STUDY, IMPLEMENTATION AND SIMULATION OF OFDM AND FBMC SYSTEMS IN MATLAB

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1 Modulation schemes candidates for 5G PHY layer

In this introduction we would look at the main modulation schemes candidates to be utilized in 5G communications.
As of today, orthogonal frequency division multiplexing (OFDM) has been the dominant technology for broadband multicarrier communications. However, in certain applications such as cognitive radios and uplink of multiuser multicarrier systems, where a subset of subcarriers is allocated to each user, OFDM may be an undesirable solution. In this article, we address the shortcomings of OFDM in these and other applications and show that filter bank multicarrier (FBMC) could be a more effective solution. Although FBMC methods have been studied by a number of researchers, and some even before the invention of OFDM, only recently has FBMC been seriously considered by a few standard committees.

1.1 Orthogonal Frequency Division Multiplexing (OFDM) basics

OFDM splits a high-rate downstream into several lower rate streams that are transmitted simultaneously over a number of subcarriers an this decrease the delay spread. ISI is eliminated introducing guard time in every symbol. The most important parameters for OFDM are: number o subcarriers, guard time, symbol duration, subcarrier spacing, modulation type per subcarrier and type of FEC. OFDM signal is a sum of subcarriers modulated with PSK or QAM. Each subcarrier is generated using IFFT block at the transmitter that is integrated with FFT block in the transceiver architecture in order to reduce complexity (they have quite similar structure and the complexity is enhanced besides 4-radix algorithm), but with the drawback of an only half duplex usage. Moreover, the IFFT block is useful to save power consumption instead of using a generator for each single subcarrier. To eliminate ICI, the symbol is cyclically extended in the guard time and the multipath signals with delay smaller than the guard time cannot cause ICI. OFDM uses raised cosine windowing for every symbol to reduce the boundary and then reduce further sidelobes power. To minimize SNR loss caused by guard time, it is desirable to have the symbol duration much larger than the guard time. A practical design choice is to make symbol duration at least 5 times the guard time which implies 1dB loss in SNR.

1.2 Further multicarrier modulation schemes considered

1.2.1 Filter Bank Multi-Carrier (FBMC)

FBMC is a multi-carrier modulation based on subcarrier filtering and adopts offset quadrature amplitude modulation (OQAM). The FBMC symbols are mapped into complex domain after quadrature amplitude modulation (QAM) which is the same as OFDM. The differences between FBMC and OFDM can be summarized as follow: i) FBMC adopts OQAM; ii) FBMC adopts filter bank instead of the IFFT and CP insertion at the transmitter and the FFT and CP removal at the receiver. At the transmitter, the real part and imaginary part of a complex symbol on each subcarrier are extracted and staggered with a time offset T/2. At the receiver, inverse procedures are operated.

1.2.2 Generalized Frequency Division Multiplexing (GFDM)

GFDM is an adaptable multiple carrier transmission method. Unlike the other schemes the carrier Orthogonality is not maintained. The out-of-band emissions can be managed in a better way and Peak to Average Power Ratio (PAPR) can be lowered. The filtering of each subcarrier is done individually. The available spectrum for each user is spread into multiple spectral segments. This concept makes it suitable to implement cognitive radio. CP is added to each character. The disadvantages are complex receiver, use of matched filter for removing interference and OQAM makes MIMO difficult.

1.2.3 Comparison in Low-Band Highly Dispersive Wireless Channels (L-BHDWC) scenario

The packet error rate is expresses as 1- (1-bit error probability)$^{\text{packet length in bit}}$ so now we will consider a packet error rate between 10 an 9, over-the-air transmission time near 100 micro seconds, 1 ms end-to-end latency, carrier frequencies below 6 GHz, bandwidth narrower than 100 MHz, we are in presence of multipath effect and we considered perfect synchronization and full channel state information at the receiver. Now we start to consider FBMC-OQAM, GFDM-OQAM and cyclic prefix-circular OQAM (WCP-COQAM), candidates for 5G in L-BHDWC scenarios, with respect to OFDM. The most important parameters in this case are: bit error rate (BER), power spectral density (PSD), spectral efficiency (SE) to represent the cost of using cyclic prefix (CP) extensions, subcarrier filtering, and windowing schemes. These three modulation schemes are all based on OQAM that consists in splitting one complex symbol into two semi-symbols, one real and one imaginary. After this operation, the whole OQAM symbol's duration remains equal to the duration of the original complex symbol, while the duration of each semi-symbol is half the whole symbol. The reason why we focus on OQAM-based schemes is that MCM systems cannot simultaneously keep good time-frequency localization, Nyquist symbol rate, and orthogonality between transmitted symbols if conventional IQ modulations are used. ISI and ICI introduced by the prototype filters of these MCM systems make it impossible to achieve perfect reconstruction of complex valued symbols transmitted at Nyquist rate. Consequently, in case matched filter receivers are used, FBMC and GFDM perform worse than OFDM in terms of BER. One solution is to send alternately real and imaginary valued symbols at twice the Nyquist rate instead of complex symbols and this technique is known as OQAM. In this scenario FBMC-OQAM performs slightly worse than the rest, in case it is uncoded, in terms of BER, FBMC-OQAM outperforms the rest for higher Eb/N0, in case it is coded, and the other have quite similar performances. FBMC-OQAM outperforms the rest (reduction of OOB radiation) in terms of PSD and thanks to the windowing, WCP-COQAM shows considerably lower OOB radiation than GFDM-OQAM. When we consider SE, OFDM performs in the same way of WCP-COQAM and GFDM-OQAM is worst. Considering the low-latency scenarios we encounter significant drawback for FBMC-OQAM compared to GFDM-OQAM and WCP-COQAM during short data transmissions.

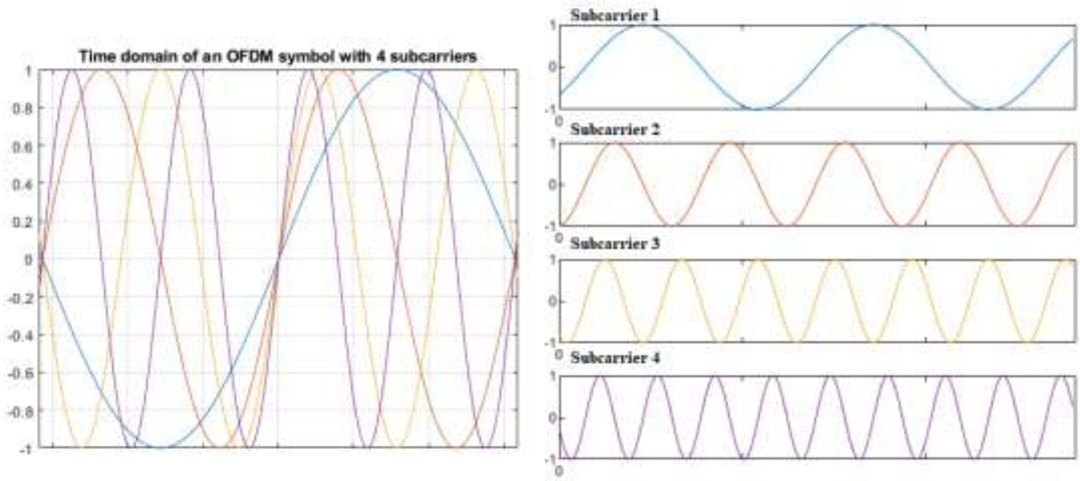
1.3 Objective of the project

The main purpose of this project would be for first the understanding of the topic in deeper manners and than to implement systems that performs properly including OFDM, OQAM-FBMC-FS (Frequency Spreading) and OQAM-FBMC-PPN (Polyphase network with PHYDYAS filter). The project start with the analysis of the OFDM system, follows the Matlab implementation step by step and the evaluation of the outcomes of the simulation o the system implemented. After the overview of the simpler OFDM starts the analysis of the FBMC in several aspects especially the computational complexity than follow the implementation of the Frequency Spreading and PHYDYAS filter techniques. Finally there are the simulations of the FBMC system implemented and some comparison with the OFDM.

2. OFDM more in details

Orthogonal Frequency Division Multiplexing or OFDM is one of the modulation formats more widely used in wireless and telecommunications standards. OFDM has been adopted in the Wi-Fi arena. It has also been chosen for LTE / LTE-A, and other standards such as WiMAX. This modulation format has also been adopted in some standard of DAB (Digital Audio Broadcasting) to the DVB (Digital Video Broadcast). Although OFDM is more complicated than earlier forms of signal format, it provides some advantages in terms of data transmission, above all where high data rates are needed along with relatively wide bandwidths.

