ORGANIZATION

- HW REMINDERS

↳ use link in README.md
to access repo through GitHub class

[you may want to UL to a non-private
repo not associated w/ GitHub class]

↳ show off your work! :)

note: HW has hard deadline

- CLASS REMINDERS

  - no class on Tue, HW still due → online!
  - occasional index cards

↳ why: so I GET FEEDBACK FROM YOU
email ahead of time if you
can't make it.

(but it's a small # of points
if you don't.)

graded on participation

TODAY: ① .1 + .1 + .1 == .3
is False → why?

② HOW DO YOU BEST LEARN?

  - USE THE BREAK TO CHAT (about course
... or, you know... whatever.)

Demo: quadratic formula

$$x_\pm = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

IF $b \gg a, c$

$$x_+ = \frac{-b + \sqrt{b^2-4ac}}{2a}$$

(small) diff of 2 big numbers.

Better way:

① $$x_+ = \frac{-b+\sqrt{b^2-4ac}}{2a} \underbrace{\frac{-b-\sqrt{b^2-4ac}}{-b-\sqrt{b^2-4ac}}}_{=1}$$

$$= \frac{\cancel{b^2-b^2}+4ac}{2a} \cdot \frac{1}{-b-\sqrt{b^2-4ac}}$$

no more difference of big #

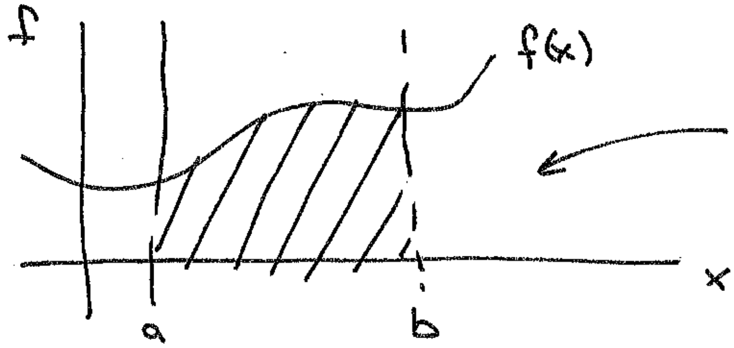
② $$x_+ = \frac{-b + b\sqrt{1-\frac{4ac}{b^2}}}{2a}$$

$$\sqrt{1-\varepsilon} = 1 - \tfrac{1}{2}\varepsilon + \cdots$$

$$\approx \frac{-b\varepsilon}{4a}$$

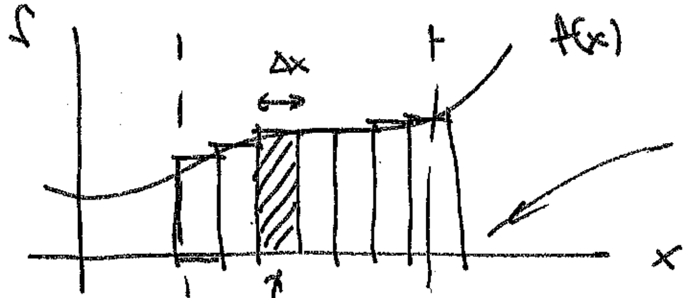
INTEGRATION

Riemann Sum



want to estim. this area.

What we can do:
- sample f @ discrete points
- sum together numbers



divide x-axis into BINS like histogram.

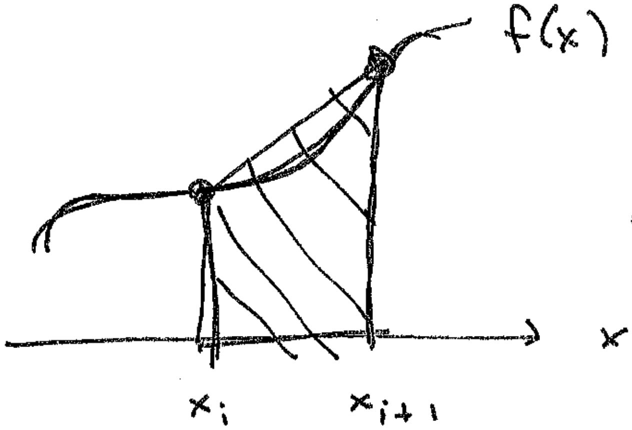
AREA OF EACH COLUMN:

$$f(x_i) \quad \Delta x$$

middle of bin

$\frac{b-a}{N} \leftarrow$ # rectangles

Better approximation: TRAPEZOIDAL RULE

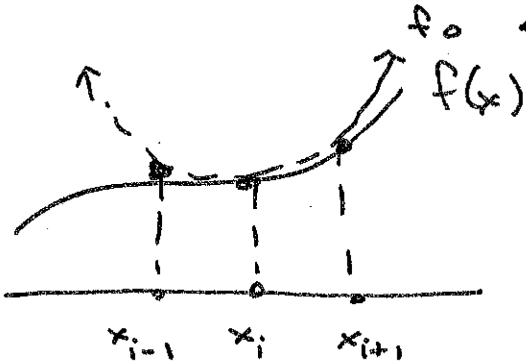


$$A_i = \frac{1}{2}(f_i + f_{i+1})\Delta x$$

convenient: no longer sampling "in between" points

EVEN BETTER APPROX : Simpson's rule

really just going from point, line, parab...



FIRST: ASSUME $x_i = 0$

$$f_0 = Ax^2 + Bx + C$$

IDEA: integrate from x_{i-1} to x_{i+1} ... hard code the answer as a func. of $f(x_{i-1}), f(x_i), f(x_{i+1})$

$$\begin{aligned} f_0(-\Delta x) &= A(\Delta x)^2 - B\Delta x + C = f(-\Delta x) \\ f_0(\Delta x) &= \quad '' \quad + B\Delta x \quad '' \quad = f(\Delta x) \\ f_0(0) &= \qquad\qquad\qquad C = f(0) \end{aligned}$$

$$f(-\Delta x) + f(\Delta x) = 2A(\Delta x)^2 + 2C$$

$$\rightarrow A = \frac{1}{2(\Delta x)^2}\left(f(-\Delta x) - 2f(0) + f(\Delta x)\right)$$

$$f(\Delta x) - f(-\Delta x) = 2B\Delta(x)$$

$$\boxed{B = \frac{1}{2\Delta x}\left(f(\Delta x) - f(-\Delta x)\right)}$$

can <u>integrate</u> the simple function. f_0:

$$\int_{-\Delta x}^{\Delta x} (Ax^2 + Bx + C)\, dx$$

$$= \frac{2}{3} A (\Delta x)^3 + 2C\Delta x$$

$$= \frac{1}{3} \frac{1}{(\Delta x)^2}\left[f(-\Delta x) - 2f(0) + f(\Delta x)\right](\Delta x)^3$$

$$+ 2f(0)\Delta x$$

$$= \frac{\Delta x}{3}\left[f(-\Delta x) + 4f(0) + f(\Delta x)\right]$$

sample 3 points.

<u>note</u>: (intuition) ORIGIN IS NOT SPECIAL

can shift all positions by x_i (change units)

$$\int_{x_i - \Delta x}^{x_i + \Delta x} f(x)\, dx \approx \frac{\Delta x}{3}\Big[f(x_i - \Delta x)$$

$$+ 4f(x_i)$$

$$+ f(x_i + \Delta x)\Big]$$