Macroeconomics 2 Presentation Equations description of "A Behavioral New Keynesian Model" by Xavier Gabaix

Gugelmo Cavalheiro Dias Paulo Mitash Nayanika

Wang Shang

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This file aims to summarise and give some all the equations of the presented article. Its goal is to serve as a complement to the presentation of the of Mai 2024 in the Macroeconomics 2 class. Although we use the same numerisation of the equations as in the article, we do present them in our own structure, following the oral presentation divided in three main sections. The sections of the article are however given as subsections, to provide some additional information for those wanting to use this document as a complementary helper to the reading of the article.

Contents

1 A Behavioral Model

Let's ignore the first two equations, since they are the same as (28) and (29), that will be explained later.

1.1 Introduction

Equation 1

$$x_t = M \cdot \mathbb{E}_t \left[x_{t+1} - \sigma(i_t - \mathbb{E}_t \left[\pi_{t+1} \right] - r_t^n) \right] \tag{1}$$

Equation 2

$$\pi_t = \beta \cdot M^f \mathbb{E}_t \left[\pi_{t+1} \right] + \kappa \cdot x_t \tag{2}$$

1.2 Basic Setup and the Household's Problem

Equation 3

$$U = \mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \right] \tag{3}$$

Equation (3) is just the flow utility of the Household, with :

- β the discount factor
- c_t the consumption of the houshold at time t
- N_t the work of the household at time t
- γ determines the concavity of the utility function with respect to the consumption, i.e. the importance of consumption in the utility function
- ϕ determines the concavity of the utility function with respect to work, i.e. the importance of work in the utility function

Equation 4

$$k_{t+1} = (1 + r_t)(k_t - c_t + y_t) \tag{4}$$

Equation (4) is the law of motion of the real financial wealth of the household, where :

- k_t is the real financial welath of the household at time t
- r_t is the real interest rate
- w_t is the real wage
- y_t is the agent's real income, defined as $y_t = w_t \cdot N_t + y_t^f$, with y_t^f the profit income (or the income from firms) at time t

Equation 5

$$\boldsymbol{X}_{t+1} = \boldsymbol{G}^{\boldsymbol{X}}\left(\boldsymbol{X}_t, \boldsymbol{\epsilon}_{t+1}\right) \tag{5}$$

Equation (5) describes the evolution of macroeconomic variables, where :

- $\boldsymbol{X}_t$ is the state vector, including several macroeconomic variables of time t, like ζ_t the aggregate TFP, and the announced actions in monetary and fiscal policy
- $\boldsymbol{G}^X$ the equilibrium transition function, i.e. the function that gives the macroeconomic variables at time $t+1$ from the macroeconomic variables at the previous period
- ϵ_t is the innovation in the economy at time t, with $\mathbb{E}_t\left[\epsilon_{t+1}\right] = 0$, that depends on the equilibrium policies of the agent and of the government

Equation 6

$$k_{t+1} = G^k(c_t, N_t, k_t, \boldsymbol{X}_t) := (1 + \bar{r} + \hat{r}(\boldsymbol{X}_t))(k_t + \bar{y} + \hat{y}(N_t, \boldsymbol{X}_t) - c_t) \tag{6}$$

Equation (6) is the application of the consideration of a set of macroeconomic variables on the law of motion of real financial wealth k_t, where :

- $\bar{r}$ is the steady state value of the real interest rate, that does not depend on time
- $\hat{r}(\boldsymbol{X}_t)$ is the value of the deviation from the steady state of the real interest rate, that depends on the state vector $\boldsymbol{X}_t$ at time t
- $\bar{y}$ is the steady state value of the agent's real income, that does not depend on time
- $\hat{y}(N_t, \boldsymbol{X}_t)$ is the deviation from the steady state of the agent's real income, that depends on the number of hours worked at time t and on the state vector at time t
- c_t is the aggregate consumption level at time t of the agent

Equation 7

$$\boldsymbol{X}_{t+1} = \boldsymbol{\Gamma}\boldsymbol{X}_t + \boldsymbol{epsilon}_{t+1} \tag{7}$$

Equation (7) describes the linear version of the equilibrium transition function, it is the linearization of the law of motion, where :

- Γ is a squared matrix that multiplies the state vector
- $\boldsymbol{X}_t$ is the state vector at time t
- ϵ_t is the innovation shock

Equation 8 (Assumption 1)

$$\boldsymbol{X}_{t+1} = \bar{m} \cdot \boldsymbol{G}^{\boldsymbol{X}}(\boldsymbol{X}_t, \boldsymbol{\epsilon}_{t+1}) \tag{8}$$

Equation (8) describes the Cognitive Discounting of the State Vector, i.e. the perception by behavioral agents of the law of motion of the macroeconomic variables, where :

- $\bar{m} \in [0n1]$ is the coginitive discount factor measuring the attention to the future

Equation 9

$$\boldsymbol{X}_{t+1} = \bar{m}(\boldsymbol{\Gamma}\boldsymbol{X}_t + \boldsymbol{\epsilon}_{t+1}) \tag{9}$$

Equation (9) is just the linearized version of the perception by behavioral agents of the law of motion of the state vector.

