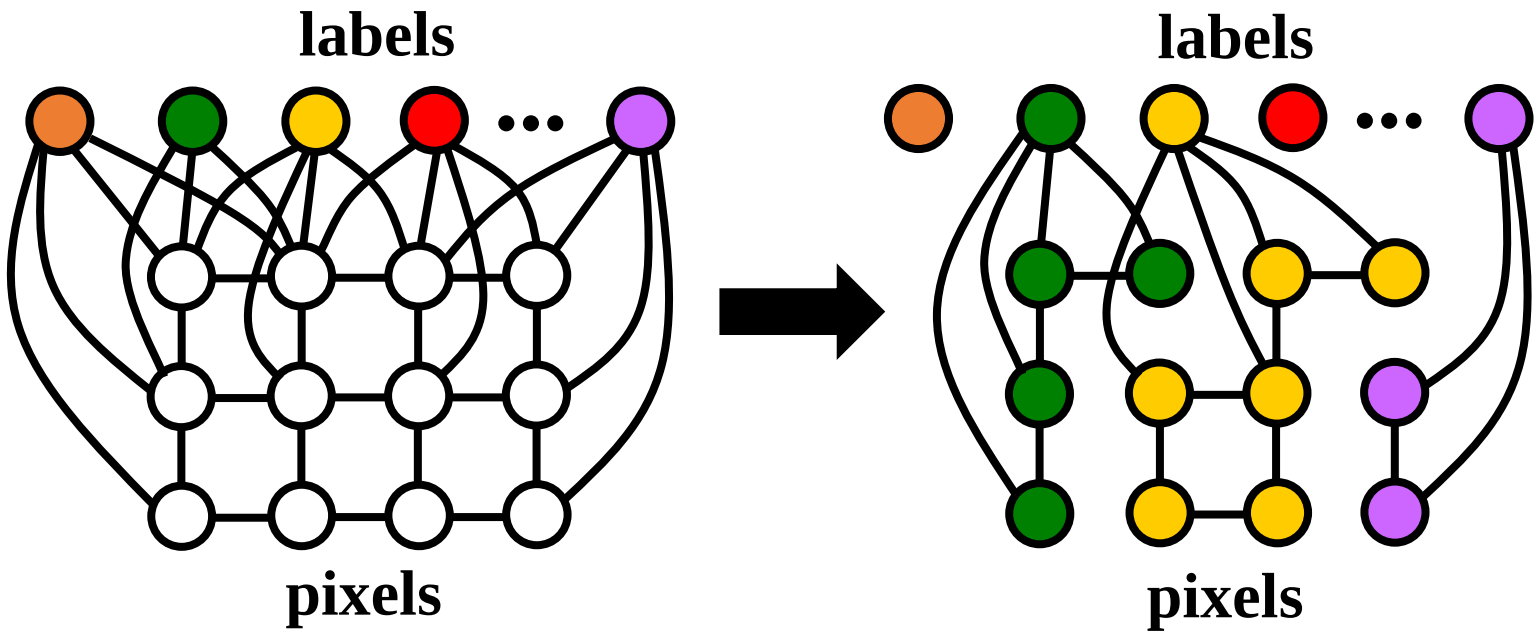


Minimizing a 2D Cost Functional

$$\text{Minimize: } \min_{\delta} E_{data} + E_{smoothness} = \sum_{p} d(p,\delta) + \sum_{\{p,q\} \in N} \kappa_{p,q} u(l_{p,q})$$

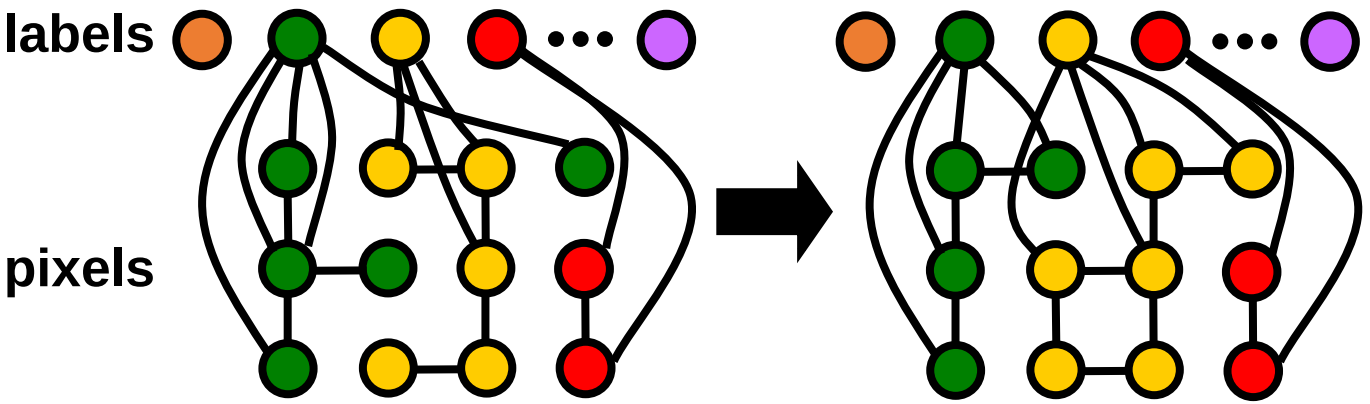


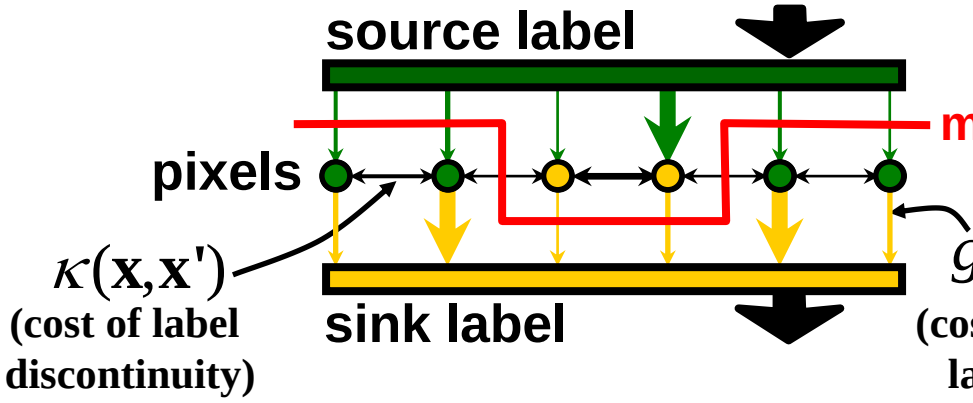
Multiway-Cut: 2D Search



[Boykov, Veksler, Zabih 1998]

Multiway-Cut Algorithm

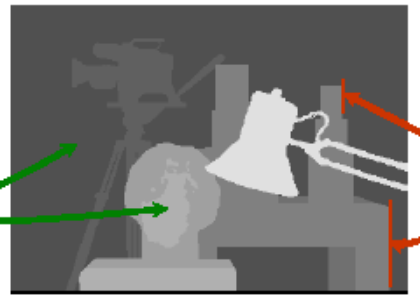




Minimizes $\sum_{\mathbf{x}} g(\mathbf{x}, f(\mathbf{x})) + \sum_{(\mathbf{x},\mathbf{x}')} \kappa(\mathbf{x},\mathbf{x}')[f(\mathbf{x}) \neq f(\mathbf{x}')]$

Energy minimization

Disparity continuous in most places,



except at depth discontinuities

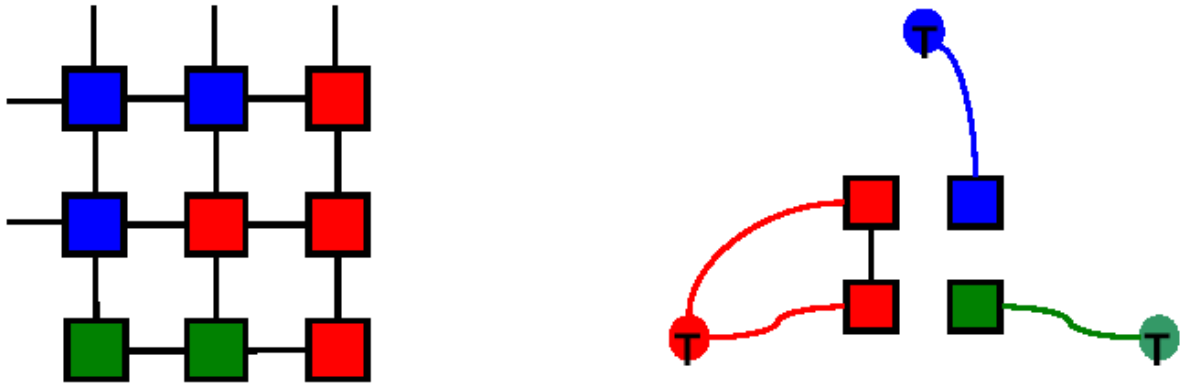
1. Matching pixels should have similar intensities.
2. Most nearby pixels should have similar disparities

→ Minimize

$$\sum [I_1(x + D(x,y), y) - I_2(x,y)]^2$$
$$+ \lambda \sum [D(x+1,y) - D(x,y)]^2$$
$$+ \mu \sum [D(x,y+1) - D(x,y)]^2$$

(Slide from Pascal Fua)

Graph Cut



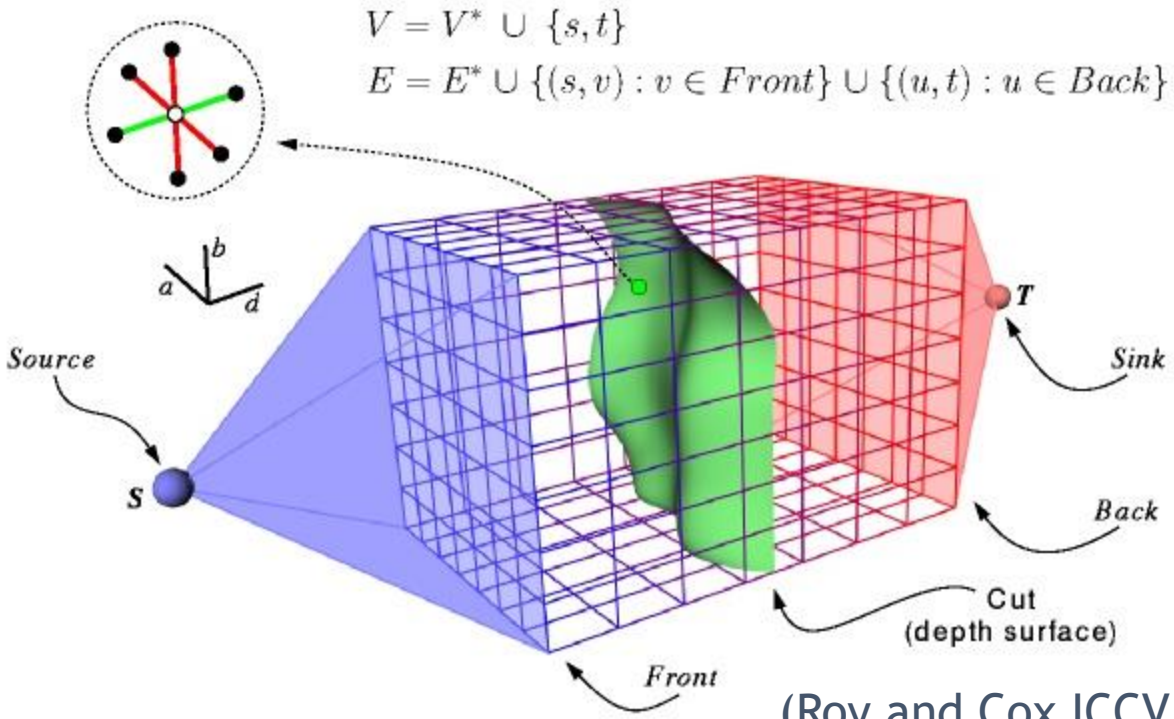
1. Stereo is a labeling problem
2. Graph cut corresponds to a labeling.

