Bearing Defect Simulation Library
Final Report Group 48
Github repository→HERE.

Pierrick Rauby

April 27th, 2021

Introduction

Bearing defect simulation is crucial when it comes to testing new defect detection methods. As explained in this project's proposal, current simulation technics either represent a localized defect or are limited to the fault with a simple geometrical shape. Thus, they do not enable the representation of complex defects. The current project attempts to address those limitations by developing a Library for bearing defect simulation where the user can specify complex defect shapes using a queuing model. As the simulation results need to be validated, it is also proposed to implement a simple model to calculate the localized faults' bearing defect frequencies.

As most of the requirements for the project were presented in the checkpoint report, Part I to III presents the motivation of the project and literature review, the conceptual model, and the proposed validations; those parts are the same as for the checkpoint. Part IV presents the results of the validations. Part V is a walk-through of the library and a tutorial to run the tests and simulations. Finally, Part VI gives the link to the repository, the work repartition, and a conclusion.

1 Description of the project

1.1 Bearing

Rolling element bearings are composed of 4 parts: an inner race, an outer race, some rolling elements (balls or rollers), and a cage that holds the rolling elements apart from one another. Figure 1 presents those different parts in a rolling element bearing.

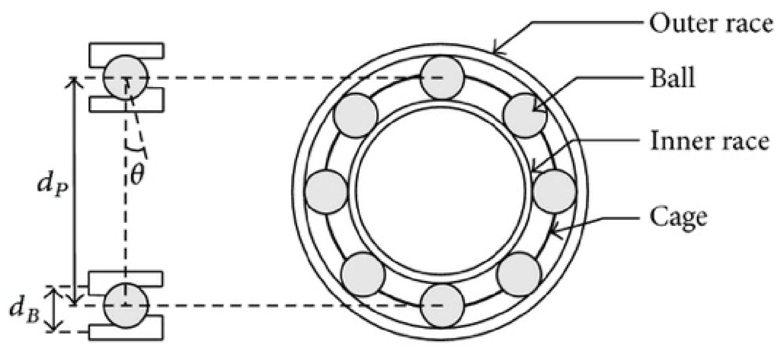
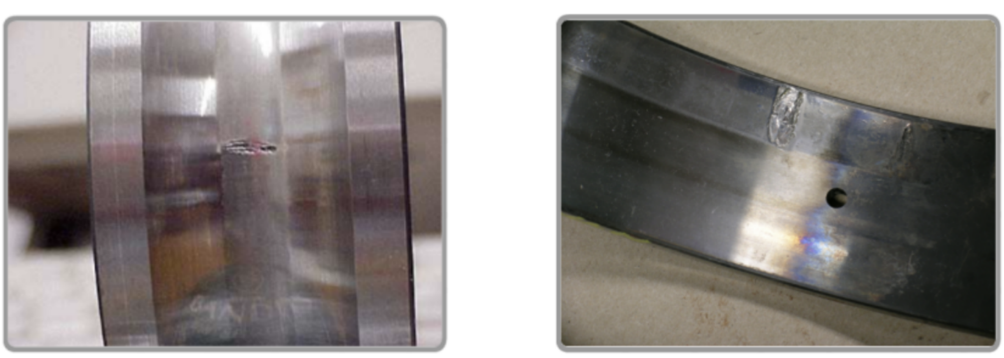


Figure 1: Components of rolling elements bearings[5]

1.2 Bearing defects

Any part of the bearing can wear out and eventually crack; the most common defects are located on the race as they are the most solicited components. Figure 2 [7] shows a defect on the outer race and a defect on the inner race. The passage of the rolling element on the defects generates an impulse that can be identified in the vibration signal.



(a) Inner-race fault

(b) Outer-race fault

Figure 2: Race defects.

1.3 Bearing defect simulation

The simplest way to simulate bearing defect is to calculate the theoretical passage frequency of any rolling element on a given point of a race [3]. These frequencies formula only depend on the rotation speed of the bearing and its geometric configuration. This signal is then mixed with some noise to create a waveform[8]. More advanced methods consider the passage frequency and consider the defect's geometrical shape to obtain a more realistic waveform. In [9] where the shaft running the bearing is considered as a 2D beam and the rest of the bearing is modeled using a FEM model and Hertzian contact Theory [2]. In [1], the entire system is modeled using FEM, the results obtain are very precise, but it is often time complicated to have all the input required to feed such a model. However, there is very little work on more complex bearing defect shapes. Additionally, there is no open-source library to model bearing defects.