The main feature of OFDM is multicarrier modulation. An OFDM signal consists of a number of closely tightened modulated carriers. When modulation is applied to a vector, the side bands expand on both sides. In general, the receiver must receive the entire signal to be able to demodulate the data successfully. Consequently, when the signals are transmitted close to each other, they must be spaced so that the receiver can separate them using a filter and there must be a guard band between them. This is not the case with OFDM. Although the side bands of each carrier overlap, they can still be received without the interference that might be expected because they are orthogonal to each other. This is obtained by setting the carrier spacing equal to the reciprocal of the symbol period.
The transmitter modulates and “filters” the subcarriers by means of the inverse fast Fourier transform operation. The iFFT works as a modulator at the transmitter side and each subcarrier is modulated with a different frequency as it is shown in the next figures obtained from Matlab project. At the receiver the FFT, conversely acts as a demodulator.



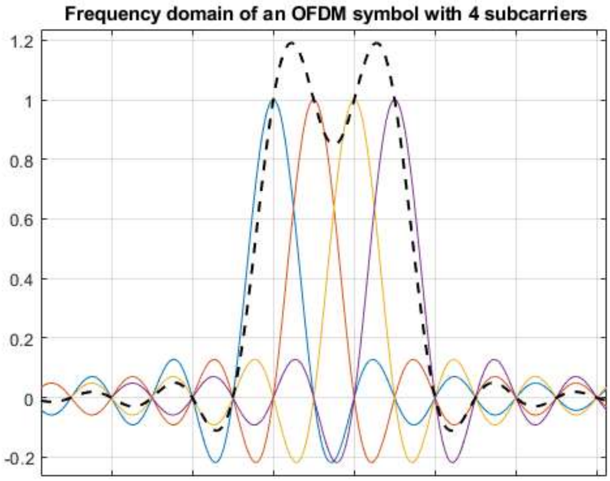


figure 1)

At the receiver each carrier is translated to direct current. Consequently, the signal is integrated over the symbol period in order to regenerate the data from that carrier. The same demodulator also demodulates the other carriers. Since the carrier spacing equal to the reciprocal of the symbol period means that they will have a whole number of cycles in the symbol period and their contribution will sum to zero, so there is no interference contribution. In OFDM non-linearity will cause interference between the carriers as a consequence of inter-modulation distortion. This will introduce undesired signals that would cause interference and compromise the orthogonality of the transmission.
The high peak to average ratio of multi-carrier systems as in the case of OFDM requires that the final amplifier on the transmitter output must handle the peaks whilst the average power is much lower, and this leads to inefficiency. This introduces distortion that results in a higher level of data errors and the system can rely on the error correction to remove them.

2.1 OFDM advantages

OFDM is used in many high data rate wireless systems because of the different advantages it provides.

- ***Immunity to selective fading:*** it divides the overall channel into multiple narrowband signals so that they are affected individually as flat fading sub-channels.
- ***Resilience to interference:*** Interference that appears on a channel may be limited by bandwidth and will not affect all sub-channels in this way, so not all data is lost.
- ***Spectrum efficiency:*** Using neighbouring overlapped subcarriers, an important advantage of OFDM is that it makes efficient use of the available spectrum..
- ***Resilient to ISI:*** it is very resistant to inter-symbol and inter-frame interference. This results from the low data rate on each of the sub-channels. The cyclic prefix helps to counteract ISI as well.
- ***Simpler channel equalisation:*** One of the problems with CDMA systems was the complexity of channel equalization that had to be applied to the whole channel. An advantage of OFDM is that by using multiple subchannels, channel equalization becomes simpler.

2.2 OFDM disadvantages

Despite OFDM is widely used, there exist still some disadvantages to take into account when this kind of system is employed.

- ***High peak to average power ratio:*** The OFDM amplitude variation is similar to the noise one and has a relatively high peak to average power ratio. This impacts the RF amplifier efficiency in fact the amplifiers need to be linear to accommodate the large amplitude variations and these factors mean the amplifier cannot operate with high efficiency.
- ***Sensitive to carrier offset and drift:*** A further disadvantage of OFDM is that it is sensitive to the offset and to the drift of the carrier frequency. Single-carrier systems are less sensitive.

2.3 Implementation an OFDM system in Matlab

The implementation started with a simple OFDM transmitter and receiver by looking at the block diagram. The Matlab code is developed according to the functionalities of each block.

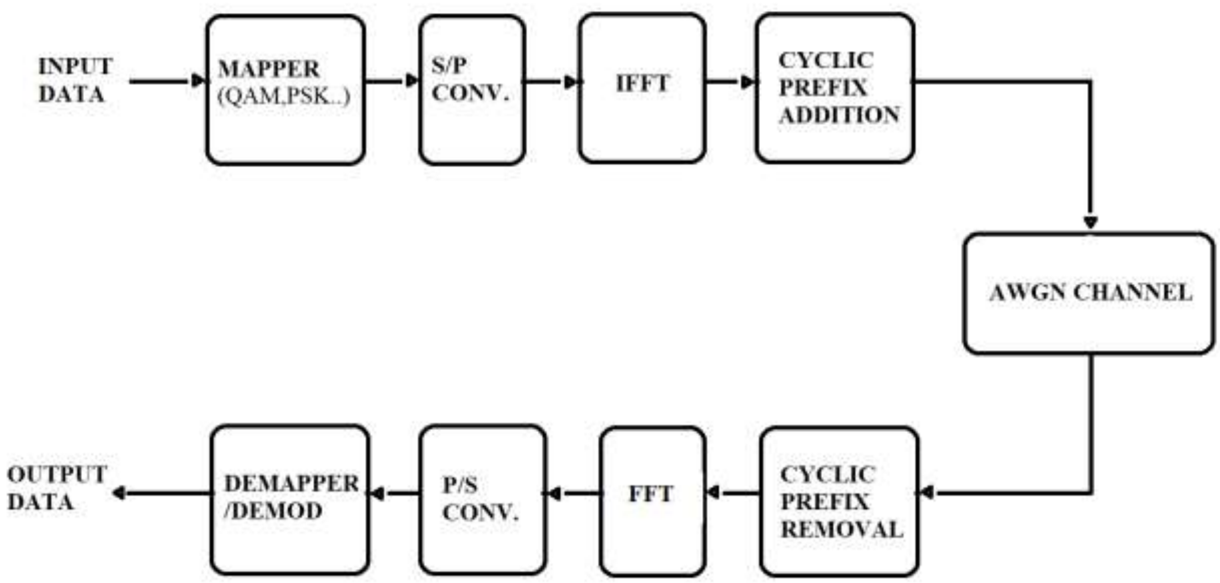


figure 2)

The first step is the setting of the main parameters the OFDM modulation will be a function of those values.

```
M = 4;                    % Modulation alphabet
k = log2(M);              % Bits/symbol
numSC = 128;              % Number of OFDM subcarriers
cpLen = 32;               % OFDM cyclic prefix length
maxNumBits = 1e15;        % Maximum number of bits transmitted
```

So, the modulation alphabet can vary depending on the type of modulation that we want to use. Now, we generate the input data (bits) then we start with the modulation (in this case M-ary).

```
inputData = randi([0,1],frameSize);  % Generate binary data

mappedData = qpskMod(inputData);       % Apply QPSK modulation
```

The `randi()` function is used to generate randomly the input bits. The value of `frameSize` is an integer value corresponding to the size of one frame to transmit. Commonly used modulation schemes are for example: BPSK (Binary Phase Shift Keying), QPSK (Quadrature Phase Shift

Keying), 8-PSK (8 Phase Shift Keying), 16-QAM (16 Quadrature Amplitude Modulation). In this case we use the second one and the bits are converted in complex symbols M by M.

We can initialize both QPSK Modulator and Demodulator at the beginning.

```
qpskMod = comm.QPSKModulator('BitInput',true);
qpskDemod = comm.QPSKDemodulator('BitOutput',true);
```

The serial to parallel converter allows to process the complex data stream partitioning it into blocks of N data symbols, that are transmitted in parallel by modulating the N = `numSC` (in this case 128) subcarriers and the iFFT (inverse Fast Fourier Transform) block is intended as the modulator due to the orthogonality of the OFDM system.

```
ifftData=ifft(qpskTx);
```

We can use the `ifft()` function or we can implement the iFFT according to the definition of iDFT (inverse Discrete Fourier Transform):

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{\frac{2\pi i}{N}kn} \qquad n = 0, \dots, N-1 \tag{1}$$

Follows the implementation where N is the number of iFFT points corresponding to the number of subcarriers.

```
function output = ifftData(input,N)

    for n=1:N
        count=0;
        for k=1:N
            count=count+(1/N)*input(k,:)*exp(1i*2*pi*(n-1)*(k-1)/N);
        end
        output(n,:)=count;
    end

end
```

The addition of the cyclic prefix consists of inverting the position of data at the end of the stream than adding them to the beginning of that one in such a way that we will obtain a circular convolution after channel convolution with the signal in time domain. Follows a Matlab plot as an example with CP length 8 and number of FFT points 32.

