

TYPES OF INFERENCES

~~1. IDENTIFYING THE POPULATION MEAN~~

~~2. IDENTIFYING THE POPULATION %~~

~~3. VERIFYING WHETHER THE POPULATION MEAN IS EQUAL
TO A CERTAIN VALUE~~

~~4. VERIFYING WHETHER THE POPULATION % IS EQUAL TO A
CERTAIN VALUE~~

5. VERIFYING WHETHER 2 POPULATION MEANS ARE DIFFERENT

6. VERIFYING WHETHER 2 POPULATION % ARE DIFFERENT

5. VERIFYING WHETHER 2 POPULATION MEANS ARE DIFFERENT

CASE STUDY: A/B TESTING

**MR. Z OWNS A LARGE
SOCIAL MEDIA COMPANY**

MR. Z OWNS A LARGE SOCIAL
MEDIA COMPANY

EVERY WEEK, **MR. Z** HOLDS
A **"STRATEGY"** MEETING



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IN ONE SUCH MEETING..

**Hey, I know!! Let's
completely rebuild the website
from scratch**




ONE SIDE THINKS

That's a great idea!



THE OTHER..



UMM.. No...User engagement is really high right now. If it ain't broke don't fix it!

Ok, then. Do an experiment. I want the results in a week!



STEP : 1

**SET UP A CONTROLLED
EXPERIMENT**

AFTER THE MEETING, THE
SVP OF WEB PRODUCTS

TOLD THE VP OF WEB PRODUCTS

WHO TOLD THE DIRECTOR OF
WEB ANALYTICS

WHO TOLD A SENIOR WEB
ANALYST

TO DO THE EXPERIMENT

TO DO THE EXPERIMENT

THE SENIOR WEB ANALYST
IS A PRO AT ALL THIS

HE GETS AN ENGINEER TO
SET UP A MOCK-UP OF THE
NEW WEBSITE

THEN HE TAKES A BUNCH OF USERS
AND RANDOMLY DIVIDES
THEM INTO 2 GROUPS

THE LUCKY DOGS

THE GUINEA PIGS

HEY, LUCKY DOGS!

YOU'RE LUCKY COZ YOUR PROFILE
PAGE IS THE SAME, JUST THE WAY
YOU LIKE IT

THE LUCKY DOGS

HEY, LUCKY DOGS!

YOU'RE LUCKY COZ YOUR
PROFILE PAGE IS THE SAME,
JUST THE WAY YOU LIKE IT

THE GUINEA PIGS

POOR GUINEA PIGS,

YOU GET THE NEW MOCK-UP,
YOU'RE NOT GOING TO FIND
ANYTHING YOU WANT FOR
A WHILE

CONTROL GROUP
~~THE LUCKY DOGS~~

HEY, LUCKY DOGS!

YOU'RE LUCKY COZ YOUR
PROFILE PAGE IS THE SAME,
JUST THE WAY YOU LIKE IT

A

TEST GROUP

~~THE GUINEA PIGS~~

POOR GUINEA PIGS,

YOU GET THE NEW MOCK-UP,
YOU'RE NOT ABLE TO FIND
ANYTHING YOU WANT FOR A
WHILE

B

OR WHEN IT'S AN
INTERWEB EXPERIMENT

CONTROL GROUP

A

TEST GROUP

B

THE ANALYST
RUNS THE A/B
TEST FOR 4 DAYS

STEP : 2

**COMPUTE SAMPLE MEANS
FOR BOTH GROUPS A, B**

CONTROL GROUP

A 5 MINS PER
VISIT

TEST GROUP

B 3 MINS PER
VISIT

THE ANALYST COMPARES THE ENGAGEMENT
(AVERAGE TIME SPENT ON THE WEBSITE)
OF THE 2 GROUPS

CONTROL GROUP

A 5 MINS PER
VISIT

TEST GROUP

B 3 MINS PER
VISIT

GASP!!