2 Conceptual Models

This part presents the conceptual models for the simple localized fault and the queuing model.

2.1 Simple localized fault

It is proposed to establish the equation for the frequency at which a the localized defect will excite the structure. In this case, we consider the bearing races and ball to be undeformable solid. We assume to know the the fundamental characteristics of the bearing:

- The number of balls n.
- The diameter of the ball d_B
- The pitch diameter, which is the diameter of the circle described by the center of the balls as they rotated around the bearing d_P.
- The contact angle θ, which is the angle between the plane of rotation of the races and the line that goes through a ball's contact point and the two races as shown in Figure 1.
- There is no slip between the balls and the races (the relative velocity between the ball and a race is zero at the contact point)
- The defect will be assumed to a be single "mathematical" point located on the outer or the inner race of the bearing.

2.2 Derivation of the bearing defect frequencies

2.2.1 Derivation

From Figure 1, the radius of the outer race is given by $\frac{d_b}{2} + \frac{d_B}{2}\cos\theta$.Denoting N_o the outer race's angular speed, the velocity of a point of the outer race is:

$$V_o = \frac{N_o}{2}\left[d_P + d_B\cos\theta\right] \tag{1}$$

The velocity in the inner can be derived similarly to equation 1 and is:

$$V_i = \frac{N_i}{2}\left[d_P - d_B\cos\theta\right] \tag{2}$$

And the velocity a the center of the rolling element can be computer as the mean between the between V_i and V_o such that $V_c = \frac{V_o+V_i}{2}$. Then we can obtain the rotational speed at the center of the rolling element with:

$$\begin{aligned} N_c &= \frac{V_o+V_i}{2}\frac{2}{d_P} \\ &= \frac{1}{2d_P}\left[N_o\left(d_P + d_B\cos\theta\right) + N_i\left(d_P - d_B\cos\theta\right)\right] \\ &= \frac{1}{2}\left[N_o\left(1+\frac{d_B}{d_P}\cos\theta\right) + N_i\left(1-\frac{d_B}{d_P}\cos\theta\right)\right] \end{aligned} \tag{3}$$

From the above computed rotationnal speed we can compute the frequency at which a single element passes at a given point of the outer race:

$$
\begin{aligned}
N_{o/b} &= N_o - N_c \\
&= N_o - \frac{1}{2}\left[N_o\left(1+\frac{d_B}{d_P}\cos\theta\right) + N_i\left(1-\frac{d_B}{d_P}\cos\theta\right)\right] \\
&= \frac{1}{2}\left(N_o - N_i\right)\left[1-\frac{d_B}{d_P}\cos(\theta)\right]
\end{aligned} \tag{4}
$$

Considering that they are n rolling elements in the bearing, we obtain the frequency of any rolling element passing on one given point of the outer race:

$$
N^n_{o/b} = \frac{n}{2}\left|N_o - N_i\right|\left[1-\frac{d_B}{d_P}\cos(\theta)\right] \tag{5}
$$

Similarly to 5, we derive the passage frequency of any rolling element on the inner-race:

$$
N^n_{i/b} = \frac{n}{2}\left|N_i - N_o\right|\left[1+\frac{d_B}{d_P}\cos(\theta)\right] \tag{6}
$$

Denoting N_r the rotational speed of a rolling element we can express V_o as in 1 but also:

$$
V_o = \frac{1}{2}N_r d_B + \frac{1}{2}N_c\left[d_P + d_B\cos(\theta)\right] \tag{7}
$$

Equating 1 and 7 and solving for N_r:

$$
N_r = \left[N_o - N_c\right]\left[\frac{d_P}{d_B} + \cos(\theta)\right] \tag{8}
$$

