5A.

1	p_1
1 1	p_2
1 2 1	p_3
1 3 3 1	p_4
1 4 6 4 1	p_5
1 5 10 10 5 1	p_6
1 6 15 20 15 6 1	p_7

[HW3]

$$p_0 = 1$$

$$p_1 = \frac{1}{6} = \left(\frac16\right)\underbrace{(1)}_{p_0}$$

$$p_2 = \left(\frac16\right)^2 + \frac16 = \left(\frac16\right)\left(\overbrace{\frac16}^{p_1} + \overbrace{1}^{p_0}\right)$$

$$p_3 = \left(\frac16\right)^3 + 2\left(\frac16\right)^2 + \frac16 = \left(\frac16\right)\left[\overbrace{\left(\frac16\right)^2 + \frac16}^{p_2} + \overbrace{\frac16}^{p_1} + \overbrace{1}^{p_0}\right]$$

$$p_4 = \left(\frac16\right)^4 + 3\left(\frac16\right)^3 + 3\left(\frac16\right)^2 + \frac16 = \frac16\left[\underbrace{\left(\frac16\right)^3 + 2\left(\frac16\right)^2 + \frac16}_{p_3} + \underbrace{\left(\frac16\right)^2 + \frac16}_{p_2} + \underbrace{\frac16}_{p_1} + \underbrace{1}_{p_0}\right]$$

We see a pretty obvious pattern emerge. We need $p_0 = 1$ and the equation to be true $\forall n \in \mathbb{Z}^+$. This can't continue forever since some n are so large a minimum number of rolls are needed.

Taking ~~Ex~~. $p_7 \neq \frac16\left(\underbrace{p_6 + p_5 + p_4 + p_3 + p_2 + p_1}_{\frac16(p_5 + p_4 + p_3 + p_2 + p_1 + p_0)} + p_0\right)$

will always need at least 2 rolls, at most 7 rolls

when multiplied by $\frac16$, yields $p_1 = \frac16$, or probability of getting 7 in one roll => invalid!

this is fine since there is a way to get a 6 from one roll; $\frac16 \cdot p_0 = \frac16$.

so keeping w/ pattern does

$$p_7 \overset{?}{=} \frac16(p_6 + p_5 + p_4 + p_3 + p_2 + p_1)$$

$$\boxed{= \left(\frac16\right)^7 + 6\left(\frac16\right)^6 + 15\left(\frac16\right)^5 + 20\left(\frac16\right)^4 + 15\left(\frac16\right)^3 + 6\left(\frac16\right)^2 + \frac16}$$

$$= \cancel{\frac16}\left(\cancel{\left(\frac16\right)^6} + \cancel{6\left(\frac16\right)^5} + \cancel{15\left(\frac16\right)^4} + \cancel{20\left(\frac16\right)^3} + \cancel{15\left(\frac16\right)^2} + \cancel{6\left(\frac16\right)}\right)$$

$$\overset{?}{=} \cancel{\frac16}\Big(\underbrace{\cancel{\frac16}}_{p_1} + \underbrace{\cancel{\frac16} + \cancel{\left(\frac16\right)^2}}_{p_2} + \underbrace{\cancel{\left(\frac16\right)^3} + \cancel{2\left(\frac16\right)^2} + \cancel{\frac16}}_{p_3} + \underbrace{\cancel{\left(\frac16\right)^4} + \cancel{3\left(\frac16\right)^3} + \cancel{3\left(\frac16\right)^2} + \cancel{\frac16}}_{p_4} +$$

$$\underbrace{\cancel{\left(\frac16\right)^5} + \cancel{4\left(\frac16\right)^4} + \cancel{6\left(\frac16\right)^3} + \cancel{4\left(\frac16\right)^2} + \cancel{\left(\frac16\right)}}_{p_5} +$$

$$\underbrace{\cancel{\left(\frac16\right)^6} + \cancel{5\left(\frac16\right)^5} + \cancel{10\left(\frac16\right)^4} + \cancel{10\left(\frac16\right)^3} + \cancel{5\left(\frac16\right)^2} + \cancel{\left(\frac16\right)}}_{p_6}\Big)$$

Yes.

$$\boxed{\begin{aligned} &p_n = \frac16(p_{n-1} + p_{n-2} + p_{n-3} + p_{n-4} + p_{n-5} + p_{n-6}), \ \forall n \in \mathbb{Z}^+ \\ &p_0 = 1 \\ &p_k = 0 \text{ for } k < 0 \end{aligned}}$$