Macroeconomics 2 Presentation

Article review :
Gabaix, Xavier. 2020. ”A Behavioral New Keynesian Model.” American Economic Review, 110(8): 2271-2327

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Sciences Po

Outline

1. Introduction
2. Baseline model of the paper
3. Consequences
4. Implications for monetary policy
5. Implications for fiscal policy
6. Behavioral Enrichments of the Model
7. Discussion of the Behavioral Assumptions, Conclusion, Critics

Introduction

1. Introduction

1.1 Goal of the paper

1.2 Method of the paper

1.3 Conclusions of the paper

1.4 Literature of the topic

Goal of the paper

This paper aims to introduce behavioral economics in the New Keynesian model.

- Behavioral Economics :
  Since Kahneman et Tversky (1979), aims to introduce deviation to the expected utility theory and the perfect rationality assumption.
- New-Keynesian framework :
  General Equilibrium with frictions, what we saw in throughout this class.

How to integrate coginitive limitations in the New Keynesian model ?

Method of the paper

- Baseline model
- Direct conclusions
- Consequences on policy
- Refinment of the baseline model
- Consequences of refinment

Conclusions of the paper

In a behavioral model :

- Forward guidance is less powerful
- The Taylor rule is changed
- The equilibrium determinacy is easier to reach
- The Zero Lower Bound (ZLB) consequences are changed
- The optimal policy are different
- Fiscal policies are more powerful
- Theoretical contradictions are solved, such as the Neo-Fisherian paradoxes

Literature of the topic

Literature on the topic :

General New Keynesian Framework : Woodford 2003b and Galí 2015

- Strength of Forward guidance : Del Negro, Giannoni, and Patterson (2015) and McKay, Nakamura, and Steinsson (2016)
- Taylor Principle and Equilibrium determinacy : Blanchard and Kahn (1980)
- ZLB and Equilibrium determinacy : Cochrane (2018) on the case of Japan
- Policy optimality : Clarida, Galí, and Gertler (1999), and rationality of firms
- Neo-Fisherian paradoxes : Cochrane (2018) on the inconsistency of the New-Keynesian model

Literature of the topic

Behavioral economics : How to incorporate cognitive limits in Macroeconomic models ?

- Limited information updating : Gabaix and Laibson (2002), Mankiw and Reis (2002)
- Related differential salience : Bordalo et al. (2018)
- Noisy signals Maċkowiak and Wiederholt (2015), Caplin, Dean, and Leahy (2017)
- Microfoundation : Gabaix (2014)
- Woodford (2013) for a literature review on the topic.

There is a lot of literature on how to model behavioral agents (check pages 5 and 6 of the articles), but the goal of the present article lies elsewhere : Exhaustivity in the model and analysis of implications on policy.

Baseline model of the paper

1. Introduction

2. Baseline model of the paper

3. Consequences

4. Implications for monetary policy

5. Implications for fiscal policy

6. Behavioral Enrichments of the Model

7. Discussion of the Behavioral Assumptions, Conclusion, Critics

2. Baseline model of the paper

2.1 Household's Problem

2.2 Firm's Problem

2.3 Solution

2.4 Synthesis Of A Behavioral New Keynesian Model

2.5 Calibration

Household's Problem

$$U = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t, N_t)\right] \tag{3}$$

With

$$u(c_t, N_t) = \frac{c^{1-\gamma} - 1}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}$$

So we have the following objective function of the household:

$$U = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c^{1-\gamma} - 1}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}\right)\right]$$

Subject to

$$k_{t+1} = (1 + r_t)(k_t - c_t + y_t) \tag{4}$$

with $y_t = w_t \cdot N_t + y_t^f$ and $Y_{it} = N_{it} e^{\zeta_t}$ where ζ_t follows an AR(1) process with mean 0

Deterministic steady state $\bar{c} = \bar{N} = \bar{w} = \bar{y} = 1$

Household's Problem

Law of motion of state vector

$$\boldsymbol{X}_{t+1} = \boldsymbol{G}^{\boldsymbol{X}}\left(\boldsymbol{X}_t, \boldsymbol{\epsilon}_{t+1}\right) \tag{5}$$

Decomposing variables as deviations from steady-state

$$r_t = \bar{r} + \hat{r}_t \qquad\qquad y_t = \bar{y} + \hat{y}_t$$

Where the deviations are functions of the State Vector

$$\hat{r}_t = \hat{r}(\boldsymbol{X}_t) \qquad \hat{y}_t = \hat{y}(N_t, \boldsymbol{X}_t) := w(\boldsymbol{X}_t)N_t + y^f(\boldsymbol{X}_t) - \bar{y}$$

Private financial wealth

$$k_{t+1} = G^k(c_t, N_t, k_t, \boldsymbol{X}_t) := (1+\bar{r}+\hat{r}(\boldsymbol{X}_t))(k_t+\bar{y}+\hat{y}(N_t, \boldsymbol{X}_t)-c_t) \tag{6}$$

Household's Problem

Linearisation of the State vector

$$\boldsymbol{X}_{t+1} = \boldsymbol{\Gamma} \boldsymbol{X}_t + \boldsymbol{\epsilon}_{t+1} \tag{7}$$

Linearisation of the deviations

$$\hat{r}(\boldsymbol{X}) = \boldsymbol{b}^r_{\boldsymbol{X}} \boldsymbol{X}$$

Household's Problem

Cognitive Discounting of the State vector (Non-Linearised and Linearised)

$$\boldsymbol{X}_{t+1} = \bar{m} \cdot \boldsymbol{G}^{\boldsymbol{X}}(\boldsymbol{X}_t, \boldsymbol{\epsilon}_{t+1}) \tag{8}$$

$$\boldsymbol{X}_{t+1} = \bar{m}(\boldsymbol{\Gamma}\boldsymbol{X}_t + \boldsymbol{\epsilon}_{t+1}) \tag{9}$$