Then we can use 8 and reinject the value of N_c from 3 to find:

$$
\begin{aligned}
N_o\left(1+\frac{d_B}{d_P}\cos\theta\right) = \\
& N_r d_B + \frac{1}{2}\left(1+\frac{d_B}{d_P}\cos\theta\right) \\
& \times\left[N_o\left(1+\frac{d_B}{d_P}\cos\theta\right) + N_i\left(1-\frac{d_B}{d_P}\cos\theta\right)\right]
\end{aligned}
$$

After solving for N_r the rotational speed of a rolling element we find:

$$
\begin{aligned}
N_r &= \frac{1}{2}\left|N_o - N_i\right|\left[\frac{d_P}{d_B} + \cos\theta\right]\left[1-\frac{d_B}{d_P}\cos\theta\right] \\
&= \frac{1}{2}\left|N_o - N_i\right|\frac{d_P}{d_B}\left[1-\left(\frac{d_B}{d_P}\cos\theta\right)^2\right]
\end{aligned} \tag{9}
$$

2.2.2 Summary of the bearing defect frequencies

Finally, we have obtained the excitation frequency for defects on the inner and outer-races, those frequency are called:

BPFO for Ball Pass Frequency Outer-race:

$$
\boxed{BPFO = \frac{n}{2}\left|N_o - N_i\right|\left[1-\frac{d_B}{d_P}\cos(\theta)\right]} \tag{10}
$$

BPFI for Ball Pass Frequency Inner-race:

$$
\boxed{BPFI = \frac{n}{2}\left|N_i - N_o\right|\left[1+\frac{d_B}{d_P}\cos(\theta)\right]} \tag{11}
$$

With:

n : the number of rolling elements in the bearing
N_o : the rotational frequency of the outer-race
N_i : the rotational frequency of the inner-race
d_B : the diameter of the rolling element
d_P : the diameter of the circle described by the rolling element
θ : the contact angle

Note: The equations derived above are present in almost any paper related to bearing defect, I am not pretending to have "discovered" it. However, in order to understand them a bit better I have done the derivations and the entire python code of the project is my own work.

The next part presents the bearing defect with a more complex shape.

2.3 Queuing model for bearing defect

This model is the project's primary objective as we want to implement a library for complex bearing defect shapes. We will use a queuing model in which the bearing race on which the defect is located will be flattened and considered a half-space where balls are rolling. We assume that the problem is a plan and that we are looking at the plan where the ball is making contact with the defective race. We completely ignore the other race and assume that the load on the ball does not change.

Finally, we are making the following complementary assumptions:

- The defect is small enough such that there is only one ball in the defect region at the time.
- All the components of the bearing are undeformable.

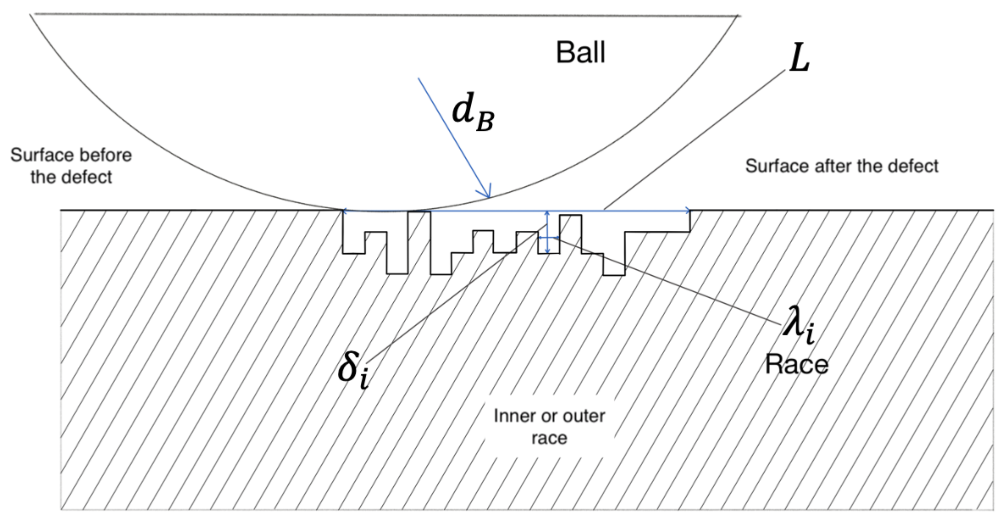


Figure 3: Modelisation of the defect.

