پاسخ سوال ۲:

$$R_1(x,y)=\frac{0.3}{(x_1,y_1)}+\frac{0.7}{(x_1,y_2)}+\frac{0}{(x_1,y_3)}+\frac{0.2}{(x_2,y_1)}+\frac{0.1}{(x_2,y_2)}+\frac{0.9}{(x_2,y_3)}+\frac{0.8}{(x_3,y_1)}+\frac{0.6}{(x_3,y_2)}+\frac{0.4}{(x_3,y_3)}$$

گسترش سیلندر R_1

$$\Rightarrow R_1(x,y,z)=\frac{0.3}{(x_1,y_1,z_1)}+\frac{0.3}{(x_1,y_1,z_2)}+\frac{0.7}{(x_1,y_2,z_1)}+\frac{0.7}{(x_1,y_2,z_2)}+\frac{0}{(x_1,y_3,z_1)}+\frac{0}{(x_1,y_3,z_2)}$$

$$+\frac{0.2}{(x_2,y_1,z_1)}+\frac{0.2}{(x_2,y_1,z_2)}+\frac{0.1}{(x_2,y_2,z_1)}+\frac{0.1}{(x_2,y_2,z_2)}+\frac{0.9}{(x_2,y_3,z_1)}+\frac{0.9}{(x_2,y_3,z_2)}+\frac{0.8}{(x_3,y_1,z_1)}+\frac{0.8}{(x_3,y_1,z_2)}$$

$$+\frac{0.6}{(x_3,y_2,z_1)}+\frac{0.6}{(x_3,y_2,z_2)}+\frac{0.4}{(x_3,y_3,z_1)}+\frac{0.4}{(x_3,y_3,z_2)}$$

$$R_2(y,z)=\frac{0.1}{(y_1,z_1)}+\frac{0.1}{(y_1,z_2)}+\frac{0.2}{(y_2,z_1)}+\frac{0.2}{(y_2,z_2)}+\frac{0.3}{(y_3,z_1)}+\frac{0.3}{(y_3,z_2)}$$

گسترش سیلندر R_2

$$\Rightarrow \frac{0.1}{(x_1,y_1,z_1)}+\frac{0.1}{(x_2,y_1,z_1)}+\frac{0.1}{(x_3,y_1,z_1)}+\frac{0.1}{(x_1,y_1,z_2)}+\frac{0.1}{(x_2,y_1,z_2)}+\frac{0.1}{(x_3,y_1,z_2)}+\frac{0.2}{(x_1,y_2,z_1)}$$

$$+\frac{0.2}{(x_2,y_2,z_1)}+\frac{0.2}{(x_3,y_2,z_1)}+\frac{0.2}{(x_1,y_2,z_2)}+\frac{0.2}{(x_2,y_2,z_2)}+\frac{0.2}{(x_3,y_2,z_2)}+\frac{0.3}{(x_1,y_3,z_1)}+\frac{0.3}{(x_2,y_3,z_1)}+\frac{0.3}{(x_3,y_3,z_1)}$$

$$+\frac{0.3}{(x_1,y_3,z_2)}+\frac{0.3}{(x_2,y_3,z_2)}+\frac{0.3}{(x_3,y_3,z_2)}$$

$$\mathrm{produce}[R_1,R_2]=\frac{0.03}{(x_1,y_1,z_1)}+\frac{0.03}{(x_1,y_1,z_2)}+\frac{0.14}{(x_1,y_2,z_1)}+\frac{0.14}{(x_1,y_2,z_2)}+\frac{0}{(x_1,y_3,z_1)}+\frac{0}{(x_1,y_3,z_2)}$$

$$+\frac{0.02}{(x_2,y_1,z_1)}+\frac{0.02}{(x_2,y_2,z_2)}+\frac{0.02}{(x_2,y_2,z_1)}+\frac{0.02}{(x_2,y_2,z_2)}+\frac{0.27}{(x_2,y_3,z_1)}+\frac{0.27}{(x_2,y_3,z_2)}+\frac{0.08}{(x_3,y_1,z_1)}$$

$$+\frac{0.08}{(x_3,y_1,z_2)}+\frac{0.12}{(x_3,y_2,z_1)}+\frac{0.12}{(x_3,y_2,z_2)}+\frac{0.12}{(x_3,y_3,z_1)}+\frac{0.12}{(x_3,y_3,z_2)}$$

$$\underset{y\in V}{\mathrm{Max}}\left[\mathrm{produce}[R_1,R_2]\right]=\frac{0.14}{(x_1,z_1)}+\frac{0.14}{(x_1,z_2)}+\frac{0.27}{(x_2,z_1)}+\frac{0.27}{(x_2,z_2)}+\frac{0.12}{(x_3,z_1)}+\frac{0.12}{(x_3,z_2)}$$

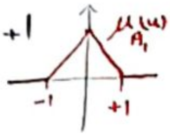
$$\Longrightarrow R_1 \circ R_2(x,y)=\begin{bmatrix}0.14 & 0.14\\ 0.27 & 0.27\\ 0.12 & 0.12\end{bmatrix}$$

پاسخ سوال ۳)

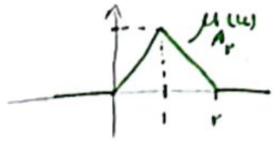
قاعده ۱: if x_1 is A_1 and x_2 is A_2, Then y is A_1

