

Parallel Computing - Notes - v1.0.0

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Preface

Every theory section in these notes has been taken from the sources:

- Validity of the single processor approach to achieving large scale computing capabilities. [1]
- Introduction to parallel algorithms, Carnegie Mellon University. [2]
- Course slides. [3]
- Reevaluating Amdahl's law. [4]
- OpenMP by Example, Johnston Hans. [5]
- Introduction to parallel computing, volume 110. [6]
- NVIDIA Tesla: A unified graphics and computing architecture. [7]
- Structured Parallel Programming: Patterns for Efficient Computation. [8]
- The critical directive, OpenMP, Microsoft. [9]
- Multithreading Architecture. [10]
- Lecture 5, Synchronization I - University of Michigan, EECS Department, Prof. Ronald Dreslinski. [11]
- Cache, Wikipedia. [12]

About:

 [GitHub repository](#)



These notes are an unofficial resource and shouldn't replace the course material or any other book on parallel computing. It is not made for commercial purposes. I've made the following notes to help me improve my knowledge and maybe it can be helpful for everyone.

As I have highlighted, a student should choose the teacher's material or a book on the topic. These notes can only be a helpful material.

During the Parallel Computing for HPC course, me and my other two colleagues [Abdullah Javed](#) and Alberto Onde, we created two projects:

 [Parallel Merge Sort & CUDA 2D Convolution](#)



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1 PRAM

1.1 Prerequisites

Before we introduce the PRAM model, we need to cover some useful topics.

- A **Machine Model** describes a “machine”. It gives a value to the operations on the machine. It is necessary because: it makes it easy to deal with algorithms; it achieves complexity bounds; it analyses maximum parallelism.
- A **Random Access Machine (RAM)** is a model of computation that describes an abstract machine in the general class of register machines. Some features are:
 - **Unbounded** number of local memory cells;
 - Each memory cell can hold an integer of **unbounded** size;
 - Instruction set includes simple operations, data operations, comparator, branches;
 - All operations take **unit time**;
 - The definition of **time complexity** is the number of instructions executed;
 - The definition of **space complexity** is the number of memory cells used.

1.2 Definition

Definition 1: PRAM

A **parallel random-access machine (parallel RAM or PRAM)** is a **shared-memory abstract machine**. As its name indicates, the PRAM is intended as the parallel-computing analogy to the random-access machine (RAM) (not to be confused with random-access memory). In the same way that the RAM is used by sequential-algorithm designers to model algorithmic performance (such as time complexity), the **PRAM is used by parallel-algorithm designers to model parallel algorithmic performance** (such as time complexity, where the number of processors assumed is typically also stated).

The PRAM model has many interesting features:

- **Unbounded collection of RAM processors** (P_0, P_1 , and so on);
- Processors don’t have tape;
- Each processor has **unbounded registers**;
- **Unbounded collection of share memory cells**;
- All **processors can access all memory cells in unit time**;
- All **communication via shared memory**.

1.3 How it works

1.3.1 Computation

A single **processor** of the PRAM, at each computation, is **composed of 5 phases** (carried out in parallel by all the processors):

1. Reads a value from one of the cells $X(1), \dots, X(N)$
 2. Reads one of the shared memory cells $A(1), A(2), \dots$
 3. Performs some internal computation
 4. May write into one of the output cells $Y(1), Y(2), \dots$
 5. May write into one of the shared memory cells $A(1), A(2), \dots$
-

1.3.2 PRAM Classification

During execution, a subset of processors may remain idle. Also, some processors can read from the same cell at the same time (not really a problem), but they could also try to write to the same cell at the same time (**write conflict**). For these reasons, PRAMs are classified according to their read/write capabilities (realistic and useful):

- **Exclusive Read (ER)**. All processors can simultaneously read from distinct memory locations.
- **Exclusive Write (EW)**. All processors can simultaneously write to distinct memory locations.
- **Concurrent Read (CR)**. All processors can simultaneously read from any memory location.
- **Concurrent Write (CW)**. All processors can write to any memory location.

❓ But what value is ultimately written?

It depends on the mode we choose:

- **Priority Concurrent Write**. Processors have priority based on which value is decided, the **highest priority is allowed to complete write**.
- **Common Concurrent Write**. All processors are allowed to complete write **if and only if all the value to be written are equal**. Any **algorithm** for this model has to **make sure that this condition is satisfied**. **Otherwise**, the algorithm is illegal and the machine state will be undefined.
- **Arbitrary/Random Concurrent Write**. One **randomly chosen processor** is allowed to complete write.

1.3.3 Strengths of PRAM

PRAM is attractive and important model for designers of parallel algorithms because:

- It is **natural**. The number of operations executed per one cycle on P processors is at most P (equal to P is the ideal case).
 - It is **strong**. Any processor can read/write any shared memory cell in unit time.
 - It is **simple**. It abstracts from any communication or synchronization overhead, which makes the complexity and correctness of PRAM algorithm easier.
 - It can be used as a **benchmark**. If a problem has no feasible/efficient solution on PRAM, it has no feasible/efficient solution for any parallel machine.
-

1.3.4 How to compare PRAM models

Consider two generic PRAMs, models A and B . Model A is **computationally stronger** than model B ($A \geq B$) if and only if any algorithm written for model B will **run unchanged** on model A in the **same parallel time** and with the **same basic properties**.

However, there are some useful metrics that can be used to compare models:

- **Time to solve problem of input size n on one processor, using best sequential algorithm:**

$$T^*(n) \quad (1)$$

- **Time to solve problem of input size n on p processors:**

$$T_p(n) \quad (2)$$

- **Speedup on p processors:**

$$SU_p(n) = \frac{T^*(n)}{T_p(n)} \quad (3)$$

- **Efficiency**, which is the work done by a processor to solve a problem of input size n divided by the work done by p processors:

$$E_p(n) = \frac{T_1(n)}{pT_p(n)} \quad (4)$$

- **Shortest run time** on any process p :

$$T_\infty(n) \quad (5)$$

- **Cost**, equal to processors and time:

$$C(n) = P(n) \cdot T(n) \quad (6)$$

- **Work**, equal to the total **number of operations**:

$$W(n) \quad (7)$$

Some properties on the metrics:

- The time to solve a problem of input n on a single processor using the best sequential algorithm *is not equal to* the time to solve a problem of input n in parallel using one of the p processors available. In other words, **the problem should not be solvable on a single processor on a parallel machine** (otherwise, what would be the point of using a parallel model?)

$$T^* \neq T_1$$

- $SU_P \leq P$
- $SU_P \leq \frac{T_1}{T_\infty}$
- $E_p \leq 1$
- $T_1 \geq T^* \geq T_p \geq T_\infty$
- $T^* \approx T_1 \Rightarrow E_p \approx \frac{T^*}{pT_p} = \frac{SU_p}{p}$
- $E_p = \frac{T_1}{pT_p} \leq \frac{T_1}{pT_\infty}$
- $T_1 \in O(C), T_p \in O\left(\frac{C}{p}\right)$
- $W \leq C$
- $p \approx \text{AREA} \quad W \approx \text{ENERGY} \quad \frac{W}{T_p} \approx \text{POWER}$

1.4 MVM algorithm

The **Matrix-Vector Multiply (MVM) algorithm** consists of four steps:

1. **Concurrent read of vector X ($1 : n$)** (transfer N elements);
2. **Simultaneous reads of different sections of the general matrix A** (transfer $\frac{n^2}{p}$ elements to each processor);
3. **Compute $\frac{n^2}{p}$ operations per processor;**
4. **Simultaneous writes** (transfer $\frac{n}{p}$ elements from each processor).

Let i be the processor index, so the MVM algorithm is simply written as:

```

1 GLOBAL READ (Z ← X)
2 GLOBAL READ (B ← Ai)
3 COMPUTE (W := BZ)
4 GLOBAL WRITE (W → yi)

```

Algorithm 1: Matrix-Vector Multiply (MVM)

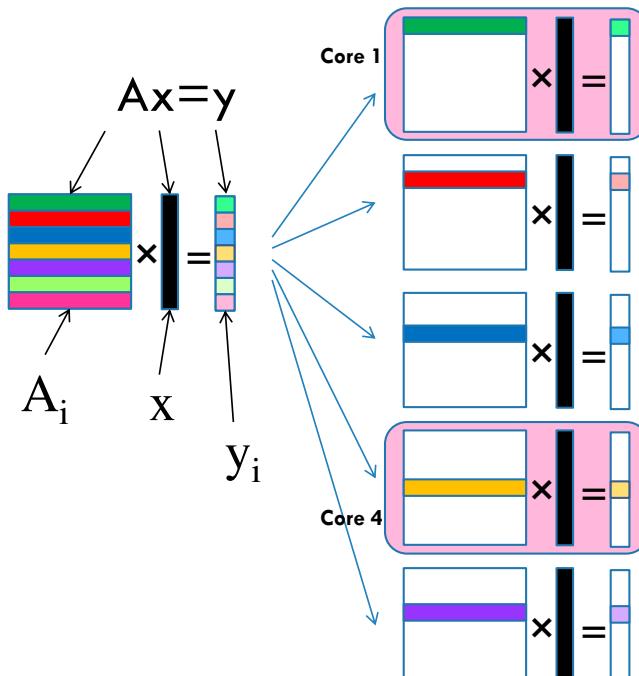


Figure 1: Example of MVM algorithm.

The performance of the MVM algorithm is as follows:

- The **time to solve** a problem of size n^2 is equal to the big O of the squared size of the problem as input divided by the number of processors available:

$$T_p(n^2) = O\left(\frac{n^2}{p}\right)$$

- The **cost** is equal to the number of processors and the time it takes to solve the problem. So it is quite trivial:

$$C = O\left(p \cdot \frac{n^2}{p}\right) = O(n^2)$$

- The **work** is equal to the cost, and the **linear power** P is equal to the ratio of work and time to solve the problem on p processors:

$$W = C \quad \frac{W}{T_p} = P$$

- The **perfect efficiency** is equal to:

$$E_p = \frac{T_1}{pT_p} = \frac{n^2}{p \frac{n^2}{p}} = 1$$

1.5 SPMD sum

The **Single Program Multiple Data (SPMD)** is a term that has been used to describe computational models for exploiting parallelism, where multiple processors work together to execute a program to get results faster.

In this section, we will see an SPMD approach on a Parallel Random Access Machine (PRAM). We will introduce one of the most common and simple mathematical operations: the sum.

The following pseudocode takes as **input an array** of size $n = 2^k$. In this case, n is a power of 2 because it ensures that the array can be evenly divided at each step of the computation. The value k is the number of iterations or levels of the summation process.

```

1 BEGIN
2   GLOBAL READ (A ← A(I))
3   GLOBAL WRITE (A → B(I))
4   FOR H = 1 : K
5     IF i ≤ n ÷ 2h THEN BEGIN
6       GLOBAL READ (X ← B(2i - 1))
7       GLOBAL READ (Y ← B(2i))
8       Z := X + Y
9       GLOBAL WRITE (Z → B(i))
10    END
11   IF I = 1 THEN
12     GLOBAL WRITE (Z → S)
13 END

```

Algorithm 2: Single Program Multiple Data (SPMD) sum

- First, read the entire input array **A** and copy the read data to another array **B**.
- Loop over **h** (**1 to k**). In each iteration, for each index i less than or equal to $n \div 2^h$, read values from array **B** at positions $2i - 1$ and $2i$; sum these values (and store the result in **Z**) and store the result (**Z**) back into **B(i)**.
- Once all iterations are complete, the final sum is stored in a variable **S**.

For example, if $n = 8$, then k would be 3, meaning that the algorithm will run for 3 iterations to sum all the elements in parallel.

h	i	adding
1	1	1,2
	2	3,4
	3	5,6
	4	7,8
2	1	1,2
	2	3,4
3	1	1,2

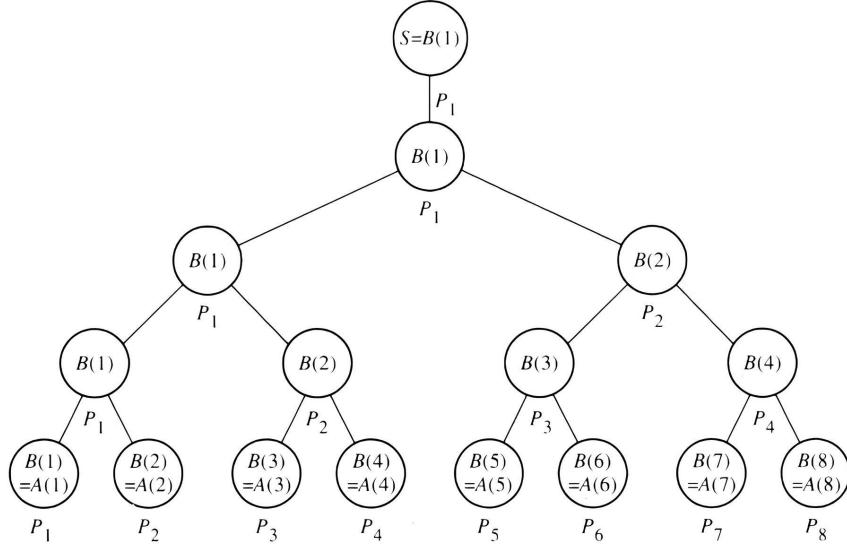
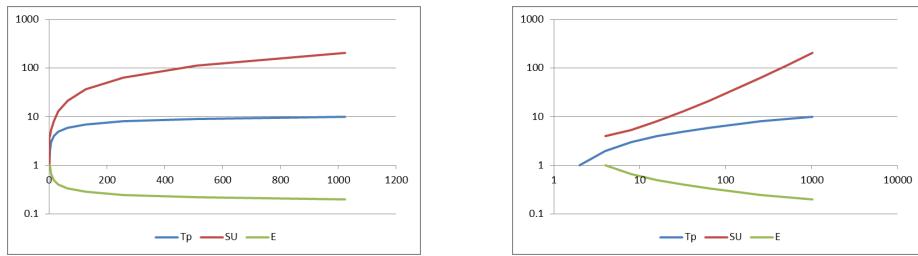


Figure 2: Computation of the sum of eight elements on a PRAM with eight processors. Each internal node represents a sum operation. The specific processor executing the operation is indicated below each node.

⌚ Performance of sum

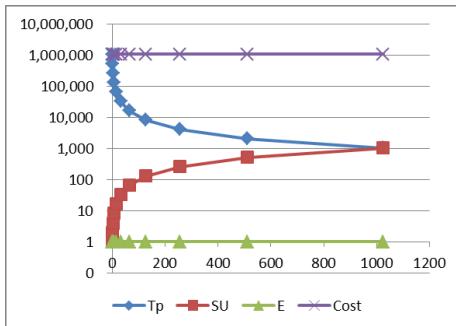
When the size of the array is equal to the number of processors ($N = P$), the **speedup and efficiency decrease**:

- $T^*(N) = T_1(N) = N$
- $T_{N=P}(N) = 2 + \log N$
- $SU_P = \frac{N}{2 + \log N}$
- $T^*(N) = P \cdot (2 + \log N) \approx N \log N$
- $E_p = \frac{T_1}{pT_p} = \frac{N}{N \log N} = \frac{1}{\log N}$



If the size of the array is much larger than the number of processors ($N \gg P$), the **speedup and power are linear**, the **cost is fixed** and the **efficiency is maximum (equal to 1)**:

- $T^*(N) = T_1(N) = N$
- $T_p(N) = \frac{N}{p} + \log p$
- $SU_P = \frac{N}{\frac{N}{p} + \log p} \approx P$
- $\text{COST} = p \left(\frac{N}{p} + \log p \right) \approx N$
- $\text{WORK} = N + P \approx N$
- $E_p = \frac{T_1}{pT_p} = \frac{N}{p \left(\frac{N}{p} + \log p \right)} \approx 1$



$$n = 1'000'000$$

Example 1

Refer to Figure 2 (page 14), the performance metrics are:

- $T_8 = 5$
- $C = 8 \cdot 5 = 40$ (could do 40 steps)
- $W = 2n = 16$ (16 on 40, wasted 24)
- $E_p = \frac{2}{\log n} = \frac{2}{\log 40} \approx 0.67$
- $\frac{W}{C} = \frac{16}{40} = 0.4$

There is also the **Prefix Sum**, which takes **advantage of idle processors in the sum**. It computes all prefix sums:

$$S_i = \sum_1^i a_j \quad a_1, \quad a_1 + a_2, \quad a_1 + a_2 + a_3$$

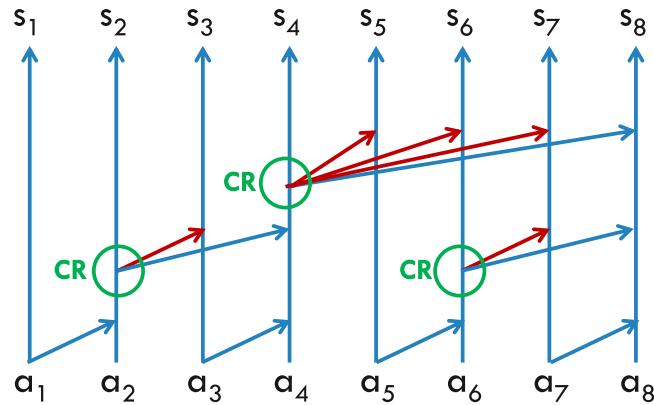


Figure 4: Prefix sum.

1.6 MM algorithm

The **Matrix Multiply (MM) algorithm** consists of three steps:

1. **Compute the two matrices** $A_{i,l}$ and $B_{l,j}$, so use the concurrent read.
2. **Make the sum.**
3. **Store** the result using exclusive write.

```

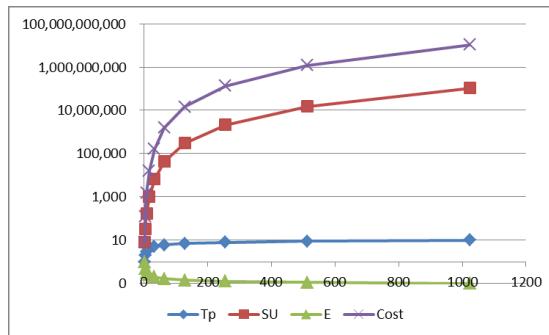
1 BEGIN
2    $T_{i,j,l} = A_{i,l}B_{l,j}$ 
3   FOR  $= H = 1 : K$ 
4     IF  $l \leq n \div 2^h$  THEN
5        $T_{i,j,l} = T_{i,j,2l-1} + T_{i,j,2l}$ 
6     IF  $l = 1$  THEN
7        $C_{i,j} = T_{i,j,1}$ 
8 END

```

Algorithm 3: Matrix Multiply (MM)

⌚ Performance of MM

- $T_1 = n^3$
- $T_{p=n^3} = \log n$
- $SU = \frac{n^3}{\log n}$
- Cost = $n^3 \log n$
- $E_p = \frac{T_1}{pT_p} = \frac{1}{\log n}$



1.7 PRAM variants and Lemmas

The PRAM model presented here is one of the most commonly used. However, there are other important variants:

- PRAM model with a **limited number of shared memory cells** (small memory PRAM). If the input data set exceeds the capacity of the shared memory, the I/O values can be evenly distributed among the processors.
- PRAM model with **limited number of processors** (small PRAM). If the number of execution threads is higher, processors can interleave multiple threads.
- PRAM model with **limited size of one machine word**.
- PRAM model with **access conflicts handling**. These are restrictions on simultaneous access to shared memory cells.

Lemma 1. *Assume $P' < P$ and same size of shared memory. Any problem that can be solved for a P processor PRAM in T steps can be solved in a P' processor PRAM in:*

$$T' = O\left(\frac{TP}{P'}\right) \quad (8)$$

Proof. Partition P simulated processors into P' groups of size $\frac{P}{P'}$ each. Associate each of the P' simulating processors with one of these groups. Each of the simulating processors simulates one step of its group of processors by:

- Executing all their read and local computation substeps first;
- Executing their write substeps then.

QED

Lemma 2. *Assume $M' < M$. Any problem that can be solved for a P processor and M -cell PRAM in T steps can be solved on a max (P, M') -processors M' -cell PRAM in $O\left(\frac{TM}{M'}\right)$ steps.*

Proof. Partition M simulated shared memory cells into M' continuous segments S_I , of size $\frac{M}{M'}$ each. Each simulating processor P'_I ($1 \leq I \leq P$), will simulate processor P_I of the original PRAM. Each simulating processor P'_I ($1 \leq I \leq M'$), stores the initial contents of S_I into its local memory and will use $M'[I]$ as an auxiliary memory cell for simulation of accesses to cell of S_I .

Simulation of one original read operation:

```

1 EACH  $P'_I$  ( $I = 1, \dots, \max(P, M')$ ) REPEATS FOR  $K = 1, \dots, \frac{M}{M'}$ 
2   WRITE THE VALUE OF THE  $K$ -TH CELL OF  $S_I$  INTO  $M'[I]$  ( $I = 1, \dots, M'$ )
3   READ THE VALUE WHICH THE SIMULATED PROCESSOR  $P_I$  ( $I = 1, \dots, P$ )
      WOULD READ IN THIS SIMULATED SUBSTEP, IF IT APPEARED IN THE
      SHARED MEMORY

```

The local computation substep of P_I ($I = 1, \dots, P$) is simulated in one step by P'_I . Simulation of one original write operation is analogous to that of read.

QED

1.8 PRAM implementation

The PRAM is an ideal model for creating parallel algorithms. Now we look at “*is it really implementable?*” The short answer is yes.

The longest answer is the following. There are already some examples of PRAM being converted to real machine models, such as [Explicit Multi-Threading \(XMT\)](#), Rigel, Tilera, etc. If conversion is not easy or possible, the implementation can be “*direct*”:

- The concurrent read is implemented as a detect-and-multicast technique.
- The concurrent write is implemented depending on the end result we want to achieve. Fetch-and-operate and prefix-sum are examples of serialized writing; otherwise, the CRCW technique is used:
 - Common CRCW: detect and merge
 - Priority CRCW: detect-and-priorities
 - Arbitrary CRCW: arbitrary

Example 2: Boolean DNF (sum of products) common CRCW

A logical formula is considered to be in DNF if it is a disjunction of one or more conjunctions of one or more literals.

Consider X as the sum of products of AND/OR operations:

$$X = a_1 b_1 + a_2 b_2 + \dots$$

The PRAM code, with X initialized to 0 and task index equal to \$, is:

```
if (a$b$) X = 1;
```

The common result is that not all processors write X and those that do write 1. The time complexity is $O(1)$. It works on common, priority and arbitrary CRCW.

Despite the previous example, exists also the PRAM SoP for the concurrent write. Let boolean X as:

$$X = a_1 b_1 + a_2 b_2 + \dots$$

The PRAM algorithm is:

```
if (aibi) X = 1;
```

Where all cores which write into X , **write the same value**.

✓ PRAM advantages

- Large body of algorithms.
- Easy to think about it.
- Sync version of shared memory. It eliminates sync and common issues, allows focus on algorithms, but allows adding these issues and allows conversion to async versions.
- Exists architectures for both synch (PRAM) and async (SM) model.
- PRAM algorithms can be mapped to other models.

1.9 Amdahl's and Gustafson's Laws

The **Amdahl's Law** is a formula which gives the **theoretical speedup in latency of the execution of a task at fixed workload that can be expected of a system whose resources are improved**. The law can be stated as:

Definition 2: Amdahl's Law

The overall performance improvement gained by optimizing a single part of a system is limited by the fraction of time that the improved part is actually used.

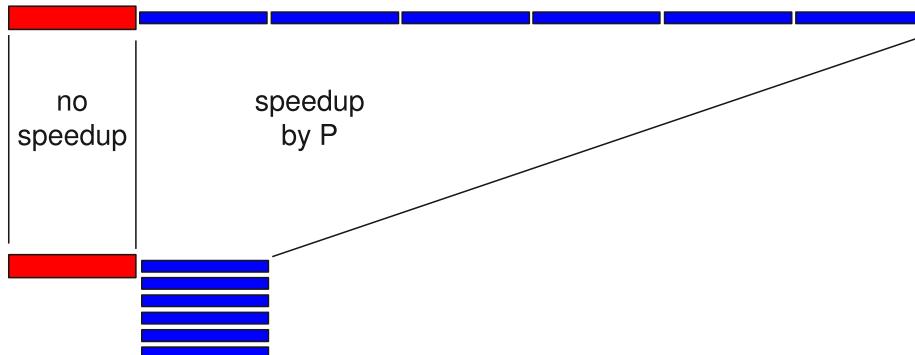
In practice, Amdahl's law says that the computation consists of interleaved segments of two types:

1. **Serial segments** (which cannot be parallelized);
2. **Parallelizable segments**.

Therefore, the metrics we can obtain are the time on P processors metric, that it is greater than the fraction of time on a processor divided by the processors P , and the speedup metric, that it is less than the number of processors P :

$$T_P > \frac{T_1}{P} \quad SU < P$$

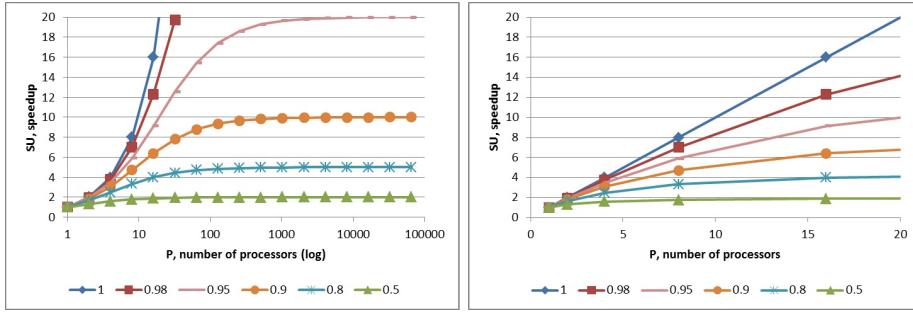
Graphically, we can see a fixed part of the line, which is the **serial segment** (no speedup), and a set of instructions that can be **parallelized** (the sum of these segments is equal to the unit time 1).



Furthermore, if we identify the parallelizable segment as f and the serial segment as $1 - f$, we obtain the following expressions:

$$\begin{aligned} SU(P, f) &= \frac{T_1}{T_P} = \frac{T_1}{T_1 \cdot (1-f) + \frac{T_1 \cdot f}{P}} = \frac{1}{(1-f) + \frac{f}{P}} \\ \lim_{P \rightarrow \infty} SU(P, f) &= \frac{1}{1-f} \end{aligned} \quad (9)$$

In the following figure we can see the speedup with parameter f . Note the pessimism: for a problem with inherent $f = 90\%$, there is no point in using more than 10 processors.

Figure 5: Amdahl's law, $SU(P)$, parameter f .

The original paper presenting Amdahl's Law [1] can be viewed by clicking on the link below or by scanning the QR code.



Amdahl's law applies only to the cases where the problem size is fixed. In practice, as more computing resources become available, they tend to get used on larger problems (larger datasets), and the time spent in the parallelizable part often grows much faster than the inherently serial work. In this case, **Gustafson's law gives a less pessimistic and more realistic assessment of the parallel performance.** [8]

Gustafson's Law gives the speedup in the execution time of a task that theoretically gains from parallel computing, using a hypothetical run of the task on a single-core machine as the baseline. To put it another way, it is the **theoretical "slowdown"** of an already parallelized task if running on a serial machine.

Against Amdahl's law, Gustafson suggests the following ideas:

- Portion f is not fixed;
- The absolute serial time is fixed;
- Parallel problem size is increased to exploit more processors;
- Fixed serial time (s of total) and fixed parallel time ($1 - s$ of total) are invariants;
- **Fixed time model** and not fixed size model (as Amdahl's law):

$$SU(P) = \frac{T_1}{T_P} = \frac{s + P \cdot (1 - s)}{s + (1 - s)} = s + P \cdot (1 - s) \quad (10)$$

Gustafson's law suggests a **linear speedup** and is **empirically applicable to highly parallel algorithms**.

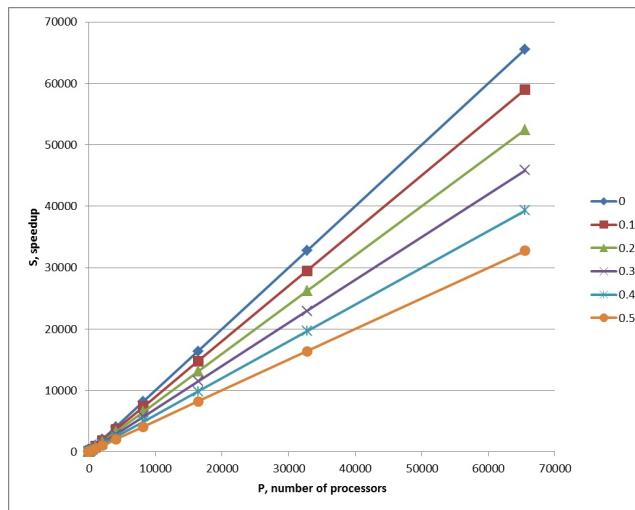


Figure 6: Gustafson's law.

Amdahl's Law states that as computing power increases, computational requirements remain the same. In other words, the **analysis of the same data will take less time with more computing power**.

Gustafson, on the other hand, argues that **more computing power leads to more careful and complete analysis of the data**. Where it would not have been possible or practical to simulate the impact of nuclear detonation on every building, car, and their contents (including furniture, structural strength, etc.) because such a calculation would have taken more time than was available to provide an answer, the increase in computing power will prompt researchers to add more data to more fully simulate more variables, giving a more accurate result.

The original paper presenting Gustafson's Law [4] can be viewed by clicking on the link below or by scanning the QR code.

[Gustafson's Law](#)



2 Fundamentals of architecture

The main purpose of this chapter is to introduce some basics of parallel computing theory. It will introduce the simplest and trivial processor and the more complex and efficient variants. The topics introduced are explained in a simple way and without any deepening, because it is only an introduction. For those who have studied computer science, this chapter might be a little boring and you might notice that some topics are explained in a simplistic way.

2.1 Introduction

2.1.1 Simplest processor

Inside a computer, a processor executes instructions.

- **Fetch/Decode:** Determine which instruction to run next;
- **ALU (execution unit):** Performs the operation described by an instruction, which may change values in the processor's registers or the computer's memory;
- **Registers:** maintain program state, store values of variables used as inputs and outputs to operations.

The simplest and most basic processor executes **one instruction per clock cycle**.

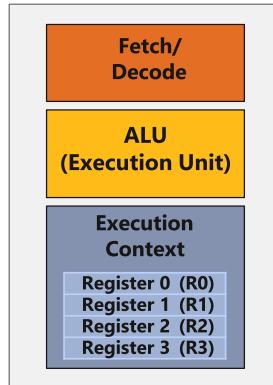


Figure 7: The simplest and most basic processor.

2.1.2 Superscalar processor

A more “complex” and realistic model is the **superscalar processor**. This **processor can decode and execute up to two instructions per clock**. The execution is slightly different from the simplest processor. The **processor automatically finds independent instructions in an instruction sequence and can execute them in parallel on multiple execution units**.

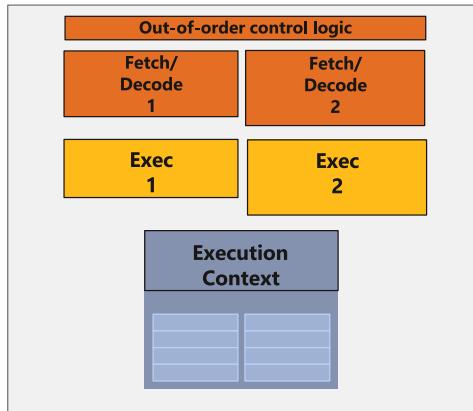


Figure 8: The superscalar processor.

The superscalar processor takes advantage of **Instruction-Level Parallelism (ILP)**¹ within an instruction stream.

- Processing **different instructions** from the same instruction stream **in parallel (within a core)**.
- **Parallelism is automatically detected by the hardware during execution.**

¹Instruction-level parallelism (ILP) is the parallel or simultaneous execution of a sequence of instructions in a computer program. More specifically ILP refers to the average number of instructions run per step of this parallel execution.

2.1.3 Single Instruction, Multiple Data (SIMD) processor

Adding execution units (ALUs) to the simplest processor can increase compute capability. Amortize the cost/complexity of managing an instruction stream across many ALUs using **Single Instruction, Multiple Data (SIMD)** processing. Therefore, the **same instruction is sent to all ALUs**. This **operation is performed in parallel on all ALUs**.

✓ Advantages

- **Efficient for data-parallel workloads:** amortize control costs over many ALUs.
- Vectorization done by:
 - Compiler (**explicit SIMD**): parallelism is explicitly requested by the programmer through intrinsics, conveyed through parallel language semantics, and inferred through loop dependency analysis by the “auto-vectorizing” compiler. In other words, the **SIMD parallelization is done at compile time, and when we inspect the program binary, we can see the SIMD instructions**.
 - At runtime by hardware (**implicit SIMD**): the **compiler generates a binary with scalar instructions**, but n instances of the program are always executed together on the processor. The **hardware** (not the compiler) is **responsible for the simultaneous execution of the same instruction by multiple program instances on different data on SIMD ALUs**.

2.1.4 Multi-Core Processor

A **Multi-Core Processor (MCP)** is a **microprocessor** on a single integrated circuit (IC) with **two or more separate central processing units (CPUs)**, called **cores** to emphasize their multiplicity (e.g., *dual-core* or *quad-core*). Each core reads and executes program instructions, specifically ordinary CPU instructions (such as add, move data, and branch). However, the MCP can **execute instructions on separate cores simultaneously**, increasing overall speed for programs that support multithreading or other parallel computing techniques.

✓ Advantages

- Provides **thread-level parallelism**: execute a completely different instruction stream on each core simultaneously.
- **Software creates threads to expose parallelism to hardware** (e.g., via threading API)

2.2 Accessing Memory

2.2.1 What is a memory?

A computer's memory is organized as an array of bytes. Each byte is identified by its address in memory (its position in that array).

Address	Value
0x0	16
0x1	255
0x2	14
0x3	0
0x4	0
0x5	0
0x6	6
0x7	0
0x8	32
0x9	48
0xA	255
0xB	255
0xC	255
0xD	0
0xE	0
0xF	0
0x10	128
:	:
0x1F	0

Table 1: Example illustration of the program's memory address space of 32 bytes, range from 0x0 to 0x1F.

From the processor's point of view, loading an instruction to access the contents present in memory is done with the `ld` assembly instruction. For example, `ld R0 ← mem[R2]` means “take the value from register `R2` and put that value into register `R0`”.

Before we introduce new concepts, let us take a moment to explain some important **terminology**:

- **Memory Access Latency**, is the time it takes for the **memory system to deliver data to the processor**.
- **Processor Stall**. A processor stalls when it **cannot execute the next instruction in an instruction stream because of a dependency on a previous instruction that has not been completed**. Accessing memory is a major source of stalling, which is one of the main reasons why memory accesses should be limited.

For **example**, in the following three assembler instructions, the add has to wait for the loading of R2 and R3 values, making parallelization more complicated:

```
ld r0 mem[r2]
ld r1 mem[r3]
add r0, r0, r1
```

- **Memory Bandwidth**, is the **rate at which the memory system can provide data to a processor**.

Bandwidth is the critical resource in modern computing.

High-performance parallel programs will:

1. **Organize computation to fetch data from memory less frequently**. For example, reuse data previously loaded by the same thread (temporal locality optimizations) or share data across threads (inter-thread cooperation);
2. Prefer to **perform additional arithmetic to store/reload values**;
3. **Programs need to access memory infrequently** to take advantage of modern processors.

2.2.2 How to reduce processor stalls

2.2.2.1 Cache

One of the most common solutions is caching.

A **cache** is a hardware or software **component that stores data so that future requests for that data can be served faster**; the data stored in a cache might be the result of an earlier computation or a copy of data stored elsewhere.

- A **Cache Hit** occurs when the **requested data is found** in a cache;
- A **Cache Miss** occurs when **it cannot**.

Cache hits are served by reading data from the cache, which is faster than recomputing a result or reading from a slower data store, so the **more requests that can be served from the cache, the faster the system performs**. [12]

Many modern CPUs have logic that predicts what data will be accessed in the future and “pre-fetches” that data into caches. **Prefetching** reduces stalls because the data is resident in the cache when it is accessed. But beware, the other side of the coin is that if the **guess is wrong**, the **performance is worse than the system without prefetching!**

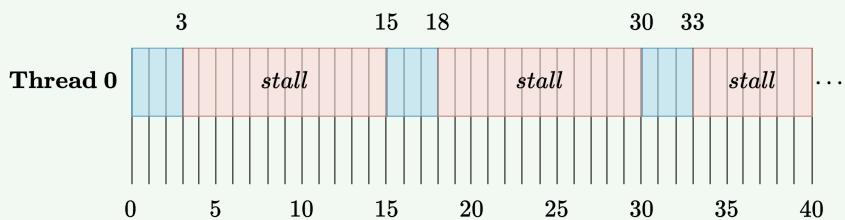
2.2.2.2 Multi-threading

A **Multithreaded Processor** is one that has the **ability to follow multiple instruction streams without software intervention**. In practice, then, this includes any machine that **stores multiple program counters (PCs) in hardware within the processor** (i.e., on chip, for microprocessor-era machines). [10]

The main idea in this architecture is to **interleave processing of multiple threads on the same core to hide stalls**. In other words, if we can't make progress on the current thread, we work on another one.

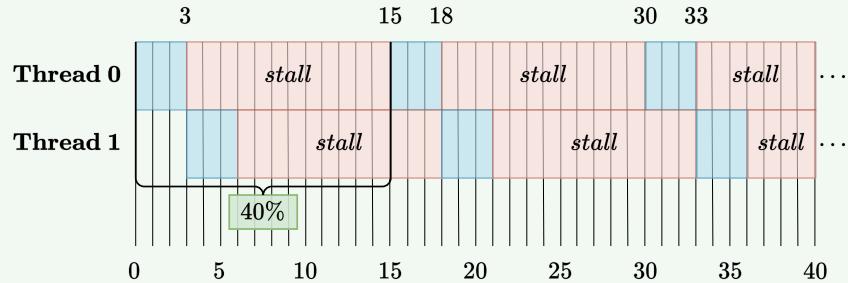
Example 1: core utilization

To better understand our explanation, suppose we are running a program where threads perform **three arithmetic instructions followed by a memory load** (with 12 cycle latency).



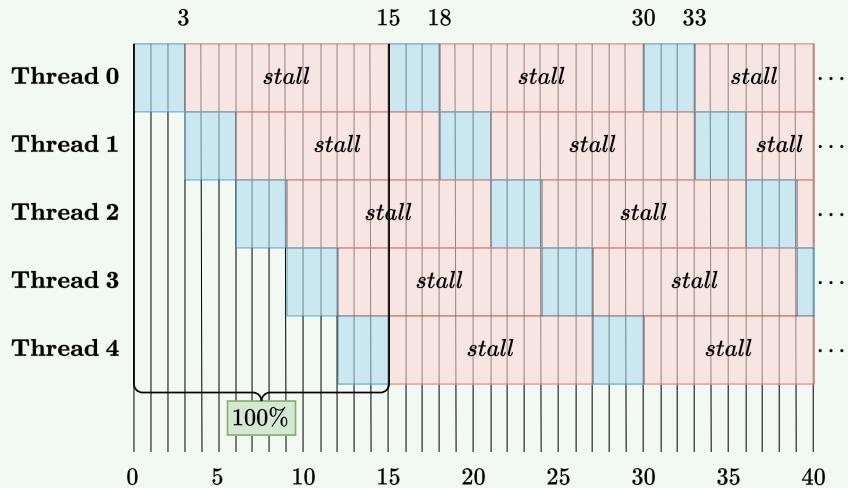
From the figure, it is clear that if we consider an arithmetic instruction and a memory stall, the core is not fully optimized to work at 100%. In

practice, we see that a single arithmetic instruction takes 3 clock cycles and the memory stall takes 12 clock cycles. This means that from this situation we are using the CPU at only 20% (3 work clock cycles on 15)! Without suggesting the final solution, try to see what happens when we add another thread.



We have gained three clock cycles, and now we are taking advantage of the 40% of the core.

Now, how many threads do we need to achieve 100% utilization? The answer is simple: the number of clock cycles of the operations to be done before the stall plus the clock cycles of the stall divided by the working operations (operations that are not stalls). In our case: $15 \div 3 = 5$.



Note that if we add more threads, there will be no benefit because the CPU is already at 100%.

✓ Multithreaded Processor benefits

- A processor with multiple hardware threads has the **ability to avoid stalls** by executing instructions from other threads when one thread must wait for a long latency operation to complete. The latency of the memory operation is not changed by multithreading, it just no longer causes reduced processor utilization.
- A **multithreaded processor hides memory latency** by performing

arithmetic from other threads. Program that feature more arithmetic per memory access need fewer threads to hide memory stalls.

■ Type of hardware-supported multithreading

- **Core manages execution contexts for multiple threads.** This type still has the same number of ALU resources: multi-threading only helps to use them more efficiently in the face of high latency operations such as memory access. The **processor decides which thread to run each clock cycle.**
- **Coarse-Grain Multithreading**, also called **Block Multithreading** or **Switch-On-Event Multithreading**, has multiple hardware contexts associated with each processor core. A hardware context is the program counter, register file, and other data required to enable a software thread to execute on a core. However, only one hardware context has access to the pipeline at a time. [10]
- **Fine-Grain Multithreading (FGMT)**, also known as **Interleaved Multithreading** or **Temporal Multithreading**, is the type just described on the previous pages, as in the example on page 29.
- **Simultaneous Multithreading (SMT)** has multiple hardware contexts associated with each core. In a simultaneous multithreaded processor, instructions from multiple threads are available to be issued on any cycle. Therefore, all hardware contexts, and in particular all register files, must be accessible to the pipeline and its execution resources. [10]

In other words, **each clock, the core selects instructions from multiple threads to execute on ALUs.**

3 Programming models

3.1 Implicit SPMD Program Compiler (ISPC)

Before introducing the ISPC compiler, we give the definition of SPMD.

Definition 1: Single Program, Multiple Data (SPMD)

Single Program, Multiple Data (SPMD) is a term that has been used to refer to computational models for exploiting parallelism, where **multiple processors work together to execute a program to achieve faster results**.

The difference between *SPMD* and *SIMD* (page 26) is that in SPMD parallel execution, **multiple autonomous processors simultaneously execute the same program at independent points**, rather than in SIMD it is vectorization at the instruction level so that **each CPU instruction processes multiple data elements**.

In other words:

- SPMD: is the **programming abstraction**, because the programmer has to think; the program is written in terms of this abstraction.
- SIMD: in general, the compilers (ISPC) issue special vector instructions that execute the logic performed by each parallel instance created (ISPC gang spawned). In addition, the compilers handle the mapping of conditional control flow to vector instructions.

The difference and the terminology used by ISPC will become clearer in the following pages. We suggest that finish this section and come back here in a moment.

Definition 2: Implicit SPMD Program Compiler (ISPC)

Implicit SPMD Program Compiler (ISPC) is a **compiler for a variant of the C programming language**, with extensions for *Single Program, Multiple Data (SPMD)* programming. Under the SPMD model, the programmer writes a program that generally appears to be a regular serial program, though the execution model is actually that a number of program instances execute in parallel on the hardware. In other words, the **ISPC gives the programmer some API to do parallelization on the code; it also generates high quality SIMD code to increase performance**.

The definition, implementation, and other details are explained in the official [Intel GitHub repository](#).

⌚ How it works?

Let us take a general main program; when we call an `ispc` function, it causes a **spawn of gang of ISPC program instances upon return, all instances have completed**. These **instances execute the same ISPC code simultaneously, and each instance has its own copy of local variables**. Take the following ISPC code as an example:

```

1  export void ispc_sinx(
2      uniform int N,
3      uniform int terms,
4      uniform float* x,
5      uniform float* result
6  ){
7      // assume N % programCount = 0
8      for (uniform int i=0; i<N; i+=programCount) {
9          int idx = i + programIndex;
10         float value = x[idx];
11         float numer = x[idx] * x[idx] * x[idx];
12         uniform int denom = 6; // 3!
13         uniform int sign = -1;
14         for (uniform int j=1; j<=terms; j++) {
15             value += sign * numer / denom;
16             numer *= x[idx] * x[idx];
17             denom *= (2*j+2) * (2*j+3);
18             sign *= -1;
19         }
20         result[idx] = value;
21     }
22 }
```

In the example, the `programCount` (row 8) and `programIndex` (row 9) variables, `uniform` (row 2, and so on) data type tell us:

- `programIndex` gives the index of the SIMD-lane being used for running each program instance (in other words, it's a varying **integer value that has value zero for the first program instance, and so forth**).
- `programCount` gives the **total number of instances in the gang**.
- A variable that is declared with the `uniform` qualifier represents a **single value that is shared across the entire gang**.

Together, these can be used to uniquely map executing program instances to input data (`programIndex` and `programCount`, `uniform` data type).

With the ISPC analogy, the **SPMD programming model** should be clear:

1. **Single thread of control** (typically a main program);
2. **Invoke the SPMD function** (in the previous example, the `ispc_sinx` function);
3. **SPMD execution**, then **multiple instances of the function run in parallel** (multiple logical threads of control);
4. **Returns and resumes a single thread of control**.

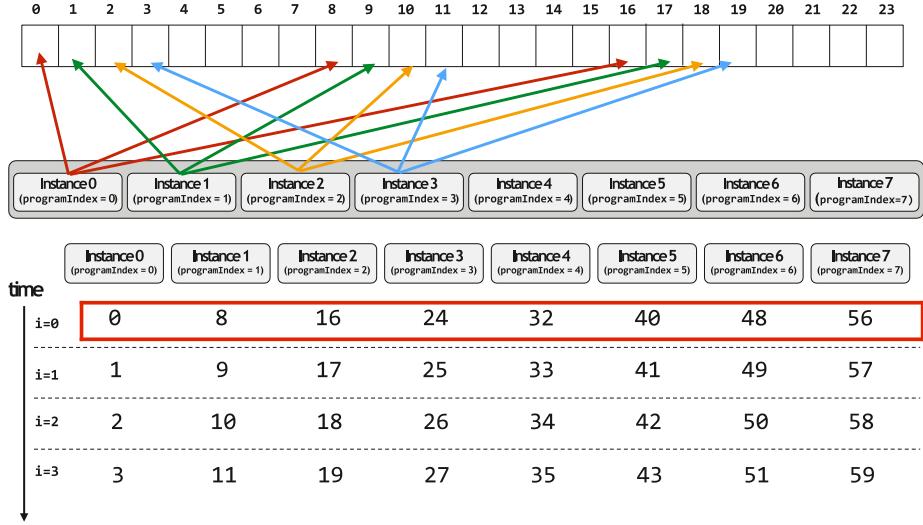


Figure 9: Example of execution with 8 instances (programCount equal to 8). For all program instances, there are eight non-contiguous values in memory. A special instruction called `gather` is needed to implement this, but unfortunately it is a more complex and expensive SIMD instruction rather than a contiguous implementation.

Figure 9 shows a possible execution of the ISPC function using 8 instances. The result is obtained and all is well. But there is one interesting observation. **Each ISPC instance writes each value in a non-contiguous way.** This can be done better:

```

1  export void ispc_sinx_v2(
2      uniform int N,
3      uniform int terms,
4      uniform float* x,
5      uniform float* result
6  ){
7      // assume N % programCount = 0
8      uniform int count = N / programCount;
9      int start = programIndex * count;
10     for (uniform int i=0; i<count; i++) {
11         int idx = start + i;
12         float value = x[idx];
13         float numer = x[idx] * x[idx] * x[idx];
14         uniform int denom = 6; // 3!
15         uniform int sign = -1;
16         for (uniform int j=1; j<=terms; j++) {
17             value += sign * numer / denom
18             numer *= x[idx] * x[idx];
19             denom *= (j+3) * (j+4);
20             sign *= -1;
21         }
22         result[idx] = value;
23     }
24 }
```

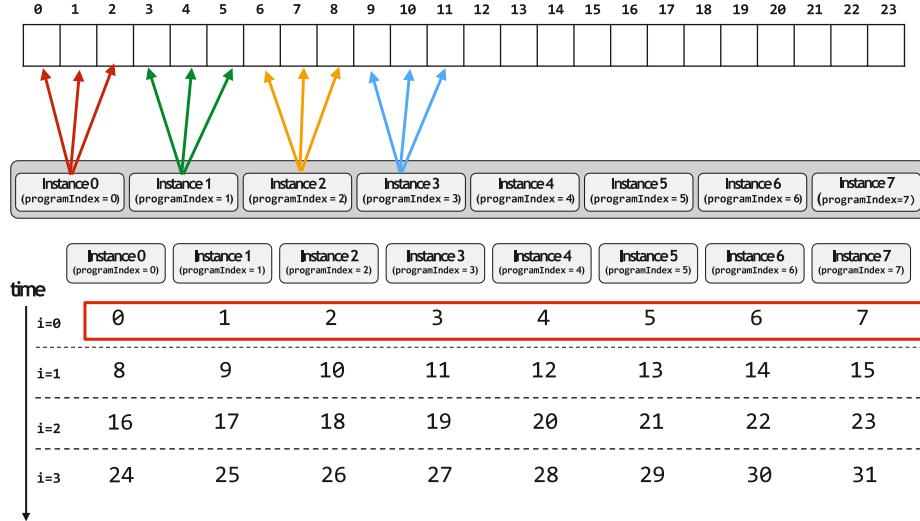


Figure 10: Example of execution with 8 instances (programCount equal to 8). A single “packed vector load” instruction efficiently implements this. For all program instances, since the eight values are contiguous in memory.

3.2 Shared Address Space Model

We give a general introduction to memory in the chapter 2.2. In parallel computing theory, **each thread communicates with other threads using read/write operations**. These instructions operate on a special area called the **Shared Address Space** (also called **Shared Variables**).

