Monte Carlo Simulations of Elastic Proton-Proton Scattering

Lorenzo Cazon Boado[a,*] Marios A. Kagarlis[b]

[a]*Departamento de Fisica de Particulas, Universidade de Santiago de Compostela, 15706 Santiago de Compostela, Spain*

[b]*Gesellschaft für Schwerionenforschung (GSI), 64220 Darmstadt, Germany*

Abstract

Differential cross sections of elastic proton-proton scattering in the center of mass are fitted with a global function $F(\theta_{cm}, m_{pp})$ of the scattering angle and the total center-of-mass energy up to 4.2 GeV/c^2. A simple parametrization is obtained, suitable for fast sampling in MC simulations.

I. Motivation

Elastic proton-proton (pp) scattering cross sections have been systematically measured over many years, making this a reaction suitable for studies of detector properties. The HADES di-electron spectrometer at GSI is currently in the commissioning phase, and its constituent detectors will be calibrated via elastic pp scattering experiments [1]. Monte Carlo (MC) simulations of this process are therefore of interest in preparation for these experiments, requiring empirical parametrizations of angular-distribution spectra over the range of accessible proton energies.

Phase-shift analyses of pp elastic scattering cross sections are available (e.g. Refs. [2,3]), yielding elaborate parametrizations that are generally in good agreement with the data. Nonetheless, the complexity of such analyses makes them cumbersome to use for fast MC simulations. Within this context, a few-parameter global function $F(\theta_{cm}, m_{pp})$ of the center-of-mass (cm) scattering angle θ_{cm} and the total cm energy $m_{pp} = \sqrt{s}$ is most convenient [4].

II. Fitting Procedure

In lieu of experimental spectra cm angular distributions from the program SAID were obtained, implementing a comprehensive phase-shift analysis that encompasses the world elastic pp-scattering data [3]. A mesh of 60×24 differential cross sections (mb/sr) was used, covering 60 scattering angles between 0-180 deg in 3-deg steps, and 24 total cm energies between 1.9-4.2 GeV/c^2 in 100 MeV increments. The fitting procedure to be described was facilitated via macros for ROOT [5], an analysis package incorporating the fitting algorithms of MINUIT [6]. No uncertainties other than those resulting from the fits are considered. The invariant-mass range extends to the limit of validity for the SAID parametrization (~GeV/c^2), set by the availability of experimental data. This range well covers proton energies expected to be available in future HADES experiments at GSI.

II.a Fitting the angular distribution spectra

A global function of the scattering angle was constructed, involving the least number of free coefficients that yielded a satisfactory fit for each of the angular-distribution spectra of fixed invariant mass. The SAID angular distributions are symmetric about 90 deg in the center of mass frame as the elastic data, but show two artificial sharp peaks located around θ_{cm} =90±86.5 deg that become less prominent with increasing invariant mass, as a relatively flat valley in-between acquires more features. The peaks are due to excluding the Coulomb potential at forward (backward) angles, in order to avoid diverging cross sections. This is of no consequence for HADES simulations since very forward angles are inaccessible. The SAID spectra are in excellent agreement with the data for finite angles up to about 3 GeV incident proton laboratory kinetic energies.

The two peaks are fitted with the symmetric Gaussian-like function

$$g_p = \alpha_0 \left[e^{-\left(\frac{90-\theta_{cm}-\alpha_1}{\alpha_2}\right)^2} + e^{-\left(\frac{\theta_{cm}-90-\alpha_1}{\alpha_2}\right)^2} \right] \qquad (1)$$

requiring three independent coefficients, α_{0-2}.

The intermediate region was fitted with a sixth-order even polynomial whose rank was optimized to minimize the reduced χ^2 function:

$$g_v(\theta_{cm}) = \alpha_3 + \alpha_4(\theta_{cm} - 90)^2 + \alpha_5(\theta_{cm} - 90)^4 + \alpha_6(\theta_{cm} - 90)^6 \qquad (2)$$

To prevent Eq. (2) from interfering with the fitting of the peaks by Eq. (1), an "envelope" function was devised as a weight for the former:

$$g_w = e^{-\left(\frac{\theta_{cm}-90}{78}\right)^{20}} \qquad (3)$$

This is effectively a step function, equal to unity over the ``flat'' region of the spectrum, and decaying rapidly near the onset of the peaks. The cutoff parameter of 78 deg, once determined by trial-and-error, was fixed.

