

The Dirac delta function $\delta(x)$ satisfies:

1.

$$\delta(x) = 0 \text{ for all } x \neq 0$$

2.

$$\int_{-\infty}^{\infty} \delta(x) dx = 1$$

From these properties, it follows that for all function f ,

$$\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0).$$