→ **Assign edge weights cleverly so that the min-weight cut gives the minimum energy!**

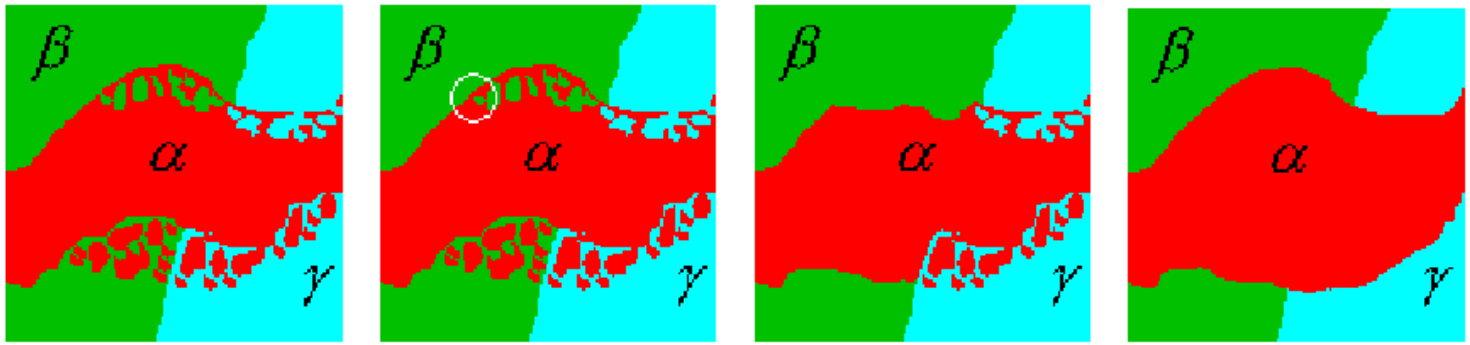
(general formulation requires multi-way cut!)

(Slide from Pascal Fua)

Simplified graph cut



(Roy and Cox ICCV'98)

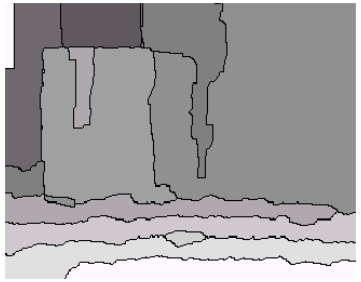


(a) initial labeling (b) standard move (c) α-β-swap (d) α-expansion

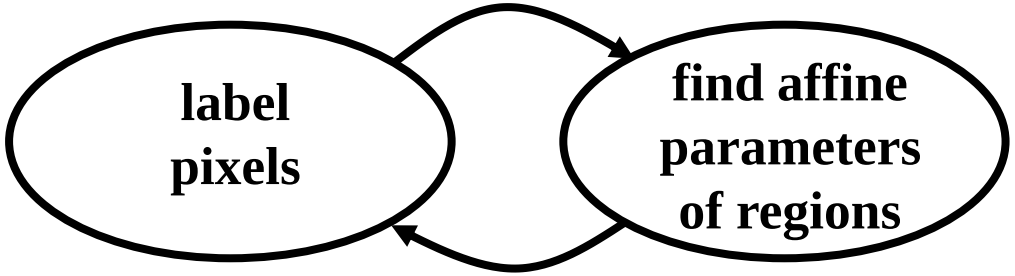
(Boykov et al ICCV'99)

Correspondence as Segmentation

- **Problem:**
  disparities (fronto-parallel) O(Δ)
  surfaces (slanted) $O(\Delta \sigma^2 n)$
  => computationally intractable!

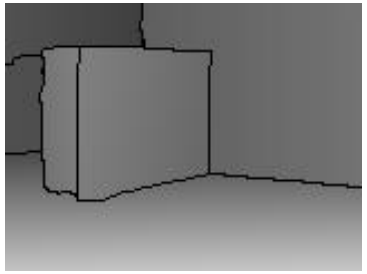


- **Solution: iteratively determine which labels to use**

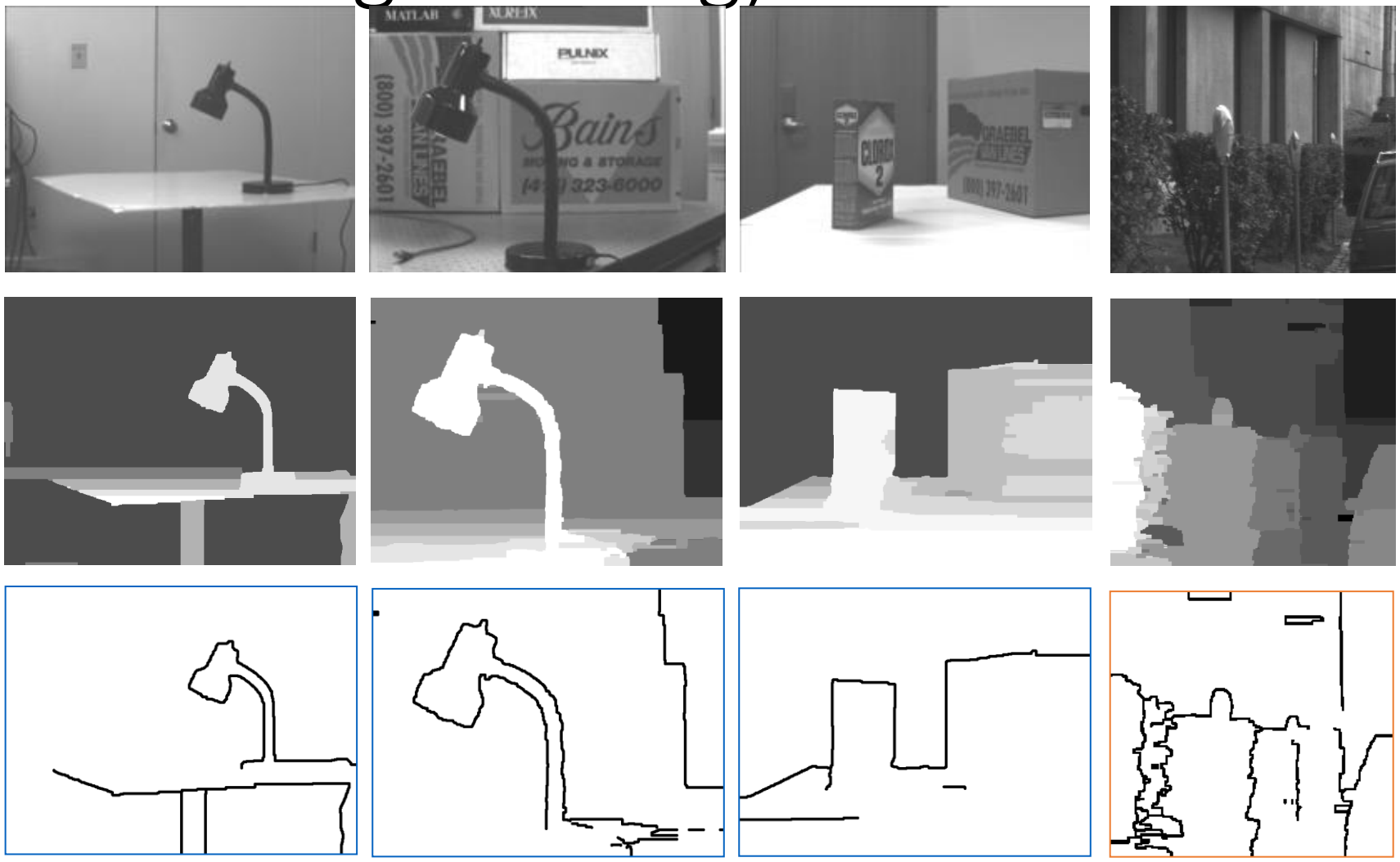


multiway-cut (Expectation)

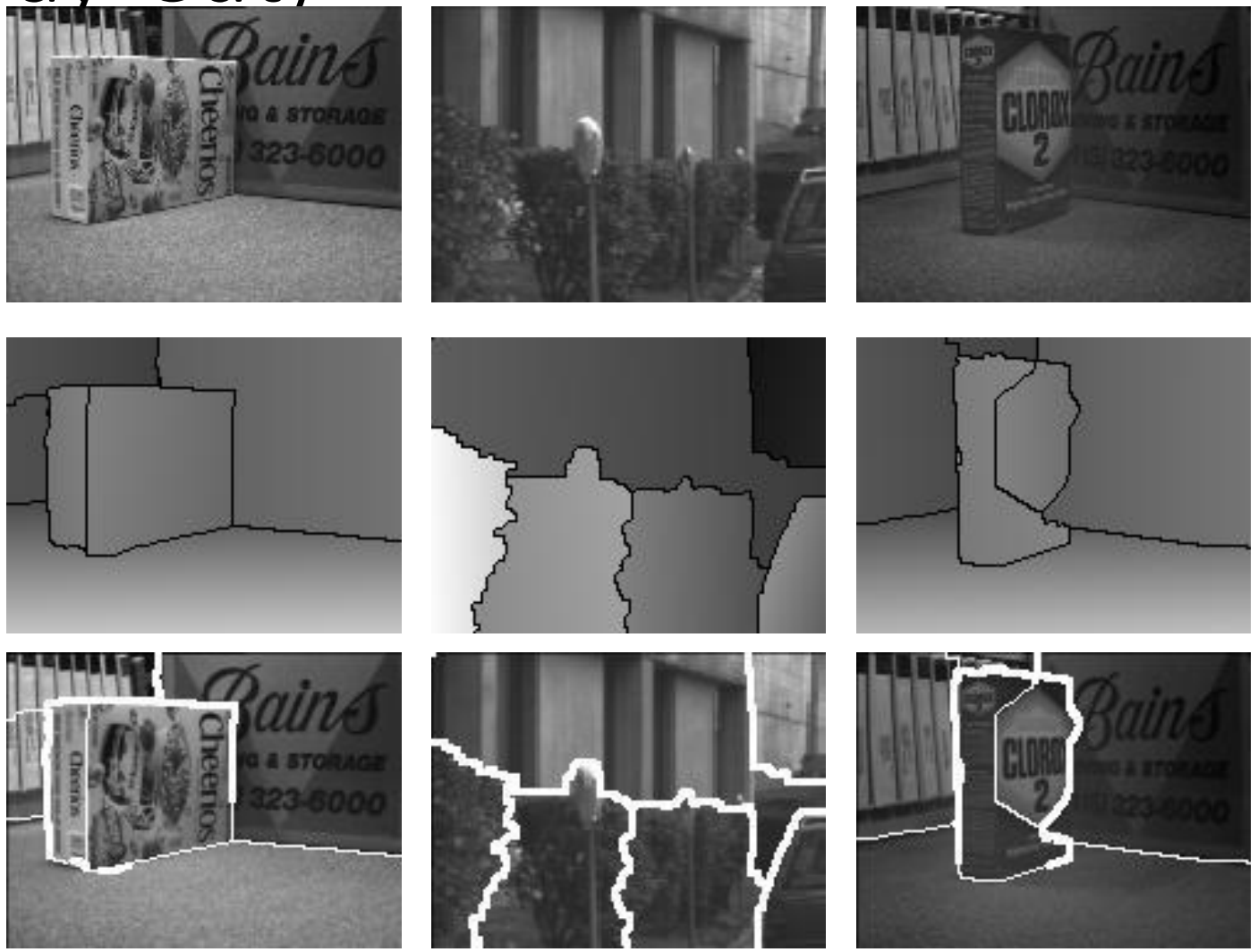
Newton-Raphson (Maximization)



