PHYS 177
winter 2019

lecture 9 : calculations w/ probability

last time we defined probability distributions $P(X)$, w/ X discrete or continuous

learning goals
- perform simple computations with probability distributions
- use empirical averages and measure their errors

how do we compute with a probability distribution?

$$\langle f(X) \rangle = \sum_i P(X_i) f(X_i)$$ expected value (average) of $f(X)$

example : die has 6 sides , $P(X=1) = P(X=2) = \ldots = 1/6$

the average roll is $\langle X \rangle = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$

what about the sum of two rolls? if the dice are independent,

$$P(X_1, X_2) = P(X_1) P(X_2) \leftarrow P(X_1,X_2) \text{ is a } \underline{\text{product distribution}}$$

$P(X_1, X_2)$ ↑ joint distribution, $P(X_1)$ ↑ marginal distribution

with a product distribution, the average of the sum of two variables is the sum of the averages :

$$\langle X_1 + X_2 \rangle = \sum_{i,j} P(X_1 = i, X_2 = j)(i+j) = \sum_{i,j} P(X_1=i) P(X_2=j)(i+j)$$

$$= \sum_i \sum_j \left[i P(X_1=i) P(X_2=j) + j P(X_1=i) P(X_2=j) \right]$$

$$= \sum_i i P(X_1=i) + \sum_j j P(X_2=j)$$

independent of j , sum of probabilities $=1$

$$= \langle X_1 \rangle + \langle X_2 \rangle = 7$$

how do we approach the average?

$$\left\langle \left(\frac{X_1+X_2}{2} - \langle X \rangle \right)^2 \right\rangle = \frac{1}{4} \langle (X_1 + X_2 - 2\langle X \rangle)^2 \rangle$$

$$= \frac{1}{4} \langle X_1^2 + X_2^2 + 2X_1X_2 + 4\langle X \rangle^2 - 4(X_1+X_2)\langle X \rangle$$

$$= \frac{1}{4}\langle X_1^2 \rangle + \frac{1}{4}\langle X_2^2 \rangle + \frac{1}{2}\langle X_1 X_2 \rangle - \langle X \rangle^2$$

$\nwarrow \sum_i i^2 P(i)$ $\qquad \nwarrow \sum_{i,j} ij P(i) P(j) = \frac{1}{2}\langle X \rangle^2$

$$= \frac{1}{2}\left(\langle X^2 \rangle - \langle X \rangle^2 \right)$$

* example notebook, use numpy sum