German University in Cairo
Media Engineering and Technology
Optimization Algorithms

Final Project
FRANCHISE OPTIMISATION

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by

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1 Business Problem

1.1 Problem

Michael Ball wants to build an American cuisine franchise in Cairo. The problem is that demand in Cairo varies with the location and is unpredictable. He has a lot of options of places to serve the food and places to prepare the meals. He has to keep in mind that his choice will affect the profit of his franchise.

1.2 Objective

We divide Cairo into a set of areas where each area has a demand. Moreover, Cairo has a set of units that could be utilized as a kitchen or as a restaurant. For each unit we decide whether Micheal should rent or not and whether the chosen unit will be a restaurant or kitchen. Each kitchen will be serving meals to restaurants and restaurants will serve the customers.

1.3 Inputs

- The total yearly budget
- The cost per distance
- List of areas where each area has:
  - Position (x, y)
  - Demand
  - Fixed radius
- List of units where each unit has the following:
  - Position (x, y)
  - Yearly rent.
  - Maximum number of customers served if the unit is a restaurant.
  - Maximum number of meals produced if the unit is a kitchen.

    - Initial cost of starting a restaurant.
    - Initial cost of starting a kitchen.

- The optimisation ratio

  This is to adjust the model's sensitivity to cost and customers. Choosing $r = 0$ will dismiss the customers and only minimises the cost, which is to be avoided as it will not buy anything. Choosing higher values for r will give more priority for customers making the model more leaning towards maximising the number of customers served. Notice that for two solutions with the same number of customers, the model will always choose the solution with the lower cost as this has nothing to do with the value of r.

1.4 Outputs

Which units will be kitchens and which restaurants will be served by each kitchen.

1.5 Features

- Customers served should not exceed the capacity of the restaurant.
- Meals produced should not exceed the capacity of the kitchen.
- The total number of customers served by all units inside any area should not exceed the demand for this area.
- For each pair of units (i, j), if i is transporting meals to j , then i is a kitchen and j is a restaurant
- For all units that are being provided with meals from some kitchen k, the sum of their customers is equal to the meals produced by that kitchen.
- Each restaurant is only associated with one kitchen.

1.6 Key Metrics

Maximizing the number of customers served and minimizing the total cost.

2 Research Problem

2.1 Problem Analysis and Definition

Michael Ball wants to open a American-cuisine restaurant franchise in Cairo. He has a list of units in Cairo that could either be a restaurant or kitchen. This could be considered as an optimization problem. The literature solves multiple of similar problems where there is a chain of stores for a certain enterprise with an objective to maximize the profit and satisfy the customers. The profit is usually interpreted as the received income minus the cost spent by the company. In our case, the cost is due to the transportation between the kitchen and the restaurants. Moreover, the construction cost is also considered. We found what is called the location-allocation problem or Facility Location problem where there are known customer locations and a number of facilities need to be located. Furthermore, the optimizer must decide which facility will serve which customer. Usually, the target is to minimize the distance between the customers and their corresponding facilities while satisfying all customers. There are different variations that might appear in the different literature models. For example, the demand of the customer might be deterministic [1,2,3] or stochastic [1,4], the capacity of each facility might be finite[2] or infinite[1]. Moreover, the number of facilities of the chain might be defined[1] or decided by the optimizer[2,4]. The next section will present the models proposed by different research papers with their corresponding solutions.

2.2 Modeling Approaches and solutions

Location-allocation Problem

General location allocation model This paper introduces a general location allocation (LA) problem. In the LA model we try to find facilities locations to satisfy the demand of some customers given their positions. Moreover, we have to decide which customer should be served by which facility while taking into consideration the demand of each customer. This paper proposes multiple models. The simplest one has the following assumptions.