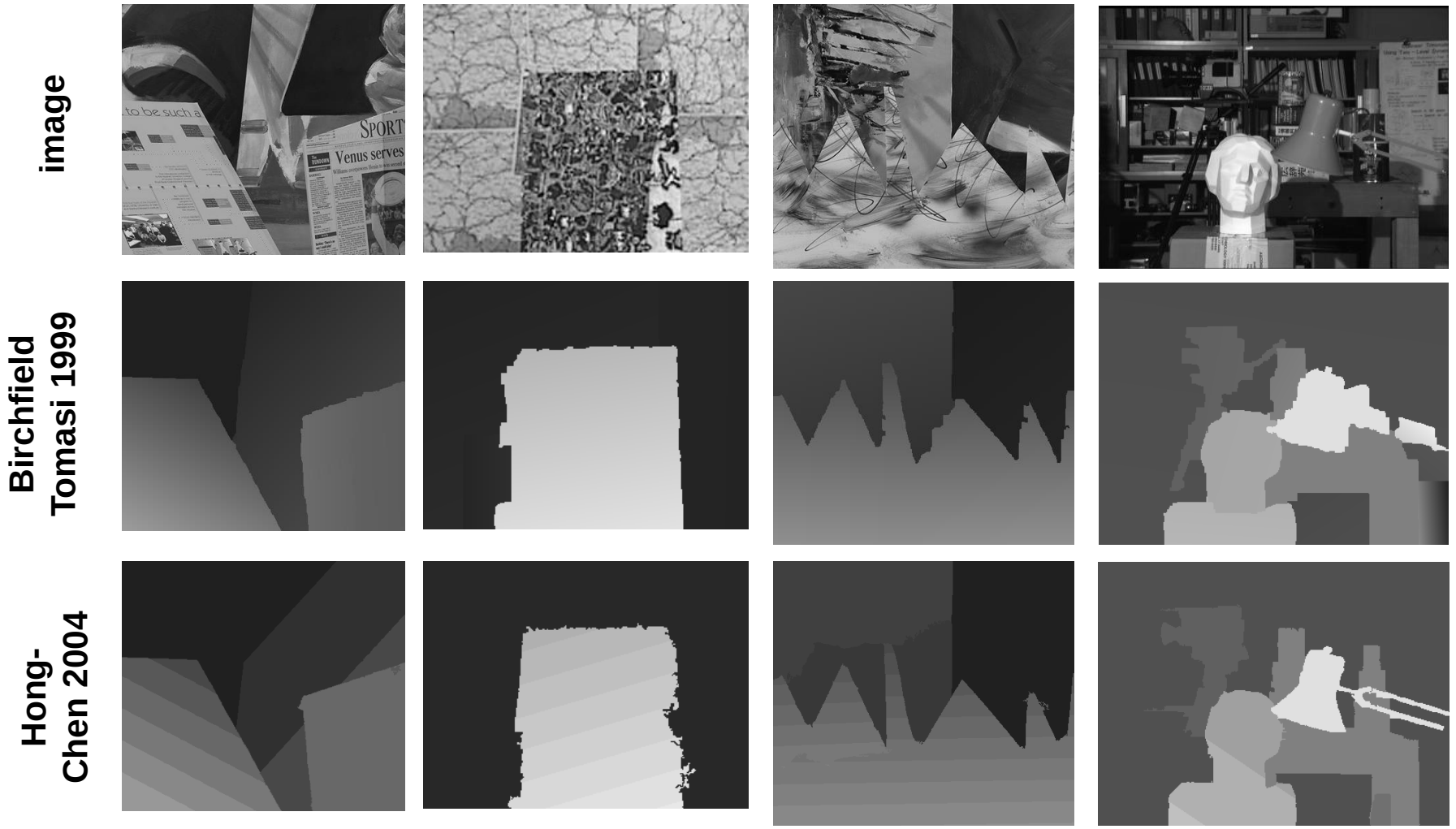
Stereo Results (Dynamic Programming)



Stereo Results (Multiway-Cut)



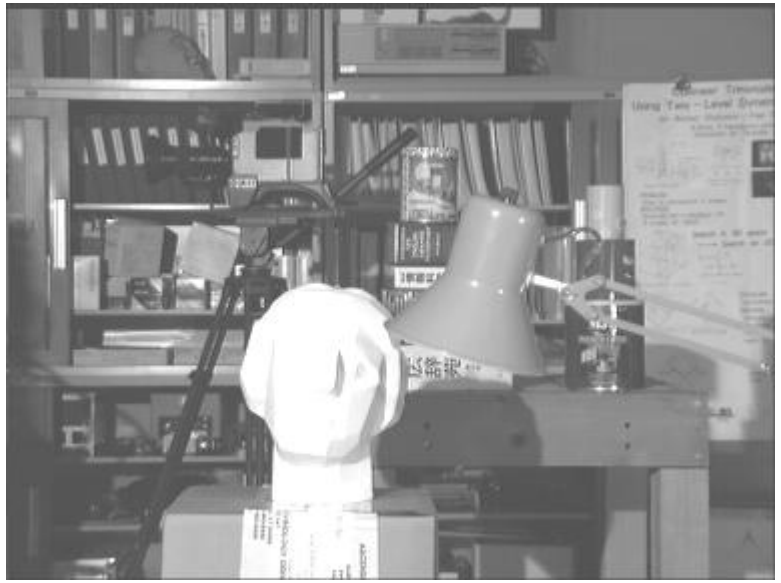
Stereo Results on Middlebury Database



Untextured regions remain a challenge

Dynamic programming **Multiway-cut**

Results: dynamic programming

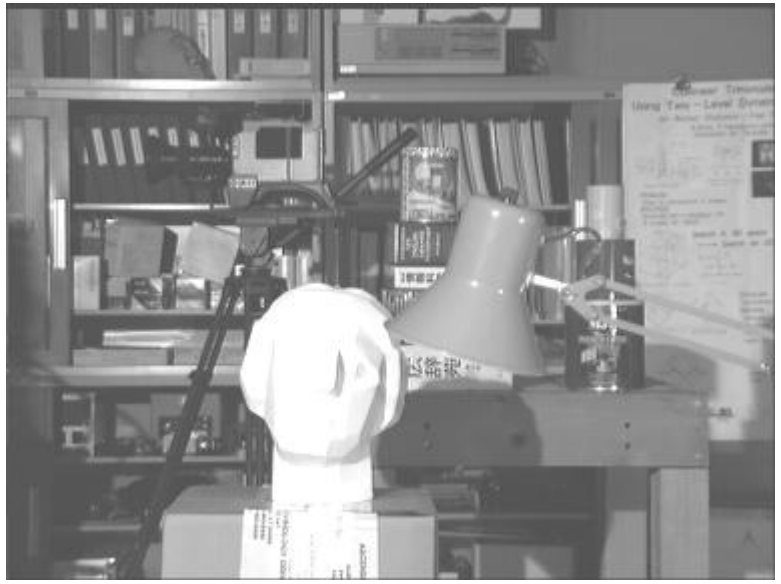


left

disparity map

[Bobick & Intille]

Results: multiway cut



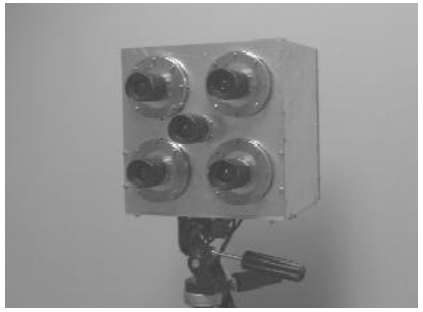
left

disparity map

[Kolmogorov & Zabih]

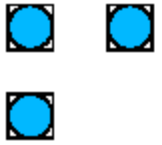
Results: multiway cut (untextured)

Multi-camera configurations



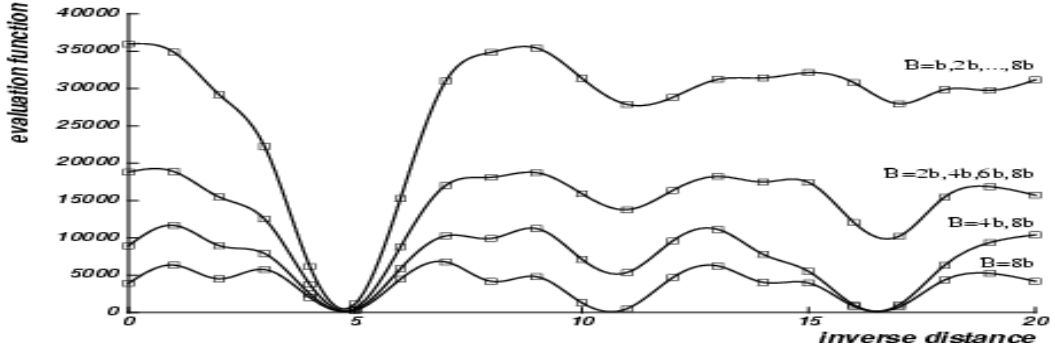
3 cameras give both robustness and precision

4 cameras give additional redundancy



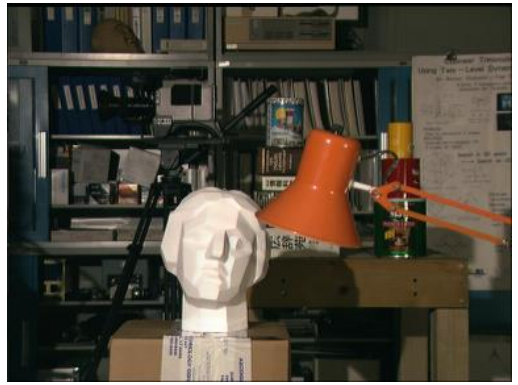
3 cameras in a T arrangement allow the system to see vertical lines.