Definition 3: Shared Address Space

The **Shared Address Space** view of a parallel platform supports a common data space that is accessible to all processors. Processors interact by modifying data objects stored in this shared-address-space. [6]

Now the first and trivial question should be: a powerful tool is the possibility to allow communication between threads, but *how can we guarantee that two or more threads accessing the same resource do not create well known problems, such as race condition?*

This property, commonly called **mutual exclusion** or **atomic operation**, can be guaranteed with some techniques:

- **Lock/Unlock mutex around a critical section:**

```

1 Lock lock_variable;
2
3 // some operations, such as spawn of threads
4
5 lock_variable.lock();
6 // critical section
7 lock_variable.unlock();

```

- Some languages have first-class support for atomicity of code blocks:

```

1 atomic {
2     // critical section
3 }

```

- Intrinsics for hardware-supported atomic read-modify-write operations:

```

1 atomicAdd(x, 10);

```

The shared address space requires **hardware support to be efficiently implemented**. The main idea is that **each processor can directly reference the contents of any memory location**. Some interesting examples that can be explored in depth are: [SUN Niagara 2](#), [Knights landing \(KNL\): 2nd Generation Intel Xeon Phi processor](#).

3.3 Message Passing model of communication

In parallel computing, the **message passing model** allows threads (or processes) to **communicate** by sending and receiving messages. This model is used to **facilitate data exchange between threads running in their own private address spaces**.

Each thread operates within its private address space, meaning they **do not share memory directly**. When they need to communicate with a specific thread without using a *shared address space model*, they use the *message passing model* to send (or receive) the data.

- Usually, the **sender** specifies
 - the recipient;
 - the buffer to be transmitted;
 - an optional message identifier (a sort of tag).
- Meanwhile, the **receiver** specifies:
 - who the sender is;
 - the buffer where to store data;
 - an optional message identifier (again, a sort of tag).

The message passing model is the **only way to exchange data between threads** because it guarantees three main advantages:

- ✓ **Data Isolation**: Using message passing, **each thread maintains its address space**, reducing the risk of race conditions.
- ✓ **Scalability**: Message passing **scales well with distributed systems and multi-core architectures**, making it suitable for large-scale parallel computing.
- ✓ **Flexibility**: It allows for **explicit control over data exchange and synchronization**, providing **Flexibility in parallel program design**. Explicit control refers to the fact that in message passing, each communication action is explicitly defined by send and receive operations. This means we can precisely dictate which data is sent, when, and to whom it is sent. Furthermore, the synchronization is managed very well because message passing naturally incorporates synchronization. When a process sends a message, it can block until it is received, ensuring that the sender and receiver are synchronized.

❷ Why message passing is preferable to the shared address space model

Unlike shared memory systems that require complex hardware mechanisms to implement system-wide load and store operations, message passing systems do not need this capability. They **only need to communicate messages between nodes**. It can be considered as a great advantage for message passing models, which for this reason is very much used in the supercomputers and clusters. Finally, this model has the ability to **connect commodity hardware systems together to form a large parallel machine**. This means we can use off-the-shelf components to build powerful computing clusters.

3.4 Data-Parallel model

In parallel computing, the **Data-Parallel model** is characterized by **applying the same operation simultaneously across multiple data points**. This model is particularly effective for tasks involving large datasets where the same computation needs to be performed on each data element.

This model organizes **computation as operations on sequences of elements** (e.g., perform the same function on all elements of a sequence). In programming languages, the basic data type used for this purpose is called **Sequence** (e.g., C++). Despite the name of the datatype used in the languages, the definition is that it **is an ordered collection of elements, where each element can be accessed and manipulated using various sequence operators**. The most common operators are:

- **Map.** The map function is a **higher-order function**² that **operates on sequences**. It applies a **side-effect-free unary function**³ $f : a \rightarrow b$ to **all elements of an input sequence**, producing an **output sequence of the same length**.

Example 1: Python Analogy

For a better understanding, we provide a very simple Python code to see how a `map` function works. In Python there is a `map` function that does exactly what we say.

```

1 # Define a trivial function that squares a number
2 def square(x):
3     return x * x
4
5 # Create a sample list
6 numbers = [1, 2, 3, 4, 5]
7
8 # Use the map function to apply the 'square' function
9 # to each element in the 'numbers' list
10 squared_numbers = map(square, numbers)
11
12 # Convert the result to a list and print it
13 print(list(squared_numbers))

```

And the result is:

```
1 [1, 4, 9, 16, 25]
```

Now the main idea is: since the `map` is a function without side effects, we can **apply it to all elements of the sequence in any order without changing the output of the program**. This allows reordering or parallel processing of sequence elements to **optimize performance**.

²A **higher-order function** is a function that can do one or both of the following:

- Take other functions as arguments (parameters).
- Returns a function or value as its result.

³A function that takes only one argument and doesn't suffer from side effects.

- **Reduction.** The reduction born from the need to make parallel operations of iterations. For example, in a `for` loop, if we need a progressive sum of an element, we should implement some kind of mechanism to manage the synchronization. Using the reduction strategies, the compiler will do this job. We suggest reading the Reduction section (5.3.1.1, page 66) in the OpenMP chapter to understand what we mean.
- Others like `scan` and `shift`.

4 Parallel Programming Models and pthreads

4.1 How to create parallel algorithms and programs

Although *parallel algorithms* and *parallel programs* are in the same father set, the parallel computing topic, these two arguments are a little different.

Definition 1: Parallel Algorithms

A **parallel algorithm**, as opposed to a traditional serial algorithm, is an **algorithm which can do multiple operations in a given time**.

Definition 2: Parallel Programs

A **parallel program** is a **program that uses multiple CPU cores, with each core performing a task independently**.

However, designing *parallel algorithms* is not an easy task because there is no heuristic for designing *parallel algorithms*. There are some rules that help in the design. The same reasoning applies to *parallel programs*, because they depend on the chosen language and architecture.

Furthermore, there is no single correct solution, but several possible parallel solutions. A **good first approach is to start with machine-independent issues (concurrency) and delay target-specific issues as much as possible**.

Design a parallel algorithm	Design a parallel program
Understand the problem to be solved	Analyze the target architecture(s)
Analyze data dependencies	Choose the best parallel programming model and language
Partition the solution	Analyze the communications (cost, latency, bandwidth, visibility, synchronization, etc.)

Table 2: Design parallel algorithms and parallel programs.

The **PCAM (Partitioning, Communication, Agglomeration, Mapping)** methodology described by [Argonne National Laboratory](#) is intended to promote an exploratory **approach to design** in which **machine independent issues**, such as *concurrency*, are **considered early** and **machine specific aspects of design are deferred until late in the design process**. In other words, we immediately consider the machine-independent issues (e.g., concurrency) at the beginning of the design approach, and all machine-specific aspects are postponed to an advanced stage of the design process.

This methodology structures the design process into **four distinct stages**:

1. **Partitioning.** The **computation** that is to be performed and the **data** operated on by this computation are **decomposed into small tasks**. Practical issues such as the number of processors in the target computer are ignored, and attention is focused on recognizing opportunities for parallel execution.
2. **Communication.** The communication required to **coordinate task execution** is determined, and appropriate **communication structures and algorithms are defined**.
3. **Agglomeration.** The task and **communication structures** defined in the first two stages of a design are **evaluated with respect to performance requirements and implementation costs**. If necessary, tasks are combined into larger tasks to improve performance or to reduce development costs.
4. **Mapping.** Each task is assigned to a processor in a manner that attempts to satisfy the competing goals of **maximizing processor utilization** and **minimizing communication costs**. Mapping can be specified statically or determined at runtime by load-balancing algorithms.

In the **first two stages**, we focus on **concurrency** and **scalability** and seek to **discover algorithms with these qualities**. In the **third and fourth stages**, attention shifts to **locality** and **other performance-related issues**.

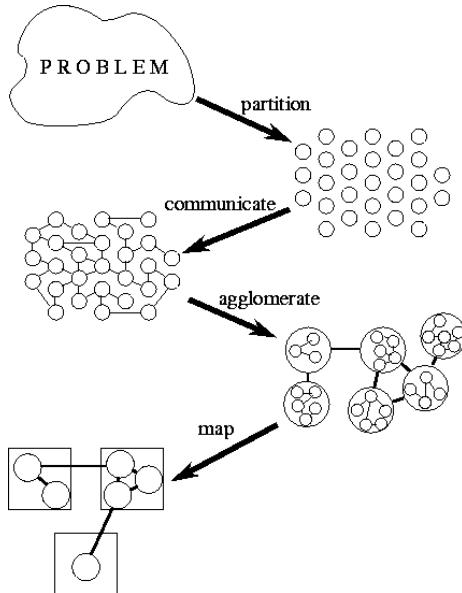


Figure 11: PCAM design methodology for parallel programs. Starting with a problem specification, develop a partition, determine communication requirements, agglomerate tasks, and finally map tasks to processors.

4.2 Analyze parallel algorithms

Whether we want to analyze our parallel algorithm created with the PCAM model or evaluate a general parallel algorithm, we need some metrics.

The classical **metrics** needed to evaluate a parallel algorithm are:

- **Time complexity**: quantifies the amount of **time required to produce a solution**.
- **Resource complexity**: quantifies how many **resources are needed to produce the solution in that time**.

In general, to analyze a **parallel algorithm**, we can consider its **structure as a directed acyclic graph (DAG)**⁴, where the nodes are the task and the edges are the data dependencies.

Parallel Algorithm Terminology and Metrics

- **Concurrent tasks**, each task is executed *independently*.
- **Parallel tasks**, each task is executed at the *same time* (because multiple computing resources are available).
- **Work W** is the *number of operations executed*. It may be higher than the sequential version of the algorithm due to communication overhead, etc.
- **Span S** is the *longest chain of dependencies* (i.e., the critical path) that determines the *minimum time required to execute the algorithm*. This is a *lower bound* on the running time, regardless of the number of processors. The range indicates the ability of an algorithm to get better performance on more processors.

How do we calculate the Span metric?

1. As we just said, we **represent a parallel algorithm as a DAG** graph, where nodes represent tasks and edges represent dependencies between tasks;
2. We **assign weights to each node** that represent the **time required to perform the corresponding task**;
3. We try to **find the Critical Path**. In other words, we determine the path from the start node to the end node that has the *maximum cumulative weight*;
4. Finally, the **sum of the weights of the nodes on the critical path** gives us the span value!

⁴A **Directed Acyclic Graph (DAG)** is a directed graph, i.e. with oriented edges, without cycles.

- **Parallelism P** is the *measure of efficiency in the use of resources*.

Trivially, it is the number of operations performed divided by the longest chain of dependencies:

$$P = \frac{W}{S} \quad (11)$$

It indicates **how many processors can be effectively used by the computation**. If the work is equal to the span, the parallelism is 1 and the computation is sequential. Ideally, but not necessarily, we win with polylogarithmic span, because if the work is $O(n \log n)$ and the span is $O(\log^2 n)$, then the parallelism is $O\left(\frac{n}{\log n}\right)$, which is actually quite high (and unlikely to be a bottleneck on most machines in the next 25 years). [2]

This measure is *one of the most important*. It indicates the **number of processors that are not idle**. It is obvious that a **good parallel algorithm is designed to have the lowest possible work** (less operation, then less resource usage, then less cost, and so on) **and the highest possible parallelism** (achievable by reducing the span, and this should be trivial, since the metric P is given by work divided by the span, so reducing the denominator, you can get a higher value).

As in all things, there is a **trade-off** between the lowest possible “work” and the highest possible “parallelism”. Reducing the work too much could eliminate the possibility of parallelizing our algorithm, and on the other hand, reducing the span too aggressively could cause communication/synchronization overhead.

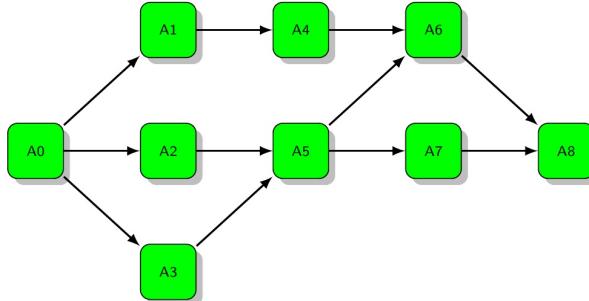


Figure 12: Example of DAG implementation with work equal to 9, span equal to 5, and parallelism equal to 1.8 ($9 \div 5$). The span calculus is not well known, has been calculated *a priori*.

Finally, we use a mathematical annotation and not only a graphical (DAG) annotation. The **Composition Rules** help determine how to combine smaller parallel tasks into a larger algorithm, while analyzing the Work and Span of the combined algorithm:

- **Single operation.** An operation takes 1 unit of work and 1 unit of span time.

$$W(op) = 1 \quad S(op) = 1$$

- **Sequential Composition.**

- The total work of executing e_1 and e_2 **sequentially** is the sum of their individual work.

$$W(e_1, e_2) = W(e_1) + W(e_2)$$

- The total span of executing e_1 and e_2 **sequentially** is the sum of their individual spans.

$$S(e_1, e_2) = S(e_1) + S(e_2)$$

- **Parallel Composition.**

- The total work of executing e_1 and e_2 in **parallel** is still the sum of their individual works.

$$W(e_1 || e_2) = W(e_1) + W(e_2)$$

- The total span of executing e_1 and e_2 in **parallel** is the **maximum of their individual spans**, since they can be executed simultaneously.

$$S(e_1 || e_2) = \max(W(e_1), W(e_2))$$

4.3 Technologies

Some famous architecture to work with parallel programming:

- Verilog/VHDL are *hardware description languages*. The target architectures are *ASIC* and *FPGA*. The **parallelism** and the **communication** are **explicit**.

✓ Pros

- Complete control on computation and memory
- No overhead introduced in the computation
- Provides access to potentially large computational power

⚠ Cons

- Requires specific hardware (e.g., ASIC or FPGA) to implement functionality
- Difficult to learn: completely different programming language and programming paradigm
- Depends on the chosen target architecture

- MPI is a *library*. The target architectures are *Multi CPUs*. The **parallelism** is **implicit** and the **communication** is **explicit**.

✓ Pros

- Can be adopted on different types of architecture
- Scalable solutions
- Synchronization and data communication are explicitly managed

⚠ Cons

- Communication can introduce significant overhead
- Programming paradigm more difficult than shared memory-based ones
- Standard does not reflect immediately advances in architecture characteristics

- PThread is a *library*. The target architectures are *Multi-core CPUs*. The **parallelism** is **explicit** and the **communication** is **implicit**.

✓ Pros

- Can be adopted on different types of architecture
- Explicit parallelism and full control over application

⚠ Cons

- Task management overhead can be significant
- Not easily scalable solutions
- Low level API
- **OpenMP** is a *C/Fortran extensions*. The target architectures are *Multi-core CPUs*. The **parallelism** is **explicit** and the **communication** is **implicit**.

✓ Pros

- Easy to learn
- Scalable solution
- Parallel applications can also be executed sequentially

⚠ Cons

- Mainly focused on shared memory homogeneous systems
- Requires small interaction between tasks

- **CUDA** is a *C extensions*. The target architectures are *CPU plus GPU(s)*. The **parallelism** is **implicit/explicit** and the **communication** is **implicit/explicit**.

✓ Pros

- Provides access to the computational power of GPUs
- Writing a CUDA kernel is quite easy
- Already optimized libraries

⚠ Cons

- Targets only NVIDIA GPUs
- Difficult to extract massive parallelism from application
- Difficult to optimize CUDA kernel

- **OpenCL** is a *C/C++ extensions and API*. The target architectures are *heterogeneous architecture*. The **parallelism** is **implicit/explicit** and the **communication** is **implicit/explicit**.

✓ Pros

- Target-independent standard
- Hides architecture details
- Same programming infrastructure for every heterogeneous architecture: CPU + GPU (and FPGA)

⚠ Cons

- Difficult programming paradigm for its heterogeneity
- Hiding of architecture details makes difficult to obtain best performances
- Gradually abandoned
- **Apache Spark** is an *API*. The target architectures are *multi CPUs*. The **parallelism is implicit** and the **communication is implicit**.

✓ Pros

- API for different languages
- Explicit parallelization and communication are not required
- Preinstalled on cloud provide VMs

⚠ Cons

- Suitable only for big data applications
- Does not (yet) fully support GPUs

Regardless of these technologies, it is quite common to mix some of them:

- **OpenMP + CUDA**: allows to exploit multi-core CPU and GPU. CUDA is used to parallelize GPU code and OpenMP is used to parallelize CPU code.
- **MPI + OpenMP**: the most common scenario are:
 1. MPI used to express coarser parallelism (multi CPU) and OpenMP used to express finer parallelism (multi core).
 2. MPI used to implement communication and OpenMP used to parallelize computation.
- **OpenCL + Verilog or VHDL**: in principle, hardware kernels (implemented for example on FPGA) can be used as accelerators; OpenMP used to describe parallelism among different processing elements; Verilog/VHDL used to describe hardware kernel. An example of target: Intel Xeon Scalable.

4.4 Threads

4.4.1 Flynn's taxonomy

Flynn's taxonomy is a **classification of computer architectures**, proposed by Michael J. Flynn. The classification system has been used as a tool in the design of modern processors and their functionalities. Since the rise of multiprocessing central processing units (CPUs), a multiprogramming context has evolved as an extension of the classification system.

The four initial classifications defined by Flynn are based upon the number of concurrent instruction (or control) streams and data streams available in the architecture:

- **Single Instruction stream, Single Data stream (SISD)**
- **Single Instruction stream, Multiple Data streams (SIMD)**
- **Multiple Instruction stream, Single Data stream (MISD)**
- **Multiple Instruction stream, Multiple Data stream (MISD)**

It is important to quote it because it is the basis for the development of many advanced technologies.

4.4.2 Definition

A UNIX process can be created by the operating system and contains information about program resources and program execution status.

Definition 3: Thread

A **thread** is an **independent stream of instructions within a process**. Threads can be scheduled by the operating system, and each thread can run concurrently with other threads. A thread also has local resources and can access the shared process resources.

In other words, a thread can be thought of as any **procedure that runs independently of its main program**. We can create each thread dynamically during execution. A good point is that a multi-threaded program is lighter than a multi-process program.

When a thread exists within a process, it shares most of the process resources, for example:

- Changes made by one thread to shared system resources (such as closing a file) will be seen by all other threads.
- Two pointers having the same value point to the same data.
- **Implicit communication** by reading and writing shared variables.
- **Reading and writing to the same memory locations requires explicit synchronization by the programmer.** If this rule is not followed, the code may suffer from a data race or race condition ⁵ problem.

The most common models for threaded programs are the manager / worker model⁶ and pipeline.

This chapter introduces the POSIX threads model.

Definition 4: pthreads

POSIX Threads, commonly known as **pthreads**, is an **execution model** that exists independently from a programming language, as well as a parallel execution model. It **allows a program to control multiple different flows of work that overlap in time**. Each flow of work is referred to as a thread, and creation and control over these flows is achieved by making calls to the POSIX Threads API.

POSIX threads and OpenMP are two **implementations of a shared memory parallel programming model using threads**. The programmer is responsible for handling parallelism and synchronization, usually through a library of subroutines or a set of compiler directives. Typically, hardware vendors have implemented their own proprietary versions of threads, but in this course we will look at POSIX threads (pthreads) and OpenMP.

⁵In parallel computing, a **Data Race** or **Race Condition** is a software problem that occurs when two threads (or processes) access the same variables, and at least one does a write. They can finish in a different order than expected.

⁶The manager/worker pattern is described as follows. The idea is that the work that needs to be done can be divided by a “manager” into separate pieces and the pieces can be assigned to individual “worker” processes. Thus the manager executes a different algorithm from that of the workers, but all of the workers execute the same algorithm. Most implementations of MPI allow MPI processes to be running different programs (executable files), but it is often convenient (and in some cases required) to combine the manager and worker code into a single program.

4.4.3 pthreads API

In 1995, the IEEE POSIX 1003.1c standard specified the API for explicitly managing threads. An **API** is a set of C language programming types and procedure calls.

- Header file to include in the main file: `pthread.h`.
- To **compile** and use it, it is necessary to add the flag `-pthread` to the `gcc` (or `g++`) options.

The API are divided by what we want to do. In general, there are two sets: thread management and thread synchronization.

- Thread Management
 - Creation (page 50)
 - Termination (page 51)
 - Joining (page 52)
 - Detaching (page 53)
 - Joining through Barriers (page 54)
- Thread Synchronization
 - Mutexes (page 55)
 - Condition variables (page 55)

4.4.3.1 Creation

Once threads are created, they are peers, and may create other threads. There is no implied hierarchy or dependency between threads. The maximum number of threads depends on the implementation.

[Doc. ▾](#)

pthread API: `pthread_create`

```

1 int pthread_create(
2     pthread_t * thread,
3     const pthread_attr_t * attr,
4     void * (* start_routine) (void *),
5     void * arg
6 )

```

- **Return value:** on success, `pthread_create()` returns 0; on error, it returns an error number, and the contents of `*thread` are undefined.
- **Arguments:**
 - `thread`: identifier for the new thread returned by the subroutine.
 - `attr`: used to set thread attributes, such as joinable, detached, scheduling and stack size.

- `start_routine`: the C routine that the thread will execute once it is created.
- `arg`: argument passed to `start_routine`. It must be passed by address as a pointer cast of type `void`.

4.4.3.2 Termination

The thread returns from its startup routine when its “life” ends. The thread makes a call to the `pthread_exit` subroutine.

[Doc.](#) 

pthread API: `pthread_exit`

```
1 void pthread_exit(void *retval)
```

- **Return value:** this function does not return to the caller.
- **Arguments:**
 - `retval`: function terminates the calling thread and returns a value via `retval`.

The thread is canceled by another thread via the `pthread_cancel` routine.

[Doc.](#) 

pthread API: `pthread_cancel`

```
1 int pthread_cancel(pthread_t thread)
```

- **Return value:** on success, `pthread_cancel()` returns 0; on error, it returns a nonzero error number.
- **Arguments:**
 - `thread`: the `pthread_cancel()` function sends a cancellation request to the thread `thread`.

4.4.3.3 Joining

The join function **blocks the calling thread until the specified thread exits.**

pthread API: pthread_join

```
1 int pthread_join(pthread_t thread, void **retval)
```

- **Return value:** on success, `pthread_join()` returns 0; on error, it returns an error number.

- **Arguments:**

- `thread`: the `pthread_join()` function waits for the thread specified by `thread` to terminate. If that thread has already terminated, then `pthread_join()` returns immediately. The thread specified by `thread` must be joinable.

If multiple threads simultaneously try to join with the same thread, the results are undefined. If the thread calling `pthread_join()` is canceled, then the target thread will remain joinable (i.e., it will not be detached).

- `retval`: if `retval` is not NULL, then `pthread_join()` copies the exit status of the target thread (i.e., the value that the target thread supplied to `pthread_exit()`) into the location pointed to by `retval`. If the target thread was canceled, then `PTHREAD_CANCELED` is placed in the location pointed to by `retval`.

4.4.3.4 Detaching

The detach function **marks a thread as detached**. When a thread is detached, its **resources are automatically released back to the system when the thread terminates**, without the need for another thread to join with it.

💡 Why would I need to detach a thread and not join it?

Good question. The answer depends on what we have to do.

- **Fire and forget tasks.** When we start a thread to perform a task that doesn't require further interaction or result processing, releasing it ensures that the resources are automatically cleaned up when the task is complete.
- **Resource management.** Detaching avoids the need for another thread to call `pthread_join()`, which can save system resources and reduce the complexity of our code. It's especially useful in a highly concurrent application with many short-lived threads.
- **Avoid deadlocks.** When we have potential circular dependencies or complex synchronization between threads, detaching threads can help avoid deadlocks by eliminating the need for one thread to wait on another.
- **Long-running background tasks.** For tasks that should run independently in the background and not block the main program flow, detaching makes sense. We make sure they clean up after themselves without having to explicitly manage their lifecycle.

[Doc.](#) 

pthread API: `pthread_detach`

```
1 int pthread_detach(pthread_t thread)
```

- **Return value:** on success, `pthread_detach()` returns 0; on error, it returns an error number.
 - **Arguments:**
 - `thread`: the `pthread_detach()` function marks the thread identified by `thread` as detached. When a detached thread terminates, its resources are automatically released back to the system without the need for another thread to join with the terminated thread.
- Attempting to detach an already detached thread results in unspecified behavior.

4.4.3.5 Joining through Barriers

The *barrier init* function initializes a **barrier object**, and the *barrier wait* function blocks a **thread** until the specified **number of threads** have called it. A **barrier object** is, in parallel computing, a synchronization tool that ensures that multiple threads reach a certain point of execution before any of them continue. It's like a **checkpoint that everyone must reach before continuing**, ensuring coordinated progress in a parallel algorithm.⁷

[Doc.](#) 

pthread API: pthread_barrier_init

```
1 int pthread_barrier_init(
2     pthread_barrier_t * barrier,
3     pthread_barrierattr_t * attr,
4     unsigned int count
5 )
```

pthread API: pthread_barrier_wait

```
1 int pthread_barrier_wait(pthread_barrier_t * barrier)
```

- **Return value:** on success, function return 0; on error, they return an error number.
- **Arguments:** the main and most important argument is **count**, which specifies the **number of threads to wait for**.

⁷For example, imagine multiple threads working on different parts of a matrix. A barrier can ensure that all threads finish their part of the computation before moving on to the next phase, such as combining results or performing subsequent operations.

4.4.3.6 Mutexes

Mutex variables are the basic **method of protecting shared data when multiple writes occur**. Only **one thread can lock a mutex variable at a time**. If multiple threads attempt to lock a mutex, only one thread will succeed. **Threads that fail to acquire the mutex are blocked**. There is also the **trylock** function, which returns immediately if the mutex is currently locked (by any thread, including the current thread). Note that a **lock function has the potential to create a deadlock situation**.

There are also **three types of mutex** that can be set using the **settype** function:

- **Normal Mutex (PTHREAD_MUTEX_NORMAL)**. A **normal mutex does not check for errors such as deadlock**. If a thread tries to lock a mutex it already owns, the thread will deadlock.
- **Error Check Mutex (PTHREAD_MUTEX_ERRORCHECK)**. Provides **error checking**. If a thread tries to lock a mutex it already owns, lock function will return an error instead of deadlocking.
- **Recursive Mutex (PTHREAD_MUTEX_RECURSIVE)**. Allows the same thread to lock the mutex multiple times without deadlocking. Each lock must have a corresponding unlock.

Doc. 

pthread API: pthread_mutex_lock

```
1 int pthread_mutex_lock(pthread_mutex_t *mutex)
```

pthread API: pthread_mutex_trylock

```
1 int pthread_mutex_trylock(pthread_mutex_t *mutex)
```

pthread API: pthread_mutex_unlock

```
1 int pthread_mutex_unlock(pthread_mutex_t *mutex)
```

4.4.3.7 Condition variables

Mutexes implement synchronization by serializing data accesses. Condition variables allow threads to synchronize explicitly by signaling the meeting of a condition. Without condition variables, the programmer would need to poll to check if the condition is met.

5 OpenMP v5.2

5.1 Introduction

OpenMP is a scalable model that gives parallel programmers a simple and flexible interface for developing portable parallel applications in C/C++ and Fortran.

- **Header file** to include in the main file: `omp.h`.
- To **compile** and use it, it is necessary to add the **flag** `-fopenmp` to the `gcc` (or `c++`) options.

The following link contains about 200 slides on OpenMP. It is an introduction, but it covers every argument required in this PoliMI course. The slides are made by a Senior Principal Engineer, Mattson Tim. He's a Senior Principal Engineer at Intel, where he's been since 1993. The [slide link](#).

✓ Benefits

- Standard across a variety of shared memory architectures and platforms.
- Supports three famous languages: Fortran, C and C++.
- Scalable from embedded systems to the supercomputer.
- The directives are intuitive, and with a limited set of directives we can implement parallel algorithms.
- Incremental parallelization of serial program.
- Coarse-grained and fine-grained parallelism. See [here](#) here an interesting difference between coarse-grained and fine-grained architecture:



❓ How it works?

OpenMP is based on the **fork-join paradigm**. A “master” thread forks a specified number of “slave” threads. Tasks are divided among the “slaves”, and each “slave” runs concurrently as the runtime allocates threads to different processors.

1. **Thread #0 born.** OpenMP programs start with a single thread;
2. **Fork.** At the start of a parallel region, the *master* creates a team of parallel worker threads (*slaves*).
3. **OpenMP code block.** Statements in the parallel block are executed in parallel by each thread.
4. **Join.** At the end of the parallel region, all threads synchronize (implicit barrier, see definition on page 54) and join the master thread. [5]

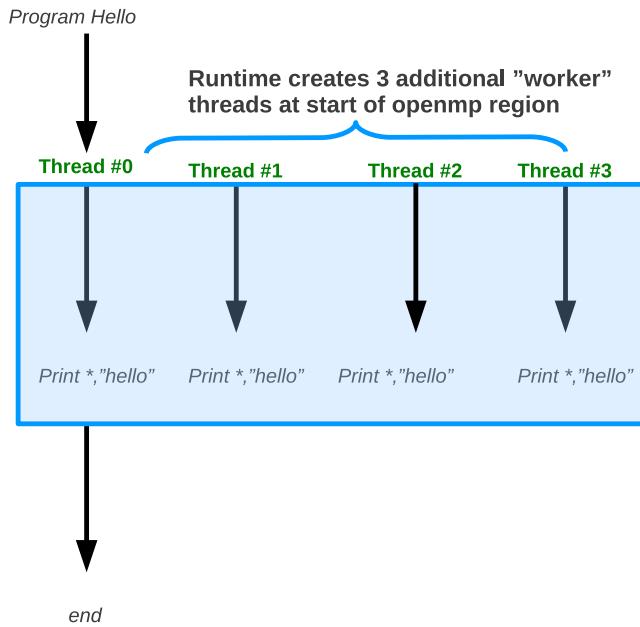


Figure 13: Example of an OpenMP program called `Hello`. At runtime, the *master* thread forks 3 additional *slave* threads to print `hello`. The example is very trivial, but here is a graphical representation of the workflow. An interesting thing to note is that **OpenMP creates a sort of region where each thread executes all the instructions in the OpenMP block**. This is important to understand. [5]

5.2 Basic syntax

We can manage OpenMP work flow using the directive syntax. We remember that the reference guide of OpenMP is available on their website:

[Reference Guide](#)



A **directive** is a combination of the base-language mechanism and a *directive-specification* (the *directive-name* followed by *optional clauses*). A construct consists of a directive and, often, additional base language code. In C++ directives are formed from either pragmas or attributes.

OpenMP: pragma omp

```
1 #pragma omp directive-specification
```

The **number of OpenMP threads** can be set using:

- At compilation time: using the environment variable `OMP_NUM_THREADS`
- At runtime: using the function

[Doc.](#)

OpenMP: omp_set_num_threads

```
1 void omp_set_num_threads(int num_threads)
```

Other useful function to get information about threads:

- The **number of threads in the current team**:

[Doc.](#)

OpenMP: omp_get_num_threads

```
1 int omp_get_num_threads()
```

The binding region for an `omp_get_num_threads` region is the innermost enclosing `parallel` region. If called from the sequential part of a program, this routine returns 1.

- The **upper bound on the number of threads** that could be used to form a new team if a `parallel` construct without a `num_threads` clause were encountered after execution returns from this routine.

[Doc. ▾](#)

OpenMP: `omp_get_max_threads`

```
1 int omp_get_max_threads()
```

- The **thread number of the calling thread**, within the current team.

[Doc. ▾](#)

OpenMP: `omp_get_thread_num`

```
1 int omp_get_thread_num()
```

- The **elapsed wall clock time in seconds**.

[Doc. ▾](#)

OpenMP: `omp_get_wtime`

```
1 double omp_get_wtime()
```

- The **precision of the timer (seconds between ticks)** used by `omp_get_wtick`.

[Doc. ▾](#)

OpenMP: `omp_get_wtick`

```
1 double omp_get_wtick()
```

OpenMP programs execute serially until they reach a `parallel` directive. As we have explained at page 57, the thread that was executing the code spawns a group of “slave” threads and becomes the “master” (thread ID 0). The code in the structured block is replicated, each thread executes a copy. At the end of the block there is an implied barrier, only the “master” thread continues.

OpenMP: `pragma omp parallel`

```
1 #pragma omp parallel optional-clauses
```

The parallel directive has **optional** clauses, the most commonly used are:

- Specify the **number of threads to spawn**:

```
1 #pragma omp parallel num_threads(int)
```

- Conditional parallelization with:

```
1 #pragma omp parallel if (condition)
```

Example 1: parallel if condition

```

1 #include <stdio.h>
2 #include <omp.h>
3
4 void test(int val)
5 {
6     #pragma omp parallel if (val != 0)
7     if (omp_in_parallel()) {
8         #pragma omp single
9         printf_s(
10             "val = %d, parallelized with %d threads\n",
11             val, omp_get_num_threads()
12         );
13     } else {
14         printf_s("val = %d, serialized\n", val);
15     }
16 }
17
18 int main( )
19 {
20     omp_set_num_threads(2);
21     test(0);
22     test(2);
23 }
```

The output will be:

```
1 val = 0, serialized
2 val = 2, parallelized with 2 threads
```

- Data scope clauses (explained in the following pages).

The **number of threads in a parallel region is determined by the following factors**, in order of priority (high to low):

1. Evaluation of the `if` clause;
2. Value of the `num_threads` clause;
3. Use of the `omp_set_num_threads()` library function;
4. Setting of the `OMP_NUM_THREADS` environment variable;
5. Implementation default, e.g., the number of CPUs on a node.

5.3 Work sharing

Work-sharing constructs divide the execution of a region of code among the team members who encounter it. A work-sharing construct must be enclosed in a parallel region for the directive to be executed in parallel. Note that the constructs do not start new threads. Also, there is no implicit barrier at the *entry* of a work-sharing construct, but there is an implicit barrier at the *exit* of a work-sharing construct.

5.3.1 For

The for directive **shares iterations of a loop across the team** (data parallelism).

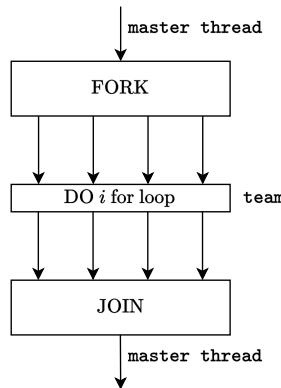


Figure 14: OpenMP for loop.

OpenMP: pragma omp for

```

1 #pragma omp parallel
2 {
3     #pragma omp for
4     /* for loop */
5 }

```

The for directive parallelize execution of iterations. The number of iteration cannot be internally modified. Some common clauses are:

- *schedule* that describes **how iterations of the loop are distributed among the threads in the team**. The schedule type can be either **dynamic**, **guided**, **runtime**, or **static**.
 - **static**. Loop iterations are divided into blocks of size **chunk** and then statically allocated to threads. If **chunk** is not specified, the iterations are divided evenly (if possible) among the threads.

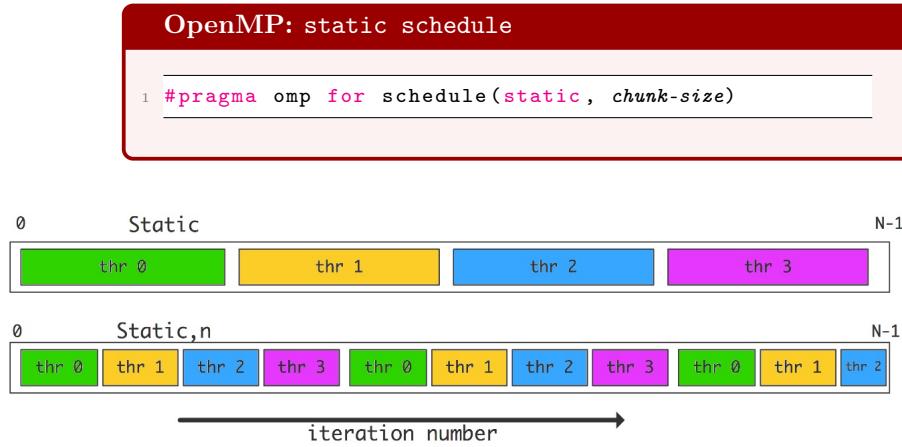


Figure 15: static schedule.

- **dynamic**. Loop iterations are divided into blocks of size `chunk-size` and distributed among the threads *at runtime*; when a thread completes one chunk, it is dynamically allocated another. The default chunk size is 1. In fact, we can see in the image that the order is not always the same.

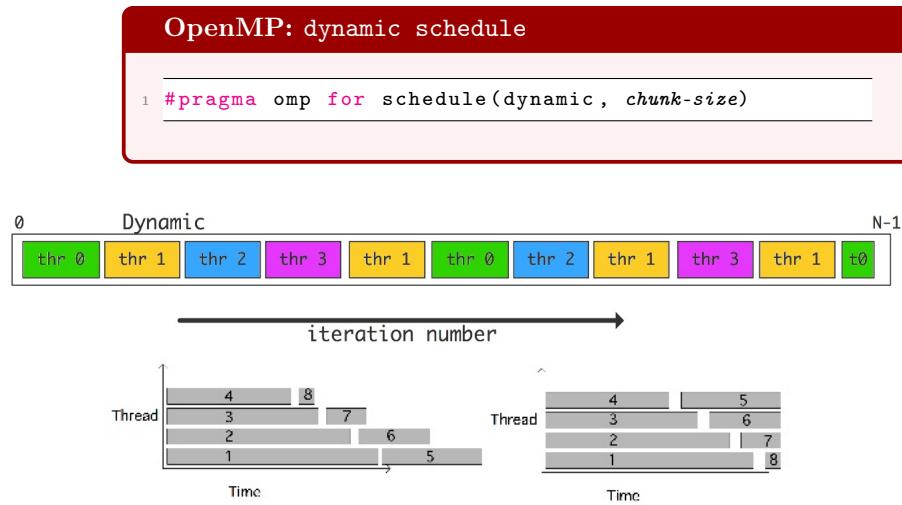
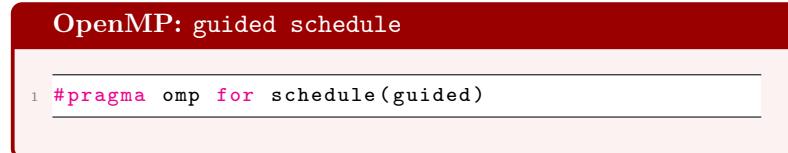
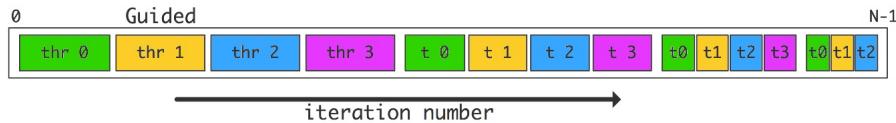


Figure 16: dynamic schedule.

- **runtime**. Depends on environment variable `OMP_SCHEDULE`.
- **guided**. Static, gradually decreases the chunk size (`chunk` specifies the smallest one).



Figure 17: `guided` schedule.

Example 2: schedule types

```

1 #include <stdio.h>
2 #include <omp.h>
3
4 #define NUM_THREADS 4
5 #define STATIC_CHUNK 5
6 #define DYNAMIC_CHUNK 5
7 #define NUM_LOOPS 20
8 #define SLEEP_EVERY_N 3
9
10 int main( )
11 {
12     int nStatic1[NUM_LOOPS],
13         nStaticN[NUM_LOOPS];
14     int nDynamic1[NUM_LOOPS],
15         nDynamicN[NUM_LOOPS];
16     int nGuided[NUM_LOOPS];
17
18     omp_set_num_threads(NUM_THREADS);
19
20     #pragma omp parallel
21     {
22         #pragma omp for schedule(static, 1)
23         for (int i = 0 ; i < NUM_LOOPS ; ++i)
24         {
25             if ((i % SLEEP_EVERY_N) == 0)
26                 Sleep(0);
27             nStatic1[i] = omp_get_thread_num( );
28         }
29
30         #pragma omp for schedule(static, STATIC_CHUNK)
31         for (int i = 0 ; i < NUM_LOOPS ; ++i)
32         {
33             if ((i % SLEEP_EVERY_N) == 0)
34                 Sleep(0);
35             nStaticN[i] = omp_get_thread_num( );
36         }
37
38         #pragma omp for schedule(dynamic, 1)
39         for (int i = 0 ; i < NUM_LOOPS ; ++i)
40         {
41             if ((i % SLEEP_EVERY_N) == 0)
42                 Sleep(0);
43             nDynamic1[i] = omp_get_thread_num( );
44         }
45
46         #pragma omp for schedule(dynamic, DYNAMIC_CHUNK)
47         for (int i = 0 ; i < NUM_LOOPS ; ++i)
48         {
49             if ((i % SLEEP_EVERY_N) == 0)
50                 Sleep(0);
51             nGuided[i] = omp_get_thread_num( );
52         }
53     }
54 }
```

```

50             Sleep(0);
51             nDynamicN[i] = omp_get_thread_num();
52         }
53
54 #pragma omp for schedule(guided)
55 for (int i = 0 ; i < NUM_LOOPS ; ++i)
56 {
57     if ((i % SLEEP_EVERY_N) == 0)
58         Sleep(0);
59     nGuided[i] = omp_get_thread_num();
60 }
61
62 printf_s(
63 "-----\n");
64 printf_s("| static | static | dynamic | dynamic | guided |\n");
65 printf_s("|      1      |      %d      |      1      |      %d      |
66           |\n",
67           STATIC_CHUNK, DYNAMIC_CHUNK);
68 printf_s("|
69           -----|\n");
70
71 for (int i=0; i<NUM_LOOPS; ++i)
72 {
73     printf_s("|      %d      |      %d      |      %d      |      %d      |
74           %d |\n",
75           nStatic1[i], nStaticN[i],
76           nDynamic1[i], nDynamicN[i], nGuided[
77           i]);
78 }

```

The result will be:

	static	static	dynamic	dynamic	guided	
	1	5	1	5		
5	0	0	0	2	1	
6	1	0	3	2	1	
7	2	0	3	2	1	
8	3	0	3	2	1	
9	0	0	2	2	1	
10	1	1	2	3	3	
11	2	1	2	3	3	
12	3	1	0	3	3	
13	0	1	0	3	3	
14	1	1	0	3	2	
15	2	2	1	0	2	
16	3	2	1	0	2	
17	0	2	1	0	3	
18	1	2	2	0	3	
19	2	2	2	0	0	

```

20 |   3   |   3   |   2   |   1   |   0   |
21 |   0   |   3   |   3   |   1   |   1   |
22 |   1   |   3   |   3   |   1   |   1   |
23 |   2   |   3   |   3   |   1   |   1   |
24 |   3   |   3   |   0   |   1   |   3   |
25 -----

```

- **`nowait`** to avoid synchronization at the end of the parallel loop. It overrides the barrier implicit in a directive.

```
1 #pragma omp for nowait
```

Example 3: nowait clause

```

1 #include <stdio.h>
2
3 #define SIZE 5
4
5 void test(int *a, int *b, int *c, int size)
6 {
7     int i;
8     #pragma omp parallel
9     {
10         #pragma omp for nowait
11         for (i = 0; i < size; i++)
12             b[i] = a[i] * a[i];
13
14         #pragma omp for nowait
15         for (i = 0; i < size; i++)
16             c[i] = a[i]/2;
17     }
18 }
19
20 int main( )
21 {
22     int a[SIZE], b[SIZE], c[SIZE];
23     int i;
24
25     for (i=0; i<SIZE; i++)
26         a[i] = i;
27
28     test(a,b,c, SIZE);
29
30     for (i=0; i<SIZE; i++)
31         printf_s("%d, %d, %d\n", a[i], b[i], c[i]);
32 }
```

The output will be:

```

1 0, 0, 0
2 1, 1, 0
3 2, 4, 1
4 3, 9, 1
5 4, 16, 2

```

5.3.1.1 Reduction

In parallel programming, sometimes there are some exceptional cases when we use the `for` statement where the variables inside the code block are not so easy to manage (memory viewpoint). For example, consider the following case:

```

1 #include "stdio.h"
2 #include "omp.h"
3 #define MAX 10
4
5
6 int main(int argc, char const *argv[])
7 {
8     double ave = 0.0;
9     double A[MAX] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};
10    int i;
11    #pragma omp parallel for
12    for(i = 0; i < MAX; i++) {
13        ave += A[i];
14    }
15    ave = ave / MAX;
16    printf("Value %f\n", ave);
17    return 0;
18 }
```

⚠ Too many threads modifying the same variable

In this case, we are combining values into a single accumulation variable (called `ave`). There is a true dependence between loop iterations that can't be trivially removed. A **continuous execution produces different results!**

```

1 $ ./example.out
2 Value 1.300000
3 $ ./example.out
4 Value 2.400000
5 $ ./example.out
6 Value 2.500000
7 $ ./example.out
8 Value 2.100000
9 $ ./example.out
10 Value 1.900000
```

This is a very common situation and a solution, which can also be used as a **synchronization technique**, is called a **reduction**.

A reduction variable in a loop **aggregates** (i.e., accumulates) a **value** that depends on each iteration of the loop and doesn't depend on the iteration order.

OpenMP: reduction

```

1 #pragma omp parallel for reduction(operator: list)
```

A reduction clause:

- It makes a local copy of each *list* variable and initialized depending on the *operator*;

- It updates occur on the local copy;
- Local copies are reduced into a single value and combined with the original global value.

Therefore, the variables in *list* must be shared in the enclosing parallel region.

Many different associative operands (*operator* value) can be used with reduction:

- + with initial value 0
- * with initial value 1
- - with initial value 0
- min with initial value as largest positive number
- max with initial value as most negative number
- & with initial value ~ 0
- | with initial value 0
- ^ with initial value 0
- && with initial value 1
- || with initial value 0

Using the **reduction** clause, the code written at the beginning of the paragraph can be fixed as follows:

```

1 #include "stdio.h"
2 #include "omp.h"
3 #define MAX 10
4
5
6 int main(int argc, char const *argv[])
7 {
8     double ave = 0.0;
9     double A[MAX] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};
10    int i;
11    // use reduction
12    #pragma omp parallel for reduction(+: ave)
13    for(i = 0; i < MAX; i++) {
14        ave += A[i];
15    }
16    ave = ave / MAX;
17    // now it prints the correct result 5.5
18    printf("Value %f\n", ave);
19    return 0;
20 }
```

✓ Avoid race condition

5.3.2 Sections

Section identifies code sections to be divided among all threads.

Sections allow to specify that the enclosed section(s) of code are to be executed in parallel. **Each section is executed once by a thread in the team.**

[Doc.](#) [☰](#)

OpenMP: sections

```

1 #pragma omp [parallel] sections [clauses]
2 {
3     #pragma omp section
4     {
5         code_block
6     }
7 }
```

The sections directive identifies a noniterative work-sharing construct that specifies a set of constructs that are to be divided among threads in a team. Each section is executed once by a thread in the team.

Each section is preceded by a **section** directive, although the **section** directive is optional for the first section. The **section** directives must appear within the lexical extent of the **sections** directive. There's an **implicit barrier** at the end of a **sections** construct, unless a **nowait** is specified.

Restrictions to the sections directive are as follows:

- A **section** directive must not appear outside the lexical extent of the **sections** directive.
- Only a single **nowait** clause can appear on a **sections** directive.

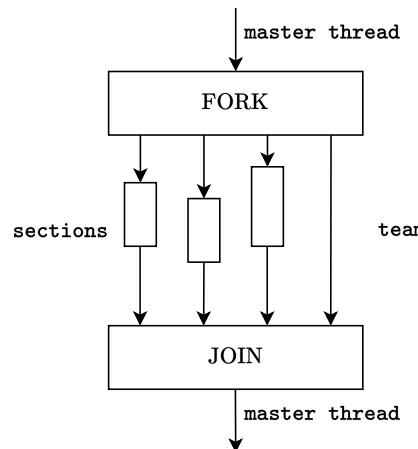


Figure 18: Breaks work into separate, discrete sections, each executed by a thread (functional parallelism).

5.3.3 Single/Master

A section (not the directive) of code should be executed on a single thread, not necessarily the main (master) thread. The `single` directive identifies a construct that specifies that the associated structured block is executed by only one thread in the team (not necessarily the master thread).

OpenMP: single and master

```

1 #pragma omp parallel
2 {
3     #pragma omp single
4     {
5         /* code section */
6     }
7     # pragma omp master
8     {
9         /* code section */
10    }
11 }
```

- `single` specifies that a section of a code is **executed only by a single thread**.
- `master` specifies that a section of a code is **executed only by the master**.

[Doc.](#) 

[Doc.](#) 

There's an **implicit barrier** after the `single` construct unless a `nowait` clause is specified.

5.3.4 Tasks

The following section has been enhanced with slides from Senior Principal Engineer Mattson Tim. He's a senior principal engineer at Intel, where he's been since 1993. His profile can be seen [here](#) and the slides are available online [here](#). He has also made an interesting [YouTube series](#) on the introduction to OpenMP.

Tasks are **independent units of work**. They consist of: *code to execute*, *data environment*, and *internal control variables* (ICV). **Threads perform the work of each task**. The runtime **system decides when to execute tasks**; each task can be deferred or executed immediately.

In other words, an OpenMP **task** is a block of code contained in a parallel region that can be executed simultaneously with other tasks in the same region.

Some useful terminology:

- **Task construct.** It identifies the **task directive plus the structured block**.
- **Task.** It is the **package of code and instructions for allocating data** created when a **thread encounters a task construct**.
- **Task region.** It is the dynamic sequence of **instructions generated by the execution of a task by a thread**.