Subjective Expectation

$$\begin{aligned}
\mathbb{E}_t^{BR}[\boldsymbol{X}_{t+1}] &= \bar{m}\boldsymbol{\Gamma}\boldsymbol{X_t} \\
\mathbb{E}_t^{BR}[\boldsymbol{X}_{t+k}] &= \bar{m}^k\boldsymbol{\Gamma}^k\boldsymbol{X_t} \\
\mathbb{E}_t^{BR}[\boldsymbol{X}_{t+k}] &= \bar{m}^k\mathbb{E}_t[\boldsymbol{X}_{t+k}] \qquad (10)
\end{aligned}$$

Household's Problem

Cognitive discounting of all Variables (LEMMA 1)

$$\mathbb{E}_t^{BR}\left[z\left(\boldsymbol{X}_{t+k}\right)\right] = \bar{m}^k \mathbb{E}_t\left[z\left(\boldsymbol{X}_{t+k}\right)\right] \tag{11}$$

For Example

$$\mathbb{E}_t^{BR}\left[\bar{r} + \hat{r}\left(\boldsymbol{X}_{t+k}\right)\right] = \bar{r} + \bar{m}^k \mathbb{E}_t\left[\hat{r}(\boldsymbol{X}_{t+k})\right] \tag{12}$$

Firm's Problem

Firm's Aggregate Price Level

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}} \tag{13}$$

Firm's Maximise their real profit

$$\nu_\tau = \left(\frac{P_{i\tau}}{P_\tau} - MC_\tau \right) \left(\frac{P_{i\tau}}{P_\tau} \right)^{-\varepsilon} c_\tau$$

Where

- $\left(\frac{P_{i\tau}}{P_\tau} \right)^{-\varepsilon} c_\tau$ is the total demand for the firm's good
- $MC_t = (1 - \tau_f)(\omega_t / e^{\zeta t}) = (1 - \tau_f) e^{-\mu_t}$ and $\mu_t := \zeta_t - \ln \omega_t$ is the labour wedge and $\tau_f = 1/\varepsilon$ which corrects the price distortions

Firm's Problem

Firm's real profit

$$\nu_\tau = \left(\frac{P_{i\tau}}{P_\tau} - MC_\tau\right)\left(\frac{P_{i\tau}}{P_\tau}\right)^{-\varepsilon} c_\tau$$

$$\nu^0(q_{i\tau}, \mu_\tau, c_\tau) := \left(e^{q_{i\tau}} - (1-\tau_f)e^{-\mu_\tau}\right) e^{-\varepsilon q_{i\tau}} c_\tau \tag{14}$$

where

- $q_{i\tau} = \ln\left(\frac{P_{i\tau}}{P_\tau}\right) = p_{i\tau} - p_\tau$ hence $\left(\frac{P_{i\tau}}{P_\tau}\right) = e^{q_{i\tau}}$

Firm's Problem

$\mathbf{X}_\tau = \left(\mathbf{X}_\tau^{\mathcal{M}}, \Pi_\tau\right)$ where $\Pi_\tau := p_\tau - p_t$ and $\mathbf{X}_\tau^{\mathcal{M}}$ is vector including macroeconomic variables.

If firm hasn't changed its price between t and τ then its real price $q_{i\tau} = q_{it} - \Pi_\tau$

Hence the flow of profit

$$\nu\left(q_{it}, \boldsymbol{X}\tau\right) := \nu^0\left(q_{it} - \Pi(\boldsymbol{X}\tau), \mu(\boldsymbol{X}\tau), c(\boldsymbol{X}_\tau)\right) \tag{15}$$

Firm's Problem

Maximisation problem for a Traditional Calvo Firm who can adjusts its prices

$$\max_{\mathrm{q}_{it}} \mathbb{E}_t \sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t} \frac{c\left(\boldsymbol{X_\tau}\right)^{-\gamma}}{\left(\boldsymbol{X_t}\right)^{-\gamma}} \nu\left(q_{it}, \boldsymbol{X}\tau\right) \tag{16}$$

Behavioural Counterpart

$$\max_{q_{it}} \mathbb{E}_t^{BR} \left[\sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t} \frac{c\left(\boldsymbol{X}_\tau^{-\gamma}\right)}{c\left(\boldsymbol{X}_t^{-\gamma}\right)} \nu\left(q_{it,\boldsymbol{X}_\tau}\right)\right] \tag{17}$$

Solution

Solving the Household's Maximisation Problem we get:

Euler

$$\hat{c}_t = \mathbb{E}_t \left[\hat{c}_{t+1} - \frac{1}{\gamma R} \hat{r}_t \right] \tag{18}$$

$$\hat{c}_t = \mathbb{E}_t \left[\hat{c}_{t+1} \right] - \sigma \hat{r}_t$$

Static First Order Condition for Labour Supply

$$N_t^{\phi} = \omega_t c_t^{\gamma} \tag{20}$$

Solution

Using Lemma 1

$$\mathbb{E}_t^{BR}\left[z\left(\boldsymbol{X}_{t+k}\right)\right] = \bar{m}^k \mathbb{E}_t\left[z\left(\boldsymbol{X}_{t+k}\right)\right]$$

We get Behavioural Euler Equation

$$\hat{c}(\boldsymbol{X_t}) = \mathbb{E}_t^{BR}\left[\hat{c}(\boldsymbol{X_{t+1}})\right] - \frac{1}{\gamma R}\hat{r}_t$$

$$\hat{c}_t = M \cdot \mathbb{E}_t\left[\hat{c}_{t+1}\right] - \sigma \hat{r}_t \tag{19}$$

here $M = \bar{m}$

Solution

Natural Rate Output without pricing frictions

$$\hat{c}_t^n = \frac{1+\phi}{\gamma+\phi}\zeta_t \tag{21}$$

$$\hat{c}_t^n = M \cdot \mathbb{E}_t \left[\hat{c}_{t+1}^n\right] - \sigma \hat{r}_t^n \tag{22}$$