(illustration from Pascal Fua)

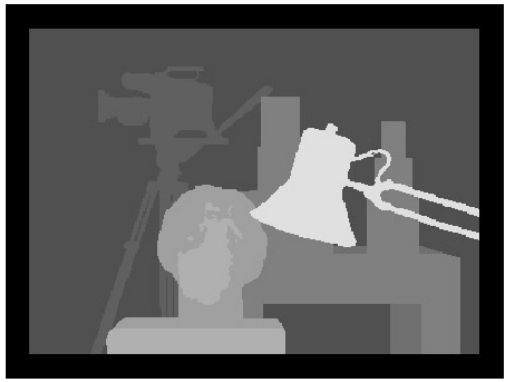


Okutami and Kanade

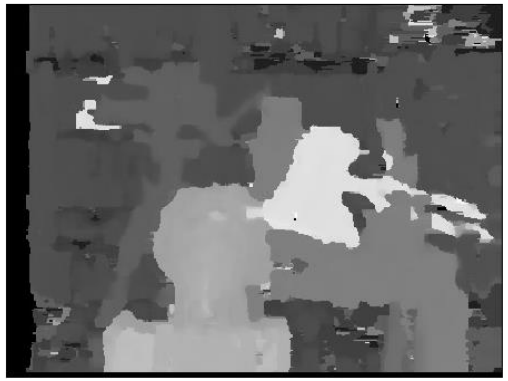
M. Pollefeys, http://www.cs.unc.edu/Research/vision/comp256fall03/



Tsukuba dataset



True disparities



16 – Fast Correlation

*2 – Dynamic progr.

*1 – SSD+MF

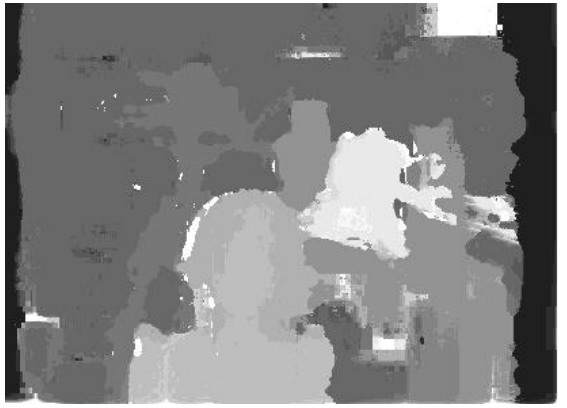
11 – GC + occlusions

19 – Belief propagation

Real-time stereo on GPU

(Yang and Pollefeys, CVPR2003)

- Computes Sum-of-Square-Differences (use pixelshader)
- Hardware mip-map generation for aggregation over window
- Trade-off between small and large support window



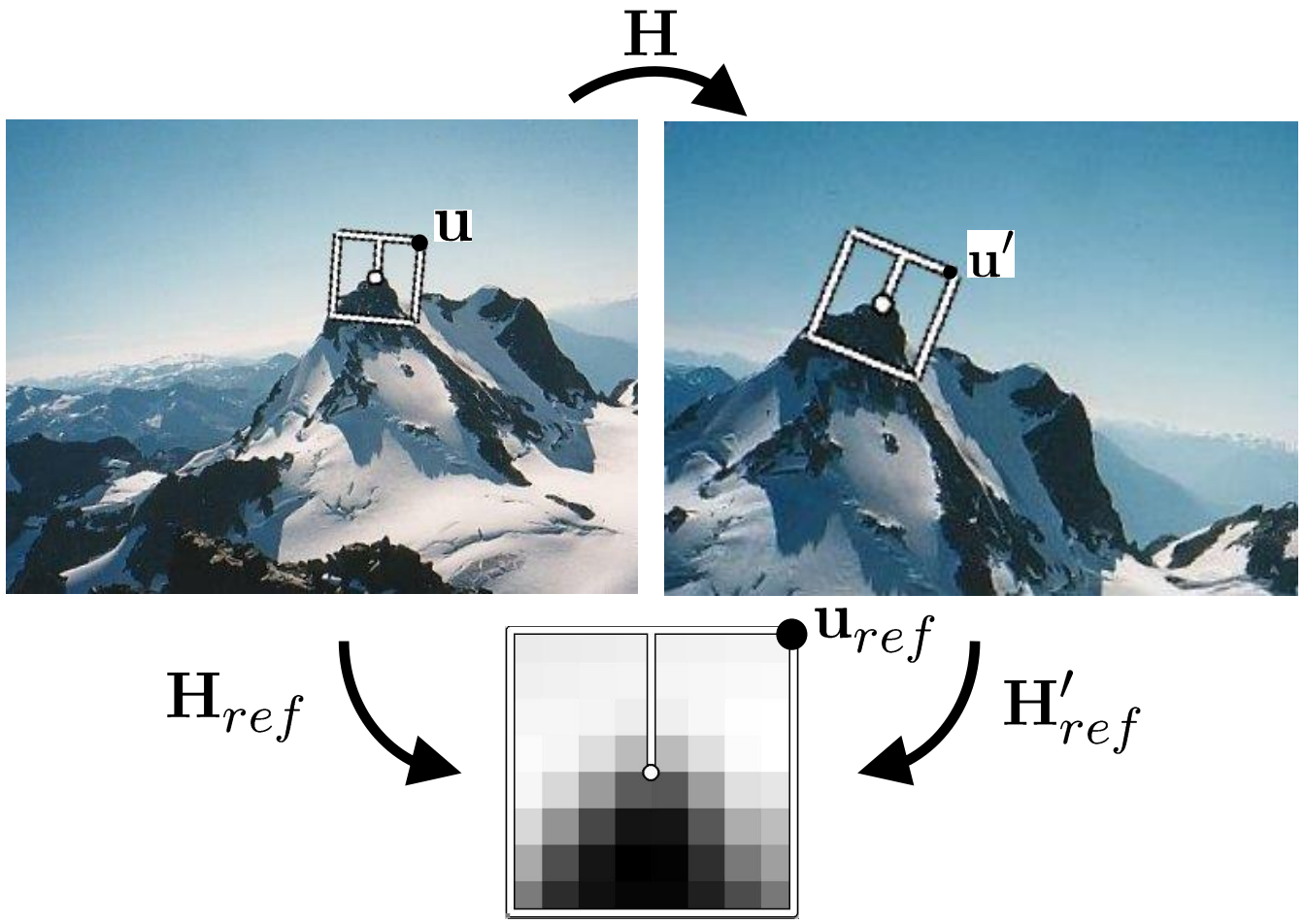
290M disparity hypothesis/sec (Radeon9800pro)
e.g. 512x512x36disparities at 30Hz

GPU is great for vision too!

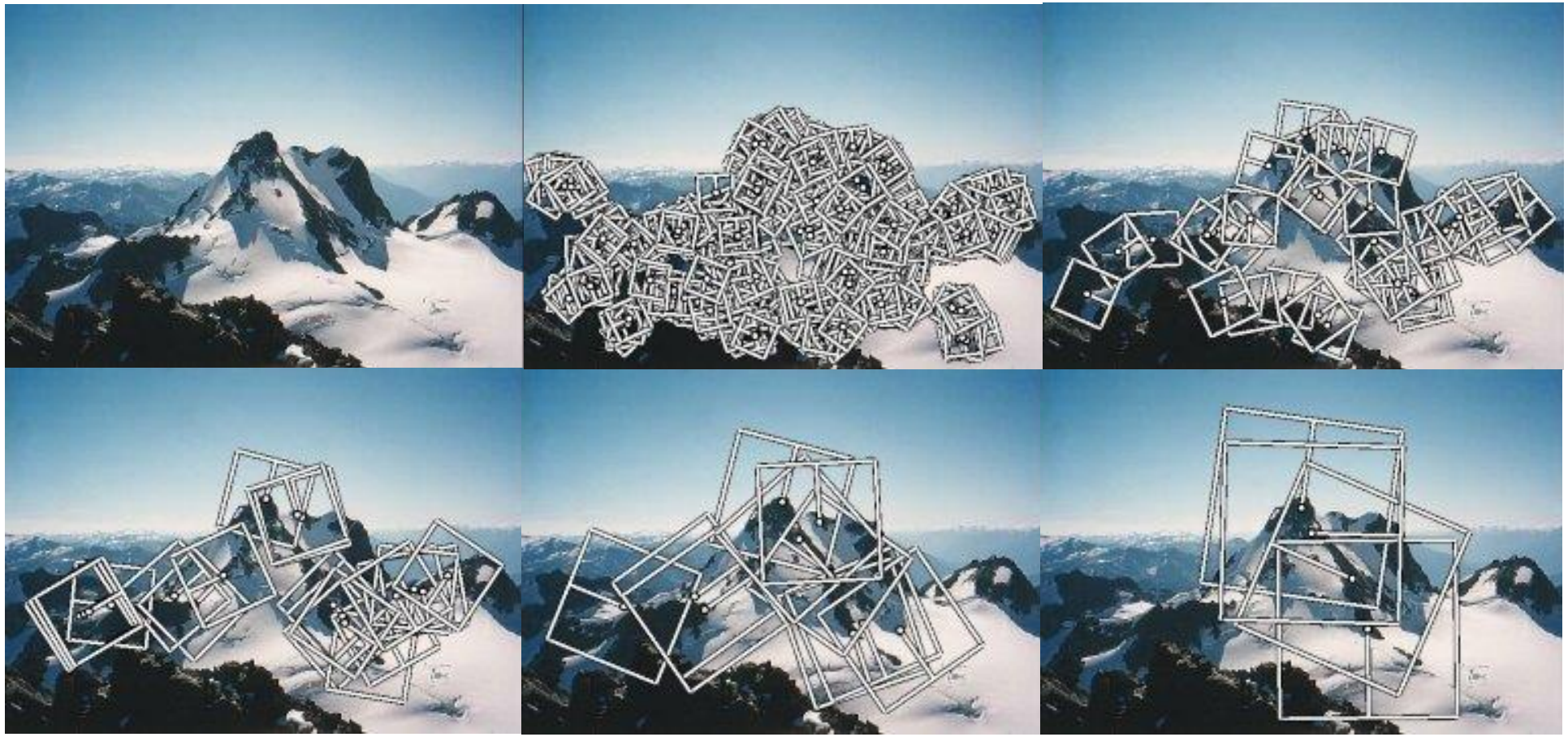
The SIFT (Scale Invariant Feature Transform) Detector and Descriptor

developed by David Lowe
University of British Columbia
Initial paper ICCV 1999
Newer journal paper IJCV 2004