Tasks are guaranteed to complete at thread barriers (using the **barrier** directive) or at task barriers (using the **taskwait** directive):

```

1 #pragma omp parallel // omp directive to parallel the code
2 {
3     #pragma omp task // multiple foo tasks created here,
4             // one for each thread
5     foo();
6     #pragma omp barrier // all foo tasks guaranteed
7             // to be completed here
8     #pragma omp single // only one thread can access to
9             // this piece of code
10    {
11        #pragma omp task // one bar task created here
12        bar();
13    }
14    // foo task guaranteed to be completed here
15 }
```

Example 4: Fibonacci with tasks

Let us see a Fibonacci example of data scoping using the tasks. In the following code, we create the Fibonacci function and we create two tasks, but each task has a private variable and these variables are also used in the return statement:

```

1 int fib(int n) {
2     int x, y;
3     if(n < 2)
```

```

4     return n;
5 #pragma omp task
6 x = fib(n-1);
7 #pragma omp task
8 y = fib(n-2);
9 #pragma omp taskwait
10    return x + y;
11 }

```

A good solution is to “share” the `x` and `y` variables because we need both values to calculate the sum.

```

1 int fib(int n) {
2     int x, y;
3     if(n < 2)
4         return n;
5     #pragma omp task shared(x)
6     x = fib(n-1);
7     #pragma omp task shared(y)
8     y = fib(n-2);
9     #pragma omp taskwait
10    return x + y;
11 }

```

✓ Main advantage

Note the following code:

```

1 // create a team of threads
2 #pragma omp parallel
3 {
4     // one thread executes the single construct
5     // and other threads wait at the implied
6     // barrier at the end of the single construct
7     #pragma omp single
8     { // block 1
9         node *p = head;
10        while(p) { // block 2
11            // the single thread creates a task
12            // with its own value for the pointer p
13            #pragma omp task firstprivate(p)
14                process(p);
15            p = p -> next; // block 3
16        }
17        // execution moves beyond the barrier
18        // once all the tasks are complete
19    }
20 }

```

The tasks have the **potential to parallelize irregular patterns and recursive function calls**. See Figure 19 (page 72) to understand how the runtime system can optimize execution using the tasks.

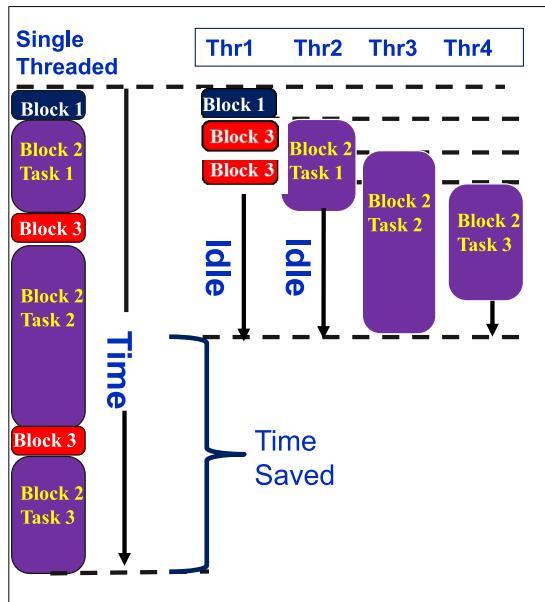


Figure 19: The main advantage of the tasks is to parallelize irregular patterns and recursive function calls.

🔗 Synchronization

When a thread encounters a **task** construct, the task is created but not immediately executed. The tasks are guaranteed to be completed:

- At a **barrier** (implicit or explicit)
- At **task synchronization points**:
 - **taskwait** construct specifies a **wait on the completion of child tasks of the current task**.

[Doc. ▾](#)

OpenMP: pragma omp taskwait

```
1 #pragma omp taskwait
```

- **taskgroup** construct specifies a **wait on the completion of child tasks of the current task** (such as **taskwait**) and their **descendent tasks** (**main difference**).

[Doc. ▾](#)

OpenMP: pragma omp taskgroup

```
1 #pragma omp taskgroup
```

5.3.4.1 Task dependences

By creating tasks instead of sections, each thread can execute any task as long as its input is ready. Since the internal scheduler decides how to manipulate the execution of tasks, there is a useful **clause to indicate a dependency between tasks**.

[Doc.](#) 

OpenMP: depend

```
1 #pragma omp task depend(dependence-type: variable)
```

The **depend** clause enforces additional constraints on the scheduling of tasks or loop iterations. These constraints establish dependencies only between sibling tasks or between loop iterations.

The *dependence-type* can be **in** or **out**.

In addition to the depend clause, we can also use the priority clause to hint that more important tasks should be executed more frequently.

The **priority** clause is a **hint for the priority of the generated task**. The priority-value is a **non-negative integer expression** that provides a hint for task execution order. Among all tasks ready to be executed, **higher priority tasks** (those with a higher numerical value in the priority clause expression) **are recommended to execute before lower priority ones**. The **default** priority-value when no priority clause is specified **is zero** (the *lowest priority*).

If a value is specified in the priority clause that is higher than the *max-task-priority-var* ICV (internal control variables) then the implementation will use the value of that ICV. A program that relies on task execution order being determined by this priority-value may have unspecified behavior.

- The `omp_get_max_task_priority` routine returns the **maximum value that can be specified in the priority clause**.

[Doc.](#) 

OpenMP: omp_get_max_task_priority

```
1 int omp_get_max_task_priority()
```

- The `OMP_MAX_TASK_PRIORITY` environment variable **controls the use of task priorities** by setting the initial value of the *max-task-priority-var* ICV. The value of this environment variable must be a non-negative integer.

[Doc.](#) 

⚠ Possible overhead

The task in general, but especially the **dependencies between tasks, can introduce overhead and reduce performance**.

Example 5: dependences

Let the following parallel code:

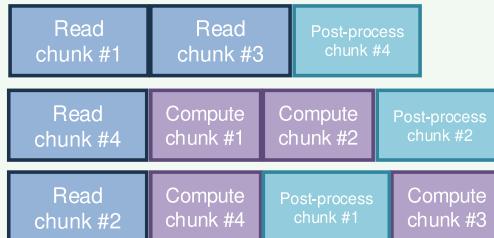
```

1 #pragma omp parallel default(none) \
2             shared(fp_read) \
3             shared(n_io_chunks) \
4             shared(n_work_chunks) \
5             shared(a, b, c) \
6             shared(status_read, status_processing) \
7             shared(status_postprocessing)
8 {
9     #pragma omp single nowait
10    {
11        for(int64_t i = 0; i < n_io_chunks; ++i) {
12            #pragma omp task depend(out: status_read[i]) \
13                           priority(20)
14            {
15                (void) read_input(
16                    fp_read, i, a, b, &status_read[i]
17                );
18            } // End of task reading in a chunk of data
19
20            #pragma omp task depend(in: status_read[i]) \
21                           depend(out: status_processing[i]) \
22                           priority(10)
23            {
24                (void) compute_results(
25                    i, n_work_chunks, a, b, c,
26                    &status_processing[i]
27                );
28            } // End of task performing the computations
29
30            #pragma omp task depend(in: status_processing[i])
31                           priority(5)
32            {
33                (void) postprocess_results(
34                    i, n_work_chunks, c,
35                    &status_postprocessing[i]
36                );
37            } // End of task postprocessing the results
38        } // End of for-loop
39    } // End of single region
40 } // End of parallel region
41

```

- Row 11. We are going to process `n_io_chunks` of data.
- Rows 12, 20. Refer to read and are compute dependence.
- Rows 21, 30. Refer to compute and are postprocess dependence.

A possible order of execution if we consider 3 threads is:



As long as the dependencies are respected, the loop iterations can be executed in any order. No explicit `flush` (page 90) are required because it is implied before and after every task.

② Custom number of iterations

Each task gets assigned a number of iterations which is the minimum between grainsize and the total number of iterations.

If a `grainsize` clause is present, the number of logical iterations assigned to each generated task is greater than or equal to the minimum of the value of the `grain-size` expression and the number of logical iterations, but less than two times the value of the `grain-size` expression.

If the `grainsize` clause has the `strict` modifier, the number of logical iterations assigned to each generated task is equal to the value of the `grain-size` expression, except for the generated task that contains the sequentially last iteration, which may have fewer iterations. The parameter of the `grainsize` clause must be a positive integer expression.

Doc. [☰](#)

OpenMP: pragma omp taskloop grainsize

```
1 #pragma omp taskloop grainsize([strict] grain-size)
```

?

Number of created tasks

To keep the number of created tasks low, the clause `num_tasks` sets the number of tasks that the runtime system can generate.

If `num_tasks` is specified, the `taskloop` construct creates as many tasks as the minimum of the `num-tasks` expression and the number of logical iterations. Each task must have at least one logical iteration. The parameter of the `num_tasks` clause must be a positive integer expression.

If the `num_tasks` clause has the `strict` modifier for a task loop with N logical iterations, the logical iterations are partitioned in a balanced manner and each partition is assigned, in order, to a generated task. The partition size is $\left\lceil \left\lceil \frac{N}{\text{num-tasks}} \right\rceil \right\rceil$ until the number of remaining iterations divides the number of

remaining tasks evenly, at which point the partition size becomes $\left\lceil \left\lceil \frac{N}{\text{num-tasks}} \right\rceil \right\rceil$.

[Doc.](#) 

OpenMP: pragma omp taskloop num_tasks

```
1 #pragma omp taskloop num_tasks([strict] num-tasks)
```

If neither a `grainsize` nor `num_tasks` clause is present, the number of loop tasks generated and the number of logical iterations assigned to these tasks is implementation defined.

5.4 Synchronization

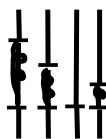
The following section has been enhanced with slides from Senior Principal Engineer Mattson Tim. He's a senior principal engineer at Intel, where he's been since 1993. His profile can be seen [here](#) and the slides are available online [here](#). He has also made an interesting [YouTube series](#) on the introduction to OpenMP.

OpenMP is a multi-threaded, shared address model. This means that threads communicate by sharing variables. Unfortunately, this can cause some problems such as race conditions (page 49). The solution is to **use synchronization to protect against data conflicts**. The good news is that synchronization can avoid data race problems, but it is also an **expensive method**. So we can use synchronization, but we **need to change the way data is accessed to minimize the need for synchronization**.

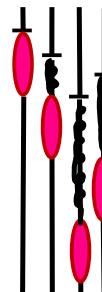
Synchronization brings **one or more threads to a well-defined and known point in their execution**. The two most common forms of synchronization are:

- **Barrier**: each thread wait at the barrier **until all threads arrive**.
- **Mutual exclusion**: define a block of code that **only one thread at a time can execute**.

The OpenMP directives are: `critical`, `atomic`, and `barrier`. These are high-level synchronization directives, but there are also low-level synchronization directives, such as `flush` and `locks`, but they are too complex at the moment, we will see them later.



(a) Barrier. Each thread wait at the barrier until all threads arrive.



(b) Mutual exclusion. Define a block of code that only one thread at a time can execute.

Barrier. Each thread waits until all threads arrive.

Doc. [≡](#)

OpenMP: barrier

```
1 #pragma omp barrier name
```

An optional name may be used to identify the **critical** region. A **thread waits at the beginning of a critical region** until no other thread is executing a critical region (anywhere in the program) **with the same name**. All unnamed critical directives map to the same unspecified name.

Example 6: synchronization with barrier directive

The following example includes several critical directives. The example illustrates a queuing model in which a task is dequeued and worked on. To guard against many threads dequeuing the same task, the dequeuing operation must be in a **critical** section. Because the two queues in this example are independent, they're protected by **critical** directives with different names, *xaxis* and *yaxis*. [9]

```
1 #pragma omp parallel shared(x, y) private(x_next, y_next)
2 {
3     #pragma omp critical ( xaxis )
4         x_next = dequeue(x);
5         work(x_next);
6     #pragma omp critical ( yaxis )
7         y_next = dequeue(y);
8         work(y_next);
9 }
10
```

Mutual exclusion. Only one thread at a time can enter a *critical* region.

Doc. [≡](#)

OpenMP: critical

```
1 #pragma omp critical
```

	omp critical	omp single
Meaning	Run code segment one by one by all threads	Run code segment once by any thread
Number of times code is executed	Number of threads	Only one
Use case	Avoid race condition	Manage control variables or signals

Table 3: **▲ omp critical vs omp single**

Example 7: synchronization with critical directive

Note the following code:

```

1 float res;
2
3 #pragma omp parallel
4 {
5     float B;
6     int i, id, nthrds, niters = big_number;
7
8     id = omp_get_thread_num();
9     nthrds = omp_get_num_threads();
10    for (i = id; i < niters; i += nthrds) {
11        B = big_job(i);
12        #pragma omp critical // threads wait their turn;
13        res += consume(B); // only one at a time
14    } // calls consume
15 }
16

```

Atomic. Provides mutual exclusion, but only when updating a memory location. It ensures that a particular memory location is accessed atomically. It is valid only for the following statement and not for a structured block.

The statement inside the atomic must be one of the following forms:

- `x binop = expr`
- `x++ or ++x`
- `x-- or --x`

Where `x` is an lvalue of scalar type and `binop` is a non-overloaded builtin operator.

[Doc. ▾](#)

OpenMP: atomic

```

1 #pragma omp atomic

```

Example 8: synchronization with atomic directive

```

1 #pragma omp parallel
2 {
3     double tmp, B;
4     B = DOIT();
5     tmp = big_ugly(B);
6     #pragma omp atomic
7     X += tmp;
8 }
9

```

5.5 Data environment

OpenMP is based on the shared memory programming model, so most **variables are shared by default**. **Global variables are also shared between threads**. But not everything is shared; for example, *stack variables* in functions called from parallel regions are **private**, as are *automatic variables* within a statement block.

Example 9: data sharing

In the following code, the variable `temp` is private (local to each thread) because it is in the stack of the function `work`; meanwhile, the variables `A`, `index`, and `count` are shared by all threads.

```

1 double A[10];
2 int main() {
3     int index[10];
4     #pragma omp parallel
5         work(index);
6     printf("%d\n", index[0]);
7 }
8
9 void work(int *index) {
10     double temp[10];
11     static int count;
12     /* other code */
13 }
```

We can refer to these arguments as **Data Scope Attribute Clauses** because the issue is really about the visibility and value of each data in each scope. Although OpenMP shares variables by default, there is (and it is a very common and *best practice*) the option to:

- Selectively **change storage attributes for construct** using the following clauses: `shared`, `private` and `firstprivate`.
- The **final value** of a private inside a parallel loop can be **transmitted to the shared variable outside the loop** with: `lastprivate`.
- The **default attributes can be overridden** with:
`default(private | shared | none)`

⚠ Note that when we say “copy” we mean the *shallow copy*, not the *deep copy*. So when the following clauses create a local copy, they create a *shallow copy*.

private clause. The statement `private(var)` creates a **new local copy** of `var` for each thread. The value of the private copies is *uninitialized* and also the original variable value remains unchanged after the region.

OpenMP: private(...)

```

1 #pragma omp parallel directive private(var1-name, var2-name, ...)
```

Example 10: private clause and *dirty memory location*

In the following code we try to use the private clause inside a parallel for. The code is very trivial, we set the number of threads to two for better understanding, so we start the parallel code with the for directive and the private clause. It has `private_test` as a private variable. So each thread will copy the variable and the initial value will be undefined.

```

1 #include <iostream>
2 #include "omp.h"
3 #define MAX 6
4
5 int main(int argc, char const *argv[])
6 {
7     int i, private_test = 10;
8     printf("Memory location of private_test: %p\n",
9            &private_test);
10    // set limit to 2 threads for better understanding
11    omp_set_num_threads(2);
12    printf("Master will execute for in parallel!\n");
13    #pragma omp parallel for private(private_test)
14    for(i = 0; i < MAX; ++i) {
15        // initialize private_test
16        private_test = i == 0 ? 0 : ++private_test;
17        printf(
18            "Thread #%i, iter_i: %d, private_test: %d\n",
19            omp_get_thread_num(), i, private_test
20        );
21    }
22    printf(
23        "private_test outside the parallel region: %d\n",
24        private_test
25    );
26    return 0;
27 }
```

Unfortunately, when we examine the output, we see a problem. Thread zero (*master*) executes the `for` statement for the first 3 iterations, while thread one (*slave*) executes the `for` statement for the last 3 iterations. The zero thread behaves as expected because we initialize the `private_test` variable to zero on the first `for` iteration (when `i` is 0). The thread one, has made a copy of the variable in another memory location and the value is unknown; so it continues to add a single value to each iteration in a *dirty memory location*. This is a trivial example that highlights the unknown values that we can find inside the private variables if we don't do any initialization.

```

1 $ g++ -fopenmp example.cpp -o example
2 $ ./example
3 Memory location of private_test: 0x7ffecdfa3aa4
4 Master will execute the for statement in parallel!
5 Thd #0, i:0, private_test:0,          mem: 0x7ffecdfa3a40
6 Thd #0, i:1, private_test:1,          mem: 0x7ffecdfa3a40
7 Thd #0, i:2, private_test:2,          mem: 0x7ffecdfa3a40
8 Thd #1, i:3, private_test:687869953, mem: 0x76a0297ffd0
9 Thd #1, i:4, private_test:687869954, mem: 0x76a0297ffd0
10 Thd #1, i:5, private_test:687869955, mem: 0x76a0297ffd0
11 private_test outside the parallel region: 10
```

firstprivate clause. The variables are initialized from the shared variable, but as in the **private** clause, the updated value doesn't leave the parallel region.

OpenMP: firstprivate(...)

```
1 #pragma omp parallel directive firstprivate(var1-name, ...)
```

Example 11: firstprivate clause

A very trivial example to see how the **firstprivate** clause works. The variable **private_test** is copied to local and initialized with the value outside the parallel region.

```
1 #include <iostream>
2 #include "omp.h"
3 #define MAX 6
4
5 int main(int argc, char const *argv[])
6     int i, private_test = 10;
7     printf("Memory location of private_test: %p\n",
8         &private_test);
9     // set limit to 2 threads for better understanding
10    omp_set_num_threads(2);
11    printf("Master will execute for in parallel!\n");
12    #pragma omp parallel for firstprivate(private_test)
13    for(i = 0; i < MAX; ++i) {
14        // initialize private_test
15        ++private_test;
16        printf(
17            "Thd #:%i, i:%d, private_test:%d, mem: %p\n",
18            omp_get_thread_num(), i,
19            private_test, &private_test
20        );
21    }
22    printf(
23        "private_test outside the parallel region: %d\n",
24        private_test
25    );
26    return 0;
27 }
```

Note that the value is not propagated outside the parallel region.

```
1 $ g++ -fopenmp example.cpp -o example
2 $ ./example
3 Memory location of private_test: 0x7ffc5cd153b0
4 Master will execute for in parallel!
5 Thd #0, i:0, private_test:11, mem: 0x7ffc5cd15350
6 Thd #0, i:1, private_test:12, mem: 0x7ffc5cd15350
7 Thd #0, i:2, private_test:13, mem: 0x7ffc5cd15350
8 Thd #1, i:3, private_test:11, mem: 0x77001d9ffdd0
9 Thd #1, i:4, private_test:12, mem: 0x77001d9ffdd0
10 Thd #1, i:5, private_test:13, mem: 0x77001d9ffdd0
11 private_test outside the parallel region: 10
```

Example 12: be careful with pointers using the `firstprivate` clause

The following code is very similar to the previous one. The difference here is that the parallel region also gets a pointer. Note that the pointer is in C style, because using the unique pointer technique (suggested in C++) will create an exception, because C++ doesn't allow the copy of a unique pointer. However, each thread creates a shallow copy of the pointer, but not a copy of the value pointed to! In this test, we use the pointer to the value of `private_test` to modify the value of `private_test`.

```

1 #include <iostream>
2 #include "omp.h"
3 #define MAX 6
4
5 int main(int argc, char const *argv[]) {
6     bool print_flag = true;
7     int i, private_test = 10;
8     // C pointer, not a good practice in C++...
9     // used only for the example
10    int *ptr_private_test = &private_test;
11    printf("Memory location of private_test : %p\n",
12          &private_test);
13    printf("Memory location of ptr_private_test: %p\n",
14          &ptr_private_test);
15    // set limit to 2 threads for better understanding
16    omp_set_num_threads(2);
17    printf("Master will execute for in parallel!\n\n");
18    #pragma omp parallel for firstprivate(private_test,
19    ptr_private_test, print_flag)
20    for(i = 0; i < MAX; ++i) {
21        if (print_flag) {
22            printf("Memory location ptr %p\n",
23                  &ptr_private_test);
24            print_flag = false;
25        }
26        // increase value pointed to by ptr
27        ***ptr_private_test;
28        // increase simple variable
29        ++private_test;
30        printf(
31            "Thread #i, i:%d\n- private_test:%d,
32            ptr_private_test:%d, mem: %p\n\n",
33            omp_get_thread_num(), i, private_test,
34            *ptr_private_test, &private_test
35        );
36    }
37    printf(
38        "private_test outside the parallel region: %d\n",
39        private_test
40    );
41    return 0;
}

```

Note an interesting observation. The variable `private_test` is incremented at each iteration; in the same way, the value pointed to by the pointer `ptr_private_test` is also incremented. Finally, the variable

`private_test` is modified because the pointer was copied in each thread and each slave, including the master, increased the value. This is a very bad practice and we want to suggest to use unique pointers of C++ or to avoid shallow copies.

```

1 Memory location of private_test      : 0x7fff3849c224
2 Memory location of ptr_private_test: 0x7fff3849c228
3 Master will execute for in parallel!
4
5 Memory location ptr 0x7fff3849c1a0
6 Thread #0, i:0
7 - private_test:11, ptr_private_test:11, mem: 0x7fff3849c198
8
9 Thread #0, i:1
10 - private_test:12, ptr_private_test:12, mem: 0x7fff3849c198
11
12 Thread #0, i:2
13 - private_test:13, ptr_private_test:13, mem: 0x7fff3849c198
14
15 Memory location ptr 0xb710cfffd0
16 Thread #1, i:3
17 - private_test:11, ptr_private_test:14, mem: 0xb710cffda8
18
19 Thread #1, i:4
20 - private_test:12, ptr_private_test:15, mem: 0xb710cffda8
21
22 Thread #1, i:5
23 - private_test:13, ptr_private_test:16, mem: 0xb710cffda8
24
25 private_test outside the parallel region: 16

```

The expected value for `private_test` should remain 10 because the `firstprivate` doesn't affect the values of the variables after the parallel region, but in this case we are using a pointer and a bad practice.

lastprivate clause. The variables **update shared variables with the value from the last iteration**, so the order of execution of the threads is important here.

OpenMP: lastprivate(...)

```

1 #pragma omp parallel directive lastprivate(var1-name, ...)
```

Example 13: lastprivate clause

In the following example, the value of the last iteration is passed (and overwritten) to the original value:

```

1 #include <iostream>
2 #include "omp.h"
3 #define MAX 6
4
5 int main(int argc, char const *argv[]) {
```

```

6     int i, private_test = 10;
7     printf("Memory location of private_test: %p\n",
8             &private_test);
9     // set limit to 2 threads for better understanding
10    omp_set_num_threads(2);
11    printf("Master will execute for in parallel!\n");
12    #pragma omp parallel for lastprivate(private_test)
13    for(i = 0; i < MAX; ++i) {
14        // initialize private_test
15        private_test = i;
16        printf(
17            "Thd #%i, i:%d, private_test:%d, mem: %p\n",
18            omp_get_thread_num(), i,
19            private_test, &private_test
20        );
21    }
22    printf(
23        "private_test outside the parallel region: %d\n",
24        private_test
25    );
26    return 0;
27 }
```

The `private_test` variable initially has a value equal to 10, but with `lastprivate` we have overwritten it.

```

1 Memory location of private_test: 0x7ffc4c82b8c0
2 Master will execute for in parallel!
3 Thd #0, i:0, private_test:0, mem: 0x7ffc4c82b860
4 Thd #0, i:1, private_test:1, mem: 0x7ffc4c82b860
5 Thd #0, i:2, private_test:2, mem: 0x7ffc4c82b860
6 Thd #1, i:3, private_test:3, mem: 0x72c7dddffdd0
7 Thd #1, i:4, private_test:4, mem: 0x72c7dddffdd0
8 Thd #1, i:5, private_test:5, mem: 0x72c7dddffdd0
9 private_test outside the parallel region: 5
```

default clause. The default storage attribute is `default(shared)`. To change the default we can write simply the value shared or none inside the brackets:

- `default(share)` is the default choice for OpenMP, so there is no need to use it except for the clause `pragma omp task`.

OpenMP: default(share)

```
1 #pragma omp parallel directive default(share)
```

Against the `private` clauses, if we want to share some variables, we can use `shared` clause.

OpenMP: shared

```
1 #pragma omp parallel directive shared(var1-name, ...)
```

- `default(private)`, each variable in the construct is made private as if specified in `private` clause.

OpenMP: default(private)

```
1 #pragma omp parallel directive default(private)
```

- `default(none)`, no default for variables in static extent. Must list storage attribute for each variable in static extent. It is a good programming practice.

OpenMP: default(none)

```
1 #pragma omp parallel directive default(none)
```

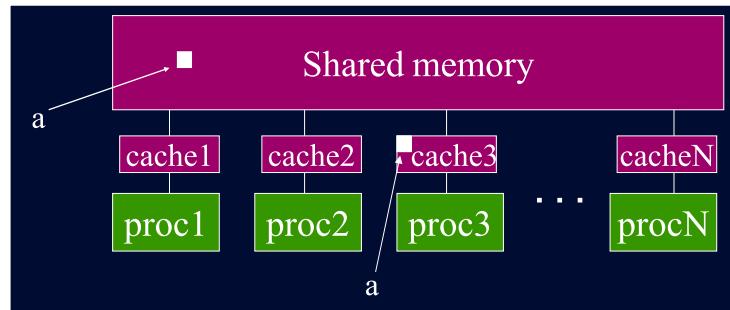
Example 14: default(none)

```
1 #include <iostream>
2 #include "omp.h"
3 #define MAX 6
4
5 int main(int argc, char const *argv[]) {
6     double private_test = 10.0;
7     int i;
8     // set limit to 2 threads for better understanding
9     omp_set_num_threads(2);
10    #pragma omp parallel for default(none) private(i,
11        private_test)
12    for(i = 0; i < MAX; i++) {
13        private_test = i;
14        printf(
15            "Thread #%, value: %f\n",
16            omp_get_thread_num(), private_test
17        );
18        printf(
19            "private_test outside the parallel region: %f\n"
20            ,
21            private_test
22        );
23    }
24 }
```

```
1 Thread #0, value: 0.000000
2 Thread #0, value: 1.000000
3 Thread #0, value: 2.000000
4 Thread #1, value: 3.000000
5 Thread #1, value: 4.000000
6 Thread #1, value: 5.000000
7 private_test outside the parallel region: 10.000000
```

5.6 Memory model

OpenMP supports a **shared memory model**. It is a model where **all threads share an address space**, but it can get complicated, for example in the following example (picture) we can see a shared space that is also in the cache of process 3; so how can we manage this situation? What are the methods to adapt? Furthermore, what happens when a thread modifies a shared address space? How does the system behave?



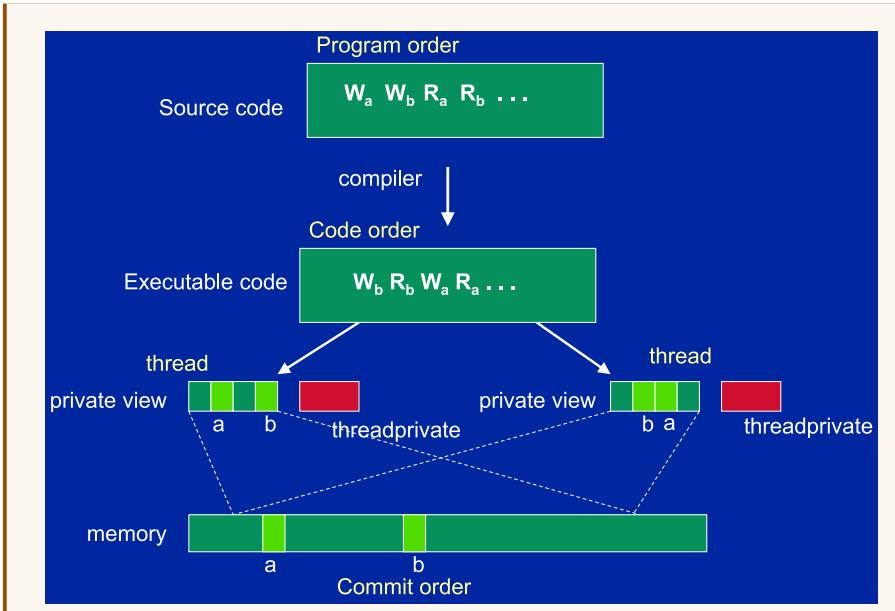
In general, a memory model is defined in terms of:

- **Coherence**: behavior of the memory system when a **single address is accessed by multiple threads**.
- **Consistency**: read, write, or synchronization (RWS) orders with **different addresses and through multiple threads**.

Remark: what the compiler does at the low level

When we write a program and ask the compiler to compile it, it does a lot of “magic” under the hood.

1. Our source code is decoded into very low level operations. These operations respect the order of the high-level operations of our code.
2. The compiler, smarter than us, tries to optimize the code while creating the executable code; so it reorders the low-level operations that are **semantically equivalent** to our program, but often allow to gain performance.
3. During the execution of the code, if it is parallel, some threads are created and each of them has a private memory address.
4. Finally, each thread writes/reads from memory, also called the commit order, using some rules.



Note that the re-ordering are made by:

- **Compile** re-orders *program order* \rightarrow *code order*.
- **Machine** re-orders *code order* \rightarrow *memory commit order*.

At any given time, the private view seen by a thread may differ from the view in shared memory. For this reason, there are **consistency models** that define constraints on the order of RWS (Reads, Writes, Synchronizations) operations.

In general, a multiprocessor adopts the **Sequential Consistency Model**. It says that given n operations (RWS), they are sequentially consistent if:

- They remain in *program order* for each processor.
- They are seen to be in the same overall order by each of the other processors.

Also, in a sequential consistency model, program order is the same as code order and commit order.

OpenMP uses a **Relaxed Consistency Model** where the compiler cannot reorder synchronization operations with read or write operations on the same thread. The consequences are:

- ✓ All threads have the same view of memory at specific points in the code, called **Consistency Points**.
- ✓ Between two consistency points, each thread has its own temporary view of memory, which may be different from the other temporary views of other threads.

- ✓ Data are read-only, this guarantee to avoid consistency issues.
- ✗ Shared data that need to be modified can create possible race conditions.

flush directive. Defines a *sequence point* at which a **thread is guaranteed to see a consistent view of memory** with respect to the flush-set. The flush-set means **all thread visible variables** for a flush construct without an argument list, and it also means a list of variables when the `flush(list)` construct is used. The action of flush is to **guarantee** that:

- All read/write operations that overlap the flush-set and occur **before** the flush will be completed before the flush is executed.
- Any read/write operations that overlap the flush-set and occur **after** the flush will not be performed until after the flush.
- Flushes with overlapping flush-sets can not be reordered.

In other words, the **flush forces data to be updated in memory so that other threads see the most recent value.**

Doc. 

OpenMP: flush

```
1 #pragma omp flush flush-set
```

A flush operation is **implied** by default by OpenMP synchronizations:

- At entry/exit of parallel/critical regions;
- At implicit and explicit barriers;
- At exit from work-sharing constructs, unless nowait is specified.

And it is **not implied**:

- At entry of work-sharing constructs;
- At entry and exit of master.

5.7 Nested Parallelism

❷ How does OpenMP create multiple threads?

OpenMP uses a **fork-join model** of parallel execution. When a thread encounters a `parallel` construct, the **thread creates a team composed of itself and some additional (possibly zero) number of threads**. The encountering thread becomes the *master* of the new team. The other threads of the team are called *slave* threads of the team.

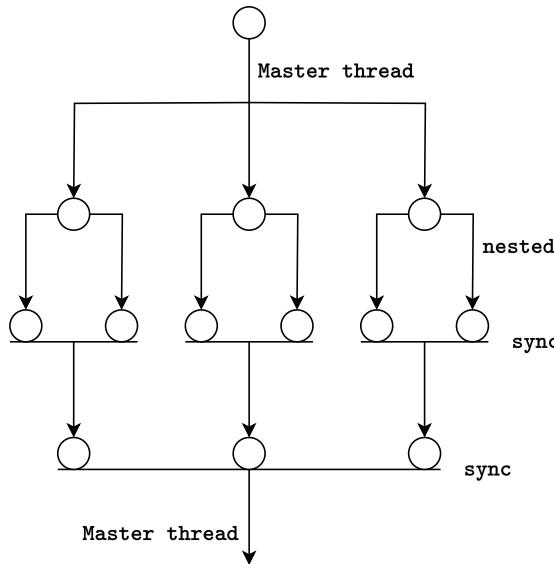


Figure 21: OpenMP fork-join model.

❷ What is the life history of each thread (*slave*) created?

All team members execute the code inside the `parallel` construct. When a thread finishes its work within the parallel construct, it waits at the implicit barrier at the end of the parallel construct. When all team members have arrived at the barrier, the threads can leave the barrier. The *master* thread continues execution of user code beyond the end of the `parallel` construct, while the *slave* threads wait to be summoned to join other teams.

OpenMP `parallel` regions can be nested inside each other. If nested parallelism is:

- **Disabled**, then the **new team** created by a thread encountering a parallel construct inside a parallel region **consists only of the encountering thread**.
- **Enabled**, then the **new team may consist of more than one thread**.

How does OpenMP manage the available threads? Thread Pool

The OpenMP runtime library maintains a pool of threads that can be used as *slave* threads in parallel regions. When a thread encounters a parallel construct and needs to create a team of more than one thread, the thread will check the pool and grab idle threads from the pool, making them *slave* threads of the team. The *master* thread might get fewer *slave* threads than it needs if there is not a sufficient number of idle threads in the pool. When the team finishes executing the parallel region, the *slave* threads return to the pool.

Summary

1. **Parallel construct.** Our main thread starts to execute our code. It encounters a `parallel` construct.
2. **Pool verification.** The OpenMP library checks its pool of threads. If within its pool of available threads have something of disposable, it allocates the *slave* (thread requested) to the *master* (thread that want to create the team). We refer to a single thread, but obviously this can be extended to multiple thread request (e.g. *master* thread requests 3 threads to OpenMP). Finally, the number of requested *slaves* cannot always be satisfied; OpenMP guarantees the best, so it continues to give the requested threads to the applicants. If it cannot satisfy the request, it returns the maximum number of *slaves* it can satisfy (e.g. *master* requests 4 threads, but OpenMP has a pool of only 2 threads available; therefore it returns 2 *slaves*).
3. **Assign and start execution.** Ideally, OpenMP returns the number of threads requested by the *master*. Otherwise, it returns the maximum number. The team now consists of the main thread, called the *master*, and its *slaves*. Each member of the team executes the code specified by the programmer within the parallel construct.
4. **End of execution of a thread.** A thread of the team finishes its work. It can finally rest, and its state changes from "running" to "waiting". It waits for its other thread friends. Each thread has an implicit barrier at the end of the parallel construct.
5. **Any thread finish.** Finally, each thread finishes its work. The *master* releases each member of the team to the OpenMP library. OpenMP updates its thread pool with the returning threads, and the *master* thread continues its life independently from the released threads.

The previous flow works if and only if nested parallelism is enabled. Otherwise, if a *master* asks to create a new team, it will ignore the request and tell the requester to continue alone.

Implementation

Nested parallelism can be enabled or disabled by passing true or false as arguments to the runtime function:

OpenMP: omp_set_nested

```
1 void omp_set_nested(int nested)
```

We can set a **default number of threads at different levels of nested parallelism** with:

```
OMP_NUM_THREADS = [list, of, integers]
```

If the nesting level is deeper than the number of entries in the list, the last value is used for all subsequent nested parallel region.

- **OMP_MAX_ACTIVATE_LEVELS** defines the upper **limit on the number of active parallel regions that may be nested**.
- **OMP_THREAD_LIMIT** avoids that recursive applications **create too many threads**.

Finally, since we are in nested parallelism, the thread number returns the thread number partially and not globally, we need other useful functions:

- Returns the maximum number of OpenMP threads available in contention group:

OpenMP: omp_get_thread_limit

```
1 int omp_get_thread_limit()
```

- Returns the maximum number of nested active parallel regions when the innermost parallel region is generated by the current task.

OpenMP: omp_get_max_active_levels

```
1 int omp_get_max_active_levels()
```

- Limits the number of nested active parallel regions when a new nested parallel region is generated by the current task.

OpenMP: omp_set_max_active_levels

```
1 void omp_set_max_active_levels(int max_levels)
```

- Returns the number of nested parallel regions on the device that enclose the task containing the call.

[Doc. !\[\]\(576c2da912b3688c853f39b2f1ebf794_img.jpg\)](#)**OpenMP: omp_get_level**

```
1 int omp_get_level()
```

- Returns the number of active, nested parallel regions on the device enclosing the task containing the call.

[Doc. !\[\]\(7ea23154e8814e92f438ec0192bc43a1_img.jpg\)](#)**OpenMP: omp_get_active_level**

```
1 int omp_get_active_level()
```

- Returns, for given nested level of the current thread, the thread number of the ancestor of the current thread.

[Doc. !\[\]\(000914e031b5b2bb557d17004e57d0c5_img.jpg\)](#)**OpenMP: omp_get_ancestor_thread_num**

```
1 int omp_get_ancestor_thread_num(int level)
```

- Returns, for a given nested level of the current thread, the size of the thread team to which the ancestor of the current thread belongs.

[Doc. !\[\]\(0f69595085646b6f8276f8ba906cdaab_img.jpg\)](#)**OpenMP: omp_get_team_size**

```
1 int omp_get_team_size(int level)
```

5.8 Cancellation

As of OpenMP 4.0, a thread can be cancelled using a combination of two commands:

- `cancel` construct cancels the innermost enclosing region of the specified type.

In other words, it **allows us to cancel the current thread**. But be careful! The directive only allows to **set the cancellation flag to a value of one**. So it is necessary to also use the **cancellation point** command or the thread must hit a barrier (implicit or explicit).

The syntax of the `cancel` construct is as follows:

[Doc. ▾](#)

OpenMP: pragma omp cancel

```
1 #pragma omp cancel construct-type-clause
```

Where `construct-type-clause` is one of the following: `parallel`, `sections`, `for`, `taskgroup`.

- The `cancellation point` construct introduces a user-defined cancellation point at which implicit or explicit tasks check whether cancellation of the innermost enclosing region of the specified type has been enabled.

In other words, **when a thread encounters the cancellation point, the cancellation flag has been checked**. The thread execution is stopped. This is an explicit cancellation request because there are other points where cancellation flag is checked, such as at another cancel region or at a barrier.

[Doc. ▾](#)

OpenMP: pragma omp cancellation point

```
1 #pragma omp cancellation point
```

Since there is an **overhead in checking for cancellation**, it can be enabled manually using the `OMP_CANCELLATION` environment variable (false by default).

⚠ Immediate cancellation is not guaranteed

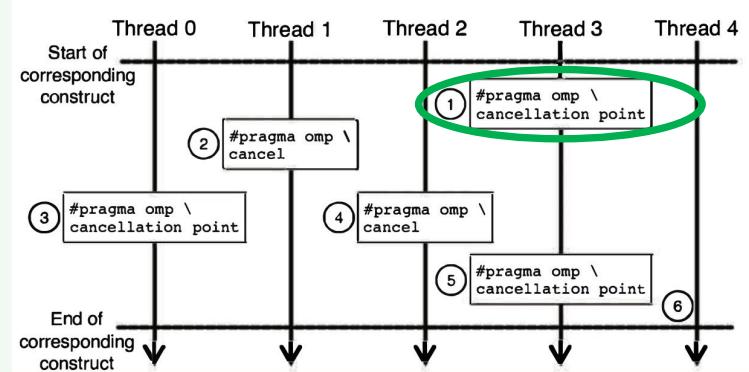
OpenMP does not guarantee that cancellation will result in immediate termination.

✓ When to use?

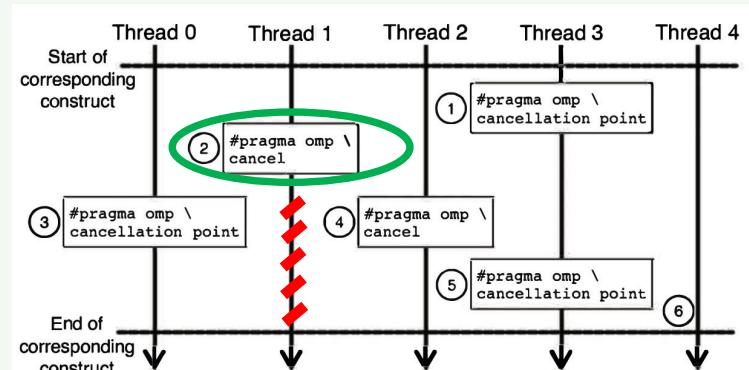
When there is a **need to stop the execution**, such as in the **divide-and-conquer algorithms** (for example, to stop a search when we find the element), or when we **need to handle some errors**.

Example 15: thread cancellation

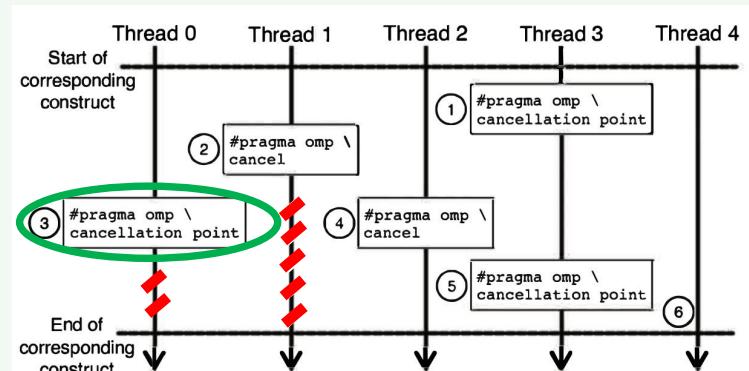
Assume that from a certain point in our algorithm there are 5 threads alive (including the *master*), and the construct pipeline is as follows.



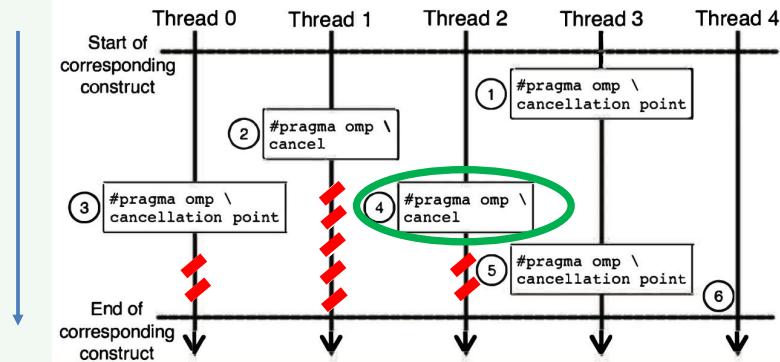
Thread 3 hits a cancellation point, but there is no cancel directive before it, so the cancellation flag is set to zero by default and the thread hasn't been broken.



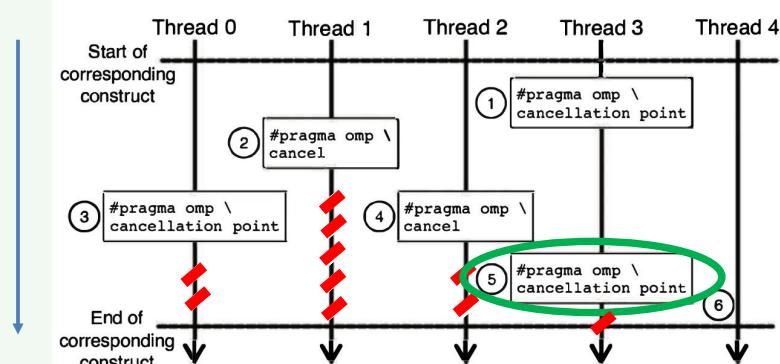
Thread 1 requests the cancellation and waits for a synchronization point, such as a barrier or an cancellation point. From that point on, it is idle and waits.



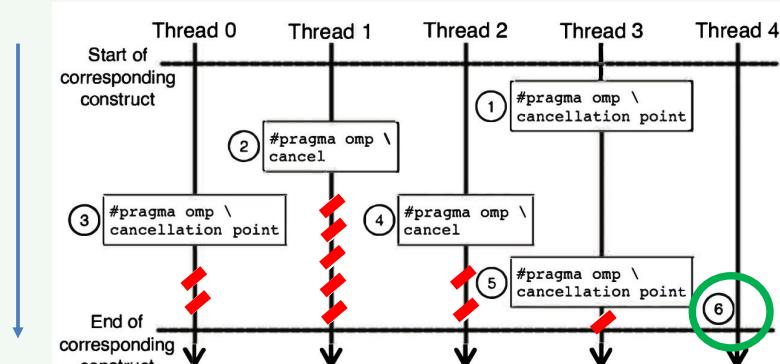
Thread 0 calls a cancellation point to check the flag and terminates because thread 1 (last step) has set the cancellation flag to 1. At this point, thread 1, which was waiting for a synchronization point, also terminates.



At the same time, thread 2 is cancelled because the cancel directive checks the cancellation flag first.



At this point the thread 3 has been terminated.



Finally, thread 4 never encounters a cancellation point and finishes execution normally.

5.9 SIMD Vectorization

A SIMD processor exploits data parallelism by providing instructions that operate on blocks of data (called **vectors**). SIMD provides **data parallelism at the instruction level** and can be combined with other OpenMP constructs to achieve multi-level parallelism.

✖ What compilers must do to understand whether a loop can be vectorized

SIMD instructions use **SIMD registers**. The compilers deal with **several issues to determine whether a loop can be vectorized by SIMD instructions**. It does:

- An analysis of the dependencies across iterations;
- An alias analysis of pointers;
- An analysis of the data layout/alignment issues;
- An analysis of conditional executions;
- Checks of the loop bounds that must not be multiple of vector length.

✖ Other compiler problems: Loop Peeling and Loop Tail

Also, the **loop iterations at the beginning and end may not be vectorized** (loop peeling, tail). This is because when a compiler tries to optimize a loop using vectorization (i.e., applying the same operation to multiple data points simultaneously to speed up execution), it often encounters problems with the iterations at the beginning and end of the loop. These iterations may not fit neatly into the vectorized operations because the total number of iterations of the loop may not be a perfect multiple of the vector length.

Essentially, the **compiler may have to split the loop into three parts**:

1. **Loop Peeling.** Handle the initial few iterations that don't align with vector boundaries.
2. **Vectorized Main Loop.** Process the majority of the loop iterations using vectorized operations.
3. **Loop Tail.** Handle the remaining iterations that can't be processed with vector operations due to their small number.

This approach ensures that the loop is as optimized as possible, even if some parts of it cannot be vectorized efficiently.

❖ Use OpenMP SIMD

The `simd` construct can be applied to a loop to indicate that the loop can be transformed into a SIMD loop (i.e., multiple iterations of the loop can be executed simultaneously using SIMD instructions).

OpenMP: pragma omp simd

```
1 #pragma omp simd
```

The loop is divided into chunks, and all iterations are executed by a single thread using SIMD vector instructions. The chunks should fit into a vector register for **performance**, and each iteration is executed by a **SIMD lane**. The compiler will generate SIMD instructions, it is **up to the user to ensure this maintains correct behavior**.

❷ Possible clauses

- Data scope clauses (page 80) can be used in a `simd` directive.
- A **collapse** clause can be used to fuse two perfectly nested loops, but the complexity can be increase.
- The `simdlen(size)` clause suggests a **preferred vector length**. Maybe the code will work better with a specific vector length, but the compiler is free to ignore it (is only a suggestion make by the programmer). It can also hurt performance but the results remain correct.
- The `safelen(size)` clause sets an **upper limit to the vector length that the compiler cannot exceed**.

❸ For SIMD

The `omp for simd` directive distributes the iterations of one or more associated loops across the threads that already exist in the team and indicates that the iterations executed by each thread can be executed concurrently using SIMD instructions.

OpenMP: pragma omp for simd

```
1 #pragma omp for simd
```

The number of threads and scheduling policy greatly affect performance. If the number of threads increases, work for each thread is smaller. Each thread should work with a chunk corresponding to the vector length. Ideally, it is correct to distribute iterations among threads in a team, then each thread uses SIMD instructions.

The clause **schedule** avoid performance degradation specifying the scheduling type and chunk size for the loop iterations. The **static** schedule divides the iterations into chunks of *chunk-size*, distributing them to the threads.

OpenMP: pragma omp declare simd

```
1 #pragma omp for simd schedule(simd:static, chunk-size)
```

Finally, we can also declare a function to be compiled for calls inside a SIMD loop.

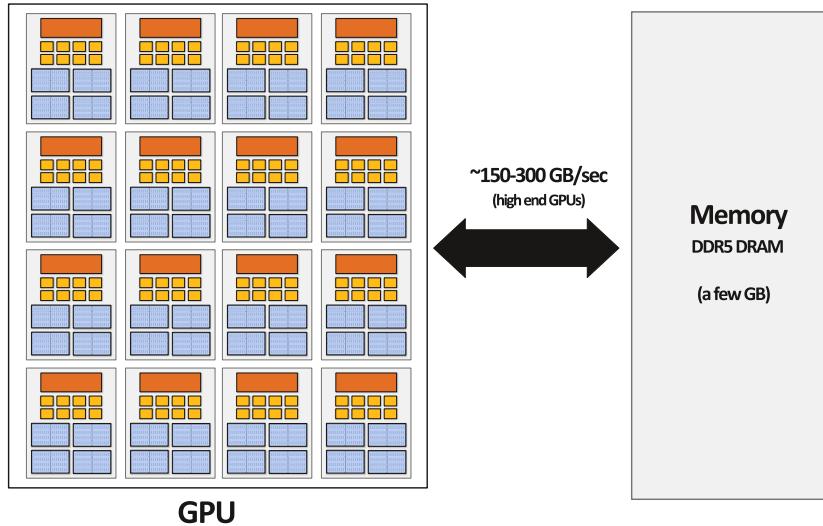
OpenMP: pragma omp declare simd

```
1 #pragma omp declare simd
2     /* function definition */
```

6 GPU Architecture

6.1 Introduction

In the image below, we can see a very basic GPU architecture:



- **GPU Structure** (left). The GPU is made up of a **grid of** smaller blocks, each representing a **core**. All the blocks form a **multi-core structure** that allows the GPU to handle many tasks simultaneously.

Within a **single core**, the GPU uses **SIMD** (Single Instruction, Multiple Data) **execution**. This means that many execution units within a core can simultaneously execute the same instruction on different pieces of data. This results in highly efficient execution of parallel tasks such as rendering graphics or running simulations.

In addition, **each core supports multi-threaded execution**, which allows multiple threads to be processed simultaneously. This further enhances the GPU's ability to perform multiple tasks simultaneously.