THE **NEW WEBSITE** WILL DROP
ENGAGEMENT BY **40%**

CONTROL GROUP

A 5 MINS PER
VISIT

TEST GROUP

B 3 MINS PER
VISIT

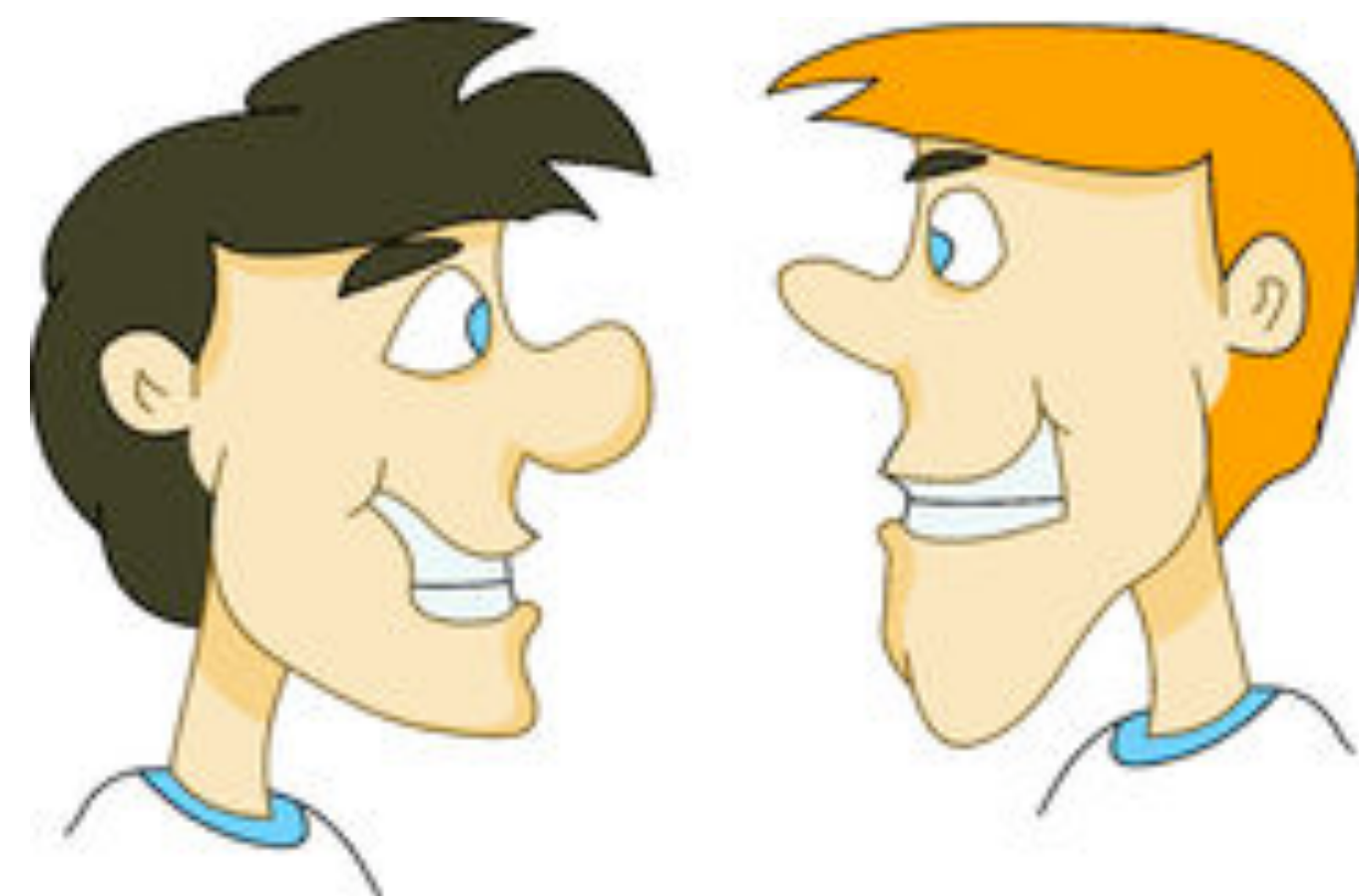
THE STRAT MEETING IS ON MONDAY

ON **FRIDAY**, THE ANALYST STAYS LATE TO
FINISH UP THE DECK WITH THESE RESULTS

THE STRAT MEETING IS ON MONDAY

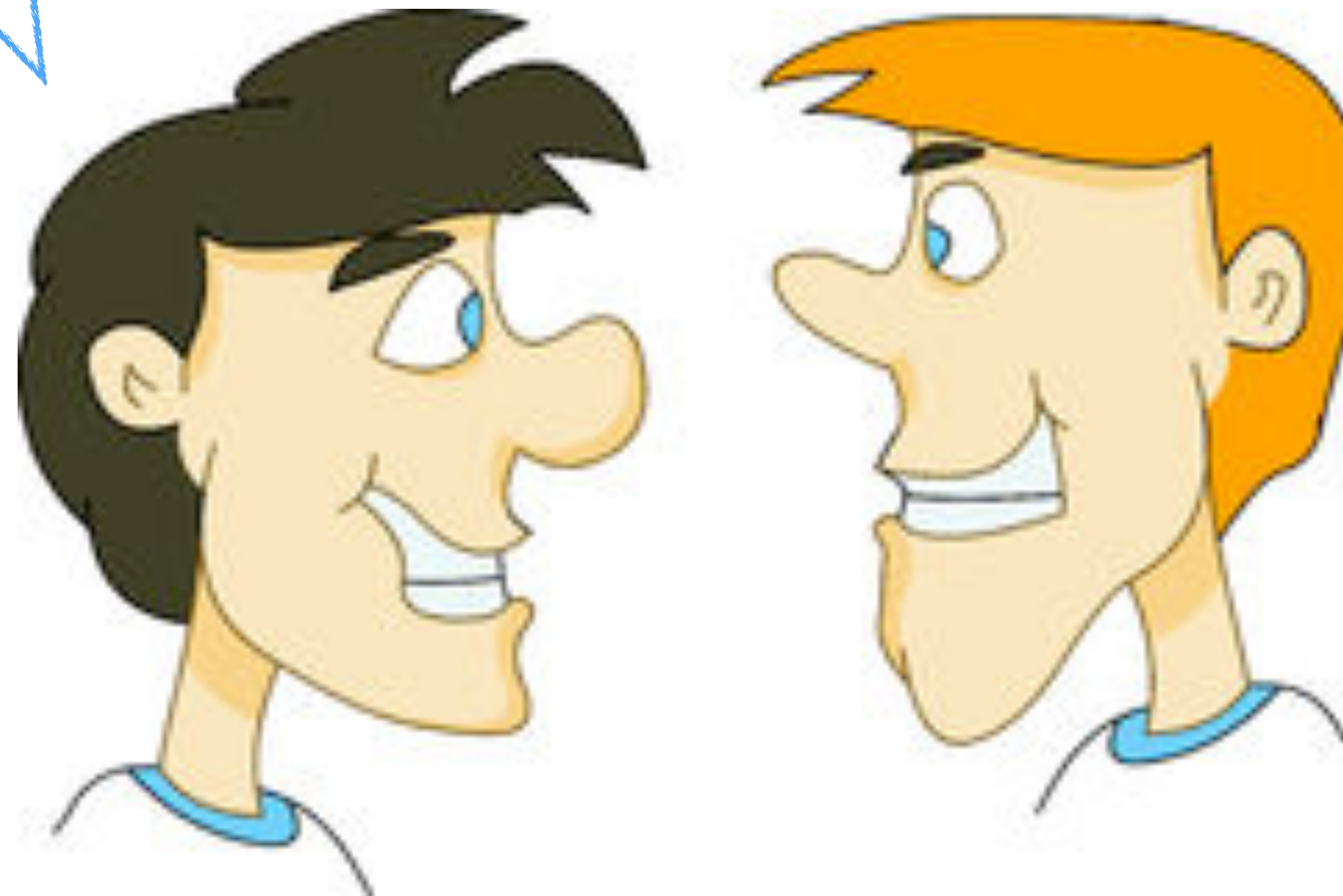
**ON FRIDAY, THE ANALYST STAYS LATE TO
FINISH UP THE DECK WITH THESE RESULTS**

ON SATURDAY, HE RUNS INTO A FRIEND



\$*@!#**

**SO, ARE YOUR RESULTS
STATISTICALLY SIGNIFICANT**



IT'S **SUNDAY**, AND THE ANALYST NEEDS TO
RUN **A TEST OF SIGNIFICANCE**

STEP : 3

**A TEST OF SIGNIFICANCE
COMPARING 2 POPULATION
MEANS**

NULL HYPOTHESIS

**ALL VARIATIONS OBSERVED ARE
DUE TO CHANCE I.E. A FLUKE**

ALTERNATIVE HYPOTHESIS

**THE VARIATIONS OBSERVED CANNOT
JUST BE EXPLAINED BY CHANCE**

NULL HYPOTHESIS **VS** ALTERNATIVE HYPOTHESIS

A TEST OF SIGNIFICANCE WILL TELL
YOU WHICH OF THESE IS BETTER

A TEST OF SIGNIFICANCE