Last, adding the quadratic function

$$g_q(\theta_{cm}) = \alpha_7(\theta_{cm} - 90)^2 \qquad (4)$$

was found to improve the fitting by smoothing out the transition between the peaks and the intermediate region.

The full function used for fitting the SAID cm angular-distribution spectra (Fig. 1) is:

$$f(\theta_{cm}) = g_p(\theta_{cm}) + g_w(\theta_{cm}) \times g_v(\theta_{cm}) + g_q(\theta_{cm}) \qquad (5)$$

II.b Fitting the coefficients

The coefficients obtained from fitting the individual angular-distribution spectra with the function (5) were subsequently fitted as functions of m_{pp}, in order to arrive at a global parametrization $F(\theta_{cm}, m_{pp})$. The two peaks rise far more steeply for the lowest few total cm energies as compared to the rest, necessitating fitting independently in two regions of m_{pp}, namely [1.9,2.1] and (2.1,4.2] GeV/c^2.

All the coefficients were fitted as polynomials f_{ni}

$$f_{n_i}(m_{pp}) = c_0 + c_1 m_{pp} + \ldots + c_{n_i} m_{pp}^{n_i} \qquad (6)$$

with rank n_i varying for each coefficient $\alpha_{i=0-7}$, with the exception of α_0 in the lower-mass region for which

$$f_0'(m_{pp}) = e^{c_0 + c_1 m_{pp}} + c_2 + c_3 m_{pp} \qquad (7)$$

was used instead. In the lower-mass region $\alpha_{0,3}$ were fitted up to 2.3 and 2.2 GeV/c^2 respectively, to supply enough data points for the free parameters, although above 2.1 GeV/c^2 the large-mass parametrization is preferred (Fig. 2).

The full parametrization $F(\theta_{cm}, m_{pp})$ (Table 1), obtained by substituting the invariant-mass dependent coefficients $\alpha_{0-7}(m_{pp})$ into Eq. (5), is in good agreement with the SAID spectra (dotted curves in Fig. 1).

Noting that $F(\theta_{cm}, m_{pp})$ is an ansatz for $d\sigma/d\Omega(\theta_{cm}, m_{pp})$, the total cross section

$$\sigma(m_{pp}) = 2\pi \int_0^\pi d\theta_{cm} \sin\theta_{cm} F(\theta_{cm}, m_{pp}) \qquad (8)$$

is also parametrized (Table 1), above 2.1 GeV/c^2 by fitting with $f_{\sigma>} = f_5$ Eq. (6), and below, with

$$f_{\sigma_<}(m_{pp}) = e^{c_0 + c_1 m_{pp}} + c_2 \ln(m_{pp}) + \frac{c_3}{m_{pp}} \qquad (9)$$

III. Monte Carlo Simulation

In simulating N_{evt} elastic pp scattering events the total cm energy $m_{pp} = \sqrt{s}$ is fixed by the beam and target kinematics, the scattering angle θ_{cm} is sampled from $F(\theta_{cm}, m_{pp})$ (Table 1), and the final spectrum is normalized by $\sigma(\sqrt{s})/N_{evt}$ (Eq. (8)) to yield differential cross sections (mb). This algorithm has been coded into the MC package Pluto^{++} [4] , which uses the Rejection Method (RM) for random sampling (see e.g. [7]). The latter establishes a criterion for sampling θ_{cm}, with the aid of two random numbers from a flat distribution, and a test function g(θ_{cm}) greater than $F(\theta_{cm}, m_{pp})$ and with an indefinite integral that is invertible in θ_{cm}.

The efficiency of the RM effectively reflects in the ratio of the area under the distribution over that of the test function. In the limit of complete overlap the minimum of two flat random number calls suffices. Most commonly a straight line over the distribution function is used as the test function, a valid but generally inefficient choice. To improve the efficiency, a test function comprising of four line segments, defined by five points (Table 2), is employed (Fig. 3). A typical MC simulation of elastic pp scattering with the parametrization presented here and Pluto^{++} is shown in Fig. 4.