Assumptions

- The solution space is continuous
- Each customer's demand can supply by several facilities ignoring the opening

cost of new facility

- Facilities are incapacitated
- Parameters are deterministic supplying all the demand
- No relationship between new facilities

Structure

Objective Function Minimizing the total transportation cost, and the transportation price depends on the quantity

Constraints All the demand of the customers should be met

LA Model Each Customer Covered by Only One Facility In this model, the same assumptions are made however another assumption is made which is each customer is only served by one facility.

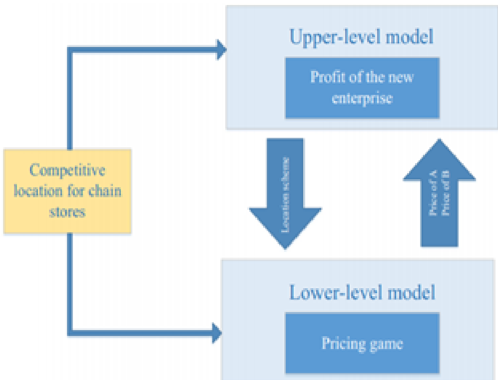
Structure: Objective Function: Same as the general model taking into consideration that the full demand of the customer is met by its facility. Constraints: 1. Each customer is served by only one facility. Solution: 1. Exact Solution: to find the optimal solution for the facilities position, we differentiate the total distance cost with respect to the x and y coordinates. However, this can only be done if the cost is only a function of the new facility. From the analysis of the problem we know that there is another variable in this problem which is: the list of facilities that serve each customer. If the number of customers and facilities are relatively small, using a modern computer we could try a facility-customer combination and find the optimal solution. However, in many industrial applications this is impossible due to the large number of variables. Therefore, the paper proposes another solution.

2. The Heuristic Approach: Heuristics are needed to quickly solve large problems and to provide good initial solutions for exact algorithms. There are two subproblems: given initial facility location, the local optimal allocation solution will be found. Then, given the found facility allocation, the local optimal location solution will be found. The algorithm will stop once the last optimal solution is the same as the new one.

- Second Paper: Optimization of competitive facility location for chain stores [2]

The paper aims to solve what is known as the facility location problem. This means , the author is trying to choose the best positions to open a chain of stores, moreover, it aims to find the price of each market in the chain depending on its location. The facility location problem could be divided into competitive location problems and non-competitive location problems. The competition takes the location and price of the competitors at each location into account in the model, while the non competitor doesn't.

i. Model: 1. Assumptions: The paper continues to make some assumptions which are: a. There are certain candidate points with known costs for the new enterprise location. b. It is assumed that the demand at each location is known. c. The existing enterprise (competitors) and the new one will sell the same products in their stores, but only with different selling prices and travel cost. Lastly, the same product might have different prices in different locations. 2. Structure: A bi-level model is introduced:



The upper-level model is given the prices of the market in chain A (the candidate) and the prices of competitor chain B, then using this data as a static data, the model tries to choose the best locations of the stores in the chain. Next, the model gives these locations to the low-level model, where, based on this data, the model will choose the prices of each store in A and the predicted prices of store B. Obviously, one can not simply choose the prices of a competitor's enterprise; however, it could be predicted from what is known as the principle of Nash equilibrium.

a. Upper Model:

i. Objective Function: Maximize the total benefit of the enterprise.

ii. Constraints:

1. The number of stores open must be within budget. 2. A point is considered to satisfy a certain demand only if it is opened by the enterprise. 3. A store can't

offer service beyond capacity.

b. Lower Model:

i. Objective Function: Minimize the distance of unit benefit (benefit divided by sales volume) of both our enterprise and the competitive one which obeys the principle of Nash equilibrium.

ii. Constraints: 1. The sales price should be higher than the marginal price. 2. A point is considered to satisfy a certain demand only if it is opened by the enterprise. 3. A store can't offer service beyond capacity.

ii. Solution:

The paper chose the Tabu Search Algorithm to solve the bi-level model problem. It does this by following a set of steps: 1. Set an initial price and generate an initial feasible solution. 2. This optimal solution will be the input for the low-level model. 3. Calculate the objective function according to the Nash equilibrium price. 4. Find in the neighborhood of the initial prices the feasible solution that results in the local minimum cost function (the cost function of the low level model). 5. Set the best neighbour as the current solution. 6. Repeat until the stopping criteria is met.

