

$$a_{11}^1 + a_{11}^2 + a_{11}^3 + a_{21}^1 + a_{21}^2 + a_{21}^3 + a_{31}^1 + a_{31}^2 + a_{31}^3 = -8e_1 \otimes f^1 \otimes f^1 - 8e_2 \otimes f^1 \otimes f^1 + 2e_3 \otimes f^1 \otimes f^1 - 24e_1 \otimes f^2 \otimes f^1 - 24e_2 \otimes f^2 \otimes f^1 + 6e_3 \otimes f^2 \otimes f^1 + 24e_1 \otimes f^3 \otimes f^1 + 24e_2 \otimes f^3 \otimes f^1 - 6e_3 \otimes f^3 \otimes f^1 = \\ = 2 \left(-(4e_1 + 4e_2 - e_3) \otimes f^1 - 3(4e_1 - 4e_2 + e_3) \otimes f^2 + 3(4e_1 + 4e_2 - e_3) \otimes f^3 \right) \otimes f^1 = 2(4e_1 + 4e_2 - e_3) \left(-f^1 - 3f^2 + 3f^3 \right) \otimes f^1$$

Второй слой идентичен первому, поэтому...

$$a_{12}^1 + a_{12}^2 + a_{12}^3 + a_{22}^1 + a_{22}^2 + a_{22}^3 + a_{32}^1 + a_{32}^2 + a_{32}^3 = -8e_1 \otimes f^1 \otimes f^2 - 8e_2 \otimes f^1 \otimes f^2 + 2e_3 \otimes f^1 \otimes f^2 - 24e_1 \otimes f^2 \otimes f^2 - 24e_2 \otimes f^2 \otimes f^2 + 6e_3 \otimes f^2 \otimes f^2 + 24e_1 \otimes f^3 \otimes f^2 + 24e_2 \otimes f^3 \otimes f^2 - 6e_3 \otimes f^3 \otimes f^2 = \\ = 2 \left(-(4e_1 + 4e_2 - e_3) \otimes f^1 - 3(4e_1 - 4e_2 + e_3) \otimes f^2 + 3(4e_1 + 4e_2 - e_3) \otimes f^3 \right) \otimes f^2 = 2(4e_1 + 4e_2 - e_3) \left(-f^1 - 3f^2 + 3f^3 \right) \otimes f^2$$

$$a_{13}^1 + a_{13}^2 + a_{13}^3 + a_{23}^1 + a_{23}^2 + a_{23}^3 + a_{33}^1 + a_{33}^2 + a_{33}^3 = 12e_1 \otimes f^1 \otimes f^3 + 12e_2 \otimes f^1 \otimes f^3 - 3e_3 \otimes f^1 \otimes f^3 + 36e_1 \otimes f^2 \otimes f^3 + 36e_2 \otimes f^2 \otimes f^3 - 9e_3 \otimes f^2 \otimes f^3 - 36e_1 \otimes f^3 \otimes f^3 - 36e_2 \otimes f^3 \otimes f^3 + 9e_3 \otimes f^3 \otimes f^3 = \\ = 3 \left((4e_1 + 4e_2 - e_3) \otimes f^1 + 3(4e_1 + 4e_2 - e_3) \otimes f^2 - 3(4e_1 + 4e_2 - e_3) \otimes f^3 \right) \otimes f^3 = -3(4e_1 + 4e_2 - e_3) \left(-f^1 - 3f^2 + 3f^3 \right) \otimes f^3$$

Тогда результирующее разложение:

$$a = 2(4e_1 + 4e_2 - e_3) \left(-f^1 - 3f^2 + 3f^3 \right) \otimes f^1 + 2(4e_1 + 4e_2 - e_3) \left(-f^1 - 3f^2 + 3f^3 \right) \otimes f^2 - 3(4e_1 + 4e_2 - e_3) \left(-f^1 - 3f^2 + 3f^3 \right) \otimes f^3 = \\ \boxed{(4e_1 + 4e_2 - e_3) \left(-f^1 - 3f^2 + 3f^3 \right) (2f^1 + 2f^2 - 3f^3)}$$