

# THE CAPITAL ASSET PRICING MODEL

$$R_i = R_f + \beta_i(R_m - R_f)$$

EXAMPLE 3: MULTIPLE LINEAR  
REGRESSION WITH AN OIL STOCK

## EXAMPLE 3: MULTIPLE LINEAR REGRESSION

WE WANT TO FIND A MODEL THAT CAN  
PREDICT THE RATE OF RETURN ON AN  
OIL STOCK LIKE EXXON MOBIL

## EXAMPLE 3: MULTIPLE LINEAR REGRESSION

$$R_i - R_f = \beta_i (R_m - R_f)$$

↑  
RETURNS OF EXXON  
MOBIL - RISK FREE RATE

THIS IS THE **CAPM MODEL** THAT WE HAVE  
ALREADY SEEN

## EXAMPLE 3: MULTIPLE LINEAR REGRESSION

$$R_i - R_f = \beta_i (R_m - R_f)$$

RETURNS OF AN INDEX THAT  
REPRESENTS THE MARKET -  
RISK FREE RATE

S&P 500

## EXAMPLE 3: MULTIPLE LINEAR REGRESSION

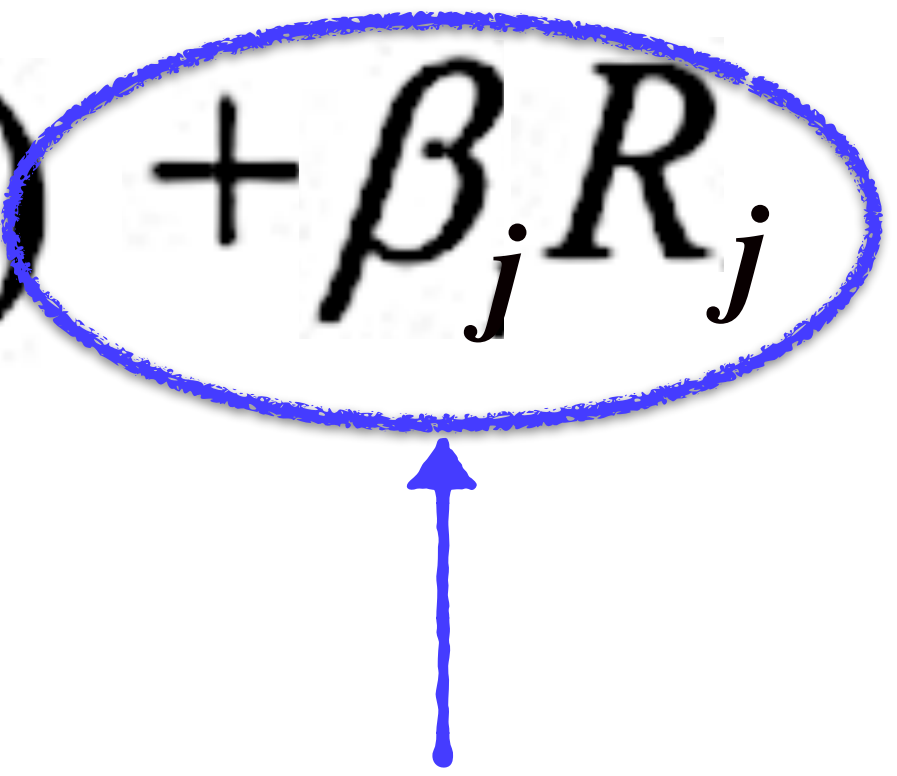
$$R_i - R_f = \beta_i (R_m - R_f)$$

THIS MODEL USES ONLY **1**  
**DEPENDENT VARIABLE**

THIS KIND OF REGRESSION IS CALLED  
**SIMPLE LINEAR REGRESSION**



## EXAMPLE 3: MULTIPLE LINEAR REGRESSION

$$R_i - R_f = \beta_i (R_m - R_f) + \beta_j R_j$$


WHAT IF WE **ADDED**  
**ANOTHER VARIABLE?**

RETURNS OF OIL PRICES

BY INCLUDING MORE VARIABLES, WE HOPE TO  
EXPLAIN **MORE OF THE VARIATION** IN THE  
DEPENDENT VARIABLE

## EXAMPLE 3: MULTIPLE LINEAR REGRESSION

$$R_i - R_f = \beta_i (R_m - R_f) + \beta_j R_j$$

THIS KIND OF REGRESSION IS CALLED  
MULTIPLE LINEAR REGRESSION

LET'S GO THROUGH THE STEPS WE'LL NEED TO DO

STEP 1: BUILD A SIMPLE LINEAR  
REGRESSION MODEL USING ONLY  
EXXON MOBIL AND S&P 500



**WE'LL DOWNLOAD THE MONTHLY PRICES FOR**  
**EXXON MOBIL (XOM)**  
**^GSPC (S&P 500)**

**FOR 6 YEARS (JAN 1, 2010 - FEB 1, 2016)**

THESE ARE THE FILE PATHS FOR THE DATA  
DOWNLOADED FROM YAHOO FINANCE

```
xomFile <- '/Users/swethakolalapudi/Desktop/Regression/xom.csv'  
snpFile <- '/Users/swethakolalapudi/Desktop/Regression/snp.csv'
```

WE'LL USE A **FUNCTION WE WROTE EARLIER**  
TO PREPROCESS THE DATA IN THESE FILES

```
names(xom)[2:3] <- c("xom.returns", "snp.returns")
```

WE'LL GIVE RETURNS COLUMNS **PROPER NAMES**

```
xom_SM <- lm(xom$xom.returns~xom$snp.returns)
```

PERFORM LINEAR REGRESSION

```
xom_SM <- lm(xom$xom.returns~xom$snp.returns)
```

LET'S SEE THE RESULTS OF THIS REGRESSION

```
summary(xom_SM)
```

BETA FOR XOM

Residuals:

Min	1Q	Median	3Q	Max
-0.076423	-0.022784	-0.003236	0.017084	0.085066

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.003908	0.003761	-1.039	0.302
xom\$snp.returns	0.868164	0.098480	8.816	5.15e-13 ***

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03186 on 71 degrees of freedom

Multiple R-squared: 0.5226, Adjusted R-squared: 0.5159

F-statistic: 77.71 on 1 and 71 DF, p-value: 5.146e-13



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THIS RELATIONSHIP  
IS STATISTICALLY  
SIGNIFICANT



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**R-SQUARE**  
**= 52%**



```
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```

LET'S SEE THE RESULTS OF THIS REGRESSION

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ADJUSTED R-SQUARE =  
51.5%

R-SQUARE = 52%

## SIMPLE LINEAR REGRESSION

ADJUSTED R-SQUARE = 51.5%

R-SQUARE = 52%

LET'S SEE WHAT HAPPENS WHEN WE **ADD**  
**ANOTHER VARIABLE** TO THE REGRESSION

**STEP 2: ADD ANOTHER  
VARIABLE TO THE REGRESSION**

## **STEP 2: ADD ANOTHER VARIABLE TO THE REGRESSION**

**WE'LL ADD THE RETURNS OF OIL PRICES**

**THESE WILL BE REPRESENTED BY THE RETURNS ON AN OIL ETF (THESE ARE FUNDS WHOSE RETURNS DEPEND HEAVILY ON OIL PRICES )**



# READ THE PRICES OF USO FROM A FILE AND CONVERT THEM TO RETURNS

```
uso <- read.table( '/Users/swethakolalapudi/Desktop/Regression/  
uso.csv', header = TRUE, sep = "," )[, c( "Date", "Adj.Close" )]  
names(uso)[2] <- "uso.returns"  
uso[, c( "Date" )] <- as.Date(uso[, c( "Date" )])  
uso <- uso[order(uso$Date, decreasing = TRUE), ]  
uso[-nrow(uso), -1] <- uso[-nrow(uso), -1] / uso[-1, -1] - 1
```

THIS IS VERY SIMILAR TO HOW WE DID  
THE PREPROCESSING EARLIER, EXCEPT WE  
ARE JUST DOING IT FOR ONE SECURITY



**MERGE THE RETURNS OF USO AND THE RETURNS WE HAVE FROM THE PREVIOUS STEP INTO ONE DATA FRAME (THIS IS SO THEY ARE ALL ALIGNED BY DATE)**

```
xom <- merge(xom, uso, by = "Date")
```

**REGRESS XOM RETURNS ON BOTH S&P AND USO**

```
xom_MLR <- lm(xom$xom.returns~xom$snp.returns + xom$uso.returns)
```

**THIS MODEL HAS 2 INDEPENDENT VARIABLES**



```
xom_MLR <- lm(xom$xom.returns~xom$snp.returns + xom$uso.returns)
```

LET'S SEE THE RESULTS OF THIS REGRESSION

```
summary(xom_MLR)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.061634	-0.020990	-0.004805	0.017007	0.085419

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.003060	0.003741	-0.818	0.4162
xom\$snp.returns	0.769195	0.112644	6.829	2.56e-09 ***
xom\$uso.returns	0.089186	0.051429	1.734	0.0873 .

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03142 on 70 degrees of freedom

Multiple R-squared: 0.5422, Adjusted R-squared: 0.5292

F-statistic: 41.46 on 2 and 70 DF, p-value: 1.325e-12

CO-EFFICIENTS  
FOR S&P AND USO



```
xom_MLR <- lm(xom$xom.returns~xom$snp.returns + xom$uso.returns)
```

LET'S SEE THE RESULTS OF THIS REGRESSION

```
summary(xom_MLR)
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BOTH OF THESE  
VARIABLES HAVE A  
STATISTICALLY  
SIGNIFICANT  
RELATIONSHIP



```
xom_MLR <- lm(xom$xom.returns~xom$snp.returns + xom$uso.returns)
```

LET'S SEE THE RESULTS OF THIS REGRESSION

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R-SQUARE  
= 54%



```
xom_MLR <- lm(xom$xom.returns~xom$snp.returns + xom$uso.returns)
```

LET'S SEE THE RESULTS OF THIS REGRESSION

```
summary(xom_MLR)
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ADJUSTED R-SQUARE =  
52.9%

R-SQUARE = 54%

LET'S COMPARE  
THIS WITH THE  
PREVIOUS RESULTS



## SIMPLE LINEAR REGRESSION

ADJUSTED R-SQUARE = 51.5%

R-SQUARE = 52%

## MULTIPLE LINEAR REGRESSION

ADJUSTED R-SQUARE = 52.9%

R-SQUARE = 54%

**R-SQUARE IMPROVED WHEN  
YOU ADDED A NEW VARIABLE**

## SIMPLE LINEAR REGRESSION

ADJUSTED R-SQUARE = 51.5%

R-SQUARE = 52%

## MULTIPLE LINEAR REGRESSION

ADJUSTED R-SQUARE = 52.9%

R-SQUARE = 54%

ADJUSTED R-SQUARE ALSO IMPROVED

## SIMPLE LINEAR REGRESSION

ADJUSTED R-SQUARE = 51.5%

R-SQUARE = 52%

## MULTIPLE LINEAR REGRESSION

ADJUSTED R-SQUARE = 52.9%

R-SQUARE = 54%

NOTICE HOW THE DIFFERENCE BETWEEN R-SQUARE AND ADJUSTED R-SQUARE INCREASED

THIS HAPPENS AS YOU ADD MORE VARIABLES

**VARIABLES THAT TAKE ONE OF A  
LIMITED SET OF VALUES ARE  
CALLED CATEGORICAL VARIABLES**

**MONTH**

**COLOR**

**QUARTER**

**GENDER**

**ARE PERFECT EXAMPLES OF CATEGORICAL VARIABLES**

CATEGORICAL VARIABLES

THESE KIND OF VARIABLES USUALLY  
DON'T HAVE ANY **NUMERIC MEANING**

HOW DO WE **INCLUDE THEM** IN A  
**LINEAR REGRESSION?**



**CATEGORICAL VARIABLES**

**HOW DO WE INCLUDE THEM  
IN A LINEAR REGRESSION?**

**WE CREATE A SET OF “DUMMY”  
VARIABLES” TO REPRESENT THE  
CATEGORICAL VARIABLE**

**THE NUMBER OF VARIABLES = NUMBER OF DISTINCT VALUES -1**

CATEGORICAL VARIABLES

HOW DO WE INCLUDE THEM  
IN A LINEAR REGRESSION?