Natural Interest Rate (here natural interest is the same as the pure natural interest rate where there are no budget deficits)

$$r_t^{n0} = \bar{r} + \frac{1+\phi}{\sigma(\gamma+\phi)} \left(M \cdot \mathbb{E}_t \left[\zeta_{t+1}\right] - \zeta_t\right) \tag{23}$$

Solution

Behavioural Discounted Euler Equation

$$x_t = M \cdot \mathbb{E}_t\left[x_{t+1}\right] - \sigma(\hat{r}_t - \hat{r}_t^n) \tag{24}$$

$$x_t = M \cdot \mathbb{E}_t\left[x_{t+1}\right] - \sigma(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - r_t^n) \tag{25}$$

where

- $\hat{r}_t = r_t - \bar{r} = (i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \bar{r})$
- $\hat{r}_t^n = r_t^n - \bar{r}$
- Therefore $\hat{r}_t - \hat{r}_t^n = (i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \bar{r})$

Iterative Version of the Behavioural Discounted Euler Equation

$$x_t = -\sigma \sum_{k\geq 0} M \cdot \mathbb{E}_t\left[\hat{r}_{t+k} - \hat{r}_{t+k}^n\right] \tag{26}$$

Solution

Optimal price for a behavioural firm resetting its price

$$p_t^* = p_t + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta\bar{m})^k \cdot \mathbb{E}_t \left[\pi_{t+1} + ... + \pi_{t+k} - \mu_{t+k}\right] \quad (27)$$

where

- $p_t^* = q_{it} + p_t$

Synthesis Of A Behavioral New Keynesian Model

Behavioural IS Curve

$$x_t = M \cdot \mathbb{E}_t \left[x_{t+1}\right] - \sigma(i_t - \mathbb{E}_t \left[\pi_{t+1}\right] - r_t^n) \tag{29}$$

Behavioural Philips Curve

$$\pi_t = \beta \cdot M^f \mathbb{E}_t \left[\pi_{t+1}\right] + \kappa \cdot x_t \tag{30}$$

Where

$$\begin{cases} M = \bar{m} \\ \sigma = \frac{1}{\gamma R} \\ M^f = \bar{m} \left(\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}}(1-\theta)\right) \end{cases} \tag{31}$$

Calibration

TABLE 1—KEY PARAMETER INPUTS

Cognitive discounting by consumers and firms	$M = 0.85$, $M^f = 0.80$
Sensitivity to interest rates	$\sigma = 0.20$
Slope of the Phillips curve	$\kappa = 0.11$
Rate of time preference	$\beta = 0.99$
Relative welfare weight on output	$\vartheta = 0.02$

Notes: This table reports the coefficients used in the model. Units are quarterly.

TABLE 2—ANCILLARY PARAMETERS

Coefficient of risk aversion	$\gamma = 5$
Inverse of Frisch elasticity	$\phi = 1$
Survival rates of prices	$\theta = 0.875$
Demand elasticity	$\varepsilon = 5.3$
Cognitive discounting	$\bar{m} = 0.85$

Notes: This table reports the coefficients used in the model to generate the parameters of Table 1. Units are quarterly.

Consequences

1. Introduction
2. Baseline model of the paper
3. Consequences
4. Implications for monetary policy
5. Implications for fiscal policy
6. Behavioral Enrichments of the Model
7. Discussion of the Behavioral Assumptions, Conclusion, Critics

3. Consequences

3.1 The Taylor Principle Reconsidered

3.2 ZLB Is Less Costly with Behavioral Agents

3.3 Forward Guidance Is Much Less Powerful

The Taylor Principle Reconsidered

Rational Model: multiple equilibria when monetary policy is passive ($\phi_\pi = \phi_x = 0$).

Behavioural Model: one unique equilibrium if consumers are myopic enough.

First organize IS and Phillips Curves into matrix form:

$$\mathbf{z}_t = \mathbf{A}\mathbb{E}_t\left[z_{t+1}\right] + \mathbf{b}a_t \tag{32}$$

where:

- $\mathbf{z}_t := (x_t, \pi_t)'$
- $\mathbf{A} = \frac{1}{1+\sigma(\phi_x+\kappa\phi_\pi)}\begin{pmatrix} M & \sigma(1-\beta^f\phi_\pi) \\ \kappa M & \beta^f(1+\sigma\phi_x)+\kappa\sigma \end{pmatrix}$
- $\mathbf{b} = \frac{-\sigma}{1+\sigma(\phi_x+\kappa\phi_\pi)}(1,\kappa)'$
- $a_t := j_t - r_t^n$

Propostion 3 (**Equilibrium Determinacy with Behavioral Agents**): There is a determinate equilibrium (i.e., all of $\mathbf{A}$'s eigenvalues are less than 1 in modulus) if and only if

$$\phi_\pi + \frac{(1-\beta M^f)}{\kappa}\phi_x + \frac{(1-\beta M^f)(1-M)}{\kappa\sigma} > 1 \tag{34}$$

When monetary policy is passive, unique equilibrium iff bounded rationality is strong enough:

$$\frac{(1-\beta M^f)(1-M)}{\kappa\sigma} > 1 \tag{35}$$

The Taylor Principle Reconsidered

Permanent Interest Rate Peg. Rational: multiple bounded equilibria since matrix A has a root greater than 1.Behavioral: a definite non-explosive equilibrium:

$$\mathbf{z}_t = \mathbb{E}_t \left[\sum_{\tau \geq t} \mathbf{A}^{\tau - t} \mathbf{b} a_\tau \right] \tag{1}$$

Long-Lasting Interest Rate Peg. Rational: very volatile economy. Behavioural: iterating $\mathbf{z}_t = \mathbb{E}_t \mathbf{A}_{ZLB} \mathbf{z}_{t+1} + \underline{\mathbf{b}}$ forward:

$$\mathbf{z}_0(T) = \left(\mathbf{I} + \mathbf{A}_{ZLB} + ... + \mathbf{A}_{ZLB}^{T-1} \right) \underline{\mathbf{b}} + \mathbf{A}_{ZLB}^{T} \mathbb{E}_0 \left[\mathbf{z}_T \right] \tag{36}$$

where:

- $\mathbf{A}_{ZLB}$: value of matrix $\mathbf{A}$ when $\phi_\pi = \phi_x = j = 0$.
- $\underline{\mathbf{b}} := (1, \kappa) \, \sigma \underline{r}$
- $\underline{r} \leq 0$ is the real interest rate that prevails during the ZLB.