A region of known size L contains the defect as presented in Figure 3, in which the ball is ""rolling" from left to right on the defect. This defect region is discretized into N intervals indexed from 0 to $N-1$, the length, and depth of interval i are respectively denoted λ_i and δ_i. The δ_i is positive when the material is missing on the race. We only consider positive depth even if, due to creeping [4](visible on the outer-race fault in Figure2), the race material could be higher than the race.

The process followed by each of the balls as it rolls on the race is as follows:

1. It starts by rolling on a region that is healthy and does not generate any vibration.
2. As soon as it enters the defect region, the ball starts to generate vibration every time it enters contact with a different interval. Hence, the simulation will first calculate which interval of the defect can enter the ball's contact; it will depend on its radius and the defect's specified geometry. For example, in Figure 3 some of the intervals cannot enter in contact with the ball as they are too deep. We will assume that the vibration's amplitude is directly related to the absolute value difference in depth between the two intervals on which the ball makes contact; that is, the higher the "step" is between the two intervals, the higher the vibration created will be.
3. Finally, when the ball exits the defect region, there is no vibration generated by the ball.

The different balls are equally spaced by a cage, as shown in Figure 1. Thus, we assume the time interval between each ball to be equal to the ball frequency as calculated in equations 10 and 11 depending on if we model an inner race or an outer race defect.

Table 1 presents the different entities and attributes for the queuing model.

Consumer: Set[n] : Ball	
Represents a ball rolling on the race of the bearing	
Attributes	Description
–	–
Queue: Unary: Waiting to go through defect	
Queue fo the balls that are waiting before going through the defect region	
Attributes	Description
REW	Number of rolling elements that still needs to go through the defect
Resource: Unary: Defect	
Represents the defect region where the balls are rolling	
Attributes	Description
DefectFree	Boolean: true if the next ball can enter the defect region

Table 1: Entities for the conceptual model.

Table **??** presents the constant and parameters of the queuing model. It should be noted that the parameters of the model all have the value *User set* as we are building a simulation library so the user can actively choose these values.

Parameters		
Name	Role	Value
t_f	Left boundary of the observable time	User set
Ω	Revolution per minute of the saft	User set
L	Length of the defect region	User set
N	Number of intervals representing the defect	User set
λ_i	List of width of the N intervals	User set
δ_i	List of depth of the N intervals	User set
Constants		
Name	Role	Value
t_0	Right boundary of the observable time	0
n	Number of rolling elments	16
d_B	Diameter of the rolling elements	8.4074mm
d_P	Diameter of circle described by the rolling elements	71.501mm
θ	Contact angle	15.17°

Table 2: Parameters and Constants for the queuing model

Table 3 present the activities for the queuing model.

Activity: OnDefect	
Models a ball rolling in through the defect region	
Precondition	defectFree is True and there is still balls in the queue
Initiating Events	defectFree=False
Duration	$\frac{2L}{\pi d_P \pm \cos\theta} \cdot \frac{n}{\text{BPFO/I}}$: the time spent by a ball in the defect zone
Terminating Event	REW-1; defectFree=True

Table 3: Activities for the queuing model.

2.4 Development platform

As explained in the proposal the main goal of the project is to create a simulation tool for bearing defect. Thus, I will not use an interactive development platform but create a Python library that can be easily be used to generate bearing vibration signal. I was initially planning to use C++ but decided to change as the library is more likely to be use if written in a higher-level language.

3 Verification and Validations plans

3.1 Verification of the conceptual models

As introduced in the proposal, the first model using calculating any rolling element's theoretical passage frequency on a given point of a race is a classical and straightforward model for simple bearing defect frequency. Thus, we will use the fact that this model is used in the literature as a verification. For example, we used the configuration of the NASA dataset to calculate the defect frequencies with the model and obtained the same as in [6].

3.2 Verification of the conceptual queuing model

As stated in the proposal, the defect generally occurs on the bearing's races, and we want to simulate complex bearing defect shapes. The conceptual model reduces the problem to multiple balls rolling on a track. The defect shape has been discretized and can represent a complex shape by setting different N intervals' depth and width representing the defect. Thus, the Conceptual Model for the queuing model correctly captures the various aspects of a bearing defect located on the inner or outer race. As explained in Lecture 16a slide 5, we have verified the conceptual model.