**“DUMMY” VARIABLES**

THE NUMBER OF VARIABLES = NUMBER OF DISTINCT VALUES - 1

EACH DUMMY VARIABLE IS **1 FOR A PARTICULAR VALUE** OF THE  
CATEGORICAL VARIABLE, **ELSE IT IS 0**

LET'S TAKE AN EXAMPLE

# EXAMPLE 4: USING A CATEGORICAL VARIABLE FOR REGRESSION



## EXAMPLE 4: USING A CATEGORICAL VARIABLE FOR REGRESSION

WE HAD EARLIER DONE AN EXERCISE TO REGRESS GOOGLE  
RETURNS AGAINST NASDAQ

LET'S INCLUDE MONTH, WHICH IS A CATEGORICAL VARIABLE

THERE ARE 12 DISTINCT VALUES FOR MONTH

LET'S INCLUDE **MONTH, WHICH IS A CATEGORICAL VARIABLE**

THERE ARE 12 DISTINCT VALUES FOR MONTH

THERE WILL BE **11 DUMMY VARIABLES**

Var1
Var2
Var3
Var4
Var5
Var6
Var7
Var8
Var9
Var10
Var11

THE VALUE OF EACH OF THESE  
DEPENDS ON **THE VALUE OF MONTH**

LET'S INCLUDE MONTH, WHICH IS A CATEGORICAL VARIABLE

M	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sep	Oct	Nov	Dec
Var1												
Var2												
Var3												
Var4												
Var5												
Var6												
Var7												
Var8												
Var9												
Var10												
Var11												

FOR 11 OF THESE MONTHS ONE VARIABLE WILL HAVE THE VALUE 1 , ALL ELSE WILL BE 0

FOR ONE OF THE MONTHS ALL THE VARIABLES WILL BE 0

LET'S INCLUDE MONTH, WHICH IS A CATEGORICAL VARIABLE

M	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sep	Oct	Nov	Dec
Var1	1	0	0	0	0	0	0	0	0	0	0	0
Var2	0	1	0	0	0	0	0	0	0	0	0	0
Var3	0	0	1	0	0	0	0	0	0	0	0	0
Var4	0	0	0	1	0	0	0	0	0	0	0	0
Var5	0	0	0	0	1	0	0	0	0	0	0	0
Var6	0	0	0	0	0	1	0	0	0	0	0	0
Var7	0	0	0	0	0	0	1	0	0	0	0	0
Var8	0	0	0	0	0	0	0	1	0	0	0	0
Var9	0	0	0	0	0	0	0	0	1	0	0	0
Var10	0	0	0	0	0	0	0	0	0	1	0	0
Var11	0	0	0	0	0	0	0	0	0	0	1	0

THIS IS CALLED ONE-HOT  
ENCODING



# LET'S SEE HOW TO DO THIS IN R

## WE ALREADY HAVE A DATA FRAME CALLED GOOG

## LET'S ADD A MONTH COLUMN

```
goog$Month = format(goog$Date, "%m")
```

Date	goog.returns	nasdaq.returns	tbonds.returns	Month
2010-02-01	-0.0287651490	0.019495859	0.02284	02
2010-03-01	0.0509375748	0.045750044	0.02560	03
2010-04-01	-0.0972357416	0.002168238	0.02420	04
2010-05-03	-0.0971921643	-0.103917642	0.02097	05
2010-06-01	-0.1017174844	-0.083434017	0.01795	06
2010-07-01	0.0736930461	0.052983210	0.01598	07
2010-08-02	-0.0852566960	-0.075809642	0.01342	08
2010-09-01	0.1555604124	0.107618795	0.01281	09
2010-10-01	0.1554058941	0.046805211	0.01179	10
2010-11-01	-0.1091324620	-0.018301121	0.01464	11
2010-12-01	0.0486888972	0.041739880	0.02016	12
2011-01-03	-0.0087618121	-0.001724194	0.01952	01
2011-02-01	0.0003502713	0.009069816	0.02137	02
2011-03-01	-0.0656800154	-0.022681285	0.02225	03
2011-04-01	-0.0924544001	0.013499781	0.01975	03

YOU CAN CREATE DUMMY VARIABLES  
USING **THE MODEL.MATRIX FUNCTION**

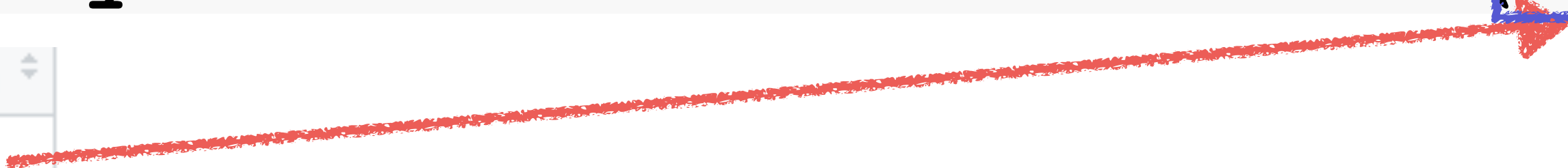
```
dummyVars <- model.matrix(~Month, goog)
```