Equation 10

$$\mathbb{E}_t^{BR}\left[\boldsymbol{X}_{t+k}\right] = \bar{m}^k \mathbb{E}_t\left[\boldsymbol{X}_{t+k}\right] \tag{10}$$

Equation (10) defines the expectation of behavioral agents in function of the rational perception of the law of motion of the state vector, where :

- $k \geq 0$ a time period in discrete context
- $\mathbb{E}_t^{BR}\left[\boldsymbol{X}_{t+k}\right]$ is the expected value of the state vector at time $t+k$ by behavioral agents (or subjective/behavioral expectation operator)
- $\bar{m}^k$ is the cognitive discounting effect at period $t+k$
- $\mathbb{E}_t\left[\boldsymbol{X}_{t+k}\right]$ is the rational expectation of the state vector at time $t+k$

Equation 11 (Lemma 1)

$$\mathbb{E}_t^{BR}\left[z\left(\boldsymbol{X}_{t+k}\right)\right] = \bar{m}^k \mathbb{E}_t\left[z\left(\boldsymbol{X}_{t+k}\right)\right] \tag{11}$$

Equation (11) defines in the general case the behavioral expectation operator, for any function of the state vector, where :

- $k \geq 0$ a time period in discrete context
- $z(\cdot)$ is a function, such that $z(0) = 0$
- $\mathbb{E}_t^{BR}\left[z\left(\boldsymbol{X}_{t+k}\right)\right]$ is the expected value of the image of the state vector by the function $z(\cdot)$ at time $t+k$ by behavioral agents
- $\bar{m}^k$ is the cognitive discounting effect at period $t+k$
- $\mathbb{E}_t\left[z\left(\boldsymbol{X}_{t+k}\right)\right]$ is the rational expectation of the image of the state vector by the function $z(\cdot)$ at time $t+k$

Equation 12

$$\mathbb{E}_t^{BR}\left[\bar{r} + \hat{r}\left(\boldsymbol{X}_{t+k}\right)\right] = \bar{r} + \bar{m}^k \mathbb{E}_t\left[\hat{r}(\boldsymbol{X}_{t+k})\right] \tag{12}$$

Equation (12) is an example of the Lemma 1 applied to the interest rate, where :

- $k \geq 0$ a time period in discrete context
- $\bar{r}$ the steady state level of the real interest rate, that does not depend on time,
- $\hat{r}(\boldsymbol{X}_{t+k})$ is the equilibrium transition function defining the value of the deviation from the steady state of the real interest rate in function of the state vector at time $t + k$
- $\bar{r} + \hat{r}(\boldsymbol{X}_t) = r_t(\boldsymbol{X}_t)$ is the value of the real interest rate at time t
- $\mathbb{E}_t^{BR}\left[\bar{r} + \hat{r}(\boldsymbol{X}_{t+k})\right]$ is the expected value of the real interest at time $t + k$ by behavioral agents
- $\mathbb{E}_t\left[\hat{r}(\boldsymbol{X}_{t+k})\right]$ is the rational expectation of value of the deviation of the real itnerest rate from the steady state at time $t + k$

1.3 The Firm's problem

Equation 13

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}} \tag{13}$$

Equation (13) describes the aggregate price level, where :

- P_t is the aggregate price level of the economy at time t
- $i \in [0, 1]$ is the firm index
- ε is the elasticity of subsititution between goods

Equation 14

$$v^0(q_{i\tau}, \mu_\tau, c_\tau) := \left(e^{q_{i\tau}} - (1 - \tau_f)e^{-\mu_\tau}\right) e^{-\varepsilon q_{i\tau}} c_\tau \tag{14}$$

Equation (14) describes the profit of the firm before the lump sum tax of the government, where :

- v is the real profit of the firm
- $q_{i\tau} = \ln\left(\frac{P_{i\tau}}{P_\tau}\right) = p_{i\tau} - p_\tau$ is the real log price at time τ
- $\tau_f = \frac{1}{\varepsilon}$ it the corrective wage subsidy from the government, funded by the lump sum tax

