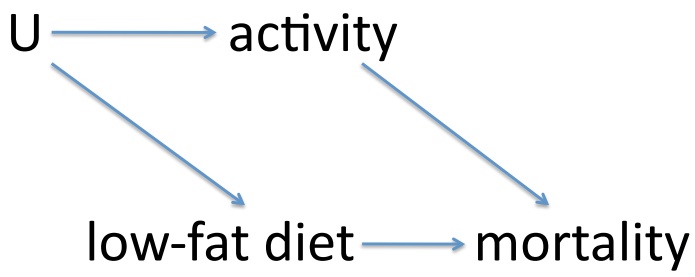
Public Health 241 - Spring 2018
Assignment #7 Solutions

1. (a) Here is the 2×2 table:

	Mortality		
	Dead	Alive	Total
Low-fat diet	59	119	178
Other diet	136	186	322

$$\begin{aligned}
\widehat{OR} &= \frac{59 \times 186}{119 \times 136} = 0.6781 \\
\widehat{var}\Big(\log(\widehat{OR})\Big) &= 1/59 + 1/136 + 1/119 + 1/186 = 0.03808 \\
95\% \text{ CI for } \log(OR) &= \log(0.6781) \pm 1.96\sqrt{0.03808} = (-0.771, -0.006) \\
95\% \text{ CI for } OR &= (\exp(-0.771), \exp(-0.006)) = (0.46, 0.99)
\end{aligned}$$

(b) The DAG would probably look like this:



Here U are other unmeasured variables that affect both low-fat diet and physical activity, such as overall health consciousness of the person. Since U is a common cause of both low-fat diet and mortality, the relationship between low-fat diet and mortality is confounded so that the crude odds will likely not be a good estimate of the causal odds ratio

(c)

$$\begin{aligned}
\widehat{OR}_0 &= \frac{17 \times 75}{75 \times 22} = 0.77 \\
\widehat{OR}_1 &= \frac{28 \times 45}{40 \times 36} = 0.88 \\
\widehat{OR}_2 &= \frac{10 \times 34}{14 \times 37} = 0.66 \\
\widehat{OR}_3 &= \frac{4 \times 32}{7 \times 24} = 0.76
\end{aligned}$$

(d) Yes, the four separate odds ratio estimates are all fairly close together.

(e) The alternative hypothesis is that $OR_0 = OR_1 = OR_2 = OR_3 \neq 1$, (with the null hypothesis being $OR_0 = OR_1 = OR_2 = OR_3 = 1$).
Here is a table summarizing the information we need for the CMH test:

Activity level	a_i	A_i	V_i
0	17	18.984	7.774
1	28	29.208	9.119
2	10	11.874	4.531
3	4	4.597	2.2704
Totals	59	64.663	23.694

$$\chi^2_{CMH} = \frac{(59 - 64.663)^2}{23.694} = 1.353$$
$$p = 0.245 \quad (\text{from } \chi^2_{(1)} \text{ distribution})$$

Based on the CMH test, we do not reject the null at the .05 level. In other words, this test does not provide support for the conjecture that low-fat diet is associated with mortality after controlling (stratifying) on activity level (assuming no multiplicative interaction).

(f) The alternative hypothesis for this test is that at least one of the stratum-specific ORs is not equal to 1 (i.e., the alternative is that there is a relationship between low-fat diet and mortality in at least one of the stratum, making no assumption regarding statistical interactions). Notice the difference between this null hypothesis and the one in (e), in particular that there is no assumption here of no multiplicative interaction so that this alternative is broader. Here is a table summarizing the information we need :

Activity level	χ^2_i
0	0.51
1	0.16
2	0.78
3	0.16
Totals	1.61

$$p = 0.81 \quad (\text{from } \chi^2_{(4)} \text{ distribution})$$

Lower power because alternative hypothesis is broader.