THE VARIABLE TO BE CONVERTED,  
THE DATA FRAME THE COLUMN IS IN

Month
02
03
04
05
06
07
08
09
10
11
12
01
02
03

# YOU CAN CREATE DUMMY VARIABLES USING **THE MODEL.MATRIX FUNCTION**

```
dummyVars <- model.matrix(~Month, goog)
```



Month
02
03
04
05
06
07
08
09
10
11
12
01
02
03

# YOU CAN CREATE DUMMY VARIABLES USING THE MODEL.MATRIX FUNCTION

```
dummyVars <- model.matrix(~Month, goog)
```

Month	(Intercept)	Month02	Month03	Month04	Month05	Month06	Month07	Month08	Month09	Month10	Month11	Month12
02	1	1	0	0	0	0	0	0	0	0	0	0
03	1	0	1	0	0	0	0	0	0	0	0	0
04	1	0	0	1	0	0	0	0	0	0	0	0
05	1	0	0	0	1	0	0	0	0	0	0	0
06	1	0	0	0	0	1	0	0	0	0	0	0
07	1	0	0	0	0	0	1	0	0	0	0	0
08	1	0	0	0	0	0	0	1	0	0	0	0
09	1	0	0	0	0	0	0	0	1	0	0	0
10	1	0	0	0	0	0	0	0	0	1	0	0
11	1	0	0	0	0	0	0	0	0	0	1	0
12	1	0	0	0	0	0	0	0	0	0	0	1
01	1	1	0	0	0	0	0	0	0	0	0	0
02	1	0	1	0	0	0	0	0	0	0	0	0
03	1	0	0	1	0	0	0	0	0	0	0	0



# YOU CAN CREATE DUMMY VARIABLES USING THE MODEL.MATRIX FUNCTION

```
dummyVars <- model.matrix(~Month, goog)
```

Month	(Intercept)	Month02	Month03	Month04	Month05	Month06	Month07	Month08	Month09	Month10	Month11	Month12
02	1	1	0	0	0	0	0	0	0	0	0	0
03	1	0	1	0	0	0	0	0	0	0	0	0
04	1	0	0	1	0	0	0	0	0	0	0	0
05	1	0	0	0	1	0	0	0	0	0	0	0
06	1	0	0	0	0	1	0	0	0	0	0	0
07	1	0	0	0	0	0	1	0	0	0	0	0
08	1	0	0	0	0	0	0	1	0	0	0	0
09	1	0	0	0	0	0	0	0	1	0	0	0
10	1	0	0	0	0	0	0	0	0	1	0	0
11	1	0	0	0	0	0	0	0	0	0	1	0
12	1	0	0	0	0	0	0	0	0	0	0	1
01	1	1	0	0	0	0	0	0	0	0	0	0
02	1	0	1	0	0	0	0	0	0	0	0	0
03	1	0	0	1	0	0	0	0	0	0	0	0

```
dummyVars <- model.matrix(~Month, goog)
```

**YOU CAN MERGE THIS WITH THE ORIGINAL DATA  
FRAME AND THEN RUN A LINEAR REGRESSION ON  
ALL 11 VARIABLES + THE NASDAQ RETURNS**

**OR**

**DIRECTLY USE THE CATEGORICAL  
VARIABLE IN THE LINEAR REGRESSION**

```
goog_MLR <- lm(goog$goog.returns~goog$nasdaq.returns+goog$Month)
```

```
goog_MLR <- lm(goog$goog.returns~goog$nasdaq.returns+goog$Month)
```