ZLB Is Less Costly with Behavioral Agents

Rational Model: Suppose ZLB lasts for T periods and $x_0(T)$ is output gap at time 0. Unboundedly intense recession as $T \to \infty$:

$$\lim_{n \to \infty} x_0(T) = -\infty$$

Behavioural Model (Propostion 4): boundedly intense recession.

$$\lim_{T \to \infty} x_0(T) = \frac{\sigma\left(1 - \beta M^f\right)}{(1-M)\left(1-\beta M^f\right) - \kappa\sigma}\underline{r}<0 \tag{38}$$

Myopia has to be stronger when agents are highly sensitive to the interest rate (high σ) and price flexibility is high (high κ). High price flexibility makes the system very reactive, and a high myopia is useful to counterbalance that.

ZLB Is Less Costly with Behavioral Agents

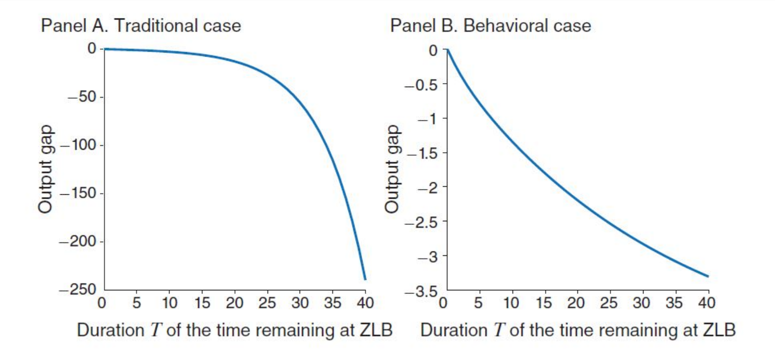

Figure 1: Rational vs Behavioural Models

This figure shows the output gap $x_0(T)$ at time 0, given that the economy will be at the ZLB for T more periods.

Forward Guidance Is Much Less Powerful

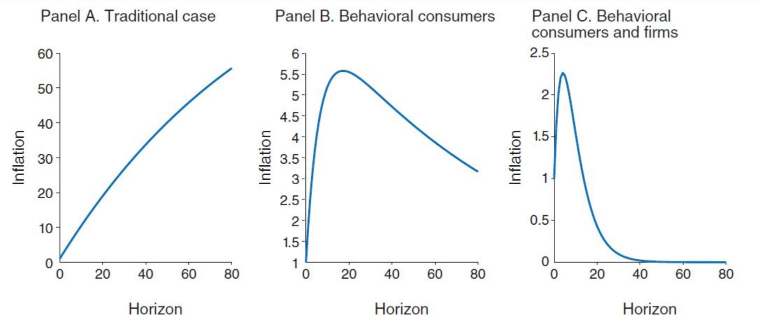

Figure 2: Rational vs Behavioural Models

This figure shows the response of current inflation to forward guidance about a one-period interest rate cut in T quarters, compared to an immediate rate change of the same magnitude.

Implications for monetary policy

1. Introduction
2. Baseline model of the paper
3. Consequences
4. Implications for monetary policy
5. Implications for fiscal policy
6. Behavioral Enrichments of the Model
7. Discussion of the Behavioral Assumptions, Conclusion, Critics

4. Implications for monetary policy

4.1 Welfare with Behavioral Agents and the Central Bank's Objective

4.2 Optimal Policy with No ZLB Constraints

4.3 Optimal Policy with Complex Trade-Offs

Welfare with Behavioral Agents and the Central Bank's Objective

Setup: The welfare loss from inflation and output gap is:

$$W = -K\mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{2}\beta^t \left(\pi_t^2 + \vartheta x_t^2\right) + W_- \tag{2}$$

where:

- $\widetilde{W} = W^* + W$. W^* is first best welfare, and W is the deviation from the first best.
- $\vartheta = \frac{\overline{\kappa}}{\epsilon}$
- $K = u_c c\left(\gamma + \phi\right)\left(\epsilon/\overline{\kappa}\right)$
- W_- is a constant.
- $\overline{\kappa}$ is the Phillips curve coefficient with rational firms.
- ϵ is the elasticity of demand.

Optimal Policy with No ZLB Constraints

Suppose productivity or discount factor shocks ⇒ changes real interest rate: $r_t^n = r_t^{n0}$.

First-best policy: zero output gap and inflation
⇒ $i_t = r_t^{n0}$ (nominal rate perfectly tracks real rate)
⇒ true with both rational and behavioral agents (as long as ZLB doesn't bind $r_t^{n0} \geq 0$)

Such shocks (**productivity and discount rate shocks**) allowed monetary policy to attain the first best.

Optimal Policy with Complex Trade-Offs

Now consider a shock that does **not** allow the monetary policy to reach the first best: "cost-push shock," i.e., a disturbance ν_t to the Phillips curve.

$\pi_t = \beta M^f \mathbb{E}t\left[\pi t + 1\right] + \kappa x_t + \nu_t$, with ν_t following a AR(1) process: $\nu_t = \rho_\nu \nu_{t-1} + \sigma_t^\nu$ with $\rho_\nu \in [0, 1)$

Examine optimal policies for **"Commitment"** policy and **"discretionary"** policy.