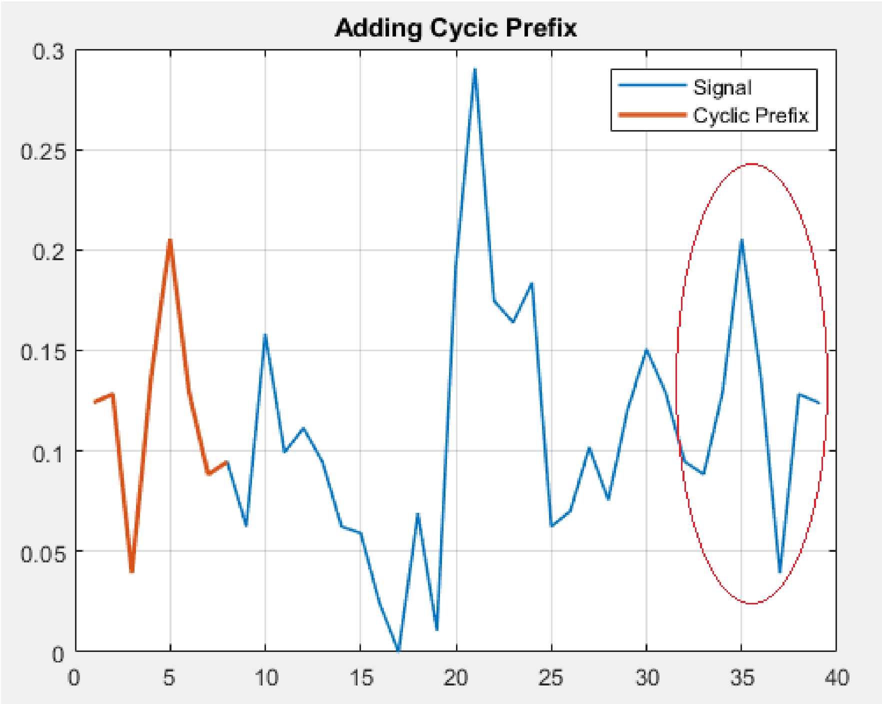


figure 3)

Than there is the implementation of the above operation.

```
dataSize = size(ifftData,1)*size(ifftData,2);           % Size of iFFT data
cyclic_prefix = ifftData(dataSize-cpLen:dataSize);      % Cyclic prefix transposed
appendPrefix = zeros(dataSize+cpLen);                   % Preallocation

    for i=1:dataSize+cpLen

        if(i<=cpLen)

            appendPrefix(i,1)=cyclic_prefix(cpLen-(i-1),1);

        else

            appendPrefix(i,1)=ifftData(i-cpLen,1);

        end

    end
```

So now the signal to be transmitted is ready then follows the AWGN (Additive White Gaussian Noise) channel that represents a simple model to simulate the transmission medium.

```
channel = comm.AWGNChannel('NoiseMethod','Variance', ...
    'VarianceSource','Input port');

snr=12;                                    % SNR in dB
powerDB = 10*log10(var(txSig));            % Calculate Tx signal power
noiseVar = 10.^(0.1*(powerDB-snr));        % Calculate the noise variance

rxSig = channel(txSig,noiseVar);           % Signal through a noisy channel
```

After the setup of the noisy channel model the signal is passed through it. Than the cyclic prefix is removed from the signal, the FFT demodulates the subcarriers and after the QPSK demodulator the bits are obtained at the receiver section.

```
rxSig(1:cpLen)=[];                              % CP removal
qpskRx=fft(rxSig);                              % FFT block (subcarrier demod)
outputData = qpskDemod(qpskRx);                 % Apply QPSK demodulation
```

The `fft()` is computed according to:

$$X_q = \mathcal{F}_d(x_n) = \sum_{k=0}^{N-1} x_k e^{-i\frac{2\pi}{N}kq} \qquad q = 0, 1, \ldots, N-1 \tag{2}$$

Alternatively:

```
function output = fftData(input,N)

    for k=1:N
        count=0;
        for n=1:N
            count=count+input(n,:)*exp(1i*2*pi*(n-1)*(k-1)/N);
        end
        output(k,:)=count;
    end

end
```

The functions of OFDM modulator and demodulator are the following: modulator and demodulator respectively.

```
function out=OFDM_Modulator(mappedData,numSC,cpLen)

%--------------------------IFFT Block---------------------------

ifftData=ifft(mappedData,numSC);

%---------------------------------------------------------------

%--------------------------ADD-CYCLIC PREFIX--------------------

cyclic_prefix = ifftData(numSC-cpLen:numSC);  % CP transposed
appendPrefix = zeros(numSC+cpLen,1);          % Preallocation

    for i=1:numSC+cpLen

        if(i<=cpLen)

            appendPrefix(i,1)=cyclic_prefix(cpLen-(i-1),1);
            cp(i,1)=cyclic_prefix(cpLen-(i-1),1);

        else

            appendPrefix(i,1)=ifftData(i-cpLen,1);

        end

    end
%---------------------------------------------------------------

out=appendPrefix;                              % Output signal

end
```

```
function out= OFDM_Demodulator(rxSig,cpLen,numSC)

%-----------------------Cyclic Prefix Removal----------------------

        rxSig(1:cpLen)=[];                      % CP removed

%------------------------------------------------------------------

%-----------------------------------FFT----------------------------

        out=fft(rxSig,numSC);                   % Output signal

%------------------------------------------------------------------

end
```

2.6 Simulation with the OFDM system implemented

After the implementation follow some simulations in order to test the performance of the system. We start with the comparison among different modulation usually adopted in the OFDM systems.

Here is shown the simulated and measured BER (Bit Error Rate) in relation to the SNR (Signal to Noise Ratio) throw the AWGN channel whit 128 subcarriers, cyclic prefix length 32. For the error probabilities the function from the communications systems toolbox are used. After this simulation we can assume that the system works correctly and performs as we expected, the simulated curves (solid) and theoretical values (dots) are similar each other enough.

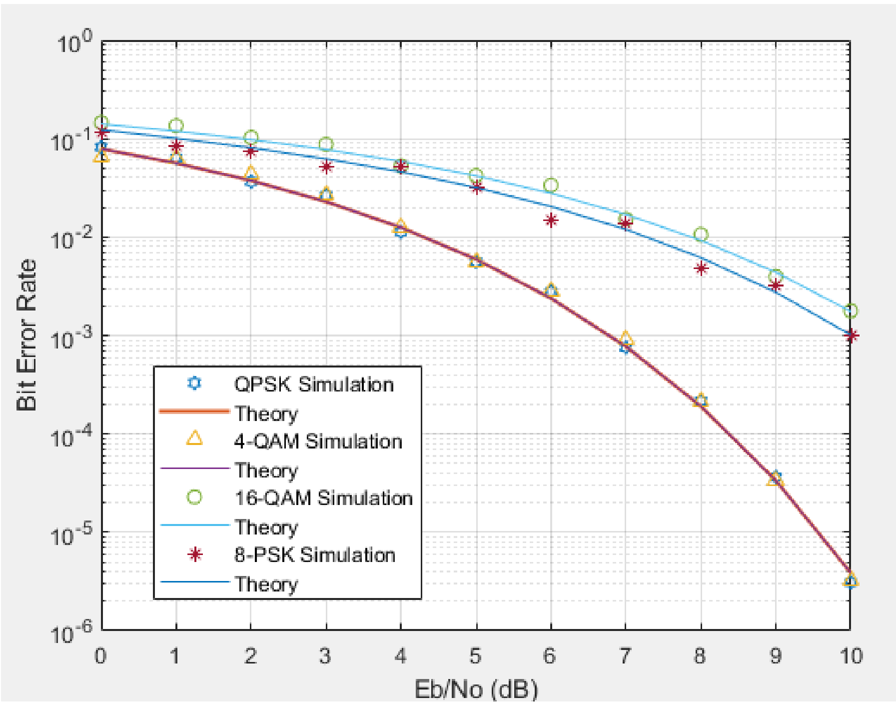


figure 4)

An important aspect of OFDM is the PAPR (Peak to Average Power Ratio) level usually high as outlined in 2.3 OFDM disadvantages. In the next simulation we will go to prove that increasing the number of FFT points so the number of subcarriers, the PAPR values will increase. The following graph in figure 5 means that the probability that the PAPR exceeds a given threshold of PAPR (abscissa) increases as the symbol length becomes higher. So, we observe this behaviour in the following figure where the complementary cumulative distribution (CCDF) function of the PAPR is shown in function of the OFDM symbol length (N=64, N=1024).

