Macroeconomics 2 Presentation
Equations description of
"A Behavioral New Keynesian Model" by Xavier Gabaix

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This file aims to summarise and give some all the equations of the presented article. Its goal is to serve as a complement to the presentation of May 3, 2024 in the Macroeconomics 2 class. Although we use the same numerisation of the equations as in the article, we do present them in our own structure, following the oral presentation divided in three main sections. The sections of the article are however given as subsections, to provide some additional information for those wanting to use this document as a complementary helper to the reading of the article.

Contents

1 A Behavioral Model

Let's ignore the first two equations since they are the same as (28) and (29), that will be explained later.

1.1 Introduction

Equation 1

$$x_t = M \cdot \mathbb{E}t \left[xt + 1 - \sigma(i_t - \mathbb{E}t \left[\pi t + 1 \right] - r_t^n) \right] \tag{1}$$

Equation 2

$$\pi_t = \beta \cdot M^f \mathbb{E}t \left[\pi t + 1 \right] + \kappa \cdot x_t \tag{2}$$

1.2 Basic Setup and the Household's Problem

Equation 3

$$U = \mathbb{E}t \left[\sum t = 0^\infty \beta^t \left(\frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \right] \tag{3}$$

Equation (3) is just the flow utility of the Household, with :

- β the discount factor
- c_t the consumption of the houshold at time t
- N_t the work of the household at time t
- γ determines the concavity of the utility function with respect to the consumption, i.e. the importance of consumption in the utility function
- ϕ determines the concavity of the utility function with respect to work, i.e. the importance of work in the utility function

Equation 4

$$k_{t+1} = (1 + r_t)(k_t - c_t + y_t) \tag{4}$$

Equation (4) is the law of motion of the real financial wealth of the household, where :

- k_t is the real financial welath of the household at time t
- r_t is the real interest rate
- w_t is the real wage
- y_t is the agent's real income, defined as $y_t = w_t \cdot N_t + y_t^f$, with y_t^f the profit income (or the income from firms) at time t

Equation 5

$$\boldsymbol{X}t+1=\boldsymbol{G}^{\boldsymbol{X}}\left(\boldsymbol{X}t,\boldsymbol{\epsilon}_{t+1}\right) \tag{5}$$

Equation (5) describes the evolution of macroeconomic variables, where :

- $\boldsymbol{X}_t$ is the state vector, including several macroeconomic variables of time t, like ζ_t the aggregate TFP, and the announced actions in monetary and fiscal policy
- $\boldsymbol{G}^X$ the equilibrium transition function, i.e. the function that gives the macroeconomic variables at time $t+1$ from the macroeconomic variables at the previous period
- ϵ_t is the innovation in the economy at time t, with $\mathbb{E}t\left[\epsilon t+1\right]=0$, that depends on the equilibrium policies of the agent and of the government

Equation 6

$$k_{t+1}=G^k(c_t,N_t,k_t,\boldsymbol{X}t):=(1+\bar{r}+\hat{r}(\boldsymbol{X}t))(k_t+\bar{y}+\hat{y}(N_t,\boldsymbol{X}t)-ct) \tag{6}$$

Equation (6) is the application of the consideration of a set of macroeconomic variables on the law of motion of real financial wealth k_t, where :

- $\bar{r}$ is the steady state value of the real interest rate, that does not depend on time
- $\hat{r}(\boldsymbol{X}t)$ is the value of the deviation from the steady state of the real interest rate, that depends on the state vector $\boldsymbol{X}t$ at time t
- $\bar{y}$ is the steady state value of the agent's real income, that does not depend on time
- $\hat{y}(N_t,\boldsymbol{X}_t)$ is the deviation from the steady state of the agent's real income, that depends on the number of hours worked at time t and on the state vector at time t
- c_t is the aggregate consumption level at time t of the agent

Equation 7

$$\boldsymbol{X}t+1=\boldsymbol{\Gamma}\boldsymbol{X}t+\boldsymbol{\epsilon}_{t+1} \tag{7}$$

Equation (7) describes the linear version of the equilibrium transition function, it is the linearization of the law of motion, where :