Optimal Policy with Complex Trade-Offs

Optimal Policy with Commitment (Propostion 5).To fight a time-0 cost-push shock, the optimal commitment policy entails, at time $t \geq 0$:

$$\pi_t = \frac{-\vartheta}{\kappa}\left(x_t - M^f x_{t-1}\mathbf{1}_{t>0}\right) \tag{40}$$

so that the (log) price level ($p_t = \sum_{\tau=0}^{t} \pi_\tau$, normalizing the initial log price level to p_{-1}=0) satisfies:

$$p_t = \frac{-\vartheta}{\kappa}\left(x_t + \left(1 - M^f\right)\sum_{\tau=0}^{t-1} x_\tau\right) \tag{41}$$

Optimal Policy with Complex Trade-Offs

Rational Model. “Price-Level Targeting” is optimal since it ensures that the price level mean-reverts to a fixed target: $p_t = (-\nu/\kappa)x_t \to 0$ in the long run.

Behavioural Model. “Price-Level Targeting” is **NOT** optimal. Price level is higher after a positive cost-push shock: optimal policy does not seek to bring price level back to baseline.

Optimal Policy with Complex Trade-Offs

This figure shows the optimal interest rate policy in response to a cost-push shock (ν_t), when the central bank follows the optimal commitment strategy.

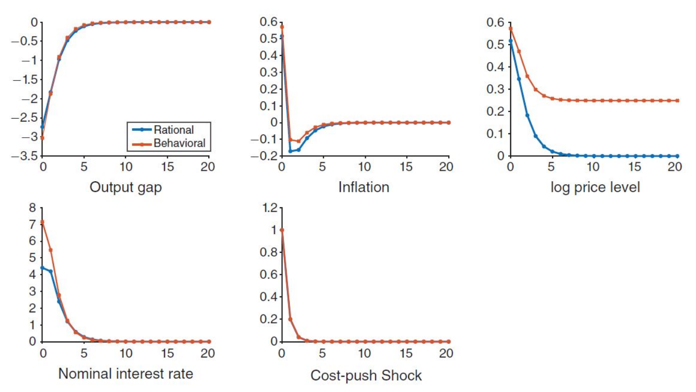

Figure 3: Rational vs Behavioural Models

Optimal Policy with Complex Trade-Offs

Optimal Discretionary Policy entails:

$$\pi_t = \frac{-\vartheta}{\kappa} x_t \tag{42}$$

so that on the equilibrium path: $i_t = K\nu_t + r_t^n$, with $K = \frac{\kappa\sigma^{-1}(1-M\rho_\nu)+\vartheta\rho_\nu}{\kappa^2+\vartheta\left(1-\beta M^f\rho_\nu\right)}$

For persistent shocks (ρ_ν>0), the optimal policy is less aggressive (K is lower) when firms are more behavioral: future cost-push shocks do not affect much the firms' pricing today, hence the central bank needs to respond less to them.

Myopia does **NOT** affect optimal trade-off between inflation and the output gap, but it it does affect the interest rate path that implements this outcome.

Implications for fiscal policy

1. Introduction
2. Baseline model of the paper
3. Consequences
4. Implications for monetary policy
5. Implications for fiscal policy
6. Behavioral Enrichments of the Model
7. Discussion of the Behavioral Assumptions, Conclusion, Critics

5. Implications for fiscal policy

5.1 Failure of Ricardian Equivalence

5.2 Consequences for Fiscal Policy

Failure of Ricardian Equivalence

The public debt evolves as:

$$B_{t+1} = B_t + Rd_t \tag{43}$$

where:

- B_t is the real value of government debt in period t, before period- t taxes.
- $d_t := \mathcal{T}_t + (r/R)B_t$. d_t is the budget deficit (after the payment of the interest rate on debt) in period t.
- $\mathcal{T}_t$ is the lump-sum transfer given by the government to the agent (so that $-\mathcal{T}_t$ is a tax).

No-Ponzi condition is the usual one, $\lim\limits_{t\to\infty} \beta^t B_t = 0$, which here takes the form $\lim\limits_{t\to\infty} \beta^t \left(\sum_{s=0}^{t-1} d_s\right) = 0$. Hence, debt does not necessarily mean-revert, and can follow a random walk.

Failure of Ricardian Equivalence

Discounted Euler Equation with Sensitivity to Budget Deficits. Non-Ricardian agents $\Rightarrow$ budget deficits temporarily increase economic activity. The IS curve becomes:

$$x_t = M\mathbb{E}_t\left[x_{t+1}\right] + b_d d_t - \sigma\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - r_t^{n0}\right) \qquad (44)$$

where:

- r_t^{n0} is the "pure" natural rate with zero deficits.
- d_t is the budget deficit.
- $b_d = \frac{\phi r R(1-\bar{(}m)}{(\phi+\gamma)(R-\bar{(}m)}$ is the sensitivity to deficits. When agents are rational, $b_d = 0$, but with behavioral agents, $b_d>0$.

Failure of Ricardian Equivalence

The behavioral IS curve holds, but with the following modified natural rate, which captures the stimulative action of deficits:

$$r_t^n = r_t^{n0} + \frac{b_d}{\sigma} d_t \tag{45}$$

Hence, bounded rationality gives both a discounted IS curve and an impact of fiscal policy: $b_d > 0$. Deficit-financed (lump-sum) tax cuts have a **"stimulative" impact on the economy**.

Consequences for Fiscal Policy

Substitutability of Monetary and Fiscal Policy. Suppose productivity or discount factor shocks (but no cost-push shocks) that alter the natural rate of interest r_t^{n0}.First best is achieved if and only if at all dates:

$$i_t = r_t^n \equiv r_t^{n0} + \frac{b_d}{\sigma} d_t \tag{46}$$

where r_t^{n0} is "pure" natural rate of interest in (23) and is independent of fiscal and monetary policy.