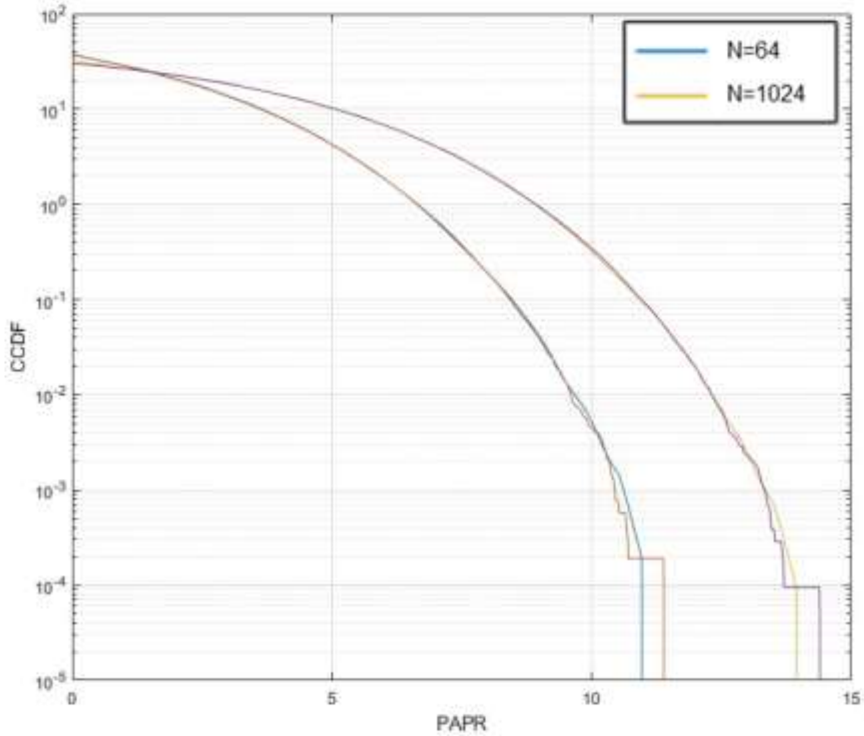


figure 5)

In this simulation 1024 and 64 FFT points are used with QPSK modulation. The figure shows theoretical and measured curves. To calculate the different PAPR values with respect to the CCDF (complementary cumulative distribution function) values the operations are performed according with

$$\mathrm{PAPR} = \frac{\max r_n^2}{E\{r_n^2\}} = \frac{\max |x_n|^2}{E\{|x_n|^2\}}, \quad 0 \leq n < N. \qquad (a)$$

Where the x_n are complex values and r_n are the moduli (amplitudes) of the complex values. The probability that a sample r_n exceeds a predefixed limit PAPR0 can then be expressed as

$$P(\mathrm{PAPR\ of\ one\ sample\ of\ } r_{1,\ldots,N} > \mathrm{PAPR}_0) = e^{-\mathrm{PAPR}_0}. \qquad (b)$$

The joint probability that the PAPR of an OFDM symbol which has N samples exceeds the threshold PAPR0 is given by.

$$P\left(\mathrm{PAPR}\left(r_{1,\ldots,N}\right) > \mathrm{PAPR}_0\right) = 1 - \left(1 - e^{-\mathrm{PAPR}_0}\right)^N. \qquad (c)$$

Which express exactly what is explained before figure 5.

3 FBMC more in details

Filter bank multicarrier aims to overcome some of the deficiencies found with OFDM, orthogonal frequency division multiplexing. One of the main shortcomings comes from the fact that OFDM requires the use of what is called a cyclic prefix. As explained in 2.3, the cyclic prefix is essentially a copy of a part of a symbol transmitted in OFDM that is added at the beginning of the next. This redundancy reduces transmission efficiency and wastes energy. A further disadvantage of OFDM is that the spectral localization of subcarriers is weak, this results in spectral losses and interference problems with unsynchronized signals. Filter bank multicarrier is a development of OFDM. Use of filter banks implemented, typically using digital signal processing techniques, FBMC. When the carriers were modulated in an OFDM system, the lateral lobes widened on both sides. With a filter bank system, the filters are used to remove these and therefore a much cleaner carrier result.

3.1 FBMC physical layer

The filter bank multicarrier leads to improve the physical layer for conventional communication networks and it is an enabling technology for the new concepts and, particularly, cognitive radio. The inverse fast Fourier transform can work as a multicarrier modulator and the fast Fourier transform can work as a multicarrier demodulator as in OFDM.
We set size of the iFFT and the FFT as M. For a set of M data samples, $d\ (mM)\ i$ with $0 \leq i \leq M-1$, the iFFT is computed. The iFFT output is expressed as folows, similarly to the *equation (1)*

$$x(n) = \sum_{i=0}^{M-1} d_i(mM) e^{j2\pi \frac{i(n-mM)}{M}}$$

(3)

The multicarrier symbol is the set of M samples obtained and m is the symbol index. A parallel-to-serial (P/S) converter is used for the transmission through the channel and $x(n)$ appear in serial form. The duration of a symbol T is the inverse of the carrier spacing, $T=M$ and the sampling frequency of the transmitted signal is unity. The carrier spacing for each of the M subacarriers is 1/M. $x(n)$ is a sine wave and the transmitted signal is a collection of sine waves such that the symbol duration contains an integer number of periods. In fact, it is the condition the so-called orthogonality condition needed for data recovery. At the receiver side, a serial-to-parallel (S/P) converter is used at the input of the FFT. The data samples are reconstructed by

$$d_i(mM) = \frac{1}{M}\sum_{n=mM}^{mM+M-1} x(n) e^{-j2\pi\frac{i(n-mM)}{M}} \quad (4)$$

The relationship between the input and output of the FFT with index $k = 0$ is:

$$y_0(n) = \frac{1}{M}[x(n-M)+....+x(n-1)] = \frac{1}{M}\sum_{i=1}^{M} x(n-i) \quad (5)$$

This equation is a low-pass linear phase FIR filter using M coefficients equal to $1/M$. Neglecting the constant delay, the frequency response is

$$I(f) = \frac{\sin \pi fM}{M \sin \pi f} \quad (6)$$

and the FFT is

$$y_k(n) = \frac{1}{M}\sum_{i=0}^{M-1} x(n-M+i) e^{-j2\pi ki/M} \quad (7)$$

Now we use a change of variables so we replace i by $M - i$, follows

$$y_k(n) = \frac{1}{M}\sum_{i=1}^{M} x(n-i) e^{j2\pi ki/M} \quad (8)$$

The filter coefficients are multiplied by $e^{j2\pi ki/M}$, which is a shift in frequency by k/M of the frequency response. To obtain bank of M filters we have to consider all the FFT outputs.

A FIR filter uses time domain or frequency domain coefficients equivalently. The two types of coefficient are correlated by the inverse discrete Fourier transform. The impulse response of the first filter in the bank in frequency domain (its DFT) consists of a singe pulse. Then the frequency coefficients are the samples of $I(f)$ (the frequency response) which is derived from the coefficients by using the interpolation formula according to the sampling theory. The first filter in the bank is called

prototype filter and it is associated with the carrier frequency zero. The number of coefficients is increased to reduce the out of band ripples in time and also in frequency domain.
The main feature of the prototype filter is the overlapping factor K (filter impulse response duration/multicarrier symbol period = K = $T_{\text{filter imp.resp.}}$ / T).

3.2 Implementation of a FBMC system using PHYDYAS filter in Matlab

Before we start the implementation, we have to design the prototype filter in such a way that the data are transmitted without any ISI despite the overlapping.

3.2.1 Prototype filter design

Digital transmissions are based on the Nyquist theory: the impulse response of the transmission filter must cross the zero axis at all the integer multiples of the symbol period.
So in frequency domain we use the symmetry condition about the cut-off frequency that corresponds to half of the symbol rate. Then, we impose the symmetry condition satisfied by the squares of the frequency coefficients. The frequency coefficients of the half-Nyquist filter obtained for K=2,3 and 4 are in the following table also used for the implementation.

