Macroeconomics 2 Presentation
Part 2 Equations

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1 Consequences of the Model

1.1 The Taylor Principle Reconsidered: Equilibria Are Determinate Even with a Fixed Interest Rate

Equation 31

$$i_t = \phi_\pi \pi_t + \phi_x x_t + j_t \tag{31}$$

The equation (31) sets the nominal interest i_t in a Taylor rule fashion. j_t is just a constant.

Equation 32

$$\mathbf{z}_t = \mathbf{A}\mathbb{E}_t\left[z_{t+1}\right] + \mathbf{b}a_t \tag{32}$$

The system of Proposition 2 can be represented as equation (32), where:

- $\mathbf{z}_t := (x_t, \pi_t)'$
- $\mathbf{A}$: see equation (33)
- $\mathbf{b} = \frac{-\sigma}{1+\sigma(\phi_x+\kappa\phi_\pi)}(1, \kappa)'$
- $a_t := j_t - r_t^n$

Equation 33

$$\mathbf{A} = \frac{1}{1+\sigma(\phi_x+\kappa\phi_\pi)}\begin{pmatrix} M & \sigma(1-\beta^f\phi_\pi) \\ \kappa M & \beta^f(1+\sigma\phi_x)+\kappa\sigma \end{pmatrix} \tag{33}$$

where:

- $\beta^f := \beta M_f$

Equation 34 - Proposition 3

Equilibrium Determinacy with Behavioral Agents: There is a determinate equilibrium (i.e., all of **A**'s eigenvalues are less than 1 in modulus) if and only if

$$\phi_\pi + \frac{(1-\beta M^f)}{\kappa}\phi_x + \frac{(1-\beta M^f)(1-M)}{\kappa\sigma} > 1 \tag{34}$$

Equation 35

In particular, when monetary policy is passive (i.e., when $\phi_\pi = \phi_x = 0$), we have a determinate equilibrium if and only if bounded rationality is strong enough, in the sense that:

$$\frac{(1-\beta M^f)(1-M)}{\kappa\sigma} > 1 \tag{35}$$

This is **Strong enough bounded rationality condition**. Condition (35) does not hold in the traditional model, where M = 1. The condition means that agents are boundedly rational enough (i.e., M is sufficiently less than 1) and the firm-level pricing or cognitive frictions are large enough.

Equation 36

Permanent Interest Rate Peg. Take the (admittedly extreme) case of a permanent peg. Then, in the traditional model, there is always a continuum of bounded equilibria, technically, because the matrix A has a root greater than 1 (in modulus) when M = 1.In the behavioral model, we do get a definite non-explosive equilibrium:

$$\mathbf{z}_t = \mathbb{E}_t\left[\sum_{\tau\geq t}\mathbf{A}^{\tau-t}\mathbf{b}a_\tau\right] \tag{36}$$

Cochrane (2018) made the point that we'd expect an economy such as Japan's to be quite volatile, if the ZLB is expected to last forever: conceivably, the economy could jump from one equilibrium to the next at each period. This is a problem for the rational model, which is solved if agents are behavioral enough, so that (35) holds.

Equation 37

Long-Lasting Interest Rate Peg. The economy is still very volatile (in the rational model) in the less extreme case of a peg lasting for a long but finite duration. We can get:

$$\mathbf{z}_0(T) = \left(\mathbf{I} + \mathbf{A}_{ZLB} + ... + \mathbf{A}_{ZLB}^{T-1}\right)\underline{\mathbf{b}} + \mathbf{A}_{ZLB}^{T}\mathbb{E}_0\left[\mathbf{z}_T\right] \tag{37}$$

where:

- $\mathbf{A}_{ZLB}$ is the value of matrix **A** in equation (33) when $\phi_\pi = \phi_x = j = 0$ in the Taylor rule.

The system (32) is, at the ZLB(t≤T): $\mathbf{z}_t = \mathbb{E}_t \mathbf{A}_{ZLB} \mathbf{z}_{t+1} + \underline{\mathbf{b}}$ with $\underline{\mathbf{b}} := (1, \kappa)\, \sigma \underline{r}$ where $\underline{r} \leq 0$ is the real interest rate that prevails during the ZLB. Iterating forward, we can get equation (37).

1.2 The ZLB Is Less Costly with Behavioral Agents

Equation 38 - Propostion 4

Call $x_0\,(T)$ the output gap at time 0, given the ZLB will lasts for T periods. In the traditional rational case, we obtain an unboundedly intense recession as the length of the ZLB increases: $\lim_{n \to \infty} x_0\,(T) = -\infty$.This also holds when myopia is mild, i.e., (35) fails. However, suppose cognitive myopia is strong enough, i.e., (35) holds. Then, we obtain a boundedly intense recession:

$$\lim_{T \to \infty} x_0\,(T) = \frac{\sigma\left(1 - \beta M^f\right)}{(1 - M)\left(1 - \beta M^f\right) - \kappa\sigma} \underline{r} < 0 \tag{38}$$

Myopia has to be stronger when agents are highly sensitive to the interest rate (high σ) and price flexibility is high (high κ). High price flexibility makes the system very reactive, and a high myopia is useful to counterbalance that.

2 Implications for Monetary Policy

2.1 Welfare with Behavioral Agents and the Central Bank's Objective

Equation 39 - Lemma 3

The welfare loss from inflation and output gap is:

$$W = -K\mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{2} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right) + W_{-} \tag{39}$$

where:

- $\vartheta = \frac{\overline{\kappa}}{\epsilon}$
- $K = u_c c\,(\gamma + \phi)\,(\epsilon / \overline{\kappa})$
- W_{-} is a constant (made explicit in equation (200) in the online Appendix).
- $\overline{\kappa}$ is the Phillips curve coefficient with rational firms (derived in Proposition 2).
- ϵ is the elasticity of demand.