- **Memory Connection** (right). The GPU is connected to DDR5 DRAM, a type of dynamic random access memory. Fast memory is essential to handle the large amounts of data that GPUs process. The more powerful the DRAM, the faster the data transfer.

Initially, GPUs were designed with a specific purpose: to render graphics quickly and efficiently. However, their role has expanded significantly over the years.

General-Purpose computing on Graphics Processing Units (GPGPU) was originally designed to **render graphics**, but GPUs have evolved to perform a wide range of computations beyond traditional graphics tasks. GPGPU takes advantage of the parallel processing capabilities of GPUs to perform computations typically handled by the CPU.

6.2 GPU compute mode

GPU compute mode refers to GPU hardware that is optimized for general-purpose computing rather than graphics rendering. This mode allows users to run non-graphics programs on the GPU's programmable cores, taking advantage of the GPU's parallel processing capabilities for tasks such as scientific simulations, data analysis, and machine learning.

Example 1: how to run code on a GPU (prior to 2007)

Now let us see how to run a simple code on a GPU. Suppose a user wants to draw a picture on a GPU:

- **GPU Shader Program Binaries.** The application, via the graphics driver, supplies the GPU with shader program binaries. These are compiled programs that the GPU will execute to perform rendering tasks.
- **Graphics Pipeline Parameters.** The application sets various parameters for the graphics pipeline, such as the output image size, to control how the rendering should be processed.
- **Vertex Buffer.** The application provides the GPU with a buffer of vertices. Vertices are data points that define the shape of the objects to be rendered.
- **Draw Command.** The application sends a draw command to the GPU using the function call `drawPrimitives(vertex_buffer)`. This command instructs the GPU to start rendering using the provided vertex data.

The stages of the graphics pipeline:

1. **Input Vertex Buffer:** The initial stage where the vertex data is input to the pipeline.
2. **Vertex Generation:** Vertices are generated or fetched from the vertex buffer.
3. **Vertex Processing:** The vertices undergo various transformations and shading calculations.
4. **Primitive Generation:** The processed vertices are used to generate geometric primitives (such as triangles).
5. **Fragment Generation (Rasterization):** The primitives are converted into fragments (potential pixels).
6. **Fragment Processing:** Fragments undergo shading and texturing calculations to determine their final color and properties.
7. **Pixel Operations:** Final operations are performed on the fragments, such as depth testing and blending.
8. **Output Image Buffer:** The processed fragments are written to the output image buffer, resulting in the final rendered image.

⌚ Some history

Before 2007 the only way to interface with GPU hardware was through the graphics pipeline. This meant that GPUs were *designed and used specifically for tasks related to graphics rendering*. The pipeline stages (such as vertex processing, fragment generation, and pixel operations) were all designed to transform vertex data into pixels displayed on the screen.

Because they were optimized to handle parallel tasks associated with rendering images, their architecture and interfaces were tightly coupled with graphics APIs.

By 2007, the concept of using GPUs for **General-Purpose computing on Graphics Processing Units (GPGPU)** was emerging. Thanks mainly to the introduction of a new architecture signed NVIDIA called Tesla and CUDA, a parallel computing platform and programming model (also OpenCL was emerging).

The **NVIDIA Tesla architecture**, introduced with the **GeForce 8800 GPU** in 2006, marked a significant shift in GPU design by unifying graphics and computing capabilities. This architecture featured a scalable parallel array of processors that could be programmed in C or via graphics APIs². The Tesla architecture enabled **flexible, programmable graphics and high-performance computing**, making it possible to use GPUs for a wide range of applications beyond traditional graphics rendering. This unification enabled massive multithreading and parallel processing, dramatically improving performance for computationally intensive tasks. [7]

From this point on, the programmable cores of the GPU are used:

- Application could allocate buffers in GPU memory and copy data to/from buffers;
- Application (via the graphics driver) provides a single kernel program binary to the GPU;
- Application tells GPU to run kernel in SPMD fashion.

6.3 CUDA

6.3.1 Basics of CUDA

⌚ What is CUDA?

Compute Unified Device Architecture (CUDA), is a parallel computing platform and Application Programming Interface (API) model created by NVIDIA. It allows developers to utilize NVIDIA GPUs for general-purpose processing, enabling them to perform a wide range of computations more efficiently than with traditional CPU processing alone.

CUDA was introduced with the NVIDIA Tesla architecture, which marked a significant shift in GPU capabilities, enabling general purpose computing on GPUs. **CUDA is designed to be similar to the C programming language**, making it familiar to many developers. It allows programmers to write code that runs on GPUs using the compute-mode hardware interface.

CUDA's abstractions are relatively low-level and closely match the performance characteristics and capabilities of modern GPUs. This design goal helps maintain a low abstraction layer, ensuring that developers can take full advantage of the hardware's potential.

Note that **Open Computing Language (OpenCL)** is an open standards version of CUDA that runs on both CPUs and GPUs from multiple vendors. **While CUDA runs only on NVIDIA GPUs, OpenCL is designed to be more versatile and work on hardware from multiple vendors.**

⌚ CUDA Thread Hierarchy

CUDA organizes threads into a hierarchical structure to efficiently manage parallel computations on GPUs. To understand how it works, let's look at it from the deepest level up:

1. **Threads.** Threads (or **CUDA Threads**) are the **smallest unit of execution** in CUDA. Each thread runs a single instance of a kernel function⁸. Threads are **identified by their unique thread IDs**, which are used to calculate memory addresses and control decisions.
2. **Thread Blocks.** Threads are grouped into blocks (called also **CUDA Blocks**). Each block can contain multiple threads that execute concurrently.

Threads within the same block can communicate and share data through shared memory and synchronization primitives like barriers and atomic operations.

The *maximum number* of threads per block is limited by the GPU architecture, typically up to 1024 threads per block.

⁸A kernel function in CUDA is a function that runs on the GPU (device) but is called from the CPU (host). These functions are executed by many parallel threads on the GPU

3. **Grids.** Blocks are organized into a grid (called also **CUDA Grids**). A grid is a **collection of blocks that execute the same kernel function**.

All threads in a grid share the same global memory space.

The grid can be multi-dimensional, allowing for flexible organization of blocks to match the problem's dimensions.

Note that CUDA uses the **x**, **y**, and **z** dimensions for threads, blocks, and grids because this **multi-dimensional structure** aligns well with many common computational problems. Many computational problems naturally fit into a multi-dimensional space. For example, image processing involves 2D data, and volumetric simulations involve 3D data.

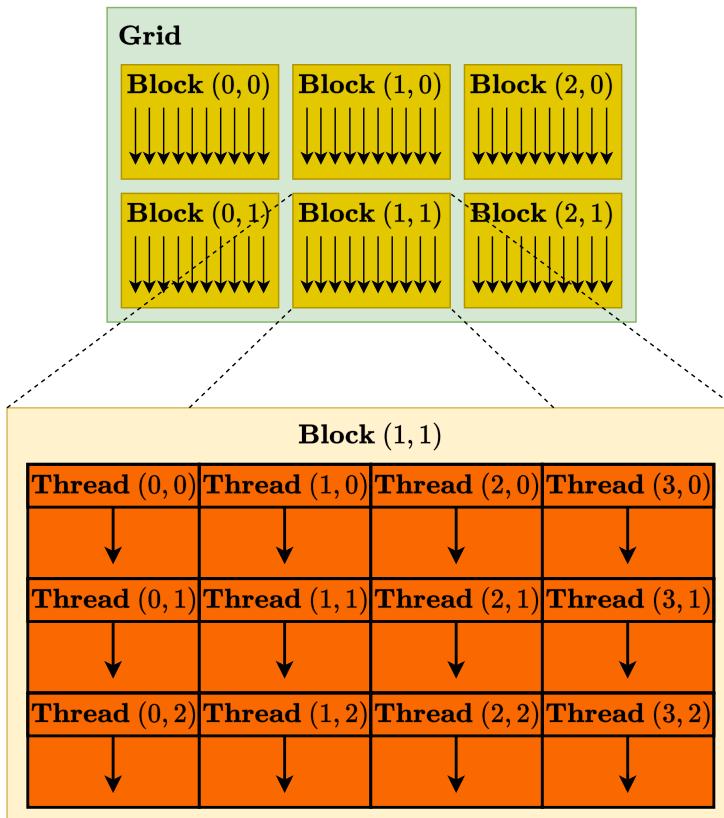


Figure 22: The figure shows an example of a CUDA grid consisting of 6 blocks, each block consisting of 12 threads. The number of threads and the number of blocks per column/row are customizable, this is just an example. In total, there are 72 CUDA threads (12 threads per block times 6 blocks in the grid).

_cuda Kernels

A **CUDA kernel** is a **function that gets executed on the GPU**. It contains the parallel portion of the application, which is executed by multiple threads in parallel. Unlike regular C/C++ functions that run on a single thread, CUDA kernels can run thousands of threads simultaneously.

Device vs Host

In CUDA, there is a term to identify the code that runs on the CPU and GPU.

Definition 1: CUDA Host (CPU)

The **CUDA Host** refers to the CPU and its associated memory. It is responsible for managing the overall program execution. This includes allocating memory, launching CUDA kernels, and transferring data between the CPU and the GPU. It is typically written in the standard programming language used (e.g., C, C++, Java, Python, etc.) and contains the logic for setting up and controlling the execution of CUDA kernels on the device.

Definition 2: CUDA Device (GPU)

The **CUDA Device** refers to the GPU and its associated memory. It is responsible for running parallel code (CUDA kernels) and performs the bulk of the computation. This takes advantage of the parallel processing power of the GPU. The device code (kernels) is written in the CUDA programming language and is designed to run on the multiple parallel cores of the GPU.

Basic CUDA syntax

Here is a sample code written in CUDA:

```

1 #include <cuda_runtime.h>
2 #include <iostream>
3 #define Nx 12
4 #define Ny 6
5
6 // Kernel definition
7 __global__ void MatAdd(
8     float A[Ny][Nx],
9     float B[Ny][Nx],
10    float C[Ny][Nx]
11 ) {
12     int i = blockIdx.x * blockDim.x + threadIdx.x;
13     int j = blockIdx.y * blockDim.y + threadIdx.y;
14     // guard against out of bounds array access
15     if (i < N && j < N)
16         C[i][j] = A[i][j] + B[i][j];
17 }
18
19 int main() {
20     // Define the hierarchy
21     dim3 threadsPerBlock(4, 3);
22     dim3 numBlocks(
23         Nx / threadsPerBlock.x,
24         Ny / threadsPerBlock.y
25     );
26     // Kernel invocation
27     // Assume matrices A, B, C are already allocated (dim Nx x Ny)
28     MatAdd<<<numBlocks, threadsPerBlock>>>(A, B, C);
29 }
```

Rows 3-4 Variables `Nx` and `define the dimensions of the matrices. In this case, they are 12×6 .`

Row 21 The `threadsPerBlock` function specifies that each block will have 4 threads in the x dimension and 3 threads in the y dimension. In other words, it defines how a block should be composed (see the figure 22).

Row 22 The `numBlocks` function defines the number of blocks required to cover the entire matrix. The number of blocks is calculated by dividing the matrix dimensions by the number of threads per block in each dimension. In other words, it defines how a grid should be composed (see the figure 22).

Row 28 The execution configuration syntax `<<<...>>>` is required to specify the number of threads per block and the number of blocks per grid (both numbers can be of type `int` or `dim3`).

The function invocation starts the `MatAdd` kernel with the given grid and block dimensions. This kernel runs on the GPU and performs the matrix addition.

Row 7 Defines a kernel function called `MatAdd` to be executed on the GPU. The `--global__` keyword indicates that this is a kernel function.

Rows 12-13 Computes the global column index `i` for each thread. This combines the block index and the thread index within the block to get the overall position.

It also does the same with the `j` index.

Row 15 The `if` condition ensures that the thread does not access out-of-bounds elements in the matrices.

Row 16 Finally, it performs the element-wise addition of matrices `A` and `B` and stores the result in matrix `C`.

As we said in Figure 22 (page 105), the example spawns 72 CUDA threads.

6.3.2 Memory model

The CUDA memory model consists of several types of memory, each with different characteristics and uses. In general, the **Host (CPU) and the Device (GPU) have different address spaces**, each one has its private memory address.

For example, the `cudaMemcpy` function in CUDA is used to **copy data** between different memory spaces, specifically **between host (CPU) memory and device (GPU) memory**.

```

1 // allocate buffer in host mem
2 float* A = new float[N];
3
4 // populate host address space pointer A
5 for (int i = 0; i < N; ++i) {
6     A[i] = (float)i;
7 }
8
9 // allocate buffer in device (GPU) address space
10 int bytes = sizeof(float) * N;
11 float* deviceA;
12 cudaMalloc(&deviceA, bytes);
13
14 // populate deviceA
15 cudaMemcpy(deviceA, A, bytes, cudaMemcpyHostToDevice);
16 // deviceA:
17 //     Destination memory address (either on the host or device).
18 //
19 // A:
20 //     Source memory address (either on the host or device).
21 //
22 // bytes:
23 //     Number of bytes to copy.
24 //
25 // cudaMemcpyHostToDevice:
26 //     Type of memory copy operation.
27 //     Copies data from host memory to device memory.

```

Note that directly accessing `deviceA[i]` is an invalid operation, because we cannot manipulate the contents of `deviceA` directly from host, since `deviceA` is not a pointer to the host's address space.

▀ Types of CUDA device memory models visible to kernels

1. **Per-thread Private Memory.** This memory is **private to each thread**. Each thread has its own memory space that other threads cannot access.
 ② **Usage.** It is **ideal for storing variables that are only relevant to individual threads** and do not need to be shared with other threads.
 ④ **Access.** It has **fast access**, limited by the number of registers available.
2. **Per-block Shared Memory.** This memory is **shared by all threads within a block**. It allows threads within the same block to cooperate by sharing data.
 ② **Usage.** It is useful for tasks where **threads within a block need to communicate or share intermediate results**. It is often used to optimize memory access patterns and reduce global memory accesses.
 ④ **Access.** It is **much faster than global memory, but limited in size**. Access is almost as fast as registers when used properly.
3. **Device Global Memory.** This memory is **accessible to all threads across all blocks**. It provides a large amount of memory, but has higher latency and lower bandwidth than shared memory.
 ② **Usage.** It is **suitable for storing large amounts of data that must be accessed by threads in different blocks**.
 ④ **Access.** It has the **slowest access** of the three types, but is necessary for large data storage and inter-block communication.

6.3.3 NVIDIA V100 Streaming Multiprocessor (SM)

The NVIDIA V100 is a powerful GPU designed for data centers, primarily used for Artificial Intelligence (AI), High Performance Computing (HPC), and data science. Meanwhile, the **NVIDIA V100 Streaming Multiprocessor (SM)** is a key component of the V100 GPU architecture.

💡 How is architecture composed?

See Figure 23 (page 111) for a graphical representation.

- **Warp Selector and Fetch/Decode:**

The *Warp Selector* and *Fetch/Decode* units are responsible for **managing the execution of warps** (groups of threads) and **decoding instructions**.

- **Functional Units:**

- **SIMD fp32 functional unit** (Yellow).

It **handles single-precision floating-point operations**. Control is shared across 16 units, allowing for 16 multiply-add (**MUL-ADD**) operations per clock cycle. This translates to one 32-wide SIMD operation every two clocks.

- **SIMD int functional unit** (Orange).

It **manages integer operations**, also shared across 16 units, with the same performance characteristics as the fp32 unit ($16 \times \text{MUL-ADD}$ per clock).

- **SIMD fp64 functional unit** (Brown).

It is **responsible for double-precision floating-point operations**. Control is shared across 8 units, allowing for 8 MUL-ADD operations per clock cycle, equating to one 32-wide SIMD operation every four clocks.

- **Tensor core unit** (Red).

It is **specialized for tensor operations**, which are crucial for deep learning and AI workloads.

- **Load/store unit** (Green).

It **handles memory operations**, such as loading data from and storing data to memory.

- **Warp Scheduler:**

The diagram below shows the scheduling of warps (Warp 0, Warp 4, Warp 60, etc.) across the functional units, indicating how different warps are processed in parallel.

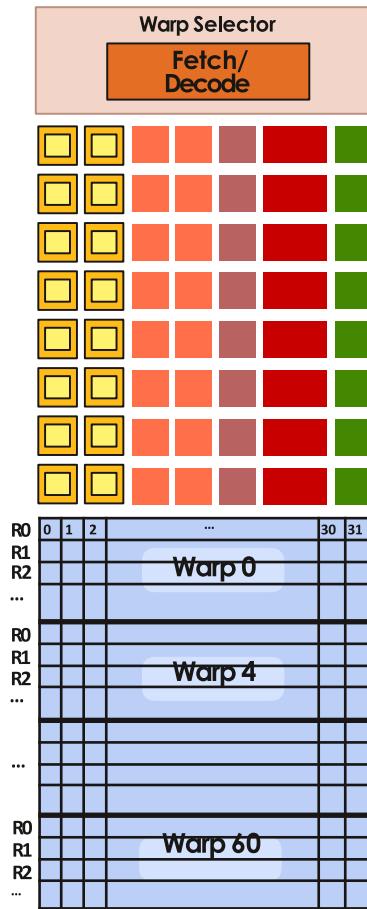


Figure 23: A “sub-core” of the NVIDIA V100 Streaming Multiprocessor (SM) architecture.

💡 What is a Warp?

A **Warp** is a group of 32 threads that execute the same instruction at the same time. Threads within a block are divided into warps. For example, a block of 256 threads would have 8 warps (256 threads / 32 threads per warp). Each SM in the V100 can schedule and interleave the execution of up to 16 warps. This means that multiple warps can be executed simultaneously, improving the overall throughput of the GPU. Finally, each warp has some registers to store the data needed by the threads.

❓ How is a Warp executed?

Threads within a warp execute the same instruction simultaneously, taking advantage of SIMD execution to improve performance.

When threads within a warp do not share the same instruction, it results in divergent execution, which can degrade performance due to the need to serialize instructions. However, the check of the same instruction by the 32 threads is done dynamically by the GPU hardware. Finally, although not part of CUDA, understanding warps is critical to optimizing CUDA programs on modern NVIDIA GPUs.

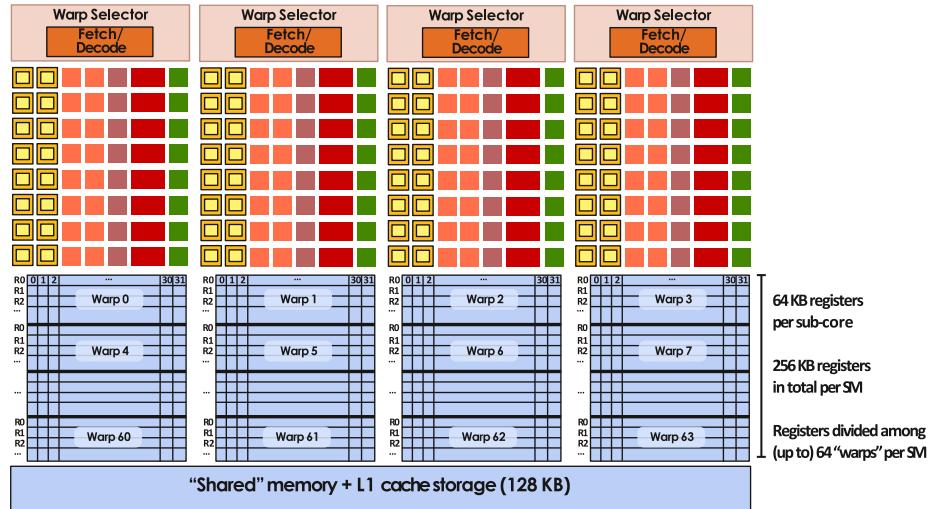


Figure 24: A NVIDIA V100 Streaming Multiprocessor (SM) architecture.

6.3.4 Running a CUDA program on a GPU

The following section is dedicated to understanding the execution of the “convolve” kernel on a fictitious dual-core GPU. In other words, we present an example of a kernel function execution (and therefore GPU-executed) that implements a convolution operation. It also aims to show why memory management is also fundamental inside the GPU.

The kernel execution requirements for this explanation are:

- Each **thread block** must execute **128 CUDA threads**.
- Each thread block must allocate $130 \times \text{sizeof}(\text{float}) = 520$ bytes of shared memory. In other words, **each thread block requires 520 bytes of shared memory**.

Let’s take an array of size N as **input** to the kernel function. When the kernel function is executed (launched), it generates thousands of thread blocks because the array size N is assumed to be very large.

```

1 #define THREADS_PER_BLK 128
2 convolve<<<(N/THREADS_PER_BLK), THREADS_PER_BLK>>>(N, input_array,
    output_array);

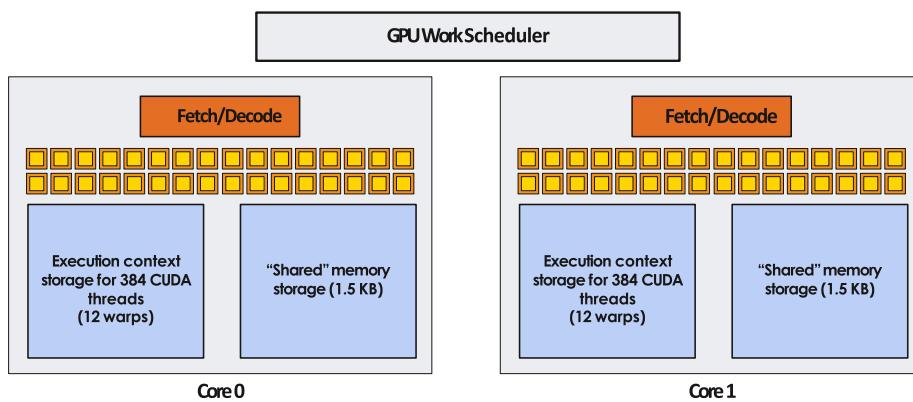
```

Where:

- $N/\text{THREADS_PER_BLK}$ is the number of blocks per grid.
- THREADS_PER_BLK is the number of threads per block, 128 CUDA threads in our case.

The **main task** of the *GPU Work Scheduler* is to **manage the two available cores** (let’s say Core 0 and Core 1), where each core has its own **Fetch/Decode** units, **execution context storage** for 384 CUDA threads (12 warps), and **shared memory storage** (1.5 KB).

Note that this architecture has fictitious cores that are smaller than V100 SM cores, with fewer execution units, less support for active warps, and less shared memory.



Execution

1. The **host (CPU)** sends a **command to the CUDA device (GPU)**, which is the execution of the kernel function.

- **Execute:** convolve (kernel function)

- **Args:**

- $N = 128'000$
- `input_array`
- `output_array`

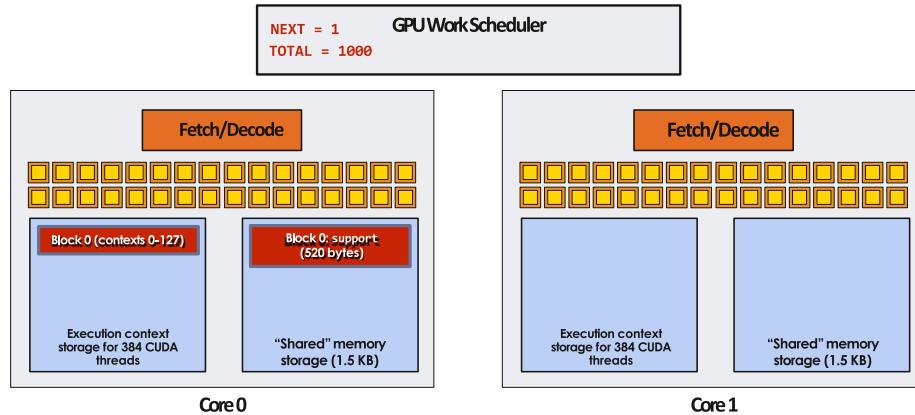
- **Number of blocks:** 1000. Note that the number of blocks is given by the formula:

$$\frac{N}{\text{THREADS_PER_BLK}}$$

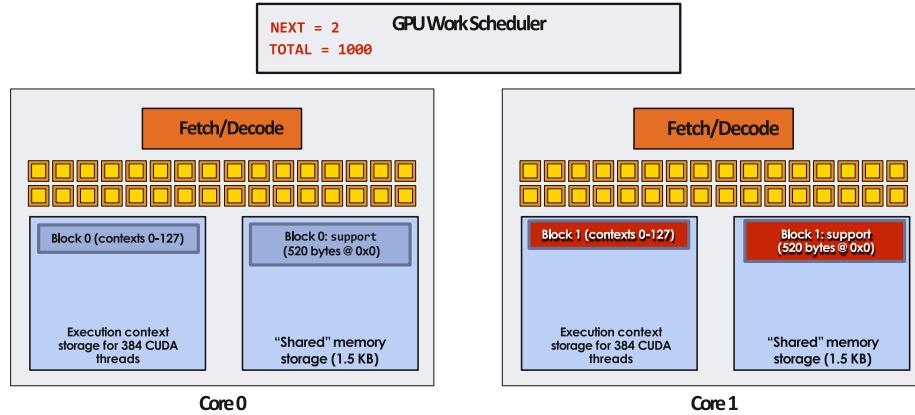
Therefore, the size of the array is easily calculated as:

$$\frac{N}{128} = 1000 \Rightarrow N = 128 \times 1000 = 128'000$$

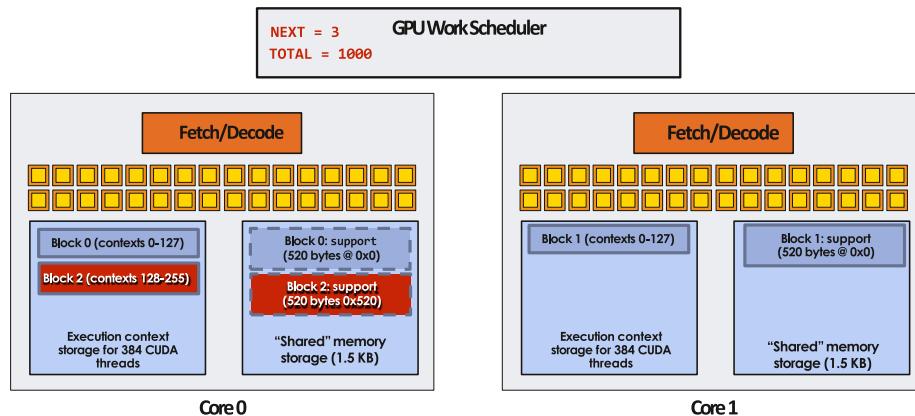
2. The *scheduler* maps *block 0* to *Core 0*, reserving execution contexts for 128 threads and 520 bytes of shared memory to meet requirements.



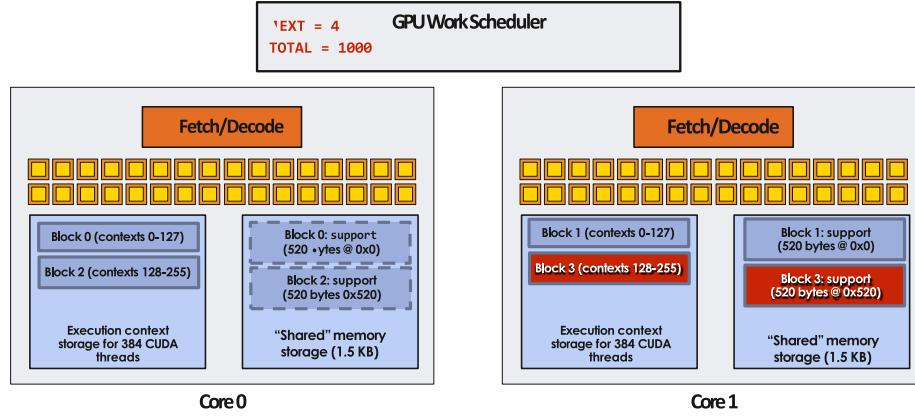
3. The *scheduler* continues to map blocks to available execution contexts, so it now maps *block 1* to *Core 1*. This shows **interleaved mapping** (where the scheduler maps blocks to available execution contexts across different cores, ensuring efficient use of resources).



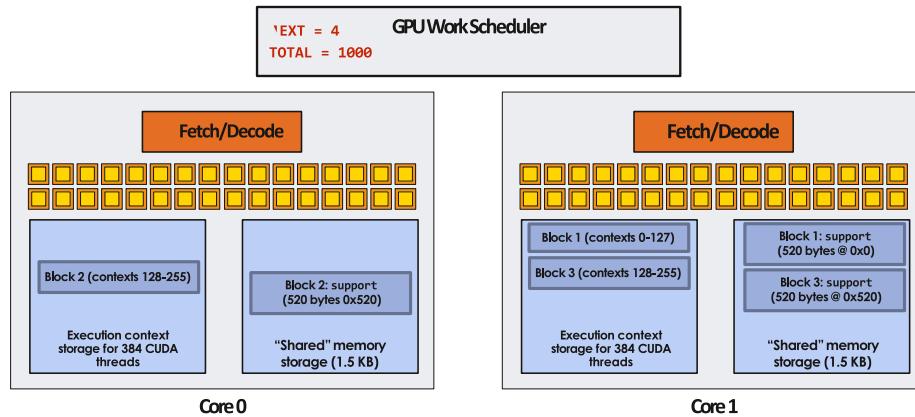
4. As in the previous step, there is a interleaved mapping phenomena. The scheduler maps *block 2* to *Core 0*. But now the shared memory (*Core 0*) is saturated because three concurrent blocks allocate $520 \text{ bytes} \times 3 = 1.56 \text{ KB} > \text{limit (1.5 KB)}$.



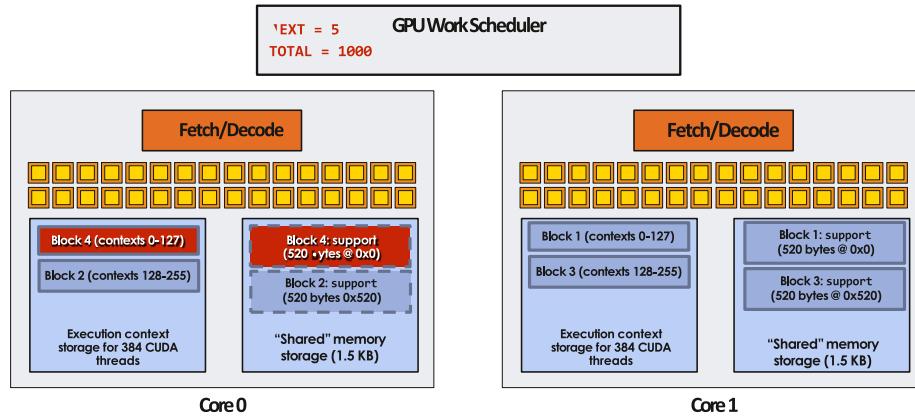
5. The scheduler assigns *block 3* to *Core 1*. But now the shared memory of *Core 1* is also **saturated**, because three concurrent blocks allocate $520 \text{ bytes} \times 3 = 1.5 \text{ KB} > \text{limit (1.5 KB)}$. So we can say that only two thread blocks fit on one core.



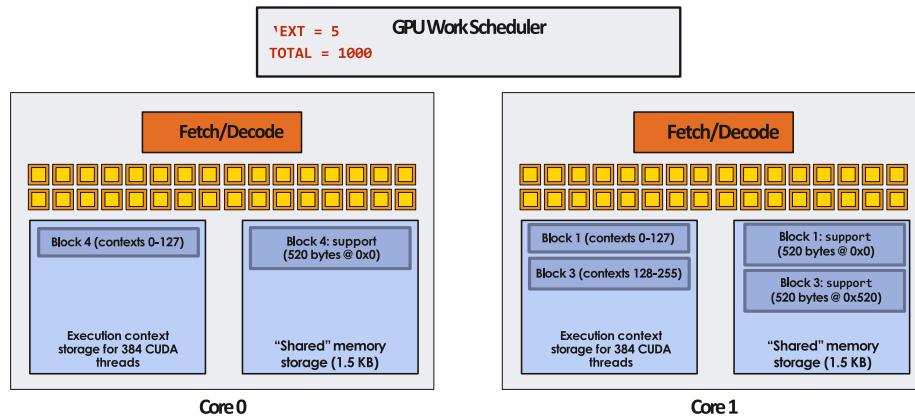
6. The scheduler waits for a task to complete on a block. The following figure shows *block 0* completing on *Core 0*. Now *Core 0* is ready to host execution blocks again.



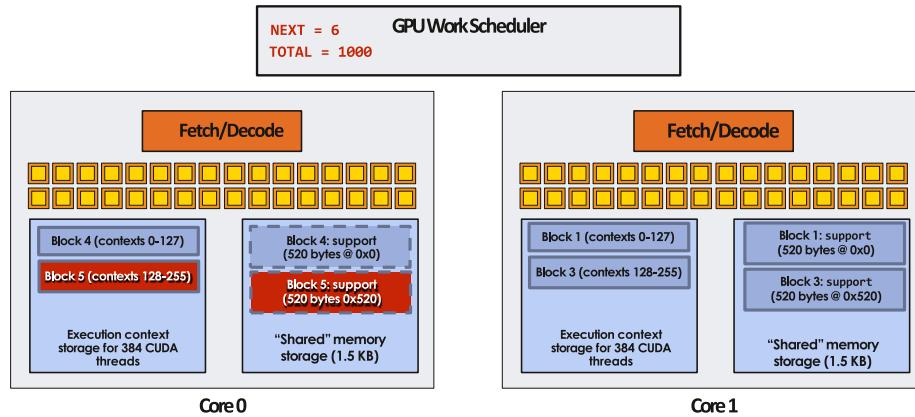
7. When the task is complete, the scheduler assigns *block 4* to *Core 0*.



8. Thread *block 2* completes on *Core 0*.



9. Finally, thread *block 5* is scheduled on *Core 0*.



The explanation is intended to illustrate a phenomenon where the GPU scheduler has to manage limited shared memory resources across multiple thread blocks.

- **Shared Memory Saturation:** When a GPU core's shared memory is fully occupied by existing thread blocks, new thread blocks cannot be scheduled until sufficient shared memory is freed up by the completion of some of the current thread blocks.
- **Idle Periods:** While the scheduler is waiting for shared memory to become available, the **GPU cores may be idle**. This doesn't mean the entire GPU is idle, but certain cores may not have new blocks to execute until resources are freed up.
- **Resource Contention:** This example shows how shared memory contention can affect the scheduling efficiency of a GPU. **Efficient use of shared memory is critical to maximizing GPU performance.**
- The concept of **resource contention**, whether it be shared memory, registers or other resources, is well known in parallel computing and GPU programming. It highlights the **importance of optimizing memory usage to avoid bottlenecks and ensure efficient execution.**

The example demonstrates how GPUs must juggle limited resources while maximizing throughput, a key aspect of parallel computing. By showing that the scheduler must wait before allocating new blocks, it emphasizes the **importance of careful resource management in kernel design and execution.**

6.3.5 Implementation of CUDA abstractions

We assume that we have a fictitious **Streaming Multiprocessor (SM)** core (Figure 23 page 111, same as NVIDIA V100, page 110) with only **four warps** of parallel execution in hardware. So there are 4 warps times 32 threads, and **128 threads** can be executed in parallel each time.

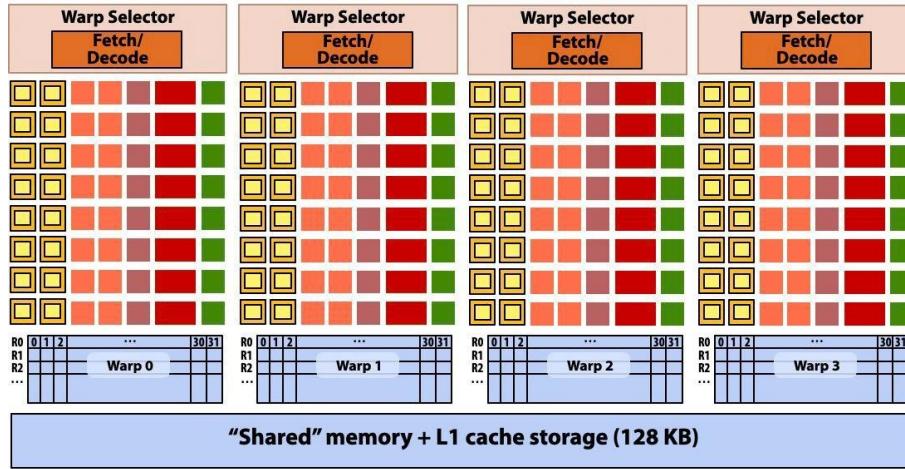


Figure 25: Streaming Multiprocessor (SM) core, with 4 warps and 128 threads in total.

❓ Why allocating all threads in a block might be inefficient

Now imagine we want to run a CUDA program where a thread **block** consists of **256 CUDA threads**, even though our GPU architecture can *only* execute 128 threads at a time. A naive implementation might execute the entire CUDA block by executing four warps (threads 0-127) to completion, and then execute the next four warps (threads 128-255) to completion. This **sequential execution of warps can lead to inefficiencies** because there may be idle periods where some warps are stalled waiting for memory accesses or synchronization points.

✅ Use interleaving execution

A good alternative is to use interleaved execution. **Interleaved execution** means that the **GPU schedules warps so that they overlap**. While some warps are waiting for memory access or synchronization, **others can execute**. This overlapping helps to **hide latencies and keep the GPU cores busy**.

However, **CUDA kernels can create dependencies between threads in a block**. To manage these dependencies, the programmer can use the function `__syncthreads()` to synchronize threads within a block. `__syncthreads()` ensures that **all threads in the block reach the synchronization point before any thread can continue**. This means that threads 128-255 cannot continue until threads 0-127 reach the synchronization point.

✓ Why interleaving execution is optimal when there are dependencies

If we run four warps to completion before starting the next four, we **may not be using shared resources** (such as shared memory and execution units) efficiently. This can also **lead to scenarios where some warps are stalled indefinitely waiting for other warps to reach synchronization points**, causing potential **deadlocks** or inefficiencies.

By interleaving the execution of warps, the GPU can better manage thread dependencies, hide latencies, and make more efficient use of its resources.

```

1 #define THREADS_PER_BLK 256
2
3 __global__ void convolve(int N, float* input, float* output)
4 {
5     __shared__ float support[THREADS_PER_BLK * 2];
6     int index = blockIdx.x * blockDim.x + threadIdx.x;
7
8     support[threadIdx.x] = input[index];
9     if (threadIdx.x < N) {
10         support[
11             THREADS_PER_BLK + threadIdx.x
12         ] = input[index + THREADS_PER_BLK];
13     }
14
15     __syncthreads();
16
17     float result = 0.0f; // thread-local
18     for (int i = 0; i < 5; i++)
19         result += support[threadIdx.x + i];
20
21     output[index] = result;
22 }
```

■ Summary

1. Thread **blocks** can be scheduled in any order by the system.
 - The system assumes no dependencies between blocks.
 - Blocks are logically concurrent, similar to ISPC tasks (Implicit SPMD Program Compiler, page 32).
2. CUDA threads in the same block run concurrently (live at the same time).
 - When a block begins executing, all threads exist and have register state allocated.
 - A CUDA thread block is an SPMD (Single Program, Multiple Data) program, similar to an ISPC gang of program instances.
 - Threads in a thread block are concurrent, cooperating “workers”.

3. CUDA implementation.

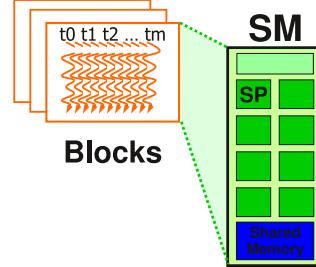
- An NVIDIA GPU warp has performance characteristics *similar* to an ISPC gang of instances.
- All warps in a thread block are scheduled onto the same SM (Streaming Multiprocessor), allowing for high-bandwidth, low-latency communication through shared memory variables.
- When all threads in a block complete, block resources (shared memory allocations, warp execution contexts) become available for the next block.

6.3.6 Advanced thread scheduling

The *main goal* of this section is to show how a CUDA kernel uses hardware execution resources: thread block allocation to execution resources, execution resource capacity constraints, and zero-overhead thread scheduling.

In general, **CUDA thread blocks execute independently and can run in any order**. The hardware is **free to assign blocks to any processor at any time**. This flexibility allows the GPU to optimize resource utilization and balance the load. A kernel (the function that runs on the GPU) scales to any number of parallel processors. This means that the same code can run efficiently on GPUs with different numbers of cores.

Thread blocks are the basic unit of work in CUDA and are assigned to SMs in block granularity (as we saw in Chapter 6.3.4). This means that a **thread block cannot be split across multiple SMs, but is executed entirely within a single SM**. Each SM has a *limit* on the number of threads and thread blocks it can support simultaneously. For example, the **Volta SM can handle up to 2048 threads**. The number of blocks an SM can hold depends on the number of threads per block:



- If a block has 256 threads, up to 8 blocks can fit ($256 \times 8 = 2048$ threads).
- If a block has 512 threads, only 4 blocks can fit ($512 \times 4 = 2048$ threads).

The **SM manages the indexes of the threads and blocks assigned to it**, enabling scheduling and execution.

■ Von Neumann model with SIMD units

The **Von Neumann Model** consists of a Control Unit, ALU (Arithmetic Logic Unit), Registers, Memory, and I/O components that work in a sequential manner. However, when we **integrate SIMD** (Single Instruction, Multiple Data) units into the model, we add the **ability to process multiple data items simultaneously using a single instruction**.

As we have explained in the section 2.1.3 page 26, SIMD allows the same operation to be performed on multiple pieces of data in parallel. This means a Control Unit (CU) sends the same instruction to multiple ALUs, each working on different data at the same time.

■ Von Neumann model with SIMD units in GPUs

The architecture uses SIMD units to execute multiple threads in parallel, making GPUs highly efficient at tasks involving large data sets, such as image processing, matrix multiplication, and other data-parallel computations.

Warps (groups of 32 threads) are executed in a SIMD fashion. All threads in a warp perform the same operation, but on different pieces of data. This is critical for speeding up computations that need to process large amounts of data in parallel. SIMD capabilities allow GPUs to efficiently handle large numbers of parallel tasks, making them far more powerful than traditional CPUs for certain workloads, such as graphics rendering and scientific computing.

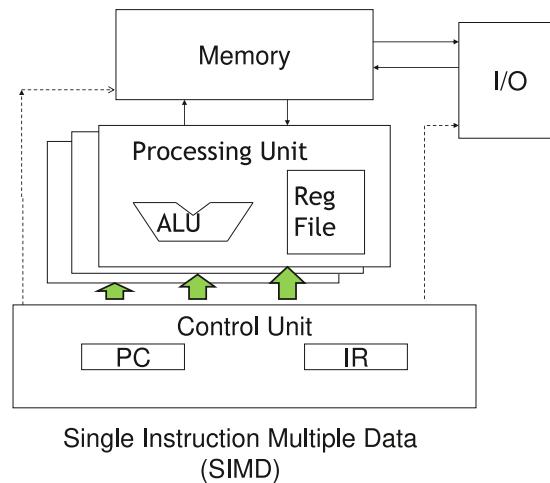


Figure 26: Von Neumann model with SIMD units.

However, these features are implementation choices, not part of the CUDA programming model. Therefore, future GPUs may have different numbers of threads in each warp.

Example 2: Warp

If 3 blocks are assigned to a Streaming Multiprocessor (SM) and each block has 256 threads, how many warps are there in an SM?

Since each warp is made up of 32 threads, if a block is made up of 256 threads, then 8 warps are required for each block:

$$\frac{\text{Threads per block}}{\text{Threads per warp}} = \frac{256}{32} = 8$$

Since the number of blocks to be allocated is 3, if each block requires 8 warps, there are $8 \times 3 = 24$ warps inside a single SM.

❑ Zero-Overhead Warp Scheduling [11]

Zero-Overhead Warp Scheduling is a feature of NVIDIA's GPUs that allows efficient management of thread execution without significant performance penalties. This mechanism is obviously implemented in the Streaming Multiprocessor architectures.

- **Warp Status.** Each warp has a state that can be:
 - **Eligible:** the warp is ready to execute its next instruction because all the *necessary operands are available* (ready for consumption).
 - **Not Eligible:** the warp cannot execute its next instruction because it is *waiting for operands or other resources*.
 - **Busy:** the warp is *currently running and executing instructions*.

Each warp also has an associated **priority value**.

- The scheduler has a **Prioritized Scheduling Policy**, so it selects:
 1. **Highest Priority and Eligible:** the scheduler first selects warps that have the highest priority and are eligible for execution.
 2. **Eligible Warps:** if no high-priority warps are eligible, the scheduler then selects from the pool of eligible warps.
- **Execution Context Switching.** After that the scheduler choose a warp to run, *how it can change the execution context from a warp to another?*
 - Before giving the answer, it is necessary to understand that the **Execution Context contains the state of all threads in a warp**, such as Program Counters, register values, and other information necessary for execution. This context is stored in hardware resources dedicated to each warp.
 - Thus zero-overhead warp scheduling takes advantage of the power of the hardware. When the warp **scheduler decides to switch** from one warp to another (to hide latency or because a warp is waiting for data), it does not need to save and restore context to and from memory. Instead, **it simply switches the execution state to the context of another warp, which is already stored in dedicated hardware**.

This means that the switching process is very fast and has virtually no overhead, hence the term “zero-overhead”!

In other words, **since all the necessary state information is already stored in the hardware, this switch is instantaneous and doesn't involve the overhead of saving/loading data to/from memory**.

❑ Advantages

- **Efficiency.** By leveraging hardware resources for context storage, CUDA ensures that switching between warps incurs virtually no overhead.
- **Latency Hiding.** This efficient scheduling helps hide latencies and keeps the GPU's computational resources fully utilized, leading to high performance.

Example 3: Matrix Multiplication on Volta Architecture

This example illustrates the impact of different block sizes on thread utilization when performing matrix multiplication using NVIDIA's Volta GPU architecture.

With the term **Block Granularity** we refer to the **number of threads per block**. Choosing the right block size is crucial for maximizing the efficiency of GPU resources.

- In general, each Streaming Multiprocessor (SM) on the Volta architecture can handle up to 2048 threads.
- 4×4 Threads per Block:
 - **Threads per Block:** 16 threads (4×4).
 - **Blocks per SM:** the GPU can accommodate up to 32 blocks per SM.
 - **Utilization:** 16 threads per block mean 2048 threads would fill 128 blocks. However, since an SM can only accommodate up to 32 blocks at a time, only 512 threads will be utilized per SM (16 threads/block \times 32 blocks), leading to under-utilization of available threads!
- 8×8 Threads per Block:
 - **Threads per Block:** 64 threads (8×8).
 - **Blocks per SM:** with 2048 threads per SM, up to 32 blocks of 64 threads each can be allocated per SM (2048 threads per SM \div 64 threads per block).
 - **Utilization:** this setup can utilize the full capacity of 2048 threads per SM, provided other resource limitations (such as shared memory or registers) are not a constraint.
- 30×30 Threads per Block:
 - **Threads per Block:** 900 threads (30×30).
 - **Blocks per SM:** with 2048 threads per SM, only two blocks of 900 threads each can be accommodated, resulting in 1800 threads (2×900), which is less than the SM's maximum capacity.
 - **Utilization:** this configuration also leads to under-utilization because it does not utilize the full 2048 thread capacity.

The example on page 125 shows some important points to emphasize:

- **Choosing Block Size:** choosing the optimal block size is critical to maximizing GPU efficiency. In the previous example, a block size of 8×8 allows full utilization of the SM's thread capacity.
- **Resource Constraints:** when choosing a block size, we must also consider other resource limitations, such as shared memory and registers, which can affect the number of threads and blocks that can be scheduled on an SM.

6.3.7 Memory and Data Locality in Depth

In CUDA programming, understanding memory and data locality is crucial for achieving high performance.

❷ Why Memory Hierarchy and Data Locality Matter

We propose a piece of code of a kernel function where there is a computation of the blur in an image:

```

1 // get the average of the surrounding 2xBLUR_SIZE x 2xBLUR_SIZE box
2 for(int blurRow = -BLUR_SIZE; blurRow < BLUR_SIZE+1; ++blurRow)
3 {
4     for(int blurCol = -BLUR_SIZE; blurCol < BLUR_SIZE+1; ++blurCol)
5     {
6         int curRow = Row + blurRow;
7         int curCol = Col + blurCol;
8         // Verify we have a valid image pixel
9         if(curRow > -1 && curRow < h && curCol > -1 && curCol < w)
10        {
11            pixVal += in[curRow * w + curCol];
12            // keep track of number of pixels
13            // in the accumulated total
14            pixels++;
15        }
16    }
17 }
18
19 // write our new pixel value out
20 out[Row * w + Col] = (unsigned char)(pixVal / pixels);

```

The **code accesses global memory for input matrix elements**. This is evident from the line 11 where `in` is likely a pointer to the global memory holding the image data. Therefore, each thread accesses global memory to read the pixel values within a `2xBLUR_SIZE x 2xBLUR_SIZE` box around the current pixel.

- Each **memory access** is 4 bytes per floating point addition.
- The **memory bandwidth** is 4 bytes per second per FLOP, 4B/s of memory bandwidth/FLOPS.

Assuming a GPU with a peak floating point rate of 1,600 GigaFLOPS and a DRAM bandwidth of 600 GB/s, then 1,600 GigaFLOPS would require 6,400 GB/s of memory bandwidth to achieve the peak FLOPS. However, the 600 GB/s memory bandwidth limits execution to 150 GFLOPS, which is only 9.3% of the peak floating point execution rate. **To get close to the 1,600 GigaFLOPS, it is necessary to drastically cut down memory accesses.**

In other words, there is an **evident Memory Bandwidth Bottleneck**. Even with a high peak floating-point rate, the actual performance can be limited by memory bandwidth (to achieve 1,600 GigaFLOPS, the kernel would require 6,400 GB/s of memory bandwidth, but the available bandwidth is only 600 GB/s).

Therefore, it is important to understand that **minimizing global memory access and optimizing data locality are essential strategies in CUDA programming to achieve high performance**. This is because accessing global memory often results in high latency, which can significantly degrade the performance of our GPU application.