K	H_0	H_1	H_2	H_3	σ^2 (dB)
2	1	$\sqrt{2}/2$	-	-	-35
3	1	0.911438	0.411438	-	-44
4	1	0.971960	$\sqrt{2}/2$	0.235147	-65

Follows the implementation.

```
H1=0; H2=0; H3=0;
% Prototype filter
switch K
    case 2
        H1=sqrt(2)/2;
    case 3
        H2=0.411438; H1=0.971960;
    case 4
        H1=0.971960; H2=sqrt(2)/2; H3=0.235147;
    otherwise
        return
end
```

We do not have out-of-band ripples, so a highly selective filter has been obtained. For K=4 the result is shown in the next figure (result from the Matlab project).

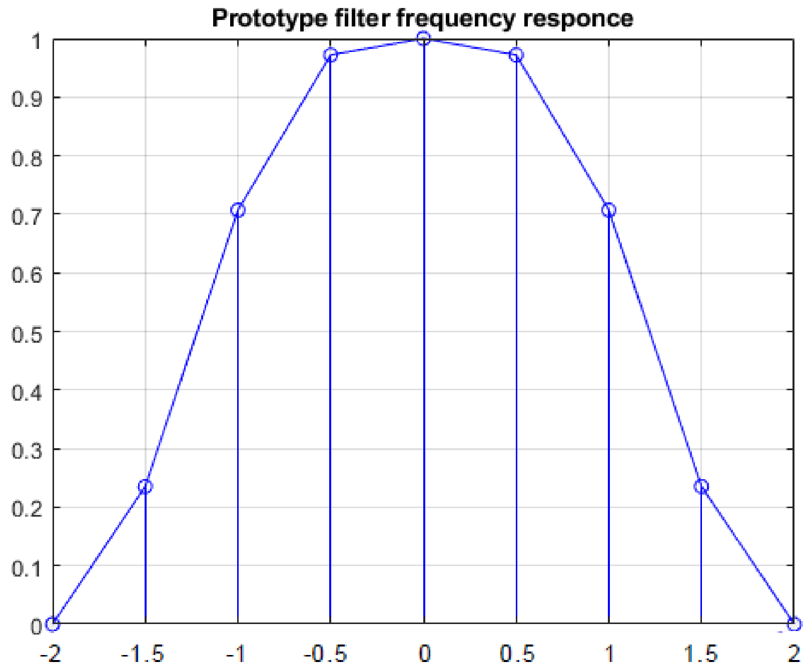


figure 6)

The impulse response $h(t)$ of the filter is obtained by the computation of the inverse Fourier transform of the pulse frequency response, that is

$$h(t) = 1 + 2\sum_{k=1}^{K-1} H_k \cos(2\pi \frac{kt}{KT}) \qquad (10)$$

Follows the Matlab implementation of the prototype filter impulse response.

```
% Prototype filter impulse response
for i=numSC/numFFT:numSC/numFFT:K*numSC-numSC/numFFT
    h(1+(i*(numFFT/numSC)))=1-2*H1*cos(pi*i/(2*numSC))...
    +2*H2*cos(pi*i/numSC)-2*H3*cos(pi*i*3/(2*numSC));
end
```

The prototype filter impulse response is shown in the following figure for the filter length L=1024, the number of sub-channels M=256 and K=4 (result from the Matlab project).

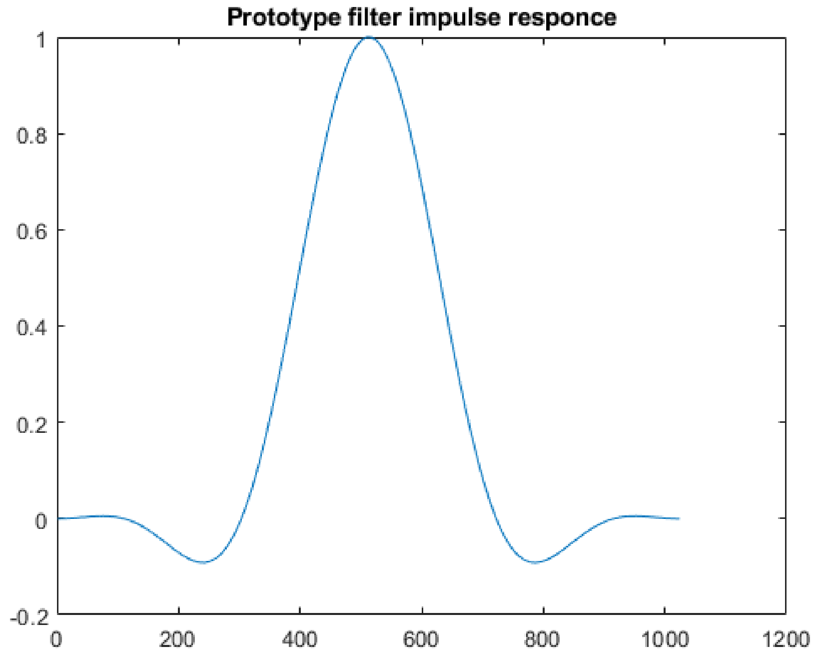


figure 7)

3.2.2 OQAM

Once the prototype filter has been designed, we obtained the filter bank by the frequency shifts *k*/*M*, as in the FFT case. Multiplying the prototype filter coefficients by $e^{j2\pi\, ki\,/M}$ we obtain the filter with index *k*. A section of the filter bank derived in that manner is shown in the next figure where the sub-channel index corresponds to the frequency axis. The subchannels with even index (odd index) do not overlap. This impact heavily on systems. In fact, a subchannel overlaps in frequency with its neighbours only as we can see the real part in blue and the imaginary part in red in *figure 8*.

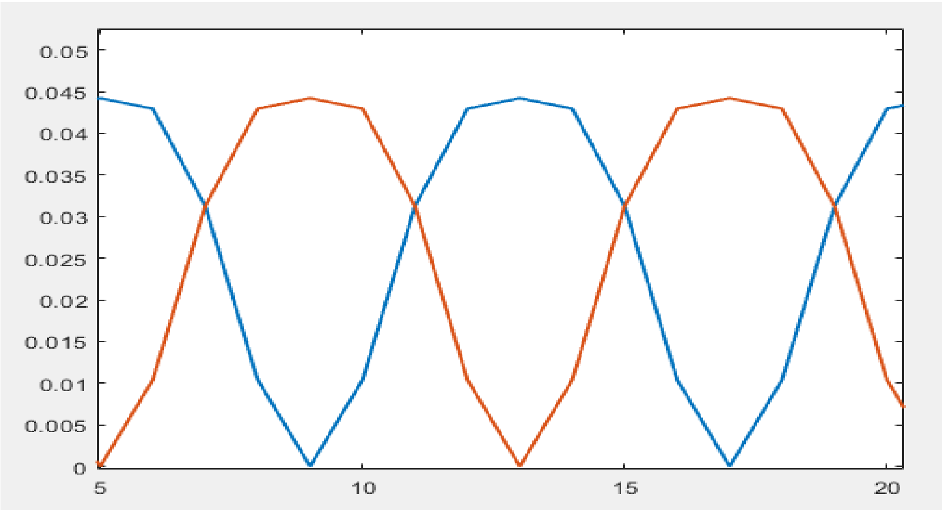


figure 8)

To reach full capacity this strategy is used. It consists on doubling the symbol rate and use alternatively the real and the imaginary part of the iFFT for each subchnnel. The imaginary part is delayed by half the symbol duration so that the real and the imaginary part of a complex data symbol are not transmitted simultaneously as in OFDM.

This is the so-called offset quadrature amplitude modulation (OQAM) and the term ‘offset’ means that the time is shifted of half the inverse of the sub-channel spacing between the real part and the imaginary part of a complex symbol. Follows the code referring to the OQAM implementation. L is the number of symbols and `dataSubCar` is used to contain the alternate the real and the imaginary parts of the previously modulated data (`modData`).

```
dataSubCar = zeros(L, 1);
% OQAM Modulator: alternate real and imaginary parts
if rem(symIdx,2)==1      % Odd symbols
    dataSubCar(1:2:L) = real(modData);
    dataSubCar(2:2:L) = 1i*imag(modData);
else                     % Even symbols
    dataSubCar(1:2:L) = 1i*imag(modData);
    dataSubCar(2:2:L) = real(modData);
end
```

3.2.3 Frequency Spreading

One of the techniques to implement FBMC is the frequency spreading where the filter bank in the transmitter can be implemented as follows

- an iFFT of size KM is used to prouce all the necessary carriers,
- a data element *d*, after the multiplication by the filter frequency coefficients, is passed to the 2K-1 inputs of the iFFT with indices $(i-1)K+1, \ldots\ldots, (i+1)K-1$. So, the data element is spread over many iFFT inputs, this operation is called “weighted frequency spreading”.