Note: The Conceptual Model does not let the user specify multiple defects. Thus, the model can simulate only one fault at a time.

3.3 Simulation model and simulation program

3.3.1 World view for the simulation model and program

The conceptual model will be translated into a Process-oriented model. I will consider the n different balls of the bearing as a different process using multi-threading. Considering that the number of balls in a bearing is limited and rarely more than 100, there should not be any issues with the number of threads.

The ball will be wrapped inside a bearing class presented in 4.

Class: Bearing		
Represent a rolling element bearing		
Attributes	Type	Description
m_n	int	Number of rolling elements of the bearing
m_dP	float	Pitch Diameter of the bearing (mm)
m_innerRace	bollean	True if we are studying the inner race
m_outerRace	bollean	True if we are studying the outer race
m_rpm	int	Rotational speed outer race vs inner race (rev/min)
m_ballList	array[n]	Array containing the balls objects
m_defect	defect	defect object
m_theta	float	contact angle (rad)

Table 4: Attributes of the class bearing

The ball object will be wrapped in a class Rolling Element as presented in the Table 5

Class : RollingElement		
Attributes	Type	Description
m_dB	float	Diameter of the ball (mm)
m_duration	float	Duration spent by the ball in the defect

Table 5: Attributes of the class Rolling Element

Finally, the defect will be described with a Defect class presented in Table 6.

3.3.2 Verification of the simulation program

As suggested by Professor Vuduk during the lectures, the code will be commented to show what aspect of the model is implemented in each function.

Class : Defect		
Attributes	Type	Description
m_L	float	Length of the defect (mm)
N	int	Number of intervals in the defect
m_lambda	array[N]	Array of N float representing the width of the intervals in mm
m_delta	array[N]	Array of N float representing the depth of the intervals in mm

Table 6: Attributes of the class defect

3.4 Plan for validation of the model

It is proposed to use face, behavior, and replicative validation on the waveform and the spectrum of both models' vibration signals to validate the model.

Face validation: Subject Matter Experts (SME)will be asked to check the signal to tell if it "looks right!"

Behavior validation: We will increase the bearing's rotational speed to validate that an increase of rotational speed leads to a rise in both models' defect frequencies.

Replicative validation: The NASA bearing defect dataset will be used for replicative validation. The test parameters will be fed into the simulation and the obtained spectrum will be compared to the spectrum of the NASA signals.

Comparison of the two models: By taking the case where the defect simulated by the queuing model is composed of only one interval of infinitely small width, we will compare the signals generated by the two simulations. Indeed, a single infinitely small interval should give the same results as the simple localized fault model.

4 Validation of the model

This part presents the different steps of validation and also explains the results obtains by the modelization.

4.1 Face validation

Luckily, I happen to know a subject matter expert Eymard Prevost, a data scientist at Georgia Pacific LLC who works daily to create the bearing defect detection model. His work also includes validating that his models classify the waveform as representing a defect or not. I showed him the simulation result presented in Figure **??** (without the title), asking him what his feeling was about the presented spectrum. I did not mention that it was a simulation and did not mention either that I was trying to represent a defect. However, I directly identified the result as representing a BPFO defect which was the correct defect to identify.