قاعده ۲: if x_1 is A_2 and x_2 is A_1, Then y is A_2

$$\mu_{A_1}(u)=\begin{cases}1-|u| & \text{if } -1\le u\le +1\\ 0 & \text{otherwise}\end{cases}$$



$$,\quad \mu_{A_2}(u)=\begin{cases}1-|u-1| & \text{if } 0\le u\le 2\\ 0 & \text{otherwise}\end{cases}$$



$(x_1^*, x_2^*) = (0.3, 0.6)$ فازی‌ساز Singleton

$$\mu_{A'}(x)=\begin{cases}1 & \text{if } x=x^*\\ 0 & \text{otherwise}\end{cases}$$

خروجی y^* تحت موتور استنتاج لوکاشویتز

$$\mu_{B'}(y)=\min_{l=1}^{M}\left\{\sup_{x\in U}\min\left[\mu_{A'}(x),\ 1-\min_{i=1}^{n}\left(\mu_{A_i^l}(x_i^*)\right)+\mu_{B^l}(y)\right]\right\}$$

و با غیرفازی‌ساز Center Average

$$y^*=\frac{\sum_{l=1}^{M}\bar{y}^l w_l}{\sum_{l=1}^{M} w_l}$$

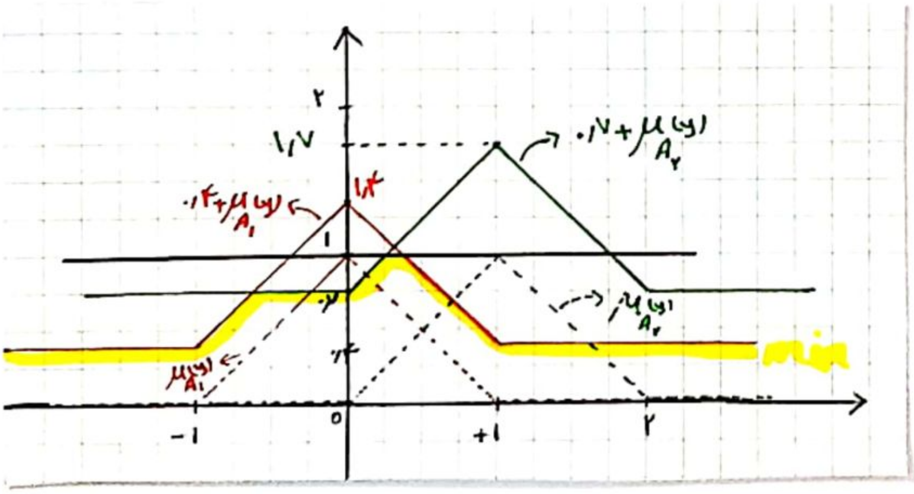
چون A' یک Singleton است، موتور استنتاج لوکاشویتز به صورت زیر ساده می‌شود

$$\mu_{B'}(y)=\min_{l=1}^{M}\left[1,\ 1-\min_{i=1}^{n}\left(\mu_{A_i^l}(x_i^*)\right)+\mu_{B^l}(y)\right]$$

$$\mu_{B'}(y)=\mathrm{Min}\Big[1,\ 1-\underbrace{\mathrm{Min}\left(\mu_{A_1}(x_1^*)+\mu_{A_2}(x_2^*)\right)}_{\text{قاعده ۱}}+\mu_{A_1}(y),\ 1-\underbrace{\mathrm{Min}\left(\mu_{A_2}(x_1^*),\mu_{A_1}(x_2^*)\right)}_{\text{قاعده ۲}}+\mu_{A_2}(y)\Big]$$

$$\Rightarrow \mu_{B'}(y)=\mathrm{Min}\Big[1,\ 1-\mathrm{Min}\big(\overset{0.7}{\mu_{A_1}(0.3)},\ \overset{0.6}{\mu_{A_2}(0.6)}\big)+\mu_{A_1}(y),\ 1-\mathrm{Min}\big(\overset{0.3}{\mu_{A_2}(0.3)},\ \overset{0.4}{\mu_{A_1}(0.6)}\big)+\mu_{A_2}(y)\Big]$$

$$\Rightarrow \mu_{B'}(y)=\mathrm{Min}\left[1,\ (1-0.6)+\mu_{A_1}(y),\ (1-0.3)+\mu_{A_2}(y)\right]\Rightarrow \mu_{B'}(y)=\mathrm{Min}\left[1,\ 0.4+\mu_{A_1}(y),\ 0.7+\mu_{A_2}(y)\right]$$



$$y_1^*=\frac{0\times 1+1\times 1}{1+1}=\frac{1}{2}=0.5$$

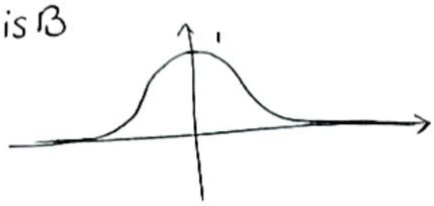
پاسخ سوال ۴

قاعده: if x_1 is A_1 and ... and x_n is A_n, Then y is B

min

گزاره فازی مقدمه

$\mu_B(y)=\exp(-y^2)$



$\mu_{A'}(x)=\begin{cases}1 & \text{if } x=x^* \\ 0 & \text{otherwise}\end{cases}$

الف) تابع عضویت خروجی $\mu_{B'}(y)$ تحت موتور استنتاج ضرب رسم کنید.