Example: if economy has a lower pure natural interest rate r_t^{n0} (hence "needs loosening"), government can: $\downarrow$ interest rates, or $\uparrow$ deficits $\Rightarrow$ Monetary and fiscal policy are perfect substitutes.

Consequences for Fiscal Policy

Fiscal Transfers as an Optimal Cure, when the ZLB Binds.

Rational Model. When the natural rate becomes negative (and with low inflation), the optimal nominal interest rate is negative ⇒ first best is not achievable and the second best policy is quite complex.

Behavioural Model. Easy first best policy:

$$\text{First best at the ZLB: } i_t = 0 \text{ and deficit: } d_t = \frac{-\sigma}{b_d} r_t^{n0} \tag{47}$$

i.e., fiscal policy runs deficits to stimulate demand.

Consequences for Fiscal Policy

Government Spending Multiplier Greater than One with Behavioral Agents. Suppose government purchases an amount G_t of aggregate good and consumes it. Utility function:

$$U(c,N,G) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi} + U(G)$$

Assumes government purchases G_0 at time 0 financed by a deficit $d_0 = G_0$ Then the fiscal multiplier is:

$$\frac{dY_0}{dG_0} = 1 + b_d \tag{3}$$

reflecting the fact that government spending has a “direct” effect of increasing GDP one-for-one, and then an “indirect” effect of making people feel richer.

Behavioral Enrichments of the Model

1. Introduction
2. Baseline model of the paper
3. Consequences
4. Implications for monetary policy
5. Implications for fiscal policy
6. Behavioral Enrichments of the Model
7. Discussion of the Behavioral Assumptions, Conclusion, Critics

6. Behavioral Enrichments of the Model

6.1 Term Structure of Consumer Attention

6.2 Flattening of the Phillips Curve via Imperfect Firm Attention

6.3 Nonconstant Trend Inflation and Neo-Fisherian Paradoxes

Term Structure of Consumer Attention

It is plausible that consumers **do not equally pay attention to all economic variables**, even in the present. We could therefore introduce attention discount factors that are variable specific, yielding **perceived variables** under Bounded Rationality :

- $\hat{r}^{BR}$ the perceived interest rate under bounded rationality
- $\hat{y}^{BR}$ the perceived income under bounded rationality

Prior to this, consumers perceived perfectly variables at the current period, now, they do not anymore.

Directly affects the consumer maximisation program.

Term Structure of Consumer Attention

The law of motion of the personal wealth of the consumer becomes thus a **perceived law of motion**

$$
\begin{aligned}
k_{t+1} &= G^k(c_t, N_t, k_t, \boldsymbol{X}_t) \\
&:= (1 + \bar{r} + \hat{r}(\boldsymbol{X}_t))(k_t + \bar{y} + \hat{y}(N_t, \boldsymbol{X}t) - c_t)
\end{aligned}
\tag{6}
$$

Turns into :

$$
\begin{aligned}
k_{t+1} &= \mathbf{G}^{k,BR}(c_t, N_t, k_t, \mathbf{X}_t) \\
&:= (1 + \bar{r} + \hat{r}^{BR}(\mathbf{X}_t))(k_t + \bar{y} + \hat{y}^{BR}(N_t, \mathbf{X}_t) - c_t)
\end{aligned}
\tag{49}
$$

What could be functional forms of $\hat{r}^{BR}$ and $\hat{y}^{BR}$?

Term Structure of Consumer Attention

The perceived values of interest rate and income are defined such that :

$$\begin{cases} \hat{r}^{BR} = m_r \cdot \hat{r}(\mathbf{X}_t) \\ \hat{y}^{BR}(N_t, \mathbf{X}_t) = m_y \cdot \hat{y}(\mathbf{X}_t) + \omega(\mathbf{X}_t)(N_t - N_t(\mathbf{X}_t)) \end{cases} \tag{50}$$

Equation (50) is a possible functional form. The main changes are the attention discount factor.
Now, what would be the expectation under behavioral expectation of those perceived values ?

Term Structure of Consumer Attention

Consumers already have a general attention discount factor $\bar{m}$, from Lemma 1 in equation (11) :

$$\mathbb{E}_t^{BR}\left[z\left(\boldsymbol{X}_{t+k}\right)\right]=\bar{m}^k \cdot \mathbb{E}_t\left[z\left(\boldsymbol{X}_{t+k}\right)\right] \tag{11}$$

Applied to the perceived interest rate and perceived income, we thus get the **Lemma 5 (Term Structure of Attention)**:

$$\begin{cases}\mathbb{E}_t^{BR}\left[\hat{r}^{BR}(\mathbf{X}_{t+k})\right]=m_r \cdot \bar{m}^k \cdot \mathbb{E}_t\left[\hat{r}(\mathbf{X}_{t+k})\right] \\ \mathbb{E}_t^{BR}\left[\hat{y}^{BR}(\mathbf{X}_{t+k})\right]=m_y \cdot \bar{m}^k \cdot \mathbb{E}_t\left[\hat{y}(\mathbf{X}_{t+k})\right]\end{cases} \tag{51}$$

Term Structure of Consumer Attention

What are consequences of this enriched attention structure term ?

