CENTER FOR
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CZECH TECHNICAL
UNIVERSITY IN PRAGUE

MASTER'S THESIS

Semidefinite Programming for Geometric Problems in Computer Vision

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Author’s declaration

I declare that I have work out the presented thesis independently and that I have listed all information sources used in accordance with the Methodical Guidelines about Maintaining Ethical Principles for Writing Academic Theses.

Prohlášení autora práce

Prohlašuji, že jsem předloženou práci vypracoval samostatně a že jsem uvedl veškeré použité informační zdroje v souladu s Metodickým pokynem o dodržování etických principů při přípravě vysokoškolských závěrečných prací.

V Praze dne ....
Podpis autora práce

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Abstrakt

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List of Algorithms

1. Introduction

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2. Semidefinite programming

2.1. Preliminaries on semidefinite programs

We introduce here some notation and preliminaries about symmetric matrices and semidefinite programs. We will introduce further notation and preliminaries later on in the text when needed.

At the beginning, let us denote the inner product for two vectors x, $y \in \mathbb{R}^n$.

$$\langle x, y \rangle = \sum_{i=1}^{n} x_i y_i \tag{2.1}$$

And the Frobenius inner product for two matrices X, $Y \in \mathbb{R}^{n \times m}$.

$$\langle X, Y \rangle = \sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij} Y_{ij} \tag{2.2}$$

2.1.1. Symmetric matrices

Let Sym_n denotes the space of $n \times n$ real symmetric matrices.

For matrix $M \in \mathrm{Sym}_n$, the notation $M \succeq 0$ means that M is positive semidefinite. $M \succeq 0$ if and only if any of the following equivalent properties holds.

1. $x^\top M x \geq 0$ for all $x \in \mathbb{R}^n$.
2. All eigenvalues of M are nonnegative.

For matrix $M \in \mathrm{Sym}_n$, the notation $M \succ 0$ means that M is positive definite. $M \succ 0$ if and only if any of the following equivalent properties holds.

1. $M \succeq 0$ and $\operatorname{rank} M = n$.
2. $x^\top M x > 0$ for all $x \in \mathbb{R}^n$.
3. All eigenvalues of M are positive.

2.1.2. Semidefinite programs

The standard (primal) form of a semidefinite program in variable $X \in \mathrm{Sym}_n$ is defined as follows:

$$\begin{array}{rcll} p^* & = & \sup\limits_{X \in \mathrm{Sym}^n} & \langle C, X \rangle \\ & & \text{s.t.} & \langle A_i, X \rangle \;=\; b_i \quad (i = 1, \ldots, m) \\ & & & \qquad X \;\succeq\; 0 \end{array} \tag{2.3}$$

where C, A_1, $\ldots$, $A_m \in \mathrm{Sym}_n$ and $b \in \mathbb{R}^m$ are given.

The dual form of the primal form is the following program in variable $y \in \mathbb{R}^m$.

$$\begin{array}{rcll} d^* & = & \inf\limits_{y \in \mathbb{R}^m} & b^\top y \\ & & \text{s.t.} & \sum\limits_{i=1}^{m} A_i y_i - C \;\succeq\; 0 \end{array} \tag{2.4}$$

2.2. State of the art review

2.3. Theoretical background

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