

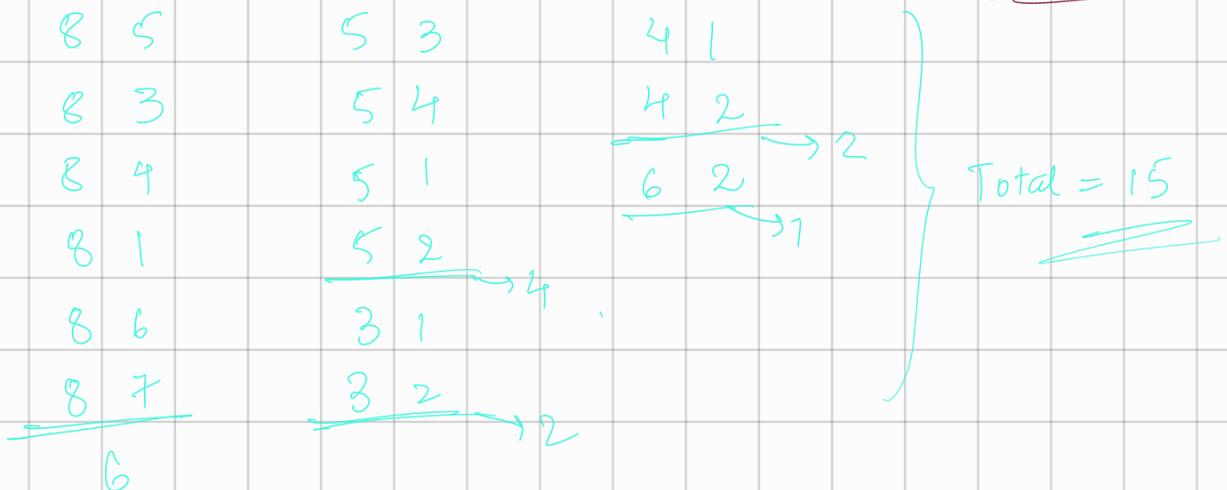
- Inversion Count → using merge sort
 - Reverse Pairs
 - Count of Smaller Elements after Self
 - Count Subarrays with 1s > 0s
 - Minimum Swaps to Sort I & II
- IMPORTANT

① Count Inversions / Inversion Count.

Eg: arr: 8 5 3 4 1 6 2
 idx: 0 1 2 3 4 5 6

Count no. of (i, j) such that $i < j$ and $\text{arr}[i] > \text{arr}[j]$

Eg: from above $(5, 8) \times \rightarrow \underline{\text{idx}[5] > \text{idx}[8]}$