- Third Paper: The uncapacitated facility location problem[3]

Model: Given n possible sites and demands at m locations, determine the optimal location of facilities to fulfil all demands such that the total cost of establishing the facilities and fulfilling the demands (distribution cost) is minimized. Objective Function: Minimizing the total cost including serving the customer and the construction cost. Constraints: 1. The facilities should be established in certain predefined locations. 2. Customers can only be served by established facilities.

Solution: The paper chose the Tabu Search Algorithm to solve the bi-level model problem. The proposed algorithm has two subproblems: 1. It finds the optimal locations of the facilities. 2. Using the giving locations from the first subproblem it finds the optimal allocation. So the algorithm is as follows : a. use any heuristic to get an initial solution (one can also use a random starting solution). b. Compute its total cost, and store it as the best solution so far. c. Using this solution, generate a set of neighboring y's. d. For every location, find the optimal allocation by the argument in section 2, and find the resulting total cost. e. Then the best improving location (or best non-tabu if no improving location solution is found) is selected and its associated attribute is stored in the tabu list. f. This procedure is repeated and controlled by tabu search until a predetermined number of iterations have been performed.

- Best Approach:

Mostly the inputs to the optimizers of the papers in the literature is the location and the demand of each customer and the optimizers try to minimize the distance between them. However, in our case we are trying to minimize the distance between the restaurants and the kitchens. So we will consider the facilities as our kitchens and the customers as our restaurants. However, we have to decide the position of the restaurants such that enough restaurants are present in each area to satisfy the demand of the customers. Moreover, similar to what is present in [1] we add a constraint that each restaurant could only be served by one kitchen.

It was found that these problems are NP problems. It is hard to find the global optimal solution, therefore, most researchers go with metaheuristics techniques. For example genetic algorithms and tabu search methods. These algorithms give satisfactory results and converge to a local minimum throughout the iterations.

2.3 References

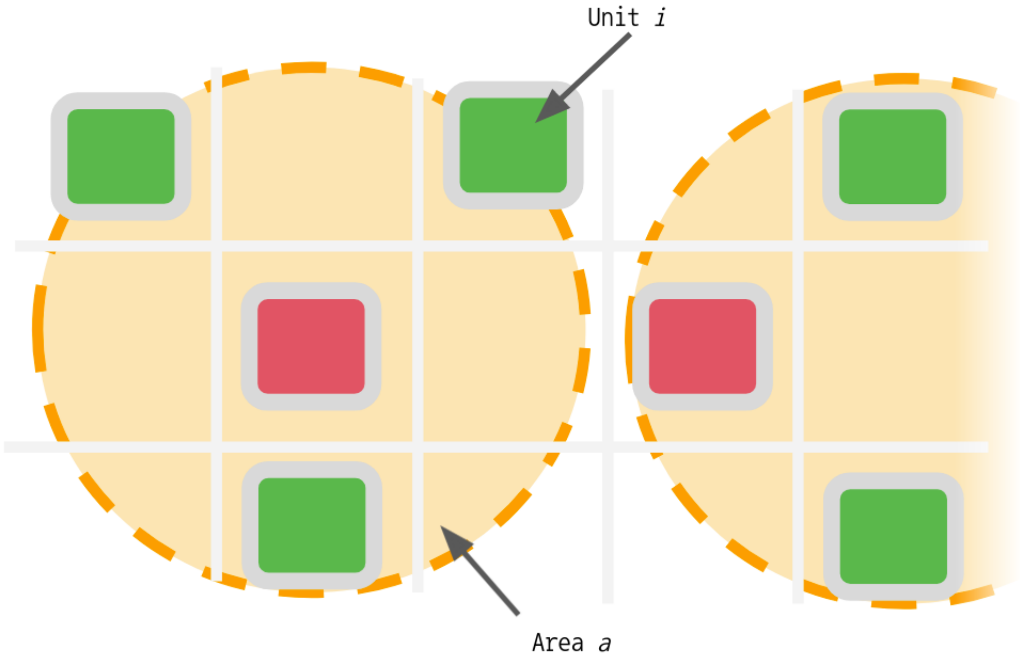
[1] Azarmand, Z., & Neishabouri, E. (2009). Location Allocation Problem. Springer Link. [2] Shan, W., Yan, Q., Chen, C., Zhang, M., Yao, B., & Fu, X. (2017). Optimization of competitive facility location for chain stores. Springer Science+Business Media, 187–205. [3] Al-Sultan, K. S., & Al-Fawzan, M. A. (1999). A tabu search approach to the uncapacitated facility location problem. Annals of Operations Research, 91–103. [4] Liu, B. L. (2008). Facility Location Problem. In Theory and Practice of Uncertain Programming (pp. 157–165). Springer-Verlag.