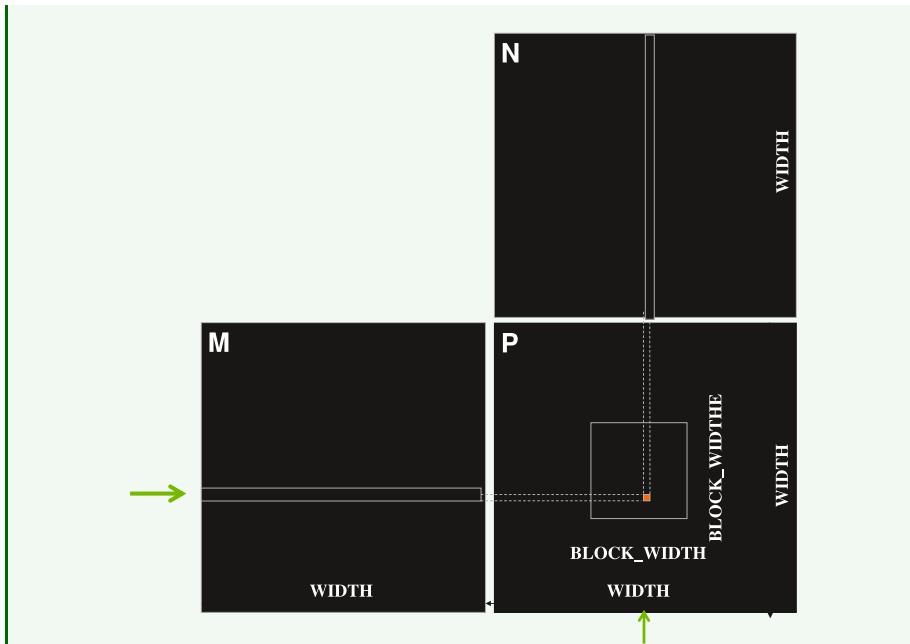
- **Global memory access.** Global memory is the largest and slowest memory available on a GPU. **Frequent accesses to global memory can cause significant performance bottlenecks due to high latency.**
- **Bandwidth and Latency.** While global memory provides high bandwidth, the latency associated with **accessing it can slow overall performance**, especially when multiple threads are accessing it simultaneously.
- **Memory hierarchy.** GPUs have a hierarchical memory structure that includes registers, shared memory, and global memory. **Using a proper memory structure** can improve performance and reduce memory bandwidth. We discussed these topics in Section 6.3.2, page 109.
- **Data Locality and Caching.** Optimizing data locality (keeping frequently accessed data close to where it is processed) is key to improving performance. **Using shared memory to cache data that is accessed multiple times by threads can significantly reduce the need for global memory accesses.**

Example 4: Matrix Multiplication

Matrices:

- Matrix M
- Matrix N
- Matrix P , the resulting matrix.

In the image below, the matrices are divided into blocks with dimensions labeled as `WIDTH` and `BLOCK_WIDTH`, indicating the block-based approach to matrix multiplication.



Arrows show the direction of data flow, indicating how elements from matrices M and N are multiplied to form matrix P .

The code used to perform the matrix multiplication is as follows:

```

1  __global__ void MatrixMulKernel(
2      float* M, float* N, float* P, int Width
3  ) {
4      // Calculate the row index of the P element and M
5      int Row = blockIdx.y * blockDim.y + threadIdx.y;
6
7      // Calculate the column index of P and N
8      int Col = blockIdx.x * blockDim.x + threadIdx.x;
9
10     int RowTimesWidth = Row * Width;
11
12     if ((Row < Width) && (Col < Width)) {
13         float Pvalue = 0;
14         // Each thread computes one element
15         // of the block sub-matrix
16         for (int k = 0; k < Width; ++k) {
17             Pvalue += M[RowTimesWidth + k] *
18                     N[k * Width + Col];
19         }
20         P[RowTimesWidth + Col] = Pvalue;
21     }
22 }
```

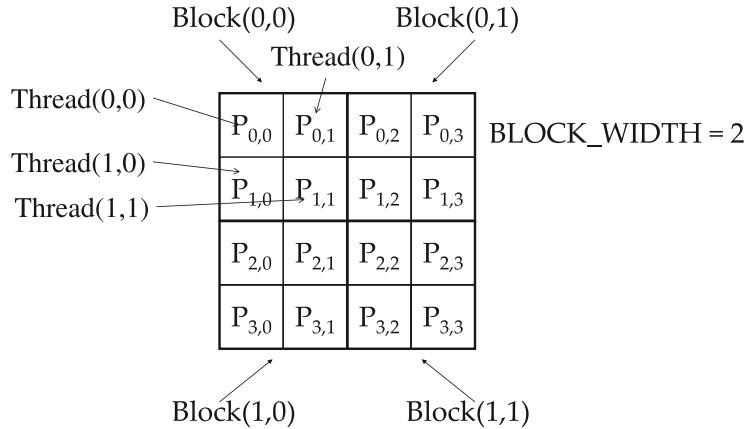
- The `MatrixMulKernel` function is defined to run on the GPU (indicated by `__global__`). It takes pointers to matrices M , N , P , and an integer `Width` representing the dimensions.
- Calculate Row Index (row 5). It calculates the row index of the current element in matrix P that the thread is responsible for.

- Calculate Column Index (row 8). It calculates the column index of the current element in matrix P .
- The if condition ensures that the thread operates within the bounds of the matrices.
- If the condition is true, initializes the `Pvalue` and iterates over the row of M and column of N . Each iteration accumulates the product of corresponding elements from M and N .
- Finally, stores the computed value in matrix P .

Since the previous example shows problems with **global memory access** (because accessing elements of matrices M , N , and P involves global memory), **optimization** (it doesn't load elements into shared memory to reduce global memory accesses), and **data locality** (because consecutive threads don't access successive memory locations), we propose an alternative.

Alternative (better) implementation of the previous example

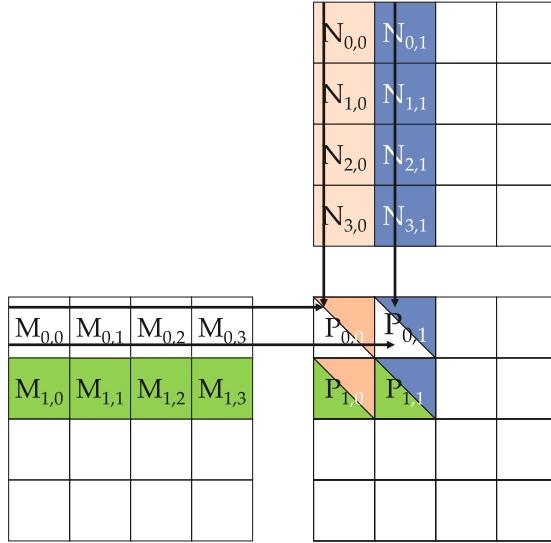
The following figure shows how threads are mapped to elements of the output matrix P during matrix multiplication. It shows how the matrix is divided into blocks and how threads handle these blocks.



- The 4×4 matrix P is divided into smaller 2×2 blocks. This division helps in managing data more efficiently and improves memory access patterns.
- The `BLOCK_WIDTH` is 2, indicating that each block contains 2×2 elements.
- Each block of the matrix P is assigned to a specific thread.
 - Block(0,0) is handled by Thread(0,0).
 - Block(0,1) is handled by Thread(0,1).
 - Block(1,0) is handled by Thread(1,0).
 - Block(1,1) is handled by Thread(1,1).

The mapping ensures that each thread is responsible for computing the values of a specific block in matrix P .

In the following figure, we illustrate an example of how elements $P_{0,0}$ and $P_{0,1}$ in matrix P are calculated using elements from matrices M and N .



- To compute $P_{0,0}$, are used elements from:
 - The first row of matrix M ($M_{0,0}, M_{0,1}, M_{0,2}, M_{0,3}$)
 - The first column of matrix N ($N_{0,0}, N_{1,0}, N_{2,0}, N_{3,0}$)

The computation involves multiplying corresponding elements and summing the results:

$$\begin{aligned} P_{0,0} = & (M_{0,0} \times N_{0,0}) + (M_{0,1} \times N_{1,0}) + \\ & (M_{0,2} \times N_{2,0}) + (M_{0,3} \times N_{3,0}) \end{aligned}$$

- To compute $P_{0,1}$, are used elements from:
 - The first row of matrix M ($M_{0,0}, M_{0,1}, M_{0,2}, M_{0,3}$)
 - The second column of matrix N ($N_{0,1}, N_{1,1}, N_{2,1}, N_{3,1}$)

The computation involves multiplying corresponding elements and summing the results:

$$\begin{aligned} P_{0,1} = & (M_{0,0} \times N_{0,1}) + (M_{0,1} \times N_{1,1}) + \\ & (M_{0,2} \times N_{2,1}) + (M_{0,3} \times N_{3,1}) \end{aligned}$$

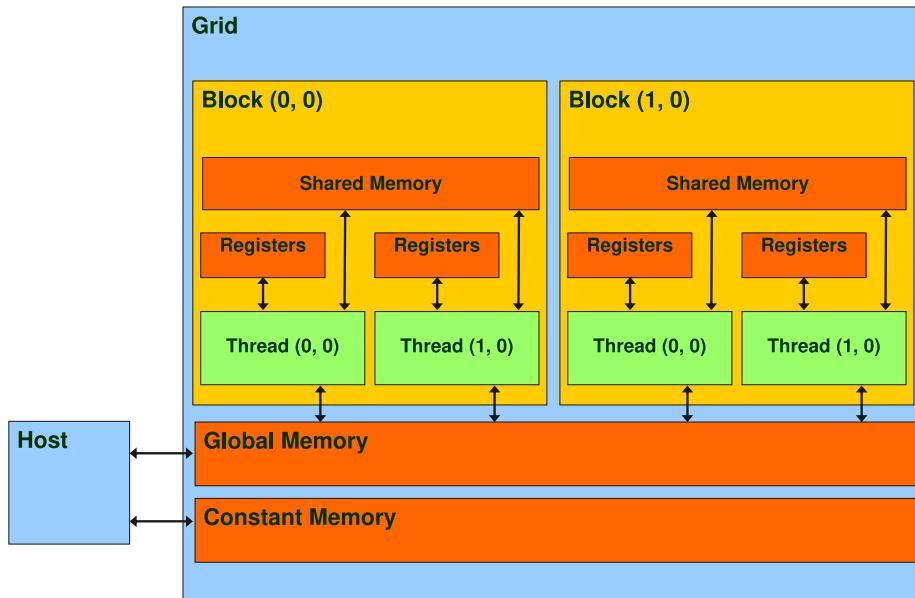
And *why is this related to storage and data locality?* For three reasons:

- ✓ **Memory Access Patterns.** By dividing the matrix into blocks and assigning specific threads to handle these blocks, the computation can take advantage of memory locality. Threads working on the same block will likely access contiguous memory locations, which improves memory access efficiency.

- ✓ **Shared Memory Usage.** Instead of repeatedly accessing global memory, threads can load the necessary block data into shared memory once. All threads in a block can then work on the data in shared memory, significantly reducing global memory access and thus improving performance.
- ✓ **Optimization for Performance.** Reducing the number of global memory accesses by optimizing data locality and using shared memory can drastically improve the performance of the matrix multiplication kernel. Efficient mapping of threads to data and making use of memory hierarchy are key strategies for achieving high performance in CUDA applications.

Memory/Data Locality are fundamental, now we go deep

Before explaining the keywords used by CUDA, it is important to understand the **programmer's view** of CUDA memory.



- **Host (CPU) Memory.** Connected to the GPU, but *distinct* from GPU memory.
- **Global Memory.** Accessible by *all threads* and the *host*. It's the *largest* but also the *slowest* type of memory.
- **Constant Memory.** Read-only memory accessible by *all* threads.
- **Grid.** Contains multiple blocks, each consisting of threads (we have already seen and discussed this in Section 2, page 104).
- **Shared Memory.** Faster than global memory, shared among threads within the same block.

- **Registers.** The fastest memory, local to each thread.

CUDA has specific declarations for different types of memory:

- **Local Variable** (`int LocalVar`): stored in registers, scope and lifetime are per thread.
- **Shared Variable** (`__shared__ int SharedVar`): stored in shared memory, scope and lifetime are per block.
- **Global Variable** (`__device__ int GlobalVar`): stored in global memory, scope is the entire grid, and the lifetime is the duration of the application.
- **Constant Variable** (`__constant__ int ConstantVar`): stored in constant memory, scope is the grid, and the lifetime is the duration of the application.

Variable Declaration	Memory	Scope	Lifetime
<code>int LocalVar</code>	Register	Thread	Thread
<code>__shared__ int SharedVar</code>	Shared	Block	Block
<code>__device__ int GlobalVar</code>	Global	Grid	Application
<code>__constant__ int ConstantVar</code>	Constant	Grid	Application

Table 4: CUDA Memory Types, Scope, and Lifetime.

There are two observations to make:

1. The `__device__` keyword is **optional** when used with `__shared__` or `__constant__`.
2. The **automatic variables** (those that are automatically managed by the compiler) **reside in the registers** because they are the fastest type of memory available to threads (very close to the processor).

However, there's an **exception for per-thread arrays**: when we declare an array inside a kernel function, this array is unique to each thread and can be relatively large. Due to their size and potential complexity, these **per-thread arrays cannot fit in registers and are stored in global memory instead**.

② And how do I decide where to put the variables?

It depends on the implementation. The good question to ask is: *can the host access the declared variable?*

- If the **host can access the variable**, it should be a **global** or **constant** variable.
 - Declared outside of a function;
 - Accessible by both host (CPU) and device (GPU).

- If the **host cannot access the variable**, it should be a **register** or **shared** variable.
 - Declared inside the kernel function;
 - If it is **shared**, it is accessible to all threads within the same block;
 - On the other hand, if it is **register** (local), it is only accessible to individual threads, suitable for frequently accessed variables.

■ In-depth analysis of shared memory in CUDA

Some features of shared memory in CUDA:

1. **Shared Memory in Each Streaming Multiprocessor (SM)**. Each SM in a CUDA GPU has its own shared memory.
Shared memory is significantly faster than global memory, both in terms of latency and throughput.
2. **Scope and Lifetime**. The scope of shared memory is limited to the block; only threads within the same block can access the same shared memory.
The lifetime of shared memory is tied to the thread block's lifetime. Once the block finishes execution, the shared memory is released.
3. **Access**. Shared memory is accessed using memory load/store instructions.
It acts as a scratchpad memory in the computer architecture, allowing threads to quickly share and exchange data.

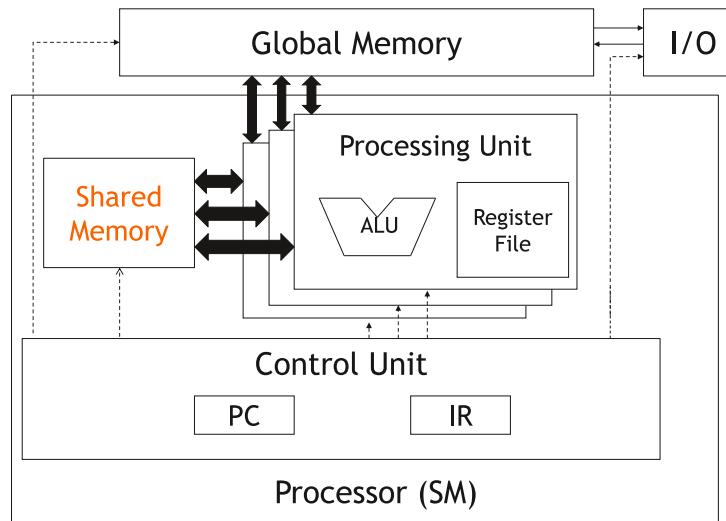


Figure 27: Hardware View of CUDA Memories.

- **Global Memory.** A large memory space accessible by all threads and the host (CPU). Used for data storage and retrieval but has higher latency compared to shared memory.
- **Shared Memory.** A smaller, faster memory space within each SM. Used for data sharing among threads in the same block.
- **Processing Unit.** Contains Arithmetic Logic Units (ALUs) and a Register File for performing computations. Registers are the fastest type of memory, used for storing per-thread data.
- **Control Unit.** Manages the execution of instructions, including the Program Counter (PC) and Instruction Register (IR).
- **I/O.** Represents input/output operations.

6.3.8 Tiling Technique

The **Tiling Technique**, also known as *blocked matrix multiplication* or *tiling*, is a strategy to **enhance the performance of matrix computations by optimizing memory access patterns**. It leverages shared memory in CUDA to reduce the number of global memory accesses, improving efficiency and throughput. A lot of parallel algorithms adopt the tiling technique.

💡 What is Tiling?

Tiling can be thought of as the **process of breaking a large matrix into smaller sub-matrices** (called *tiles* or blocks) **that can fit into faster, limited shared memory**. The main goal of tiling is to:

- Minimize slower **global memory accesses**;
- Maximize the **use of faster shared memory**.

Instead of a large matrix, we can think of the global memory contents as tiles and focus the computation of CUDA threads on one or a small number of tiles at a time.

💡 Great analogy to understand the basic concept of Tiling

Reducing the number of vehicles in a congested traffic system can significantly improve the delays experienced by all vehicles. This is analogous to carpooling for commuters. We can image:

- **Drivers**: represent *threads accessing their memory data operands*.
- **Cars**: represent *memory access requests*.

Just as carpooling reduces the number of cars on the road, tiling reduces the number of global memory accesses by loading data into shared memory. The result is a reduction in traffic (memory access requests) and an obvious improvement in overall efficiency.

Unfortunately, just like in real life, there are some problems. For example, there are the **challenges of carpooling**. In fact, some carpools are easier to organize than others because the participants need to have similar work schedules. So certain vehicles may be more suitable for carpooling. However, other **commuters may have different needs, so the number of carpools may increase and there is a risk of increasing traffic again**.

Similar challenges exist in tiling calculations. Some computations may be easier to tile based on data access patterns and the nature of the computation. **Organizing data and computations efficiently to fit into tiles may be more challenging for certain algorithms**. So what is the **general heuristic to adopt in order to use the tiling technique** or not? In general, it is:

- ✓ **Good** to use tiling when people have similar schedule. In computing, is good when threads have *similar access timing*.

✗ Bad to use tiling when people have very different schedule. In computing, is bad **when threads have very *different timing***.

But synchronization is also important. Just as workers' schedules must be aligned for effective carpooling, **threads** in parallel computing **must be synchronized** for efficient execution. Efficient data access and memory usage depend on synchronized operations to reduce latency and improve performance. The following figure shows synchronization between multiple threads.

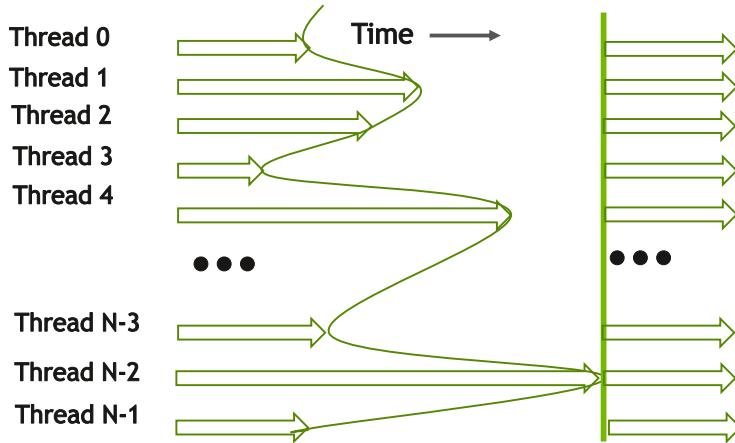


Figure 28: Barrier Synchronization for Tiling.

On the left are multiple threads (Thread 0, Thread 1, Thread 2, ..., Thread N-1) progressing over time. The wave represents when each thread reaches its barrier on the code; at that point, the thread must wait until all threads have arrived before it can continue. This ensures that all threads are synchronized at certain points during execution. Execution of all threads resumes when the last thread (in the picture, Thread N-2) reaches the barrier.

Synchronization is fundamental in the tiling technique because it ensures that all threads have their share of data loaded into shared memory and are ready to proceed before they perform any computations. After the computation, they use another synchronization barrier to ensure that all threads have completed their work before moving to the next tile.

❖ Summary - How it works

1. **Identify a Tile.** Determine a section of global memory content that multiple threads will access. Dividing the workload into smaller tiles allows for efficient memory access and utilization of shared memory.
2. **Load the Tile.** Transfer the tile from global memory to on-chip memory (shared memory). Loading data into shared memory reduces the latency associated with accessing global memory.
3. **Barrier Synchronization.** Ensures that all threads are ready to begin the computation phase. It also ensures that all threads have the necessary data before starting the computation.
4. **Access Data.** Multiple threads access their data from the on-chip memory. Threads perform computations using the data stored in shared memory, benefiting from its faster access time.
5. **Barrier Synchronization.** Ensure all threads have completed the current phase (computations).
6. **Next Tile.** Move on to the next tile and repeat the process.

✓ Advantages

- ✓ **Improved Memory Access Patterns.** By loading data into shared memory, the number of global memory accesses is reduced, leading to better performance.
- ✓ **Higher Computational Throughput.** Tiling helps achieve higher computational throughput by leveraging the faster shared memory.

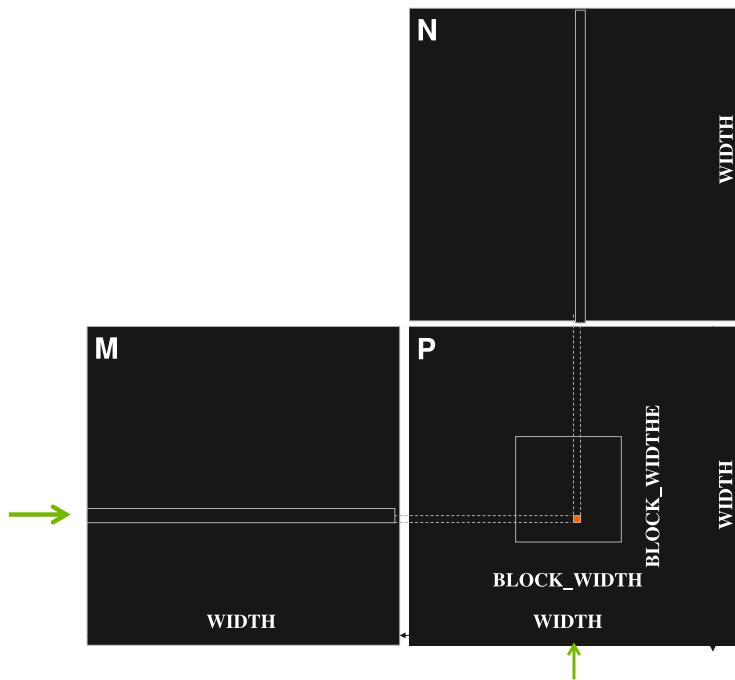
6.3.8.1 Tiled Matrix Multiplication

In the following section, we present an **example of matrix multiplication using the tiling technique** to illustrate the efficiency improvements brought about by the tiling technique.

▣ Traditional approach

Classical matrix multiplication has the following characteristics

- Each thread accesses a row of matrix M and a column of matrix N .
- Each thread block accesses a strip of matrix M and a strip of matrix N .



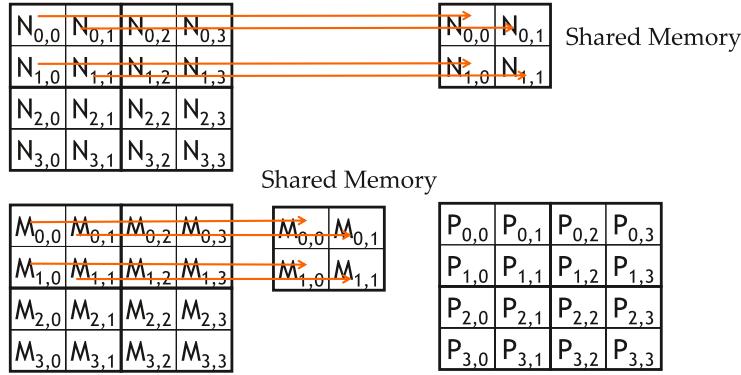
In the traditional approach, threads access data directly from global memory, which can be inefficient due to high latency.

▣ Tiled Matrix Multiplication

1. As a **first step**, since we know the problem and (ideally) how the solution is implemented, we can try to brainstorm on how to apply tiling. In general, matrix operations lend themselves well to the tiling technique. In the matrix multiplication:
 - The execution of each thread can be broken into phases.
 - The data accesses by the thread block in each phase are focused on one tile of M and one tile of N .
 - The tile is of **BLOCK_SIZE** elements in each dimension.

2. As second step, the threads in a block participate in **loading items into shared memory using the tiling technique**.

As we can see in the following figure, each thread in a block contributes to loading elements from matrices M and N into shared memory. Each thread is responsible for loading one element from the M matrix and one element from the N matrix into shared memory. This parallel loading ensures that data is moved quickly and efficiently from global memory to faster shared memory.

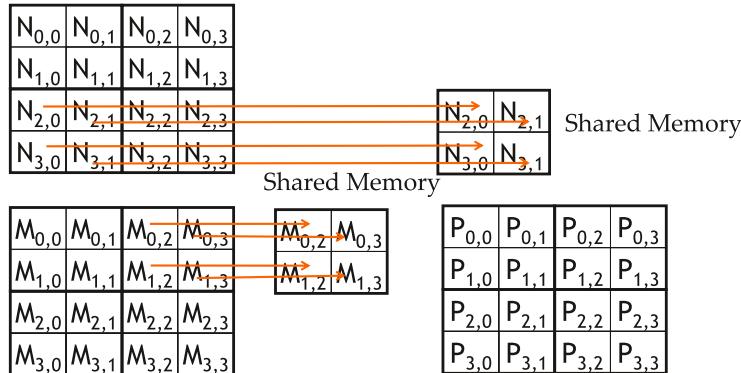


The matrices M and N are the input and P is the result matrix. Each element is indexed by i and j (e.g., M_{ij}).

- Elements of M are loaded into shared memory by the threads.
- Similarly, elements of N are loaded into shared memory.

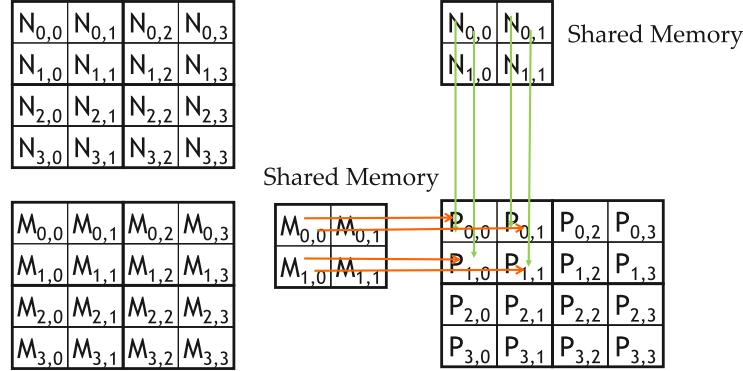
This particular phase focuses on the initial loading of elements for the tile corresponding to block (0,0). Each thread in block (0,0) loads one element from M and one from N , populating the shared memory with the necessary data for the computation.

These steps are performed for each block of the matrices. In the following figure, we can see another step of loading into shared memory.

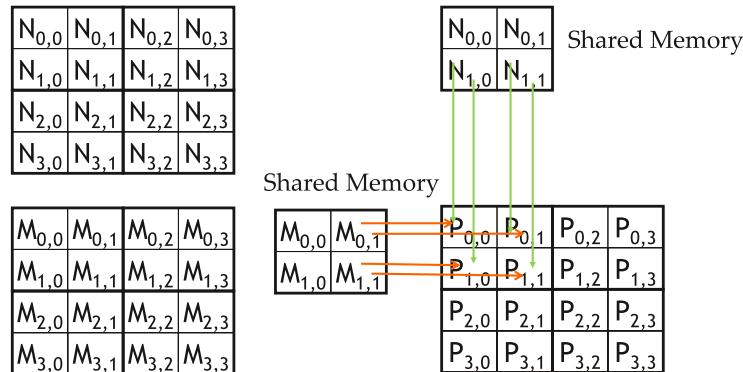


3. After loading into shared memory, there is the loading step, done in two iterations, into the result matrix. This is only a graphical representation, because in reality this step can be merged with the execution step. This step prepares the operands to be used in the execution step.

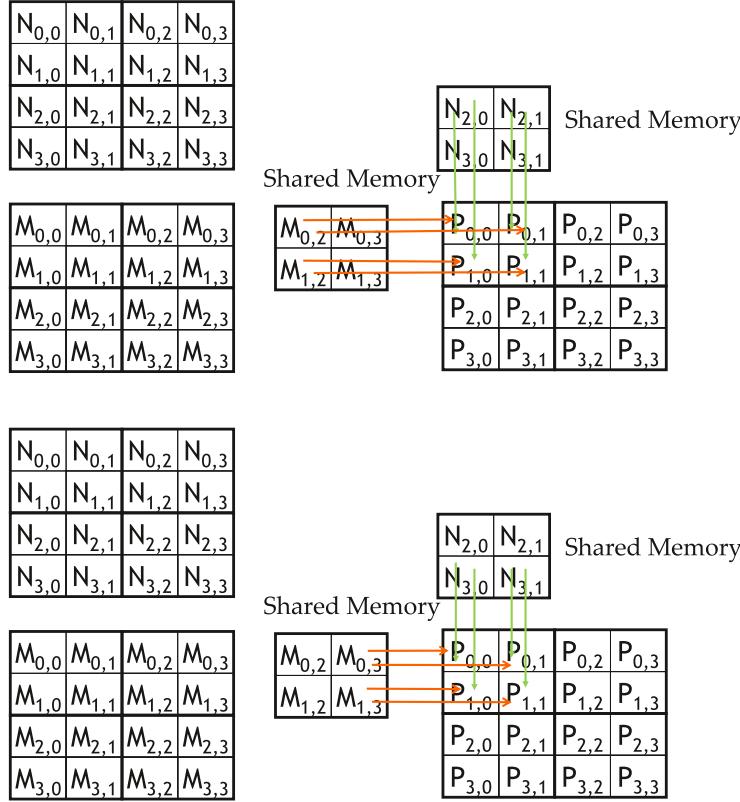
- (a) In the first iteration, the elements from the shared memory $N_{0,0}, N_{0,1}$ are used against the element from the shared memory $M_{0,0}, M_{1,0}$.



- (b) In the second iteration, the elements from the shared memory $N_{1,0}, N_{1,1}$ are used against the element from the shared memory $M_{0,1}, M_{1,1}$.



This step is done for each block of the matrices. In the figure on the next page, we can see two more steps of the load block.



4. After each load, there is the **execution phase**. In the following table, we highlight how four threads in a block perform matrix multiplication using shared memory.

	Phase 0			Phase 1		
thread _{0,0}	$\mathbf{M}_{0,0}$ ↓ $Mds_{0,0}$	$\mathbf{N}_{0,0}$ ↓ $Nds_{0,0}$	$PValue_{0,0} +=$ $Mds_{0,0} * Nds_{0,0} +$ $Mds_{0,1} * Nds_{1,0}$	$\mathbf{M}_{0,2}$ ↓ $Mds_{0,0}$	$\mathbf{N}_{2,0}$ ↓ $Nds_{0,0}$	$PValue_{0,0} +=$ $Mds_{0,0} * Nds_{0,0} +$ $Mds_{0,1} * Nds_{1,0}$
thread _{0,1}	$\mathbf{M}_{0,1}$ ↓ $Mds_{0,1}$	$\mathbf{N}_{0,1}$ ↓ $Nds_{1,0}$	$PValue_{0,1} +=$ $Mds_{0,0} * Nds_{0,1} +$ $Mds_{0,1} * Nds_{1,1}$	$\mathbf{M}_{0,3}$ ↓ $Mds_{0,1}$	$\mathbf{N}_{2,1}$ ↓ $Nds_{0,1}$	$PValue_{0,1} +=$ $Mds_{0,0} * Nds_{0,1} +$ $Mds_{0,1} * Nds_{1,1}$
thread _{1,0}	$\mathbf{M}_{1,0}$ ↓ $Mds_{1,0}$	$\mathbf{N}_{1,0}$ ↓ $Nds_{1,0}$	$PValue_{1,0} +=$ $Mds_{1,0} * Nds_{0,0} +$ $Mds_{1,1} * Nds_{1,0}$	$\mathbf{M}_{1,2}$ ↓ $Mds_{1,0}$	$\mathbf{N}_{3,0}$ ↓ $Nds_{1,0}$	$PValue_{1,0} +=$ $Mds_{1,0} * Nds_{0,0} +$ $Mds_{1,1} * Nds_{1,0}$
thread _{1,1}	$\mathbf{M}_{1,1}$ ↓ $Mds_{1,1}$	$\mathbf{N}_{1,1}$ ↓ $Nds_{1,1}$	$PValue_{1,1} +=$ $Mds_{1,0} * Nds_{0,1} +$ $Mds_{1,1} * Nds_{1,1}$	$\mathbf{M}_{1,3}$ ↓ $Mds_{1,1}$	$\mathbf{N}_{3,1}$ ↓ $Nds_{1,1}$	$PValue_{1,1} +=$ $Mds_{1,0} * Nds_{0,1} +$ $Mds_{1,1} * Nds_{1,1}$

time →

The values $Mds_{i,j}$ are the values loaded in the load step from global to shared memory of the M matrix. The same reasoning applies to Nds .

Once the data is loaded into shared memory, each thread uses the loaded values to update the partial product value $PValue$ for the resulting matrix P :

- **thread_{0,0}:**

```
1 PValue0,0 += Mds0,0 * Nds0,0 + Mds0,1 * Nds1,0
```

- **thread_{0,1}:**

```
1 PValue0,1 += Mds0,0 * Nds0,1 + Mds0,1 * Nds1,1
```

- **thread_{1,0}:**

```
1 PValue1,0 += Mds1,0 * Nds0,0 + Mds1,1 * Nds1,0
```

- **thread_{1,1}:**

```
1 PValue1,1 += Mds1,0 * Nds0,1 + Mds1,1 * Nds1,1
```

5. **Barrier Synchronization step.** To synchronize all threads in a block, we use the function `__syncthreads()`. This function acts as a barrier synchronization, ensuring that all threads in a block reach that point before any thread can proceed.

Barrier synchronization is particularly useful for coordinating the execution of tiled algorithms in phases. It is useful in:

- ✓ **Loading Phase.** Ensures that **all elements of a tile are loaded into shared memory before any computation begins**.
- ✓ **Computation Phase.** Ensures that **all threads have completed their computation on the current tile before moving to the next phase or tile**.

6.3.8.2 Implementation Tiled Matrix Multiplication

After a great explanation of how to apply the tiling technique to the matrix multiplication operation, we present the CUDA implementation. As we said, the goal of tiled matrix multiplication with CUDA is to optimize the matrix multiplication process by taking advantage of shared memory (small, fast memory space accessible to all threads within the same block).

■ Introduction to the implementation

The main core of the implementation is about **tile indexing**. It can be 1 or 2 dimensional. In the following figures, to understand the logic, we show 2 dimensional indexing, which is more natural.

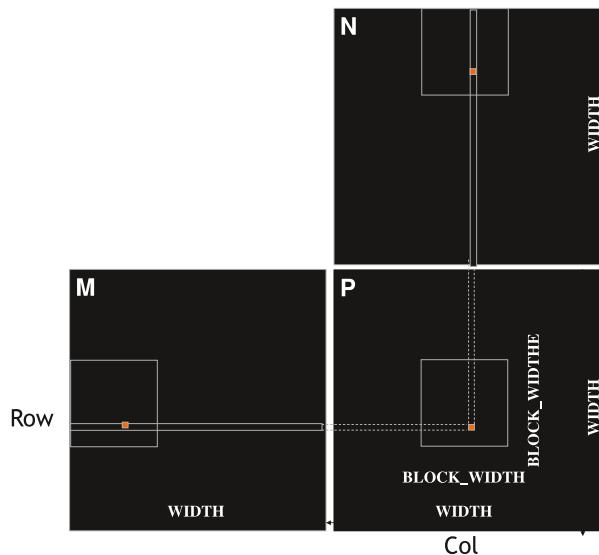
- When each thread loads the input from the original matrix (M or N), it needs an index.

```

1 int Row = by * blockDim.y + ty;
2 int Col = bx * blockDim.x + tx;
3 // 2D indexing for accessing Tile 0:
4     M[Row][tx]
5     N[ty][Col]
```

At each iteration, each thread in a block considers the same row of the matrix M and the same column of the matrix N . To distribute the workload among the threads, we assign each thread to each column of the matrix M (using the unique index tx , the index of the thread in the CUDA block), and with the same reasoning, we assign each thread to each row of the matrix N . The row in the matrix M is fixed, but the column is taken entirely by the assignment of all threads in the block (using tx).

In the following figure, we see that the row in the matrix M is fixed, but the column is fully occupied thanks to the assignment of all threads in the block (using tx).



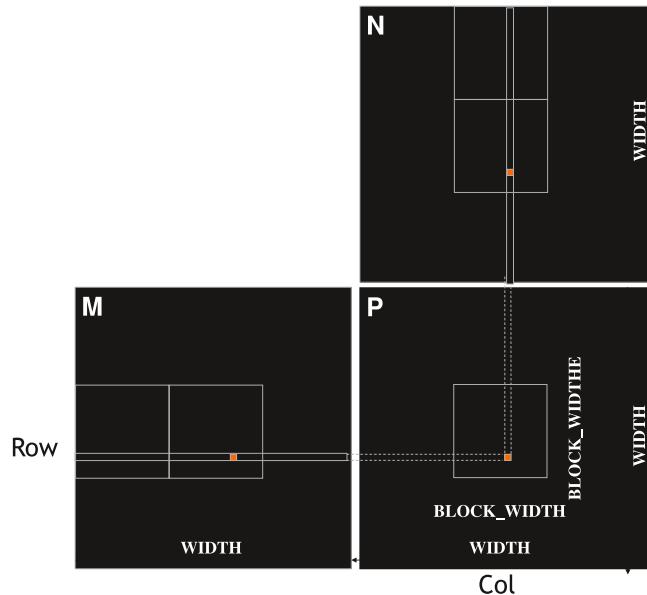
When the first phase is finished, we move on to the second tile. For this reason, we use a kind of **offset** given by the formula $p \times \text{TILE_WIDTH}$ ($\text{TILE_WIDTH} = \text{BLOCK_WIDTH}$), where p is the number of the phase (at the beginning zero, then one, and so on).

In the following image, we see that at phase 1, to load the tile 1, we use the formula:

```

1 // 2D indexing for accessing Tile 1:
2 M[Row][1 * TILE_WIDTH + tx]
3 N[1 * TILE_WIDTH + ty][Col]

```



✖ CUDA code

```

1 #define TILE_WIDTH 16
2
3 __global__ void MatrixMulKernel(
4     float* M, float* N, float* P, int Width
5 ) {
6     __shared__ float ds_M[TILE_WIDTH][TILE_WIDTH];
7     __shared__ float ds_N[TILE_WIDTH][TILE_WIDTH];
8
9     int bx = blockIdx.x; int by = blockIdx.y;
10    int tx = threadIdx.x; int ty = threadIdx.y;
11
12    int Row = by * blockDim.y + ty;
13    int Col = bx * blockDim.x + tx;
14    float Pvalue = 0;
15
16    // Loop over the M and N tiles
17    // required to compute the P element
18    int bound = Width / TILE_WIDTH;
19    for (int p = 0; p < bound; ++p) {
20        // Collaborative loading of M and N tiles
21        // into shared memory
22        ds_M[ty][tx] = M[Row * Width + p * TILE_WIDTH + tx];
23        // = M[Row][p * TILE_WIDTH + tx]
24        ds_N[ty][tx] = N[(p * TILE_WIDTH + ty) * Width + Col];
25        // = N[p * TILE_WIDTH + ty][Col]
26        __syncthreads();
27
28        for (int i = 0; i < TILE_WIDTH; ++i) {
29            Pvalue += ds_M[ty][i] * ds_N[i][tx];
30        }
31        __syncthreads();
32    }
33    P[Row * Width + Col] = Pvalue;
34 }
```

- **Index variables:**

- **bx** and **by** are the **block indices** in the *x* and *y* directions.
- **tx** and **ty** are the **thread indices** within a block.
- **Row** and **Col** are the **row and column indices of the element in the output matrix**.

- **Main Loop:** **ph** (phase) determines which tile is currently being processed.

- **Loading Tiles into Shared Memory:**

- Each thread loads one element from the current tile of *M* and *N* into shared memory.
- **ds_M[ty][tx]** loads an element from *M*.
- **ds_N[ty][tx]** loads an element from *N*.
- **__syncthreads()** is called to ensure all threads have loaded their elements before proceeding.

- **Matrix Multiplication within a Tile.** Once the tiles are loaded into shared memory, each thread computes the **partial dot product for the corresponding element** in the output matrix. The nested loop (second for loop) accumulates the product of elements from M and N .
- Finally, after all tiles have been processed, the **final value is stored** in the output matrix P .

✓ Final consideration

A bigger block is better and the reason is simple. **Bigger tiles mean more threads per block.** For example:

- TILE_WIDTH of 16 results in $16 \times 16 = 256$ threads per block.
- TILE_WIDTH of 32 results in $32 \times 32 = 1024$ threads per block.

Therefore, the *workload* per phase is:

- TILE_WIDTH = 16:
 - Each block performs 512 (2×256) float loads from global memory.
 - Executes **8192** ($256 \times (2 \times 16)$) **multiply-add operations** (16 floating point operations for each memory load).
- TILE_WIDTH = 32:
 - Each block performs 2048 (2×1024) float loads from global memory.
 - Executes **65536** ($1024 \times (2 \times 32)$) **multiply-add operations** (32 floating point operations for each memory load).

Although a larger block might be better, shared memory is not infinite. It depends on the implementation. If we take a classic Streaming Multiprocessor (SM) with 16KB of shared memory, when we do a memory usage analysis:

- TILE_WIDTH = 16:
 - Uses **2KB** ($2 \times 256 \times 4B$) of shared memory per block.
 - Allows up to **8 blocks per SM** in parallel execution (8×256 threads = 2048 threads).

This allows up to 4096 (8×512) pending loads (2 per thread, 256 per block).

- TILE_WIDTH = 32:
 - Uses **8KB** ($2 \times 32 \times 32 \times 4B$) of shared memory per block.
 - Allows up to **2 blocks per SM** in parallel execution (limited by thread count). If a GPU limits the thread count to 1536 threads per SM, the number of blocks per SM is reduced to one!

Using `__syncthreads()` ensures that all threads reach a barrier before continuing, which can **temporarily reduce the number of active threads**. More blocks per SM can be beneficial to balance memory usage and thread count.

In summary, **choosing the right tile size and efficiently managing shared memory and threads are critical to optimizing GPU performance**. This balance affects how many operations can be performed simultaneously and how effectively memory is used.

6.3.8.3 Any size matrix handling

The following paragraph focuses on the challenge of **handling matrix multiplication for matrices of arbitrary size** using tiled matrix multiplication. Often, real-world applications require support for matrices that aren't always square or multiples of the tile width (`TILE_WIDTH`). So *what is the strategy for these situations?*

A Limitations of the Tiled Matrix Multiplication presented

- The base kernel can only handle square matrices whose dimensions are multiples of `TILE_WIDTH`. This is a major limitation, since non-square matrices also exist.
- A possible solution would be to apply padding, but padding these matrices to match tile sizes can result in **significant overhead** in terms of space and data transfer time.

The basic idea is that instead of padding, a different technique is proposed to efficiently handle matrices of arbitrary size and prevent access to invalid memory locations.

Example 5: graphical example

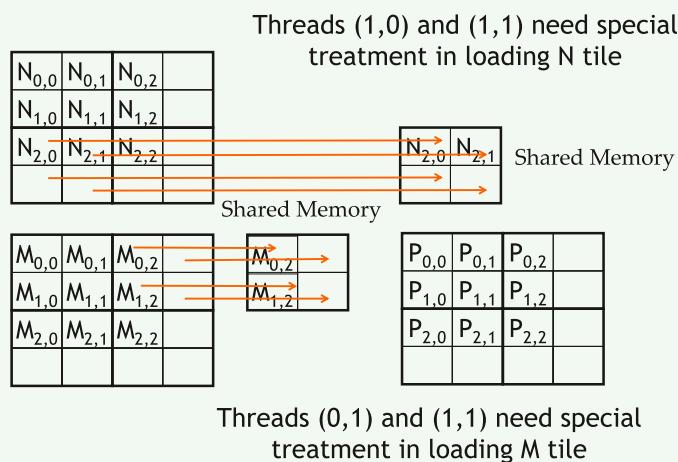
Imagine we have a 3×3 matrix and a `TILE_WIDTH` of 2. The grid and block configuration may result in some threads accessing positions outside the bounds of the 3×3 matrix.

The basic logic will be:

- Threads within the valid range of the matrix will load elements normally.
- Threads outside the valid range need conditions to avoid accessing or writing to out-of-bound elements.

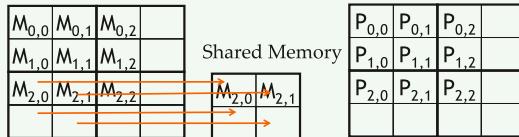
Graphically, the special cases that we need to deal with will be:

- When we load elements from the matrix into shared memory:



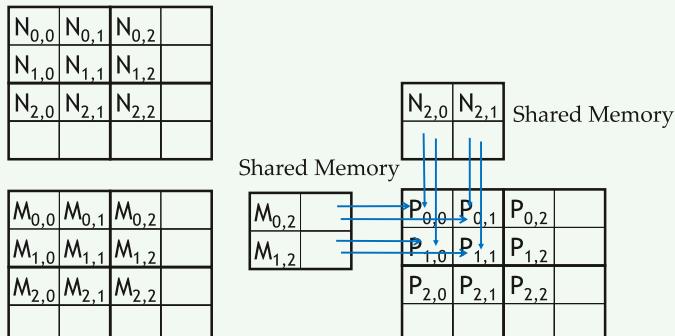
And:

Threads (0,1) and (1,1) need special treatment in loading N tile



Threads (1,0) and (1,1) need special treatment in loading M tile

- If we use the out-of-bound elements to calculate the result:



All Threads need special treatment.
None of them should introduce invalidate contributions to their P elements.

✓ A simple solution

The main simple and efficient solution is the **conditional loading**. When a thread is to load an input element, **check if the element index is within valid bounds**:

- If valid: proceed to **load the element**.
- If invalid: do not load the element, but **write a 0** instead.

Assigning a 0 value ensures that the multiply-add step does not affect the final value of the output element. Also, this simple check helps avoid errors from invalid memory access and ensures correctness.

❓ Why do we have to load a zero? We cannot skip this part directly?

1. **Safety and Validity.** When a thread loads a 0 instead of an out-of-bound element, it **ensures that the memory access is valid and does not cause runtime errors or fetch garbage data**. This is crucial for maintaining the stability of the program.
2. **Maintaining Computational Integrity.** The 0 value acts as a **neutral element in multiplication**, meaning it does not alter the final sum during the multiply-add operations. For example:

```
sum += tileM[ty][k] * tileN[k][tx]
```

If `tileM[ty][k]` or `tileN[k][tx]` is 0, the sum remains unaffected by that particular operation, preserving the integrity of the calculation.

3. **Facilitating Parallel Execution.** In CUDA, all threads in a warp (a group of 32 threads) execute the same instruction simultaneously. If some threads are out-of-bound, they still participate in the synchronization and memory access patterns, **ensuring the warp executes efficiently without branching or divergence**.

By loading 0s, these threads can reach synchronization points (such as `__syncthreads()`) together with other threads that load valid data, ensuring all threads in the block stay in sync.

A **Warp Divergence** occurs when **threads in a warp follow different execution paths due to conditional branches**. This means some threads are active while others are idle, leading to inefficiency. The consequences of warp divergence can have a **severe impact on performance**.

- *Performance Impact:*
 - Divergence causes **some threads to stall while others execute**, reducing the overall throughput.
 - **All threads in the warp must eventually reconverge to execute the same instructions again**, prolonging the execution time for that warp.
- *Synchronization Issues:*
 - Skipping `__syncthreads()` or similar synchronization points can lead to incorrect behavior. Threads that do not wait might access incomplete or inconsistent data in shared memory, leading to incorrect results.
 - **Ensuring that all threads in a warp (or block) reach synchronization points is crucial for data consistency.**

4. **Shared Memory Utilization.** All threads, including those loading 0s, contribute to filling the shared memory tiles. This **ensures that when the matrix multiplication is performed, the shared memory contains a complete tile** (with zeros in out-of-bound areas). This approach guarantees that the shared memory tiles are properly used for the block's calculations, and threads with 0 elements help maintain the structure and coherence of these tiles.

Implementation

The code snippet is as follows:

```

1 for (int p = 0; p < (Width - 1) / TILE_WIDTH + 1; ++p) {
2     if (Row < Width && p * TILE_WIDTH + tx < Width) {
3         ds_M[ty][tx] = M[Row * Width + p * TILE_WIDTH + tx];
4     } else {
5         ds_M[ty][tx] = 0.0;
6     }
7     if (p * TILE_WIDTH + ty < Width && Col < Width) {
8         ds_N[ty][tx] = N[(p * TILE_WIDTH + ty) * Width + Col];
9     } else {
10        ds_N[ty][tx] = 0.0;
11    }
12    __syncthreads();
13
14    // Ensuring valid computations and
15    // writing to the output matrix
16    if (Row < Width && Col < Width) {
17        for (int i = 0; i < TILE_WIDTH; ++i) {
18            Pvalue += ds_M[ty][i] * ds_N[i][tx];
19        }
20    }
21    __syncthreads();
22 }
23
24 if (Row < Width && Col < Width) {
25     P[Row * Width + Col] = Pvalue;
26 }
```

- Matrix M . Each thread loads:
 - An element in the position $M[\text{Row}][p * \text{TILE_WIDTH} + \text{tx}]$ if the row is less than the width of the matrix ($\text{Row} < \text{Width}$) and the tile number ($p * \text{TILE_WIDTH}$) plus the index number of the thread tx is less than the width of the matrix ($p * \text{TILE_WIDTH} + \text{tx} < \text{Width}$).
 - Otherwise, load 0.
- Matrix N . Each threads loads:
 - An element in the position $N[p * \text{TILE_WIDTH} + \text{ty}][\text{Col}]$ if the column is less than the width of the matrix ($\text{Col} < \text{Width}$) and the tile number ($p * \text{TILE_WIDTH}$) plus the index number of the thread ty is less than the width of the matrix ($p * \text{TILE_WIDTH} + \text{ty} < \text{Width}$).
 - Otherwise, load 0.
- Boundary condition $(\text{Width} - 1) / \text{TILE_WIDTH} + 1$:
 - $\text{Width} - 1$: adjusts the maximum index for a zero-based index system. Essentially, it ensures we account for the last valid index in the matrix.
 - $/ \text{TILE_WIDTH}$: divides the total width by the tile width, determining how many full tiles fit within the width.

