

Real Space wavefunctions

$$\psi_1(x, y, z) =$$

$$\sin(1x) + \sin(3x) + \sin(2y) + \sin(1z)$$

$$\psi_2(x, y, z) =$$

$$\sin(2x) + \sin(1y) + \sin(2y)$$

Reciprocal Space wavefunctions

$$\psi_1(G_x, G_y, G_z) =$$

$$\begin{aligned} & \frac{i}{2}\delta(G_x - 1)\delta(G_y)\delta(G_z) - \frac{i}{2}\delta(G_x + 1)\delta(G_y)\delta(G_z) + \frac{i}{2}\delta(G_x - 3)\delta(G_y)\delta(G_z) - \frac{i}{2}\delta(G_x + 3)\delta(G_y)\delta(G_z) \\ & + \frac{i}{2}\delta(G_x)\delta(G_y - 2)\delta(G_z) - \frac{i}{2}\delta(G_x)\delta(G_y + 2)\delta(G_z) + \frac{i}{2}\delta(G_x)\delta(G_y)\delta(G_z - 1) - \frac{i}{2}\delta(G_x)\delta(G_y)\delta(G_z + 1) \end{aligned}$$

$$\psi_2(G_x, G_y, G_z) =$$

$$\begin{aligned} & \frac{i}{2}\delta(G_x - 2)\delta(G_y)\delta(G_z) - \frac{i}{2}\delta(G_x + 2)\delta(G_y)\delta(G_z) + \frac{i}{2}\delta(G_x)\delta(G_y - 1)\delta(G_z) - \frac{i}{2}\delta(G_x)\delta(G_y + 1)\delta(G_z) \\ & + \frac{i}{2}\delta(G_x)\delta(G_y - 2)\delta(G_z) - \frac{i}{2}\delta(G_x)\delta(G_y + 2)\delta(G_z) \end{aligned}$$

Real Space Convolution Functions

$$f_1(x, y, z) =$$

$$2 + \frac{1}{2} \cos(2x) - \cos(4x) - \frac{1}{2} \cos(6x) - \frac{1}{2} \cos(4y) - \frac{1}{2} \cos(2z) + 2 \sin(1x) \sin(2y) + 2 \sin(1x) \sin(1z) \\ + 2 \sin(3x) \sin(2y) + 2 \sin(3x) \sin(1z) + 2 \sin(2y) \sin(1z)$$

$$f_2(x, y, z) =$$

$$\frac{3}{2} - \frac{1}{2} \cos(4x) + \cos(y) - \frac{1}{2} \cos(2y) - \cos(3y) - \frac{1}{2} \cos(4y) + 2 \sin(2x) \sin(y) + 2 \sin(2x) \sin(2y)$$

$$f_3(x, y, z) =$$

$$\frac{1}{2} + \cos(x) - \frac{1}{2} \cos(3x) - \frac{1}{2} \cos(5x) + \frac{1}{2} \cos(y) - \frac{1}{2} \cos(3y) - \frac{1}{2} \cos(4y) + \sin(x) \sin(2y) + \sin(2x) \sin(2y) \\ + \sin(3x) \sin(2y) + \sin(x) \sin(y) + \sin(3x) \sin(y) + \sin(2x) \sin(z) + \sin(y) \sin(z) + \sin(2y) \sin(z)$$