(g) Here is a table summarizing the information we need:

Activity level	$a_i d_i / n_i$	$b_i c_i / n_i$
0	6.746	8.730
1	8.456	9.664
2	3.579	5.453
3	1.910	2.507
Totals	20.691	26.354

$$\widehat{OR}_{MH} = \frac{20.691}{26.354} = 0.79$$

(h) Here is a table summarizing the information we need:

Activity level	$\log(\widehat{OR}_i)$	w_i
0	-0.251	7.801
1	-0.131	9.161
2	-0.406	4.548
3	-0.228	2.341
Totals		23.859

$$\begin{aligned}
\log \widehat{OR}_W &= \frac{(-0.251) \times 7.801 + \ldots + (-0.228) \times 2.341}{23.859} = -0.232 \\
\widehat{OR}_W &= \exp(-0.232) = 0.793 \\
\widehat{var}\Big(\log(\widehat{OR}_W)\Big) &= 1/23.859 = 0.0419 \\
95\% \text{ CI for } \log(OR) &= -0.232 \pm 1.96\sqrt{0.0419} = (-0.633, 0.169) \\
95\% \text{ CI for } OR &= (\exp(-0.633), \exp(0.169)) = (0.53, 1.18)
\end{aligned}$$

(i) Yes, since the adjusted estimated is substantially different from the crude estimate, there is confounding (we know from the DAG above that activity is neither a collider nor on the causal pathway so non-collapsibility and confounding are equivalent here).

(j) On the current data set you get the following:

```
. cc mort diet
                                                              Proportion
                 |   Exposed   Unexposed  |      Total     Exposed
-----------------+------------------------+------------------------
           Cases |        59         136  |        195       0.3026
        Controls |       119         186  |        305       0.3902
-----------------+------------------------+------------------------
           Total |       178         322  |        500       0.3560
                 |                        |
                 |      Point estimate    |    [95% Conf. Interval]
                 |------------------------+------------------------
      Odds ratio |         .6780771       |    .4532461    1.010482 (exact)
 Prev. frac. ex. |         .3219229       |   -.0104819    .5467539 (exact)
 Prev. frac. pop |         .1256027       |
                 +-----------------------------------------------
                              chi2(1) =     3.98  Pr>chi2 = 0.0460

. cc mort diet, by(act)

             act |       OR       [95% Conf. Interval]   M-H Weight
-----------------+-------------------------------------------------
               0 |   .7727273      .3548236   1.663488     8.730159 (exact)
               1 |       .875      .4324228   1.766572      9.66443 (exact)
               2 |   .6563707      .2285842    1.84153     5.452632 (exact)
               3 |   .7619048      .1468516   3.427209     2.507463 (exact)
-----------------+-------------------------------------------------
           Crude |   .6780771      .4532461   1.010482              (exact)
    M-H combined |   .7851281      .5228808   1.178904
-----------------------------------------------------------------------
Test of homogeneity (M-H)      chi2(3) =     0.25  Pr>chi2 = 0.9690

                  Test that combined OR = 1:
                              Mantel-Haenszel chi2(1) =      1.35
                                                Pr>chi2 =    0.2447
```

On the old (wrong) data set you get the following:

```
. cc mort diet
                                                              Proportion
                 |   Exposed   Unexposed  |      Total     Exposed
-----------------+------------------------+------------------------
           Cases |        59         139  |        198       0.2980
        Controls |       119         183  |        302       0.3940
-----------------+------------------------+------------------------
           Total |       178         322  |        500       0.3560
                 |                        |
                 |      Point estimate    |    [95% Conf. Interval]
                 |------------------------+------------------------
      Odds ratio |         .6527417       |    .4364899    .9723664 (exact)
 Prev. frac. ex. |         .3472583       |    .0276336    .5635101 (exact)
 Prev. frac. pop |         .1368336       |
                 +-----------------------------------------------
                              chi2(1) =     4.81  Pr>chi2 = 0.0282
```

```
. cc mort diet, by(act)

             act |       OR       [95% Conf. Interval]   M-H Weight
-----------------+-------------------------------------------------
               0 |   .7132867       .327589    1.53607     9.079365 (exact)
               1 |       .875      .4324228   1.766572      9.66443 (exact)
               2 |   .6563707      .2285842    1.84153     5.452632 (exact)
               3 |   .7619048      .1468516   3.427209     2.507463 (exact)
-----------------+-------------------------------------------------
           Crude |   .6527417      .4364899   .9723664              (exact)
    M-H combined |   .7647561      .5093186   1.148303
-------------------------------------------------------------------
Test of homogeneity (M-H)      chi2(3) =     0.30  Pr>chi2 = 0.9594

                   Test that combined OR = 1:
                               Mantel-Haenszel chi2(1) =      1.67
                                                Pr>chi2 =    0.1969
```