- `TILE_WIDTH + 1`: ensuring that any remaining part of the matrix that doesn't fit perfectly into the last tile is also processed. If the width isn't an exact multiple of the tile width, the `+1` ensures that an additional tile is considered to cover the remaining part of the matrix.

6.3.9 Optimizing Memory Coalescing

In this chapter, we introduce the importance of memory coalescing for effectively using memory bandwidth in CUDA.

2 Introduction

Memory bandwidth is important in CUDA because each thread may need to access memory, and efficient memory access is critical to overall performance. Ideally, when multiple threads in a CUDA program access memory, their accesses should be *coalesced*. This means that **memory accesses from multiple threads are combined into a single memory transaction**.

The memory type inside the GPU is DRAM. Since DRAM accesses are not as fast as local registers, an optimization is required. For this reason, the technique of *DRAM bursts* is introduced:

- It allows a block of data to be transferred in one go. When CUDA threads coalesce memory accesses, the entire burst can be used effectively, resulting in *fewer memory transactions and higher memory bandwidth utilization*.
- By coalescing memory accesses into fewer DRAM bursts, the *latency (delay) associated with memory access is reduced*. This is because once a burst is initiated, the additional data within the burst can be accessed quickly.
- CUDA developers use a variety of techniques to ensure that thread memory accesses are concatenated to take full advantage of DRAM burst capabilities. These include *aligning data structures and managing memory access patterns*.

For **example**, consider a CUDA kernel where each thread accesses successive memory locations. When these accesses are combined, a single DRAM burst can fetch multiple data points needed by multiple threads, *significantly speeding up the computation*.

2 What is a DRAM burst?

A **DRAM (Dynamic Random Access Memory) burst** is a way of **reading data from or writing data to memory**. When accessing DRAM, **data isn't retrieved one byte at a time, but rather in bursts**. A DRAM burst involves the **transfer of multiple bytes of data in a single, continuous sequence**, rather than in separate, discrete chunks.

- **Non-Burst Timing.** There are **gaps between data transfers**, resulting in inefficiencies.
- **Burst Timing.** Represents a **continuous sequence of data transfer**, illustrating the efficiency of burst mode. **Burst bytes are transmitted to the processor, but may be discarded if accesses are not sequential.**

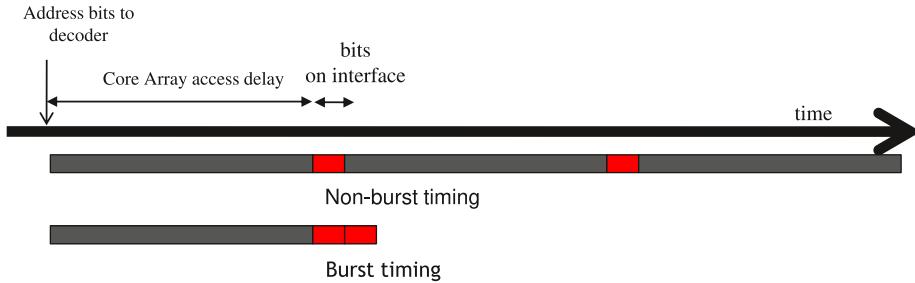


Figure 29: The figure shows a comparison of non-burst and burst timing in DRAM systems.

Burst mode minimizes the gaps between data transfers, allowing for faster and more efficient data flow. This is in contrast to non-burst mode, where data transfers are inefficient due to gaps.

In more detail, burst mode can be described by the following features:

- **Burst operation.** When accessing a specific memory address, the DRAM retrieves an entire block of adjacent data, called a **burst**. This allows faster data access than retrieving individual bytes one at a time.
- **Burst Length.** The length of the burst indicates **how many bytes are transferred in one operation**. Common burst lengths are 4, 8, or 16 bytes, but larger lengths are used depending on the system and application.
- **Memory efficiency.** This method improves memory access efficiency. Once the initial memory location is accessed, the DRAM can transfer the rest of the data in the burst with less overhead.
- **System Usage.** Burst transfers are particularly useful in applications where large blocks of data must be moved quickly, such as graphics processing where textures and images are frequently accessed.

Example 6: great analogy with burst technology

We can think of burst mode as a bookshelf, where instead of taking out one book at a time, we take out a whole section at once. This way, we get more books (data) in one go, reducing the time it takes to reach for each book individually.

The following figure shows a system view of the DRAM burst. Each address space is divided into burst sections, and when one location is accessed, all other locations in the same section are also delivered to the processor.

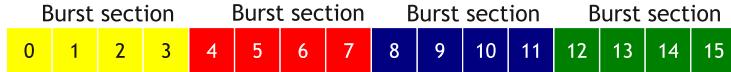


Figure 30: In the figure, a 16-byte address space is divided into 4-byte burst sections. In practice, address spaces are much larger (e.g., 4 GB) and burst sections are also larger (e.g., 128 bytes or more).

⚠️ Coalesced access vs un-coalesced access

Memory Coalescing refers to the process of combining (or *coalescing*) multiple memory accesses by different threads into a single memory transaction. This is important in the context of GPUs because they run many threads in parallel, and efficient memory access can significantly improve performance.

When threads in a warp (a group of threads running in parallel) access successive memory locations, these accesses are combined into a single memory transaction. This means that all requested data is retrieved in one go.

When threads access memory locations that are scattered or misaligned, multiple memory transactions are required, leading to inefficiencies. This phenomenon is called **Un-Coalesced Access**.

Un-Coalesced Accesses occur when the memory requests of multiple threads in a warp do not fit properly into a single memory transaction. This inefficiency results in *increased memory latency* and *lower overall performance*. It is manifested when:

- **Non-Sequential Memory Access.** When threads access memory addresses that are not contiguous, the memory controller must handle each access individually or in smaller, less efficient chunks.
- **Crossing Burst Boundaries.** When memory accesses span multiple DRAM burst sections, multiple memory transactions are required. Each burst section may only be able to handle a portion of the requested data, resulting in additional overhead.
- **Thread misalignment.** If the data being accessed by threads is misaligned with the natural boundaries of memory bursts, the accesses cannot be merged effectively. This often happens with poorly structured data layouts.

The un-coalesced accesses are a penalty for memory optimization because the memory controller cannot combine these accesses into a single transaction because they are spread out. Instead, it must handle multiple transactions, each fetching only a few bytes relevant to the threads.

The consequences of un-coalesced access are:

- **Increased DRAM transactions.** Each un-coalesced access requires a separate DRAM burst, increasing the number of memory transactions.
- **Wasted Bandwidth.** Not all bytes in a DRAM burst are utilized. For example, if each burst fetches 32 bytes and only 8 bytes are used by threads, the remaining 24 bytes are wasted.
- **Higher Latency.** More transactions means more latency because each transaction requires setup and transmission time.

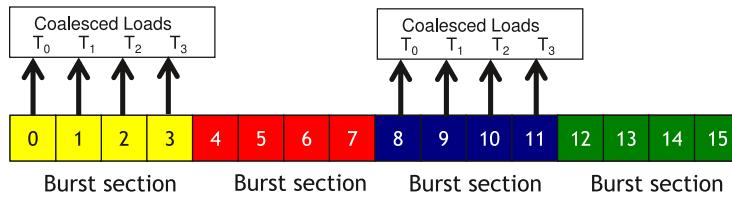


Figure 31: When all threads of a warp execute a load instruction, if all accessed locations are in the same burst section, only one DRAM request is made and the access is fully merged.

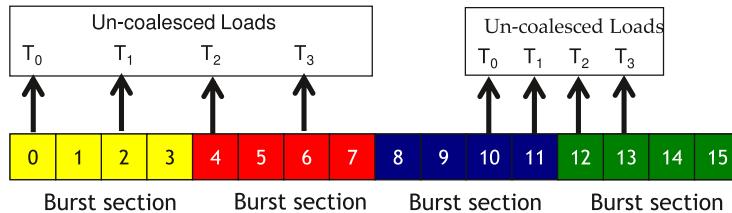


Figure 32: If the accessed locations are spread across burst boundaries, coalescing fails (un-coalesced accesses), multiple DRAM requests are made. This results in some garbage bytes that are not used by threads.

Example 7: great analogy to un-coalesced access

Think of un-coalesced access as trying to get multiple items from different aisles in a supermarket, one at a time. We end up walking back and forth more and taking longer to collect all the items than if we collected items from a single, well-organized aisle.

Example 8: difference from coalesced and un-coalesced access

Suppose a warp of 32 threads accesses an array in global memory. If each thread accesses elements sequentially (thread 0 accesses element 0, thread 1 accesses element 1, and so on), **a single memory transaction retrieves all 32 elements**.

In contrast, if the same warp accesses memory in a non-sequential manner (thread 0 accesses element 0, thread 1 accesses element 4, thread 2 accesses element 8, and so on), **multiple memory transactions are required**, reducing efficiency.

② How do we guarantee coalesced access?

Accesses in a warp are to consecutive locations **if the index in an array access is in the form of:**

$$\text{(expression with terms independent of threadIdx.x)} + \text{threadIdx.x} \quad (12)$$

This formula ensures coalesced memory access in GPU programming.

- `threadIdx.x`: represents the **thread index** within a warp.
- Expression Independent of `threadIdx.x`: this part of the formula ensures that the **base index is the same for all threads within a warp**. It's crucial for aligning memory access.

But *why does it work?* For two main reasons:

1. **Consecutive Access**: if the base index (the part of the expression that is independent of `threadIdx.x`) is the same for all threads, then adding `threadIdx.x` to that base index means that each thread is accessing a consecutive location.
2. **Memory Coalescing**: using this formula ensures that all threads within a warp are accessing adjacent memory locations, resulting in coalesced access.

Before we continue, we need to understand **how a matrix is stored in memory**. It depends on the language, the implementation, and so on, but in general, when we say *linear memory space*, a 2-dimensional matrix is stored as in Figure 33 (page 159).

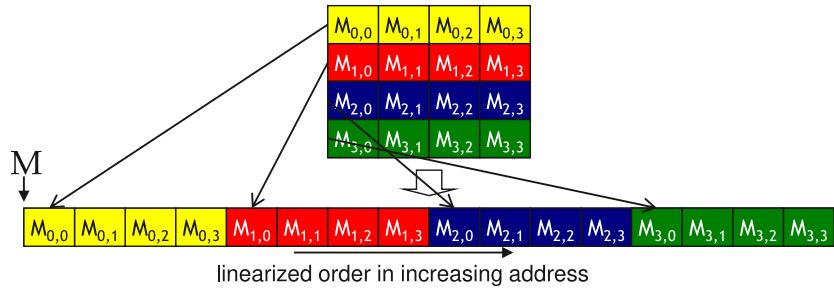


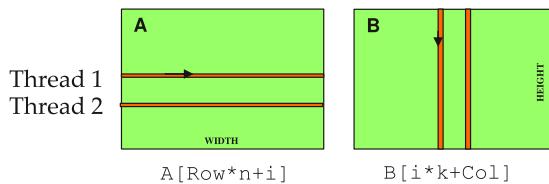
Figure 33: Linear memory space.

In GPU programming, matrix multiplication involves two main access patterns:

- Matrix A (left operator): the access pattern is $A[\text{Row} * n + i]$
 - i is the **loop counter** in the inner product loop of the kernel code;
 - **Row** is the **current row being processed**;
 - n is the **number of columns** in matrix A.
- Matrix B (right operator): the access pattern is $B[i * k + \text{Col}]$
 - i is the **loop counter**;
 - k is the **number of columns** in matrix B;
 - Col is calculated as:

`blockIdx.x * blockDim.x + threadIdx.x`

This means that **threads access elements in matrix B based on their column index**.



- Matrix B. The accesses are **Coalesced**. As we can see from the figure 34, the accesses are coalesced. This is because matrix B is stored in memory in a linear, one-dimensional array. For a 2D matrix, the elements are stored row by row.

In a warp, threads are indexed consecutively, like `threadIdx.x = 0, 1, ...`. When i is fixed for a loop iteration, successive threads access successive memory locations in matrix B. For example:

- Thread 0 accesses $B[i * k + 0]$
- Thread 1 accesses $B[i * k + 1]$
- Thread 2 accesses $B[i * k + 2]$

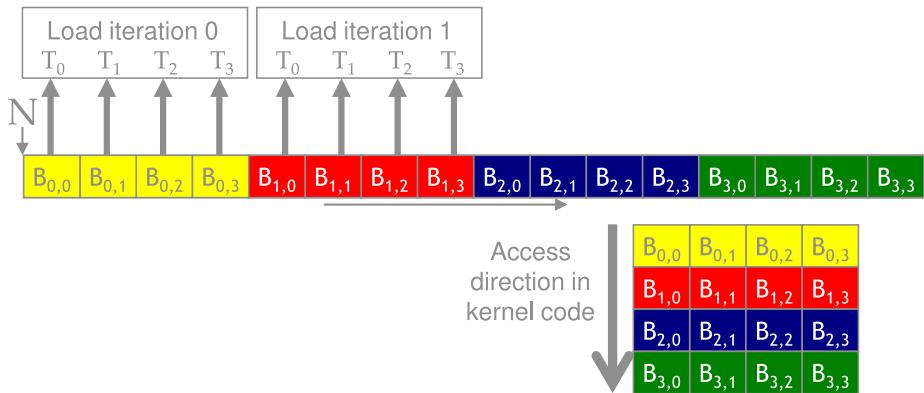


Figure 34: Coalesced accesses.

The main reason for coalesced access in matrix B is to **align thread indexes with contiguous memory addresses**. This optimizes memory transactions, making data retrieval more efficient and increasing overall performance.

- Matrix A. The accesses are **Un-Coalesced**. The figure shows why the accesses for matrix A are not coalesced.

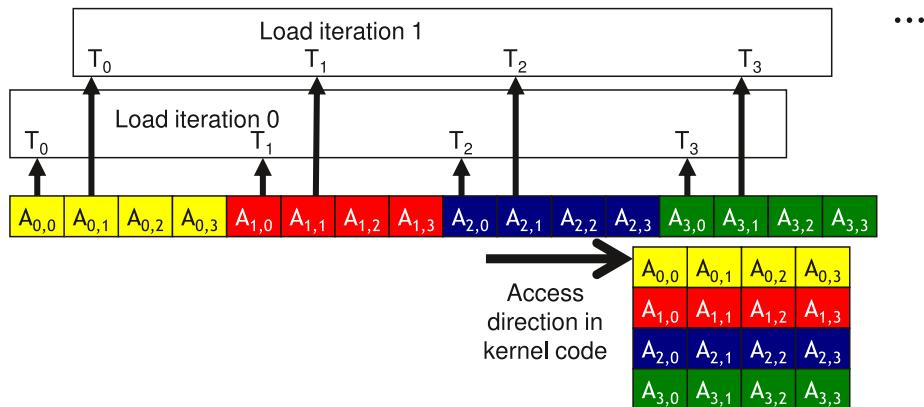


Figure 35: Un-Coalesced Accesses.

The access pattern $A[\text{Row} * n + i]$ means that each thread in a warp accesses different rows of the matrix A. If we consider consecutive threads (e.g. thread 0, thread 1, thread 2, thread 3), they access elements in different rows.

Threads in a warp access elements vertically, not horizontally. For example, if Row is fixed for a given iteration of the outer loop, i varies. This means that thread 0 accesses $A[\text{Row} * n + 0]$, thread 1 accesses $A[\text{Row} * n + 1]$, and so on. This results in non-consecutive memory locations being accessed by consecutive threads.

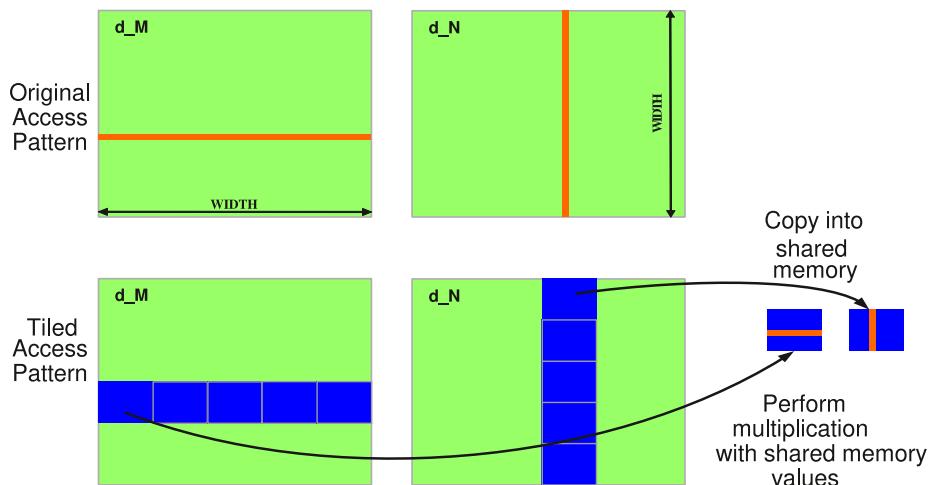
💡 How to optimize the matrix multiplication to avoid un-coalesced accesses

Corner Turning is a technique used in matrix multiplication to optimize memory access patterns, especially in parallel computing environments like GPU. The goal is to achieve memory coalescing, which improves performance by ensuring that memory accesses are aligned and contiguous.

Corner Turning works as follows:

- **Column-Major Layout.** Normally, matrices are stored in a row-major layout (elements are stored row by row). In corner turning, the **second matrix (B)** is stored in a column-major layout (elements are stored column by column).
- **Tiled Access.** The **matrices are divided into smaller submatrices or tiles**. Each thread loads a tile of the first matrix (A) and a tile of the second matrix (B) into shared memory.
- **Memory Coalescing.** By loading tiles into shared memory, threads can access consecutive memory locations within the tiles. This ensures that memory accesses are coalesced, reducing memory latency and increasing bandwidth utilization.

In practice, it works as shown in the following figure:



- Original Access Pattern:
 - Matrix d_M . The original access pattern is **horizontal**. **Each thread accesses consecutive elements in a row.**
 - Matrix d_N . The original access pattern is **vertical**. **Each thread accesses elements in different rows.**

- Tiled Access Pattern:

- Matrix d_M . The matrix is **divided into smaller horizontal tiles**. Each tile is a small submatrix that fits into shared memory.
- Matrix d_N . Similarly, this matrix is **divided into smaller vertical tiles**, which are also small enough to fit into shared memory.

- Shared Memory Operations:

- **Loading Tiles into Shared Memory.** Both tiles from d_M and d_N are loaded into shared memory. This allows for *more efficient access patterns*.
- **Matrix Multiplication.** The **multiplication of these tiles is performed using the values stored in shared memory**. This step ensures that the data access is coalesced and reduces latency.

By dividing matrices into tiles and loading them into shared memory, **threads can access successive memory locations within a tile**. This *ensures that memory accesses are concatenated*, reducing memory transactions and increasing efficiency.

In addition, using shared memory for these tiles reduces the need to repeatedly access global memory, which is slower. Shared memory access is much faster, further improving the performance of matrix multiplication.

From a code point of view, we see:

- Original Pattern:

```

1 Matrix d_M (Row-major):
2 Thread 0: M[0][0], M[0][1], ...
3 Thread 1: M[1][0], M[1][1], ...
4
5 Matrix d_N (Column-major):
6 Thread 0: N[0][0], N[1][0], ...
7 Thread 1: N[0][1], N[1][1], ...

```

- Tiled Pattern:

1. Load Tiles into Shared Memory:

```

1 Tile from d_M: [[M00, M01], [M10, M11]]
2 Tile from d_N: [[N00, N10], [N01, N11]]

```

2. Perform Multiplication. Threads use values from shared memory for the multiplication, ensuring coalesced access and reduced memory latency.

7 CUDA

Although the CUDA topic has already been discussed in section 6.3.1 (page 104), here we introduce more technical topics, a kind of CUDA laboratory. It is done because in the Parallel Computing course we have done a CUDA theory part and a CUDA practice part.

7.1 Introduction

CUDA, which stands for **Compute Unified Device Architecture**, is a parallel computing platform and application programming interface (API) model created by NVIDIA. It allows developers to use the power of GPUs (Graphics Processing Units) for general-purpose processing, which enables substantial performance improvements for computationally intensive tasks.

❓ Why CUDA?

GPUs, originally designed to render graphics, have evolved into **highly efficient and powerful processors capable of handling thousands of threads simultaneously**. This transformation has made GPUs, and by extension CUDA, incredibly valuable for applications requiring massive parallelism, such as scientific simulations, machine learning, and deep learning.

❓ Why can we not just use the CPU?

Understanding the fundamental differences between CPU and GPU architectures is key to appreciating CUDA's advantages:

- CPU (Central Processing Unit):
 - Designed for sequential processing.
 - Features powerful Arithmetic Logic Units (ALUs) with low latency.
 - Utilizes large hierarchical caches to optimize access to frequently used data.
 - Employs advanced control mechanisms, such as branch prediction and data forwarding, to minimize delays.
- GPU (Graphics Processing Unit):
 - Optimized for parallel processing.
 - Contains a large number of simpler, pipelined ALUs designed for high-throughput computations, despite having longer latency.
 - Relies on smaller caches to facilitate high memory throughput.
 - Uses simpler control mechanisms, enabling efficient context switching and handling many threads concurrently.

CUDA leverages these GPU characteristics to execute programs with a parallel-first approach, breaking down tasks into smaller, manageable pieces and processing them simultaneously. This approach leads to significant speedups compared to traditional CPU-only processing, making CUDA a pivotal technology for high-performance computing.

■ GPGPU programming paradigms

CUDA helped bring General Purpose computing on Graphics Processing Units (GPGPU). Initially designed to accelerate rendering and processing of graphics, GPUs have evolved into versatile processors capable of handling a wide range of computational tasks beyond graphics. This paradigm shift has allowed GPUs to be used for general-purpose computing, providing significant performance improvements for applications requiring high parallelism and computational power (the definition and introduction of GPGPU can be found on page 101).

The GPU (called a device) acts as a co-processor for the CPU (host):

- CPU (**host**):
 - *General-purpose* processor.
 - Handles diverse tasks, running an operating system, and executing a sequence of stored instructions.
 - Efficient for tasks requiring low-latency access to cached data.
- GPU (**device**):
 - Specialized for *high-throughput computation*.
 - Contains a large number of processing elements for parallel tasks.
 - Relies on dedicated, high-bandwidth memory.
 - Best for data-parallel tasks, hiding memory latency effectively.

The relationship of these two concepts:

- Both CPU and GPU have their own memory spaces.
- Optimal performance achieved through cooperation.
- CPU handles complex logic and serial tasks; GPU manages parallel processing.

About the architecture, the differences are:

- CPU (**host**):
 - Composed of control units, ALUs, cache, and DRAM.
 - Efficient for tasks requiring complex control and low-latency data access.
- GPU (**device**):
 - Made up of DRAM and a grid of processing units.
 - High-throughput design for parallel processing tasks.

GPGPU (General-Purpose computing on Graphics Processing Units) **programming paradigms** are various approaches or models used to leverage the GPU for general-purpose computation. These paradigms allow programmers to write code that runs efficiently on GPUs. The main paradigms are:

1. **Applications.** End-user programs that take advantage of GPU performance. Designed to take advantage of the GPU's processing power to accelerate performance (such as video processing software, scientific simulations, and machine learning models).
2. **Libraries.** Pre-built functions and routines optimized for GPUs. In other words, a collection of pre-written code and routines that are optimized to run on GPUs. Using these libraries, developers can avoid writing low-level GPU code. Examples of libraries are cuBLAS (for linear algebra), cuFFT (for Fast Fourier Transforms), and TensorFlow (for machine learning).
3. **Compiler Directives.** Special instructions embedded in the source code that guide the compiler on how to optimize and parallelize the code for GPU execution. OpenACC and OpenMP are commonly used directives.
4. **Programming Languages.** Languages or extensions of languages specifically designed for GPGPU programming. They provide the syntax and constructs needed to write code that executes on the GPU. Examples are CUDA and OpenCL (Open Computing Language).

Given an application code, in general:

- **Serial Parts run on the CPU (host).** These are the parts of the code that need to be executed sequentially and can benefit from the sophisticated control mechanisms of the CPU.
- **Computation-Intensive and Data-Parallel Parts offloaded to the GPU (device).** These parts can be executed in parallel, making use of the GPU's many cores for faster computation.

The flow inside the GPU can be described in three general steps:

1. **Copy Input Data from CPU Memory to GPU Memory.** The data is transferred over the PCI bus. This step is critical to prepare the data so that the GPU can access and process it.
2. **Load GPU Program and Execute, Caching Data on Chip for Performance.** The GPU program (kernel) is loaded and executed. Data is cached on the GPU chip to improve performance during execution. This step takes advantage of the parallel processing power of the GPU.
3. **Copy Results from GPU Memory to CPU Memory.** After calculation, the results are transferred back to the CPU memory via the PCI bus. This step is necessary to integrate the results into the larger application or for further processing on the CPU.

❓ Common GPGPU issues and solutions

- Data Movement between CPU and GPU is the main Bottleneck.
 - **Low Bandwidth.** Data transfers between the CPU and GPU are relatively slow because they use the PCI Express (PCIe) bus, which has a bandwidth of about 12-14GB/s. This is much lower compared to the internal memory bandwidth of the CPU and GPU.
 - **Relatively High Latency.** Data transfer times can be significant, introducing delays that affect overall performance.
 - **Data Transfer can take more time than the actual computation.** In some cases, moving data between the CPU and GPU can take longer than the time it takes to perform the actual computations on the GPU. This is a critical point to consider when designing GPU-accelerated applications.
- Issues Porting CPU Applications to GPGPU.
 - **Ignoring Data Movement can destroy GPU Performance Benefits.** When converting CPU applications to run on a GPU, it's crucial to manage data transfers efficiently. Failing to do so can negate the performance advantages of using a GPU.
 - **Solutions to Hide/Automate Data Transfer.** Some programming techniques and tools can help hide or automate the data transfer process. These solutions can optimize performance by reducing the manual overhead of managing data movement.

In other words, efficient data management is the key to performance. In fact, the performance of GPGPU applications depends heavily on how well data is managed between the CPU and the GPU. Efficient data transfer strategies are essential to unleash the full computing power of GPUs.

7.2 CUDA Basics

CUDA, developed by NVIDIA, is a parallel computing platform and programming model that enables developers to harness the immense computational power of NVIDIA GPUs (Graphics Processing Units).

CUDA allows programmers to write code for GPUs using familiar programming languages like C, C++, and Fortran. This accessibility lowers the learning curve, enabling more developers to take advantage of GPU acceleration without requiring extensive knowledge of graphics programming.

The CUDA programming model evolves with the underlying hardware architecture, ensuring that developers can maximize performance gains from the latest GPU advancements. By writing a program for a single thread and instantiating it across many parallel threads, developers can achieve significant speedups for data-parallel tasks.

❖ The most important function: the kernel

A **kernel** is a function that runs on the GPU. When a kernel is launched, **thousands of threads execute its code simultaneously**. The programmer specifies the number of threads, each acting independently on different data elements. This approach leverages Single Instruction, Multiple Data (SIMD) and Single Program, Multiple Data (SPMD) parallelism.

Example 1: comparison between C and CUDA C program

```

1 void vsum(int* a, int* b, int* c) {
2     int i;
3     for (i=0; i<N; i++) {
4         c[i] = a[i] + b[i];
5     }
6 }
7
8 void main() {
9     int va[N], vb[N], vc[N];
10    ...
11    vsum(va, vb, vc);
12    ...
13 }
14
15
16 __global__ void vsum(int* a, int* b, int* c) {
17     int i = ... // get unique thread ID
18     c[i] = a[i] + b[i];
19 }
20
21 void main() {
22     int va[N], vb[N], vc[N];
23     ...
24     vsum<<<N, 1>>>(va, vb, vc);
25     ...
26 }
```

In the CUDA version, the `vsum` function is defined as a kernel using the `__global__` keyword, and it is launched on the GPU with a specified number of threads. Each thread processes a different element of the input arrays independently, showcasing CUDA's ability to handle parallel tasks efficiently.

7.2.1 GPGPU Best Practices

NVIDIA provides a guide to help developers get the most out of NVIDIA CUDA GPUs:

[CUDA C++ Best Practices Guide](#)



Among the suggestions, NVIDIA explains a process for accelerating an application with NVIDIA GPUs called the *Cyclical Process for Accelerating Applications with NVIDIA GPUs*.

The **Cyclical Process for Accelerating Applications with NVIDIA GPUs** is a systematic approach designed to optimize and enhance the performance of applications by leveraging the computational power of NVIDIA GPUs. This process ensures that applications can take full advantage of the parallel processing capabilities of GPUs for improved performance and efficiency. The process consists of four main stages: Assess, Parallelize, Optimize, and Deploy.

1. **Assess.**

- (a) **Locate Bottlenecks.** Identify the parts of our application where performance is limited. This involves profiling our application to understand where most of the computation time is spent.
- (b) **Estimate Parallelization Benefits.** Determine the potential performance improvements from parallelizing the application. Evaluate how much of the workload can be efficiently offloaded to the GPU.

2. **Parallelize.**

- (a) **Apply Libraries, Compiler Directives, or CUDA.** Utilize GPU-optimized libraries (such as cuBLAS for linear algebra operations), apply compiler directives (like OpenACC) to guide the parallelization, or write custom CUDA kernels to parallelize the identified computational bottlenecks.

3. **Optimize.**

- (a) **Apply Optimizations.** Fine-tune our GPU code to improve performance. This can include optimizing memory access patterns to reduce latency, ensuring efficient data transfer between the CPU and GPU, and maximizing thread utilization to fully exploit the GPU's parallel architecture.
- (b) **Measure Performance Improvements.** Continuously profile and benchmark our application to monitor the impact of the optimizations and ensure that they lead to significant performance gains.

4. Deploy.

- (a) **Compare with Performance Estimations.** Verify that the performance improvements achieved through parallelization and optimization meet the initial estimations. This step ensures that the application performs as expected in a real-world environment.
- (b) **Move to Production.** Once satisfied with the performance, deploy the optimized application to production. This involves integrating the GPU-accelerated code into the main application and ensuring it runs efficiently in the production environment.

The process is depicted as a *continuous cycle*, indicating that **optimization and assessment are ongoing activities**. This **iterative approach ensures that the application remains optimized and continues to benefit from GPU acceleration over time**.

By following this cyclical process, developers can systematically **identify performance bottlenecks**, **efficiently parallelize** and **optimize their applications**, and **achieve significant performance improvements using NVIDIA GPUs**. This methodical approach helps ensure that the application takes full advantage of GPU capabilities, leading to faster, more efficient computing.

7.2.2 Compilation

The compilation of a CUDA file follows a structured process that involves both the CPU and GPU components.

1. **.cu File**. The source file containing CUDA kernels and the rest of the application code is usually saved with a .cu extension.
2. **CUDA Kernels**. The CUDA-specific code within the .cu file is processed by the NVIDIA CUDA Compiler (NVCC). This includes all the functions intended to run on the GPU.
2. **Rest of the Application**. The non-CUDA parts of the code, intended to run on the CPU, are processed by the host CPU compiler.
3. **NVCC Compiler**. NVCC compiles the CUDA kernels into CUDA object files, which are specific to the GPU.
3. **CPU Compiler**. The CPU compiler processes the remaining application code into CPU object files, which are specific to the CPU.
4. **Linker**. The linker combines the CUDA object files and CPU object files into a single executable that can run on both the CPU and GPU.

An example of a compilation is the command:

```
1 nvcc -arch=sm_70 -o hello-gpu 01-hello/01-hello-gpu.cu -run
```

That demonstrates how to compile and execute a CUDA file using NVCC. The **-arch=sm_70** flag specifies the architecture of the GPU, in this case, the Volta architecture (**sm_70**).

Useful Compilation Flags for NVCC:

Flags	Description
-x	Treat all input files as .cu files.
-Xcompiler	Pass a host compiler flag that is not supported by NVCC.
-g	Include host debugging symbols.
-G	Include device debugging symbols.
-lineinfo	Include line information with symbols.

Table 5: Some compilation flags.

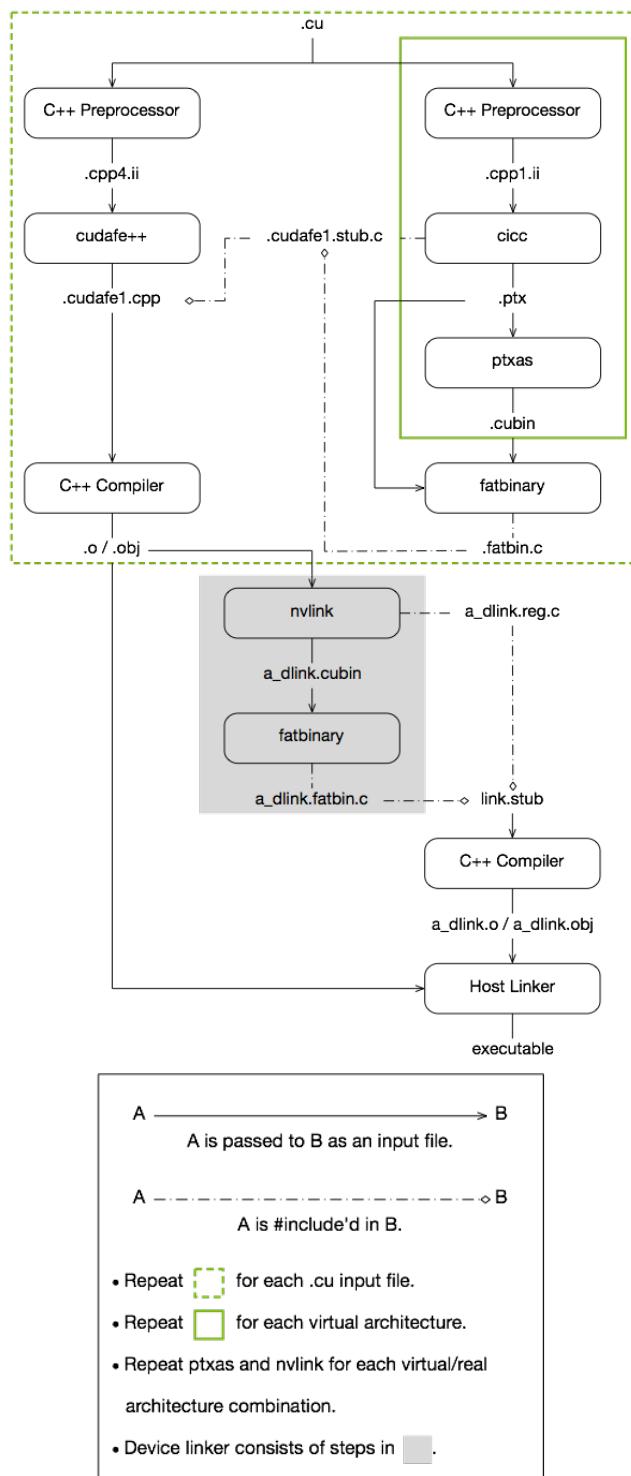


Figure 36: CUDA Compilation Trajectory, source: [NVIDIA CUDA Compiler Driver NVCC](#)

7.2.3 Debugging

Given the complexity and parallel nature of GPU programming, effective debugging tools are essential to ensure code correctness and optimize performance. NVIDIA provides a suite of **debugging tools** specifically designed for CUDA applications, helping developers identify and resolve issues in their code.

- `cuda-memcheck` is a comprehensive tool that detects memory-related errors in CUDA applications. It helps identify issues such as memory leaks, out-of-bounds accesses, and race conditions. Its capabilities:
 - **Memory Leaks.** Finds memory that is allocated but not freed, preventing unnecessary resource consumption.
 - **Memory Errors.** Detects accesses to invalid memory locations, which can cause unpredictable behavior.
 - **Race Conditions.** Identifies scenarios where multiple threads access shared data concurrently, potentially leading to incorrect results.
 - **Illegal Barriers.** Spots improper use of synchronization barriers in parallel code.
 - **Uninitialized Memory.** Flags the use of uninitialized memory, which can lead to unreliable outcomes.
- `cuda-gdb` is an extension of the GNU Debugger (GDB) tailored for debugging CUDA applications. It offers a familiar debugging environment for those already comfortable with GDB. Its capabilities:
 - **Setting Breakpoints.** Allows developers to pause execution at specific points in the code to inspect the state of the program.
 - **Inspecting Memory.** Enables examination of variables and memory contents to ensure they hold expected values.
 - **Stepping Through Code.** Provides the ability to step through code line by line, both on the CPU and GPU, to trace the execution flow.

With the debugging tools, there are also some profiling tools provided by NVIDIA.

CUDA profiling⁹ is the process of measuring various aspects of a CUDA program's performance to **identify inefficient code segments, resource bottlenecks, and opportunities for optimization**. Profiling helps developers understand how their applications are utilizing GPU resources and make informed decisions to enhance performance.

- **NVPROF** is a command-line profiler provided by NVIDIA that **gives detailed timing information for each CUDA kernel and memory operation**. Its capabilities:

⁹Profiling is a process used in software development to analyze and measure the performance characteristics of a program. The goal of profiling is to identify parts of the code that are consuming the most resources or taking the most time, so that developers can optimize these areas and improve the overall performance of the application.

- **Kernel Execution Time.** Measures the time taken by each CUDA kernel to execute.
- **Memory Transfers.** Monitors the time spent on data transfers between the CPU and GPU.
- **API Calls.** Tracks the duration of CUDA API calls.
- **Occupancy.** Analyzes the utilization of GPU resources, helping to identify potential underutilization or over-subscription of resources.

Developers use NVPROF to run their applications and generate detailed profiling reports, which can then be analyzed to pinpoint performance bottlenecks.

- **NVIDIA Visual Profiler (NVVP)** is a **graphical profiling tool** that provides a visual representation of the application's performance, making it easier to identify and address bottlenecks. Its capabilities:

- **Timeline Visualization.** Displays a timeline of kernel executions, memory transfers, and API calls, allowing developers to see the sequence and overlap of events.
- **Detailed Metrics.** Offers in-depth metrics on kernel performance, memory usage, and other critical aspects of CUDA applications.
- **Optimization Suggestions.** Provides recommendations for optimizing code based on the profiling results.

NVVP is particularly useful for visualizing complex interactions within CUDA applications and understanding how different parts of the code affect overall performance.

- **NSIGHT** is an integrated development environment that includes **advanced profiling and debugging features** for CUDA applications. Its capabilities:

- **Advanced Profiling.** Combines the features of NVPROF and NVVP, offering a comprehensive set of profiling tools within a single interface.
- **Interactive Analysis.** Allows developers to interactively analyze performance data and make real-time adjustments to their code.
- **Unified Development.** Integrates debugging and profiling, providing a seamless environment for developing and optimizing CUDA applications.

NSIGHT is ideal for developers who need a powerful, all-in-one tool for debugging and profiling their CUDA applications.

- **Third-Party Profiling Tools:**

- **TAU** (Tuning and Analysis Utilities). A performance analysis tool that can be used to profile CUDA applications along with other types of programs. **TAU provides detailed performance data and visualization capabilities.**

- **VampirTrace.** Captures performance data and visualizes it to help optimize parallel applications. It supports a range of profiling features for CUDA and other programming models.

Another powerful profiling system is Nsight. **Nsight Systems** offers both a **command-line profiler and a graphical user interface (GUI)**, providing comprehensive insights into the execution of CUDA applications. It helps identify performance bottlenecks, understand GPU utilization, and improve overall application efficiency.

- **Nsight Command-Line Profiler.** `nsys` can be used to profile an accelerated application by launching it and gathering detailed statistics about its performance. The information reported are:
 - **GPU Activity.** Insights into how the GPU is utilized during the application's execution.
 - **CUDA API Calls.** Details on the usage of CUDA API functions.
 - **Memory Activity.** Information about memory operations, including transfers between CPU and GPU.

An example command is:

```
1 nsys profile --stats=true -o vector-add-no-prefetch-report ./vector-add-no-prefetch
```

This command profiles the application `vector-add-no-prefetch` and generates a report with detailed statistics.

- **Nsight Systems GUI.** Provides a visual representation of the profiling data collected by `nsys`, making it easier to analyze and interpret the results. Its capabilities:
 - **Timeline View.** Displays a comprehensive timeline of CPU and GPU activities, showing how different tasks are executed over time.
 - **Detailed Metrics.** Offers in-depth metrics on kernel performance, memory usage, and other critical aspects.
 - **Interactive Analysis.** Allows developers to zoom into specific regions of the timeline and examine detailed activities.

7.2.4 CUDA Kernel

Launching a CUDA kernel involves **defining a function that will run on the GPU and then executing this function from the host (CPU) code.**

1. Defining functions.

- CPU Function:

```
1 void CPUFunction() {
2     printf("This function is defined to run on the CPU.");
3 }
```

This function runs on the CPU and prints a message.

- GPU Function:

```
1 __global__ void GPUFunction() {
2     printf("This function is defined to run on the GPU.");
3 }
```

This function is defined to run on the GPU, indicated by the `__global__` qualifier, and it prints a message when executed.

2. Launching the Kernel.

In the main function, we launch the GPU function using a special syntax:

```
1 int main() {
2     CPUFunction();
3
4     // Launch the GPU kernel
5     // with 1 block and 1 thread
6     GPUFunction<<<1, 1>>>();
7     cudaDeviceSynchronize();
8 }
9
```

- The `CPUFunction()` is called normally, as it's a CPU function.
- The `GPUFunction<<<1, 1>>>()` syntax launches the GPU kernel with a specified execution configuration: `<<1, 1>>` means 1 block and 1 thread per block.
- `cudaDeviceSynchronize()` is called to ensure that the CPU waits for the GPU to finish executing the kernel before continuing.

Just like with variables (table 4, page 133), CUDA provides specific keywords that define the scope and lifetime of functions.

Function Declaration	Executed on	Callable from
<code>__device__ type DeviceFunc()</code>	Device	Device
<code>__global__ void KernelFunc()</code>	Device	Host
<code>__host__ type HostFunc()</code>	Host	Host

Table 6: CUDA function qualifiers.

💡 How can we customize the kernel?

Kernel configuration in CUDA involves specifying **how many blocks and threads will be used** to execute a kernel on the GPU. The key parameters are:

- **Number of Blocks.** Specifies **how many blocks** of threads will be launched on the GPU.
- **Number of Threads per Block.** Specifies **how many threads** will be in each block.

For a *1D hierarchy*, we use simple integer numbers to specify these values because a 1D grid or block can be represented by a single dimension:

```
1 // Example of launching a kernel with 1 block and 256 threads in 1D
2 KernelFunc<<<1, 256>>>();
```

For *higher dimensions* (2D or 3D), CUDA uses the **dim3 type to represent the grid and block dimensions**. **dim3** is a CUDA-specific type that can hold three unsigned integer values, representing the dimensions in the *x*, *y*, and *z* directions. This provides a flexible way to configure more complex thread hierarchies:

```
1 // Example of launching a kernel with a 2D grid and blocks
2 dim3 gridDim(16, 16); // 16 blocks in x and y dimensions
3 dim3 blockDim(16, 16); // 16 threads per block in x and y
4 KernelFunc<<<gridDim, blockDim>>>();
```

⌚ CPU-GPU Synchronization

In CUDA, **kernel executions are asynchronous**, meaning that the **CPU can continue to execute other instructions while the GPU is running the kernel**. However, there are times when synchronization is required to ensure that the CPU is waiting for the GPU to complete its tasks.

- **Asynchronous Kernel Launch.** When a kernel is launched, the CPU can proceed with subsequent instructions without waiting for the GPU to finish.

```
1 KernelFunc<<<1, 256>>>();
2 // The CPU immediately continues executing the next
  instructions
```

- **Synchronizing CPU and GPU.** To synchronize, CUDA provides runtime functions such as `cudaDeviceSynchronize()`, which ensures that the CPU waits until the GPU has completed all preceding tasks.

```
1 KernelFunc<<<1, 256>>>();
2 // Wait for the GPU to finish
3 cudaDeviceSynchronize();
```

Error handling

The errors in CUDA can be caught in two ways:

- Return type: most **CUDA runtime functions return a variable of type `cudaError_t`**. This type is an enumerated value that indicates the success or failure of the function call.
- Kernels return `void`: CUDA kernel functions return `void`, meaning they do not provide direct feedback about errors. Instead, **errors are checked using specific functions**, such as: `cudaGetLastError()`. It returns the last error that occurred during the execution of a CUDA function.

```

1 cudaError_t err = cudaGetLastError();
2 if (err != cudaSuccess) {
3     printf("CUDA error: %s\n", cudaGetErrorString(err));
4 }
5

```

Asynchronous errors (during kernel execution) are reported as error states of `cudaDeviceSynchronize()`. It **blocks the CPU thread until the GPU has completed all previously requested tasks**. This includes kernel executions, memory transfers, and other asynchronous operations issued.

```

1 kernel<<<blocks, threads>>>();
2 cudaError_t err = cudaDeviceSynchronize();
3 if (err != cudaSuccess) {
4     printf("CUDA error: %s\n", cudaGetErrorString(err));
5 }

```

Example 2: error handling

```

1 cudaError_t err1, err2;
2 err1 = cudaMallocManaged(&a, N);
3 SomeKernel<<<1, -1>>>(); // Invalid thread number
4 err2 = cudaGetLastError();
5 if (err1 != cudaSuccess || err2 != cudaSuccess) {
6     printf("Error 1: %s\n", cudaGetErrorString(err1));
7     printf("Error 2: %s\n", cudaGetErrorString(err2));
8 }

```

- `err1 = cudaMallocManaged(&a, N);` attempts to allocate managed memory and returns an error code if it fails.
- `cudaGetErrorString(err1)` converts the error code to a readable string.
- `SomeKernel<<<1, -1>>>();` intentionally uses an invalid thread configuration to generate an error.
- `err2 = cudaGetLastError();` captures the error from the kernel launch.
- `cudaGetErrorString(err2)` converts the kernel launch error code to a readable string.

7.3 Execution Model

The **CUDA execution model** defines how parallel computing tasks are organized and executed on NVIDIA GPUs. It consists of both software and hardware components that work together to efficiently process large amounts of data in parallel.

❖ Software Hierarchy

- **Grid.** A grid is a **collection of thread blocks that execute a given kernel function**. Each grid is associated with a specific kernel launch. Grids can be one-dimensional, two-dimensional, or three-dimensional, depending on the problem's complexity.
- **Thread Block.** A thread block is a **group of threads that execute together and can cooperate by sharing data through shared memory**. Each block is independent, allowing the scheduler to execute blocks in any order. Thread blocks can also be organized in one, two, or three dimensions.
- **Thread.** The **smallest unit of execution in CUDA**. Each thread executes the **same code but operates on different data elements**. Threads within a block can communicate and synchronize with each other, enabling collaborative computation.

▀▀ Hardware Hierarchy

- **GPU.** The **physical hardware that executes the parallel tasks**. A GPU consists of **multiple Streaming Multiprocessors (SMs)**.
- **Streaming Multiprocessor (SM).** Each SM can **execute multiple threads concurrently**. Thread blocks are assigned to SMs for execution. SMs manage and execute warps of threads.
- **GPU Core.** Individual **cores within the SMs that perform the actual computation**. Cores execute threads in groups called **warps**, typically comprising 32 threads.

⚠ What happens if the kernel startup doesn't match the input dataset?

Execution configuration mismatches occur when the number of threads configured for a kernel launch does not perfectly match the size of the dataset being processed. This can lead to *inefficiencies* or the need for *additional handling in the kernel code*.

- **Optimal Thread Configuration:** **multiple of 32 threads**. It's generally desirable to configure blocks with a multiple of 32 threads for performance reasons. This is because GPUs execute threads in groups of 32, known as warps. Ensuring the number of threads per block is a multiple of 32 can prevent under-utilization of GPU resources.

As the warp dimension changes, the optimal thread configuration should also change.

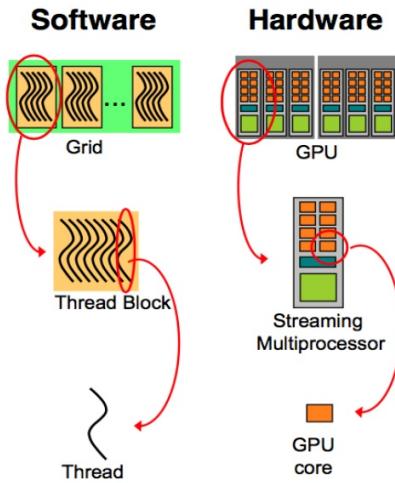


Figure 37: Graphical representation of the CUDA execution model.

- **Dealing with Non-Multiple of 32 threads.** If the dataset size (e.g., $N=1000$) is not a multiple of 32, we can create more than 1000 threads and use an extra check within the kernel to handle the excess threads.

1. We pass the dataset size N as an argument to the kernel.
2. Inside the kernel, we check if the thread ID is greater than or equal to N . If it is, those threads should not perform any operations.

```

1  __global__ void kernelFunction(float* data, int N) {
2      int tid = blockIdx.x * blockDim.x + threadIdx.x;
3      if (tid < N) {
4          // Perform operations only if thread ID
5          // is within the dataset size
6          data[tid] = data[tid] * 2.0f;
7      }
8  }
```

- **Insufficient threads.** In some cases, there may be fewer threads than data elements to process, either for performance reasons or because the dataset is very large.

