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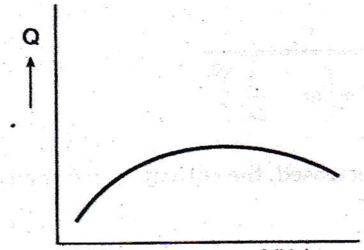

Fig. 3.EM14(vi)

Here r is
s and the

The initial increase of Q with frequency f is due to the fact that the inductive reactance increases linearly with f while the resistance due to the skin effect does not rise faster than $f^{1/2}$. At high frequencies, the dielectric losses in the coil format and the insulating material used are greater than the losses due to skin effect. As the resistance due to dielectric losses rises with frequency as f^3, the Q falls at high frequencies. At intermediate frequencies, the increase of the resistance R due to skin effect and dielectric losses is such that $Q(=\omega L/R)$ remains almost constant.

24. Which element an inductor or a capacitor—does possess a higher Q at a given frequency?

 Ans. Capacitor, because the losses in a capacitor are much smaller.

25. Is the coil resistance r the same at all frequencies?

 Ans. No. At low frequencies, r is the dc resistance of the wire of the coil. As frequency increases, in the radio frequency range, r increases owing to the skin effect, eddy current losses, and dielectric losses in the coil format. If the core is ferromagnetic, hysteresis losses will also enhance r.

26. How can you reduce the eddy current for a magnetic core?

 Ans. By laminating the core or using a ferrite core.

[See also the 'Aids to the Viva Voce' for Expt. No. EM12 and EM13 and the questions on electronic voltmeters at the end of Part 2.]

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Study of response curve of a series LCR circuit and

Experiment No. EM15

INVESTIGATIONS ON A SERIES ~~RESONANT CIRCUIT~~

determination of its resonant frequency, impedance at resonance, quality factor and band width

Object :

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1. To draw the resonance curve of a series LCR circuit for various values of R and L/C, and to find the Q factor in each case.

2. To find the Q factor from the ratio V_c/V_i at resonance, where V_c and V_i are the capacitor voltage and the source voltage, respectively.

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3. To draw the phasor diagram of voltages at series resonance.

Theory :

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In Fig 3.~~EM15(i)~~, a coil of self inductance L and resistance R_L is connected in series with an external resistance r, and a capacitance C. The total resistance in the circuit is $R = R_s + R_L + r$, where R_S is the internal resistance of the ac voltage source. This circuit is known as the series LCR circuit. If V_i is the driving voltage, a current I will flow in the circuit. At an angular frequency ω of the ac source, the magnitude of the current is

considerable
EM14(vi)].

$$I = \frac{V_i}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{... (i)}$$

As the frequency $f\left(=\frac{\omega}{2\pi}\right)$ is increased, the current at first increases, attains a maximum, and then falls, as shown in Fig. ~~3.EM15(ii)~~. 2

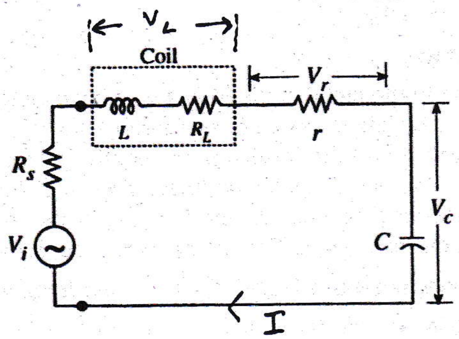


Fig. ~~3.EM15(i)~~ 1

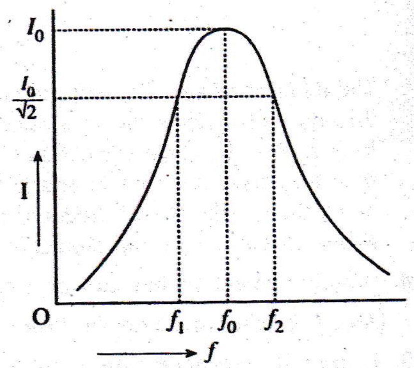


Fig. ~~3.EM15(ii)~~ 2

The I versus f curve [Fig. ~~3.EM15(ii)~~ 2] is known as the *resonance curve*. The maximum current I_o is obtained at a frequency f_o, called the series resonant frequency. It is given by