- Γ is a squared matrix that multiplies the state vector
- $\boldsymbol{X}_t$ is the state vector at time t
- ϵ_t is the innovation shock

Equation 8 (Assumption 1)

$$\boldsymbol{X}t+1=\bar{m}\cdot\boldsymbol{G}^{\boldsymbol{X}}(\boldsymbol{X}t,\boldsymbol{\epsilon}_{t+1}) \tag{8}$$

Equation (8) describes the Cognitive Discounting of the State Vector, i.e. the perception by behavioural agents of the law of motion of the macroeconomic variables, where :

- $\bar{m}\in[0n1]$ is the cognitive discount factor measuring the attention to the future

Equation 9

$$\boldsymbol{X}t+1=\bar{m}(\boldsymbol{\Gamma}\boldsymbol{X}t+\boldsymbol{\epsilon}_{t+1}) \tag{9}$$

Equation (9) is just the linearized version of the perception by behavioral agents of the law of motion of the state vector.

Equation 10

$$\mathbb{E}t^{BR}\left[\boldsymbol{X}t+k\right]=\bar{m}^k\mathbb{E}t\left[\boldsymbol{X}t+k\right] \tag{10}$$

Equation (10) defines the expectation of behavioral agents in function of the rational perception of the law of motion of the state vector, where :

- $k\geq 0$ a time period in discrete context
- $\mathbb{E}t^{BR}\left[\boldsymbol{X}t+k\right]$ is the expected value of the state vector at time $t+k$ by behavioral agents (or subjective/behavioral expectation operator)
- $\bar{m}^k$ is the cognitive discounting effect at period $t+k$
- $\mathbb{E}t\left[\boldsymbol{X}t+k\right]$ is the rational expectation of the state vector at time $t+k$

Equation 11 (Lemma 1)

$$\mathbb{E}t^{BR}\left[z\left(\boldsymbol{X}t+k\right)\right]=\bar{m}^k\mathbb{E}t\left[z\left(\boldsymbol{X}t+k\right)\right] \tag{11}$$

Equation (11) defines in the general case the behavioral expectation operator, for any function of the state vector, where :

- $k\geq 0$ a time period in discrete context
- $z(\cdot)$ is a function, such that $z(0)=0$
- $\mathbb{E}t^{BR}\left[z\left(\boldsymbol{X}t+k\right)\right]$ is the expected value of the image of the state vector by the function $z(\cdot)$ at time $t+k$ by behavioural agents
- $\bar{m}^k$ is the cognitive discounting effect at period $t+k$
- $\mathbb{E}t\left[z\left(\boldsymbol{X}t+k\right)\right]$ is the rational expectation of the image of the state vector by the function $z(\cdot)$ at time $t+k$

Equation 12

$$\mathbb{E}t^{BR}\left[\bar{r}+\hat{r}\left(\boldsymbol{X}t+k\right)\right]=\bar{r}+\bar{m}^{k}\mathbb{E}t\left[\hat{r}(\boldsymbol{X}t+k)\right] \tag{12}$$

Equation (12) is an example of the Lemma 1 applied to the interest rate, where :

- $k \geq 0$ a time period in discrete context
- $\bar{r}$ the steady state level of the real interest rate, that does not depend on time,
- $\hat{r}(\boldsymbol{X}_{t+k})$ is the equilibrium transition function defining the value of the deviation from the steady state of the real interest rate in the function of the state vector at time $t+k$
- $\bar{r}+\hat{r}(\boldsymbol{X}t) = rt(\boldsymbol{X}_t)$ is the value of the real interest rate at time t
- $\mathbb{E}t^{BR}\left[\bar{r}+\hat{r}(\boldsymbol{X}t+k)\right]$ is the expected value of the real interest at time $t+k$ by behavioural agents
- $\mathbb{E}t\left[\hat{r}(\boldsymbol{X}t+k)\right]$ is the rational expectation of the value of the deviation of the real interest rate from the steady state at time $t+k$

1.3 The Firm's problem

Equation 13

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}} \tag{13}$$

Equation (13) describes the aggregate price level, where :