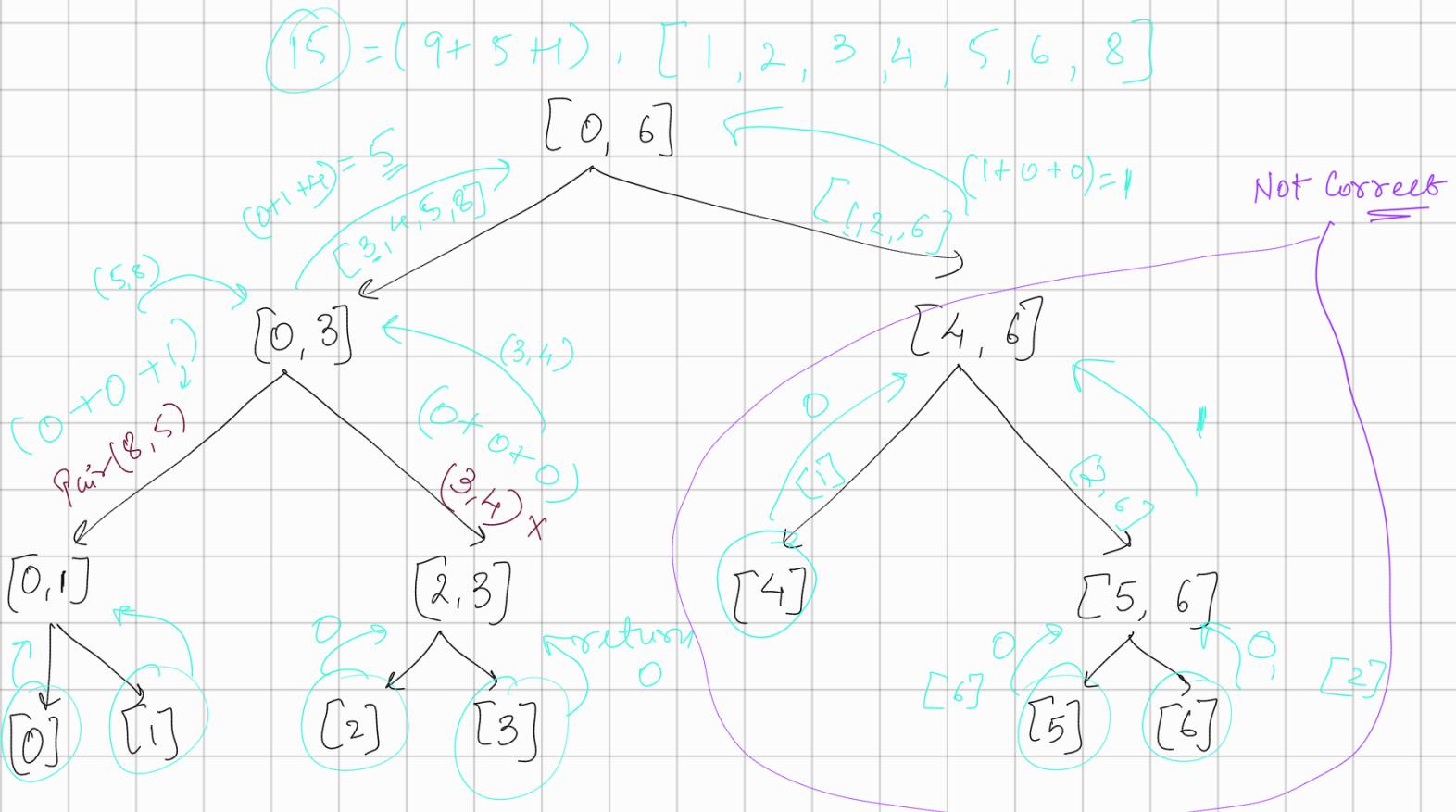
Brute force $\Rightarrow O(N^2)$

Optimized approaches

- Merge Sort $\rightarrow O(N \log N)$ Easy
- Fenwick Tree
- Policy Based Data Structure
- Self Balancing BST, Red - Black Tree

* Using Merge Sort

8 5 3 4 1 6 2
 0 1 2 3 4 5 6



at $[0, 3]$ \rightarrow [5, 8] [3, 4]

$low = 1$, $high = 0$

at $idx = 0, 0 \rightarrow$ we can tell no. of possible pairs
 arr: [5, 3] & next of 5 $>= 5$, so one more pair = [8, 3]

∴ This we can conclude at
 the first index of both arrays.

after this, $\because 3 < 5$, pt_2++

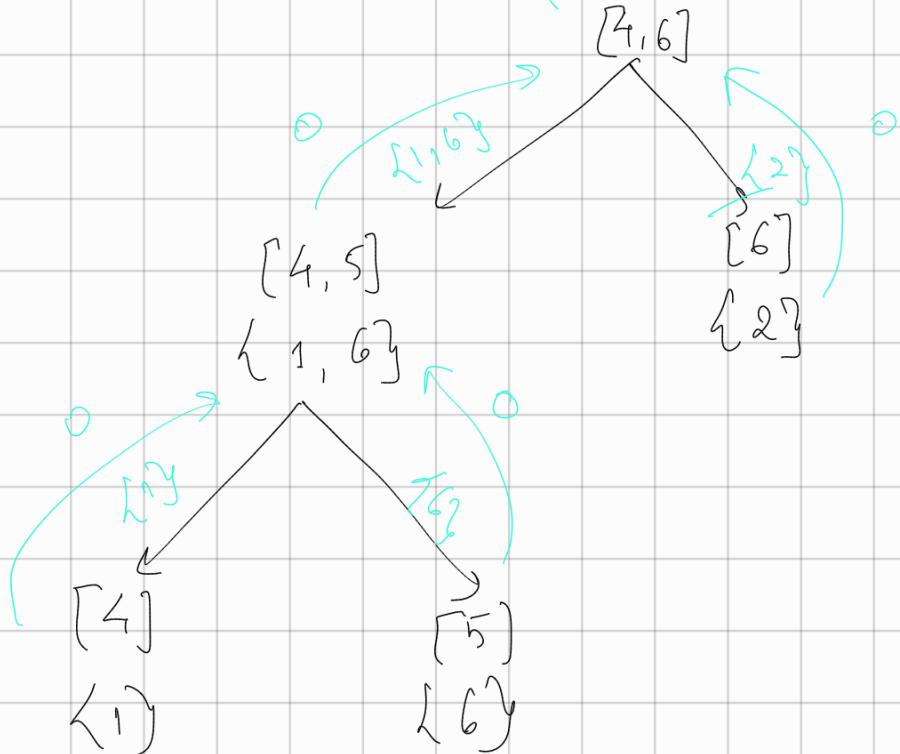
\Rightarrow [5, 8] [3, 4]
 \uparrow \uparrow
 $pt_{\alpha 1}$ $pt_{\alpha 2}$

