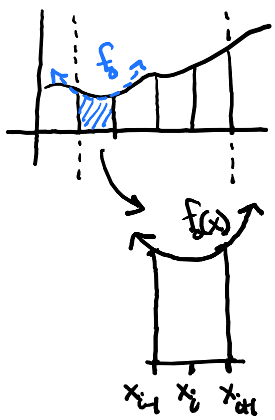
PHYS 177
winter 2019

lecture 4: numerical integration 2

learning goals
- recognize more advanced integration methods
- estimate error on integrals

last time we built a simple integrator, can we do better?

Simpson's rule



rather than straight lines, make locally <u>quadratic</u> approx.

this is a <u>universal theme</u>: finer Δx, higher orders in perturbations

assume $x_{i\pm1} = x_i \pm \Delta x$, then

$$f_0(x_{i-1}) = a(-\Delta x)^2 + b(-\Delta x) + c$$

$$f_0(x_i) = c$$

$$f_0(x_{i+1}) = a(\Delta x)^2 + b(\Delta x) + c$$

$c = f(x_i)$, sum other eqs. to get $f(x_{i-1}) + f(x_{i+1}) = 2a(\Delta x)^2 + \underset{f(x_i)}{2c}$

$$\rightarrow a = \frac{1}{2(\Delta x)^2}\underbrace{\left(f(x_{i-1}) - 2f(x_i) + f(x_{i+1})\right)}_{\text{discrete Laplacian operator!}}$$

$$b = \underbrace{\frac{1}{2\Delta x}\left(f(x_{i+1}) - f(x_{i-1})\right)}_{\text{discrete derivative!}}$$

then $A = \int dx\,(ax^2 + bx + c) = \frac{\Delta x}{3}\left(f(x_{i-1}) + 4f(x_i) + f(x_{i+1})\right)$

* notebook example with plot

plot with multiple choices of resolution, compare vs. trapezoid

compare on arbitrary function and on a quadratic one

prepare trapezoidal integral for comparison, use numpy

test help() function to get info

test multiple plots