Review: Matt Brown's Canonical Frames



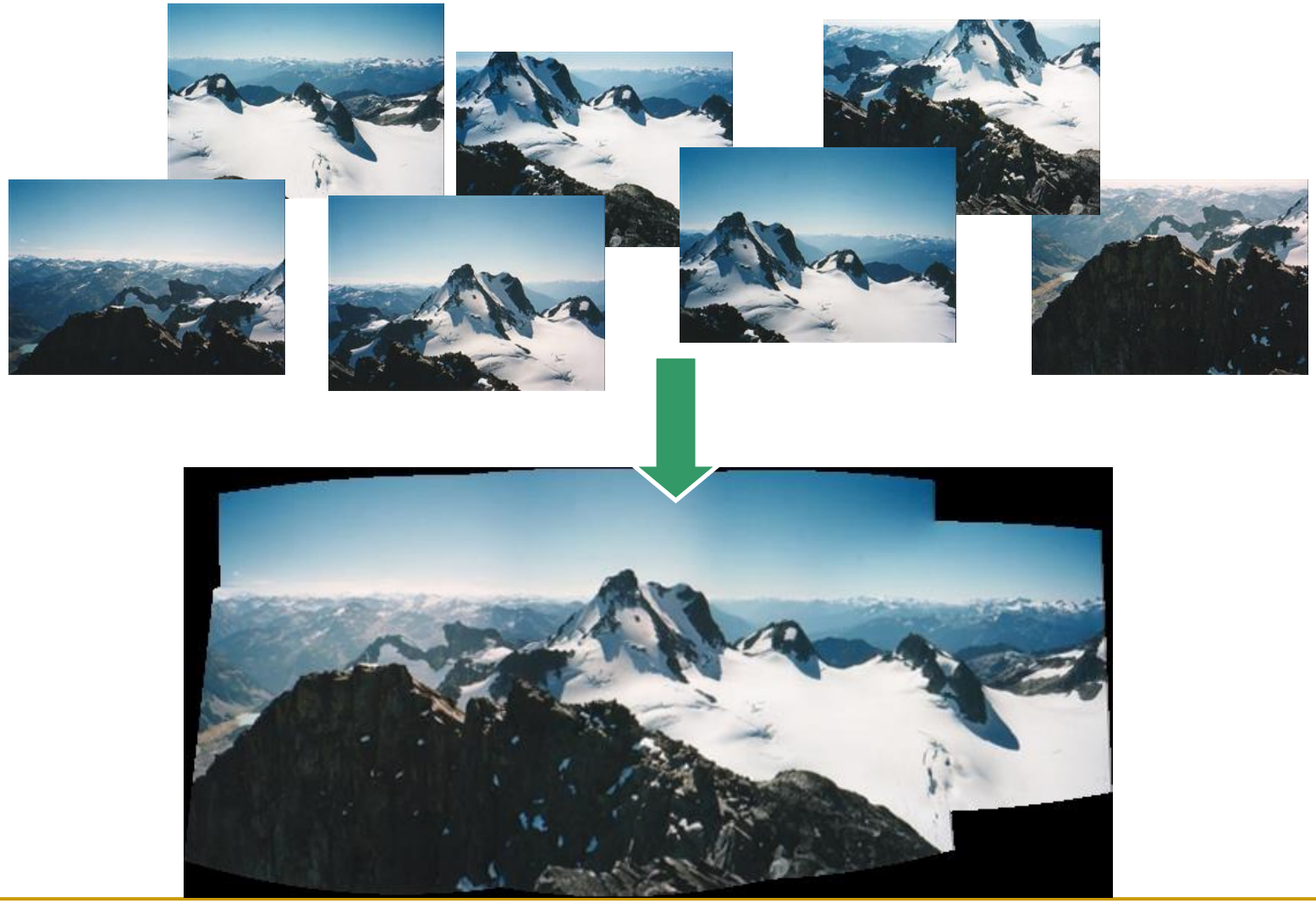
Multi-Scale Oriented Patches



- Extract oriented patches at multiple scales

[Brown, Szeliski, Winder CVPR 2005]

Application: Image Stitching



[Microsoft Digital Image Pro version 10]

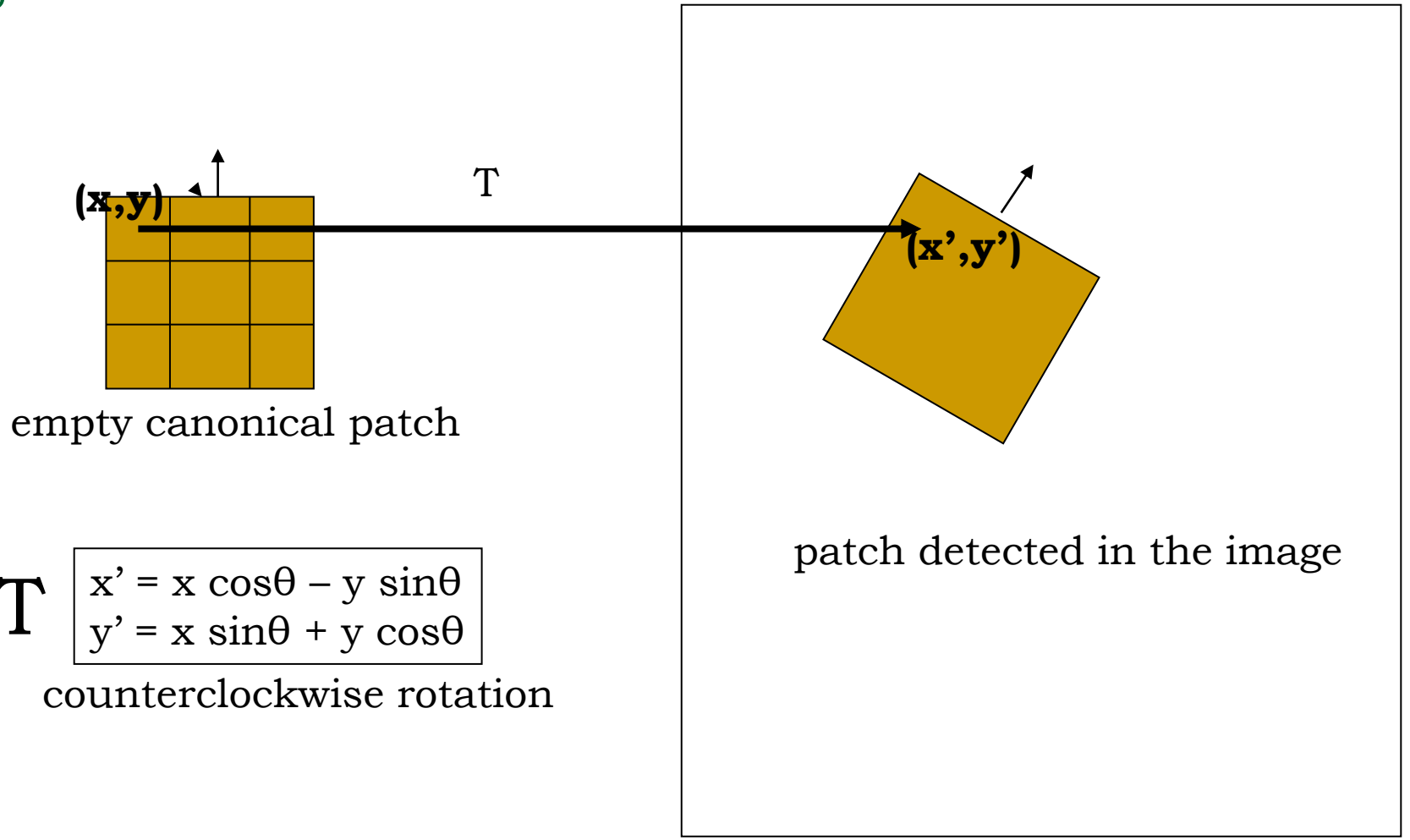
Ideas from Matt's Multi-Scale Oriented Patches

- 1. Detect an interesting patch with an interest operator. Patches are translation invariant.
- 2. Determine its dominant orientation.
- 3. Rotate the patch so that the dominant orientation points upward. This makes the patches rotation invariant.
- 4. Do this at multiple scales, converting them all to one scale through sampling.
- 5. Convert to illumination "invariant" form

Implementation Concern: How do you rotate a patch?

- Start with an “empty” patch whose dominant direction is “up”.
- For each pixel in your patch, compute the position in the detected image patch. It will be in floating point and will fall between the image pixels.
- Interpolate the values of the 4 closest pixels in the image, to get a value for the pixel in your patch.

Rotating a Patch

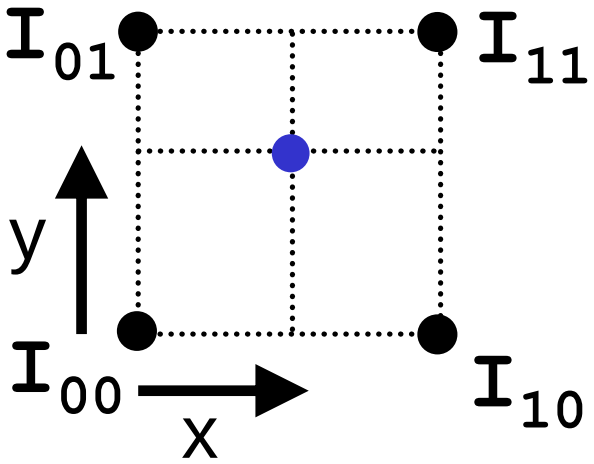


$$T \quad \begin{aligned} x' &= x\cos\theta - y\sin\theta \\ y' &= x\sin\theta + y\cos\theta \end{aligned}$$

counterclockwise rotation

Using Bilinear Interpolation

- Use all 4 adjacent samples



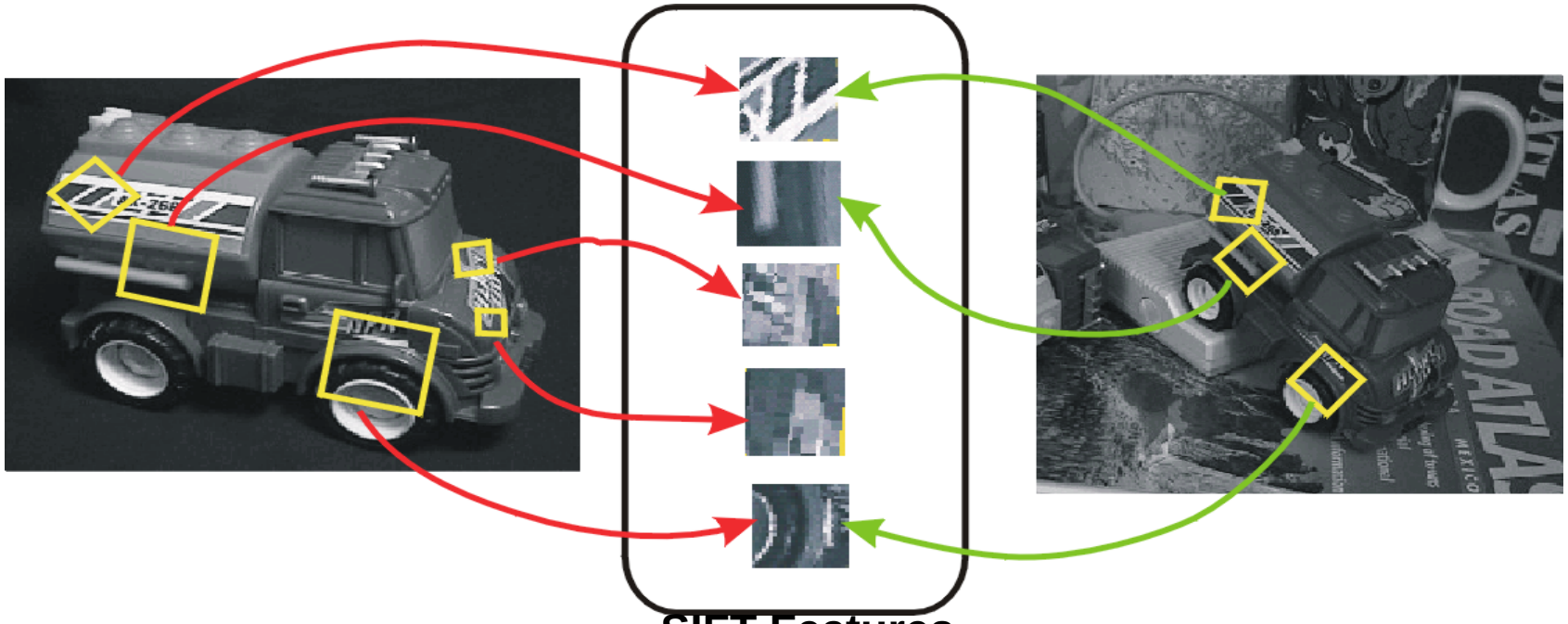
SIFT: Motivation

- The Harris operator is not invariant to scale and correlation is not invariant to rotation[1].
- For better image matching, Lowe's goal was to develop an interest operator that is invariant to scale and rotation.
- Also, Lowe aimed to create a descriptor that was robust to the variations corresponding to typical viewing conditions. The descriptor is the most-used part of SIFT.

