

## ① Introduction

- Print Increasing
- Print Decreasing
- Print Increasing Decreasing
- Power - linear
- Power - logarithmic
- Power - zig zag
- Tower of Hanoi

## ② Recursion and arrays

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Principle of Mathematical Induction (e.g.  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ )

① Trivial case  $\equiv$  Base case

$$\sum_{i=0}^0 i = 0, \quad \sum_{i=1}^1 i = 1 \quad \checkmark$$

② Assume formula is valid for  $k$  \*

$$\sum_{i=1}^k i = \frac{k \times (k+1)}{2}$$

③ Prove for  $k+1$

$$\sum_{i=1}^{k+1} i = (k+1) \left[ \frac{(k+1)+1}{2} \right] ? \text{ Prove.}$$

If we can prove, then formula is correct.

Similarly, we expect from recursive function to solve upto k and we solve for K+1

## ① Print Increasing

### a) Expectation

void printInc (int n) : → 1, 2, 3, ... n

### b) Keep faith on recursion for smaller values.

void printInc (n-1) : → 1, 2, ..., n-1 ✓✓

Assume it is correct, do not question the little one

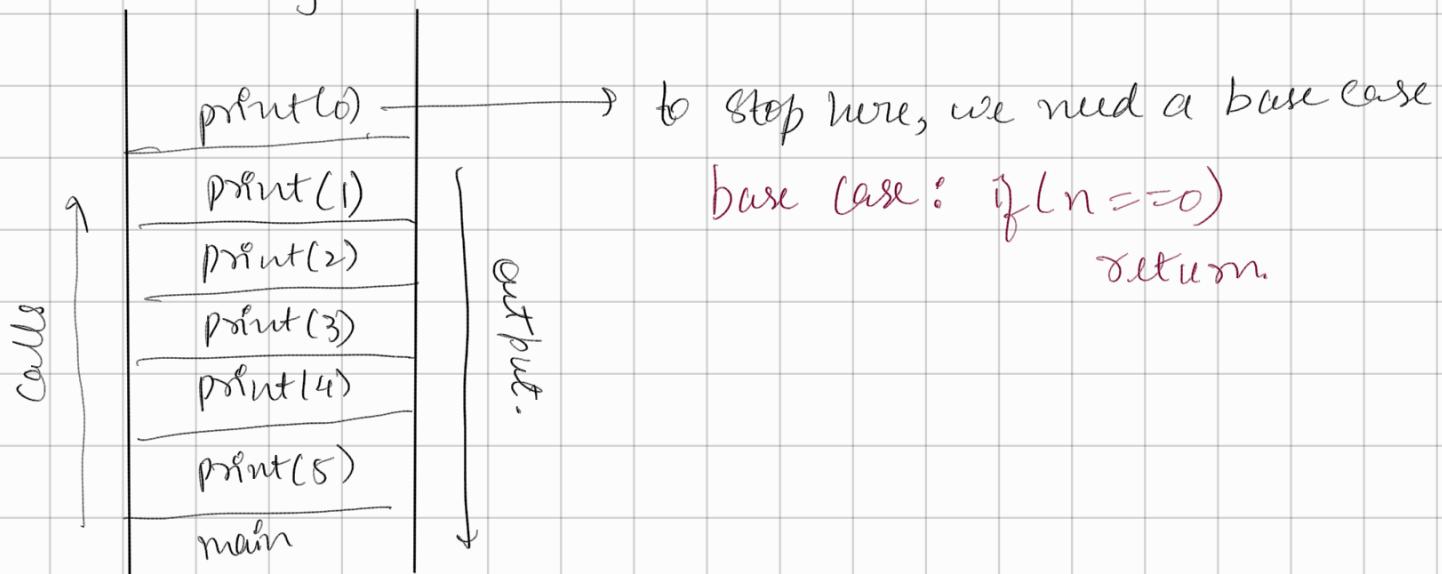
### c) Meet expectations with faith

Print (n).