$\min[\mu_{A_1}(x_1^*),\dots,\mu_{A_n}(x_n^*)]=\mu_{A_p}(x_p^*)$ (1)

$\mu_{B'}(y)=\max_{l=1}^{M}\left[\sup_{x\in U}\left(\mu_{A'}(x)\times\prod_{i=1}^{n}(\mu_{A_i^l}(x_i)\mu_{B^l}(y))\right)\right]$ ، چون $M=1 \Rightarrow$ نیازی به Max گیری نداریم

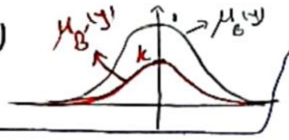
چون از فازی ساز Singleton استفاده می‌کنیم، موتور استنتاج به صورت زیر ساده می‌شود. $\exp(-y^2)$

$\Rightarrow \mu_{B'}(y)=\cancel{\sup_{x\in U}}\left(\mu_{A'}(x)\times\prod_{i=1}^{n}\mu_{A_i^l}(x_i)\mu_{B^l}(y)\right)$

$\mu_{B'}(y)=\prod_{l=1}^{n}\mu_{A_i^l}(x^*)\mu_{B^l}(y) \Rightarrow \mu_{B'}(y)=\mu_{A_1}(x_1^*)\times\mu_{A_2}(x_2^*)\times\dots\times\mu_{A_n}(x_n^*)\times\mu_B(y)$

$0<k<1$ ← k عدد ثابت $\mu_{A_p}(x_p^*)$

$\Rightarrow \mu_{B'}(y)=k\exp(-y^2)$



ب) تابع عضویت خروجی $\mu_{B'}(y)$ را تحت موتور استنتاج زاده رسم کنید.

$\mu_{B'}(y)=\min_{l=1}^{M}\left\{\sup_{x\in U}\min\left[\mu_{A'}(x),\max\left(\min(\mu_{A_1^l}(x_1),\dots,\mu_{A_n^l}(x_n),\mu_{B^l}(y)),1-\min_{i=1}^{n}(\mu_{A_i^l}(x_i))\right)\right]\right\}$

چون از فازی کننده Singleton استفاده شده، موتور استنتاج زاده به صورت زیر ساده می‌شود

$\mu_{B'}(y)=\min_{l=1}^{M}\left\{\max\left[\min\left(\boxed{\mu_{A_1^l}(x_1^*),\dots,\mu_{A_n^l}(x_n^*)},\mu_{B^l}(y)\right),1-\boxed{\min_{i=1}^{n}(\mu_{A_i^l}(x_i^*))}\right]\right\}$ ، چون $M=1 \Rightarrow$ نیازی به min گیری نداریم

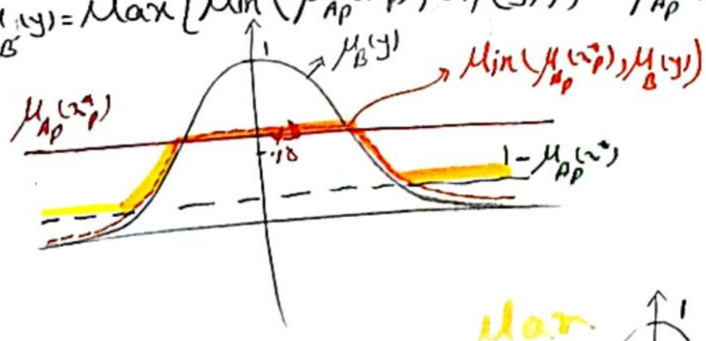
(1) (1)

$\exp(-y^2)$

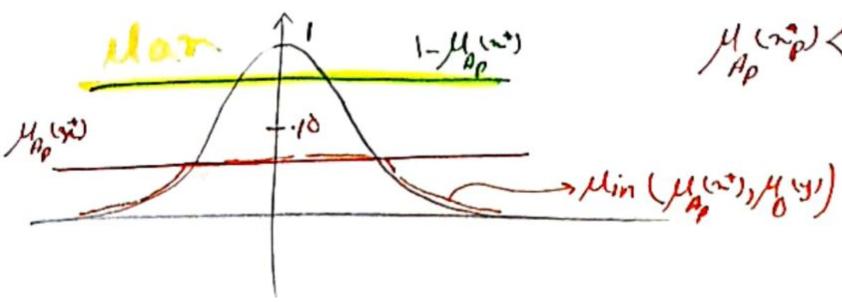
$\Rightarrow \mu_{B'}(y)=\max\left[\min\left(\mu_{A_p}(x_p^*),\mu_B(y)\right),1-\mu_{A_p}(x_p^*)\right]$