\rightarrow 2 possible pairs. [5, 4] &
 \because next of 5 $>= 5$, [8, 4],

So, we get the correct count at $[0, 3]$ which is 4.
 But, we also have to add the $(\text{Inv} + \text{St})$ to the answer. So at $[0, 3]$, the answer will be:

$$(1 + 0 + 4) = \boxed{5}$$

$$(0+0+1)=1, \{1, 2, 6\}$$



∴ CODE is same as mergeSort

except when $(\text{num}[ptx2] > \text{num}[ptx1])$

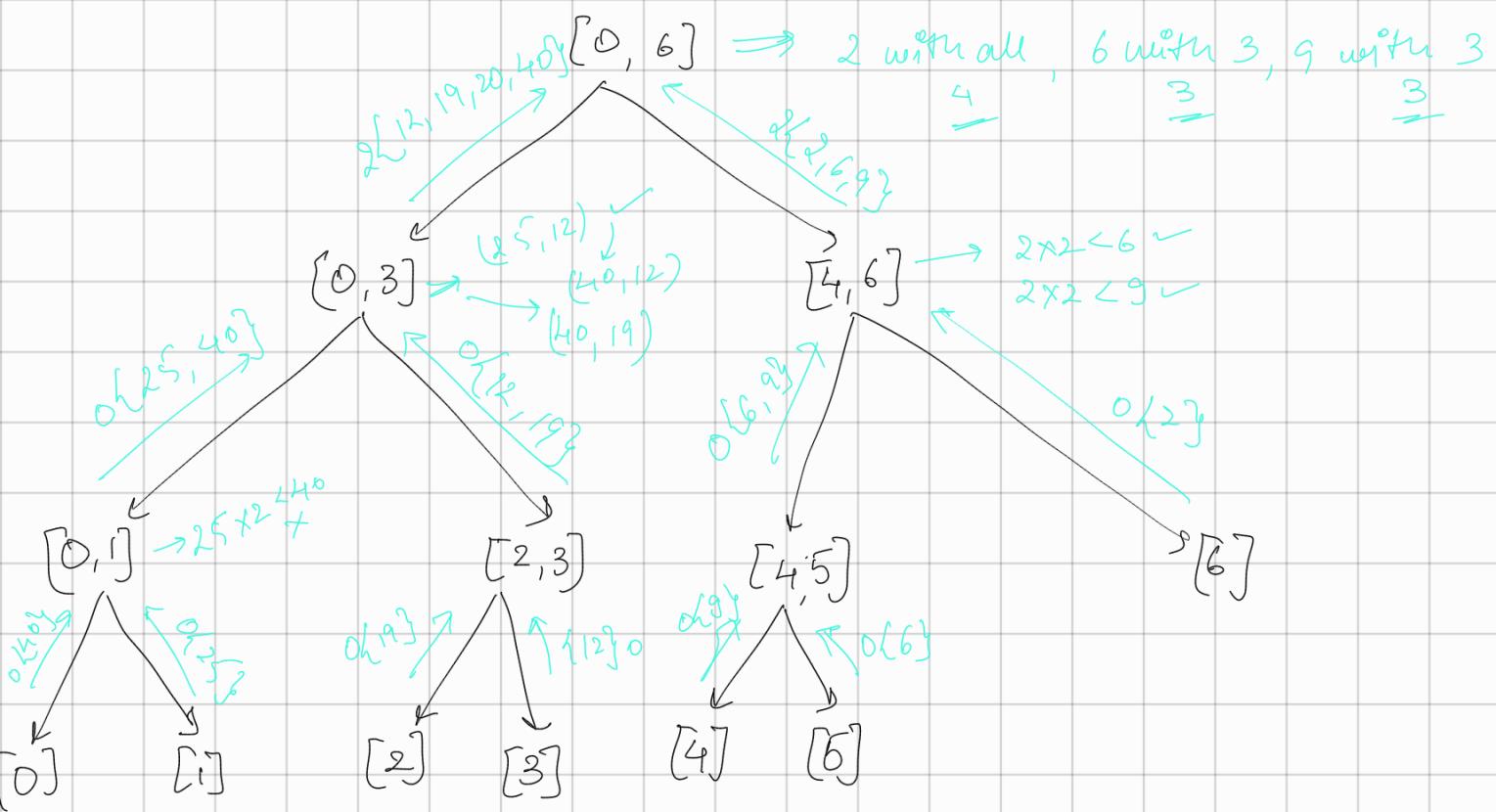
$\text{Count} += (\text{aix} - \text{ptx1} + \text{ld})$;