NULL HYPOTHESIS **VS** ALTERNATIVE HYPOTHESIS

THIS INVOLVES

1) COMPUTING A **TEST STATISTIC**

SOME VARIABLE WHOSE PROBABILITY
DISTRIBUTION IS KNOWN

A TEST OF SIGNIFICANCE

NULL HYPOTHESIS **VS** ALTERNATIVE HYPOTHESIS

THIS INVOLVES

1) COMPUTING A TEST STATISTIC

2) COMPUTE THE **PROBABILITY** IF THE
NULL HYPOTHESIS IS TRUE

A TEST OF SIGNIFICANCE

NULL HYPOTHESIS **VS** ALTERNATIVE HYPOTHESIS

THIS INVOLVES

1) COMPUTING A TEST STATISTIC

2) COMPUTE THE PROBABILITY IF THE
NULL HYPOTHESIS IS TRUE

**3) IF THE PROBABILITY IS TOO LOW,
REJECT THE NULL HYPOTHESIS, ELSE
ACCEPT IT**

A

5 MINS PER VISIT

B

3 MINS PER VISIT

SAMPLE MEANS
ARE DIFFERENT

NULL HYPOTHESIS

THE DIFFERENCE BETWEEN A AND B IS
DUE TO CHANCE

ALTERNATIVE HYPOTHESIS

THE DIFFERENCE BETWEEN A AND B IS REAL

STEP : 3 PERFORM A TEST OF SIGNIFICANCE

STEP : 3A

COMPUTE A TEST STATISTIC

WE'LL USE THE **Z-STATISTIC**

**Z-STATISTIC FOR THE NULL
HYPOTHESIS**

SAMPLE MEAN A

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

WE'LL USE THE **Z-STATISTIC**

**Z-STATISTIC FOR THE NULL
HYPOTHESIS**

$$Z = \frac{(\bar{X}_1 - \boxed{\bar{X}_2})}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

SAMPLE MEAN B

WE'LL USE THE **Z-STATISTIC**

**Z-STATISTIC FOR THE NULL
HYPOTHESIS**

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

STANDARD ERROR A

WE'LL USE THE **Z-STATISTIC**

**Z-STATISTIC FOR THE NULL
HYPOTHESIS**

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

STANDARD ERROR B

WE'LL USE THE **Z-STATISTIC**

**Z-STATISTIC FOR THE NULL
HYPOTHESIS**

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

**THIS IS WHY IT'S IMPORTANT TO CALCULATE
STANDARD ERROR TOO - NOT JUST THE MEANS**

A 5 MINS PER VISIT STANDARD ERROR : 0.5

B 3 MINS PER VISIT STANDARD ERROR : 0.4

$$Z = \frac{\overset{5}{(\bar{X}_1 - \bar{X}_2)}}{\sqrt{\underset{0.5}{\sigma_{x_1}^2} + \underset{0.4}{\sigma_{x_2}^2}}} = 3.12$$