Reciprocal Space Convolution Functions

$$f_1(G_x, G_y, G_z) =$$

$$2\delta(G_x)\delta(G_y)\delta(G_z) - \frac{1}{4}\delta(G_x - 6)\delta(G_y)\delta(G_z) - \frac{1}{2}\delta(G_x - 4)\delta(G_y)\delta(G_z) - \frac{1}{4}\delta(G_x)\delta(G_y - 4)\delta(G_z) + \frac{1}{4}\delta(G_x - 2)\delta(G_y)\delta(G_z) \\ - \frac{1}{2}\delta(G_x - 3)\delta(G_y - 2)\delta(G_z) - \frac{1}{2}\delta(G_x - 1)\delta(G_y - 2)\delta(G_z) + \frac{1}{2}\delta(G_x + 1)\delta(G_y - 2)\delta(G_z) + \frac{1}{4}\delta(G_x + 2)\delta(G_y)\delta(G_z) \\ + \frac{1}{2}\delta(G_x - 3)\delta(G_y + 2)\delta(G_z) + \frac{1}{2}\delta(G_x - 1)\delta(G_y + 2)\delta(G_z) - \frac{1}{2}\delta(G_x + 1)\delta(G_y + 2)\delta(G_z) + \frac{1}{2}\delta(G_x + 3)\delta(G_y - 2)\delta(G_z) \\ - \frac{1}{2}\delta(G_x + 3)\delta(G_y + 2)\delta(G_z) - \frac{1}{2}\delta(G_x + 4)\delta(G_y)\delta(G_z) - \frac{1}{4}\delta(G_x)\delta(G_y + 4)\delta(G_z) - \frac{1}{4}\delta(G_x + 6)\delta(G_y)\delta(G_z) \\ - \frac{1}{4}\delta(G_x)\delta(G_y)\delta(G_z - 2) - \frac{1}{2}\delta(G_x - 3)\delta(G_y)\delta(G_z - 1) - \frac{1}{2}\delta(G_x)\delta(G_y - 2)\delta(G_z - 1) - \frac{1}{2}\delta(G_x - 1)\delta(G_y)\delta(G_z - 1) \\ + \frac{1}{2}\delta(G_x + 1)\delta(G_y)\delta(G_z - 1) + \frac{1}{2}\delta(G_x - 3)\delta(G_y)\delta(G_z + 1) + \frac{1}{2}\delta(G_x)\delta(G_y - 2)\delta(G_z + 1) + \frac{1}{2}\delta(G_x - 1)\delta(G_y)\delta(G_z + 1) \\ - \frac{1}{2}\delta(G_x + 1)\delta(G_y)\delta(G_z + 1) + \frac{1}{2}\delta(G_x)\delta(G_y + 2)\delta(G_z - 1) - \frac{1}{2}\delta(G_x)\delta(G_y + 2)\delta(G_z + 1) - \frac{1}{4}\delta(G_x)\delta(G_y)\delta(G_z + 2) \\ + \frac{1}{2}\delta(G_x + 3)\delta(G_y)\delta(G_z - 1) - \frac{1}{2}\delta(G_x + 3)\delta(G_y)\delta(G_z + 1)$$

$$f_2(G_x, G_y G_z) =$$

$$\begin{aligned} & \frac{3}{2}\delta(G_x)\delta(G_y)\delta(G_z) - \frac{1}{4}\delta(G_x-4)\delta(G_y)\delta(G_z) - \frac{1}{4}\delta(G_x)\delta(G_y-4)\delta(G_z) - \frac{1}{2}\delta(G_x)\delta(G_y-3)\delta(G_z) - \frac{1}{4}\delta(G_x)\delta(G_y-2)\delta(G_z) \\ & - \frac{1}{2}\delta(G_x-2)\delta(G_y-2)\delta(G_z) + \frac{1}{2}\delta(G_x)\delta(G_y-1)\delta(G_z) - \frac{1}{2}\delta(G_x-2)\delta(G_y-1)\delta(G_z) + \frac{1}{2}\delta(G_x)\delta(G_y+1)\delta(G_z) \\ & + \frac{1}{2}\delta(G_x-2)\delta(G_y+1)\delta(G_z) + \frac{1}{2}\delta(G_x+2)\delta(G_y-2)\delta(G_z) + \frac{1}{2}\delta(G_x+2)\delta(G_y-1)\delta(G_z) - \frac{1}{2}\delta(G_x+2)\delta(G_y+1)\delta(G_z) \\ & - \frac{1}{4}\delta(G_x)\delta(G_y+2)\delta(G_z) + \frac{1}{2}\delta(G_x-2)\delta(G_y+2)\delta(G_z) - \frac{1}{2}\delta(G_x+2)\delta(G_y+2)\delta(G_z) - \frac{1}{2}\delta(G_x)\delta(G_y+3)\delta(G_z) \\ & - \frac{1}{4}\delta(G_x+4)\delta(G_y)\delta(G_z) - \frac{1}{4}\delta(G_x)\delta(G_y+4)\delta(G_z) \end{aligned}$$