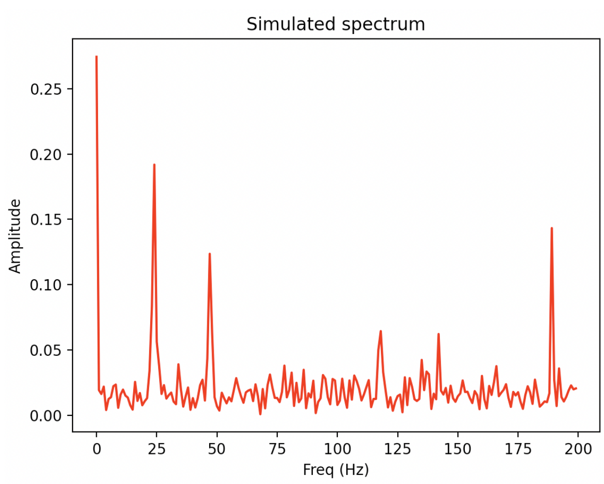
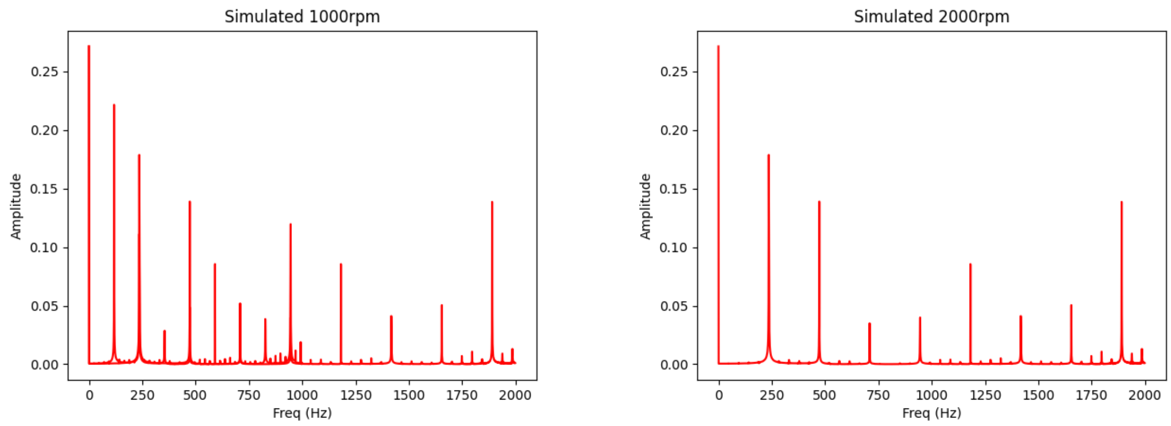


Figure 4: Simulation presented to the SME.

4.2 Behavior validation

As discussed above, we can check different rotational speeds and see the influence on the simulated bearing defect frequencies. Figure 5 presents the bearing of the NASA data with two different rotational speeds of 2000rpm and 1000rpm. It can be seen that on the 1000rpm case, the bearing defect frequencies are divided by 2. The other parameters of the model have a very complex influence on the behavior. It is pretty challenging to analyze the influence of each of them over the general behavior of the model.



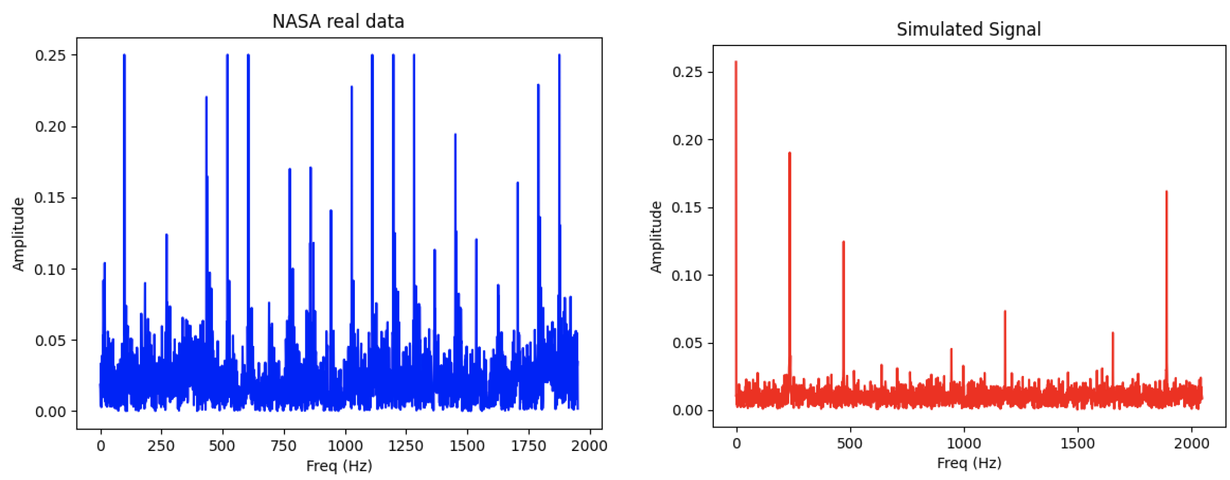
(a) Rotational speed of 1000rpm

(b) Rotational speed of 2000rpm

Figure 5: Influence of the bearing rotational speed over the defect frequencies.

