$$\sum_{k=0}^{n+1}\left(\frac{n-k+1}{n+1}a_k+\frac{k}{n+1}a_{k-1}\right)\cdot B_k^{n+1}(t)=\sum_{k=0}^{n+1}\frac{n-k+1}{n+1}a_k\cdot B_k^{n+1}(t)+\sum_{k=0}^{n+1}\frac{k}{n+1}a_{k-1}\cdot B_k^{n+1}(t)=$$

$$=\sum_{k=0}^{n+1}\frac{n-k+1}{n+1}\cdot\binom{n+1}{k}\cdot t^k\cdot(1-t)^{n+1-k}\cdot a_k+\sum_{k=0}^{n+1}\frac{k}{n+1}\cdot\binom{n+1}{k}\cdot t^k\cdot(1-t)^{n+1-k}\cdot a_{k-1}=$$

$$=\sum_{k=0}^{n+1}\frac{n-k+1}{n+1}\cdot\frac{(n+1)!}{(k!)\cdot(n+1-k)!}\cdot t^k\cdot(1-t)^{n+1-k}\cdot a_k+\sum_{k=0}^{n+1}\frac{k}{n+1}\cdot\frac{(n+1)!}{(k!)\cdot(n+1-k)!}\cdot t^k\cdot(1-t)^{n+1-k}\cdot a_{k-1}=$$

$$=\sum_{k=0}^{n+1}\frac{n!}{k!\cdot(n-k)!}\cdot t^k\cdot(1-t)^{n+1-k}\cdot a_k+\sum_{k=0}^{n+1}\frac{n!}{(k-1)!\cdot(n+1-k)!}\cdot t^k\cdot(1-t)^{n+1-k}\cdot a_{k-1}=$$

$$=\sum_{k=0}^{n+1}\binom{n}{k}\cdot t^k\cdot(1-t)^{n-k}\cdot a_k\cdot(1-t)+\sum_{k=0}^{n+1}\binom{n}{k-1}\cdot t^{k-1}\cdot(1-t)^{n+1-k}\cdot a_{k-1}\cdot t=$$

$$=\cancel{\sum_{k=0}^{n+1}}=\sum_{k=0}^{n}\binom{n}{k}\cdot t^k\cdot(1-t)^{n-k}\cdot a_k\cdot(1-t)+\sum_{k=1}^{n+1}\binom{n}{k-1}\cdot t^{k-1}\cdot(1-t)^{n+1-k}\cdot a_{k-1}\cdot t=$$

$$=\sum_{k=0}^{n}B_k^n(t)\cdot a_k\cdot(1-t)+\sum_{k=0}^{n}\binom{n}{k}\cdot t^k\cdot(1-t)^{n-k}\cdot a_k\cdot t=$$

$$=\sum_{k=0}^{n}B_k^n(t)\cdot a_k\cdot(1-t)+\sum_{k=0}^{n}B_k^n(t)\cdot a_k\cdot t=$$

$$=\sum_{k=0}^{n}B_k^n(t)\cdot a_k\cdot(1-t+t)=\sum_{k=0}^{n}B_n^k(t)\cdot a_k \quad\blacksquare$$