The output of the iFFT is a block of KM samples and K consecutive iFFT outputs will overlap in the time domain since the symbol rate is 1/M. Then, the filter bank output is provided by an overlap and sum operation. Follows the block diagram.

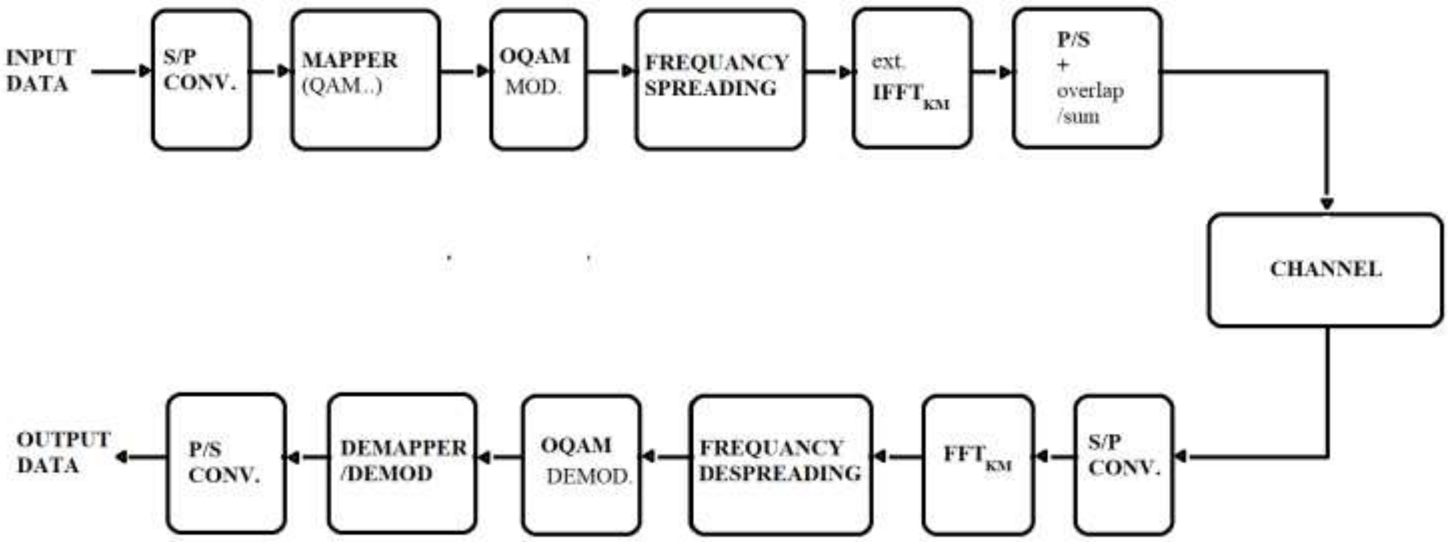


figure 9)

The implementation of the receiver is based on an extended FFT, with KM points. In this case we have classical sliding window situation because the FFT input blocks overlap.

A weighted despreading operation is used to recover the data elements at the output of the FFT. For the data recovery holds the following property of the frequency coefficients of the Nyquist filter

$$\frac{1}{K}\sum_{k=-K+1}^{K-1}\left|H_k\right|^2 = 1 \qquad (11)$$

As in OFDM implementation we initialize the channel function and we set the main parameters for the whole system.

```
numFFT = 1024;            % Number of FFT points
numGuards = 212;          % Guard bands on both sides
K = 4;                    % Overlapping symbols, one of 2, 3, or 4
numSymbols = 100;         % Simulation length in symbols
bitsPerSubCarrier = 2;    % 2: 4QAM, 4: 16QAM, 6: 64QAM, 8: 256QAM
```

The modulator and demodulator.

```
% QAM symbol mapper
qamMapper = comm.RectangularQAMModulator(...
    'ModulationOrder', 2^bitsPerSubCarrier, ...
    'BitInput', true, ...
    'NormalizationMethod', 'Average power');

% QAM demodulator
qamDemod = comm.RectangularQAMDemodulator(...
    'ModulationOrder', 2^bitsPerSubCarrier, ...
    'BitOutput', true, ...
    'NormalizationMethod', 'Average power');
```

We design the prototype filter.

```
Hk = [fliplr([H1 H2 H3]) 1 [H1 H2 H3]];
```

The generation of random bits and the modulation are obtained in the same manner as in the OFDM implementation. Than the complex data are processed by the OQAM modulator (pre-processing) as described above.

Follows the addition of the guard bands at the beginning and at the end of the stream of data exiting from the OQAM modulator.

```
OQAMdata = [zeros(numGuards*K,1); dataSubCarUp; zeros(numGuards*K,1)];
```

Now we have to filter this stream and then apply the extended iFFT of size KM.

```
% Apply filtering FS
filteredData = filter(Hk, 1, OQAMdata);
% Compute IFFT of length KM for the transmitted symbol
txSymb = ifft(filteredData);
```

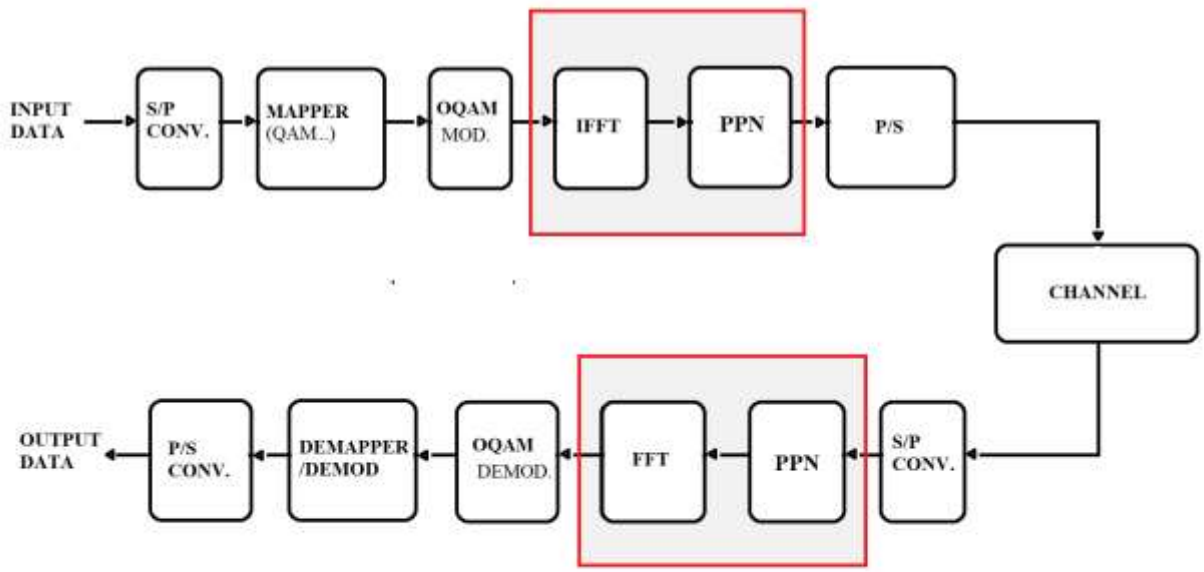
In this implementation we set M = number of FFT points. The `txSymb` are grouped in a matrix each as serial data than transmitted. So, the transmitted signal is passed through the noisy channel as in OFDM project than arrives to the receiver.

```
% Perform FFT
fftData = fft(rxSig);

% Matched filtering with prototype filter
filteredData = filter(Hk, 1, fftData);

% Remove guards
rxNoGuard = filteredData(numGuards*K+1:end-numGuards*K);


    % OQAM post-processing
    if rem(symIdx, 2)
        % Imaginary part is K samples after real one
        r1 = real(rxNoGuard(1:2*K:end));
        r2 = imag(rxNoGuard(K+1:2*K:end));
        modData = complex(r1, r2);
    else
        % Real part is K samples after imaginary one
        r1 = imag(rxNoGuard(1:2*K:end));
        r2 = real(rxNoGuard(K+1:2*K:end));
        modData = complex(r2, r1);
    end

% Demapper: Perform hard decision
outputData = qamDemod(modData);
```

The FFT of size KM is performed on the received signal than follows the frequency despreading, the guard bands are removed. The OQAM post processing is applied depending on the symbol index that corresponds to the position in the matrix of the transmitted serial streams, effectively the received data are processed as serial to parallel streams according to the receiver part of the block diagram, the use of a matrix is a simplification.