$$f_3(G_x, G_y G_z) =$$

$$\begin{aligned} & \frac{1}{2}\delta(G_x)\delta(G_y)\delta(G_z) - \frac{1}{4}\delta(G_x-5)\delta(G_y)\delta(G_z) - \frac{1}{4}\delta(G_x)\delta(G_y-4)\delta(G_z) - \frac{1}{4}\delta(G_x-3)\delta(G_y)\delta(G_z) - \frac{1}{4}\delta(G_x)\delta(G_y-3)\delta(G_z) \\ & - \frac{1}{4}\delta(G_x-3)\delta(G_y-2)\delta(G_z) - \frac{1}{4}\delta(G_x-2)\delta(G_y-2)\delta(G_z) + \frac{1}{2}\delta(G_x-1)\delta(G_y)\delta(G_z) - \frac{1}{4}\delta(G_x-1)\delta(G_y-2)\delta(G_z) \\ & + \frac{1}{4}\delta(G_x)\delta(G_y-1)\delta(G_z) - \frac{1}{4}\delta(G_x-3)\delta(G_y-1)\delta(G_z) - \frac{1}{4}\delta(G_x-1)\delta(G_y-1)\delta(G_z) + \frac{1}{2}\delta(G_x+1)\delta(G_y)\delta(G_z) \\ & + \frac{1}{4}\delta(G_x+1)\delta(G_y-2)\delta(G_z) + \frac{1}{4}\delta(G_x+1)\delta(G_y-1)\delta(G_z) + \frac{1}{4}\delta(G_x)\delta(G_y+1)\delta(G_z) + \frac{1}{4}\delta(G_x-3)\delta(G_y+1)\delta(G_z) \\ & + \frac{1}{4}\delta(G_x-1)\delta(G_y+1)\delta(G_z) - \frac{1}{4}\delta(G_x+1)\delta(G_y+1)\delta(G_z) + \frac{1}{4}\delta(G_x+2)\delta(G_y-2)\delta(G_z) + \frac{1}{4}\delta(G_x-3)\delta(G_y+2)\delta(G_z) \\ & + \frac{1}{4}\delta(G_x-2)\delta(G_y+2)\delta(G_z) + \frac{1}{4}\delta(G_x-1)\delta(G_y+2)\delta(G_z) - \frac{1}{4}\delta(G_x+1)\delta(G_y+2)\delta(G_z) - \frac{1}{4}\delta(G_x+2)\delta(G_y+2)\delta(G_z) \\ & - \frac{1}{4}\delta(G_x+3)\delta(G_y)\delta(G_z) + \frac{1}{4}\delta(G_x+3)\delta(G_y-2)\delta(G_z) + \frac{1}{4}\delta(G_x+3)\delta(G_y-1)\delta(G_z) - \frac{1}{4}\delta(G_x+3)\delta(G_y+1)\delta(G_z) \\ & - \frac{1}{4}\delta(G_x+3)\delta(G_y+2)\delta(G_z) - \frac{1}{4}\delta(G_x)\delta(G_y+3)\delta(G_z) - \frac{1}{4}\delta(G_x)\delta(G_y+4)\delta(G_z) - \frac{1}{4}\delta(G_x+5)\delta(G_y)\delta(G_z) \\ & - \frac{1}{4}\delta(G_x-2)\delta(G_y)\delta(G_z-1) - \frac{1}{4}\delta(G_x)\delta(G_y-2)\delta(G_z-1) - \frac{1}{4}\delta(G_x)\delta(G_y-1)\delta(G_z-1) + \frac{1}{4}\delta(G_x)\delta(G_y+1)\delta(G_z-1) \\ & + \frac{1}{4}\delta(G_x-2)\delta(G_y)\delta(G_z+1) + \frac{1}{4}\delta(G_x)\delta(G_y-2)\delta(G_z+1) + \frac{1}{4}\delta(G_x)\delta(G_y-1)\delta(G_z+1) - \frac{1}{4}\delta(G_x)\delta(G_y+1)\delta(G_z+1) \\ & + \frac{1}{4}\delta(G_x+2)\delta(G_y)\delta(G_z-1) - \frac{1}{4}\delta(G_x+2)\delta(G_y)\delta(G_z+1) + \frac{1}{4}\delta(G_x)\delta(G_y+2)\delta(G_z-1) - \frac{1}{4}\delta(G_x)\delta(G_y+2)\delta(G_z+1) \end{aligned}$$