## LET'S SEE THE RESULTS OF THE REGRESSION

```
summary(goog_MLR)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.167102	-0.027855	0.004201	0.034741	0.121227

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.001029	0.022894	-0.045	0.9643
goog\$nasdaq.returns	0.810490	0.163540	4.956	6.21e-06 ***
goog\$Month02	0.001253	0.031531	0.040	0.9684
goog\$Month03	-0.023391	0.032359	-0.723	0.4726
goog\$Month04	-0.048513	0.032311	-1.501	0.1385
goog\$Month05	0.002568	0.032431	0.079	0.9371
goog\$Month06	-0.014763	0.032375	-0.456	0.6500
goog\$Month07	0.074377	0.032477	2.290	0.0255 *
goog\$Month08	-0.011228	0.032519	-0.345	0.7311
goog\$Month09	0.030690	0.032339	0.949	0.3464
goog\$Month10	0.048443	0.033136	1.462	0.1490
goog\$Month11	-0.012551	0.032330	-0.388	0.6992
goog\$Month12	0.025718	0.032315	0.796	0.4293

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05596 on 60 degrees of freedom

Multiple R-squared: 0.5187, Adjusted R-squared: 0.4225

F-statistic: 5.389 on 12 and 60 DF, p-value: 4.221e-06

NOTICE HOW LM  
AUTOMATICALLY  
CREATED 11  
DUMMY VARIABLES  
FROM MONTH



Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.001029	0.022894	-0.045	0.9643
goog\$nasdaq.returns	0.810490	0.163540	4.956	6.21e-06 ***
goog\$Month02	0.001253	0.031531	0.040	0.9684
goog\$Month03	-0.023391	0.032359	-0.723	0.4726
goog\$Month04	-0.048513	0.032311	-1.501	0.1385
goog\$Month05	0.002568	0.032431	0.079	0.9371
goog\$Month06	-0.014763	0.032375	-0.456	0.6500
goog\$Month07	0.074377	0.032477	2.290	0.0255 *
goog\$Month08	-0.011228	0.032519	-0.345	0.7311
goog\$Month09	0.030690	0.032339	0.949	0.3464
goog\$Month10	0.048443	0.033136	1.462	0.1490