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}}. \quad \text{... (ii)}$$

At resonance, the current I is in phase with the voltage V_i. The Q factor of the circuit is

$$Q = \frac{\omega_o L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}. \quad \text{... (iii)}$$

The Q factor denotes the sharpness of resonance. The higher the Q, the sharper the fall of I on either side of resonance. Let the frequencies where the current drops to $I_o/\sqrt{2} = 0.707 I_o$ be f_1 and f_2. The bandwidth of the circuit is $(f_2 - f_1)$ and the Q factor can be written as

$$Q = \frac{f_o}{f_2 - f_1} \quad \text{... (iv)}$$

Equation (iii) shows that Q depends on R and L/C. As R decreases, Q as well as the peak current at resonance I_o (= V_i/R) increases, as shown in Fig. 3.EM15(iii). For a fixed R, if L/C increases, Q also increases, but I_o remains fixed [Fig. ~~3.EM15(iv)~~ 4].

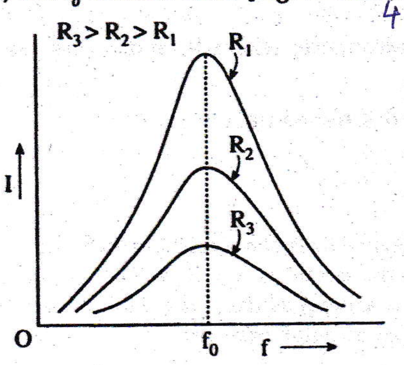


Fig. ~~3.EM15(iii)~~ 3

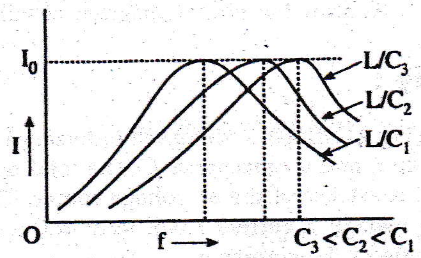


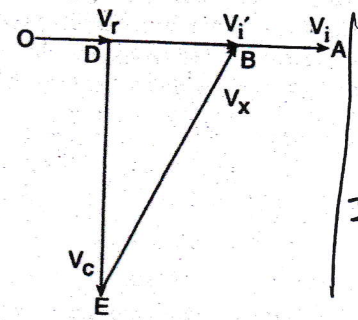
Fig. ~~3.EM15(iv)~~ 4

For a given L, L/C can be increased by taking smaller values of C.

The capacitor voltage at resonance is given by

$$V_c = QV_i. \qquad \text{... (v)}$$

Thus Q can also be found from the ratio V_c/V_i at resonance.



Impedance across C

$Z_c = \frac{V_c}{I} = \frac{1}{2\pi f C}$

Impedance across L

$Z_L = \frac{V_L}{I} = 2\pi f L$

Fig. 3.EM15(v)

not required now

Let V_i' be the voltage at resonance across the resonant circuit when it is connected to the source. At resonance, I and V_i are in phase. So, V_i' (= $V_i - IR_s$) is in phase with V_i. If V_r is the voltage drop across r, we have $V_r = Ir$, so that V_r is also in phase with V_i. Let the straight lines OA, OB, and OD denote the phasors V_i, V_i', and V_r, respectively [Fig. 3.EM15(v)]. The phasor DB then represents the phasor sum of the voltages V_C (≡ DE) and V_x (≡ EB), V_x being the voltage drop across the coil (consisting of L and R_L in series).

