$f(x) = \cos\left(\frac{x}{2}\right)$

$f'(x) = -\sin\left(\frac{x}{2}\right) \cdot \frac{1}{2}$

$f''(x) = -\cos\left(\frac{x}{2}\right) \cdot \frac{1}{4}$

$\vdots$

$f^{(n)}(x) = \frac{1}{2^n} \cdot (-1)^{\left\lceil \frac{n}{2} \right\rceil} \cdot \sin\left(\frac{x}{2}\right)$ dla $n \,\%\, 2 == 1$

$f^{(n)}(x) = \frac{1}{2^n} \cdot (-1)^{\left\lceil \frac{n}{2} \right\rceil} \cdot \cos\left(\frac{x}{2}\right)$ dla $n \,\%\, 2 == 0$

$$\max_{x \in [-1,1]} |f(x) - L_n(x)| = \max_{x \in [-1,1]} \left| \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot P_n(x) \right| \leq \frac{\frac{1}{2^{n+1}} \cdot 1 \cdot 1}{(n+1)!} \cdot \frac{1}{2^n}$$

$$= \frac{\frac{1}{2^{2n+2}}}{(n+1)!} = \frac{1}{2^{2n+2}(n+1)!} \leq \frac{1}{10^3}$$

dla $n = 7$

0 → +
1 −
2 −
3 +
4 +
−
−
+
+

4
−

1 −
2 +
3 −
3 4 +

2

3 ? F
4 ? F
6 ? F
7 ? T