goog\$Month11	-0.012551	0.032330	-0.388	0.6992
goog\$Month12	0.025718	0.032315	0.796	0.4293

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05596 on 60 degrees of freedom

Multiple R-squared: 0.5187, Adjusted R-squared: 0.4225

F-statistic: 5.389 on 12 and 60 DF, p-value: 4.221e-06

**CO-EFFICIENTS FOR  
EACH OF THE VARIABLES**



Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.001029	0.022894	-0.045	0.9643
goog\$nasdaq.returns	0.810490	0.163540	4.956	6.21e-06 ***
goog\$Month02	0.001253	0.031531	0.040	0.9684
goog\$Month03	-0.023391	0.032359	-0.723	0.4726
goog\$Month04	-0.048513	0.032311	-1.501	0.1385
goog\$Month05	0.002568	0.032431	0.079	0.9371
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goog\$Month07	0.074377	0.032477	2.290	0.0255 *
goog\$Month08	-0.011228	0.032519	-0.345	0.7311
goog\$Month09	0.030690	0.032339	0.949	0.3464
goog\$Month10	0.048443	0.033136	1.462	0.1490
goog\$Month11	-0.012551	0.032330	-0.388	0.6992
goog\$Month12	0.025718	0.032315	0.796	0.4293

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05596 on 60 degrees of freedom

Multiple R-squared: 0.5187, Adjusted R-squared: 0.4225

F-statistic: 5.389 on 12 and 60 DF, p-value: 4.221e-06

STATISTICAL  
SIGNIFICANCE

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.001029	0.022894	-0.045	0.9643
goog\$nasdaq.returns	0.810490	0.163540	4.956	6.21e-06 ***
goog\$Month02	0.001253	0.031531	0.040	0.9684
goog\$Month03	-0.023391	0.032359	-0.723	0.4726
goog\$Month04	-0.048513	0.032311	-1.501	0.1385
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goog\$Month06	-0.014763	0.032375	-0.456	0.6500
goog\$Month07	0.074377	0.032477	2.290	0.0255 *
goog\$Month08	-0.011228	0.032519	-0.345	0.7311
