Chair for Algebraic and Logical
Foundations of Computer Science
TU Dresden

Summer Term 2022

Advanced Logics – Assignment 12

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Due: Thursday, 14. July 2022, 9:20

The stars indicate the difficulty of an exercise (no star - easiest, one star - harder, two stars - hardest).

Exercise 12.1 [FOL-sentences for star-free languages]

Let $\Sigma = \{a, b\}$. Provide an FOL-sentence ϕ over $Voc_{\Sigma,\leqslant}$ that defines the language over $\Sigma = \{a, b\}$ given by the following star-free expression:

$$\overline{b\overline{\emptyset}a\overline{\emptyset}}$$

Exercise 12.2 [From FOL to star-free and regular expressions]

Let $\Sigma = \{a, b\}$. Provide star-free and regular expressions for the star-free languages defined by the following FOL-sentences ϕ_i over $Voc_{\Sigma,\leqslant}$:

$$\phi_1 = \forall x \exists y.(\, P_a(x) \wedge P_b(y)\,)$$

$$\phi_2 = \forall x.(\, P_a(x) \rightarrow \exists y \exists z.(\, E(y,x) \wedge E(z,y) \wedge P_b(z)\,)$$

Exercise 12.3 [Star-freedom of a regular language?]

Is the regular language given by the regular expression

$$(\,a\,b\,)^*$$

star-free? Justify your answer.

Exercise 12.4 [Regular language associated with an MSO-formula]

Provide a characterization of $\mathfrak{L}(\phi)$ for the MSO-formula over $Voc_{\Sigma,\text{graph}}$ for $\Sigma = \{a, b\}$

$$\begin{aligned} \phi(z, Y) &= \psi_1 \wedge \psi_2, \text{ where} \\ \psi_1 &= P_a(z) \rightarrow z \notin Y \\ \psi_2 &= \exists X.\big(\mathit{first} \in X \wedge z \in X \wedge \forall y \forall y'.(E(y,y') \rightarrow (y \in X \leftrightarrow y' \notin X))\big) \end{aligned}$$

and depict an NFA for $\mathfrak{L}(\phi)$.

Exercise 12.5 [REG2EMSO (*)]

Provide an alternative proof for the EMSO-definability of regular languages by defining a direct transformation from regular expressions to EMSO-sentences without using the equivalence of regular expressions and finite automata.

Exercise 12.6 [Hamilton path problem and MSO/ESOL-definability]

(a) Show that there is no MSO-sentence ϕ over Voc_{graph} such that for all finite undirected graphs $\mathcal{G}$:

$$\mathcal{G} \models \phi \quad \text{if and only if} \quad \mathcal{G} \text{ has a Hamilton path.}$$

Remind: A Hamilton path in a graph $\mathcal{G} = (V, E)$ is a path $v_0 v_1 \dots v_n$ s.t. $v_i \neq v_j$ for $0 \leqslant i < j \leqslant n$ and $V = \{v_0, v_1, \dots, v_n\}$

(b) Does there exists an ESOL-sentence ϕ over Voc_{graph} s.t. for all finite undirected graphs $\mathcal{G}$:

$$\mathcal{G} \models \phi \quad \text{if and only if} \quad \mathcal{G} \text{ has a Hamilton path?}$$