Week 1

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March 7, 2024

0.1 Exercise 1

球体积公式为 $V = \frac{4}{3}\pi r^3$，令半径的准确值为 R，测量值为 $\hat{R}$，则球体积的相对误差为 $e_r(\hat{V}) = \frac{\hat{V}-V}{V} = \frac{\frac{4}{3}\pi\hat{R}^3-\frac{4}{3}\pi R^3}{\frac{4}{3}\pi R^3} = \frac{\hat{R}^3-R^3}{R^3} = (\frac{\hat{R}}{R})^3 - 1$，由题意得 $|(\frac{\hat{R}}{R})^3 - 1| \leq 1\%$，则 $0.99 \leq (\frac{\hat{R}}{R})^3 \leq 1.01$，即有 $-0.00334 \leq e_r(\hat{R}) = \frac{\hat{R}-R}{R} \leq 0.00332$
故误差限 $\varepsilon_r(\hat{R}) \approx 0.00332$

0.2 Exercise 2

(1) 绝对误差为 $\sin(x+h) - \sin x \approx h \cdot \sin'(x) = h\cos x$
(2) 相对误差为 $\frac{\sin(x+h)-\sin x}{\sin x} \approx \frac{h\cos x}{\sin x} = h\cot x$

0.3 Exercise 4

令 $\hat{Y}_n$ 为计算得到的值，则有 $\hat{Y}_0 = Y_0 = 28$，$\hat{Y}_n = \hat{Y}_{n-1} - \frac{1}{100} \times 27.982$
有 $Y_{100} = Y_0 - \sqrt{783}, \hat{Y}_{100} = \hat{Y}_0 - 27.982$ 则绝对误差为 $e(\hat{Y}_{100}) = \hat{Y}_{100} - Y_{100} = \sqrt{783} - 27.982 = 0.000137$，
相对误差为 $e_r(\hat{Y}_{100}) = \frac{e(\hat{Y}_{100})}{Y_{100}} = 0.00767847$

0.4 补充题

$d_0 = 3$，又有 $0.1\% = 0.001 \leq \frac{5}{d_0+1} \times 10^{-3} = 0.00125$，则前三位一定是对的
（含“四舍五入”后与准确值的结果相同）