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goog\$Month10	0.048443	0.033136	1.462	0.1490
goog\$Month11	-0.012551	0.032330	-0.388	0.6992
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---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05596 on 60 degrees of freedom

Multiple R-squared: 0.5187, Adjusted R-squared: 0.4225

F-statistic: 5.389 on 12 and 60 DF, p-value: 4.221e-06

**ONLY MONTH07 IS  
STATISTICALLY SIGNIFICANT  
AND IT'S CO-EFFICIENT VALUE IS LOW**



Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.001029	0.022894	-0.045	0.9643
goog\$nasdaq.returns	0.810490	0.163540	4.956	6.21e-06 ***
goog\$Month02	0.001253	0.031531	0.040	0.9684
goog\$Month03	-0.023391	0.032359	-0.723	0.4726
goog\$Month04	-0.048513	0.032311	-1.501	0.1385
goog\$Month05	0.002568	0.032431	0.079	0.9371
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goog\$Month07	0.074377	0.032477	2.290	0.0255 *
goog\$Month08	-0.011228	0.032519	-0.345	0.7311
goog\$Month09	0.030690	0.032339	0.949	0.3464
goog\$Month10	0.048443	0.033136	1.462	0.1490
goog\$Month11	-0.012551	0.032330	-0.388	0.6992
goog\$Month12	0.025718	0.032315	0.796	0.4293

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05596 on 60 degrees of freedom

Multiple R-squared: 0.5187, Adjusted R-squared: 0.4225

F-statistic: 5.389 on 12 and 60 DF, p-value: 4.221e-06

**THERE IS A LESSON HERE**  
**ADDING MORE VARIABLES IS NOT ALWAYS A  
GOOD THING!!**

# EXAMPLE 5: DEALING WITH OUTLIERS WITH RLM()

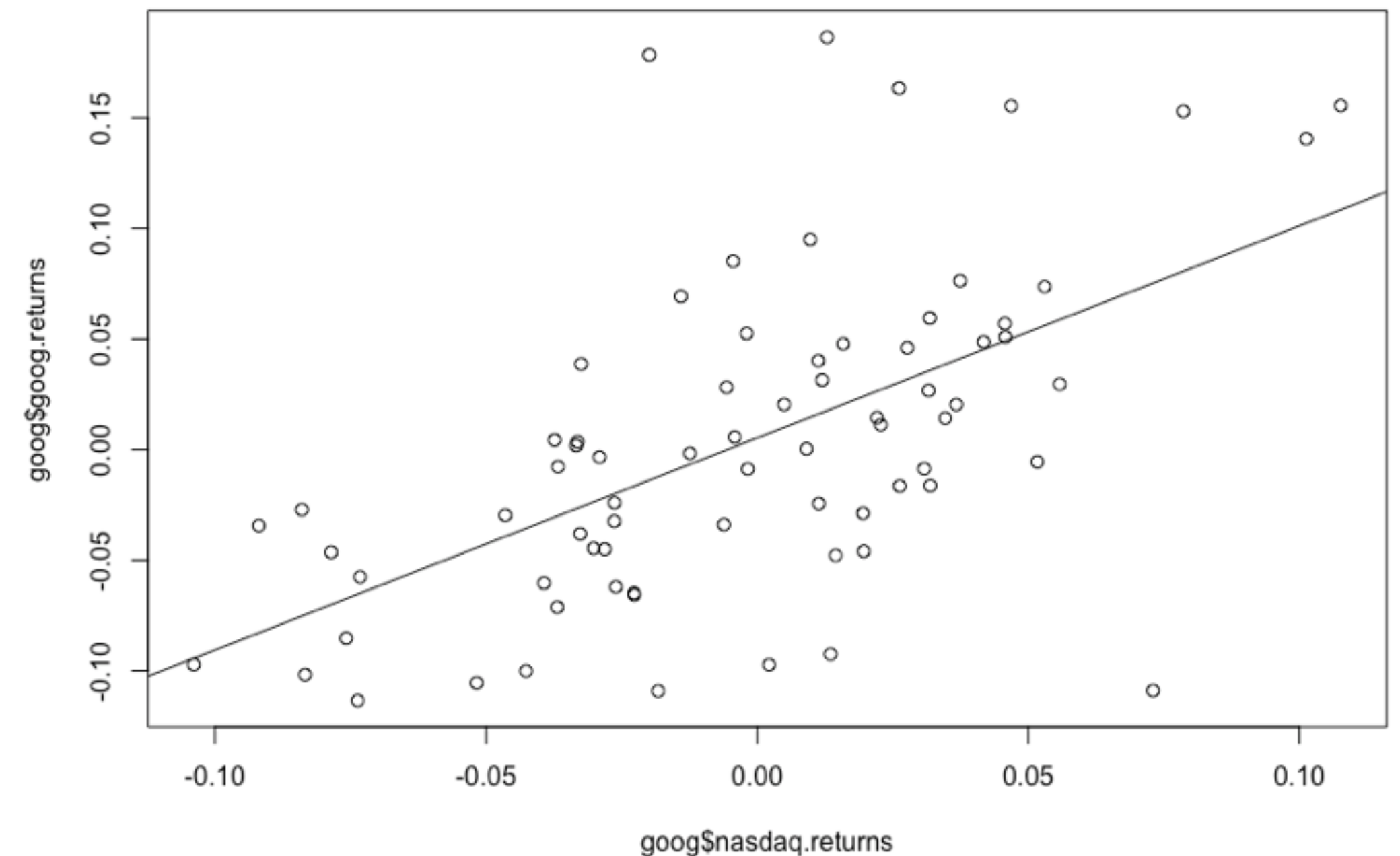
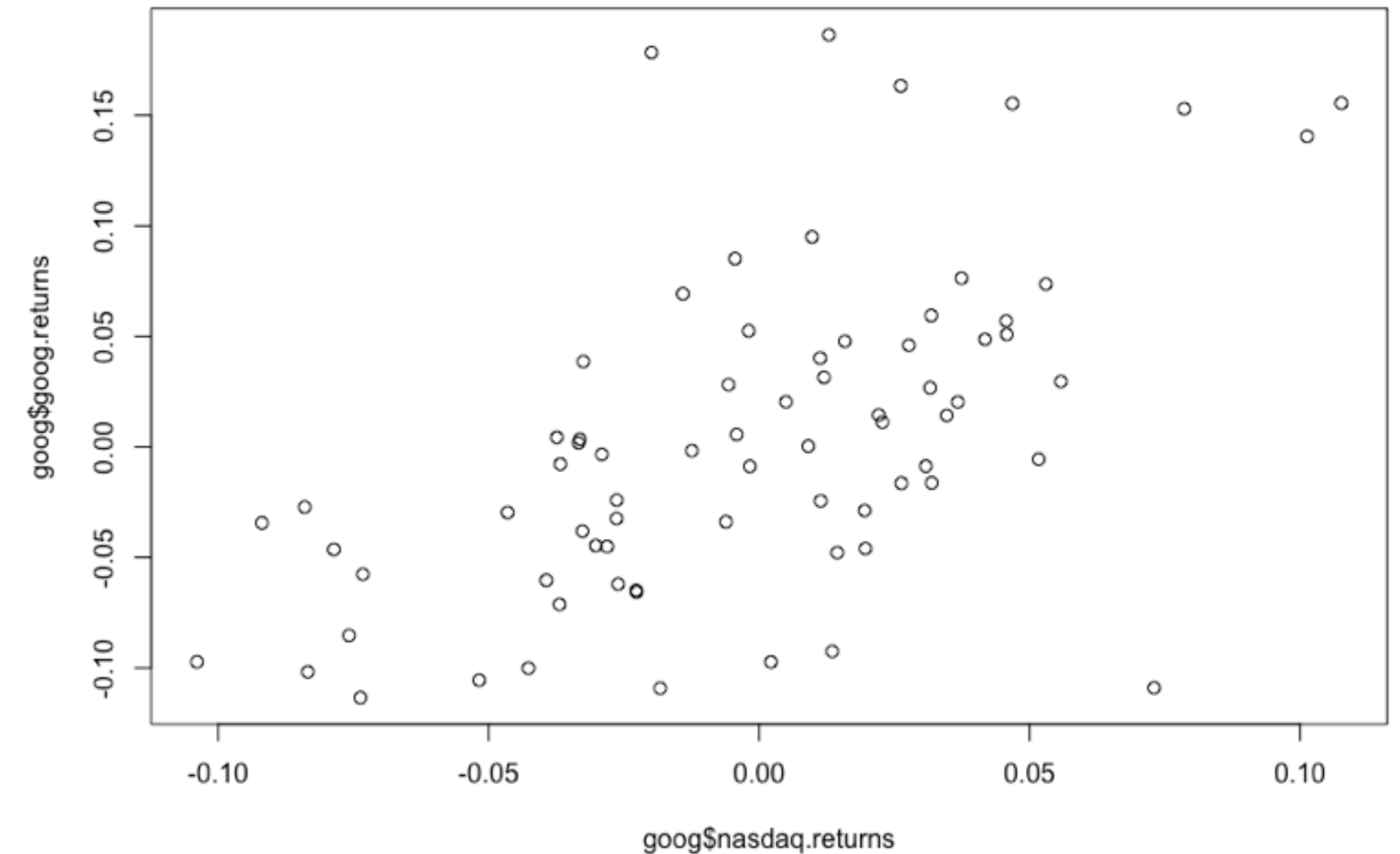


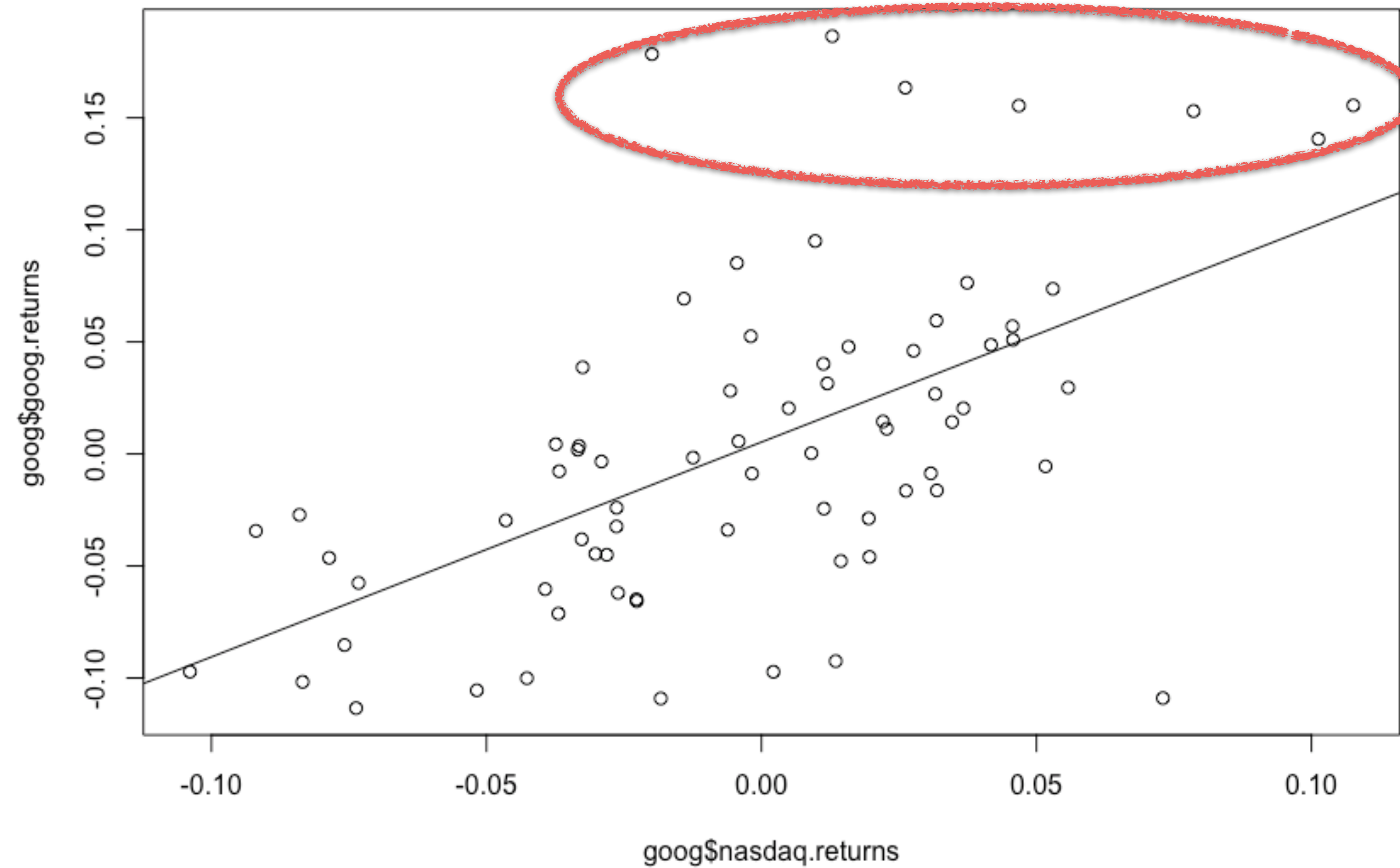
# HERE IS THE PLOT FOR GOOGLE VS NASDAQ RETURNS