$\Rightarrow \mu_{B'}(y)=\max\left[\min\left(\mu_{A_p}(x_p^*),\exp(-y^2)\right),1-\mu_{A_p}(x_p^*)\right]$

if $\mu_{A_p}(x_p^*)\geq 0.5$



if $\mu_{A_p}(x_p^*)<0.5$



پاسخ سوال ٦

الف) QoQ

$$\text{فرض} : \mu_Q(u,u)=1 \; *$$

$$\mu_{PoQ}(x,z)=\underset{y\in V}{Max}\, Min\left[\mu_P(x,y),\mu_Q(y,z)\right]$$

$$\Rightarrow \mu_{QoQ}(u,u)=\underset{u\in U}{Max}\, Min\left[\underbrace{\overset{1}{\mu_Q(u,u)}}_{\text{فرض}*},\underbrace{\overset{1}{\mu_Q(u,u)}}_{*\text{فرض}}\right]\Rightarrow \mu_{QoQ}(u,u)=1$$

$\Rightarrow$ ترکیب QoQ یک رابطه Reflexive

ب)

$$\left.\begin{aligned}&\mu_{QoQ}(u,u)=1\\&\mu_Q(u,u)\leq 1\end{aligned}\right\}\Rightarrow Q\subseteq QoQ$$

پاسخ سوال ۷

فرض می‌کنیم A , B دو مجموعه فازی محدب هستند

$$\mu_A[\lambda x_1+(1-\lambda)x_2]\geqslant Min[\mu_A(x_1),\mu_A(x_2)]$$

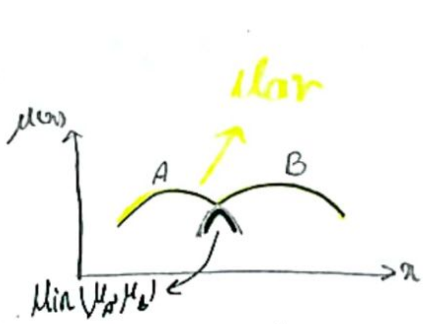
$$\mu_B[\lambda x_1+(1-\lambda)x_2]\geqslant Min[\mu_B(x_1),\mu_B(x_2)]$$

$$\mu_{A\cap B}[\lambda x_1+(1-\lambda)x_2]\geqslant Min\left[Min(\mu_A(x_1),\mu_A(x_2)),\,Min(\mu_B(x_1),\mu_B(x_2))\right]$$

$$\Rightarrow \mu_{A\cap B}[\lambda x_1+(1-\lambda)x_2]\geqslant Min\left[Min(\mu_A(x_1),\mu_B(x_1)),\,Min(\mu_A(x_2),\mu_B(x_2))\right]$$

$$\Rightarrow \mu_{A\cap B}[\lambda x_1+(1-\lambda)x_2]\geqslant Min\left[\mu_{A\cap B}(x_1),\mu_{A\cap B}(x_2)\right]$$

$$\Rightarrow \mu_{A\cap B}[\lambda x_1+(1-\lambda)x_2]\geqslant Min\left[\mu_{A\cap B}(x_1),\mu_{A\cap B}(x_2)\right]\;\checkmark$$



اما Union این دو مجموعه محدب فازی، یک مجموعه محدب فازی نیست.

پاسخ سوال ۸:

$V=[y_{۱},y_{۲}]$ $\qquad$ $U=[x_{۱},x_{۲},x_{۳}]$

قاعده: if x is A, Then y is B

مشاهده: x is A'

نتیجه y is B'

$$A=\frac{۰/۲}{x_{۱}}+\frac{۰/۸}{x_{۲}}+\frac{۰/۴}{x_{۳}}$$

$$B=\frac{۰/۶}{y_{۱}}+\frac{۰/۷}{y_{۲}} \longrightarrow B\mathcal{E}$$

$$A'=\frac{۰/۷}{x_{۱}}+\frac{۰/۹}{x_{۲}}+\frac{۱}{x_{۳}}$$

الف استلزام Dienes_Rescher

$$G.M.P: \mu_{B'}(y)=\underset{x\in U}{Sup}\; t\,[\underbrace{\mu_{A'}(x)}_{\text{مشاهده}},\underbrace{\mu_{A\to B}(x,y)}_{\text{استلزام}}]$$

$\mu_{A'E}(x,y)$

استلزام: $\mu_{QD}(x,y)=Max[1-\mu_A(x),\mu_B(y)] \Rightarrow \mu_{QD}(x,y)=\frac{۰/۸}{(x_{۱},y_{۱})}+\frac{۰/۸}{(x_{۱},y_{۲})}+\frac{۰/۶}{(x_{۲},y_{۱})}+\frac{۰/۷}{(x_{۲},y_{۲})}+\frac{۰/۶}{(x_{۳},y_{۱})}+\frac{۰/۷}{(x_{۳},y_{۲})}$

$$\mu_{B'}(y)=\underset{x\in U}{Max}\;Min[\mu_{A'}(x),\mu_{QD}(x,y)]$$

$$\Rightarrow \mu_{B'}(y)=\underset{x\in U}{Max}\left[\frac{۰/۷}{(x_{۱},y_{۱})}+\frac{۰/۷}{(x_{۱},y_{۲})}+\frac{۰/۶}{(x_{۲},y_{۱})}+\frac{۰/۷}{(x_{۲},y_{۲})}+\frac{۰/۶}{(x_{۳},y_{۱})}+\frac{۰/۷}{(x_{۳},y_{۲})}\right]$$

$$\Rightarrow \mu_{B'}(y)=\frac{۰/۷}{y_{۱}}+\frac{۰/۷}{y_{۲}}$$

ب) استلزام Zadeh

$$G.M.P:\ \mu_{B'}(y)=\sup_{x\in U} t\big[\overset{\text{مشاهده}}{\mu_{A'}(x)},\ \overset{\text{استلزام}}{\mu_{A\to B}(x,y)}\big]$$

