РГР №1

Овчинников П.А.
368606, 22.1
Вариант №6

1. $z=\cos^3\left(xy+\sqrt{\frac{x+y}{x-y}}\right)+e^{-x/y}$

$$\frac{\partial z}{\partial x}=-3y\cos^2\left(xy+\sqrt{\frac{x+y}{x-y}}\right)\sin\left(xy+\sqrt{\frac{x+y}{x-y}}\right)\left(1-\frac{1}{\sqrt{\frac{x+y}{x-y}}\,(x^2-2xy+y^2)}\right)-\frac{e^{-x/y}}{y}=$$

$$=\boxed{-3y\cos^2\left(xy+\sqrt{\frac{x+y}{x-y}}\right)\sin\left(xy+\sqrt{\frac{x+y}{x-y}}\right)\left(1-\frac{1}{\sqrt{x+y}}\right)-\frac{e^{-x/y}}{y}}$$

$$\frac{\partial z}{\partial y}=-3x\cos^2\left(xy+\sqrt{\frac{x+y}{x-y}}\right)\sin\left(xy+\sqrt{\frac{x+y}{x-y}}\right)\left(1+\frac{1}{\sqrt{\frac{x+y}{x-y}}\,(y^2-2xy+x^2)}\right)+\frac{xe^{-x/y}}{y^2}=$$

$$=\boxed{x\left(-3\cos^2\left(xy+\sqrt{\frac{x+y}{x-y}}\right)\sin\left(xy+\sqrt{\frac{x+y}{x-y}}\right)\left(1+\frac{1}{\sqrt{x+y}}\right)+\frac{e^{-x/y}}{y^2}\right)}$$

2. $u=(\sin x)^{y+z}=\sin^y x\cdot\sin^z x$

$$du=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy+\frac{\partial u}{\partial z}dz=\boxed{(\sin x)^{y+z-1}(y+z)\cos x\,dx+(\sin x)^{y+z}\ln(\sin x)\,dy+(\sin x)^{y+z}\ln(\sin x)\,dz}$$

$$\frac{\partial u}{\partial x}=y\sin^{y-1}x\cdot\cos x\cdot\sin^z x+z\sin^y x\cdot\sin^{z-1}x\cdot\cos x=(\sin x)^{y+z-1}(y+z)\cos x$$

$$\frac{\partial u}{\partial y}=\sin^y x\cdot\ln(\sin x)\cdot\sin^z x=(\sin x)^{y+z}\ln(\sin x)$$

$$\frac{\partial u}{\partial z}=\sin^y x\cdot\sin^z x\cdot\ln(\sin x)=(\sin x)^{y+z}\ln(\sin x)$$

3. $u=\frac{e^{ax}(y-z)}{a^2}$

$$\frac{du}{dx}=\frac{\partial u}{\partial y}\cdot\frac{dy}{dx}+\frac{\partial u}{\partial z}\cdot\frac{dz}{dx}=\boxed{\frac{e^{ax}}{a^{\cancel{2}}}\cancel{a}\cos x-\frac{e^{ax}}{a^2}\left(\frac{3\sin(3/x)}{x^2}+\frac{1}{2\sin^2(x/2)}\right)=\frac{e^{ax}}{a}\left(\cos x-\frac{3\sin(3/x)}{ax^2}+\frac{1}{2a\sin^2(x/2)}\right)}$$

$$\frac{\partial u}{\partial y}=\left(e^{ax}(y-z)a^{-2}\right)'_y=\frac{e^{ax}}{a^2}\qquad \frac{\partial u}{\partial z}=\frac{-e^{ax}}{a^2}$$

$$\frac{dy}{dx}=a\cos x\qquad \frac{dz}{dx}=\frac{3\sin(3/x)}{x^2}+\frac{1}{2\sin^2(x/2)}$$