- P_t is the aggregate price level of the economy at time t
- $i \in [0,1]$ is the firm index
- ε is the elasticity of substitution between goods

Equation 14

$$v^0(q_{i\tau},\mu_\tau,c_\tau) := \left(e^{q_{i\tau}}-(1-\tau_f)e^{-\mu_\tau}\right)e^{-\varepsilon q_{i\tau}}c_\tau \tag{14}$$

Equation (14) describes the profit of the firm before the lump sum tax of the government, where :

- v is the real profit of the firm
- $q_{i\tau} = \ln\left(\frac{P_{i\tau}}{P_\tau}\right) = p_{i\tau} - p_\tau$ is the real log price at time τ
- $\tau_f = \frac{1}{\varepsilon}$ it the corrective wage subsidy from the government, funded by the lump sum tax

- $\mu_\tau = \zeta_t - \ln(\omega_t)$ is the labor wedge, which is zero at efficiency
- ε is the elasticity of subsititution between goods
- c_τ is the aggregate level of consumption

Equation 15

$$v\left(q_{it}, \boldsymbol{X}\tau\right) := v^0\left(q_{it} - \Pi(\boldsymbol{X}\tau), \mu(\boldsymbol{X}\tau), c(\boldsymbol{X}_\tau)\right) \tag{15}$$

Equation 15 describes the flow profit of the firm in function of the real log price and of the extended macro state vector, where :

- $q_{it} = \ln\left(\frac{P_{it}}{P_t}\right) = p_{it} - p_t$ is the real log price
- $\boldsymbol{X}\tau = (\boldsymbol{X}^{\mathcal{M}}\tau, \Pi_\tau)$ is the extended macro state vector, with $\boldsymbol{X}^{\mathcal{M}\tau}$ the vector of macro variables, including $\zeta\tau$ and possible announcements about future policy
- $\Pi\left(\boldsymbol{X}\tau\right) := p\tau - p_t = \pi_{t+1} + ... + \pi_\tau$ is the inflation between times t and τ
- $q_{it} - \Pi(\boldsymbol{X}\tau) = qi\tau$ is the real price of the firm if they didn't change its price between t and τ
- $\mu\left(\boldsymbol{X}_\tau\right)$ is the labor wedge in function of the extended state vector at time t
- $c(\boldsymbol{X}_\tau)$ is the aggregate consumption level in function of the extended state vector at time t

Equation 16

$$\max_{\mathrm{q}it} \mathbb{E}t \sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t} \frac{c\left(\boldsymbol{X_\tau}\right)^{-\gamma}}{\left(\boldsymbol{X_t}\right)^{-\gamma}} v\left(q_{it}, \boldsymbol{X}\tau\right) \tag{16}$$

Equation (16) describes the maximisation program of the firm given that they have a Calvo-like probability of θ of being able to change their price at each period, where :

- t is the initial period
- τ is the time period index
- q_{it} is the real log price of the firm at time t
- β is the discount factor
- θ is the Calvo like probability that the firm can change its price at any period
- $\frac{c\left(\boldsymbol{X}\tau^{-\gamma}\right)}{c(\boldsymbol{X}t^{-\gamma})}$ is the adjustment in the stochastic discount factor/pricing kernel between times t and τ

Equation 17

$$\max_{q_{it}} \mathbb{E}t^{BR}\left[\sum \tau = t^{\infty} (\beta\theta)^{\tau - t} \frac{c\left(\boldsymbol{X}\tau^{-\gamma}\right)}{c\left(\boldsymbol{X}t^{-\gamma}\right)} v\left(q_{it,\boldsymbol{X}_\tau}\right)\right] \tag{17}$$

Equation 17 describes the maximisation program of the behavioral firm, i.e. such that it is maximisation the behavioral expectation operator of the flow profit, where :

- $\mathbb{E}_t^{BR}$ is the behavioral/subjective expected value operator
- t is the initial period
- τ is the time period index
- q_{it} is the real log price of the firm at time t
- β is the discount factor
- θ is the Calvo like probability that the firm can change its price at any period
- $\frac{c\left(\boldsymbol{X}\tau^{-\gamma}\right)}{c\left(\boldsymbol{X}t^{-\gamma}\right)}$ is the adjustment in the stochastic discount factor between times t and τ and it is approximately 1 when linearised around deterministic steady state.