② Reverse Pairs

Count pairs where $i < j$ & $a[i] > 2 \times a[j]$

Here, find the pairs first & then merge

| | | | | | | | |
|-----|----|----|----|----|---|---|---|
| Eg: | 40 | 25 | 19 | 12 | 9 | 6 | 2 |
| idx | 0 | 1 | 2 | 3 | 4 | 5 | 6 |



- ③ Count winning streaks → GOOGLE. — Same as ↴
 ④ Count subarrays with more 0's than 1's (LeetCode 2031)

revision $O(2^n) \rightarrow O(N^3) \rightarrow O(N^2) \rightarrow N \log N$

3 loops

optimized

inv. count

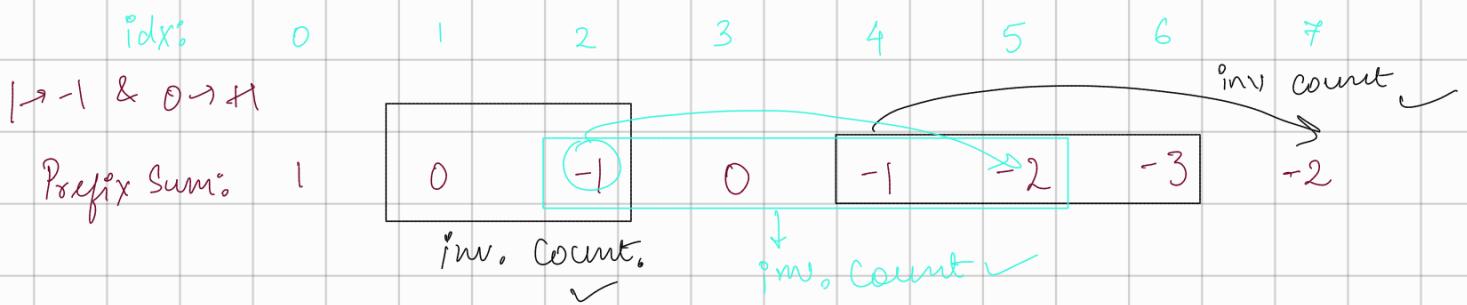
subarrays
(2 loops)

