

a) BAZA:  $v_0 = 1, v_1 = x, v_2 = x^2$

$$u_0 = 1$$

$$u_1 = x - \frac{\langle v_1, v_0 \rangle}{\langle v_0, v_0 \rangle} \cdot v_0 = x - \frac{\langle x, 1 \rangle}{\langle 1, 1 \rangle} \cdot 1 = x - \frac{-2 \cdot 1 + 0 \cdot 1 + 1 \cdot 2}{1+1+1+1+1} = x$$

$$u_2 = x^2 - \frac{\langle v_2, v_0 \rangle}{\langle v_0, v_0 \rangle} \cdot v_0 - \frac{\langle v_2, v_1 \rangle}{\langle v_1, v_1 \rangle} \cdot v_1 = x^2 - \frac{\langle x^2, 1 \rangle}{\langle 1, 1 \rangle} \cdot 1 - \frac{\langle x^2, x \rangle}{\langle x, x \rangle} \cdot x =$$

$$= x^2 - \frac{4+1+0+1+4}{5} - \frac{4 \cdot (-2) + 1 \cdot 0 + 1 \cdot 8}{4+1+0+1+4} \cdot x = x^2 - 2 - \frac{4}{5}x$$

b)  $w_2^*(x) = \sum_{k=0}^2 a_k \cdot p_k(x)$

$$a_k = \frac{\langle w_1, p_k \rangle}{\langle p_k, p_k \rangle}$$

$$a_0 = \frac{4 \cdot 1 + 1 + 1 + 1 + 4}{1+1+1+1+1} = \frac{11}{5}$$

$$a_1 = \frac{4 \cdot (-2) + 1 + 1 + 8}{10} = 0$$

~~$$a_2 = \frac{\langle w_1, p_2 \rangle}{\langle p_2, p_2 \rangle} = \frac{4 \cdot (2) + (-1) + (-2) + (-1) + 4 \cdot 2}{4+1+4+1+4} = \frac{8-4+8}{14} = \frac{12}{14} = \frac{6}{7}$$~~

$$a_2 = \frac{\langle w_1, p_2 \rangle}{\langle p_2, p_2 \rangle} = \frac{4 \cdot (2) + (-1) + (-2) + (-1) + 4 \cdot 2}{4+1+4+1+4} = \frac{8-4+8}{14} = \frac{12}{14} = \frac{6}{7}$$

$$w_2^*(x) = \frac{11}{5} \cdot 1 + 0 \cdot x + \frac{6}{7} \cdot (x^2 - 2) = \frac{6}{7}x^2 - \frac{12}{7} + \frac{11}{5} = \frac{6}{7}x^2 + \frac{14}{35}$$