$$f(x) = \sin\frac{x}{2}$$

$$f'(x) = \frac{1}{2}\cos\frac{x}{2}$$

$$f''(x) = -\frac{1}{4}\sin\frac{x}{2}$$

$$f'''(x) = -\frac{1}{8}\cos\frac{x}{2}$$

$$\vdots$$

$$f^{(n)}(x) = \frac{1}{2^n}\cdot\sin\frac{x}{2}\cdot(-1)^{\lfloor\frac{n}{2}\rfloor}$$ DLA n PARZYSTEGO

$$f^{(n)}(x) = \frac{1}{2^n}\cdot\cos\frac{x}{2}\cdot(-1)^{\lfloor\frac{n}{2}\rfloor}$$ DLA n NIEPARZYSTEGO

$$\left|f^{(n+1)}(t)\right| \leq \frac{1}{2^{n+1}}$$

$|P_{n+1}| \leq \frac{1}{2^n}$ DLA WĘZŁÓW CZYBYSZEWA DLA $x \in \langle -1, 1\rangle$

~~DLA~~

$$|(x-x_0)(x-x_1)\ldots(x-x_n)| \leq \frac{1}{2^n} \quad /:2^{n+1}$$

$$\left|\left(\frac{x}{2}-\frac{x_0}{2}\right)\left(\frac{x}{2}-\frac{x_1}{2}\right)\ldots\left(\frac{x}{2}-\frac{x_n}{2}\right)\right| \leq \frac{1}{2^{2n+1}} \qquad \frac{x}{2} \in \left\langle -\frac{1}{2}, \frac{1}{2}\right\rangle$$

$$\left|\left(\frac{x}{2}+\frac{1}{2}-\left(\frac{x_0}{2}+\frac{1}{2}\right)\right)\left(\frac{x}{2}+\frac{1}{2}-\left(\frac{x_1}{2}+\frac{1}{2}\right)\right)\ldots\left(\frac{x}{2}+\frac{1}{2}-\left(\frac{x_n}{2}+\frac{1}{2}\right)\right)\right| \leq \frac{1}{2^{2n+1}} \qquad \frac{x}{2}+\frac{1}{2} \in \langle 0, 1\rangle$$

DLA $0 \leq i \leq n$

$$\frac{x_i}{2}+\frac{1}{2} = \frac{\cos\left(\frac{2k+1}{2n+2}\pi\right)}{2}+\frac{1}{2} = \frac{1}{2}\cos\left(\frac{2k+1}{2n+2}\pi\right)+\frac{1}{2} \Leftarrow$$ WĘZŁY Z ZADANIA

DLA NASZEGO ZADANIA ZACHODZI $|P_{n+1}| \leq \frac{1}{2^{2n+1}}$, CZYLI

$$\max_{x\in\langle 0,1\rangle}|f(x)-L_n(x)| \leq \max_{x\in\langle 0,1\rangle}\left|\frac{f^{(n+1)}(\xi)}{(n+1)!}\cdot P_{n+1}\right| \leq \frac{1}{2^{n+1}(n+1)!}\cdot\frac{1}{2^{2n+1}} = \frac{1}{2^{3n+2}\cdot(n+1)!} \leq 10^{-15}$$

DLA $n=9$