- $\mu_\tau = \zeta_t - \ln(\omega_t)$ is the labor wedge, which is zero at efficiency
- ε is the elasticity of subsititution between goods
- c_τ is the aggregate level of consumption

Equation 15

$$v\left(q_{it}, \boldsymbol{X}_\tau\right) := v^0\left(q_{it} - \Pi(\boldsymbol{X}_\tau), \mu(\boldsymbol{X}_\tau), c(\boldsymbol{X}_\tau)\right) \tag{15}$$

Equation 15 describes the flow profit of the firm in function of the real log price and of the extended macro state vector, where :

- $q_{it} = \ln\left(\frac{P_{it}}{P_t}\right) = p_{it} - p_t$ is the real log price
- $\boldsymbol{X}_\tau = (\boldsymbol{X}_\tau^{\mathcal{M}}, \Pi_\tau)$ is the extended macro state vector, with $\boldsymbol{X}^{\mathcal{M}_\tau}$ the vector of macro variables, including ζ_τ and possible announcements about future policy
- $\Pi\left(\boldsymbol{X}_\tau\right) := p_\tau - p_t = \pi_{t+1} + ... + \pi_\tau$ is the inflation between times t and τ
- $q_{it} - \Pi(\boldsymbol{X}_\tau) = q_{i\tau}$ is the real price of the firm if they didn't change its price between t and τ
- $\mu\left(\boldsymbol{X}_\tau\right)$ is the labor wedge in function of the extended state vector at time t
- $c(\boldsymbol{X}_\tau)$ is the aggregate consumption level in function of the extended state vector at time t

Equation 16

$$\max_{q_{it}} \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t} \frac{c\left(\boldsymbol{X}_\tau^{-\gamma}\right)}{c\left(\boldsymbol{X}_t^{-\gamma}\right)} v\left(q_{it,\boldsymbol{X}_\tau}\right)\right] \tag{16}$$

Equation (16) describes the maximisation program of the firm given that they have a Calvo-like probability of θ of being able to change their price at each period, where :

- t is the initial period
- τ is the time period index
- q_{it} is the real log price of the firm at time t
- β is the discount factor
- θ is the Calvo like probability that the firm can change its price at any period
- $\frac{c\left(\boldsymbol{X}_\tau^{-\gamma}\right)}{c\left(\boldsymbol{X}_t^{-\gamma}\right)}$ is the adjustment in the stochastic discount factor between times t and τ

Equation 17

$$\max_{q_{it}} \mathbb{E}_t^{BR} \left[\sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t} \frac{c\left(\boldsymbol{X}_\tau^{-\gamma}\right)}{c\left(\boldsymbol{X}_t^{-\gamma}\right)} v\left(q_{it}, \boldsymbol{X}_\tau\right) \right] \tag{17}$$

Equation 17 describes the maximisation program of the behavioral firm, i.e. such that it is maximisation the behavioral expectation operator of the flow profit, where :

- $\mathbb{E}_t^{BR}$ is the behavioral/subjective expected value operator
- t is the initial period
- τ is the time period index
- q_{it} is the real log price of the firm at time t
- β is the discount factor
- θ is the Calvo like probability that the firm can change its price at any period
- $\frac{c\left(\boldsymbol{X}_\tau^{-\gamma}\right)}{c\left(\boldsymbol{X}_t^{-\gamma}\right)}$ is the adjustment in the stochastic discount factor between times t and τ

1.4 Model solution

Equation 18

$$\hat{c}_t = \mathbb{E}_t \left[\hat{c}_{t+1} - \frac{1}{\gamma R} \hat{r}_t \right] \tag{18}$$

Equation (18) is the linearized version of the Euler equation obtained from the presented model. It is also called the investment-savings (IS) curve, where :

- $\hat{c}_t$ is the value of the deviation from the steady state of the aggregate consumption at time t
- $\mathbb{E}_t\left[\hat{c}_{t+1}\right]$ is the rational expectation of the value of the deviation from the steady state of the aggregate consumption at time $t+1$
- γ is the factor of the importance of consumption
- $R := 1 + \bar{r}$ is defined from the real intereste rate at the steady state (cf. page 7 of the article)

Equation 19

$$\hat{c}_t = M \cdot \mathbb{E}_t \left[\hat{c}_{t+1} - \sigma \hat{r}_t\right] \tag{19}$$

Equation (19) is the application of Lemma 1 (equation (11)) on the previous Euler equation, i.e. a cognitively discounted aggregate Euler equation, where :