COMPUTE THE VALUE FOR THE
NULL HYPOTHESIS

STEP : 3 PERFORM A TEST OF SIGNIFICANCE

STEP : 3B

**COMPUTE THE PROBABILITY IF
THE NULL HYPOTHESIS IS TRUE**

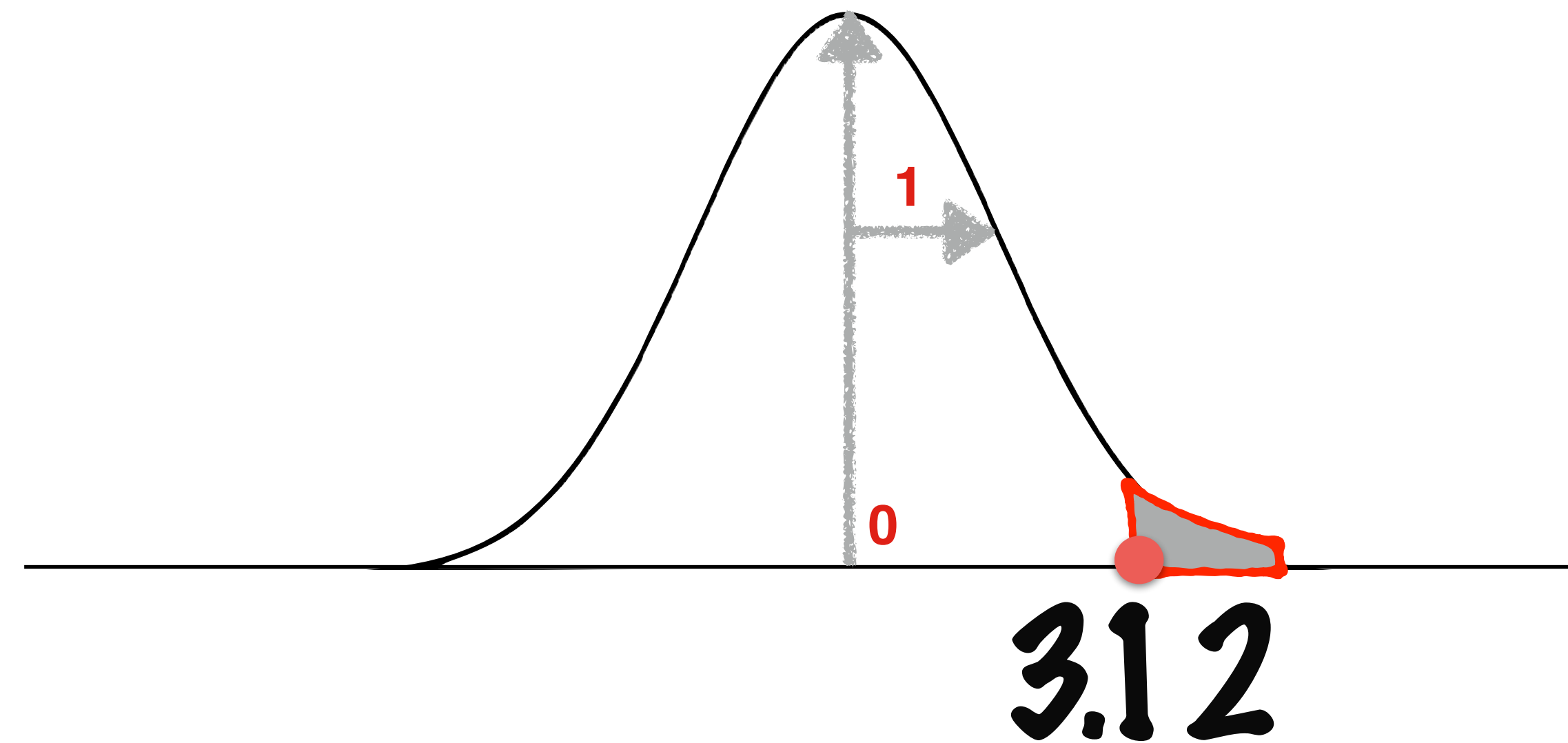
Z-STATISTIC FOR THE NULL HYPOTHESIS ≈ 3.12