1.4 Model solution

Equation 18

$$\hat{c}t = \mathbb{E}t\left[\hat{c}t + 1 - \frac{1}{\gamma R}\hat{r}t\right] \tag{18}$$

Equation (18) is the linearized version of the Euler equation obtained from the presented model. It is also called the investment-savings (IS) curve, where :

- $\hat{c}_t$ is the value of the deviation from the steady state of the aggregate consumption at time t
- $\mathbb{E}t\left[\hat{c}t + 1\right]$ is the rational expectation of the value of the deviation from the steady state of the aggregate consumption at time $t + 1$
- γ is the factor of the importance of consumption
- $R := 1 + \bar{r}$ is defined from the real interest rate at the steady state (cf. page 7 of the article)

Equation 19

$$\hat{c}t = M \cdot \mathbb{E}t\left[\hat{c}t+1\right] - \sigma\hat{r}t \tag{19}$$

Equation (19) is the application of Lemma 1 (equation (11)) on the previous Euler equation, i.e. a cognitively discounted aggregate Euler equation, where :

- M is the macro parameter of attention, such that $M = \bar{m}$ here
- $\sigma = \frac{1}{\gamma R}$

Equation 20

$$N^{\phi}t = \omega t c_t^{\gamma} \tag{20}$$

Equation (20) the result of the static First Order Condition for labor supply, where :

- N_t is the quantity of labor provided at time t
- ω_t is the real wage at time t
- c_t is the aggregate quantity of consumption at time t
- γ is the consumption importance in the utility

Equation 21

$$\hat{c}t^n = \frac{1+\phi}{\gamma+\phi}\zeta t \tag{21}$$

Equation (21) is the equation giving us the consumption in a natural economy where there are no frictions in prices i.e. flexible price economy. where :

- $\hat{c}_t^n$ is the flexible price/natural economy consumption
- γ is the the elasticity of substitution between goods.
- ϕ is the Frisch elasticity of labour
- ζ_t is the Total Factor prodcutivity

  We calculate this natural economy consumption the same way we calculated the flexible price economy consumption using the log-linearised version of the natural economy price, first-order condition for labour supply, and market clearing condition for consumption and income.

is the flexible price/natural economy consumption

Equation 22

$$\hat{c}^n t = M \cdot \mathbb{E}t\left[\hat{c}^n t + 1\right] - \sigma \hat{r}^n t \tag{22}$$

Equation (22) is the application of Lemma 1 (equation (11)) on the previous Euler equation, i.e. a cognitively discounted aggregate Euler equation for the natural economy where there are no price frictions. Here :

- M is the macro parameter of attention, such that $M = \bar{m}$ here
- $\sigma = \frac{1}{\gamma R}$
- $\hat{r}_t^n$ is the real interest rate for the natural economy

Equation 23

$$r^{n0} t = \bar{r} + \frac{1+\phi}{\sigma(\gamma+\phi)}\left(M \cdot \mathbb{E}t\left[\zeta_{t+1}\right] - \zeta_t\right) \tag{23}$$

Equation (23) is the equation for the pure natural rate of interest: this is the interest rate that prevails in an economy without pricing frictions, and undisturbed by government policy (in particular, budget deficits). In this equation :

- r_t^{n0} is pure natural rate of interest
- γ is the the elasticity of substitution between goods.
- ϕ is the Frisch elasticity of labour
- $\sigma = \frac{1}{\gamma R}$
- M is the macro parameter of attention, such that $M = \bar{m}$ here
- ζ_t and ζ_{t+1} is the Total Factor productivity in time t and $t+1$ respectively

This pure natural interest rate is calculated by isolating the $\hat{r}_t^n$ in Equation (22) and then replacing $\hat{c}t^n = \frac{1+\phi}{\gamma+\phi}\zeta t$, $\hat{c}t + 1^n = \frac{1+\phi}{\gamma+\phi}\zeta t + 1$ and $\hat{r}_t^n = r_t^n - \bar{r}$ and by assuming $r_t^n = r_t^{n0}$ which holds when there are no budget deficits.