3.2.4 Implementation of a Polyphase Network

An issue for FBMC systems is how to reduce the complexity of this typology of filters. A significant amount of redundancy is present in the computations because of the overlapping in the time domain of the iFFT outputs and FFT inputs. An efficient method to reduce this redundancy is the so-called PPN-FFT scheme. There are two of the most famous type of implementation of polyphase network filters: PHYDYAS and IOTA which have similar performance so now we will focus only on PHYDYAS. *figure 10)*



This PPN approach is the frequency domain vision of the frequency spreading, in fact in the block diagram above we inverted the sequence of the iFFT (FFT) block and the modulator (demodulator) filter bank block. So an equivalent time domain vision is described, with the objective to reduce the computational complexity. In fact, the number of the FFT points can be kept to M, but we need some additional processing, called , indeed, polyphase network (PPN). The relationship between input and output sequences of the prototype filter, in the time domain, is defined by a set of coefficients, and holds

$$y(n) = \sum_{i=0}^{L-1} h_i x(n-i) \qquad (12)$$

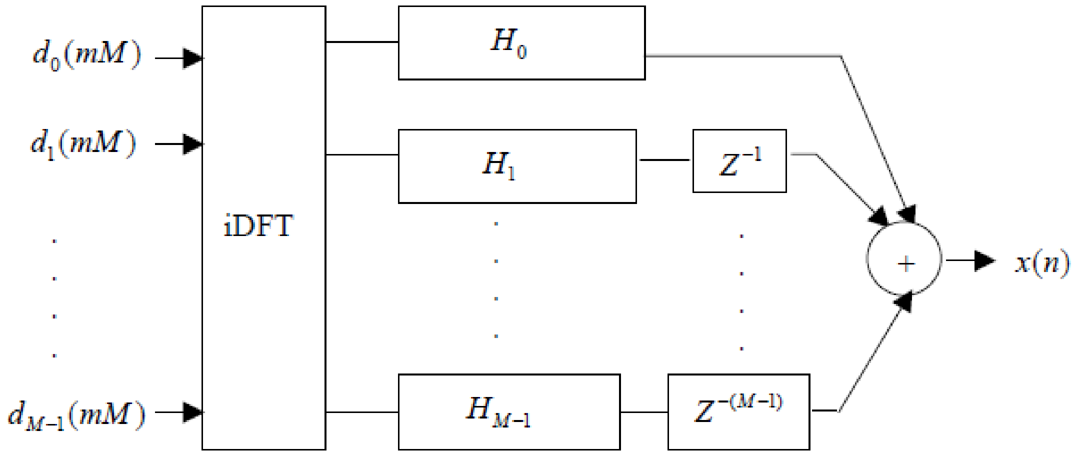
The sequence of coefficients h_i $(0 \le i \le L-1)$ is the filter impulse response of length L and the frequency response is

$$H(f) = \sum_{i=0}^{L-1} h_i e^{-j2\pi if} \qquad (13)$$

Where the sampling frequency is assumed to be unity. Now, if we assume that the filter length is $L = K*M$, we can decompose the sequence of filter coefficients into M interleaved sequences of K coefficients and the transfer function can be expressed as a double summation.

$$H(f) = \sum_{p=0}^{M-1} \sum_{k=0}^{K-1} h_{kM+p} \, e^{-j2\pi(kM+p)f} \qquad (14)$$

So, each filter element has the frequency response of a phase shifter, this is the reason of the name polyphase decomposition, and polyphase network for the complete set. In the implementation the transmitter output is the sum of the outputs of the filters of the bank. The data are processed by the filter elements after the summation which is performed by the iDFT. Finally, the structure for the implementation of the filter bank in the transmitter is shown in the following figure.



PPN-iFFT implementation of the transmitter filter bank *figure 11)*

Each section of the PPN has K multiplications for K=4 in terms of complexity and the complete PPN requires KM multiplications, which is less than the iFFT as soon as the number M of sub-channels becomes large. Follows the implementation.

```
%PPN DECOMPOSITION type 1
H=zeros(KM,1);

km=1;

for p=1:M

    for k=1:K

        H(km)=h((k-1)*M+p)*exp(-1i*2*pi*(k-1)*M)*exp(-1i*2*pi*(p-1));
        km=km+1;

    end

end

filteredData=d.*H;

```

The same scheme is applied to the filter bank in the receiver with the difference that the frequency shifts are multiples of –1/M and the discrete Fourier transform replaces the iFFT.
For each subchannel, in fact, the signal of interest is shifted around the zero frequency and filtered.

3.3 Simulation of FBMC system

After the implementation follow some simulations in order to test the performance of the system. We start with the comparison among different modulation usually adopted in the OFDM systems. Here is shown the simulated and measured BER in relation to the SNR throw the AWGN channel whit 128 subcarriers.

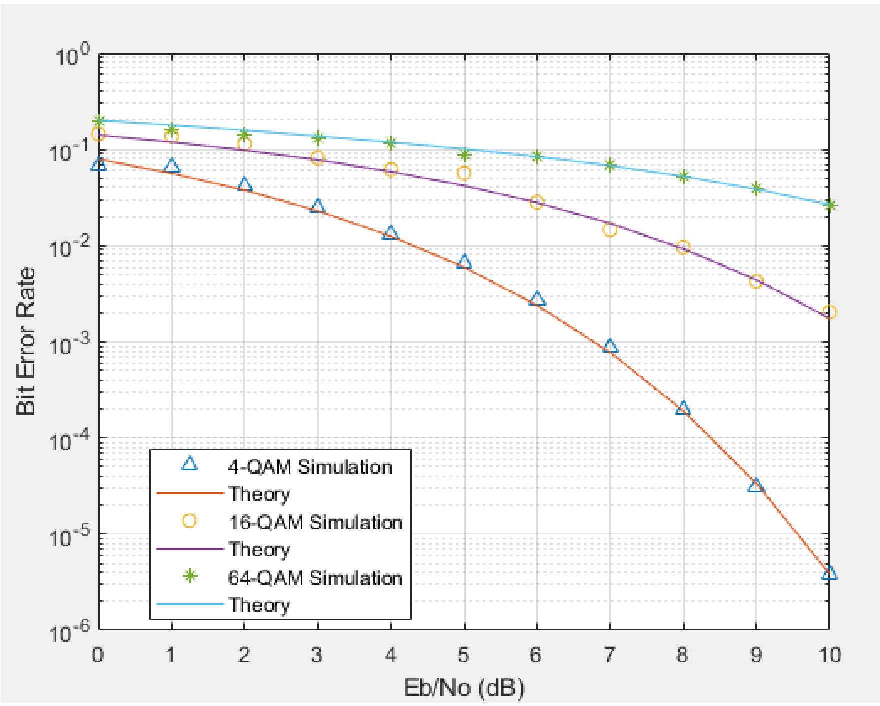


figure 12)

For the error probabilities the function from the communications systems toolbox are used. After this simulation we can assume that the system works correctly and performs as we expected, the simulated points and theoretical curves agree.

Now we want to analyse the PSD (power spectral density) and the magnitude of an FBMC signal with respect the three different values of the overlapping factor K with number of subacarriers 1024, guard band length 212, 4-QAM mapping we transmit 100 symbols.

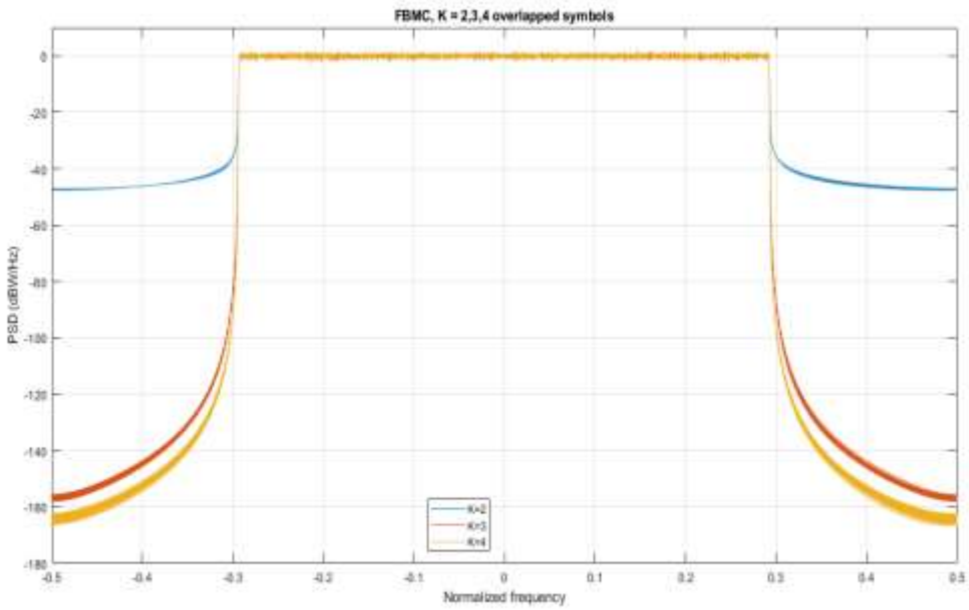


figure 13)

It is easy to understand that increasing the overlapping factor we avoid the waste of power visible in the right and the left sides in fact the most used overlapping factors are 3 and 4. This behaviour is more visible analysing the magnitude response as well.