4.3 Replicative validation

In order to go even further in the validation of the model, I compared the simulation results with an actual BPFO defect presented in the NASA dataset. The results are presented in Figure 6. It can be seen that the simulated signal is quite different from the actual signal. We can find different cause for the difference in the



(a) Real spectrum of the NASA dataset.

(b) Simulated spectrum

Figure 6: Real and simulated spectrum for the same bearing configuration.

two signal:

- The current model requires to provide an accurate shape of the defect, which is unknown in the case of the NASA dataset. Thus, a simple defect shape was used for this validation but there is no way to know of close this simulated defect is to the real one.
- The real signal is actually composed by multiple vibration signal from the 4 bearing on the shaft because of the experimental setup of the NASA dataset.
- Even if the model gives a great flexibility in term of defect shape description, it is nonetheless a simple model. We have made strong assumption when developing it, here we see its limitations.

4.4 comparison of two models

We have implemented both a simple frequency calculation model and a more complex one. Setting the defect in the queueing model to one minimal interval makes this model very close to the simple frequency calculation model. Figure 7 presents where the calculated frequencies lines-up of the queuing model. We represent the first three harmonics of the BPFO frequencies. It can be seen that the frequency aligns perfectly, which is an excellent sign that both the model we have developed is valid but also that we have implemented it correctly and that no programming mistake was made.

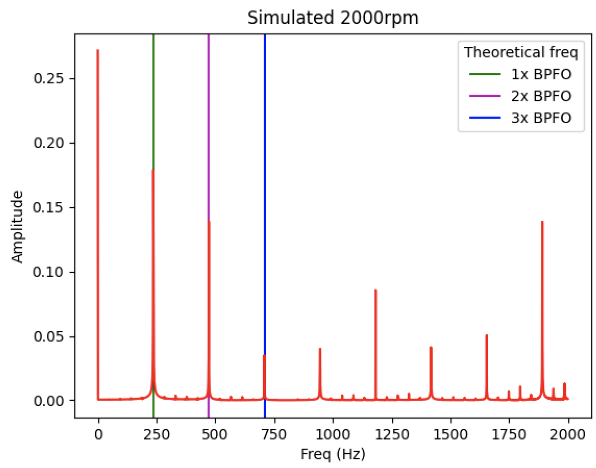


Figure 7: Comparison of the simulated spectrum and the calculated defect frequencies.

5 Tutorial

This section presents the tutorial of the project. It is was tested with Python 3.9.1

5.1 Structure of the repository

The repository contains the following folders:

Bearing_defect_simulation: It contains 2 folders:

- **Bearing** This folder contains the classes:
  - **Bearing** implementation of the bearing object as in 4
  - **Defect** implementation of the 6
  - **RollingElement** implementation of the 5
- **DES** The folder where the simulation engine lives with in the class Simulation, 2 other classes were also added:
  - **Acquisition** which manages the time function and time interval.
  - **Signal** The Signal class is where the results of the simulation are stored.

test contains the test for validation of the project. When run, each code will to recreatese one of the Figures 5,6 or 7. It also contains three .csv files that contain the data for 2 BPFO defects and one healthy signal from the NASA dataset.

docs contains the different reports of the project

requirements.txt: the requirement to install

simulation.py the main code to run with command line argument if you want to test the project.

README.MD, LICENSE and .gitignore I will not offend you by detailing the purpose of these files.