IV. Summary

A convenient parametrization $F(\theta_{cm}, m_{pp})$ of elastic pp scattering differential cross sections has been obtained, in terms of the cm scattering angle and the total cm energy, which is valid over the entire range of available data for incident proton momenta up to ~7.2 GeV/c. This provides an alternative to the complex phase-shift analyses, suitable for use with fast MC simulation codes. The SAID spectra are for most angles reproduced to within 10-20%, comparable to experimental uncertainties, with larger uncertainties near minima. Simulations of elastic pp scattering are of interest in conjunction with spectrometer studies and detector calibration, due to the well-known systematics of this process.

V. References

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VI. Tables

Table 1: The parametrization $F(\theta_{cm}, m_{pp})$ tabulated below yields cm differential cross sections (mb/sr) of elastic proton-

proton scattering in terms of the cm scattering angle θ_{cm} (deg) and the total cm energy m_{pp} (GeV/c^2) up to 4.2 GeV/c^2. The coefficients $\alpha_{0\text{-}7}$ of Eq. (5) are fitted by the $n+1$ and 4-parameter functions $f_n(m_{pp})$ and $f'_0(m_{pp})$, Eqs. (6,7) respectively. The total cross section $\sigma(m_{pp})$ (mb) is used for the absolute normalization of the spectra, and it is fitted with $f_n(m_{pp})$ for n=5, and $f_{\sigma<}$ of Eq. (9), for the two invariant-mass regions.

Region I. 1.9 < m_{pp} < 2.1 GeV/c^2 (28 parameters)

$\alpha(m_{pp})$	f_n	c_0	c_1	c_2	c_3
α_0	f'_0	90.16161	-42.54657	250.6459	-82.64069
α_1	f_1	81.46621	2.442514		
α_2	f_1	-1.613531	1.424601		
α_3	f_3	222.9465	-125.6755	-23.42175	15.85506
$\alpha_4 \times 10^2$	f_2	1.675475	-1.718991	0.4285591	
$\alpha_5 \times 10^4$	f_2	-2.464428	2.413957	-0.5908636	
$\alpha_6 \times 10^8$	f_2	3.266005	-3.199807	0.7833496	
$\alpha_7 \times 10$	f_2	3.624606	-3.551761	0.8713306	
$\sigma \times 10^{-1}$	$f_{\sigma<}$	8.62782	-4.02031	-1584.07	2923.57

Region II. 2.1 < m_{pp} < 4.2 GeV/c^2 (55 parameters)

$\alpha(m_{pp})$	f_n	c_0	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
α_0	f_2	208.6132	-85.75014	9.606524							
$\alpha_1 \times 10^2$	f_3	9014.328	-249.0616	26.35260	3.374141						
$\alpha_2 \times 10$	f_1	1.403383	4.544666								
$\alpha_3 \times 10^2$	f_9	816000.1	-1924544.	1962241.	-1126850.	397754.5	-88173.33	11948.51	-896.5965	27.03873	.1756284
$\alpha_4 \times 10^5$	f_9	-535576.2	1179326.	-1056764.	478358.4	-100020.3	-1995.474	5989.377	-1358.871	135.3221	-5.257673
$\alpha_5 \times 10^7$	f_7	6069.216	-15344.59	15584.70	-8343.774	2567.415	-458.0696	44.27438	-1.805952		
$\alpha_6 \times 10^{11}$	f_5	4072.224	-5742.969	3032.050	-734.3411	78.63649	-2.664602				
$\alpha_7 \times 10^4$	f_5	470.0798	-1108.074	924.0620	-362.0080	70.10400	-5.363294				
$\sigma \times 10^{-2}$	f_5	-264.737	402.894	-146.488	-12.4763	14.8093	-1.90930				

Table 2: The five reference points $\theta_{cm}, d\sigma/d\Omega(\theta_{cm}, m_{pp})$ below define the test function $g(\theta_{cm})$ of Fig. (3). The prefactor in the second column is to be multiplied by $F(\theta_{cm}, m_{pp})$ (Table 1) evaluated at the angle of the first column, with $m_{pp} = \sqrt{s}$ fixed by the beam and target kinematics.