[1]But Schmid and Mohr developed a rotation invariant descriptor for it in 1997.

Idea of SIFT

- Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



Claimed Advantages of SIFT

- **Locality:** features are local, so robust to occlusion and clutter (no prior segmentation)
- **Distinctiveness:** individual features can be matched to a large database of objects
- **Quantity:** many features can be generated for even small objects
- **Efficiency:** close to real-time performance
- **Extensibility:** can easily be extended to wide range of differing feature types, with each adding robustness

Overall Procedure at a High Level

1. Scale-space extrema detection

   Search over multiple scales and image locations.

2. Keypoint localization

   Fit a model to detrmine location and scale.
   Select keypoints based on a measure of stability.

3. Orientation assignment

   Compute best orientation(s) for each keypoint region.

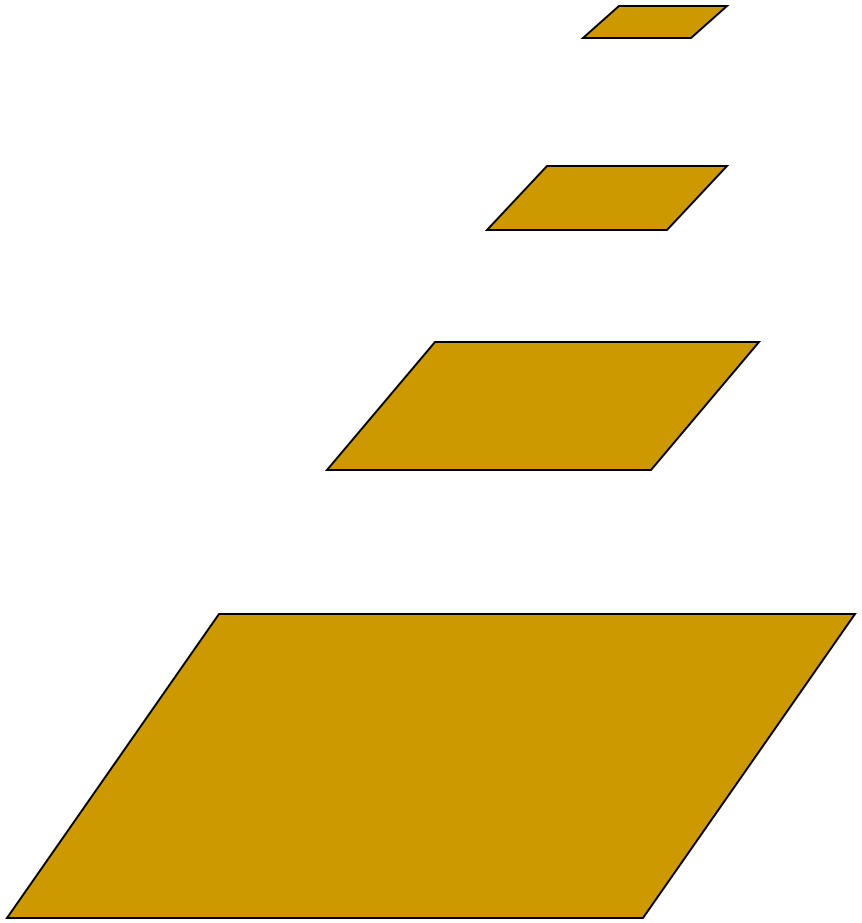
4. Keypoint description

   Use local image gradients at selected scale and rotation to describe each keypoint region.

1. Scale-space extrema detection

- Goal: Identify locations and scales that can be repeatably assigned under different views of the same scene or object.
- Method: search for stable features across multiple scales using a continuous function of scale.
- Prior work has shown that under a variety of assumptions, the best function is a Gaussian function.
- The scale space of an image is a function $L(x,y,\sigma)$ that is produced from the convolution of a Gaussian kernel (at different scales) with the input image.

Aside: Image Pyramids



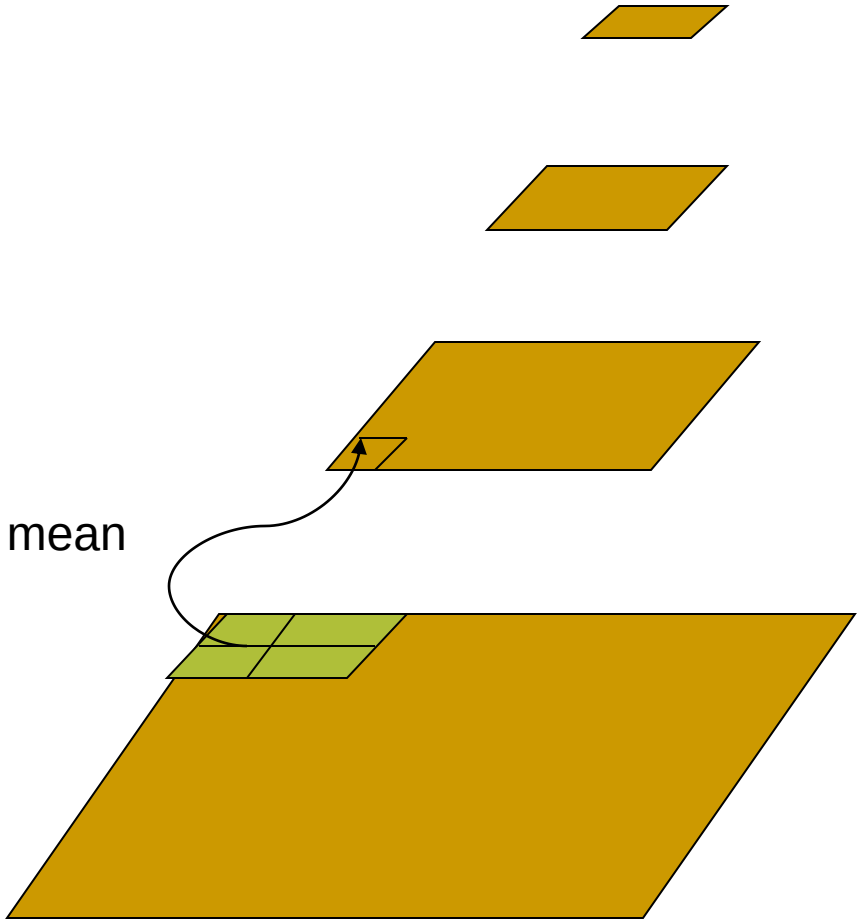
And so on.

3rd level is derived from the 2nd level according to the same funtion

2nd level is derived from the original image according to some function

Bottom level is the original image.

Aside: Mean Pyramid



And so on.

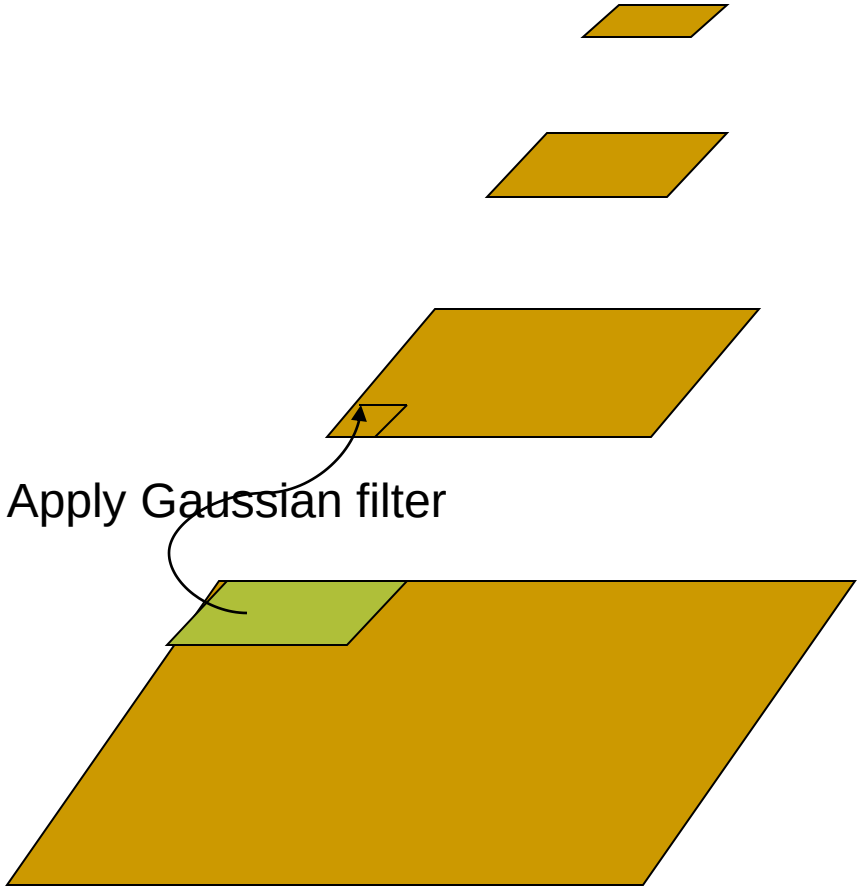
At 3rd level, each pixel is the mean of 4 pixels in the 2nd level.

At 2nd level, each pixel is the mean of 4 pixels in the original image.

Bottom level is the original image.

Aside: Gaussian Pyramid

At each level, image is smoothed and reduced in size.

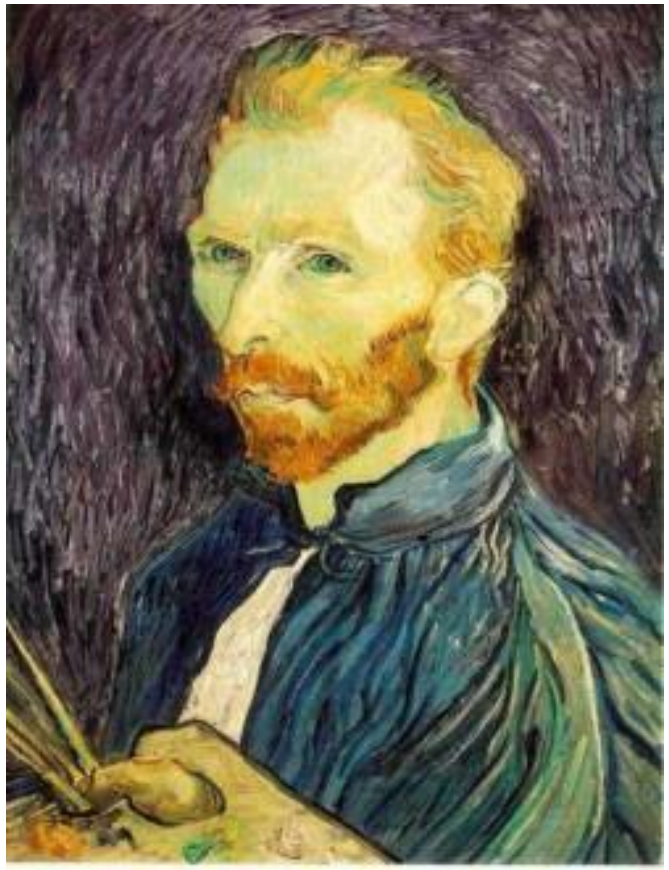


And so on.