P-VALUE

$$P(Z > 3.12)$$

= (AREA UNDER THE CURVE ABOVE 3.1)

= ?



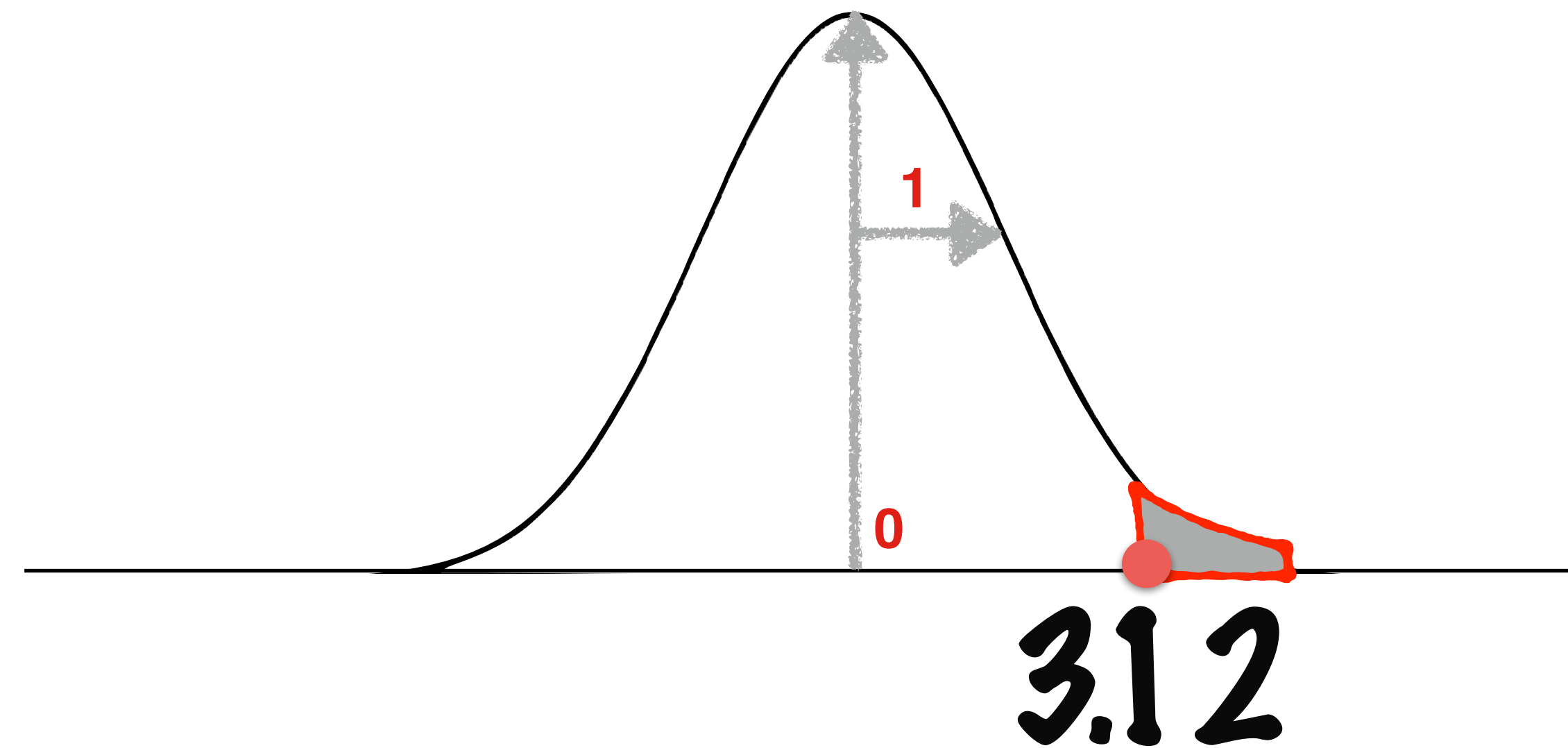
1 SIDED TEST (SINCE WE ARE CHECKING $A > B$ NOT JUST INEQUALITY)