θ_{cm} (deg)	$\times F(\theta_{cm}, m_{pp})$ (mb/sr)
0.0	1.1
3.7	1.1
10.5	1.2
36.0	1.3
90.0	1.4

VII. Figures

Fig 1: Angular distributions of proton-proton scattering in the center of mass are shown for six total cm energies, in intervals of 400 MeV/c^2. The SAID [3] differential cross sections are plotted as open circles (green), while the fits to $f(\theta_{cm})$ of Eq. (5), and the spectra reproduced from the parametrization $F(\theta_{cm}, m_{pp})$ of Table 1 are shown as solid (blue) and dotted (red) curves respectively.

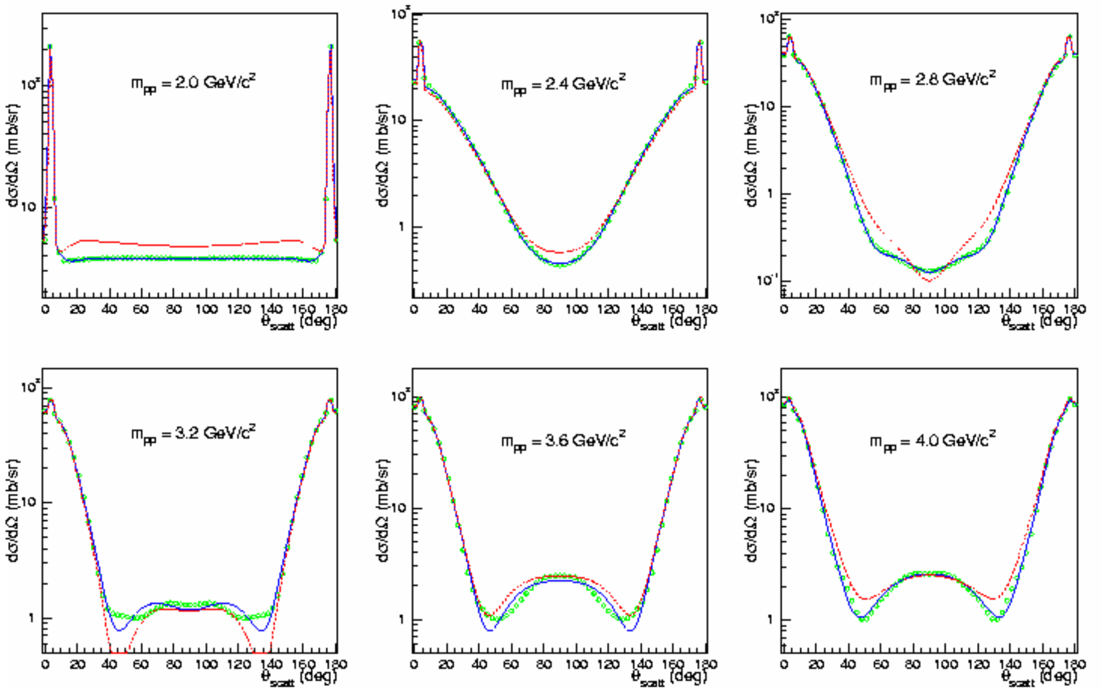


Fig 2: The coefficients $\alpha_{0\text{-}7}$ are derived from fitting the differential cross section spectra to Eq. (5) (open blue circles). The (red) curves result from fitting these coefficients to the invariant-mass dependent functions of Table 1. The index of each coefficient, as well as the number of required parameters for the two itotal cm-energy regions, are indicated in each panel. The units are consistent with differential cross sections in mb/sr, for angles in deg and $m_{pp} = \sqrt{s}$ in GeV/c^2.