Apparatus and Accessories :

(i) An audio oscillator (of output resistance $R_s \simeq 10$ ohm), (ii) the given coil (typically $L \simeq 10$ mH, $R_L \simeq 100$ ohm or less), (iii) three capacitors (typically 0.05, 0.1 and 0.15 μF), (iv) three resistances (typically 30, 50 and 70 ohm), and (v) an electronic voltmeter (EVM).

Procedure :

1. Use the minimum value of C ($\simeq 0.05\ \mu\text{F} = C_1$) and a resistance r (say, 30 ohm = r_1). Set up the circuit as in Fig. 3.EM15(i). Here $R = R_1 = R_s + R_L + r_1$.

2. Switch on the ac source and set its output voltage V_i to a convenient value (say, 3V rms). Note that V_i is the open-circuit voltage across the source terminals, i.e. the voltage with the network disconnected. Keep V_i constant as the frequency is varied.

3. At different frequencies measure the voltage V_r across r by the EVM, and calculate the current I from the relationship $I = V_r/r$.

4. Plot I against the frequency f. From the plot, find the resonant frequency f_o and the bandwidth $(f_2 - f_1)$. Calculate Q from Eq. (iv).

5. Set the oscillator frequency at the resonant frequency f_o and measure the voltage V_c across the capacitor. Calculate Q from Eq. (v) and compare it with the value determined in step 4.

6. Repeat steps 2 to 5 for two other values of r (say, $r_2 = 50\Omega$, and $r_3 = 70\Omega$).

7. Keep r fixed at its minimum value ($\simeq$ 30 ohm) and repeat steps 2 to 5 for two other values of C (say, $C_2 = 0.1\ \mu F$ and $C_3 = 0.15\ \mu F$).

8. At resonance, draw the phasor diagram [Fig. 3.EM15(v)] by, first drawing, to a chosen scale the straight line OA representing V_i. From OA, cut off the segment OB to represent V_i', and the segment OD to represent V_r. With centre at D, draw a circular arc of radius representing V_c. With centre at B draw another circular arc of radius corresponding to V_x. Let the two arcs intersect at E. Then DE and EB will represent V_c and V_x, respectively. (Note that the capacitor voltage lags the current, and so the arcs must be drawn on the lower side of OA.) Measure ∠BDE and ∠DBE. Since the capacitor losses are negligible, ∠BDE is expected to be nearly 90°, showing the 90° phase lead of the current over V_c. However, inductors are imperfect and so ∠DBE will be less than 90°, reflecting that the voltage across the coil leads the current by less than 90°.

Experimental Results :

TABLE 1

Data for the resonance curve

1A. $C = \ldots\ \mu F (= C_1)$, $r = \ldots \Omega (= r_1)$.

Take 3 sets

L C R1
L C R2
L C' R1

Source frequency f (Hz)	Input voltage V_i (volt)	Voltage across r V_r (volt)	Current I (= V_r/r) (A)
...		...	...
...	...	...	...
...		...	...
etc.		etc.	etc.

Add two more columns to measure Vc and VL

Make similar tables 1B, 1C etc. for other given values of r and C.

TABLE 2

Q from the resonance curve

Value of C (μF)	Value of r (ohm)	Resonant freq. f_o from graph (Hz)	Bandwidth $(f_2 - f_1)$ from graph (Hz)	$Q = \dfrac{f_0}{f_2 - f_1}$
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...
etc.	etc.	etc.	etc.	etc.

. PHYSICS

other values

o a chosen scale, , and the segment With centre at B, t E. Then DE and urrent, and so the apacitor losses are current over V_c. the voltage across

t I

(A)

$Q = \frac{f_0}{f_2 - f_1}$

...

...

...

etc.

TABLE 3

Q from the ratio V_c/V_i at resonance

Input voltage $V_i = \ldots$ volt.