In this case, the solution is using a **Stride Factor**. We can use a stride factor **corresponding to the total number of threads**. The stride factor is calculated as $\text{gridDim.x} * \text{blockDim.x}$, representing the **total number of threads in the grid**. Each thread processes multiple elements by iterating with a step size equal to the stride.

```

1  __global__ void kernelFunctionWithStride(float* data, int N) {
2      int tid = blockIdx.x * blockDim.x + threadIdx.x;
3      int stride = gridDim.x * blockDim.x;
4      for (int i = tid; i < N; i += stride) {
5          data[i] = data[i] * 2.0f;
6      }
7  }
```

7.4 Querying Device Properties

Different GPUs have varying capabilities and configurations, such as the number of Streaming Multiprocessors (SMs), maximum thread dimensions, memory size, etc. Knowing these properties helps developers optimize their code to run efficiently on different hardware. The main properties are:

- **Get Device ID:** `cudaGetDevice`; to retrieve the current device ID.

```
1 int deviceId;
2 cudaGetDevice(&deviceId);
```

- **Query Device Properties:** `cudaGetDeviceProperties`; to get the properties of the specified device.

```
1 cudaDeviceProp props;
2 cudaGetDeviceProperties(&props, deviceId);
```

- **Understanding the `cudaDeviceProp` Structure.** This structure contains numerous attributes of the device. Some key attributes include:

- `name`. Name of the device.
- `totalGlobalMem`. Total global memory available on the device.
- `sharedMemPerBlock`. Shared memory available per block.
- `regsPerBlock`. Number of registers available per block.
- `warpSize`. Number of threads in a warp.
- `maxThreadsPerBlock`. Maximum number of threads per block.
- `maxThreadsDim`. Maximum dimension sizes of a thread block (x, y, z).
- `maxGridSize`. Maximum dimension sizes of a grid (x, y, z).
- `clockRate`. Clock frequency of the device.
- `multiProcessorCount`. Number of streaming multiprocessors on the device.

Example 3: common queries

Some common queries are:

- Maximum Dimensions of a Thread Block:

```
1 printf("Max Thread Dimensions: %d x %d x %d\n",
2     props.maxThreadsDim[0], props.maxThreadsDim[1],
3     props.maxThreadsDim[2]);
```

- Maximum Number of Threads in a Block:

```
1 printf("Max Threads per Block: %d\n",
2     props.maxThreadsPerBlock);
```

- Maximum Size of GPU Global Memory:

```
1 printf("Total Global Memory: %zu bytes\n",
2         props.totalGlobalMem);
```

- Warp Size:

```
1 printf("Warp Size: %d\n", props.warpSize);
```

- Number of Streaming Multiprocessors (SM):

```
1 printf("Number of SMs: %d\n",
2        props.multiProcessorCount);
```

7.5 Thread hierarchy

The **CUDA thread hierarchy** is a fundamental concept in CUDA programming that allows for organizing and managing parallel computation tasks efficiently on NVIDIA GPUs. As we presented in Section 6.3.1, on page 104, the hierarchy consists of:

1. Threads

- **Basic Execution Unit.** A thread is the smallest unit of execution in CUDA. Each thread executes a portion of the overall computation.
- **Unique Thread ID.** Each thread within a block has a unique ID, which can be accessed using built-in variables like `threadIdx`.

2. Thread Blocks

- **Group of Threads.** Threads are grouped into blocks. A thread block can contain up to 1024 threads, depending on the GPU architecture.
- **Cooperation and Communication.** Threads within the same block can cooperate and share data through shared memory, and they can synchronize using `__syncthreads()`.
- **Unique Block ID.** Each block within a grid has a unique ID, which can be accessed using `blockIdx`.

3. Grids

- **Collection of Blocks.** Blocks are grouped into a grid. A grid can contain up to $2^{31} - 1$ blocks¹⁰.
- **Grid Dimensions.** Grids can be one-dimensional, two-dimensional, or three-dimensional. Each block within a grid has a unique position within the grid, identified using `blockIdx.x`, `blockIdx.y`, and `blockIdx.z`.

GPU hierarchy

This hierarchy maps directly to the NVIDIA GPU architecture (**hardware hierarchy**):

1. Streaming Multiprocessor (SM).

- **Block Execution.** Each Streaming Multiprocessor (SM) on a GPU executes one or more thread blocks.
- **Warps.** Threads within a block are executed in groups of 32, known as warps. Warps are the basic unit of execution on the GPU.

¹⁰The limit of $2^{31} - 1$ blocks in CUDA grid comes from the maximum range of signed 32-bit integer. CUDA uses signed 32-bit integers to represent the indices of blocks within a grid. A signed 32-bit integer can represent values from -2^{31} to $2^{31} - 1$. Since negative block indices don't make sense in the context of a grid, CUDA only uses positive values. Therefore, the maximum positive value that can be used is $2^{31} - 1$, which equals 2'147'483'647.

2. GPU Core.

- **Thread Execution.** Each GPU core executes a single thread from a warp. Multiple GPU cores within an SM work together to execute all the threads in a warp.

3. Execution of Grids.

- **Multiple Grids.** A GPU can execute one or more grids of thread blocks. Each grid corresponds to a single kernel launch.
- **Interleaved Execution.** If the number of blocks exceeds the number of SMs, the execution of blocks is interleaved among the SMs.

② How to use the hierarchy

The CUDA thread hierarchy (and hardware hierarchy) provides a structured way to manage and organize parallel computation on the GPU. Unconsciously, when we coordinate parallel threads to execute a task, we leverage this hierarchy to maximize performance and resource utilization:

- **Kernel Launch Configuration.** When launching a kernel, we specify the grid and block dimensions. For example:

```

1 // 256 threads per block
2 dim3 dimBlock(256);
3 // Grid size to cover all elements
4 dim3 dimGrid((N + 256 - 1) / 256);
5 MyKernel<<<dimGrid, dimBlock>>>(a, b, N);

```

This configuration ensures that the kernel has enough threads to process all elements in parallel.

- **Mapping Threads to Data.** Each thread calculates its unique index based on its position within the block and grid, allowing it to process a specific element of the data array. For example:

```

1 int i = blockIdx.x * blockDim.x + threadIdx.x;
2 if (i < N) {
3     // Each thread processes a unique element
4     a[i] = a[i] + b;
5 }

```

- **Handling Data Beyond Block Size.** If the data size exceeds the number of threads per block, the grid dimension ensures all data is covered. For example:

```

1 dim3 dimGrid((N + blocksize - 1) / blocksize);

```

This calculation ensures that even if N is not a multiple of the block size, the entire dataset is processed.

- **Scalability.** The hierarchy allows the program to scale across different GPU architectures by adjusting the grid and block sizes to match the hardware capabilities (e.g., number of SMs, maximum threads per block).

✖ Thread hierarchy Variables

CUDA provides several built-in variables to help manage and identify the hierarchy of threads in our parallel computation. These variables are used within kernel functions to determine the unique identity of each thread, block, and grid:

- **Grid and Block Dimensions:**

- `gridDim.x`: represents the number of blocks in the grid along the *x*-axis.
- `blockDim.x`: represents the number of threads in a block along the *x*-axis.

- **Block and Thread Indices:**

- `blockIdx.x`: the block index along the *x*-axis. It ranges from 0 to `gridDim.x - 1`.
- `threadIdx.x`: the thread index within a block along the *x*-axis. It ranges from 0 to `blockDim.x - 1`.

- **Global Thread Index Calculation:** to identify the **unique index of each thread across the entire grid**, we can use the formula:

$$\text{globalID} = \text{blockIdx.x} \times \text{blockDim.x} + \text{threadIdx.x} \quad (13)$$

This formula computes the global thread index by combining the block and thread indices.

- **2D/3D Structures:** in more complex applications, such as image processing or solving partial differential equations, threads use their IDs to decide which data to process. This simplifies memory addressing when dealing with multidimensional data. Example of thread index calculations in a 2D grid:

$$\begin{aligned} \text{globalID} = & (\text{blockIdx.y} \times \text{gridDim.x} + \text{blockIdx.x}) \times \\ & (\text{blockDim.x} \times \text{blockDim.y}) + \\ & (\text{threadIdx.y} \times \text{blockDim.x} + \text{threadIdx.x}) \end{aligned}$$

- **Threads Per Block and Thread Number in Block:**

- `threadsPerBlock`: total **number of threads in a block**.

$$\text{threadsPerBlock} = \text{blockDim.x} \times \text{blockDim.y} = 8$$

- `threadNumInBlock`: unique **thread number** within a block.

$$\text{threadNumInBlock} = \text{threadIdx.x} + \text{blockDim.x} \times \text{threadIdx.y}$$

- `blockNumInGrid`: unique **block number** within the grid.

$$\text{blockNumInGrid} = \text{blockIdx.x} + \text{gridDim.x} \times \text{blockIdx.y}$$

- `tid` (Global Thread ID): unique **thread ID across the entire grid**.

$$\text{tid} = \text{blockNumInGrid} \times \text{threadsPerBlock} + \text{threadNumInBlock}$$

7.6 Memory hierarchy

The memory hierarchy in CUDA was explained in Section 6.3.2, page 108. Here is a small summary:

- **Per-thread** (registers):
 - **Scope:** Per thread.
 - **Lifetime:** Duration of the thread.
 - **Speed:** Fastest memory in the CUDA memory hierarchy.
 - **Usage:** Used for storing frequently accessed variables and temporary data.
 - **Limitations:** Limited in size; excessive usage can lead to spilling into local memory, which is slower.
- **Per-block** (shared memory, cache):
 - **Scope:** Per block.
 - **Lifetime:** Duration of the block.
 - **Speed:** Much faster than global memory, similar to L1 cache.
 - **Usage:** Used for data sharing among threads within the same block, enabling efficient inter-thread communication.
 - **Configuration:** Typically user-managed, allowing for explicit control over data placement.
 - **Limitations:** Limited in size; excessive usage can limit the number of active blocks per SM.
- **Global Memory** (off-chip DRAM):
 - **Scope:** All threads.
 - **Lifetime:** Duration of the application.
 - **Speed:** Significantly slower than shared memory and registers due to higher latency.
 - **Usage:** Used for data that needs to be accessed by multiple threads or blocks.
 - **Characteristics:** Large in size, but high latency; optimizing access patterns (coalesced accesses) can improve performance.

Memory Accessibility

- **Per-thread:**

- **Host:** cannot directly access registers.
- **Device:** registers are accessed exclusively by the thread that owns them.
- **Operations:**
 - * **Host:** none. The host cannot allocate, deallocate, or access registers directly.
 - * **Device:** each thread can read from and write to its own registers. But registers are private to the thread, meaning *no other thread can access another thread's registers*.

- **Per-block:**

- **Host:** cannot directly access shared memory, but can specify shared memory size for each kernel launch.
- **Device:** can read from and write to shared memory within a block.
- **Operations:**
 - * **Host:** none directly (can specify size in kernel launch).
 - * **Device:** Read/Write.

- **Global Memory:**

- **Host:** can allocate and deallocate memory using `cudaMalloc` and `cudaFree`. Can copy data to/from global memory using `cudaMemcpy`.
- **Device:** can read from and write to global memory directly.
- **Operations:**
 - * **Host:** `cudaMalloc`, `cudaFree`, `cudaMemcpy`, `cudaMemset`.
 - * **Device:** Read/Write.

Memory Management APIs

- **cudaMalloc:**

- **Purpose:** allocates memory on the GPU.
- **Usage:** used when we need to allocate space for variables or arrays that the GPU will use.
- **Function signature:**

```
1 cudaError_t cudaMalloc(void** devPtr, size_t size);
```

- **Parameters:**

- * **devPtr:** pointer to the allocated device memory.
- * **size:** size in bytes of the allocated memory.

- **Example:**

```
1 float* d_array;
2 cudaMalloc((void**)&d_array, N * sizeof(float));
```

- **cudaMallocHost:**

- **Purpose:** allocates pinned memory on the host.
- **Usage:** used for host memory that can be asynchronously copied to the device, improving transfer efficiency.

- **Function signature:**

```
1 cudaError_t cudaMallocHost(void** ptr, size_t size);
```

- **Parameters:**

- * **ptr:** pointer to the allocated host memory.
- * **size:** size in bytes of the allocated memory.

- **Example:**

```
1 float* h_array;
2 cudaMallocHost((void**)&h_array, N * sizeof(float));
```

- **cudaFree:**

- **Purpose:** frees memory that was allocated on the GPU.
- **Usage:** used to deallocate memory that was previously allocated with `cudaMalloc`.

- **Function signature:**

```
1 cudaError_t cudaFree(void* devPtr);
```

- **Parameters:**

- * **devPtr:** pointer to the memory to be freed.

- **Example:**

```
1 float* d_array;
2 cudaFree(d_array);
```

- **cudaFreeHost:**

- **Purpose:** frees pinned memory that was allocated on the host.
- **Usage:** used to deallocate memory that was previously allocated with `cudaMallocHost`.
- **Function signature:**

```
1 cudaError_t cudaFreeHost(void* ptr);
```

- **Parameters:**

- * `ptr`: pointer to the memory to be freed.

- **Example:**

```
1 float* h_array;
2 cudaFreeHost(h_array);
```

- **cudaMemcpy:**

- **Purpose:** copies data between host and device, or between different regions of device memory.
- **Usage:** used for transferring data to and from the GPU, or between different memory regions on the GPU.
- **Function signature:**

```
1 cudaError_t cudaMemcpy(void* dst, const void* src, size_t
count, cudaMemcpyKind kind);
```

- **Parameters:**

- * `dst`: destination pointer.
- * `src`: source pointer.
- * `count`: size in bytes of the memory to be copied.
- * `kind`: type of copy operation (e.g., `cudaMemcpyHostToDevice`, `cudaMemcpyDeviceToHost`, `cudaMemcpyDeviceToDevice`)

- **Example:**

```
1 cudaMemcpy(d_array, h_array, N * sizeof(float),
cudaMemcpyHostToDevice);
```

⚠ cudaMallocHost/cudaFreeHost can reduce CPU performance

Pinned memory (or page-locked memory) is a **region of system memory that is “locked” and cannot be paged out by the operating system**. This type of memory provides faster data transfer rates between the CPU and GPU because it allows for direct memory access (DMA) without the need for the CPU to perform intermediate steps. Since `cudaMallocHost` and `cudaFreeHost` work on this type of memory, the possible worst-case scenarios are:

- ✖ Limited System Resources.** **Pinned memory consumes physical RAM.** When we allocate a large amount of pinned memory, it reduces the available RAM for other system tasks, which can lead to increased memory pressure and reduced performance for other applications.

✖ **Increased Memory Allocation Time.** Allocating pinned memory (`cudaMallocHost`) is generally more time-consuming than allocating pageable memory because the operating system must ensure that the allocated memory pages are locked in RAM and cannot be paged out.

Similarly, freeing pinned memory (`cudaFreeHost`) involves **unlocking these pages, which can also be a time-consuming operation.**

✖ **Memory Fragmentation.** Frequent allocations and deallocations of pinned memory can lead to **memory fragmentation**. Over time, this fragmentation can make it harder for the operating system to find contiguous blocks of free memory, which can slow down memory allocation and deallocation operations.

✖ **Reduced Cache Efficiency.** Pinned memory allocations can affect the CPU's cache efficiency. When a large portion of memory is pinned, it may reduce the effectiveness of the CPU's memory caching mechanisms, leading to increased memory access times for other processes.

Is suggest to use the pinned memory when:

- ✓ **Performance-Critical Data Transfers.** Use pinned memory for data transfers that are critical for performance, where the speedup from faster data transfer rates outweighs the potential downsides.
- ✓ **Async Operations.** Pinned memory is particularly beneficial for asynchronous data transfers between the host and device, enabling overlap of computation and data transfer for better performance.

Example 4: Manual Memory Management - Vector Addition

```

1 % allocate h_A, h_B, h_C
2 void vecAdd(float *h_A, float *h_B, float *h_C, int n) {
3     int size = n * sizeof(float);
4     float *d_A, *d_B, *d_C;
5
6     // Allocate memory on the GPU (device)
7     // for arrays d_A, d_B, d_C
8     cudaMalloc((void**)&d_A, size);
9     cudaMalloc((void**)&d_B, size);
10    cudaMalloc((void**)&d_C, size);
11
12    // Copy data from host arrays h_A and h_B
13    // to device arrays d_A and d_B
14    cudaMemcpy(d_A, h_A, size, cudaMemcpyHostToDevice);
15    cudaMemcpy(d_B, h_B, size, cudaMemcpyHostToDevice);
16
17    // Kernel invocation code (not shown in the image)
18    // Example kernel launch:
19    // vecAddKernel<<<blocks, threads>>>(d_A, d_B, d_C, n);
20
21    // Copy the result from device array
22    // d_C back to host array h_C
23    cudaMemcpy(h_C, d_C, size, cudaMemcpyDeviceToHost);
24

```

```

25 // Free the allocated memory on the device
26 cudaFree(d_A);
27 cudaFree(d_B);
28 cudaFree(d_C);
29 }
30 % free h_A, h_B, h_C

```

- **Allocate Host Memory.** Before the `vecAdd` function is called, host memory for arrays `h_A`, `h_B`, and `h_C` is allocated. This memory is used to store the input data and the result on the host (CPU).
- **Function Definition.** The `vecAdd` function performs the steps in the code.
- **Memory Allocation on Device.** `cudaMalloc` allocates memory on the GPU for `d_A`, `d_B`, and `d_C`. The size of each array is determined by `n * sizeof(float)`.
- **Copy Data from Host to Device.** `cudaMemcpy` copies data from the host arrays `h_A` and `h_B` to the device arrays `d_A` and `d_B` using the `cudaMemcpyHostToDevice` flag. This step transfers the input data from the CPU to the GPU for processing.
- **Kernel Invocation.** The CUDA kernel (not shown in the image) is invoked to perform vector addition on the device. Each thread on the GPU computes one element of the result vector.
- **Copy Data from Device to Host.** `cudaMemcpy` copies the result from the device array `d_C` back to the host array `h_C` using the `cudaMemcpyDeviceToHost` flag. This step transfers the computed result from the GPU to the CPU.
- **Free Allocated Memory.** `cudaFree` frees the allocated memory on the GPU for `d_A`, `d_B`, and `d_C`. This ensures that no memory leaks occur on the GPU.

❷ How can we allocate memory that is shared between CPU and GPU?

Unified Memory in CUDA is a single memory space that is accessible by both the host (CPU) and the device (GPU). It abstracts away the complexities of explicit memory management, making it easier to develop CUDA applications. Its features:

- **Single Address Space.** Both the **CPU and GPU can access the same memory location**, *eliminating the need for explicit data transfers between them*. This means that the same pointer can be used on both the host and the device.
- **Automatic Data Migration.** The CUDA runtime system handles the movement of data between the host and device automatically. When the

GPU accesses data that resides in the host memory, the CUDA runtime migrates the data to the GPU memory and vice versa.

- **On-Demand (Page) Migration.** Unified Memory uses a demand-paging mechanism similar to virtual memory in operating systems. **When a page of memory is accessed by the CPU or GPU, and it is not currently resident in the respective memory, it is migrated on demand.**

✓ Pros

- ✓ **Simplified Memory Management.** Developers do not need to worry about explicit memory copies (`cudaMemcpy`). The **CUDA runtime manages data movement automatically**.
- ✓ **Ease of Programming.** Writing CUDA programs becomes easier, especially for those new to CUDA, as **the same pointer can be used for both host and device operations**.
- ✓ **Efficient Use of Memory.** Reduces code complexity, as there is **no need for separate host and device memory allocations and management**.

✗ Cons

- ✗ **Potential Performance Overhead.** The **automatic data migration can introduce runtime overhead due to page faults¹¹** when data is accessed.

Unified Memory uses a similar (to operating system) paging mechanism to manage memory across the CPU and GPU. When a memory page is accessed by the CPU or GPU and is not present in the respective memory (host or device), a page fault occurs. The CUDA runtime automatically migrates the page from the current memory location (host to device or device to host) to the memory where the access occurred.

This can be less efficient compared to explicitly managed memory transfers.

- ✗ **Limited Control.** Developers have less control over when and where data is moved. For performance-critical applications, **manual memory management can sometimes yield better performance**.

- ✗ **Memory Contention.** Sharing the same memory space between the CPU and GPU can lead to contention, **potentially impacting performance if both are trying to access the same memory simultaneously**.

¹¹Modern operating systems use a virtual memory system where the physical memory (RAM) is abstracted as a larger virtual memory space. This allows programs to use more memory than physically available by using disk storage (paging). A **page fault** occurs when a program tries to access a part of the memory that is not currently in the physical memory (RAM). The operating system intervenes to load the required memory page from disk into RAM, allowing the program to continue execution.

`cudaMallocManaged` is a function in CUDA that **allocates managed memory, which is accessible by both the host (CPU) and the device (GPU)**. This unified memory simplifies the programming model by allowing both the CPU and GPU to share a single memory space.

```
1 cudaError_t cudaMallocManaged(void **devPtr, size_t size);
```

- `devPtr`: pointer to the allocated unified memory.
- `size`: size in bytes of the allocated memory.

✓ How to avoid page faults with `cudaMallocManaged`

Asynchronous memory prefetching is a technique used to proactively move data between the host and device to avoid page faults and improve performance. This is particularly useful in Unified Memory, where data can be accessed by both the CPU and GPU.

Instead of waiting for a page fault to occur (which would trigger an on-demand data transfer, a scenario we would avoid due to performance overhead), we prefetch the required data to the desired memory location before it is needed. This **operation is done asynchronously**, meaning that it **does not block the execution of other operations**. This allows the program to continue running while the data is being transferred in the background.

In CUDA, this technique is used with the function `cudaMemPrefetchAsync`:

```
1 cudaError_t cudaMemPrefetchAsync(
2     void *devPtr, size_t count, int dstDevice, cudaStream_t stream
3     = 0
4 );
```

- `devPtr`: pointer to the memory to be prefetched.
- `count`: size of the memory to be prefetched, in bytes.
- `dstDevice`: the destination device (can be a GPU device ID or the result of the function `cudaCpuDeviceId` for the host).
- `stream`: the stream to perform the operation in (optional).

An **example** of use:

```
1 int deviceId;
2 cudaGetDevice(&deviceId);
3
4 // Prefetch data to the device
5 cudaMemPrefetchAsync(pointerToSomeUMData, size, deviceId);
6
7 // Prefetch data back to the host
8 cudaMemPrefetchAsync(pointerToSomeUMData, size, cudaCpuDeviceId);
```

7.7 Streams

CUDA Streams are **sequences of operations** (like kernel launches or memory copies) that **execute in order**. By *default*, all **operations in CUDA run in the default stream**, which is *blocking* and *sequential*.

Is important to understand the CUDA streams, because by changing the default order, we can gain performance:

- Concurrency. By using multiple streams, we can **perform operations in parallel**, leading to better performance.
- Overlap Computation and Data Transfer. For example, **while one stream is running a kernel on the GPU, another stream can be copying data to or from the GPU**.

⚠ Default vs Non-Default Streams

- Default Stream:
 - Executes operations **sequentially**.
 - **Blocks all other streams** until its operations are complete.
- Non-Default Streams:
 - Allow for **concurrent** execution of operations.
 - Operations within the **same stream execute sequentially**, but **operations in different streams can run in parallel**.

✖ CUDA implementation

1. **Creating Streams.** We can create streams using `cudaStreamCreate`.

```

1 cudaStream_t stream1, stream2;
2 cudaStreamCreate(&stream1);
3 cudaStreamCreate(&stream2);

```

2. **Launching Kernels in Streams.** Launch kernels in different streams to execute them concurrently.

```

1 // Launching a kernel in stream1
2 kernel1<<<blocks, threads, 0, stream1>>>(...);
3
4 // Launching a kernel in stream2
5 kernel2<<<blocks, threads, 0, stream2>>>(...);

```

3. **Synchronizing Streams.** We can synchronize individual streams or all streams.

```
1 // sync specific stream:  
2 cudaStreamSynchronize(stream1);  
3  
4 // sync all streams  
5 cudaDeviceSynchronize();
```

4. **Destroying Streams.** Once we're done with the streams, it's important to destroy them to free up resources.

```
1 cudaStreamDestroy(stream1);  
2 cudaStreamDestroy(stream2);
```

Example 5

Imagine we have two kernels to execute, `kernel1` and `kernel2`. If we use the default stream, they will execute one after the other:

1. Launch `kernel1`;
2. Wait for `kernel1` to complete;
3. Launch `kernel2`;
4. Wait for `kernel2` to complete.

This sequential execution means the GPU is not fully utilized. Using two non-default streams, we can launch both kernels to execute concurrently:

1. Create two streams:

```
1 cudaStream_t stream1, stream2;  
2 cudaStreamCreate(&stream1);  
3 cudaStreamCreate(&stream2);
```

2. Launch `kernel1` in `stream1` and `kernel2` in `stream2`:

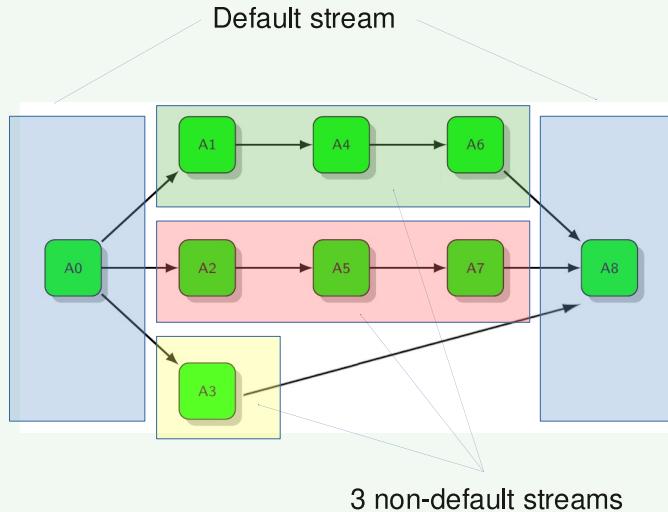
```
1 kernel1<<<blocks, threads, 0, stream1>>>(...);  
2 kernel2<<<blocks, threads, 0, stream2>>>(...);
```

Now, `kernel1` and `kernel2` can run in parallel, allowing the GPU to be used more efficiently.

Example 6: DAG

The following image shows a series of tasks (A1 through A8) organized in a DAG. The arrows between tasks indicate dependencies, meaning one task must complete before another can start.

- Default Stream: contains tasks A0, and A8.
- Non-Default Streams: there are three non-default streams that contain tasks A1, A4, A6, A2, A5, A7, and A3.



The execution flow is:

- Initial Tasks.
 - Task A0 starts in the default stream.
 - Concurrently, tasks A1, A2, and A3 run in non-default streams (`stream1`, `stream2`, and `stream3`).
- Dependent Tasks.
 - After A1, A2, and A3 complete, A4, A5, A6, and A7 start in the respective non-default streams.
- Final Task.
 - After all dependent tasks are complete, task A8 runs in the default stream.

The implementation is the following:

1. Task A0 in Default Stream.

```
1 kernelA0<<<blocks, threads>>>(...); // Default stream
```

2. Tasks A1, A2, and A3 in Different Streams:

```

1 // Stream 1
2 kernelA1<<<blocks, threads, 0, stream1>>>(...);
3 // Stream 2
4 kernelA2<<<blocks, threads, 0, stream2>>>(...);
5 // Stream 3
6 kernelA3<<<blocks, threads, 0, stream3>>>(...);

```

3. Synchronize Streams Before Dependent Tasks:

```

1 cudaStreamSynchronize(stream1);
2 cudaStreamSynchronize(stream2);
3 cudaStreamSynchronize(stream3);

```

4. Tasks A4, A5, A6, and A7 in Non-Default Streams:

```

1 // Stream 1
2 kernelA4<<<blocks, threads, 0, stream1>>>(...);
3 // Stream 2
4 kernelA5<<<blocks, threads, 0, stream2>>>(...);
5 // Stream 1
6 kernelA6<<<blocks, threads, 0, stream1>>>(...);
7 // Stream 2
8 kernelA7<<<blocks, threads, 0, stream2>>>(...);

```

5. Synchronize Streams Again Before Final Task:

```

1 cudaStreamSynchronize(stream1);
2 cudaStreamSynchronize(stream2);
3 cudaStreamSynchronize(stream3);

```

6. Task A8 in Default Stream:

```

1 kernelA8<<<blocks, threads>>>(...); // Default stream

```

7. Destroying Streams:

```

1 cudaStreamDestroy(stream1);
2 cudaStreamDestroy(stream2);
3 cudaStreamDestroy(stream3);

```

Using concurrent CUDA streams helps manage complex task dependencies in DAG-like applications. By running independent tasks in different streams, we can achieve parallel execution and optimize the performance of our CUDA applications. The example shows how to create streams, start kernels in these streams, synchronize them, and finally destroy the streams to free up resources. Finally, **non-default streams and manual memory management allow for overlapping data transfers and computations.**

7.8 CUDA and OpenMP or MPI

7.8.1 Motivations

In today's computing landscape, **achieving high performance and efficiency in complex applications** often requires the power of parallel processing. This need has given rise to several parallel computing frameworks, each designed to address specific aspects of parallelism. Among these, CUDA, OpenMP, and MPI stand out as key technologies that enable efficient utilization of computing resources, whether on a single machine with multiple GPUs or across distributed clusters.

⌚ Systems with multiple GPUs - CUDA + OpenMP

- **OpenMP**

- **Purpose:** used to dispatch parallel tasks across multiple GPUs.
- *Why OpenMP?*
 - * **Simplified Parallelism.** OpenMP provides a straightforward way to parallelize tasks using compiler directives.
 - * **Task Scheduling.** It allows for efficient scheduling and management of tasks across multiple GPUs.
 - * **Shared Memory Model.** OpenMP works well with shared memory systems, making it easier to manage and synchronize tasks.

- **CUDA**

- **Purpose:** used for programming NVIDIA GPUs to perform parallel computations.
- *Why CUDA?*
 - * **Fine-Grained Control.** CUDA provides detailed control over GPU resources and performance optimization.
 - * **Massive Parallelism.** Enables the execution of thousands of threads in parallel, maximizing GPU utilization.
 - * **Optimized Performance.** Designed specifically for NVIDIA GPUs, ensuring optimal performance and efficiency.

OpenMP is particularly useful for tasks that benefit from shared memory parallelism, making it easier to write efficient and portable code for multi-core processors. It is often used in conjunction with CUDA to manage the CPU-side parallelism, thereby optimizing the performance of the entire system.

2. Distributed Clusters with Multiple Nodes Containing GPUs - CUDA + MPI

MPI (Message Passing Interface) is a **standardized and portable message-passing system designed for parallel computing in distributed memory environments, such as clusters of computers**. MPI excels in managing communication between nodes in a distributed system, allowing for efficient coordination and data exchange. When combined with CUDA, MPI enables developers to scale their applications across multiple nodes, each leveraging the power of GPUs, thus tackling larger and more complex problems than would be possible on a single machine.

- **MPI**

- **Purpose:** used for communication across nodes in a distributed system.
- *Why OpenMP?*
 - * **Scalability.** MPI is designed for high-performance communication in distributed memory systems, allowing for efficient parallel computing across multiple nodes.
 - * **Inter-Node Communication.** Provides robust mechanisms for data exchange between nodes, ensuring coordination and synchronization in distributed environments.
 - * **Flexibility.** Works with various hardware and software configurations, making it versatile for different parallel computing scenarios.

- **CUDA**

- **Purpose:** used within each node's GPU to perform parallel computations.
- *Why CUDA?*
 - * **Performance.** Ensures that computations within each GPU are optimized for maximum throughput.
 - * **Integration with MPI.** CUDA can be effectively combined with MPI to handle local computations within each node, while MPI manages communication between nodes.

Example 7: Scenario CUDA + MPI

A distributed cluster with multiple nodes, each containing one or more GPUs. MPI processes (e.g., MPI Process N, MPI Process N+1) manage communication across nodes, while CUDA handles computations within each GPU.

Technology	When?
------------	-------

OpenMP

- Ideal for managing parallel tasks within a single system with multiple GPUs.
- Simplifies task scheduling and synchronization in shared memory environments.

MPI

- Essential for communication in distributed clusters with multiple nodes containing GPUs.
- Provides scalable and efficient data exchange between nodes.

CUDA

- Used for fine-grained parallel computations within each GPU.
 - Ensures optimized performance and maximum GPU utilization.
-

Table 7: When to use each technology.

Batch processing vs Cooperative patterns

The computation patterns in a Multi-GPU environment are divided into batch and cooperative patterns.

- **Batch Processing.** Batch processing is ideal for scenarios where the same task needs to be performed multiple times with different data sets. It's a simple way to maximize GPU utilization. In other words, run the same independent task multiple times with different data. Steps involved:

1. **Identify the number of available GPUs.** Use CUDA APIs to determine how many GPUs are available for processing.
2. **Initialize and allocate memory.** Allocate necessary memory on each GPU. This step involves copying the data required for computation from the host (CPU) to the device (GPU).
3. **Create CUDA streams.** CUDA streams enable concurrent execution of kernels. Creating multiple streams allows us to run multiple tasks in parallel on the GPU.
4. **Launch the kernel.** Dispatch the kernels for execution. Each GPU runs its own copy of the kernel, processing a different subset of the data.

5. **Synchronize.** Ensure all GPU tasks are complete. Use synchronization mechanisms to manage dependencies and ensure data integrity.
6. **Retrieve results.** Copy the results from the device back to the host.

 **Pros**

- ✓ **High throughput.** Efficiently handles repetitive tasks.
- ✓ **Scalability.** Easy to scale by adding more GPUs.

 **Cons**

- ✗ **Limited to independent tasks.** Not suitable for tasks that require inter-GPU communication.
- **Cooperative Patterns.** Tasks need to work together to achieve a common goal. These tasks are interdependent and require communication between GPUs.

Steps involved:

1. **Task division.** Split the task into smaller sub-tasks that can be distributed across GPUs.
2. **Memory allocation and initialization.** Allocate memory for each sub-task and initialize the required data.
3. **Kernel execution.** Launch kernels that perform the sub-tasks. Unlike batch processing, these kernels may need to communicate with each other.
4. **Inter-GPU communication.** Implement communication protocols to share data and results between GPUs. This can involve techniques like GPU direct RDMA (Remote Direct Memory Access).
5. **Synchronization and coordination.** Use synchronization mechanisms to coordinate the execution of sub-tasks and ensure they are completed in the correct order.
6. **Result aggregation.** Combine the results from all GPUs to produce the final output.

 **Pros**

- ✓ **Solves complex tasks.** Suitable for problems that require collaborative computation.
- ✓ **Efficient use of resources.** Leverages the combined power of multiple GPUs.

 **Cons**

- ✗ **Complex implementation.** Requires careful design and management of inter-GPU communication.
- ✗ **Potential bottlenecks.** Synchronization and communication can introduce delays.

The main differences are:

- **Task Independence vs. Interdependence.** Batch processing excels with independent tasks, whereas cooperative patterns manage interdependent tasks.
- **Communication Needs.** Batch processing minimizes communication between tasks, while cooperative patterns facilitate necessary inter-task communication.
- **Implementation Complexity.** Batch processing is generally simpler to implement, whereas cooperative patterns require more sophisticated design and coordination.

By dividing the computation patterns this way, it becomes easier to choose the most appropriate approach for different types of workloads, ensuring optimal performance and efficient use of GPU resources.

7.8.2 CUDA API for Multi-GPUs

In high-performance computing, the use of multiple GPUs can significantly increase computational throughput and efficiency. **CUDA provides several API calls that allow developers to effectively manage and utilize multiple GPUs.** The most important of these are:

- The `cudaSetDevice()` function is used to **specify which GPU device should be active for subsequent CUDA operations on the host thread**. By setting the desired GPU device, developers can control where kernel executions, memory allocations, and other device-specific calls take place. This feature is especially important when working with systems that have multiple GPUs, as it allows fine-grained control over which device is used for specific tasks.

– **Function:**

```
1 cudaSetDevice(int device)
```

– **Description:**

- * Sets the GPU device to use for subsequent CUDA operations on the active host thread.
- * This function specifies which GPU to use for operations like kernel launches, memory allocations, and other device-specific calls.

– **Parameters:**

- * `device`: the integer ID of the GPU to be set as the active device.

– **Usage:**

- * Does not affect other host threads, meaning the device setting is local to the thread where this function is called.
- * Does not affect previous asynchronous calls.

– **Example:**

```
1 cudaSetDevice(0); // Sets GPU 0 as the active device
```

CUDA Runtime Calls Affected by `cudaSetDevice()`

If `cudaSetDevice()` is called before:

- **Kernel Launch.** The kernel will execute on the specified active device.
- **Memory Allocations.** All memory allocations will be made on the specified active device.
- **Stream Creation.** The stream will be associated with the specified active device.
- **Synchronization Functions.** These functions will synchronize tasks on the specified active device and active host thread only.

- The `cudaGetDevice()` function retrieves the ID of the GPU currently being used by the active host thread. This function is useful for confirming which GPU is set as active, especially in debugging scenarios. Knowing the active device ensures that operations are being performed on the intended GPU, preventing potential errors and resource conflicts.

– **Function:**

```
1 cudaGetDevice(int *device)
```

– **Description:**

- * Retrieves the ID of the GPU currently being used by the active host thread.
- * Helps to confirm which GPU is currently set as active.

– **Parameters:**

- * `device`: a pointer to an integer where the ID of the active device will be stored.

– **Usage:**

- * This function is useful for debugging and ensuring that the correct GPU is being utilized.

– **Example:**

```
1 int device;
2 // Gets the ID of the current active GPU
3 // and stores it in 'device'
4 cudaGetDevice(&device);
```

- The `cudaGetDeviceCount()` function is used to determine the number of CUDA-capable GPUs available in the system. This function is typically called during the initialization phase to understand the GPU resources available. By knowing the number of GPUs, developers can design their applications to efficiently distribute workloads across multiple devices, maximizing performance and resource utilization.

– **Function:**

```
1 cudaGetDeviceCount(int *count)
```

– **Description:**

- * Determines the number of CUDA-capable GPUs available in the system.
- * Helps in discovering the total number of GPUs that can be utilized for computation.

– **Parameters:**

- * `count`: a pointer to an integer where the number of CUDA-capable devices will be stored.

– Usage:

- * This function is typically called during the initialization phase to understand the GPU resources available.

– Example:

```
1 int deviceCount;
2 // Gets the number of CUDA-capable GPUs
3 // and stores it in 'deviceCount',
4 cudaGetDeviceCount(&deviceCount);
```

7.8.3 Memory Management with Multiple GPUs

In systems with multiple GPUs, **efficient memory management is critical to optimizing performance and ensuring smooth operation**. There are two primary approaches to memory management in such environments: manual memory management and unified memory.

- **Manual Memory Management** requires **setting the active device using the `cudaSetDevice()`** (page 203) **function before any memory allocation or kernel launch**. This ensures that subsequent operations are directed to the specified GPU.

For **example**, setting GPU 0 as the active device allows for memory allocation on that particular GPU. This method provides fine-grained control over which GPU handles specific tasks, allowing for optimized resource utilization.

- **Unified Memory**, on the other hand, offers a **more automated approach**. If the flag `cudaDevAttrConcurrentManagedAccess`:

- Is **set for all devices, memory can be allocated using the function `cudaMallocManaged()` without the need to set the active device**. This simplifies the process by allowing all GPUs to access the allocated memory automatically.
- Is **not set, but devices can still access each other's memory, it becomes necessary to set the active device before memory allocation**. This setup ensures that all other devices can access the data through PCIe, facilitating efficient data transfer and resource sharing.

Example 8: vector addition with multiple GPUs

This example demonstrates how to perform vector addition using multiple GPUs.

```

1 float *m_A0, float *m_B0, *m_A1, float *m_B1, int n;
2 int size = n * sizeof(float);
3
4 // Set the active device to GPU 0
5 cudaSetDevice(0);
6 // Allocate memory on GPU (device) 0
7 cudaMalloc((void**) &m_A0, size);
8 cudaMalloc((void**) &m_B0, size);
9
10 // Set the active device to GPU 1
11 cudaSetDevice(1);
12 // Allocate memory on GPU (device) 1
13 cudaMalloc((void**) &m_A1, size);
14 cudaMalloc((void**) &m_B1, size);
15
16 // Memory initialization on the host (CPU) and memory
17 // transfers
18
19 // Set the device for kernel execution
20 cudaSetDevice(0);
21 // Launch kernel on GPU 0

```

```
21 vecAdd<<<gridDim, blockDim>>>(m_A0, m_B0);
22
23 // Set the device for kernel execution
24 cudaSetDevice(1);
25 // Launch kernel on GPU 1
26 vecAdd<<<gridDim, blockDim>>>(m_A1, m_B1);
27
28 // Free memory on GPU 0 and GPU 1
29 cudaFree(m_A0); cudaFree(m_B0);
30 cudaFree(m_A1); cudaFree(m_B1);
```

- Row 5. Setting the Active Device: it sets GPU 0 as the active device. Any subsequent CUDA operations, such as memory allocation or kernel launches, will be directed to GPU 0.
- Rows 7-8. Memory Allocation on GPU 0: memory is allocated on GPU 0 for two arrays, `m_A0` and `m_B0`, each of size `n * sizeof(float)`.
- Row 11. Setting the Active Device to GPU 1: it changes the active device to GPU 1, directing subsequent operations to this device.
- Rows 13-14. Memory Allocation on GPU 1: similar to GPU 0, memory is allocated on GPU 1 for two arrays, `m_A1` and `m_B1`.
- Row 16. Memory Initialization and Transfers: the comment mentions initializing memory on the host and transferring it to the devices, although the specific code for this is not shown. Typically, we would use `cudaMemcpy()` to transfer data from the host to the device.
- Rows 19-21. Kernel Execution: after setting GPU 0 as the active device, the `vecAdd` kernel is launched to perform vector addition on the arrays `m_A0` and `m_B0`.
- Row 24-26. Kernel Execution on GPU 1: similarly, GPU 1 is set as the active device, and the `vecAdd` kernel is launched on `m_A1` and `m_B1`.
- Row 29-30. Freeing Allocated Memory: after the computations are complete, the allocated memory on both GPUs is freed to avoid memory leaks.

Memory transfers between GPUs

In CUDA, there are several methods for transferring data between GPUs, each providing a different level of control and ease of use. The three main methods for transferring memory regions between GPUs are:

- **Fully Explicit Transfer.** In this method, we **explicitly specify both the source and destination devices for the memory transfer**. This approach gives us full control over the transfer process, but requires more detailed management of the transfer operation.

How it works:

1. Set the source device using `cudaSetDevice()`.
2. Allocate memory and perform the necessary operations on the source device.
3. Set the destination device using `cudaSetDevice()`.
4. Allocate memory on the destination device.
5. Use `cudaMemcpyPeerAsync()` to transfer data asynchronously between the two devices.

Example 9: Fully Explicit Transfer

```

1 int srcDevice = 0;
2 int dstDevice = 1;
3 float *d_src, *d_dst;
4 size_t size = n * sizeof(float);
5
6 cudaSetDevice(srcDevice); // Set source device
7 cudaMalloc(&d_src, size); // Allocate memory on source
    device
8
9 cudaSetDevice(dstDevice); // Set destination device
10 cudaMalloc(&d_dst, size); // Allocate memory on
    destination device
11
12 // Transfer data from source to destination device
13 cudaMemcpyPeerAsync(d_dst, dstDevice, d_src, srcDevice,
    size, stream);

```

`cudaMemcpyPeerAsync()` is a CUDA runtime API function used to **perform asynchronous memory transfers between two devices**. This function is particularly useful in multi-GPU systems, where it allows for non-blocking data transfers, enabling concurrent execution of other operations. Function Prototype:

```

1 cudaError_t cudaMemcpyPeerAsync(
2 void* dst,           // Destination pointer
3 int dstDevice,       // Destination device ID
4 const void* src,     // Source pointer
5 int srcDevice,       // Source device ID
6 size_t count,        // Size of the memory to be transferred in
    bytes
7 cudaStream_t stream // CUDA stream (optional)
8 );

```

Parameters:

- **dst**: pointer to the destination memory on the destination device.
- **dstDevice**: ID of the destination device.
- **src**: pointer to the source memory on the source device.
- **srcDevice**: ID of the source device.
- **count**: size of the memory to be transferred in bytes.
- **stream**: CUDA stream in which the transfer should be performed (optional). If no stream is specified, the default stream is used.

- **Partially Explicit Transfer.** This method uses unified addressing, which simplifies the transfer process by allowing us to use a unified address space. We only need to specify one of the devices explicitly, while the CUDA runtime handles the rest.

How it works:

1. Check if unified addressing is supported using:

```
cudaDeviceGetAttribute()
```

2. Allocate memory on one device (e.g., the source device).
3. Use `cudaMemcpy()` with the `cudaMemcpyDefault` flag to transfer data to the destination device.

Example 10: Fully Explicit Transfer

```

1 int srcDevice = 0;
2 int dstDevice = 1;
3 float *d_src;
4 size_t size = n * sizeof(float);
5
6 // Check if unified addressing is supported
7 int unifiedAddressing = 0;
8 cudaDeviceGetAttribute(&unifiedAddressing,
9     cudaDevAttrUnifiedAddressing, srcDevice);
10
11 if (unifiedAddressing) {
12     cudaSetDevice(srcDevice); // Set source device
13     cudaMalloc(&d_src, size); // Allocate memory on
14     source device
15
16     // Transfer data to destination device using unified
17     // addressing
18     cudaMemcpy(d_dst, d_src, size, cudaMemcpyDefault);
19 }
```

- **Implicit Transfer.** In this method, the **CUDA driver automatically handles the transfer of memory between devices without explicit intervention from the programmer**. This is the simplest method in terms of coding effort, as the driver takes care of the details.

How it works:

- The CUDA driver internally manages memory transfers between GPUs. This typically involves:
 1. Allocate memory (unified memory) on the source device using `cudaMallocManaged()`. This memory is accessible to all GPUs and the host, allowing CUDA to implicitly handle data transfers.
 2. Using unified memory or other mechanisms to handle the data transfer transparently. In other words, access the managed memory from any GPU by setting the active device and performing operations on it. CUDA internally manages the data transfer to ensure the memory is available on the device that needs it.

```

1 cudaSetDevice(0); // Set device 0 and use the memory
2 kernel<<<gridDim, blockDim>>>(d_managed);
3
4 cudaSetDevice(1); // Set device 1 and use the memory
5 kernel<<<gridDim, blockDim>>>(d_managed);

```

Example 11: Fully Explicit Transfer

```

1 float *d_managed;
2 size_t size = n * sizeof(float);
3
4 // Allocate managed memory accessible by both devices
5 cudaMallocManaged(&d_managed, size);
6
7 // Now d_managed can be accessed by any device as needed
8 cudaSetDevice(0); // Set device 0 and use the memory
9 kernel<<<gridDim, blockDim>>>(d_managed);
10
11 cudaSetDevice(1); // Set device 1 and use the memory
12 kernel<<<gridDim, blockDim>>>(d_managed);

```

❓ Communication Channels

There are two main types of communication channels connecting GPUs together:

- **Standard PCIe 3.0 Link**

- **Speed:** 32 GB/s
- **Description:** PCIe (Peripheral Component Interconnect Express) is a standard communication interface used to connect GPUs to the CPU and other peripherals. While widely used, PCIe 3.0 has limitations in terms of bandwidth, which can impact performance in data-intensive applications.

- **NVLink**

- **Speed:** 300 GB/s (on the Tesla V100)
- **Description:** NVLink is a high-speed interconnect technology developed by NVIDIA. It provides significantly higher bandwidth compared to PCIe, allowing for faster data transfer between GPUs. This can greatly improve performance in applications that require rapid communication and data exchange between multiple GPUs.

7.8.4 Batch Processing and Cooperative Patterns with OpenMP

In this section, we show how to implement batch processing and cooperative patterns using OpenMP.

Batch Processing OpenMP

The code snippet shows how to manage multiple GPU devices using OpenMP. The process includes:

```

1 #include <cuda_runtime.h>
2 #include <omp.h>
3 #include <vector>
4
5 __global__ void kernel(float* data, int size) {
6     // Kernel code here
7 }
8
9 int main() {
10     int deviceCount;
11     cudaGetDeviceCount(&deviceCount);
12     std::vector<cudaStream_t> streams(deviceCount);
13
14     // Initialize devices and streams in parallel
15     #pragma omp parallel for num_threads(deviceCount)
16     for(int dev = 0; dev < deviceCount; ++dev) {
17         cudaSetDevice(dev);
18         cudaStreamCreate(&streams[dev]);
19         // Allocate memory and initialize data
20     }
21
22     // Launch kernels in parallel
23     #pragma omp parallel for num_threads(deviceCount)
24     for(int dev = 0; dev < deviceCount; ++dev) {
25         cudaSetDevice(dev);
26         kernel<<<gridDim, blockDim, 0, streams[dev]>>>(* arguments
27         /* */);
28     }
29
30     // Synchronize streams and clean up
31     #pragma omp parallel for num_threads(deviceCount)
32     for(int dev = 0; dev < deviceCount; ++dev) {
33         cudaSetDevice(dev);
34         cudaStreamSynchronize(streams[dev]);
35         cudaStreamDestroy(streams[dev]);
36         // Free allocated memory
37     }
38
39     return 0;
}

```

1. Rows 10-12. **Getting the Device Count.** This part of the code retrieves the number of CUDA-capable devices (GPUs) in the system using `cudaGetDeviceCount()`. It then creates a vector of CUDA streams, with each stream corresponding to a GPU.
2. Rows 15-20. **Allocating Memory and Initializing Streams.** This OpenMP parallel for loop iterates over the number of devices. For each device, it sets the current device using `cudaSetDevice()`, creates a CUDA

stream using `cudaStreamCreate()`, and then allocates memory and initializes it. By using `num_threads(deviceCount)`, the loop is parallelized to run on multiple threads, each handling a different GPU.

3. Rows 23-27. **Launching Kernels.** This OpenMP parallel for loop is similar to the previous one. It sets the current device and launches a kernel on each GPU using the corresponding CUDA stream. The function:

```
kernel<<gridDim, blockDim, streams[dev]>>(...)
```

Specifies the grid and block dimensions, along with the stream to be used for the kernel execution.

Here, the OpenMP code creates some tasks associated with CUDA streams.

Cooperative Patterns OpenMP

The following enumeration extends the concept of cooperative patterns using OpenMP. The code snippet shows how to synchronize CUDA streams using OpenMP:

1. Initialization of Streams:

```
1 std::vector<cudaStream_t> streams;
2 // Initialization of the streams on each device
3 // ...
```

2. Launching Kernels:

```
1 #pragma omp parallel
2 {
3     // Launch the different kernels on the streams.
4 }
```

3. Synchronizing Streams:

```
1 #pragma omp for num_threads(streams.size())
2 for(auto& stream : streams)
3     cudaStreamSynchronize(stream);
```

4. Barrier Synchronization:

```
1 #pragma omp barrier
```

Instead of creating each task independently, OpenMP creates streams but waits for each to finish using a barrier.