3 Modeling

3.1 Describing the Model

We have a number of units, denoted with i each belonging to a different area a. The core of the problem is to find which units to buy, as well as which units are going to be a *kitchen* and which units are going to be a *restaurant*. Our goal is to increase the number of customers the franchises serves, while keeping the operating cost at minimum.

Each unit i has some properties which are:



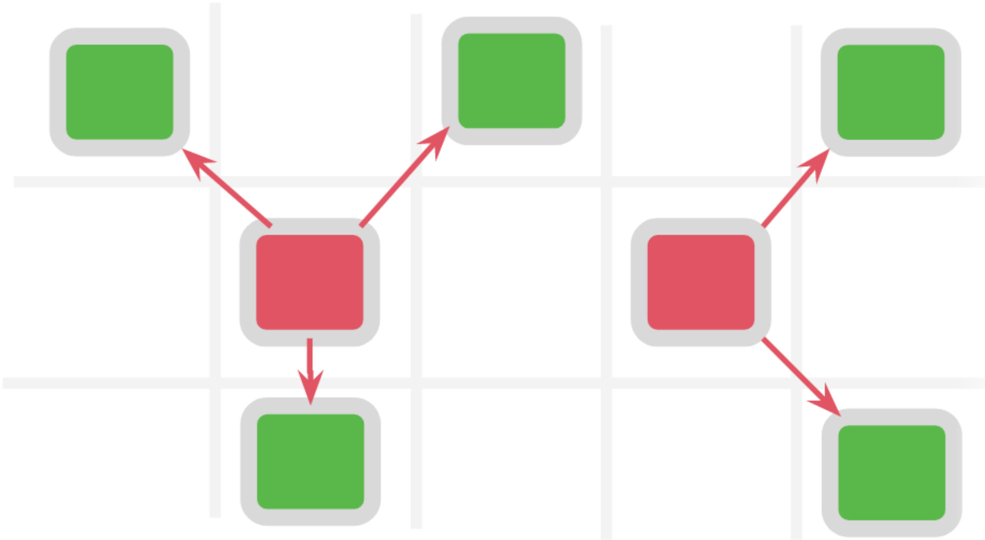
- Yearly rent, $rent_i$.
- Maximum number of customers served if the unit is a restaurant, $capacity_i^R$.
- Maximum number of meals produces if the unit is a kitchen, $capacity_i^K$.
- Initial cost of starting a restaurant, $initial_i^R$.
- Initial cost of starting a kitchen, $initial_i^K$.

Each area a has a certain demand within a fixed radius, d_a

We can then define the following decision variables:

$$\begin{aligned} R_i &\Rightarrow \text{Whether unit } i \text{ will be a restaurant or not} \\ K_i &\Rightarrow \text{Whether unit } i \text{ will be a kitchen or not} \\ Y_{i,a} &\Rightarrow \text{The number of customers served by restaurant } i \text{ that will go towards the demand of area } a \end{aligned}$$

A formal definition will be provided in the next section. Each kitchen needs to provide nearby restaurants with meals so they can serve their customers, however each transport will incur a cost. We can then define the following decision variables:



$T_{i,j} \Rightarrow$ The number of meals kitchen i will be a supplying restaurant j
$Z_{i,j} \Rightarrow$ Whether kitchen i will be a supplying restaurant j

3.2 Mathematical Model

Inputs

- The total yearly budget, $budget$
- The cost per distance, cpd
- The optimisation ratio, r.