$\mu_{A'E}(x,y)$

استلزام: $$\mu_{Q_Z}(x,y)=\mathrm{Max}\big[\mathrm{Min}(\mu_A(x),\mu_B(y)),\ 1-\mu_A(x)\big]$$

$$\mathrm{Min}(\mu_A(x),\mu_B(y))=\frac{0.2}{(x_1,y_1)}+\frac{0.2}{(x_1,y_2)}+\frac{0.6}{(x_2,y_1)}+\frac{0.7}{(x_2,y_2)}+\frac{0.4}{(x_3,y_1)}+\frac{0.4}{(x_3,y_2)}$$

$$\mu_{QZ}(x,y)=\frac{0.8}{(x_1,y_1)}+\frac{0.8}{(x_1,y_2)}+\frac{0.6}{(x_2,y_1)}+\frac{0.7}{(x_2,y_2)}+\frac{0.6}{(x_3,y_1)}+\frac{0.6}{(x_3,y_2)}$$

$$\mu_{B'}(y)=\mathrm{Max}_{x\in U}\ \mathrm{Min}\big[\mu_{A'}(x),\mu_{QZ}(x,y)\big]$$

$$\Rightarrow \mu_{B'}(y)=\mathrm{Max}_{x\in U}\Big[\frac{0.7}{(x_1,y_1)}+\frac{0.7}{(x_1,y_2)}+\frac{0.6}{(x_2,y_1)}+\frac{0.7}{(x_2,y_2)}+\frac{0.6}{(x_3,y_1)}+\frac{0.6}{(x_3,y_2)}\Big]$$

$$\mu_{B'}(y)=\frac{0.7}{y_1}+\frac{0.7}{y_2}$$

پاسخ سوال ۹)

قاعده ۱: if x is A, Then y is B

قاعده ۲: if x is very A, Then y is More or less B

مشاهده: x is A'

نتیجه: y is B'

قسمت ب) استنتاج بر مبنای قواعد مجزا

استلزام Dienes_Rescher

ابتدا قاعده ۱ را با مشاهده GMP می‌زنیم

مشاهده　　استلزام

$$G.M.P \quad \mu_{B'}(y)=\sup_{x\in U} t\left[\mu_{A'}(x), \mu_{A\to B}(x,y)\right]$$

استلزام: $\mu_{Q_D}(x,y)=Max\left[1-\mu_A(x), \mu_B(y)\right] \Rightarrow \mu_{Q_D}(x,y)=\frac{0.8}{(x_1,y_1)}+\frac{0.8}{(x_1,y_2)}+\frac{0.6}{(x_2,y_1)}+\frac{0.7}{(x_2,y_2)}+\frac{0.6}{(x_3,y_1)}+\frac{0.7}{(x_3,y_2)}$

$$\mu_{B'_1}(y)=\underset{x\in U}{Max}\, Min\left[\mu_{A'}(x), \mu_{Q_D}(x,y)\right]$$

$$\Rightarrow \mu_{B'_1}(y)=\underset{x\in U}{Max}\left[\frac{0.7}{(x_1,y_1)}, \frac{0.7}{(x_1,y_2)}+\frac{0.6}{(x_2,y_1)}+\frac{0.7}{(x_2,y_2)}+\frac{0.6}{(x_3,y_1)}+\frac{0.7}{(x_3,y_2)}\right]$$

$$\mu_{B'_1}(y)=\frac{0.7}{y_1}+\frac{0.7}{y_2}$$

سپس قاعده ۲ را با مشاهده GMP می‌زنیم

$$Very\ A=\frac{0.04}{x_1}+\frac{0.64}{x_2}+\frac{0.16}{x_3}$$

$$More\ or\ less\ B=\frac{0.77}{y_1}+\frac{0.84}{y_2}$$

استلزام: $\mu_{Q_D}(x,y)=Max\left[1-\mu_A^2(x), \mu_B^{0.5}(y)\right] \Rightarrow \mu_{Q_D}(x,y)=\frac{0.96}{(x_1,y_1)}+\frac{0.96}{(x_1,y_2)}+\frac{0.77}{(x_2,y_1)}+\frac{0.84}{(x_2,y_2)}+\frac{0.84}{(x_3,y_1)}+\frac{0.84}{(x_3,y_2)}$

$$\mu_{B'_2}(y)=\underset{x\in U}{Max}\, Min\left[\mu_{A}(x), \mu_{Q_D}(x,y)\right]$$

$$\Rightarrow \mu_{B'_2}(y)=\underset{x\in U}{Max}\left[\frac{0.7}{(x_1,y_1)}+\frac{0.7}{(x_1,y_2)}+\frac{0.77}{(x_2,y_1)}+\frac{0.84}{(x_2,y_2)}+\frac{0.84}{(x_3,y_1)}+\frac{0.84}{(x_3,y_2)}\right]$$

$$\mu_{B'_2}(y)=\frac{0.84}{y_1}+\frac{0.84}{y_2}$$

$$1)\ \mu_{B'}(y)=\mu_{B'_1}(y)\underbrace{+}_{max}\mu_{B'_2}(y) \qquad \mu_{B'}(y)=\frac{0.84}{y_1}+\frac{0.84}{y_2}$$

$$2)\ \mu_{B'}(y)=\mu_{B'_1}(y)\underbrace{*}_{T\text{norm}\to min}\mu_{B'_2}(y) \qquad \mu_{B'}(y)=\frac{0.7}{y_1}+\frac{0.7}{y_2}$$