At 2nd level, each pixel is the result of applying a Gaussian mask to the first level and then subsampling to reduce the size.

Bottom level is the original image.

Example: Subsampling with Gaussian pre-filtering



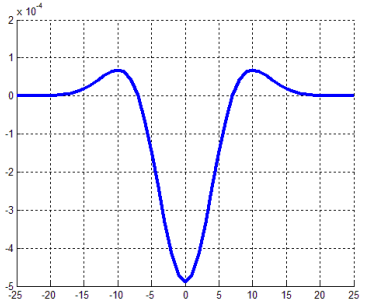
Gaussian 1/2

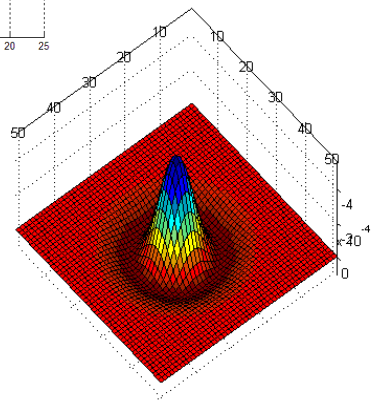
G 1/4

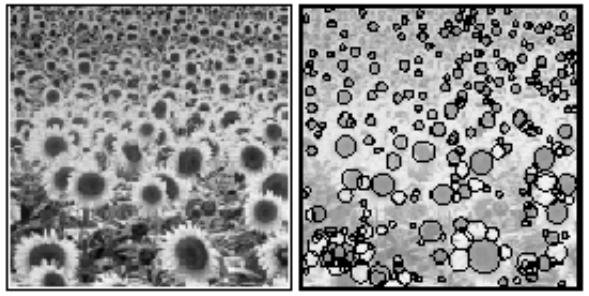
G 1/8

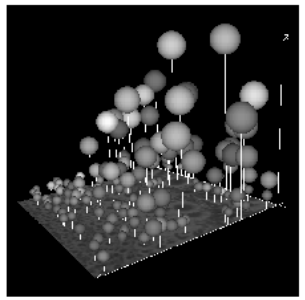
Lowe's Scale-space Interest Points

- **Laplacian of Gaussian** kernel
  - Scale normalised (x by scale^2)
  - Proposed by Lindeberg
- Scale-space detection
  - Find local maxima across scale/space
  - A good "blob" detector







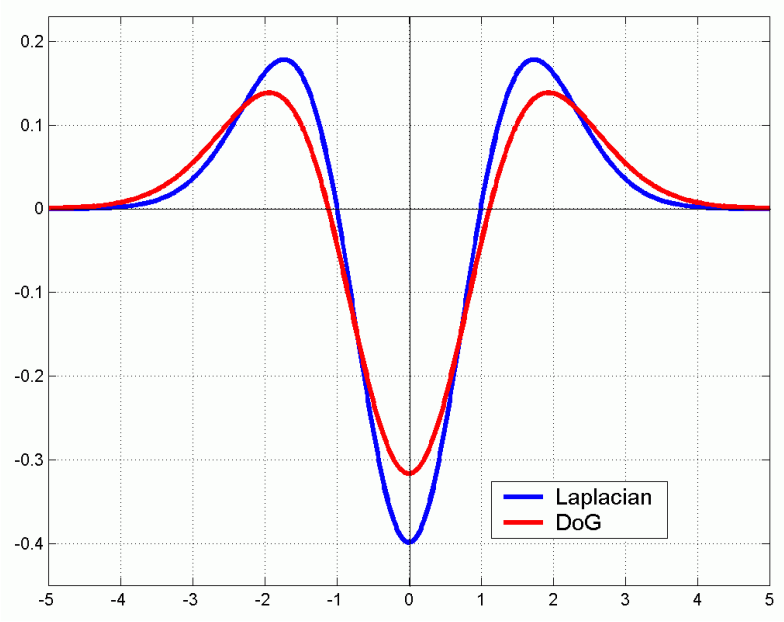


$$G(x, y, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{x^2+y^2}{\sigma^2}}$$

$$\nabla^2 G(x, y, \sigma) = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}$$

[T. Lindeberg IJCV 1998]

Lowe's Scale-space Interest Points: Difference of Gaussians



- Gaussian is an ad hoc solution of heat diffusion equation
- He

$$\frac{\partial G}{\partial \sigma} = \sigma \nabla^2 G.$$

$$G(x, y, k\sigma) - G(x, y, \sigma) \approx (k-1)\sigma^2 \nabla^2 G.$$

- k is not necessarily very small in practice

Lowe's Pyramid Scheme

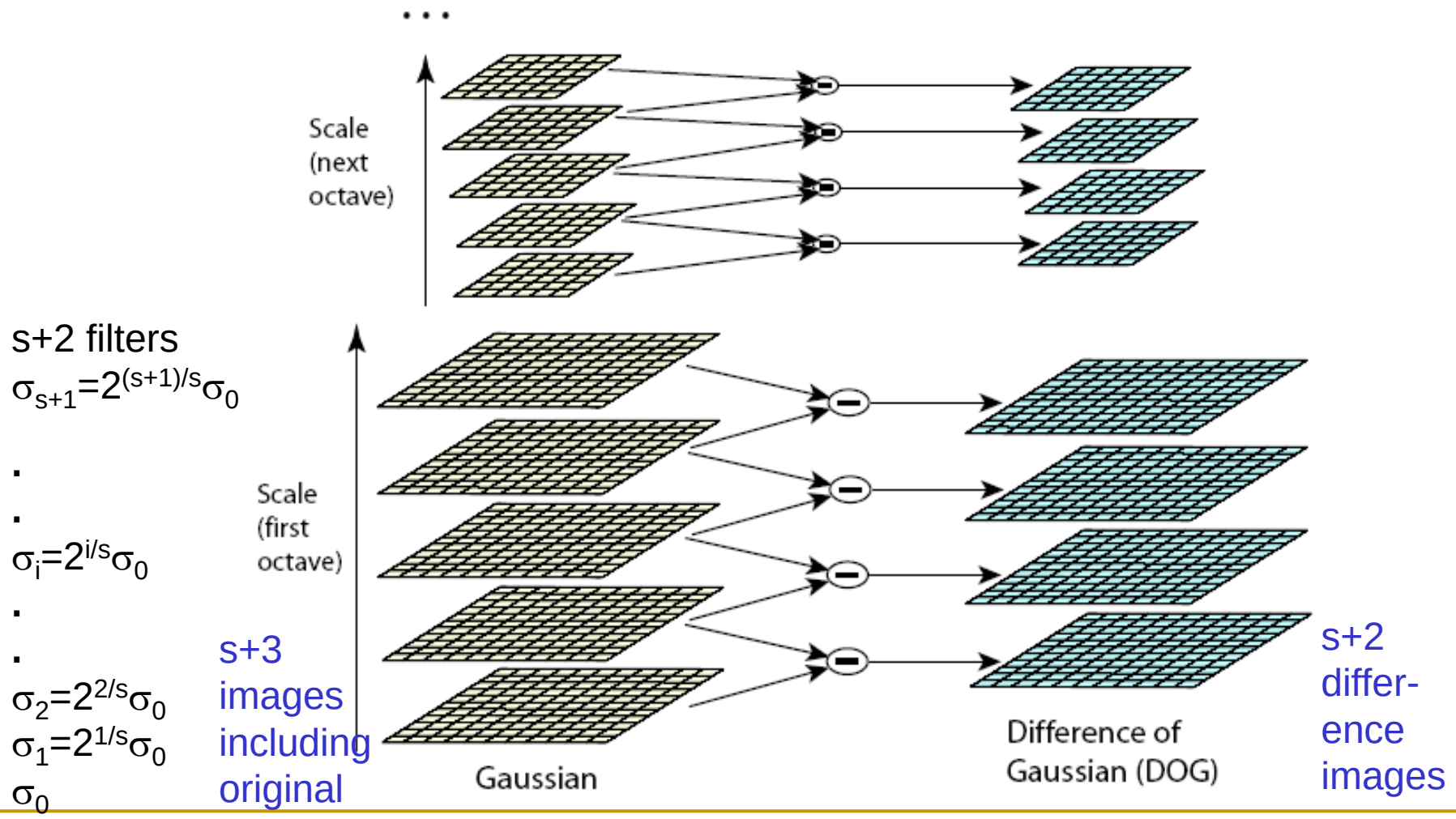
- Scale space is separated into octaves:
  - Octave 1 uses scale σ
  - Octave 2 uses scale 2σ
  - etc.

- In each octave, the initial image is repeatedly convolved with Gaussians to produce a set of scale space images.

- Adjacent Gaussians are subtracted to produce the DOG

- After each octave, the Gaussian image is down-sampled by a factor of 2 to produce an image ¼ the size to start the next level.

Lowe's Pyramid Scheme



s+2 filters

$\sigma_{s+1}=2^{(s+1)/s}\sigma_0$

.

.

$\sigma_i=2^{i/s}\sigma_0$

.

.

$\sigma_2=2^{2/s}\sigma_0$

$\sigma_1=2^{1/s}\sigma_0$

σ_0

s+3 images including original

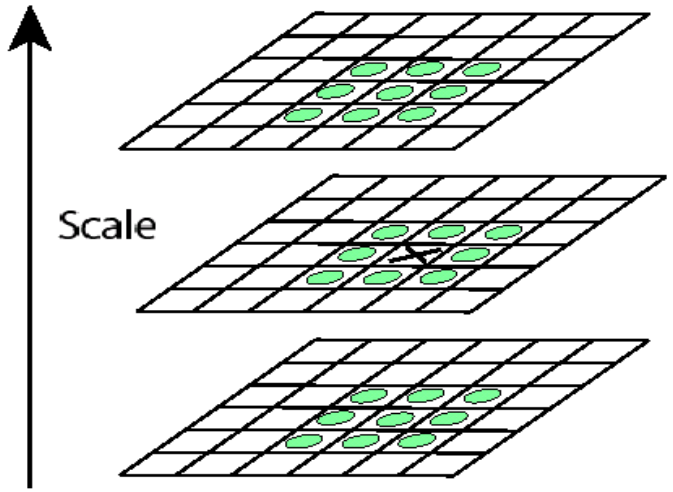
s+2 differ-ence images

The parameter **s** determines the number of images per octave.

Key point localization

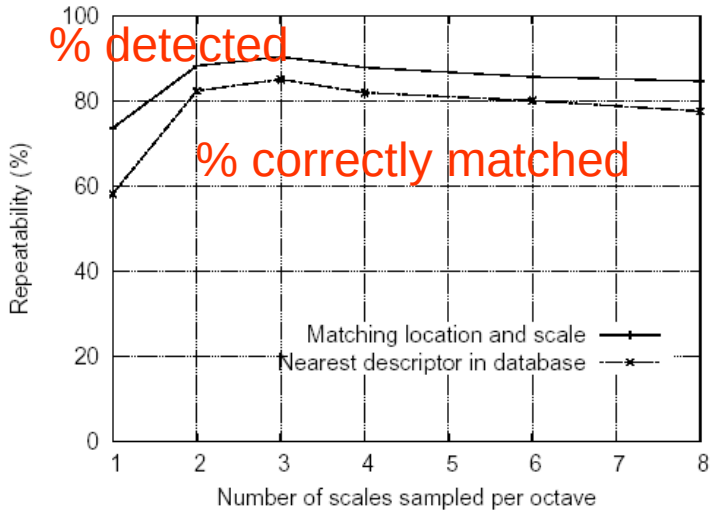
s+2 difference images.
top and bottom ignored.
s planes searched.