Equation 24

$$x_t = M \cdot \mathbb{E}t\left[xt + 1\right] - \sigma(\hat{r}t - \hat{r}^n t) \tag{24}$$

Equation (24) is the equation for the behavioural discounted Euler equation where :

- x_t is the output gap at period t
- x_{t+1} is the output gap at period $t+1$
- M is the macro parameter of attention, such that $M = \bar{m}$ here

- $\sigma = \frac{1}{\gamma R}$
- $\hat{r}_t$ is the deviation of the real interest rate from the steady state.
- $\hat{r}^n_t$ is the deviation of the real interest rate from the steady state for the natural economy.

This is calculated by subtracting the expression $\hat{c}t = M \cdot \mathbb{E}t\,[\hat{c}t+1] - \sigma\hat{r}t$ i.e. Equation (19) and $\hat{c}^n t = M \cdot \mathbb{E}t\,[\hat{c}^n t+1] - \sigma\hat{r}^n t$ i.e. Equation (22)

Equation 25

$$x_t = M \cdot \mathbb{E}t\,[xt+1] - \sigma(i_t - \mathbb{E}t\,[\pi t+1] - r^n_t) \tag{25}$$

Equation (25) is the behavioural discounted Euler equation after replacing the equation for Fisher equation where $\hat{r}_t = r_t - \bar{r} = (i_t - \mathbb{E}t\,[\pi t+1] - r^n_t)$

- i_t is the nominal interest rate
- $\mathbb{E}t\,[\pi t+1]$ is the expected inflation in the future
- $M = \bar{m}$
- $\sigma = \frac{1}{\gamma R}$

Equation 26

$$x_t = -\sigma \sum_{k\geq 0} M \cdot \mathbb{E}t\left[\hat{r}t+k - \hat{r}^n_{t+k}\right] \tag{26}$$

Equation (26) iteratively using Equation (24) reduces to this equation. It indicates that changes in the interest rate in the 1000^{th} period will have a discounted impact on the output gap. Therefore, the effect of changes in interest rates on the output gap diminishes over time, with changes in the 1000^{th} period having a smaller impact compared to changes in the 1^{st} period.

Equation 27

$$p^*t = pt + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta\bar{m})^k \cdot \mathbb{E}t\,[\pi t+1+...+\pi t+k - \mu_{t+k}] \tag{27}$$

Equation (27) ..., where :

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Equation 28 - Proposition 2, the first equation

$$x_t = M \cdot \mathbb{E}t\,[xt+1] - \sigma(i_t - \mathbb{E}t\,[\pi t+1] - r^n_t) \tag{28}$$

Equation (28) is the Behavioural IS Curve. Essentially we are representing the Euler equation in terms of the output gap and also using the Fisher Equation.

Equation 29 - Proposition 2, second equation

$$\pi_t = \beta \cdot M^f \mathbb{E}t\left[\pi t + 1\right] + \kappa \cdot x_t \tag{29}$$

Equation (29) is the Behavioural Phillips curve where:

- M^f is the aggregate attention parameter for firms and $M^f = \bar{m}\left(\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}}(1-\theta)\right)$
- $\kappa = \widetilde{\kappa}$ is the slope of the Phillips curve and $\widetilde{\kappa} = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\gamma + \phi)$ which is the slope obtained from fully rational firms.

Equation 30

$$\begin{cases} M = \bar{m} \\ \sigma = \frac{1}{\gamma R} \\ M^f = \bar{m}\left(\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}}(1-\theta)\right) \end{cases} \tag{30}$$

Equation (30) defines the parameters in Proposition 2.