5.2 How to run the project?

Once you have cloned the project, you should install the requirements of the project:

```
python3 -m pip install -r requirements.txt
```

Then you can run the simulation by doing:

```
python3 simulation.py
```

This will run the default settings for the simulation, the simulation.py code also come with and command line argument parser with the following option:

Command line argument			
Argument	Type	Fonction	Default
–n	int	Number of rolling elements	16
–dP	float	Pitch diameter of the bearing (mm)	71.501
–race	str	Race affected by the defect either ”inner” or ”outer”	”outer”
–rpm	int	Rotational speed of the cage one relative to the other (rpm)	2000
–dB	float	The Rolling Element Diameter (mm)	8.4074
–theta	float	The contact angle of the bearing (deg)	15.17
–L	float	The length of the defect region (mm)	3.8
–N	int	The number of interval of the defect.	5
–a_lambda	list	The different lengths of the intervals representing the defect (mm)	[0.7,0.7,0.8,0.8,0.8]
–a_delta	list	The different depth of the intervals representing the defect (mm)	[0.5,0,0.5,0,0.7]
–duration	float	The duration of the simulation (s)	1
–frequency	float	The time resolution of the simulation (Hz)	20000
–noise	float	The ration of noise to add in the simulation [0.0,0.9]	0.1

Note1: The *duration* command-line argument does not represent the program's runtime but the duration for which the ball roll in the bearing.

Note2: When playing with the different command-line arguments, one should be very careful that what the program is asked for actually makes sense. This program is intended to simulate actual situations, and the validity of the argument does not check. For example, the program will try to run (and will undoubtedly crash) if one asks for a defect length longer than the circumference of the race or a negative number of rolling elements, but it does not make any sense in real life. The time resolution and duration of acquisition should be chosen carefully to satisfy the Nyquist–Shannon sampling theorem as the simulation engine behaves like if it is sampling the analog vibration generated by the ball rolling on the races.

6 Division of the work and Conclusion

6.1 Division of the work

This is a team of one. I am doing everything.

6.2 Github Repository of the project

The project repository can be found at this address:
`https://github.gatech.edu/prauby3/Bearing_defect_simulation`.
The repository is private, and I have added all the Teaching assistants as collaborators to it.
Alternatively, the project can also be found at: `https://github.com/PierrickRauby/Bearing_defect_simulation`.

6.3 Limitate and conclusion

This project was an excellent opportunity to apply the simulation concept introduced in the class to a subject that I work on every day: rolling-element bearing defect. My Ph.D. research is focused on identifying such defects, and the tool that I have implemented during the class will come in very handy to check my algorithm. Of course, some strong assumptions were made, and the simulator comes with some limitations. As explained earlier, a global idea of the defect geometry is required to obtain meaningful results. In addition, the value of the amplitudes generated by the simulator does not mean anything physically as they only satisfy a proportionality relationship one to the other. Finally, I have implemented the FFT using the NumPy FFT, but this comes with significant limitations as my results suffer from some windowing issues. I will fix that in the coming weeks with a more proper FFT.

References

[1] D. Stronbergsson et al. Multi-body simulation and validations from rolling-element bearings. *Journal of Engineering Tribology*, 09 2020.

[2] G. Inglebert et al. Theorie du contact de hertz, contacts ponctuel ou lineiques. *Mecanique, frotement et lubrification*, 09 2011.

[3] Ghalamchi et al. Modeling and dynamic analysis of spherical roller bearing with localized defects: Analytical formulation to calculate defect depth and stiffness. *Shock and Vibration*, 04 2016.

[4] S. Alhasia et al. A study of the dynamics of the rolling element and its effect on outer race creep. *SAE Technical Paper*, 01 2016.

[5] Guillaume Gautier, Roger Serra, and Jean-Mathieu Mencik. Roller bearing monitoring by new subspace-based damage indicator. *Shock and Vibration*, 2015, 08 2015.

[6] William Gousseau, François Girardin, and Julien Griffaton. Analysis of the rolling element bearing data set of the center for intelligent maintenance systems of the university of cincinnati. *Safran Aircraft Engine*, 10 2016.

[7] Mobius Institute. Mobius institute - rolling element bearings. Mobius institute website.

[8] Yi Wang, Guanghua Xu, Ailing Luo, Lin Liang, and Kuosheng Jiang. An online tacholess order tracking technique based on generalized demodulation for rolling bearing fault detection. *Journal of Sound and Vibration*, 367:233–249, 2016.

[9] Yizhou Yang, Wenguang Yang, and Dongxiang Jiang. Simulation and experimental analysis of rolling element bearing fault in rotor-bearing-casing system. *Engineering Failure Analysis*, 92:205–221, 2018.