```
plot(goog$nasdaq.returns, goog$goog.returns)
```

# HERE IS THE PLOT OF THE FITTED LINE AFTER REGRESSION

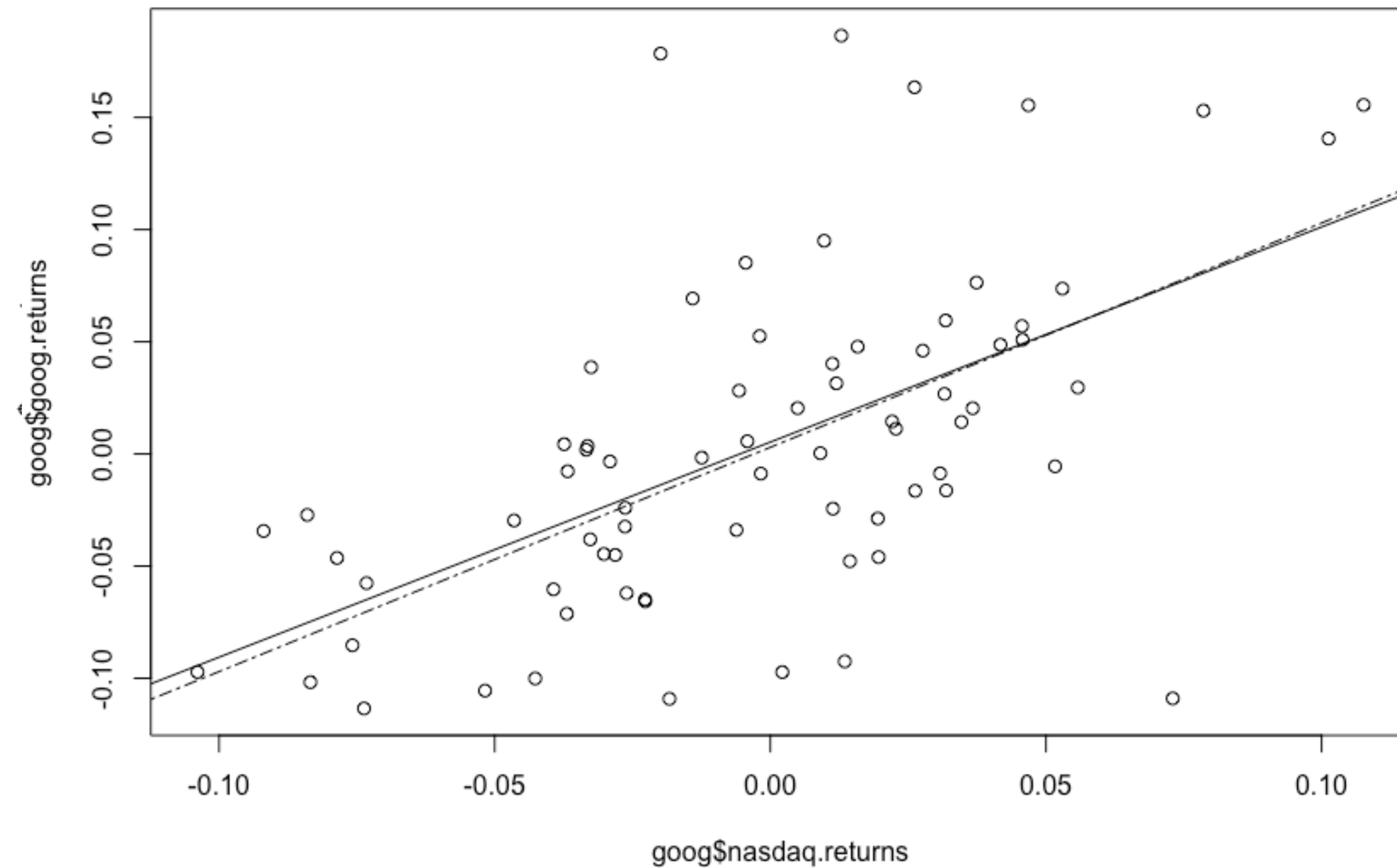
```
abline ( googM )
```





**YOU CAN SEE THAT THERE ARE SOME  
OUTLIERS IN THIS PLOT WHICH  
MIGHT BE DISTORTING THE LINE**

**RLM() FITS A LINE  
AFTER TAKING INTO  
ACCOUNT THE  
EFFECT OF OUTLIERS**

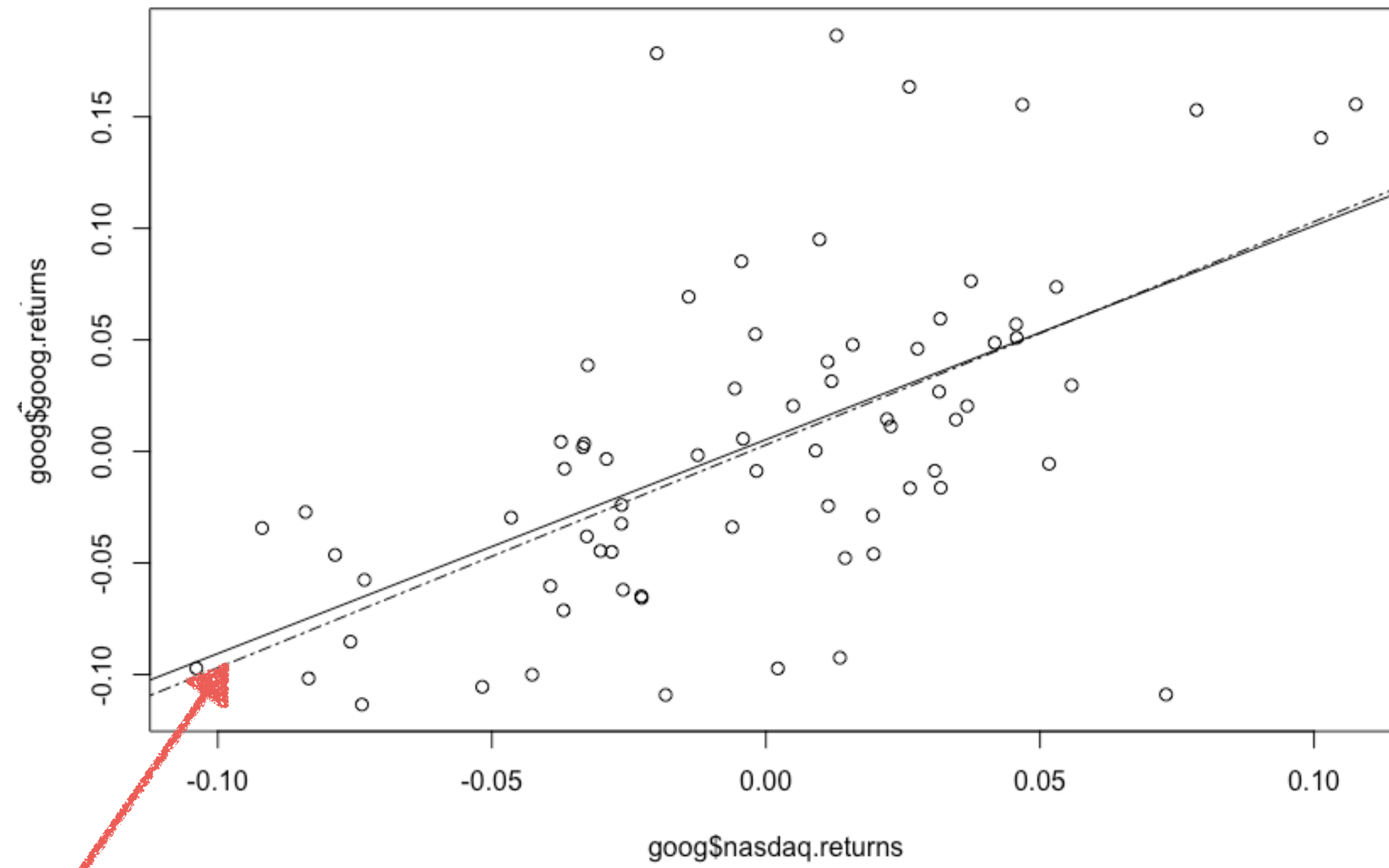


**RLM() IS A  
FUNCTION IN THE  
MASS PACKAGE**

```
require(MASS)  
googRLM <- rlm(goog$goog.returns~goog$nasdaq.returns)
```

```
abline(googRLM, lty = 'twodash')
```

**LET'S SEE THE RESULTS  
AFTER USING RLM**



**THE DOTTED LINE IS THE RESULT  
OF RLM() WHICH HAS BEEN  
SLIGHTLY ADJUSTED FOR THE  
EFFECT OF OUTLIERS**