2 Consequences of the Model

$$A = B \tag{31}$$

This is a description

3 Behavioral Enrichments to the Model

3.1 Term Structure of Consumer Attention

Equation 49

$$\begin{aligned} k_{t+1} &= \mathbf{G}^{k,BR}(c_t, N_t, k_t, \mathbf{X}_t) \\ &:= (1 + \bar{r} + \hat{r}^{BR}(\mathbf{X}_t))(k_t + \bar{y} + \hat{y}^{BR}(N_t, \mathbf{X}_t) - c_t) \end{aligned} \tag{49}$$

Equation (49) translates the perception of the law of motion of the personal wealth k, where :

- k_{t+1} is the personal wealth at time $t + 1$
- $\mathbf{G}^{k,BR}$ is the transition function of personal wealth under bounded rationality, i.e. the perception of the transition function under bounded rationality
- c_t is the consumption at time t
- N_t is work at time t
- $\bar{r}$ the interest rate at the steady state

- $\hat{r}^{BR}$ the perception under bounded rationality of the deviation of the interest rate from the steady state
- $\mathbf{X}_t$ the state vector
- $\bar{y}$ the income at the steady state
- $\hat{y}^{BR}(N_t, \mathbf{X}_t)$ the perception under bounded rationality of the deviation of the income from the steady state

Equation 50

$$\begin{cases} \hat{r}^{BR} = m_r \cdot \hat{r}(\mathbf{X}_t) \\ \hat{y}^{BR}(N_t, \mathbf{X}_t) = m_y \cdot \hat{y}(\mathbf{X}_t) + \omega(\mathbf{X}_t)(N_t - N_t\mathbf{X}_t) \end{cases} \tag{50}$$

The equation (50) defines the perceived values under bounded rationality of the interest rate and of the income, where :

- $\hat{r}^{BR}$ is the perception of the deviation of the interest rate from the steady state under bounded rationality
- $m_r \in [0, 1]$ is the attention discount factor for the interest rate
- $\hat{r}_t(\mathbf{X}_t)$ is the objective value of the deviation of the interest rate from the steady state
- $\hat{y}^{BR}(N_t, \mathbf{X}_t)$ is the perception of the deviation of the personal from the steady state under bounded rationality, which is function of the number of hours worked, and of the macroeconomics state vector at time t
- $m_y \in [0, 1]$ is the attention discount factor for the personal income
- $\omega(\mathbf{X}_t)$ is the real wage, which is function of the macroeconomic state vector
- $N(\mathbf{X}_t)$ is the aggregate labor supply

The perceptions of the laws of motion are thus further refined by differentiating the attention level in function of the economic variable considered. Taken together, equations (49) and (50) can be taken as more comple budget constraints, that are to take into account in the maximisation process of the utility function. Note that both discount factors m_r and m_y apply to the period t, meaning they are contemporaneous discount factor.

Equation 51, Lemma 5 (Term Structure of Attention)

$$\begin{cases} \mathbb{E}_t^{BR}\left[\hat{r}^{BR}(\mathbf{X}_{t+k})\right] = m_r \bar{m}^k \mathbb{E}_t\left[\hat{r}(\mathbf{X}_{t+k})\right] \\ \mathbb{E}_t^{BR}\left[\hat{y}^{BR}(\mathbf{X}_{t+k})\right] = m_r \bar{m}^k \mathbb{E}_t\left[\hat{y}(\mathbf{X}_{t+k})\right] \end{cases} \tag{51}$$

Equation (51) is the application of Lemma 1 (equation (11)) on the perceived values just defined in equations (49) and (50), where :

- $\mathbb{E}_t^{BR}$ is the operator of the expected value under bounded rationality, i.e. the behavioral expectation defined in equation (11)

- $\hat{y}^{BR}(N_t, \mathbf{X}_t)$ is the perception of the deviation of the personal from the steady state under bounded rationality, which is function of the number of hours worked, and of the macroeconomics state vector at time t

- $\hat{r}^{BR}$ is the perception of the deviation of the interest rate from the steady state under bounded rationality

- $m_r \in [0, 1]$ is the attention discount factor for the interest rate

- $m_y \in [0, 1]$ is the attention discount factor for the personal income

- $\bar{m}$ is the general discount factor applicable for all future variables

Equation (51) is the main consequence of the refinment previously done in equations (49) and (50) : not only are the agents generally myopic to the future, but they are also not fully attentive to present variables, depending on the variables. It can indeed seem plausible to say that they pay more attention to the income than to the interest rate, even when they have the information about the current period. This equation, coupled with the next one, argues that one is not fully rational even when they have acces to the infomraiont of their present income and personal Euler equation, because they have this present attention deficiency to income m_y.