- M is the macro parameter of attention, such that $M = \bar{m}$ here
- $\sigma = \frac{1}{\gamma R}$

Equation 20

$$N_t^{\phi} = \omega_t c_t^{\gamma} \tag{20}$$

Equation (20) the result of the static First Order Condition for labor supply, where :

- N_t is the quantity of labor provided at time t
- ω_t is the real wage at time t
- c_t is the aggregate quantity of consumption at time t
- γ is the consumption importance in the utility

Equation 21

$$\hat{c}_t^n = \frac{1+\phi}{\gamma+\phi} \zeta_t \tag{21}$$

Equation (21) ..., where :

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Equation 22

$$\hat{c}_t^n = M \cdot \mathbb{E}_t \left[\hat{c}_{t+1}^n\right] - \sigma \hat{r}_t^n \tag{22}$$

Equation (22) ..., where :

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Equation 23

$$r_t^{n0} = \bar{r} + \frac{1+\phi}{\sigma(\gamma+\phi)} \left(M \cdot \mathbb{E}_t \left[\zeta_{t+1}\right] - \zeta_t\right) \tag{23}$$

Equation (23) ..., where :

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Equation 24

$$x_t = M \cdot \mathbb{E}_t [x_{t+1}] - \sigma(\hat{r}_t - \hat{r}_t^n) \tag{24}$$

Equation (24) ..., where :

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Equation 25

$$x_t = M \cdot \mathbb{E}_t [x_{t+1}] - \sigma(i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^n) \tag{25}$$

Equation () ..., where :

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Equation 26

$$x_t = -\sigma \sum_{k \geq 0} M \cdot \mathbb{E}_t \left[\hat{r}_{t+k} - \hat{r}_{t+k}^n\right] \tag{26}$$

Equation (26) ..., where :

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Equation 27

$$p_t^* = p_t + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta\bar{m})^k \cdot \mathbb{E}_t [\pi t + 1 + ... + \pi_{t+k} - \mu_{t+k}] \tag{27}$$

Equation (27) ..., where :

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1.5 A Behavioral New Keynesian Model

Equation 28 - Proposition 2, first equation

$$x_t = M \cdot \mathbb{E}_t [x_{t+1}] - \sigma(i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^n) \tag{28}$$

Equation (28) ..., where :

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Equation 29 - Proposition 2, second equation

$$\pi_t = \beta \cdot M^f \mathbb{E}_t [\pi_{t+1}] + \kappa \cdot x_t \tag{29}$$

Equation (29) ..., where :

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Equation 30

$$\begin{cases} M = \bar{m} \\ \sigma = \frac{1}{\gamma R} \\ M^f = \bar{m}\left(\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}}(1-\theta)\right) \end{cases} \tag{30}$$

Equation (30) ..., where :

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2 Consequences of the Model

3 Behavioral Enrichments of the Model

$$\begin{aligned} k_{t+1} &= \mathbf{G}^{k,BR}(c_t, N_t, k_t) \\ &:= (1 + \bar{r} + \hat{r}^{BR}(\mathbf{X}_t))(k_t + \bar{y} + \hat{y}^{BR}(N_t, \mathbf{X}_t) - c_t) \end{aligned} \tag{49}$$

With :

- k_{t+1} the capital at time $t+1$
- $\mathbf{G}^{k,BR}$ the ... ?
- c_t the consumption at time t
- N_t work at time t
- k_t capital at time t
- $\bar{r}$ the ... ?
- $\hat{r}^{BR}$ the ... ?
- $\mathbf{X}_t$ the ... ?
- $\bar{y}$ the ... ?
- $\hat{y}^{BR}(N_t, \mathbf{X}_t)$ the ... ?
- $\mathbf{X}_t$ the ... ?

$$\begin{cases} \hat{r}^{BR}(\mathbf{X}_t) = m_r \hat{r}(\mathbf{X}_t) \\ \hat{y}^{BR}(N_t, \mathbf{X}_t) = m_y \hat{y}(\mathbf{X}_t) \end{cases} \tag{49}$$

With :

- k_{t+1} the capital at time $t+1$
- $\mathbf{G}^{k,BR}$ the ... ?

- c_t the consumption at time t
- N_t work at time t
- k_t capital at time t
- $\bar{r}$ the ... ?
- $\hat{r}^{BR}$ the ... ?
- $\mathbf{X}_t$ the ... ?
- $\bar{y}$ the ... ?
- $\hat{y}^{BR}(N_t, \mathbf{X}_t)$ the ... ?
- $\mathbf{X}_t$ the ... ?