Value of C (μF)	Value of r (ohm)	Resonant freq. f_o from graph (Hz)	Voltage across the capacitor V_c (volt)	$Q = \frac{Vc}{V_i}$ (= Q_1, say)	Q from Table 2 (= Q_2, say)	Remarks
...	...	...	...	...	...	Q_1 and Q_2 are nearly equal
...	...	...	...	...	...	
...	...	...	...	...	...	
etc.	etc.	etc.	etc.	etc.	etc.	

TABLE 4

To draw the phasor diagram at resonance

A. Choice of scale for the phasor diagram

Resonant frequency $f_0 = \ldots$ Hz.

1 cm represents ... volt.

Therefore, $V_i = \ldots$ volt $\equiv \ldots$ cm = OA, $V_i' = \ldots$ volt $\equiv \ldots$ cm = OB, $V_r = \ldots$ volt $\equiv \ldots$ cm = OD, $V_c = \ldots$ volt $\equiv \ldots$ cm = DE, and $V_x = \ldots$ volt $\equiv \ldots$ cm = EB.

B. Inference from the phasor diagram

∠BDE	∠DBE	Inference
...	...	(i) I leads V_c by 90° showing that the capacitor is a good one. (ii) V_x leads I by ... which is less than 90°. So the inductor is imperfect.

not required now

Table 4

Impedances across C and L (do for set I only)

f	I	V_c	V_L	$Z_c = V_c/I$	$Z_L = V_L/I$

Note : From the resonant frequency f_0, the inductance L can be calculated if the capacitance C is known from the relationship

$$L = \frac{1}{4\pi^2 f_0^2 C} \quad \ldots \text{(vi)}$$

Using different known capacitors, L can be found from which the mean L is determined. By measuring input voltage V to the LCR circuit at resonance, its resistance R can be found from the relation

$$R = \frac{V}{I} = \frac{Vr}{V_r} \quad \ldots \text{(vii)}$$

* Plot Z_L vs f and Z_c vs f in a single graph paper

Plot V_L vs f

*Plot $1/Z_c$ vs f

Q is then found from

$$Q=\frac{1}{R}\sqrt{\frac{L}{C}}$$

and compared with the values of Q obtained in TABLES 2 and 3.

The following table can be used for the above determination of L and Q.

* from the slope of Z_L vs f graph find L.
* from the slope of $\frac{1}{Z_C}$ vs f graph find C

TABLE 5

Determination of L and Q (r = ... ohm)

f_0 (from resonant curves) (Hz)	C (F)	L [from Eq. (vi)] (H)	Mean L (H)	Input voltage V (volt)	V_r (volt)	R [from Eq. (vii)] (ohm)	Q $\left(=\frac{1}{R}\sqrt{\frac{L}{C}}\right)$
...	...	...			...	...	...
...	...	...	...	...	...	...	...
etc.	etc.	etc.			etc.	etc.	etc.

Percentage Error in Q :

We have $\qquad Q=\left|\frac{V_C}{V_i}\right|.$

Hence, if δQ is the error in Q, we have

$$\frac{\delta Q}{Q}=\frac{\delta V_C}{V_C}+\frac{\delta V_i}{V_i},$$

where δV_c and δV_i are the smallest divisions of the scales used for measurements of V_c and V_i respectively. $\frac{\delta Q}{Q}$ is the proportional error; multiplying it by 100 we get the percentage error in Q.

Discussions :

1. The capacitance C must be much larger than the input capacitance C_i of the EVM.
2. The effect of the self-capacitance of the coil can be neglected over the frequency range of interest here.
3. Capacitors are more or less ideal while inductors are not. Hence the voltage is measured across the capacitor, and not across the inductor, to find Q.
4. The measured voltages are the effective or r.m.s. values.
5. If the source resistance R_s is not sufficiently small, the output voltage of the audio oscillator can be fed to a unity gain buffer. The output of the buffer supplies power to the given network. In this case, the effective source resistance becomes very small.
6. The connecting wires must be straight and short.
7. If the amplitude of the output voltage of the oscillator changes with frequency, it must be adjusted.