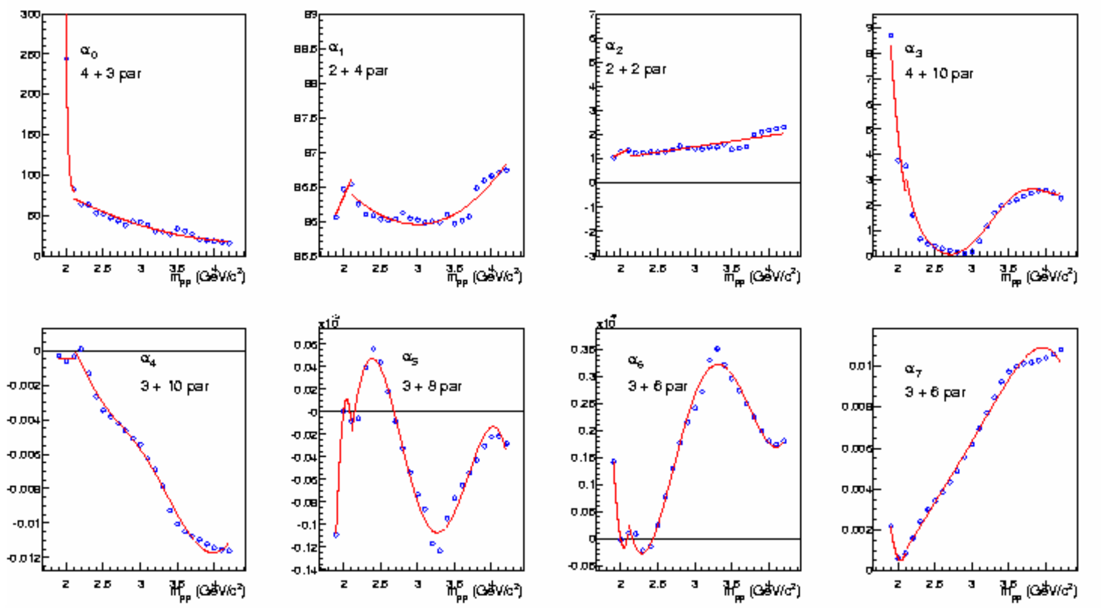


Fig 3: The distribution function $F(\theta_{cm}, m_{pp})$, with m_{pp}=2.994 GeV/c^2 corresponding to beam protons of T_{lab}=2.9 GeV, is shown (dotted red line) with the test function g(θ_{cm}) (solid green curve) defined by the five points of Table 2 as discussed in the text. Due to the symmetry about θ_{cm} =90 deg only half of the spectrum is shown. Although the sampling efficiency depends on θ_{cm} and m_{pp}, the ratio of the areas under the two curves is indicative of the efficiency averaged over angles, for the case in hand 67.9%. In practive, this means that 67.9% of the time the first attempt at sampling is successful.