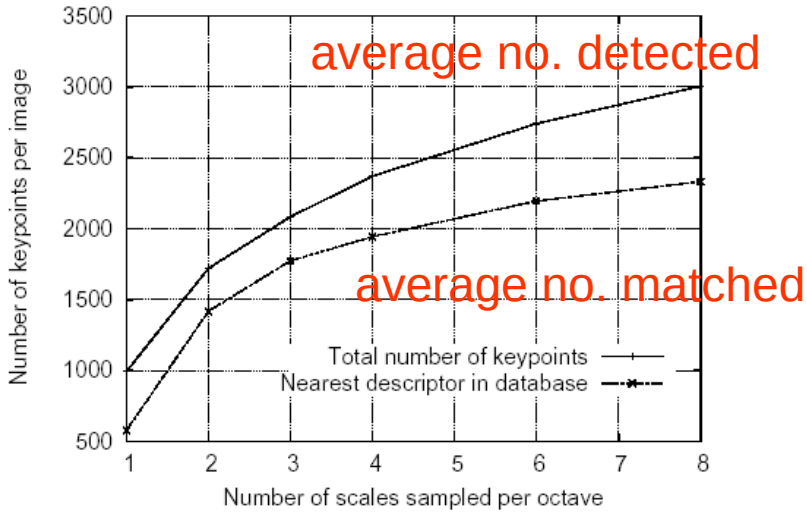
- Detect maxima and minima of difference-of-Gaussian in scale space
- Each point is compared to its 8 neighbors in the current image and 9 neighbors each in the scales above and below



For each max or min found, output is the **location** and the **scale**.

Scale-space extrema detection: experimental results over 32 images that were synthetically transformed and noise added.





- Sampling in scale for efficiency
  - How many scales should be used per octave? S=?
    - More scales evaluated, more keypoints found
    - S < 3, stable keypoints increased too
    - S > 3, stable keypoints decreased
    - S = 3, maximum stable keypoints found

Keypoint localization

- Once a keypoint candidate is found, perform a detailed fit to nearby data to determine
  - location, scale, and ratio of principal curvatures
- In initial work keypoints were found at location and scale of a central sample point.
- In newer work, they fit a 3D quadratic function to improve interpolation accuracy.
- The Hessian matrix was used to eliminate edge responses.

Eliminating the Edge Response

- Reject flats:
  - $|D(\hat{\mathbf{x}})|$ < 0.03
- Reject edges:

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

Let α be the eigenvalue with larger magnitude and β the smaller.

$$\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$\mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

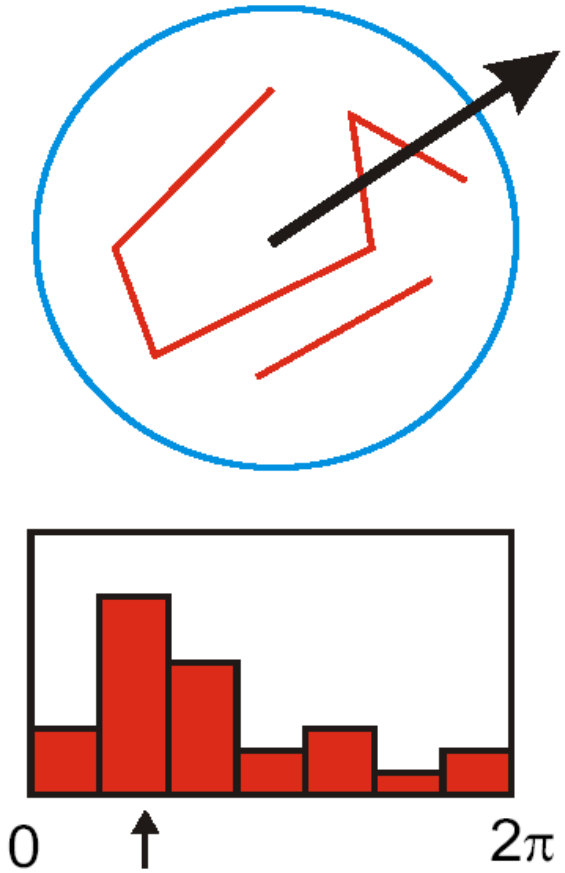
Let $r = \alpha/\beta$. So $\alpha = r\beta$

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r},$$

$(r+1)^2/r$ is at a min when the 2 eigenvalues are equal.

  - r < 10
- What does this look like?

3. Orientation assignment



- Create histogram of local gradient directions at selected scale
- Assign canonical orientation at peak of smoothed histogram
- Each key specifies stable 2D coordinates (x, y, scale,orientation)

If 2 major orientations, use both.

Keypoint localization with orientation

233x189

832

initial keypoints

729

keypoints after
gradient threshold

536

keypoints after
ratio threshold

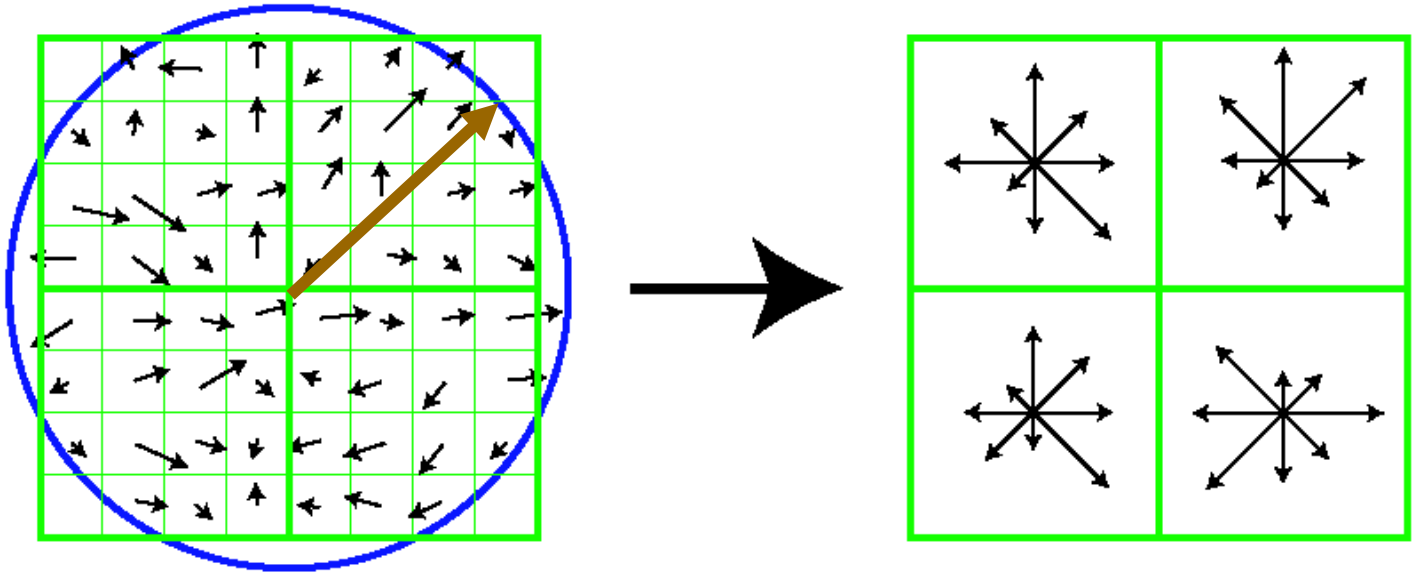
4. Keypoint Descriptors

- At this point, each keypoint has
  - location
  - scale
  - orientation
- Next is to compute a descriptor for the local image region about each keypoint that is
  - highly distinctive
  - invariant as possible to variations such as changes in viewpoint and illumination

Normalization

- Rotate the window to standard orientation
- Scale the window size based on the scale at which the point was found.

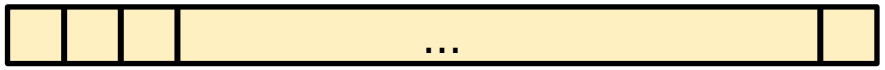
Lowe's Keypoint Descriptor (shown with 2 X 2 descriptors over 8 X 8)



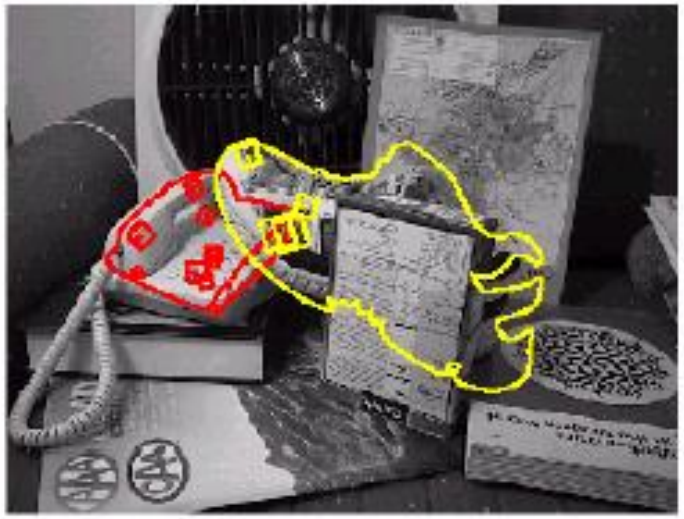
In experiments, 4x4 arrays of 8 bin histogram is used, a total of 128 features for one keypoint

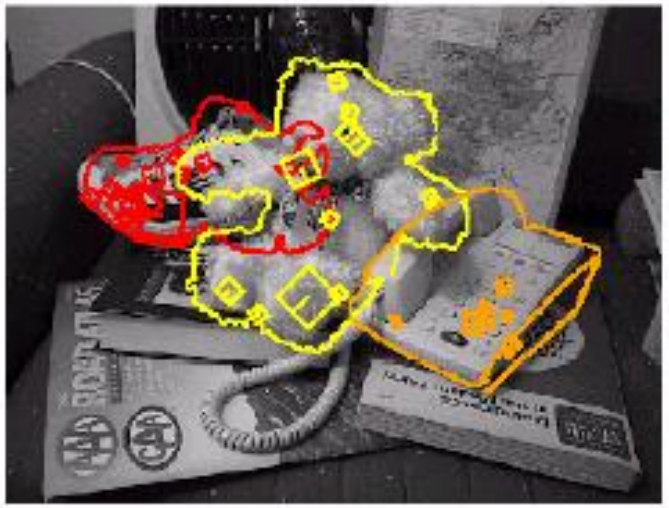
Lowe's Keypoint Descriptor

- use the normalized region about the keypoint
- compute gradient magnitude and orientation at each point in the region
- weight them by a Gaussian window overlaid on the circle
- create an orientation histogram over the 4 X 4 subregions of the window
- 4 X 4 descriptors over 16 X 16 sample array were used in practice. 4 X 4 times 8 directions gives a vector of 128 values.



Using SIFT for Matching “Objects”





Uses for SIFT

- Feature points are used also for:
  - Image alignment (homography, fundamental matrix)
  - 3D reconstruction (e.g. Photo Tourism)
  - Motion tracking
  - Object recognition
  - Indexing and database retrieval
  - Robot navigation
  - … many others

[Photo Tourism: Snavely et al. SIGGRAPH 2006]