When we solve for consumption, we get **Proposition 8** (Behavioral Consumption Function) :

$$\hat{c}_t = \mathbb{E}_t \left[\sum_{\tau \geq t} \frac{\bar{m}^{\tau-t}}{R^{\tau-t}} \Big(b_r m_r \hat{r}(\mathbf{X}_\tau) + m_Y \frac{\bar{r}}{R} \hat{y}(\mathbf{X}_\tau) \Big) \right] \tag{52}$$

With :

$$\begin{cases} c_t = c_t^d + \hat{c}_t \\ c_t^d = \bar{y} + b_k \cdot k_t \\ b_k := \frac{\bar{r}}{R} \cdot \frac{\phi}{\phi+\gamma} \end{cases} \qquad \begin{cases} m_Y = \frac{\phi \cdot m_y + \gamma}{\phi+\gamma} \\ b_r := -\frac{1}{\gamma \cdot R^2} \end{cases}$$

Term Structure of Consumer Attention

Interest rate has **direct** and **indirect** effects on consumption.
For a consumer, a decrease in future interest rate :

- increases their present consumption, because it is more profitable to consume right now (direct effect)
- increases other consumers future consumption, increasing their future income, increasing their current consumption (indirect effect)

Therefore, the aggregate consumption multiplies the positive effect on consumption of a decrease in future interest rate.
What does this behavioral model imply for this multiplicator ?

Term Structure of Consumer Attention

In the **rational consumer** case :

If we derive from equation (52), we get the direct effect :

$$\Delta^{\text{direct}} := \left.\frac{\partial \hat{c}_0}{\partial \hat{r}_\tau}\right|_{(y_t)_{t\geq 0} \text{ held constant}} = -\alpha \cdot \frac{1}{R^\tau}$$

If we derive from equation (26), we get the indirect effect :

$$\Delta^{GE} := \frac{\partial \hat{c}_0}{\partial \hat{r}_\tau} = -\alpha R$$

Put together :

$$\frac{\Delta^{GE}}{\Delta^{\text{direct}}} = R^{\tau+1} \tag{53}$$

In the **behavioral consumer** case :

If we derive from equation (52), we get the direct effect :

$$\Delta^{\text{direct}} := \left.\frac{\partial \hat{c}_0}{\partial \hat{r}_\tau}\right|_{(y_t)_{t\geq 0} \text{ held constant}} = -\alpha \cdot m_r \cdot \bar{m}^\tau \frac{1}{R^\tau}$$

If we derive from equation (26), we get the indirect effect :

$$\Delta^{GE} := \frac{\partial \hat{c}_0}{\partial \hat{r}_\tau} = -\alpha m_r \cdot M^\tau \frac{R}{R - r \cdot m_Y} R$$

Put together :

$$\frac{\Delta^{GE}}{\Delta^{\text{direct}}} = \left(\frac{R}{R - r m_Y}\right)^{\tau+1} \in \left[1, R^{\tau+1}\right] \tag{54}$$

Term Structure of Consumer Attention

In a behavioral framework, the multiplicative effect is dampened by bounded rationality.
An attention discount factor that is variable specific allows to explain why forward guidance is not as strong as what theory predicts.
What about variable specific attention deficiency for firms now ?

Flattening of the Phillips Curve via Imperfect Firm Attention

If we introduce variable specific inattention for firms, equation (15), defining the real profit of the firm :

$$v\left(q_{it}, \boldsymbol{X}\tau\right) := v^0\left(q_{it} - \Pi(\boldsymbol{X}\tau), \mu(\boldsymbol{X}\tau), c(\boldsymbol{X}_\tau)\right) \tag{15}$$

Turns into a perceived real profit of the firm :

$$v^{BR}(q_{it}, (\mathbf{X}_\tau)) := v^0\left(q_{it} - m^f_\pi \cdot \Pi(\mathbf{X}_\tau), m^f_x \cdot \mu(\mathbf{X}_\tau), c(\mathbf{X}_\tau)\right) \tag{55}$$

Where :

- m^f_π is the attention deficit to inflation
- m^f_x is the attention deficit to marginal cost

Flattening of the Phillips Curve via Imperfect Firm Attention

The maximisation program of equation (16) :

$$\max_{q_{it}} \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t} \frac{c\left(\boldsymbol{X_\tau}\right)^{-\gamma}}{\left(\boldsymbol{X_t}\right)^{-\gamma}} v\left(q_{it}, \boldsymbol{X}\tau\right) \right] \tag{16}$$

turns into :

$$\max_{q_{it}} \mathbb{E}_t^{BR} \left[\sum_{\tau=t}^{\infty} (\beta\theta)^{\tau-t} \frac{c(\mathbf{X}_\tau)^{-\gamma}}{c(\mathbf{X}_t)^{-\gamma}} v^{BR}(q_{it}, \mathbf{X}_\tau) \right] \tag{56}$$

Flattening of the Phillips Curve via Imperfect Firm Attention

Solving it yields :

$$p_t^* = p_t + (1 - \beta\theta) \cdot \sum_{k=0}^{\infty} (\beta\theta\bar{m})^k \, \mathbb{E}_t \left[m_\pi^f (\pi_{t+1} + ... + \pi_{t+k}) - m_x^f \mu_{t+k} \right] \tag{57}$$

In comparison, we had in the baseline model :

$$p_t^* = p_t + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta\bar{m})^k \cdot \mathbb{E}_t \left[\pi_{t+1} + ... + \pi_{t+k} - \mu_{t+k} \right] \tag{27}$$

Flattening of the Phillips Curve via Imperfect Firm Attention

We also get :

$$M^f = \bar{m}\left(\theta + m_\pi^f \cdot (1-\theta) \cdot \frac{1-\beta\cdot\theta}{1-\beta\cdot\theta\cdot\bar{m}}\right) \in [0,1]$$

$$\kappa = m_x^f \bar{\kappa} \tag{58}$$

Where

- M^f is the general attention factor of the firm
- m_x^f is the attention deficiency to the output gap
- $\kappa = m_x^f \cdot \bar{\kappa}$, is the perceived value of the importance of outputgap on inflation

Flattening of the Phillips Curve via Imperfect Firm Attention

If we solve the Phillips curve, the equation (29) :

$$\pi_t = \beta \cdot M^f \cdot \mathbb{E}_t \left[\pi_{t+1}\right] + \kappa \cdot x_t \tag{29}$$