{prefix}

{prefix}

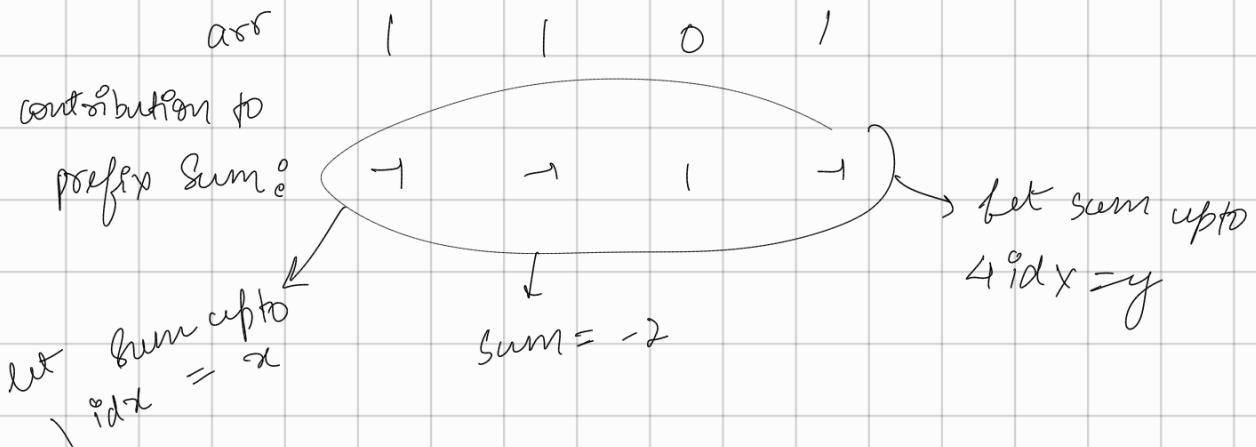
To check, $\text{count}(1) - \text{count}(0) > 0$

Eg: $[0 \quad | \quad 1 \quad 1 \quad 0 \quad | \quad 1 \quad 1 \quad 1 \quad 0]$



* If we can find an inversion count in prefix sum, the same index elements in the original array will be a valid subarray