الف) استنتاج بر مبنا ترکیب قواعد

استلزام قاعده ۱ : $\mu_{QD_1}(x,y)=Max\left[1-\mu_A(x),\mu_B(y)\right] \Rightarrow \mu_{Q_{D_1}}(x,y)=\frac{0.8}{(x_1,y_1)}+\frac{0.8}{(x_1,y_2)}+\frac{0.6}{(x_2,y_1)}+\frac{0.7}{(x_2,y_2)}+\frac{0.6}{(x_3,y_1)}+\frac{0.7}{(x_3,y_2)}$

استلزام قاعده ۲ : $\mu_{QD_2}(x,y)=Max\left[1-\mu_A'(x)-\mu_B''(y)\right] \Rightarrow \mu_{Q_{D_2}}(x,y)=\frac{0.96}{(x_1,y_1)}+\frac{0.96}{(x_1,y_2)}+\frac{0.77}{(x_2,y_1)}+\frac{0.84}{(x_2,y_2)}+\frac{0.84}{(x_3,y_1)}+\frac{0.84}{(x_3,y_2)}$

ترکیب استلزام این دو قاعده را : $Q_M=\bigcup_{l=1}^{M}R_u^{(l)}$ Union → Max

$$\mu_{Q_M}(x,y)=\frac{0.96}{(x_1,y_1)}+\frac{0.96}{(x_1,y_2)}+\frac{0.77}{(x_2,y_1)}+\frac{0.84}{(x_2,y_2)}+\frac{0.84}{(x_3,y_1)}+\frac{0.84}{(x_3,y_2)}$$

$$G.M.P=\mu_{B'}(y)=\underset{x\in U}{Max}\ Min\left[\mu_{A'}(x),\mu_{Q_M}(x,y)\right]$$

$$\mu_{B'}(y)=\underset{x\in U}{Max}\left[\frac{0.7}{(x_1,y_1)}+\frac{0.7}{(x_1,y_2)}+\frac{0.77}{(x_2,y_1)}+\frac{0.84}{(x_2,y_2)}+\frac{0.84}{(x_3,y_1)}+\frac{0.84}{(x_3,y_2)}\right]$$

$$\Rightarrow \mu_{B'}(y)=\frac{0.84}{y_1}+\frac{0.84}{y_2}$$

ترکیب گودل : $Q_G=\bigcap_{l=1}^{M}R_u^{(l)}$ Intersection → Min

$$\mu_{Q_G}(x,y)=\frac{0.8}{(x_1,y_1)}+\frac{0.8}{(x_1,y_2)}+\frac{0.6}{(x_2,y_1)}+\frac{0.7}{(x_2,y_2)}+\frac{0.6}{(x_3,y_1)}+\frac{0.7}{(x_3,y_2)}$$

$$GMP:\mu_{B'}(y)=\underset{x\in U}{Max}\ Min\left[\mu_{A'}(x),\mu_{Q_G}(x,y)\right]$$

$$\mu_{B'}(y)=\underset{x\in U}{Max}\left[\frac{0.7}{(x_1,y_1)}+\frac{0.7}{(x_1,y_2)}+\frac{0.6}{(x_2,y_1)}+\frac{0.7}{(x_2,y_2)}+\frac{0.6}{(x_3,y_1)}+\frac{0.7}{(x_3,y_2)}\right]\Rightarrow \mu_{B'}(y)=\frac{0.7}{y_1}+\frac{0.7}{y_2}$$

پاسخ سوال ۱۰: چون رابطه فازی در فضای n بُعدی قرار دارد و هر بُعد یک projection دارد پس n projection متمایز دارد.

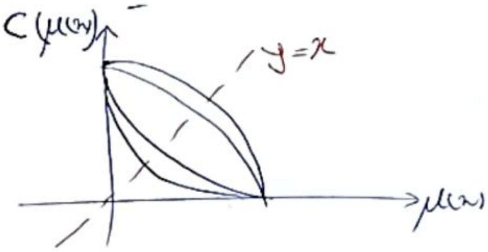
پاسخ سوال ۱۱: اگر $\mu(x)$ تمامی توابع complement را رسم کنیم به صورت غیرصعودی خواهند بود زیرا طبق دو شرط زیر

شرط اول: $C(0)=1 \;,\; C(1)=0$

شرط دوم: $a,b \in [0,1]$ if $a<b$ then $C(a) \geq C(b)$

غیرصعودی بودن آنها اثبات خواهد شد. که این منحنی ها غیرصعودی با خط $y=x$ تلاقی دارند.