Turns into a Phillips Curve with Behavioral Firms, allowing for imperfect attention to inflation and costs (**Proposition 10**) :

$$\pi_t = \beta \cdot \bar{m} \left(\theta + m_\pi^f \cdot (1-\theta) \cdot \frac{1-\beta \cdot \theta}{1-\beta \cdot \theta \cdot \bar{m}} \right) \cdot \mathbb{E}_t \left[\pi_{t+1}\right] + m_x^f \cdot \bar{\kappa} \cdot x_t$$

Nonconstant Trend Inflation and Neo-Fisherian Paradoxes

Now, let's change the way we consider inflation. Instead of a steady state, agents perceive a default value. The article proposes the following functional form :

$$\pi_t^d = (1-\zeta)\bar{\pi}_t + \zeta\bar{\pi}_t^{CB} \tag{59}$$

The IS curve is unchanged, and is the same as equation (28) :

$$x_t = M\mathbb{E}_t\left[x_{t+1}\right] - \sigma\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - r_t^n\right) \tag{60}$$

Nonconstant Trend Inflation and Neo-Fisherian Paradoxes

The Phillips curve of equation (29) :

$$\pi_t = \beta \cdot M^f \mathbb{E}_t \left[\pi_{t+1}\right] + \kappa \cdot x_t \tag{29}$$

Turns into :

$$\hat{\pi}_t = \beta \cdot M^f \cdot \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \kappa \cdot x_t \tag{61}$$

The only difference is through $\hat{\pi}_t$, with $\pi_t = \pi_t^d + \hat{\pi}_t$.

Nonconstant Trend Inflation and Neo-Fisherian Paradoxes

Finally, the equilibrium condition changes through ζ the weight given to the Central Bank declaration. Equation (34) in the Baseline model :

$$\phi_\pi + \frac{(1-\beta M^f)}{\kappa}\phi_x + \frac{(1-\beta M^f)(1-M)}{\kappa\sigma} > 1 \tag{34}$$

Turns into :

$$\phi_\pi + \zeta\frac{(1-\beta M^f)}{\kappa}\phi_x + \zeta\frac{(1-\beta M^f)(1-M)}{\kappa\sigma} > 1 \tag{62}$$

The term ζ can be considered as the weight of central bank guidance.

- What if it is 0 ?
- What are the consequences for Central Bankers ?

<u>**Neo-fisherian neutrality**</u> : In the long-run, inflation (money) should not affect output.

In the New-Keynesian framework, it does not work : if inflation is permanently higher, then output is permanent higher. Let $M^f = 1$, then :

$$\pi_t = \beta \cdot M^f \cdot \mathbb{E}_t \left[\pi_{t+1}\right] + \kappa \cdot x_t \tag{29}$$

Turns in the steady state into :

$$\pi = \beta \cdot \pi + \kappa \cdot x$$

$$\iff$$

$$\pi \cdot \frac{1-\beta}{\kappa} = x$$

Nonconstant Trend Inflation and Neo-Fisherian Paradoxes

In the current extension, it is not the case :

Trend in inflation :

$$\pi_t = \pi_t^d + \hat{\pi}_t$$

Perception of the trend :

$$\pi_t^d = (1-\zeta) \cdot \bar{\pi}_t + \zeta \cdot \bar{\pi}_t^{CB} \tag{59}$$

Phillips curve :

$$\hat{\pi}_t = \beta \cdot M^f \cdot \mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \kappa \cdot x_t \tag{61}$$

IS curve :

$$x_t = M \cdot \mathbb{E}_t\left[x_{t+1}\right] - \sigma\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - r_t^n\right) \tag{60}$$

We find $\implies i = r^n + \bar{\pi}$ and no impact of long run inflation over output.

Conclusion of those enrichments :

- Variable specific attention deficiency allows for more detail in the specification.
- Allows to highlight even more the conclusions of the general model
- Answer both empirical problems and pure theoretical questions

Discussion of the Behavioral Assumptions, Conclusion, Critics

1. Introduction

2. Baseline model of the paper

3. Consequences

4. Implications for monetary policy

5. Implications for fiscal policy

6. Behavioral Enrichments of the Model

7. Discussion of the Behavioral Assumptions, Conclusion, Critics

7. Discussion of the Behavioral Assumptions, Conclusion, Critics

7.1 Behavioural enrichments

7.2 Conclusion

7.3 Critics

Behavioural enrichments

Theoretical Microfoundations: evidence exists for the inattention to small parameters, but no direct evidence on cognitive discounting. More empirical research on this is required despite the presence of extant evidence. Literature exists on cognitive discounting closely resembling hyperbolic discounting

Lucas Critique: Attention is endogenised in the appendix to reflect the fact that the attention becomes more intense when the volatility/deviation in the environment is more. Lucas critique hence becomes relevant for large changes.

Behavioural enrichments

Long Run Learning: Learning and Attention is costly and require effort. Hence the agent might not learn anything in the long run.

Parsimony and new variants: The current model is quite parsimonious and with just one standard parameter; $\bar{m}$. The term structure of attention enriches the model and the measurement of such an attention parameters is better. There is scope for improving the measurement of macro parameter of attention. Reasonable variations are possible for this model and the author chooses an *"a happy balance between tractability, parsimony, and psychological and macroeconomic realism"*

Conclusion

1. This parsimonious model can be expanded examples include capital accumulation, a more frictional labor market, distortionary taxes, and agents that are heterogeneous in wealth or rationality.
2. Requirement for more empirical work to estimate attention to current variables and future variables.
3. Introduce survey designs which measures people's subjective view of the world.

Critics

Overall the paper acknowledges that the model presented is parsimonious and more variations to it can be added. It more or less exhaustively covers the variations while discussing the possible behavioural enrichments.

However, a few critiques
1. While modeling welfare for behavioral agents, the paper does not use subjective expectation.
2. It is possible that agents possess heterogeneous beliefs and can have different ways of expressing their partial myopia.
3. There is a need for more empirical testing for behavioral parameters, which the paper also acknowledges.