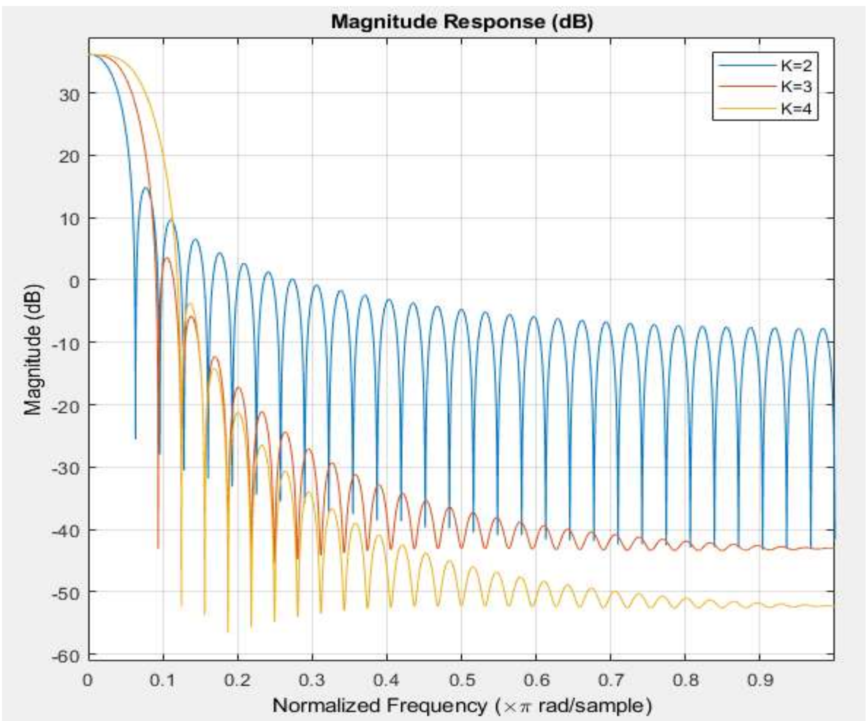


figure 14)

3.4 Comparison between OFDM an OQAM-FBMC

FBMC is an evolution of OFDM, they are very similar but the FBMC doesn't need cyclic prefix so it doesn't waste band neither power and it uses filters and not only the filtering effect of the iFFT and FFT blocks, so this will be reflected on the frequency selectivity. In the following simulations we will see the difference between the out of band sidelobes radiation of the two modulation schemes

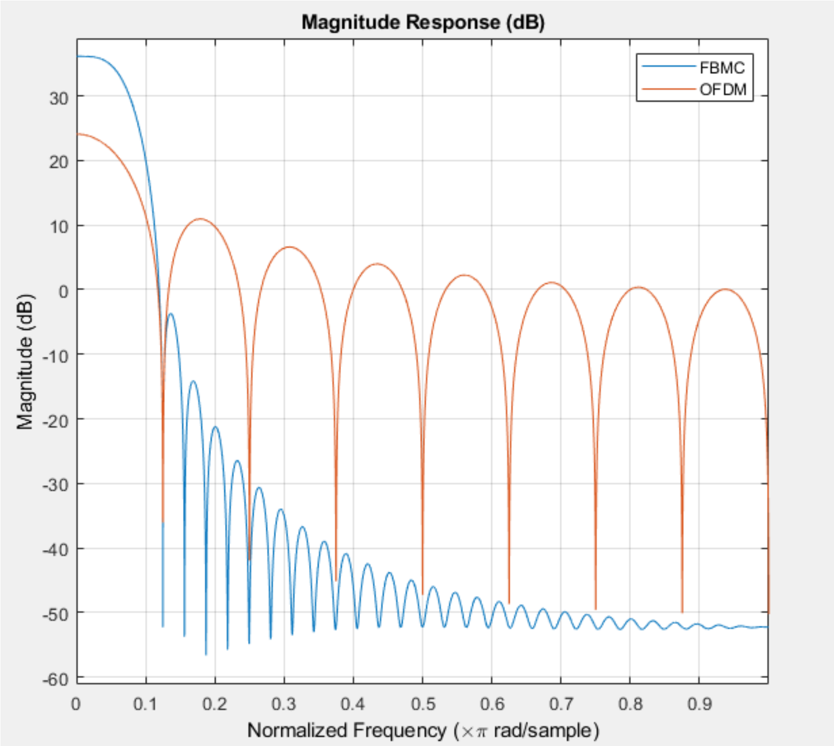


figure 15)

We can observe that the sidelobes are smaller in the case of the FBMC so there is a smaller waste of power and a more efficient use of the spectrum.

In a similar way we obtain the same result from the PSD (Power Spectral Density) comparison as follows.

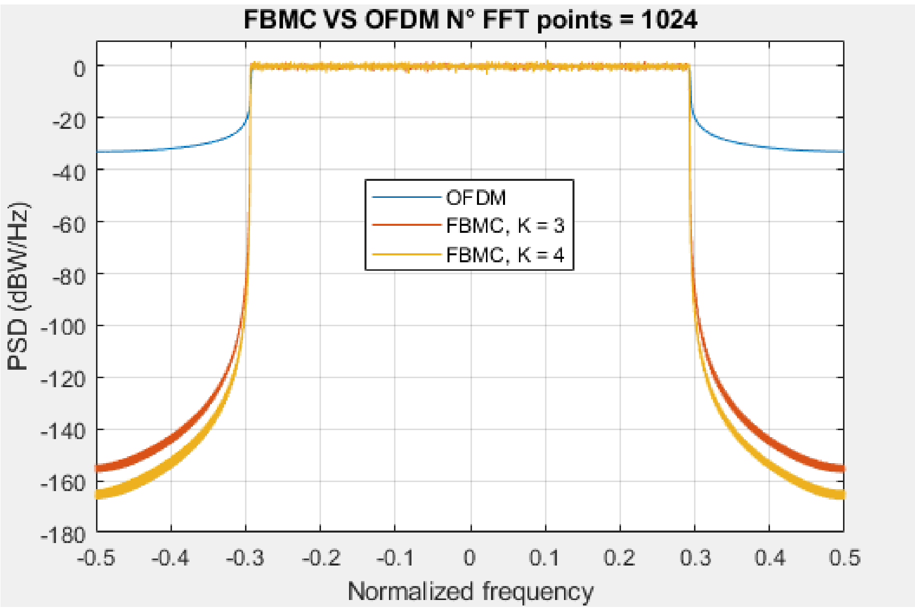


figure 16)

4 Conclusions

In conclusion we can say that FBMC has better performance with respect to OFDM for the very low out-of-band radiation as noticeable from the simulations above and in term of spectrum efficiency because FBMC does not need to use the cyclic prefix as the OFDM.

FBMC, however, presents some drawback in terms of complexity that becomes higher with respect to the OFDM due to the proportionality with the number of operations performed for the modulation (demodulation) and filtering.

Finally, the goal of the project has been achieved, the implemented systems work correctly. As future works the study of the system in multipath environment (in Rayleigh channel) and the implementation of coding schemes to improve the performance of the system will be considered.

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https://it.mathworks.com/matlabcentral/fileexchange/

The equations a), b), c) at page 13 refer to the following documentation:
https://home.mht.bme.hu/~kollar/web/Diploma_Kollar.pdf

The equations:
3), 4), 5) at page 14;
6), 7), 8) at page 15;
10) at page 16;
11) at page 19;
12), 13), 14) and *figure 11* at page 21 refer to the following documentation:
http://www.ict-phydyas.org/teamspace/internal-folder/FBMC-Primer_06-2010.pdf

The equations:
1), 2) at page 9 and 10 respectively;
refer to the following documentation:
https://it.wikipedia.org/wiki/Trasformata_di_Fourier_veloce

The results shown have been achieved during the development of this project in collaboration and with the supervision of BME, BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS, https://www.bme.hu/?language=en.