Equation 52

$$\hat{c}_t = \mathbb{E}_t \left[\sum_{\tau \geq t} \frac{\bar{m}^{\tau - t}}{R^{\tau - t}} \left(b_r m_r \hat{r}(\mathbf{X}_\tau) + m_Y \frac{\bar{r}}{R} \hat{y}(\mathbf{X}_\tau) \right) \right] \tag{52}$$

Where :

- $\hat{c}_t$ is the value of the deviation from the steady state of consumption

- τ ...

Equation 53

$$\frac{\Delta^{GE}}{\Delta^{\text{direct}}} = R^{\tau+1} \tag{53}$$

To understand this equation, we have to add :

$$\begin{cases} \Delta^{\text{direct}} := \left. \frac{\partial \hat{c}_0}{\partial \hat{r}_\tau} \right|_{(y_t)_{t \geq 0} \text{ held constant}} = -\alpha \cdot m_r \cdot \bar{m}^\tau \frac{1}{R^\tau} \\ \Delta^{GE} := \frac{\partial \hat{c}_0}{\partial \hat{r}_\tau} = -\alpha m_r \cdot M^\tau \frac{R}{R - r \cdot m_Y} R \end{cases}$$

Where :

- $M = \frac{\bar{m}}{R - r \cdot m_Y}$
- Δ^{direct}
- Δ^{GE}

Equation 54

$$\frac{\Delta^{GE}}{\Delta^{\text{direct}}} = \left(\frac{R}{R - rm_Y}\right)^{\tau+1} \in \left[1, R^{\tau+1}\right] \tag{54}$$

3.2 Flattening of the Phillips Curve via Imperfect Firm Attention

Equation 55

$$v^{BR}(q_{it}, (\mathbf{X}_\tau)) := v^0 \left(q_{it} - m_\pi^f \Pi(\mathbf{X}_\tau), m_x^f \mu(\mathbf{X}_\tau), c(\mathbf{X}_\tau)\right) \tag{55}$$

Equation 56

$$\max_{q_{it}} \mathbb{E}_t^{BR} \left[\sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t} \frac{c(\mathbf{X}_\tau)^{-\gamma}}{c(\mathbf{X}_\tau)^{-\gamma}} v^{BR}(q_{it}, \mathbf{X}_\tau)\right] \tag{56}$$

Equation 57

$$p_t^* = p_t + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta\bar{m})^k \, \mathbb{E}_t \left[m_\pi^f (\pi_{t+1} + ... + \pi_{t+k}) - m_x^f \mu_{t+k}\right] \tag{57}$$

Equation 58

$$\kappa = m_x^f \bar{\kappa} \tag{58}$$

3.3 Nonconstant Trend Inflation and Neo-Fisherian Paradoxes

Equation 59

$$\pi_t^d = (1-\zeta)\bar{\pi}_t + \zeta \bar{\pi}_t^{CB} \tag{59}$$

Where :

- $\bar{\pi}_t$ is the moving average of past inflation
- $\bar{\pi}_t^{CB}$ is the inflation guidance

Equation 60

$$x_t = M\mathbb{E}_t \left[x_{t+1}\right] - \sigma \left(i_t - \mathbb{E}_t \left[\pi_{t+1}\right] - r_t^n\right) \tag{60}$$

Equation 61

$$\pi_t = \beta \cdot M^f \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \kappa \cdot x_t \tag{61}$$

Equation 62, Proposition 12

$$\phi_\pi + \zeta \frac{(1 - \beta M^f)}{\kappa} \phi_x + \zeta \frac{(1 - \beta M^f)(1 - M)}{\kappa\sigma} > 1 \tag{62}$$