2.2 Optimal Policy with Complex Trade-Offs: Reaction to a Cost-Push Shock

Equation 40 & 41 - Proposition 5

Optimal Policy with Commitment: Suboptimality of Price-Level Targeting: To fight a time-0 cost-push shock, the optimal commitment policy entails, at time $t \geq 0$:

$$\pi_t = \frac{-\vartheta}{\kappa}\left(x_t - M^f x_{t-1}\mathbf{1}_{t>0}\right) \tag{40}$$

so that the (log) price level ($p_t = \sum_{\tau=0}^{t} \pi_\tau$, normalizing the initial log price level to p_{-1}=0) satisfies:

$$p_t = \frac{-\vartheta}{\kappa}\left(x_t + \left(1 - M^f\right)\sum_{\tau=0}^{t-1} x_\tau\right) \tag{41}$$

With rational firms ($M^f = 1$) , the optimal policy involves "price-level targeting". it ensures that the price level mean- reverts to a fixed target ($p_t = (-\nu/\kappa)x_t \rightarrow 0$ in the long run). However, with behavioral firms, the price level is higher (even in the long run) after a positive cost-push shock: the optimal policy does not seek to bring the price level back to baseline.

Equation 42 - Propostion 6

Optimal Discretionary Policy: The optimal discretionary policy entails:

$$\pi_t = \frac{-\vartheta}{\kappa} x_t \tag{42}$$

so that on the equilibrium path: $i_t = K\nu_t + r_t^n$. where:

- $K = \frac{\kappa\sigma^{-1}(1-M\rho_\nu)+\vartheta\rho_\nu}{\kappa^2+\vartheta(1-\beta M^f\rho_\nu)}$

For persistent shocks (ρ_ν>0), the optimal policy is less aggressive (K is lower) when firms are more behavioral.

3 Implications for Fiscal Policy

3.1 Cognitive Discounting Generates a Failure of Ricardian Equivalence

Equation 43

The public debt evolves as:

$$B_{t+1} = B_t + Rd_t \tag{43}$$

where:

- B_t is the real value of government debt in period t, before period- t taxes.
- $d_t := \mathcal{T}_t + (r/R)B_t$. d_t is the budget deficit (after the payment of the interest rate on debt) in period t.
- $\mathcal{T}_t$ is the lump-sum transfer given by the government to the agent (so that $-\mathcal{T}_t$ is a tax).

No-Ponzi condition is the usual one, $\lim_{t\to\infty} \beta^t B_t = 0$, which here takes the form $\lim_{t\to\infty} \beta^t \left(\sum_{s=0}^{t-1} d_s\right) = 0$. Hence, debt does not necessarily mean-revert, and can follow a random walk.

Equation 44 & 45 - Propostion 7

Discounted Euler Equation with Sensitivity to Budget Deficits: Because agents are not Ricardian, budget deficits temporarily increase economic activity. The IS curve (24) becomes

$$x_t = M\mathbb{E}_t [x_{t+1}] + b_d d_t - \sigma \left(i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^{n0}\right) \tag{44}$$

where:

- r_t^{n0} is the "pure" natural rate with zero deficits (derived in (23)).
- d_t is the budget deficit.
- $b_d = \frac{\phi r R(1-\bar(m)}{(\phi+\gamma)(R-\bar(m)}$ is the sensitivity to deficits. When agents are rational, $b_d = 0$, but with behavioral agents, $b_d>0$.

In the sequel, we will write this equation by saying that the behavioral IS curve (25) holds, but with the following modified natural rate, which captures the stimulative action of deficits:

$$r_t^n = r_t^{n0} + \frac{b_d}{\sigma} d_t \tag{45}$$

Hence, bounded rationality gives both a discounted IS curve and an impact of fiscal policy: $b_d>0$. Deficit-financed (lump-sum) tax cuts have a "stimulative" impact on the economy.

3.2 Consequences for Fiscal Policy

Equation 46 - Lemma 4

First Best: When there are shocks to the natural rate of interest, the first best is achieved if and only if at all dates:

$$i_t = r_t^n \equiv r_t^{n0} + \frac{b_d}{\sigma} d_t \tag{46}$$

Where:

- r_t^{n0} is the "pure" natural rate of interest given in (23) and is independent of fiscal and monetary policy.

This condition pins down the optimal sum of monetary and fiscal policy (i.e., the value of $i_t - (b_d/\sigma)d_t$), but not their precise values, as the two policies are perfect substitutes.

Equation 47

With behavioral agents, there is an easy first best policy:

$$\text{First best at the ZLB: } i_t = 0 \text{ and deficit: } d_t = \frac{-\sigma}{b_d r_t^{n0}} \tag{47}$$

i.e., fiscal policy runs deficits to stimulate demand.

Equation 48

Suppose that the government purchases at time 0 an amount G_0, financed by a deficit $d_0 = G_0$, and the central bank does not change the nominal rate at time 0 (we keep future deficits at 0 for t ¿ 0, so that debt is permanently higher). Then the fiscal multiplier is:

$$\frac{dY_0}{dG_0} = 1 + b_d \tag{48}$$

reflecting the fact that government spending has a "direct" effect of increasing GDP one-for-one, and then an "indirect" effect of making people feel richer.