Z-STATISTIC FOR THE NULL HYPOTHESIS = 3.12

P-VALUE

$P(Z > 3.12)$

In R the value can be
calculated in 2 ways



```
> 1 - (pnorm(3.12))  
[1] 0.0009042552
```

```
> pnorm(-3.12)  
[1] 0.0009042552
```

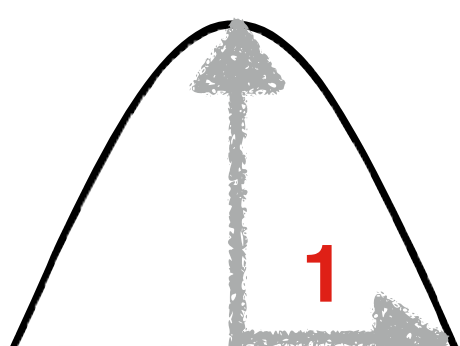
Z-STATISTIC FOR THE NULL HYPOTHESIS = 3.12

P-VALUE

$P(Z > 3.12)$

$= 0.00009$

We can even calculate the same using a Normal distribution table

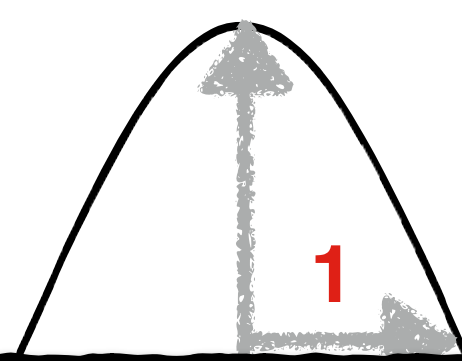


STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786

Z-STATISTIC FOR THE NULL HYPOTHESIS =3.12

P-VALUE



$P(Z > 3.12)$
 $=0.0009$

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

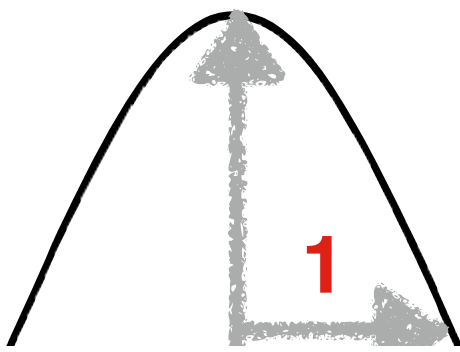
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
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-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
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-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639

Z-STATISTIC FOR THE NULL HYPOTHESIS = 3.12

P-VALUE

$$P(Z > 3.12)$$

$$= 0.0009$$



Values in this table
represent the area to the
left of the Z statistic

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
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-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
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-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
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-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
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-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786

STEP : 3 PERFORM A TEST OF SIGNIFICANCE

STEP : 3C

IF THE PROBABILITY IS TOO
LOW, REJECT THE NULL
HYPOTHESIS, ELSE ACCEPT IT

Z-STATISTIC FOR THE NULL HYPOTHESIS = 3.12

P-VALUE = 0.0009

PHEW!

SINCE THE P-VALUE IS TOO LOW
THE NULL HYPOTHESIS IS
REJECTED

So, the experiment showed
that the new website **brings
down** engagement, and this result is
statistically significant with **99%
confidence**

