

1. For each of the following parts, draw an arbitrary graph on 5 vertices satisfying the specified properties, and give both its adjacency list and adjacency matrix representations.
  - (a) A complete undirected graph.
  - (b) A complete directed graph.
  - (c) An undirected cycle graph (note that a cycle graph is a connected graph in which every vertex has degree exactly 2).
  - (d) A directed cycle graph.
  - (e) A binary tree, with the edges oriented from parent to child.
  - (f) An undirected graph, with two connected components, respectively, with 2, 3 vertices.
2. Given a graph  $G$  with  $m$  edges, show that
  - (a) the adjacency matrix representation of  $G$  contains exactly  $2m$  1s.
  - (b) the adjacency list representation of  $G$  contains exactly  $2m$  linked list nodes.
3. For each of the following questions, write a function to perform the specified operation on a graph  $G$ , represented using an adjacency matrix. For each of the functions, give the best bound you can on their worst case asymptotic complexity.
  - (a) Compute the degree of the node that has the most neighbors.
  - (b) Check if  $G$  is an empty graph.
  - (c) Check if  $(u, v)$  is in  $E(G)$ , where  $u, v \in V(G)$ .
  - (d) Check if the graph contains a triangle. A *triangle* is a set of three vertices such that the induced subgraph formed by them is complete.
  - (e) Add an edge  $(u, v)$  to  $E(G)$ , where  $u, v \in V(G)$ .
  - (f) Delete the edge  $(u, v) \in E(G)$ .
  - (g) Subdivide the edge  $(u, v) \in E(G)$ . The *subdivide* operation introduces a new vertex  $w$  such that the resultant graph contains edges  $(u, w), (w, v)$  instead of the edge  $(u, v)$ .
  - (h) Output the complement of the graph. The *complement* of a graph  $G(V, E)$  is the graph  $G'(V, E')$ , where  $E' = \{(u, v) : (u, v) \notin E \text{ and } u \neq v\}$ .
  - (i) Check if the graph is *Eulerian*. A graph is Eulerian if it contains a circuit that passes through every edge.
4. Repeat each part of the preceding question assuming that the graph  $G$  is represented using adjacency lists.
5. Question 22.1-6 in CLRS.