$$\Rightarrow C(a) = a$$



پاسخ سوال ۱۲ -

$A = \{(1, 0.5), (2, 1), (3, 0.3)\}$

$B = \{(2, 0.4), (3, 0.4), (4, 1)\}$

$|1-2| = 1,\ |1-3| = 2,\ |1-4| = 3$

$|2-2| = 0,\ |2-3| = 1,\ |2-4| = 2$

$|3-2| = 1,\ |3-3| = 0,\ |3-4| = 1$

$0, 1, 2, 3$ — کمترین فاصله: 0 ، بیشترین فاصله: 3

$$\mu_{d(A,B)}(\delta) = \underset{\delta = d(a,b)}{Max}\left[Min\left(\mu_A(a), \mu_B(b)\right)\right]$$

قدر مطلق

$\delta \in d(a,b)$	$a \in A$	$b \in b$	$\mu_A(a)$	$\mu_B(b)$	Min	$Max(\mu_A(a), \mu_B(b))$
0	2	2	1	0.4	0.4	0.4
	3	3	0.3	0.4	0.3	
1	1	2	0.5	0.4	0.4	0.4
	2	3	1	0.4	0.4	
	3	2	0.3	0.4	0.3	
	3	4	0.3	1	0.3	
2	1	3	0.5	0.4	0.4	1
	2	4	1	1	1	
3	1	4	0.5	1	0.5	0.5

$$\Rightarrow d(A,B) = \{(0, 0.4), (1, 0.4), (2, 1), (3, 0.5)\}$$

پاسخ سوال ۱۳

$$\tilde{F} = \{(f_1, 0.4), (f_2, 0.7), (f_3, 0.4)\} \quad , \quad X = [\underset{a}{1}, \underset{b}{2}]$$

بازه انتگرال

$$f_1(x) = x$$

$$f_2(x) = x^2$$

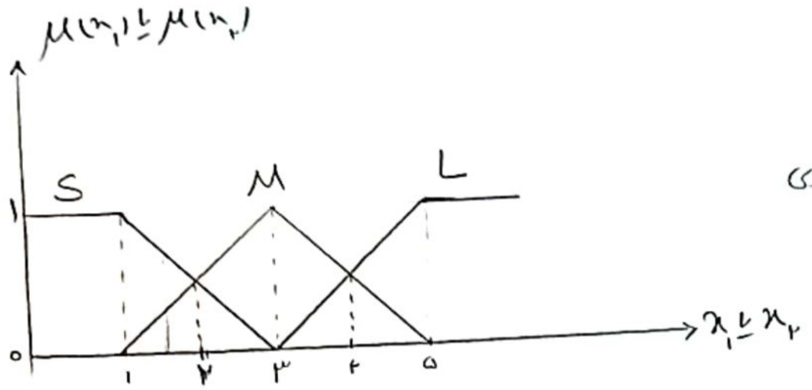
$$f_3(x) = x + 1$$

$$\alpha = 0.4 : \int_1^2 f_{0.4}(x)\,dx \Rightarrow \begin{cases} \int_1^2 f_1(x)\,dx = \int_1^2 x\,dx = \frac{x^2}{2}\Big|_1^2 = \frac{4}{2} - \frac{1}{2} = 2 - 0.5 = 1.5 \Rightarrow \tilde{I}_{0.4}(1,2) = \{(1.5, 0.4)\} \\ \int_1^2 f_3(x)\,dx = \int_1^2 (x+1)\,dx = (\frac{x^2}{2} + x)\Big|_1^2 = \overset{4}{\cancel{(\frac{4}{2}+2)}} - \overset{\frac{3}{2}}{\cancel{(\frac{1}{2}+1)}} = \frac{5}{2} = 2.5 \Rightarrow \tilde{I}_{0.4}(1,2) = \{(2.5, 0.4)\} \end{cases}$$

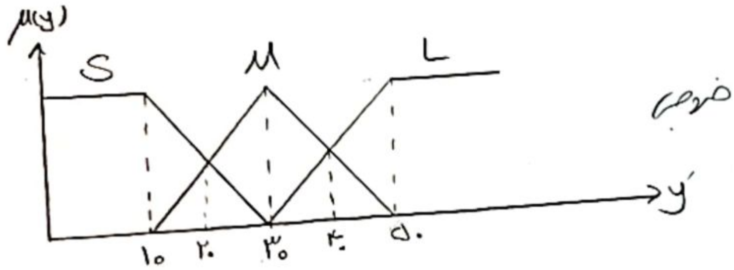
$$\alpha = 0.7 : \int_1^2 f_{0.7}(x)\,dx \Rightarrow \int_1^2 f_2(x)\,dx = \int_1^2 x^2\,dx = \frac{x^3}{3}\Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \Rightarrow \tilde{I}_{0.7}(1,2) = \{(\overset{2.3}{\frac{7}{3}}, 0.7)\}$$

$$\Rightarrow \tilde{I}(1,2) = \{(1.5, 0.4), (\frac{7}{3}, 0.7), (2.5, 0.4)\}$$

پاسخ سوال ۱۵ :