4. $u=\frac{1-e^{xy}}{\sqrt{x}+\sqrt{y}}$; $x=r\cos\varphi$, $y=r\sin\varphi$

$$\frac{\partial u}{\partial x}=\frac{-ye^{xy}(\sqrt{x}+\sqrt{y})-(1-e^{xy})\frac{1}{2\sqrt{x}}}{x+2\sqrt{xy}+y}\qquad \frac{\partial u}{\partial y}=\frac{-xe^{xy}(\sqrt{x}+\sqrt{y})-(1-e^{xy})\frac{1}{2\sqrt{y}}}{x+2\sqrt{xy}+y}$$

$$\frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial r}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial r}=\boxed{\frac{\left(\frac{e^{xy}-1}{2\sqrt{x}}-ye^{xy}(\sqrt{x}+\sqrt{y})\right)\cos\varphi+\left(\frac{e^{xy}-1}{2\sqrt{y}}-xe^{xy}(\sqrt{x}+\sqrt{y})\right)\sin\varphi}{x+2\sqrt{xy}+y}}$$

$$\frac{\partial x}{\partial r}=\cos\varphi\qquad \frac{\partial y}{\partial r}=\sin\varphi$$

$$\frac{\partial u}{\partial \varphi}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial \varphi}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial \varphi}=\boxed{\frac{r\left(-\left(\frac{e^{xy}-1}{2\sqrt{x}}-ye^{xy}(\sqrt{x}+\sqrt{y})\right)\sin\varphi+\left(\frac{e^{xy}-1}{2\sqrt{y}}-xe^{xy}(\sqrt{x}+\sqrt{y})\right)\cos\varphi\right)}{x+2\sqrt{xy}+y}}$$

$$\frac{\partial x}{\partial \varphi}=-r\sin\varphi\qquad \frac{\partial y}{\partial \varphi}=r\cos\varphi$$

5. $F(x;y;z)=x^{2/3}+y^{2/3}+z^{2/3}-a^{2/3}=0$

$$\frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}=-\frac{\frac{2}{3\sqrt[3]{x}}}{\frac{2}{3\sqrt[3]{z}}}=\boxed{-\sqrt[3]{\frac{z}{x}}}$$

Тогда в контексте одинаковых степеней и коэффициентов перед переменными x и y очевидно, что:

$$\frac{\partial z}{\partial y}=\boxed{-\sqrt[3]{\frac{z}{y}}}$$

6. $u=\ln\frac{x^2-y^2}{xy}$ $\quad \frac{\partial^3 u}{\partial x^2\partial y}$ — ?

$$\frac{\partial u}{\partial x}=\frac{\cancel{xy}}{x^2-y^2}\cdot\frac{2x^2y-(x^2-y^2)y}{x^{\cancel{2}}y^{\cancel{2}}}=\frac{x^2+y^2}{x^3-xy^2}$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{2x(x^3-xy^2)-(x^2+y^2)(3x^2-y^2)}{(x^3-xy^2)^2}=\frac{2x^4-2x^2y^2-3x^4-2x^2y^2+y^4}{(x^3-xy^2)^2}=\frac{-x^4-4x^2y^2+y^4}{(x^3-xy^2)^2}$$

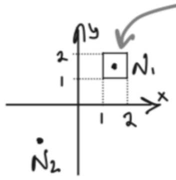
$$\frac{\partial^3 u}{\partial x^2\partial y}=\frac{(4y^3-8x^2y)(x^3-xy^2)^{\cancel{2}}-4(x^4+4x^2y^2-y^4)\cancel{(x^3-xy^2)}xy}{(x^3-xy^2)^{\cancel{4}3}}=\frac{4(x^3y^3-\cancel{xy^5}-2x^5y+2x^3y^3-x^5y-4x^3y^3+\cancel{xy^5})}{(x^3-xy^2)^3}=$$

$$=\frac{4x^{\cancel{3}}y(-3x^2-y^2)}{\cancel{x^3}(x^2-y^2)^3}=\boxed{\frac{-4y(3x^2+y^2)}{(x^2-y^2)^3}}$$

7. $z=x^3-3xy^2+18y$ extr — ? внутри $\begin{cases}x=1\\y=1\\x=2\\y=2\end{cases}$

$$\begin{cases}\frac{\partial z}{\partial x}=3x^2-3y^2\\ \frac{\partial z}{\partial y}=18-6xy\end{cases}\Leftrightarrow\left[\begin{array}{l}\begin{cases}x=\sqrt3\\y=\sqrt3\end{cases} N_1(\sqrt3;\sqrt3)\\ \begin{cases}x=-\sqrt3\\y=-\sqrt3\end{cases} N_2(-\sqrt3;-\sqrt3)\end{array}\right.$$