Eg: Subarray idx [1, 4] from above



$$\text{then } y - x = -2$$

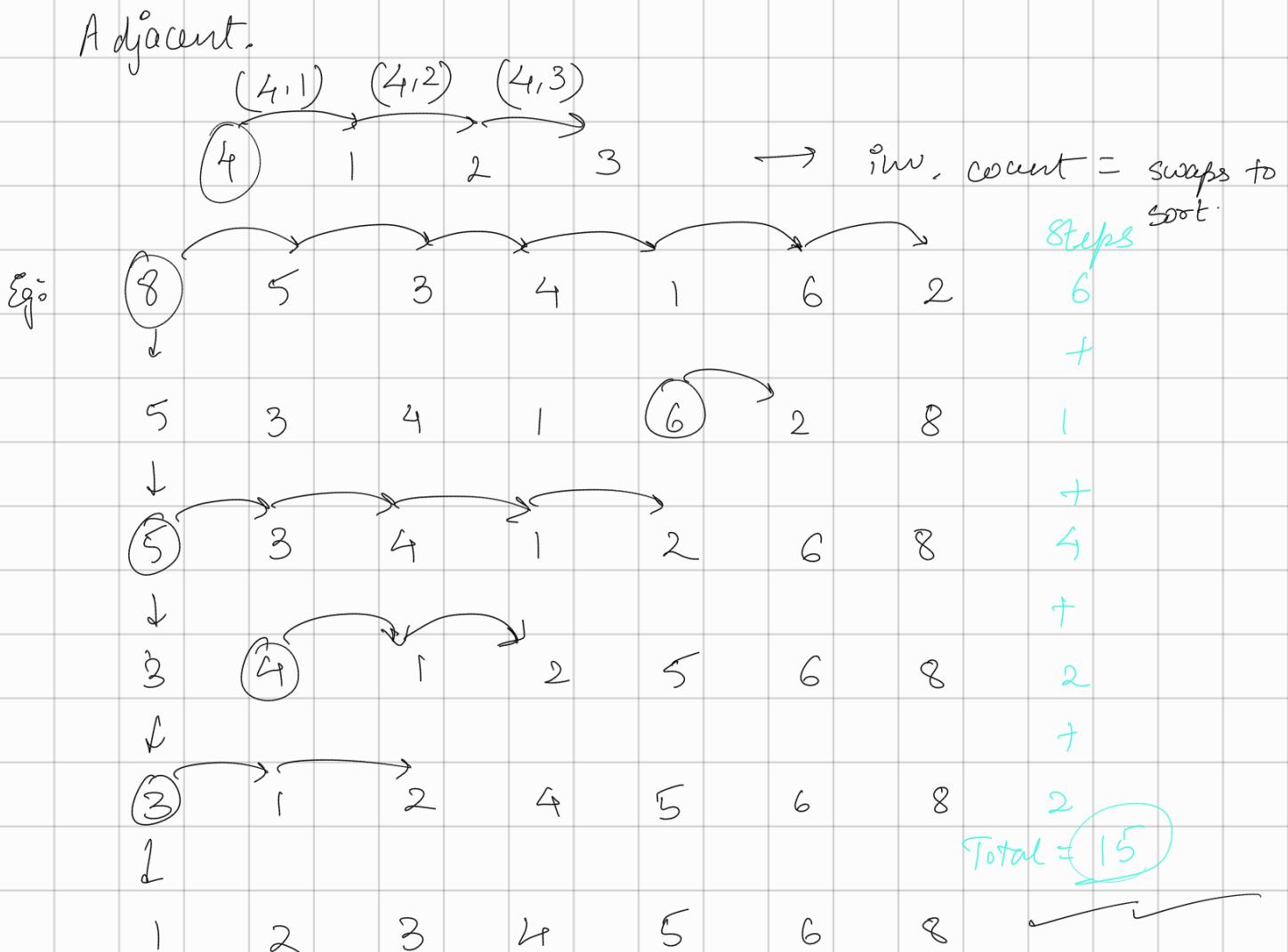
$$\text{or } y - x = 2$$

So, here whenever $y - x < 0$ / $y > x$
we can find a valid subarray

So, in essence, we only need to find the inversion count in the prefix sum array

1 0

(4) Minimum swaps To Sort. (Adjacent)



(5) Minimum swaps To Sort (Not Only Adjacent)

Eg: 8 5 3 4 1 6 2

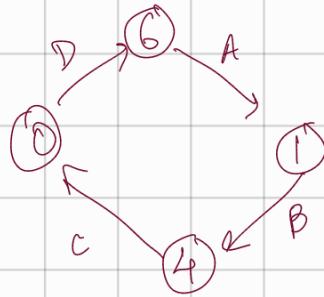
idx 0 1 2 3 4 5 6

Correct Position 1 2 3 4 5 6 8

On comparing given array & correct array

| Number | 8 | 5 | 3 | 4 | 1 | 6 | 2 | already in place → do not touch |
|-----------------|--------|--------|---|---|--------|---|--------|------------------------------------|
| is at | 0 | 1 | 2 | 3 | 4 | 5 | 6 | |
| Needs to be at: | 6 | 4 | 2 | 3 | 0 | 5 | 1 | |
| | Edge D | Edge B | | | Edge C | | Edge A | |

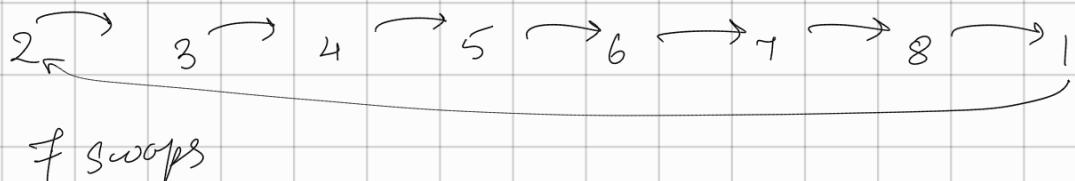
Graph for elements not at correct position



→ Edge A B C D
form a cycle

Here, total number of minimum swaps = no. of edges for cycle

Corner Case I



Corner case II

| elem | from | to |
|------|------|----|
| 40 | 0 | 3 |
| 50 | 1 | 4 |
| 30 | 2 | 2 |
| 60 | 3 | 5 |
| 20 | 4 | 1 |
| 70 | 5 | 6 |
| 10 | 6 | 0 |
| 80 | 7 | 80 |

40 : 0 → 3 ✓

50 : 1 → 4

30 : 2 → 2

60 : 3 → 5 ✓

20 : 4 → 1

70 : 5 → 6 ✓

10 : 6 → 0 ✓

80 : ? → ?

0 → 3
10 ↑ ↓ 60

6 ← 5
70

1 → 4
20
50

Cycle 1

4 swaps

Cycle 2

2 swaps

Total = 6 swaps

Consider all elements which are not at their correct place.

This is a $O(N \log_2 N + N)$ approach