ورودی



خروجی

if x_1 is Small and x_2 is Medium Then y is Small ✓

if x_1 is Large and x_2 is Medium Then y is Large ✓

if x_1 is Medium and x_2 is Small Then y is Medium ✓

if x_1 is Medium and x_2 is Large Then y is Large ✓

if x_1 is Small and x_2 is Large Then y is Medium ✓

~~if x_1 is Medium and x_2 is Large Then y is Large~~ ✓

یکی می‌توانند

if x_1 is Large and x_2 is Small Then y is Medium

if x_1 is Small and x_2 is Large Then y is Large ✓

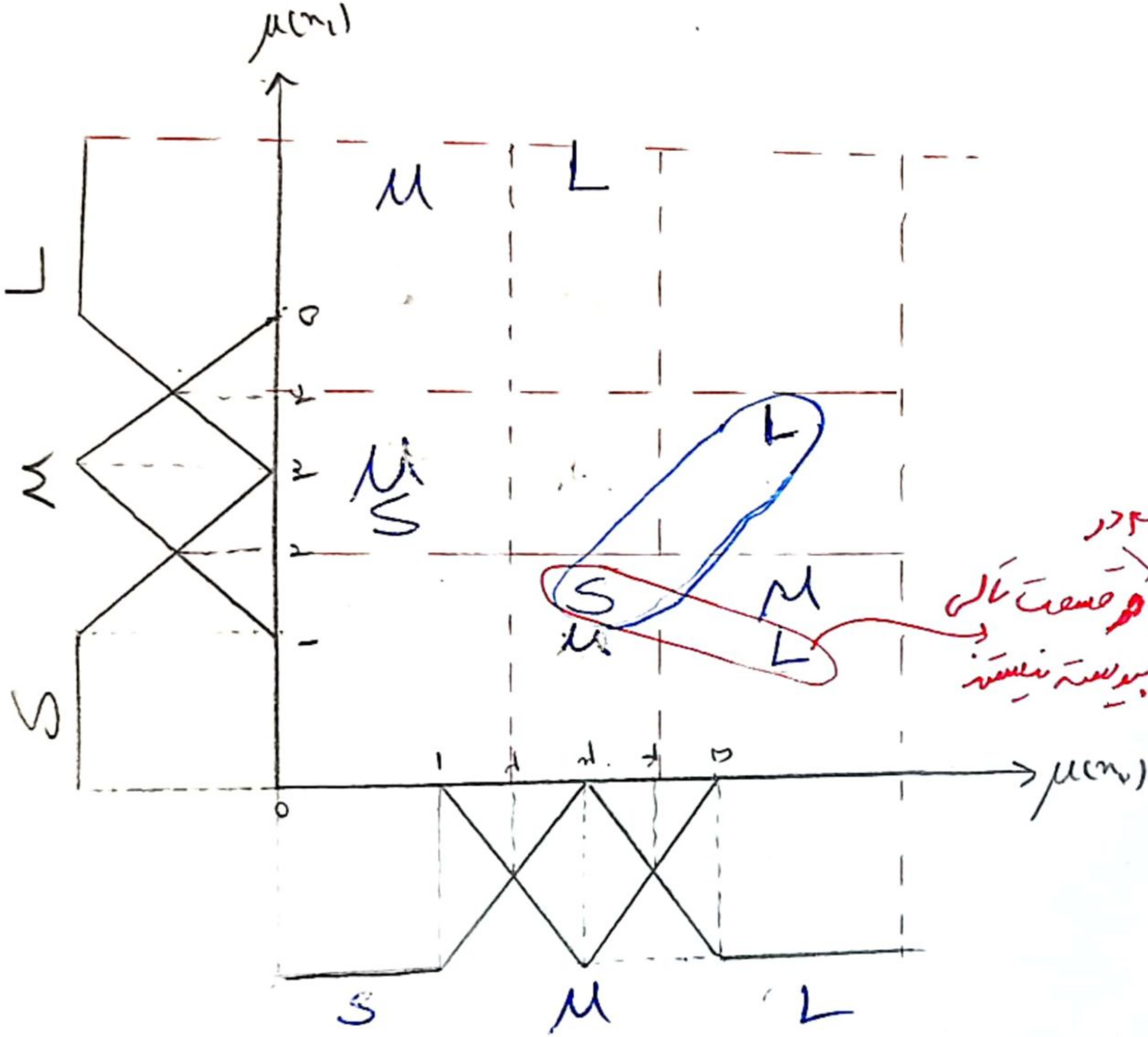
~~if x_1 is Large and x_2 is Medium Then y is Large~~ ✓

یکی می‌توانند

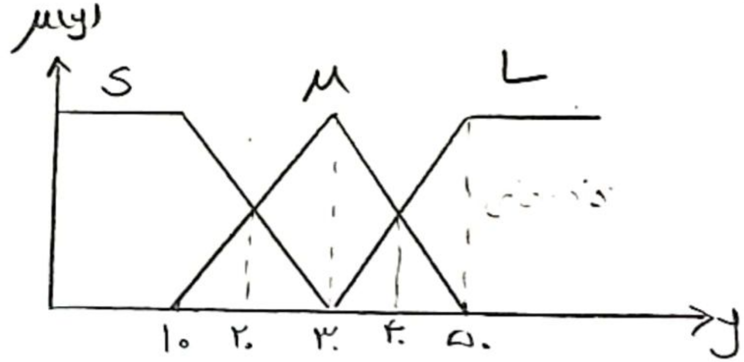
if x_1 is Small and x_2 is Medium Then y is Medium ✓

~~if x_1 is Small and x_2 is Large Then y is Large~~ ✓

if x_1 is Medium and x_2 is Small Then y is Small ✓



با هم در
اشتراکی قسمت تالی
ندارند ← پیوسته نیستند



قواعد ناسازگار و ناپیوسته اند.