PHYS 177
winter 2019

lecture 15: convex optimization

learning goals
- formulate an optimization problem
- determine if a function is convex

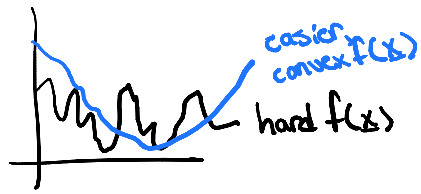
many examples in physics where we want to find the minimum of some function

$$\min_{\underline{x}} f(\underline{x})$$, perhaps subject to constraints on parameters $\underline{x}$

examples: configuration of a system w/ lowest energy, minimizing action, brachistochrone

other common examples include best fit

in general, this is a <u>hard problem</u>



we have some hope if the objective function is <u>convex</u>:

$$f(\alpha \underline{x} + \beta \underline{y}) \leq \alpha f(\underline{x}) + \beta f(\underline{y})$$

convex functions have strictly non-negative second derivatives
they also can't have multiple local minima, any local min is a global min

algorithms

starting at some value of the parameters $\underline{x}_0$, we can try to move in the direction that the function is decreasing most rapidly

$$\boxed{\underline{s} = -\nabla f(\underline{x}_0)}$$ ← step direction

steepest descent method

set $\underline{x}_1 = \underline{x}_0 + \underline{s}$, continue iterating until $\underbrace{|\nabla f(\underline{x})| < \epsilon}_{\text{termination condition}}$

* notebook example, simple polynomials w/ good and bad naive convergence