замкнутая обл. — квадрат



$$A=\frac{\partial^2 z}{\partial x^2}\Big|_{N_1}=(3x^2-3y^2)'_x=6x=6\sqrt3$$

$$B=\frac{\partial^2 z}{\partial x\partial y}\Big|_{N_1}=(3x^2-3y^2)'_y=-6y=-6\sqrt3$$

$$C=\frac{\partial^2 z}{\partial y^2}\Big|_{N_1}=(18-6xy)'_y=-6x=-6\sqrt3$$

$$AC-B^2=6\sqrt3\cdot(-6\sqrt3)-(-6\sqrt3)^2=-108-108=-216<0$$

$\Downarrow$

N_1 не является extr

$\Downarrow$

$\boxed{\text{в заданной области нет extr}}$

8. $x^2+y^2+z^2=169 \Leftrightarrow \frac{x^2}{169}+\frac{y^2}{169}+\frac{z^2}{169}=1$ $\qquad M(3;4;12)$

$$\frac{\partial F}{\partial x}\Big|_M=\frac{2x}{169}\Big|_M=\frac{6}{169}$$

$$\frac{\partial F}{\partial y}\Big|_M=\frac{2y}{169}\Big|_M=\frac{8}{169}$$

$$\frac{\partial F}{\partial z}\Big|_M=\frac{2z}{169}\Big|_M=\frac{24}{169}$$

Касательная: $\frac{6(x-x_0)}{169}+\frac{8(y-y_0)}{169}+\frac{24(z-z_0)}{169}=0$

Нормаль: $\frac{169(x-x_0)}{6}=\frac{169(y-y_0)}{8}=\frac{169(z-z_0)}{24}$

9. $z=\ln(e^x+e^y+e^z)$ $\quad O(0;0;0)$ $\quad$ grad $z(0)$ — ? $\quad \bar l\left(\frac{1}{\sqrt2};\frac12;\frac12\right)=\frac{\sqrt2}{2}\bar i+\frac12\bar j+\frac12\bar k$

$$\frac{\partial z}{\partial x}\Big|_0=\frac{e^x}{e^x+e^y+e^z}\Big|_0=\frac13 \qquad \frac{\partial z}{\partial y}\Big|_0=\frac{e^y}{e^x+e^y+e^z}\Big|_0=\frac13 \qquad \frac{\partial z}{\partial z}\Big|_0=\frac{e^z}{e^x+e^y+e^z}\Big|_0=\frac13$$

$$\text{grad}\, z(0)=\frac{\partial z}{\partial x}\Big|_0\bar i+\frac{\partial z}{\partial y}\Big|_0\bar j+\frac{\partial z}{\partial z}\Big|_0\bar k=\boxed{\frac{\bar i}{3}+\frac{\bar j}{3}+\frac{\bar k}{3}}$$

$$\frac{\partial z}{\partial \bar l}\Big|_0=\frac{\partial z}{\partial x}\Big|_0\cos\alpha+\frac{\partial z}{\partial y}\Big|_0\cos\beta+\frac{\partial z}{\partial z}\Big|_0\cos\gamma=\frac13\cdot\frac{\sqrt2}{2}+\frac13\ \frac12+\frac13\cdot\frac12=\frac{\sqrt2}{6}+\frac26=\boxed{\frac{\sqrt2+2}{6}}$$

$(\cos\alpha;\cos\beta;\cos\gamma)$ — един. вектор, сонаправленный с $\bar l$

$$\bar l_0=\frac{\bar l}{|\bar l|}=\frac{\frac{\sqrt2}{2}\bar i+\frac12\bar j+\frac12\bar k}{\sqrt{\frac24+\frac14+\frac14}}=\frac{\sqrt2}{2}\bar i+\frac12\bar j+\frac12\bar k \Rightarrow \begin{cases}\cos\alpha=\sqrt2/2\\ \cos\beta=1/2\\ \cos\gamma=1/2\end{cases}$$