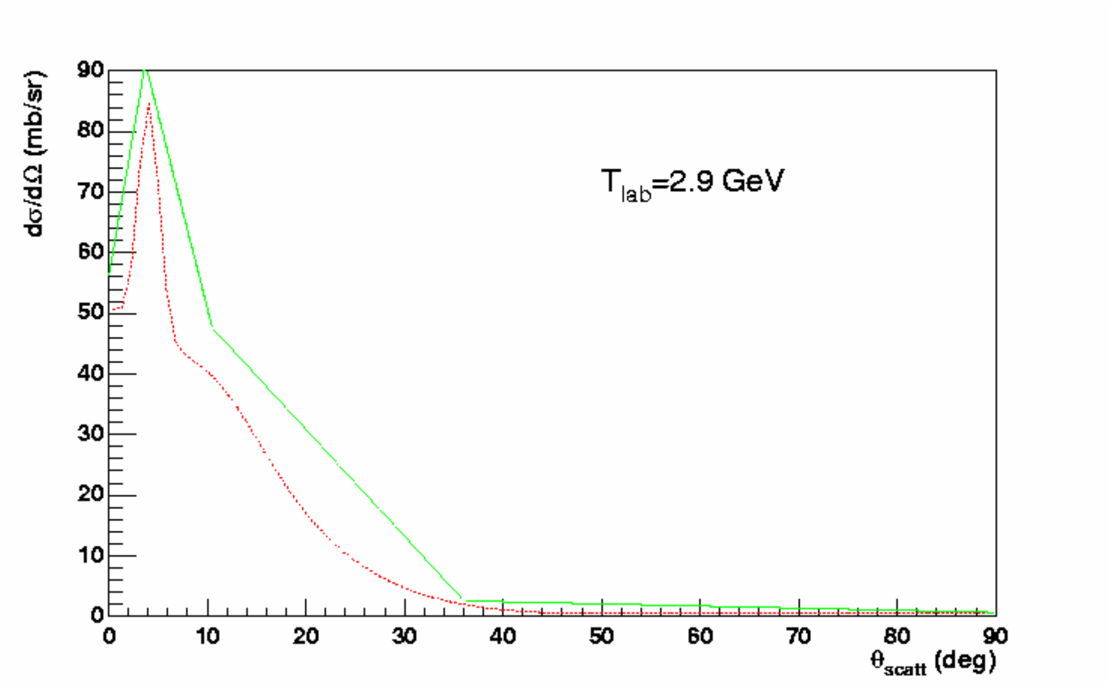
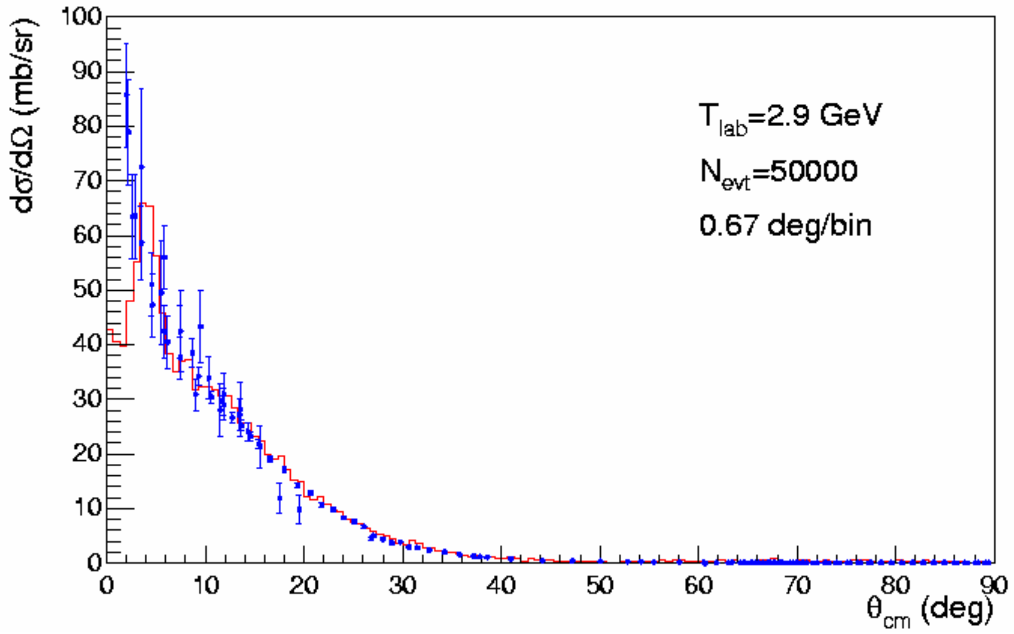


Fig 4: A MC simulation of N_{evt}=50k elastic pp scattering events is shown (red histogram), for m_{pp}=2.994 GeV/c^2 corresponding to incident protons of T_{lab}=2.9 GeV, with the parametrization of Table 1 implemented in the code Pluto^{++} [4]. The calculation shown requires 11.2 CPU seconds on a 200 MHz Linux PC. The plotted data are a compilation from Refs. [8-12] for T_{lab}=2.8 - 3.0 GeV. The binning is chosen for optimum matching in the display of the simulation and the data, with the number of bins (135) roughly equal to that of the available data points (145). The simulation is scaled by the factor $\sigma\left(\sqrt{s}\right)/N_{evt}$ (see Eqs. (8-9) and the related discussion).


$d\sigma/d\Omega$ (mb/sr)
T_{lab}=2.9 GeV
N_{evt}=50000
0.67 deg/bin
θ_{cm} (deg)
0
10
20
30
40
50
60
70
80
90
100