### d) Base Case. (Eg, if $n=0$ , then don't print only return)

CODE (POST ORDER)

```
void printInc (int n) {
    if (n == 0) return;           // Base call
    printInc (n-1);             // Call for smaller problem
    Sysc (n);                  // Print current n
}
```



## function call stack

```

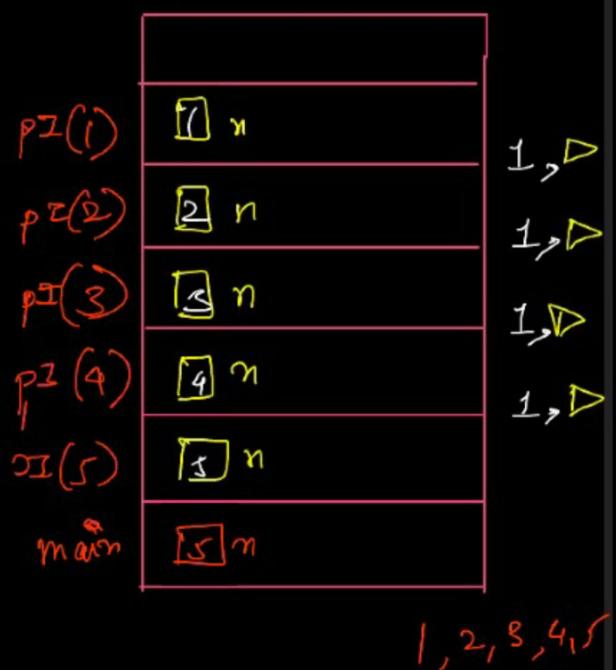
void printInc(int n) {
    Base case → ③ if (n == 0) return;
    Faith → ① printInc(n-1);
    Meeting expectation → ② System.out.println(n);
    { }
}

```

```

main() {
    printInc(5);
}

```



## ② Print Decreasing

expectation

a) printDecreasing(int n): n, n-1, ... 1

b) printDec(n-1): n-2, n-3, ..., 1 *faith*

## c) mult expectation & print n

print first  
then call n-1

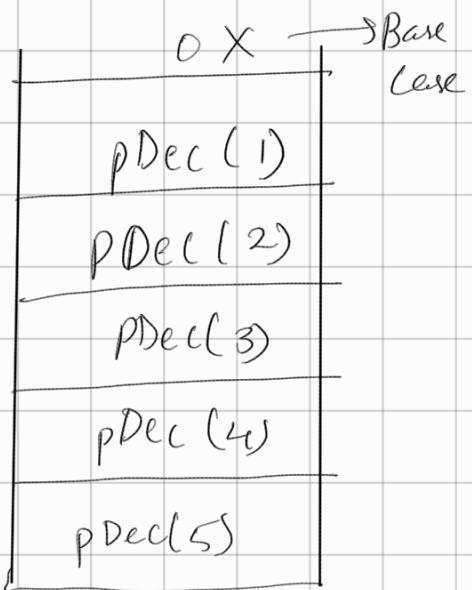
Base case: if (n == 0)  
return

CODE (PRE ORDER)

```

if (n == 0) return;
System.out.println(n);
printDec(n-1);

```



③ Point Increasing Decreasing

① Expectation:

$pdi(n) \Rightarrow n, n+1 \dots 1 \dots n-1, n$

② Faith:

$pdi(n-1) \Rightarrow n-1 \dots 1 \dots n-1$

③ meeting expectation

$Sys0(n)$  preorder  
 postorder

CODE:

```
if (n == 0) return;  
Sys0(n);  
pdi(n-1);  
Sys0(n);  
y
```

④ factorial.

a) Expectation:  $fac(n) \rightarrow 1 \times 2 \times \dots \times n$

b) faith :  $fac(n-1) \rightarrow 1 \times 2 \times \dots \times n-1$

c) work : return  $n \times fac(n-1)$

CODE:

```
int fact(int n){  
    if (n <= 1)  
        return 1;  
    return n * fact(n-1);  
}
```

⑤ Power linear ( $O(n)$ )

- (a) Expect:  $\text{pow}(x, n) \rightarrow x^n$
- (b) Faith:  $\text{pow}(x, n-1) \rightarrow x^{n-1}$
- (c) work:  $x \times \text{pow}(x, n-1);$

CODE:

```
pow(x, n) {
    if (n == 0)
        return 1;
    return x * pow(x, n-1);
}
```

⑥ Power logarithmic ( $O(\log_2 n)$ )

```
pow(x, n) {
    if (n == 0) return 1;
    int ans = pow(x, n/2);
    ans *= ans;
    if (n%2 == 1)
        ans *= x;
    return ans;
}
```

① expect:  $\text{pow}(x, n) \rightarrow x^n$

② faith:  $\text{pow}(x, n/2) \rightarrow x^{n/2}$   ~~$\star$~~  Important

③ work:  $ans = \text{pow}(x, n/2); ans *= ans;$   
 $\text{if } (n \% 2 == 1) ans *= x;$