  This is to adjust the model's sensitivity to cost and customers. Choosing $r = 0$ will dismiss the customers and only minimises the cost, which is to be avoided as it will not buy anything. Choosing higher values for r will give more priority for customers making the model more leaning towards maximising the number of customers served. Notice that for two solutions with the same number of customers, the model will always choose the solution with the lower cost as this has nothing to do with the value of r.

- The set of all units available with their properties, $i \in I$.

- The set of all areas with their demand, $a \in A$.

Calculated Variables

We can construct some variables from the inputs to enable us to state our objective functions and constraints.

$$\theta_{i,a} \Rightarrow \text{Whether unit } i \text{ is in area } a$$
$$\Delta_{i,j} \Rightarrow \text{The euclidean distance between unit } i \text{ and } j$$

Optimisation Problem

We can define the total cost as:

$$cost = \sum_{i}(K_i * (rent_i + initial_i^K) + R_i * (rent_i + initial_i^R)) + (T_{cost} * 365)$$

$$T_{cost} = \sum_{i,j} Z_{i,j} * \Delta_{i,j} * cpd$$

Next, we can define the total number of customers as:

$$customers = \sum_{i,a} Y_{i,a} * 365$$

Finally, we can construct our objective functions and constraints:

$$
\begin{aligned}
\max \quad & \mathit{customers} * r - \mathit{cost} \\
\text{subject to} \quad & R_i + K_i \leqslant 1 \quad \text{for each } i \\
& \text{Each unit } i \text{ cannot be both a restaurant and a kitchen.} \\
& T_{i,j} \leqslant M * Z_{i,j} \quad \text{for each } i, j \\
& Z_{i,j} \leqslant M * T_{i,j} \quad \text{for each } i, j \\
& \text{Linking } Z_{i,j} \text{ with } T_{i,j}.\ Z_{i,j} \text{ will be zero when } Z_{i,j} \text{ is zero and one when} \\
& T_{i,j} \text{ is non-zero.} \\
& \sum_a Y_{i,a} \leqslant d_a \quad \text{for all } i \\
& \text{The total number of customers served by restaurants in area } a, \\
& \text{don't exceed } a\text{'s demand.} \\
& \sum_a Y_{j,a} = \sum_i T_{i,j} \quad \text{for all } j \\
& \text{The total number of customers served in all areas equals} \\
& \text{the total number of meals supplied by all kitchens to its restaurants.} \\
& Y_{i,a} \leqslant R_i * \mathit{capacity}_i^R \quad \text{for all } i, a \\
& \text{For all areas and restaurants, the total number of customers} \\
& \text{served by a restaurant } i\text{, in area } a \text{ should not exceed restaurant } i \text{ capacity.} \\
& \sum_j T_{i,j} \leqslant \mathit{capacity}_i^K \quad \text{for all } i \\
& \text{The total number of meals supplied by any kitchen} \\
& \text{to it's restaurants shouldn't exceed the kitchen's capacity.} \\
& \sum_i T_{i,j} \leqslant \mathit{capacity}_j^R \quad \text{for all } j \\
& \text{The total number of meals delivered to any restaurant by any kitchen} \\
& \text{ shouldn't exceed the restaurant's capacity.} \\
& \sum_i T_{i,j} \leqslant K_i * M \quad \text{for all } j \\
& \text{Only kitchens can supply.} \\
& \sum_i T_{i,j} \leqslant R_j * M \quad \text{for all } i \\
& \text{Only restaurants can receive.}
\end{aligned}
$$