7.8.5 OpenMP for heterogeneous architectures

Recent versions of OpenMP support parallel execution on heterogeneous architectures, including:

- **Host CPU:** the main processor of the system.
- **Attached Accelerators:** additional hardware such as GPUs, FPGAs, DSPs, etc. Useful for handling specific tasks to increase performance.

In other words, **OpenMP allows the same code to run on both the host (CPU) and the target device (accelerator)**. This means that different types of hardware can be used together to perform parallel computation more effectively.

To offload the execution of code to an accelerator device, such as a GPU, we use the **target** directive in OpenMP.

[Doc. ▾](#)

OpenMP: pragma omp target

```
1 #pragma omp target
```

- **Offloading Execution.** The **code within the region is executed on the accelerator device if one is present.**
 - If no accelerator device is available, the code continues to run on the host (CPU).
 - If there is an **if** clause that evaluates to false, the code also continues execution on the host.
- **Device Thread Execution.** A **thread on the target device (e.g., GPU) executes the code.**
- **Synchronous Execution.** By default, the **host thread blocks (waits) until the device thread has completed execution.** To avoid this blocking, we can use the **nowait** clause.

map clause

OpenMP: map

```
1 #pragma omp target map(map-type : variables)
```

The **map** clause in OpenMP is used to **specify how variables are mapped from the host to the target device (such as a GPU) and back**. This is crucial for ensuring that the data required by the target region is available on the target device and that any results are brought back to the host. It is used mainly to:

- **Copying Variables to Device:** the variables listed in the `map` clause are copied from the host to the target device.
- **Copying Variables Back to Host:** after the `target` region execution, the updated variables can be copied back to the host.

There are several types of `map` clauses that are used to give information to the compiler to avoid unnecessary data transfers:

- **to clause.**
 - Syntax: `map(to: lst)`
 - Description: **copies variables (list) from the host to the target device.**
 - Summary: Host → Target Device
- **from clause.**
 - Syntax: `map(from: lst)`
 - Description: **copies variables (list) from the target device to the host after the target region execution.**
 - Summary: Host ← Target Device
- **tofrom clause.**
 - Syntax: `map(tofrom: lst)`
 - Description: **copies variables from the host to the target device before the target region and back from the device to the host after execution.**
 - Summary: Host ↔ Target Device

The default value is `tofrom`. Because these operations copy memory from one device to another, it is **important to limit their use**. For example, in the following code:

```

1 #pragma omp target map(a, b, c, d)
2 {
3     #pragma parallel for
4     for (i = 0; i < N; ++i) {
5         a[i] = b[i] * c + d;
6     }
7 } // End of target

```

At `map-enter`, there is a copy of the variables in device memory (expensive). There is a computation phase where the device memory uses the variables. Finally, at the `map-exit`, the device copies the variables to host memory (expensive).

Example 12: SAXPY operation

This example demonstrates how to perform a SAXPY operation (Single-Precision A·X Plus Y) using OpenMP. The SAXPY operation is a basic linear algebra operation, and here it is implemented to run on a target device like a GPU.

```

1 void saxpy() {
2     double a, x[SZ], y[SZ];
3     double t = 0.0;
4     double tb, te;
5     tb = omp_get_wtime();
6     #pragma omp target map(to:x[0:SZ]) \
7         map(tofrom:y[0:SZ])
8     for (int i = 0; i < SZ; i++) {
9         y[i] = a * x[i] + y[i];
10    }
11    te = omp_get_wtime();
12    t = te - tb;
13    printf("Time of kernel: %lf\n", t);
14 }
```

- Variables Initialization.
 - `a`, `x[SZ]` and `y[SZ]` are the main variables involved in the SAXPY operation.
 - `t`, `tb`, and `te` are used for timing the operation.
- Timing the Operation.
 - `tb = omp_get_wtime();` records the start time before the computation begins.
 - `te = omp_get_wtime();` records the end time after the computation.
- OpenMP Target Directive.
 - `map(to:x[0:SZ]):` copies the array `x` from the host to the target device.
 - `map(tofrom:y[0:SZ]):` copies the array `y` to the target device and copies it back to the host after execution.
- SAXPY Operation. The `for` loop performs the SAXPY computation on the target device.
- Printing the Time. Finally, calculates the elapsed time for the kernel execution and prints the time.

7.8.6 MPI-CUDA applications

❷ What is MPI?

MPI stands for **Message Passing Interface**. It is a standardized and portable message-passing system designed to function on a wide variety of parallel computing architectures.

- **Process Communication:** MPI allows **multiple processes to communicate with each other by sending and receiving messages**. This is essential for coordinating tasks in parallel computing environments.
- **Distributed Computing:** MPI is commonly **used in distributed computing**, where processes run on different nodes (computers) in a network and need to work together on a common task.
- **Parallel Execution:** Each process in an MPI program runs **independently** and can perform computations on different parts of the data, leading to efficient parallel execution.
- **Scalability:** MPI can scale **from a few processes** on a single computer to thousands of processes on a **supercomputer**.

In the real-world scenario, it is common to use MPI and CUDA together to solve a computational problem, specifically focusing on wave propagation and heat transfer.

On the next page we see the Wave Propagation Stencil. It is a computational pattern to solve wave propagation problems, such as those found in seismic imaging or heat transfer simulations. It involves calculating the value of a point in a grid based on the values of its neighbors, typically using finite difference methods.

The calculation involves a weighted sum of values from neighboring points in the volume, which is typical in numerical simulations of physical processes such as heat transfer. The volume is divided into subdomains (D1, D2, D3, D4) to parallelize the computation. Domain decomposition is a technique that divides the computational domain (the volume) into smaller parts that can be processed simultaneously by different processors or GPUs. This approach increases computational efficiency by distributing the workload across multiple processing units.

❷ How is the workload shared between MPI and CUDA?

- MPI is used for communication between different processes handling different subdomains. It processes exchange boundary data to ensure continuity and accuracy of the simulation.
- CUDA is used for performing the calculations on each subdomain. Each GPU handles the computations for its assigned subdomain, performing the weighted sum calculations at each time step.

❖ Implementation

- **Main program:** initialize communication, execute process, finalize communication.

```

1 MPI_Init(&argc, &argv);
2 MPI_Comm_rank(MPI_COMM_WORLD, &pid);
3 MPI_Comm_size(MPI_COMM_WORLD, &np);
4
5 if(np < 3) {
6     if(0 == pid)
7         printf("Needed 3 or more processes.\n");
8     MPI_Abort(MPI_COMM_WORLD, 1);
9     return 1;
10 }
11
12 if(pid < np - 1)
13     compute_node(dimx, dimy, dimz / (np - 1), nreps);
14 else
15     data_server(dimx, dimy, dimz, nreps);
16
17 MPI_Finalize();

```

The program starts by initializing MPI, getting the rank of the process (its ID), and the total number of processes.

- Initializes the MPI environment.
- Gets the rank (ID) of the calling process.
- Gets the number of processes.

So it checks if there are at least 3 processes. If not, it prints an error message and aborts. Furthermore, the program decides based on the process ID (pid) whether the process will be a compute node or a data server. Finally, the program closes the MPI environment.

- **Server process:** partition and distribute data.

```

1 /* Find number of MPI processes */
2 /* Allocate input data */
3 /* Initialize input data */
4 /* Calculate number of points per compute process */
5 for (int process = 1; process <= last_node; process++) {
6     MPI_Send(send_address, int_num_points, MPI_FLOAT, process,
7             DATA_DISTRIBUTE, MPI_COMM_WORLD);
8     send_address += dimx * dimy * (dimz / num_comp_nodes);
9 }
10 /* Wait for nodes to compute */
11 MPI_Barrier(MPI_COMM_WORLD);
12
13 /* Collect output data */
14 MPI_Status status;
15 for (int process = 0; process < num_comp_nodes; process++) {
16     MPI_Recv(output + process * num_points / num_comp_nodes,
17             num_points / num_comp_nodes, MPI_FLOAT, process,
18             DATA_COLLECT, MPI_COMM_WORLD, &status);
19 }

```

The server process first initializes MPI and determines the total number of MPI processes and the number of compute nodes. It also allocates and initializes the input data and calculates the number of points per compute process.

After that, the server process uses a loop to send data to each compute node.

It calls `MPI_Barrier` to synchronize all processes. This ensures that all compute nodes have completed their calculations before continuing.

Finally, the server process uses another loop to receive the computed results from each compute node.

- **Compute process:** receive data and offload computation to the GPU.

```

1  /* Part 1 */
2
3  /* Alloc host memory */
4  float *h_input = (float *)malloc(num_bytes);
5  /* Alloc device memory for input and output data */
6  float *d_input = NULL;
7  cudaMalloc((void **)&d_input, num_bytes);
8  /* Receive data and move it to device memory */
9  float *rcv_address = h_input + num_halo_points * (0 == pid);
10 MPI_Recv(rcv_address, num_points, MPI_FLOAT, server_process,
11           MPI_ANY_TAG, MPI_COMM_WORLD, &status);
12 cudaMemcpy(d_input, h_input, num_bytes, cudaMemcpyHostToDevice
13            );
14 /* Part 2 */
15 void launch_kernel(float *next, float *in, float *prev, float
16                   *velocity, int dimx, int dimy, int dimz)
17 {
18     dim3 Gd, Bd, Vd;
19     Vd.x = dimx; Vd.y = dimy; Vd.z = dimz;
20
21     Bd.x = BLOCK_DIM_X; Bd.y = BLOCK_DIM_Y; Bd.z = BLOCK_DIM_Z
22     ;
23
24     Gd.x = (dimx + Bd.x - 1) / Bd.x;
25     Gd.y = (dimy + Bd.y - 1) / Bd.y;
26     Gd.z = (dimz + Bd.z - 1) / Bd.z;
27
28     wave_propagation<<<Gd, Bd>>>(next, in, prev, velocity, Vd)
29     ;
30 }
31 /* Part 3 */
32 MPI_Status status;
33 int left_neighbor = (pid > 0) ? (pid - 1) : MPI_PROC_NULL;
34 int right_neighbor = (pid < np - 2) ? (pid + 1) :
35 MPI_PROC_NULL;
36 /* Upload stencil coefficients */
37 /* Calculate offsets */
38 MPI_Barrier( MPI_COMM_WORLD );
39 /* Compute boundary values needed by other nodes first */
40 launch_kernel(d_output + left_stage1_offset, d_input +
left_stage1_offset, dimx, dimy, 1/2, stream0);

```

```
41 launch_kernel(d_output + right_stage1_offset, d_input +
    right_stage1_offset, dimx, dimy, 1/2, stream0);
42 /* Compute the remaining points */
43 launch_kernel(d_output + stage2_offset, d_input +
    stage2_offset, dimx, dimy, dimz, stream1);
44 /* Copy the data needed by other nodes to the host */
45 cudaMemcpyAsync(h_left_boundary, d_output + num_halo_points,
    num_halo_bytes, cudaMemcpyDeviceToHost, stream0);
46 cudaMemcpyAsync(h_right_boundary, d_output +
    right_stage1_offset + num_halo_points, num_halo_bytes,
    cudaMemcpyDeviceToHost, stream0);
47 cudaStreamSynchronize(stream0);
48
```

1. Part 1: Receiving Data and Allocating Memory. This part shows how the compute process receives data and prepares for computation on the GPU.
2. Part 2: Launching the Kernel. This part shows how to set up and launch a CUDA kernel for computation.
3. Part 3: Managing Neighbors and Synchronizing Streams. This part shows how to handle boundary conditions and synchronize computations.

8 Memory Consistency

8.1 Coherence vs Consistency

💡 Why Memory Consistency?

Memory consistency models are fundamental to parallel computing because they define the rules that govern the visibility and ordering of memory operations across multiple threads. Unlike sequential computing, where the order of operations is straightforward, parallel computing introduces complexity due to concurrent execution. This concurrency can lead to race conditions, making it difficult to determine the “latest” value of a shared memory location.

💡 Why Memory Models are Fundamental to Parallel Computing

In an ideal scenario, loads (reads) should return the *most recent* value written to a memory location. However, defining and achieving this “most recent” value in a parallel environment is **complicated because multiple threads may be interacting with the same memory at the same time**. This is where memory consistency models play a critical role.

When we talk about memory, there are two fundamental concepts to explore: coherence and consistency.

- **Memory Coherence** ensures that all processors see a consistent view of a single memory location.

It defines the requirements for the observed behavior of reads and writes to the same memory location:

- All processors must agree on the order of reads/writes to a single memory location (X). This means that if one processor writes a value to a memory location, all other processors should eventually see that value.
- A timeline of operations involving the same memory location. It is possible to create a timeline such that the observations of all processors are consistent with that timeline. This ensures that each processor sees the most recent write to a memory location in the correct order.

- **Memory Consistency** extends this concept to the entire memory, ensuring an apparent ordering of operations, which dictates how memory operations performed by one thread become visible to other threads.

It defines the behavior of reads/writes to different memory locations:

- **Coherence guarantees eventual propagation.** Coherence ensures that writes to a single memory location (X) will eventually propagate to other processors.
- **Consistency deals with the timing of propagation.** Consistency addresses when writes to one memory location (X) propagate

to other processors relative to reads and writes to other memory locations. This means that consistency ensures a predictable order for operations across multiple memory locations, considering their interactions.

Example 1: Chronology of Operations

Imagine the following chronology of operations on a memory address (X):

$t = 1$. Write. $P_0 \xrightarrow{5} X$

$t = 2$. Read. $P_1 \xleftarrow{5} X$

$t = 3$. Write. $P_2 \xrightarrow{10} X$

$t = 4$. Write. $P_2 \xrightarrow{11} X$

$t = 5$. Read. $P_1 \xleftarrow{11} X$

This example demonstrates how:

- Memory *coherence* ensures that **all processors eventually agree on the order of operations for the same memory location**.
- Memory *consistency* ensures a **predictable interaction between operations on different memory locations**.

② What happens if there is a cache system?

Modern parallel computing systems rely heavily on caches to improve performance. Therefore, **ensuring that all caches maintain a consistent view of memory (cache coherence) becomes critical to the proper operation of the system**. While memory coherence is about maintaining a consistent view of memory, cache coherence is about implementing this consistency in systems where each processor has its own local cache. So here we define again the concepts of consistency and coherence in the cache environment.

- **Cache Coherence.** This is a more specific implementation of memory coherence. In general, caches are used to store copies of frequently accessed data to speed up processing. However, when multiple processors modify their cached copies of the same memory location, inconsistencies can occur.

Therefore, the main goal is to ensure that the memory system in a parallel computer behaves as if the caches were not there. This is similar to how a memory system in a uni-processor system behaves as if the cache were not there.

In a system without caches, there would be no need for cache coherence. Cache coherence **ensures that all processors see a consistent view of memory, even though each processor may have a local cache**. This means that any changes made to a memory location by one processor will eventually be reflected in the caches of the other processors.

- **Memory Consistency** (in the cache environment). Defines the allowed behavior of loads (reads) and stores (writes) to different addresses in a parallel system. This behavior should be specified whether or not caches are present. A memory consistency model specifies the rules for the order in which memory operations (loads and stores) become visible to other threads. It ensures that all processors in the system observe memory operations in a predictable and coherent manner, regardless of the presence of caches.

In other words, it is the same definition presented on the previous page, but we emphasize that model should also work with the cache.

8.2 Definition

After a brief discussion of the difference between coherence and consistency terms, and why it is important to understand these concepts, here we present a deeper view of memory consistency. This is because consistency deals with the broader and more complex issue of how all memory operations (across all memory locations) are ordered and observed in a parallel system. It is also necessary to understand the high-level rules that govern the interactions between processors and memory in a multiprocessor environment.

Definition 1: Memory Consistency

Memory Consistency models define **how memory operations** (loads and stores) **performed by different processors are ordered** and **become visible to one another in a multiprocessor system.**

❷ Why the order decided by the memory consistency model is important

One main reason:

- **Performance.** In multiprocessor systems, memory operations can be reordered to optimize performance. Unfortunately, this **reordering can result in behavior that seems counterintuitive or unusual from the perspective of a programmer** who expects sequential execution. However, it allows optimizations such as overlapping memory accesses with computations and reducing memory access latency.

Most application programmers don't have to deal directly with the effects of memory reordering, because higher-level constructs and synchronization mechanisms handle them. However, understanding memory consistency is critical to writing correct and efficient parallel programs. Developers of system software, such as operating systems and compilers, must deal with these issues to ensure that their low-level code conforms to the hardware's memory consistency model and maintains the correct order of memory operations.

⊜ Memory Operation Ordering

A program defines a sequence of loads and saves. There are **four types of memory operation sequences**:

1. $W_X \rightarrow R_Y$: Write to X must commit before a subsequent read from Y .

This means that if a write to memory location X occurs before a read from another memory location Y in the program order, the write must be complete and visible before the read.

2. $R_X \rightarrow R_Y$: Read from X must commit before a subsequent read from Y .

This order ensures that if a read from X occurs before a read from Y in the program, the first read must be completed before the second read occurs.

3. $R_X \rightarrow W_Y$: Read from X must commit before a subsequent write to Y .

This means that if a read from X occurs before a write to Y in the program order, the read must be completed before the write is performed.

4. $W_X \rightarrow W_Y$: Write to X must commit before a subsequent write to Y .

This ordering ensures that if a write to X happens before a write to Y in the program, the first write must be completed before the second write is performed.

The word “subsequent” means that each left operation must be completed and its result visible before the right operation can occur.

8.3 Sequential Consistency Model

❑ Sequential Consistency Model

Sequential Consistency is a *memory consistency model* concept introduced by Lamport in 1976, which ensures that all operations in a multi-threaded system are executed in some sequential order, as if they were manipulating a single shared memory. This means that each thread's operations occur in program order, maintaining the illusion of a single, consistent timeline of memory operations. In other words, a sequentially consistent memory system maintains all four orders of memory operations (the order of operations seen in the enumerated list on page 225).

Sequential Consistency is the **strongest model**, because it requires all memory operations to appear as if they are executed in a strict sequential order. The degree of reordering of memory operations determines the strength of a model's weakness.

❑ Relaxing Memory Method

Relaxed Memory method allows certain memory operation orders to be violated to improve performance. It primarily aim to improve performance by **hiding memory latency**. By allowing some flexibility in the order of memory operations, these models enable systems to execute instructions more efficiently. So operations like "Write X then Read Y" can be done out of order if they are independent.

▲ Problems with Sequential Consistency Model

There are **four main problems** associated with the sequential consistency model in parallel computing systems:

- **Performance Overhead.** Requires all operations to appear in a strict sequential order.

Can introduce significant performance overhead due to the necessity of maintaining order, especially when memory writes take a long time (hundreds of cycles).

- **Instruction Dependency.** Requires waiting for earlier instructions to finish, even if they don't conflict.

Limits the ability of the system to execute independent instructions in parallel.

- **Latency Issues.** Memory operations, especially writes, can have high latency.

Delays in writing data mean that processors must wait before executing subsequent instructions, leading to inefficiencies.

- **Limited Parallelism.** Limits the potential for parallel execution of independent instructions.

Reduces the system's ability to effectively use parallelism, which affects overall performance.

Fix Sequential Consistency Model

The **Write Buffer** method is a technique used in multiprocessor systems to enhance performance and reduce latency. It involves **temporarily storing write operations in a buffer before propagating them to the main memory**. This allows processors to continue executing subsequent instructions without waiting for the write to complete.

The write buffer method helps in maintaining sequential consistency while **reducing the performance overhead and latency issues**. By buffering writes and allowing processors to read from their own write buffers, this method enhances the efficiency and parallelism of the system.

Note: The write buffer is **only a technique**. The model that will use this feature will be present in the following pages.

How does Write Buffer work?

- **Structure.** Each processor has *its own write buffer*.
- **Buffered Writes.** When a processor writes to a memory location, the write is placed in the processor's write buffer instead of being immediately propagated to the shared memory.
- **Read from Buffer.** When the processor reads a memory location, it first checks its write buffer before accessing the shared memory.

Example 2: Write Buffer method

Initial Setup

Processor 0 and Processor 1 both interact with shared memory. They initialize $A = 0$ and $B = 0$.

Instructions

- Processor 0 executes:
 1. $A = 1$
 2. $r1 = B$
- Processor 1 executes:
 1. $B = 1$
 2. $r2 = A$

Question

Can $r1$ and $r2$ both be 0 after executing these instructions?

Answer

- **Sequential Consistency.** Under sequential consistency, all operations must appear to be executed in a strict sequential order. r_1 cannot be 0 if Processor 1 has written $B = 1$. Similarly, r_2 cannot be 0 if Processor 0 has written $A = 1$.

Therefore, no, r_1 and r_2 cannot both be 0, because the write from one processor should be visible to the read from the other processor.

- **With Write Buffers.** Each processor temporarily stores its write operations in its own write buffer before committing them to main memory.

– Processor 0:

1. Writes $A = 1$ to its write buffer.
2. Reads B from the main memory (which is still 0 because Processor 1's write is buffered).

– Processor 1:

1. Writes $B = 1$ to its write buffer.
2. Reads A from the main memory (which is still 0 because Processor 0's write is buffered).

The result is that r_1 and r_2 can both be 0 because each processor reads from main memory before the other's buffered write is committed. So yes, with the buffered write technique, r_1 and r_2 can both be 0.

Summary

This example shows how write buffers allow processors to perform and cache writes independently, resulting in scenarios where reads may not immediately reflect recent writes from other processors. This can optimize performance by reducing write latency, but introduces complexity in ensuring memory consistency.

⌚ Performance: Sequential Consistency vs. Write Buffer

The Figure 38 shows three different benchmarks that were run to compare the Sequential Consistency and Write Buffer models. The benchmarks executed are: MP3D (Matterport 3D), LU (Master Lu), PTHOR.

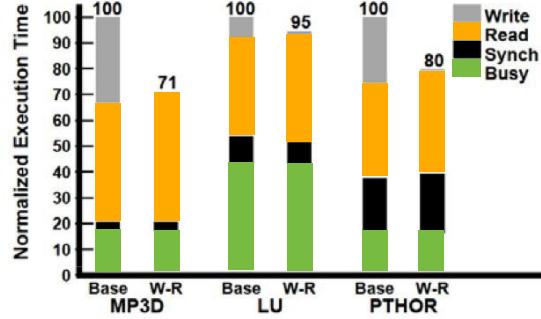


Figure 38: Write Buffer Performance.

- MP3D:
 - Base (Sequential Consistency): 100 (normalized execution time)
 - W-R (Write Buffer): 71 (normalized execution time)
 - Performance Improvement: 29% reduction in execution time.
- LU:
 - Base (Sequential Consistency): 100 (normalized execution time)
 - W-R (Write Buffer): 95 (normalized execution time)
 - Performance Improvement: 5% reduction in execution time.
- PTHOR:
 - Base (Sequential Consistency): 100 (normalized execution time)
 - W-R (Write Buffer): 80 (normalized execution time)
 - Performance Improvement: 20% reduction in execution time.

These results indicate that the write buffer method can significantly improve performance in multiprocessor systems, especially in scenarios where write latency is a critical factor.

8.4 Memory Models with Relaxed Ordering

The following models are **grouped based on the types of memory operation reordering** they allow. Each model applies the relaxing memory models, so they allow certain memory operation orders to be violated to improve performance.

Before introducing each memory model, we will talk about problems and possible solutions for memory models with relaxed order.

⚠ Problems with Memory Reordering

Reordering of memory operations can lead to **inconsistencies** and **unpredictable behavior** in multiprocessor systems.

✓ Solutions: Synchronization Primitives

- ✓ **Memory Fence Instructions (Barriers)**: Prevent reorderings by ensuring that all memory operations before the fence complete before any new operations begin.

Fence instructions can be costly in terms of performance but are essential for maintaining order and consistency.

- ✓ **Read-Modify-Write**: Ensures atomic read and write operations to a memory location.
- ✓ **Compare-and-Swap**: Atomically compares a memory location's value and swaps it if it matches a specified value.
- ✓ **Transactional Memory**: Allows a group of memory operations to be executed atomically.

8.4.1 Allowing Reads to Move Ahead of Writes

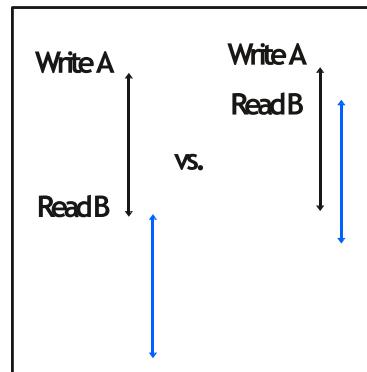
❷ Which memory operation is relaxed?

Given the four memory operation sequences (see detailed explanation on page 225), the relaxed operations are marked as cancel:

- ✓ *Relaxed $W_X \rightarrow R_Y$* : Write to X must commit before a subsequent read from Y .

This means that if a write to memory location X occurs before a read from another memory location Y in the program order, the write must be complete and visible before the read.

- ✗ $R_X \rightarrow R_Y$: Read from X must commit before a subsequent read from Y .
- ✗ $R_X \rightarrow W_Y$: Read from X must commit before a subsequent write to Y .
- ✗ $W_X \rightarrow W_Y$: Write to X must commit before a subsequent write to Y .



Models

- **Total Store Ordering (TSO)**

How does it work?

It allows a processor to read a variable (e.g., B) before its write to another variable (e.g., A) is visible to all processors ($W_A \rightarrow R_B$).

This enables the processor to hide the latency of write operations by allowing reads to proceed independently.

Implications

Other processors cannot see the new value of A (the value written to register A) until the write to A is observed by all processors.

Only $W_A \rightarrow R_B$ order is relaxed, while $W_A \rightarrow W_B$ constraints still exist, meaning writes by the same thread occur in program order. Other processors see these writes in the correct order, but there might be a delay before the writes are visible due to buffering.

- **Processor Consistency (PC)**

How does it work?

Similar to TSO, but with slightly more flexibility.

Any processor can read the new value of A before the write is observed by all processors, allowing reads to be even more independent of writes.

Implications

Like TSO, only $W_A \rightarrow R_B$ order is relaxed, and $W_A \rightarrow W_B$ constraints remain, ensuring that writes by the same thread occur in program order.

Main differences between TSO and PC

1. **Read Flexibility**

- **TSO:** Reads by a processor can move ahead of its own writes.
- **PC:** Reads by any processor can observe new values of writes *before they are globally visible*, allowing even more flexibility.

2. **Write Order**

- **Both Models:** Maintain the order of writes within the same processor, ensuring that $W_A \rightarrow W_B$ order is preserved.

3. **Performance vs. Complexity**

- **TSO:** Strikes a *balance between performance and simplicity* by allowing limited reordering of reads and writes.
- **PC:** Offers *greater flexibility in read operations*, potentially improving performance but at the *cost of slightly increased complexity*.

8.4.2 Allowing writes to be reordered

❷ Which memory operation is relaxed?

Given the four memory operation sequences (see detailed explanation on page 225), the relaxed operations are marked as cancel:

- ✓ *Relaxed $W_X \rightarrow R_Y$* : Write to X must commit before a subsequent read from Y .

This means that if a write to memory location X occurs before a read from another memory location Y in the program order, the write must be complete and visible before the read.

- ✗ $R_X \rightarrow R_Y$: Read from X must commit before a subsequent read from Y .
- ✗ $R_X \rightarrow W_Y$: Read from X must commit before a subsequent write to Y .
- ✓ *Relaxed $W_X \rightarrow W_Y$* : Write to X must commit before a subsequent write to Y .

This ordering ensures that if a write to X happens before a write to Y in the program, the first write must be completed before the second write is performed.

■ Models

• Partial Store Ordering (PSO)

❖ How does it work?

PSO allows more **aggressive reordering of write operations** compared to TSO (page 232) and PC (page 232).

⚠ Implications

- Thread 1 on Processor 1 (P1)

```
1 A = 1;
2 flag = 1;
```

- Thread 2 on Processor 2 (P2)

```
1 while (flag == 0);
2 print A;
```

P2 may observe the change to `flag` before the change to `A`. This shows that PSO allows writes to `A` and `flag` to be reordered, improving write performance but potentially complicating program reasoning.

✔ Benefits

- $W_X \rightarrow W_Y$. **Write Buffering:** Processors can reorder write operations in a write buffer. For example, one write might be a cache miss while another is a cache hit, and reordering them can optimize performance.
- $R_X \rightarrow R_Y, R_X \rightarrow W_Y$. **Instruction Reordering:** Processors can reorder independent read and write instructions within the instruction stream, leveraging out-of-order execution to maximize efficiency.

These reorderings and optimizations are particularly **effective in single instruction streams**, where dependencies between instructions are well understood and managed.

8.4.3 Allowing all reorderings

❷ Which memory operation is relaxed?

Given the four memory operation sequences (see detailed explanation on page 225), the relaxed operations are marked as cancel:

- ✓ ***Relaxed $W_X \rightarrow R_Y$*** : Write to X must commit before a subsequent read from Y .

This means that if a write to memory location X occurs before a read from another memory location Y in the program order, the write must be complete and visible before the read.

- ✓ ***Relaxed $R_X \rightarrow R_Y$*** : Read from X must commit before a subsequent read from Y .

This order ensures that if a read from X occurs before a read from Y in the program, the first read must be completed before the second read occurs.

- ✓ ***Relaxed $R_X \rightarrow W_Y$*** : Read from X must commit before a subsequent write to Y .

This means that if a read from X occurs before a write to Y in the program order, the read must be completed before the write is performed.

- ✓ ***Relaxed $W_X \rightarrow W_Y$*** : Write to X must commit before a subsequent write to Y .

This ordering ensures that if a write to X happens before a write to Y in the program, the first write must be completed before the second write is performed.

█ Models

- **Weak Ordering (WO)**

❖ How does it work?

Operations are divided into critical and non-critical sections. Reordering is allowed outside of critical sections to maximize performance.

Example: Can reorder any non-critical operations to enhance efficiency, executing them independently.

- **Release Consistency (RC)**

❖ How does it work?

Divides operations into acquire and release categories, allowing extensive reordering within these categories.

Example: Reorders operations within acquire or release phases, enabling significant performance gains.

In these models, there are **no strict guarantees about the order of operations on data**. Essentially, everything can be reordered.

❷ Why allow all reorders?

By overlapping multiple reads and writes, the **system can execute reads as early as possible and delay writes as late as possible**, effectively **hiding memory latency and increasing performance**.

8.5 Languages Need Memory Models Too

Memory models are critical not only for hardware systems, but also for programming languages. They provide a framework that defines how memory operations (reads and writes) can be ordered and observed by multiple threads. This chapter discusses the importance of memory models in programming languages and how they help maintain consistency and predictability in multi-threaded programs.

❖ Compiler Optimizations

- **Invisible Optimizations.** Compilers can optimize code in ways that are not directly visible to the programmer, such as reordering instructions to enhance performance.

Example 3: optimization *not* visible to programmer

Code written by the programmer:

```

1 X = 0
2 for i = 0 to 100:
3     X = 1
4     print X

```

Code optimized by the compiler:

```

1 X = 1
2 for i = 0 to 100:
3     print X

```

These optimizations can lead to unexpected behaviors in multi-threaded programs if not properly managed.

- **Visible Optimizations.** When optimizations are visible to the programmer, it becomes essential to understand how these changes affect the execution order and consistency of memory operations.

Example 4: optimization *is* visible to programmer

Code written by the programmer:

– Thread 1

```

1 X = 0
2 for i = 0 to 100:
3     X = 1
4     print X

```

– Thread 2

```

1 X = 0

```

– Expected result: 111111111...

– Result obtained: 111110111...

Code optimized by the compiler:

- Thread 1

```
1 X = 1
2 for i = 0 to 100:
3     print X
```

- Thread 2

```
1 X = 0
```

- Expected result: 111111111...

- Result obtained: 111110000...

❑ Need for Language-Level Memory Models

- **Contract to Programmers.** Memory models provide a contract to programmers, ensuring that **memory operations will be reordered by the compiler in a way that maintains consistency**. For example, there should be no reordering of shared memory operations that could lead to race conditions.
- **Guarantees.** Modern languages like C11, C++11, and Java 5 guarantee sequential consistency for data-race-free programs, meaning that the program's behavior will be predictable if there are no data races.

▲ Importance of Synchronization

- **Data Races.** Without proper synchronization, data races can occur, leading to non-deterministic and buggy behavior. Programs with data races do not have any guarantees and can produce unpredictable results.
- **Synchronization Libraries.** Using synchronization primitives like locks, barriers, and memory fences ensures that memory operations are executed in the correct order, preventing data races and maintaining program correctness.

❑ Summary

Memory models at the language level are essential for maintaining consistency and predictability in multi-threaded programs. They help manage compiler optimizations and provide a framework for programmers to ensure that their code behaves correctly, even when memory operations are reordered.

8.6 Implementing Locks

8.6.1 Introduction

Locks are essential in concurrent programming for managing access to shared resources and preventing data races. A common mistake in implementing locks is using a simple Load-Test-Store lock sequence, which can lead to severe issues such as data races due to its non-atomic nature.

⚠ Simple but Incorrect Lock Implementation: Load-Test-Store lock

The idea behind the **Load-Test-Store lock** is to **check if the lock is available** (i.e., not held by any other processor) **and then acquire it by setting it**. This process involves loading the current value of the lock, checking its state, and storing a new value if it's free. Here's the pseudo-code for the implementation:

```

1  lock:
2      // Load the value at address into register R0
3      ld  R0, mem[addr]
4
5      // Compare the value in R0 to 0
6      cmp R0, #0
7
8      // If R0 is not zero, jump back to 'lock' (retry)
9      bnz lock
10
11     // Store 1 at the address to indicate the lock is acquired
12     st  mem[addr], #1
13
14 unlock:
15     // Store 0 at the address to release the lock
16     st  mem[addr], #0

```

In this implementation, a processor repeatedly loads and checks a memory address, acquiring the lock by storing a value to indicate it is locked. However, **this sequence is not atomic**, leading to a **potential data race**:

- Processor 0 (P0) attempts to acquire the lock:
 - `ld R0, mem[addr]`: P0 loads the value from memory address `addr` into register `R0`. Suppose the value is 0 (lock is *free*).
 - `cmp R0, #0`: P0 compares `R0` with 0. Since `R0` is 0, the comparison is true.
 - `st mem[addr], #1`: P0 stores 1 at `mem[addr]`, indicating **it has acquired the lock**.
- Processor 1 (P1) attempts to acquire the lock simultaneously:
 - `ld R1, mem[addr]`: P1 loads the value from memory address `addr` into register `R1`. Suppose the value is still 0 (before P0 updates the lock).
 - `cmp R1, #0`: P1 compares `R1` with 0. Since `R1` is 0, the comparison is true.

- `st mem[addr]`, #1: P1 stores 1 at `mem[addr]`, indicating **it has acquired the lock**.

Both P0 and P1 believe they have acquired the lock because they both observed the lock as free (0) and updated it to 1. This leads to a **data race where two processors think they have the lock, causing inconsistent states and potential conflicts in accessing the critical section.**

❓ Why does Load-Test-Store suffer from Data Race?

- **Non-Atomic Operations:** The sequence of load, compare, and store operations are not atomic. During this sequence, *other processors can interrupt and perform their own operations*, leading to race conditions.
- **Lack of Synchronization:** There is no mechanism to ensure that the load-test-store sequence is executed as a single, uninterruptible operation, which is necessary to prevent data races.

✓ Solution: Advanced Locking Techniques

To address these issues, more sophisticated lock implementations are necessary. The following sections will explore *Test-and-Set Based Lock* (page 241) and *Test-and-Test-and-Set Lock* (page 245), which provide reliable and efficient synchronization mechanisms to prevent data races and ensure proper access control.

8.6.2 Test-and-Set based lock

The **Test-and-Set instruction** is a fundamental **atomic operation used for synchronization** in multiprocessor systems. It **checks the value of a lock variable and sets it to a new value in one uninterruptible action**. The operation ensures that no other processors can modify the lock variable simultaneously, preventing race conditions.

💡 How does it work?

It performs **two actions in one uninterruptible step**:

1. It loads the value from a memory address into a register;
2. Then sets the memory address to a specified value (usually indicating the lock is acquired).

```
1 ts R0, mem[addr]      // load mem[addr] into R0
2                      // if mem[addr] is 0, set mem[addr] to 1
```

🔒 Lock Implementation

To implement a lock using the test-and-set instruction, we must **repeatedly attempt to acquire the lock until it is free**.

```
1 lock:
2     ts  R0, mem[addr]    // Attempt to acquire the lock
3     bnz R0, lock         // If the lock was not free (R0 ≠ 0),
4                           // retry
```

- **ts R0, mem[addr]**: Atomically loads the value at **mem[addr]** into register **R0** and sets **mem[addr]** to 1 if it was 0.
- **bnz R0, lock**: If **R0** is non-zero (meaning the lock was already held), it branches back to the start and retries.

🔓 Unlock Implementation

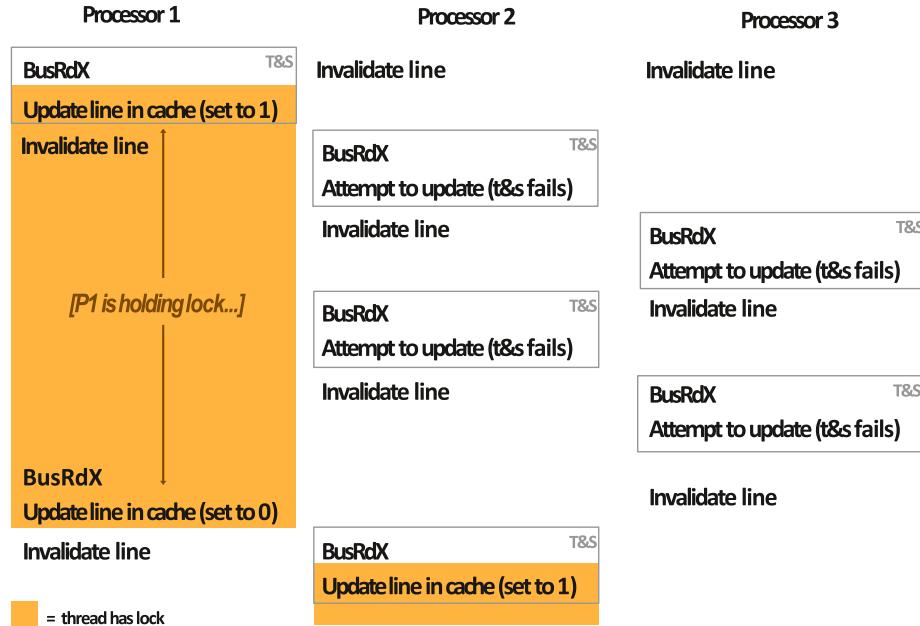
To release the lock, simply we **set the lock variable back to 0**.

```
1 unlock:
2     st  mem[addr], #0      // Release the lock by storing 0
```

- **st mem[addr], #0**: Stores 0 at the memory address **addr**, indicating the lock is now free.

⚠ Coherence Protocol

Using test-and-set based locking is fundamental to **maintaining consistency** across multiple processor **caches** by ensuring that all processors have the same view of the lock variable. So it involves invalidating and updating cache lines across processors as the lock state changes.



- Processor 1 (P1):

- **Acquiring Lock:**

- * **BusRdX Request:** P1 sends a Bus Read Exclusive (BusRdX) request to acquire the lock.
 - * **Cache Update:** P1 sets the lock value to 1 in its cache, indicating the lock is held.
 - * **Invalidation:** Other processors' caches are invalidated for the lock variable.

- **Releasing Lock:**

- * **BusRdX Request:** P1 sends another BusRdX request to release the lock.
 - * **Cache Update:** P1 sets the lock value to 0 in its cache, indicating the lock is free.
 - * **Invalidation:** Other processors' caches are invalidated for the lock variable.

- Processor 2 (P2) and Processor 3 (P3):

- **Attempting to Acquire Lock:**

- * **Invalidation:** Their cache lines for the lock are initially invalidated.
 - * **BusRdX Request:** Both processors send BusRdX requests to acquire the lock.
 - * **Retry Loop:** If the lock is not free, they continuously retry, causing repeated BusRdX requests.

② Implications of Coherence Traffic

- **Contention.** High contention occurs when multiple processors compete for the same lock at the same time.

Each processor continuously attempts to execute the test-and-set instruction, resulting in repeated BusRdX requests. This competition generates significant traffic on the memory bus and forces processors to repeatedly invalidate each other's cache lines for the lock variable.

The result is a high level of contention that can cause delays and inefficiencies as processors wait for the lock to become available.

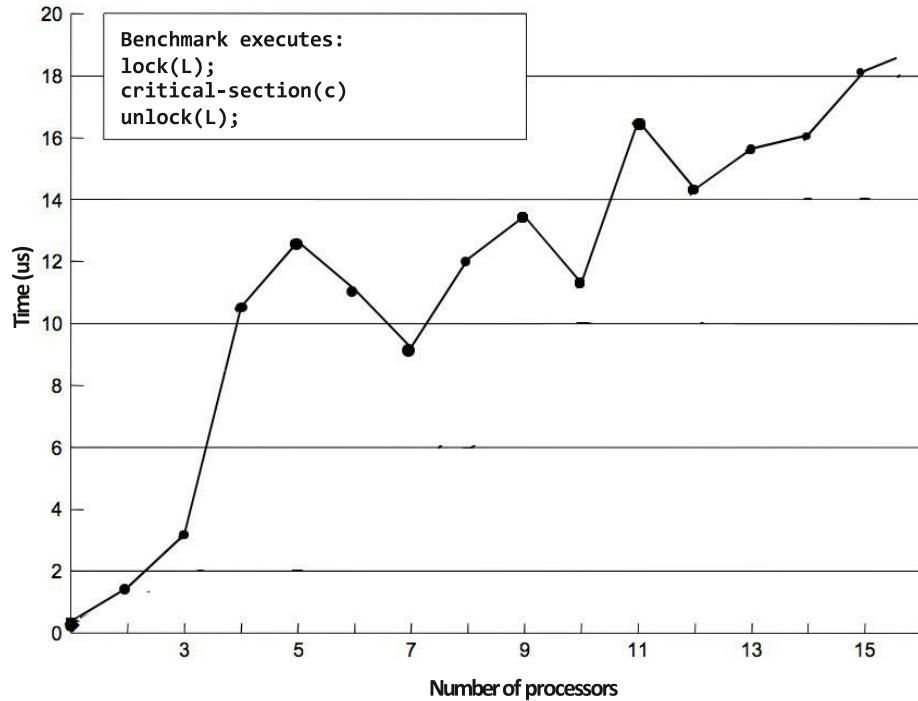
- **Performance Impact.** The frequent BusRdX requests and resulting cache invalidations can significantly degrade system performance, especially as the number of processors increases.

Each BusRdX request incurs a cost in terms of memory bus usage and latency. As more processors contend for the lock, the cumulative effect of these requests becomes more pronounced, leading to:

- ✖ **Increased Latency:** The time taken for processors to acquire and release the lock increases, slowing down the overall execution of the program.
 - ✖ **Memory Bus Saturation:** The memory bus becomes saturated with coherence traffic, reducing the available bandwidth for other memory operations.
 - ✖ **Inefficiency:** Processors spend more time waiting and retrying to acquire the lock, leading to inefficient use of processing resources.

Test-and-Set Performance

The following graph plots the performance of the test-and-set lock in terms of time (in microseconds) versus the number of processors. It specifically measures the time taken to acquire and release the lock as the number of processors increases.



- **X-Axis:** Number of processors (ranging from 0 to 16).
- **Y-Axis:** Time in microseconds (ranging from 0 to 20).
- **Initial Increase:** As the number of processors increases from 1 to 4, the time taken to acquire and release the lock increases significantly.
- **Fluctuations:** There is a noticeable dip in time around 7 processors, followed by fluctuations as the number of processors continues to increase.
- **Overall Trend:** Despite the fluctuations, the **general trend shows an upward increase in time with more processors**, indicating **higher contention**.

The performance of the test-and-set lock shows that with an **increasing number of processors**, the **time to acquire and release the lock increases due to contention and coherence traffic**.

8.6.3 Test-and-Test-and-Set lock

The **Test-and-Test-and-Set lock** mechanism improves upon the simple Test-and-Set lock by **reducing interconnect traffic and contention**. It **combines an initial test phase with a subsequent atomic Test-and-Set operation**.

Implementation

```

1 void Lock(int* lock) {
2     while (1) {
3         // Spin-wait until the lock appears free
4         // (assume *lock is NOT register allocated)
5         while (*lock != 0);
6
7         // Try to acquire the lock atomically
8         if (test_and_set(*lock) == 0)
9             return;
10    }
11 }
12
13 void Unlock(int* lock) {
14     *lock = 0; // Release the lock
15 }
```

- **Initial Spin Phase:**

- **Purpose:** Processors continuously **check the value of the lock variable** (`*lock`) in a loop **until it becomes 0** (free). This spin-wait phase helps reduce contention on the interconnect by delaying the atomic test-and-set operation until the lock appears free.
- **Implementation:** `while (*lock != 0);` causes the processor to repeatedly **check the lock value without sending any bus requests**.

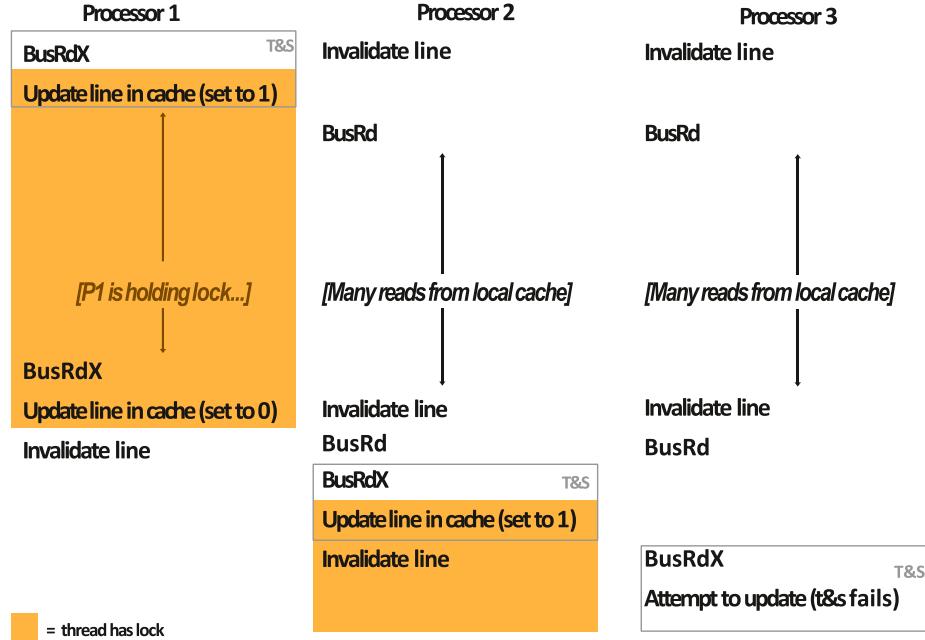
- **Atomic Test-and-Set Operation:**

- **Purpose:** Once the lock appears free, the **processor attempts to acquire it using the atomic test_and_set instruction**. This operation is performed in one uninterruptible step to ensure that only one processor can acquire the lock at a time.
- **Implementation:** The line of code `if (test_and_set(*lock) == 0) return;` **checks if the lock was successfully acquired** (i.e., the function `test_and_set` returns 0) and **exits the loop if it was**.

- **Unlock Operation:**

- **Purpose:** To **release the lock**, the processor sets the lock variable back to 0, indicating that the lock is free.
- **Implementation:** `*lock = 0;` simply sets the lock value to 0.

⚠ Coherence Traffic



- Processor 1 (P1)
 - **BusRdX:** P1 sends a Bus Read Exclusive (BusRdX) request to acquire the lock.
 - **Update Line in Cache:** P1 updates its cache line, setting the lock value to 1, indicating that it has acquired the lock.
 - **[P1 is holding lock...]:** P1 holds the lock and performs its critical section work.
 - **BusRdX (Release):** P1 sends another BusRdX request when it releases the lock.
 - **Update Line in Cache (Release):** P1 updates its cache line, setting the lock value to 0, indicating that the lock is now free.
 - **Invalidate Line:** Other processors' caches are invalidated for the lock variable.
- Processor 2 (P2)
 - **Invalidate Line:** Initially, P2's cache line for the lock is invalidated.
 - **BusRd:** P2 reads the lock value.
 - **[Many Reads from Local Cache]:** P2 spins, repeatedly reading the lock value from its local cache until it becomes free.
 - **Invalidate Line:** When the lock is acquired/released, P2's cache line is invalidated again.
 - **BusRd:** P2 reads the updated lock value.

- Processor 3 (P3)
 - **Invalidate Line:** Initially, P3’s cache line for the lock is invalidated.
 - **BusRd:** P3 reads the lock value.
 - **[Many Reads from Local Cache]:** P3 spins, repeatedly reading the lock value from its local cache until it becomes free.
 - **Invalidate Line:** When the lock is acquired/released, P3’s cache line is invalidated again.
 - **BusRd:** P3 reads the updated lock value.
 - **BusRdX:** P3 sends a BusRdX request to acquire the lock.
 - **Update Line in Cache:** P3 updates its cache line, setting the lock value to 1, indicating that it has acquired the lock.
 - **Invalidate Line:** Other processors’ caches are invalidated for the lock variable.
 - **BusRdX (Failed Attempt):** If P3’s test-and-set operation fails (because another processor acquires the lock), it continues to spin and retry.

Comparison with Test-and-Set

- **Slightly Higher Latency than Test-and-Set in Uncontented Cases**
 - The test-and-test-and-set lock involves an **initial test phase** before the atomic test-and-set operation. This extra step **can introduce slightly higher latency when there is no contention**, as processors spend additional time in the spin-wait phase.
- **Generates Much Less Interconnect Traffic**
 - **One Invalidation per Waiting Processor per Lock Release:** Each waiting processor experiences one cache invalidation per lock release, resulting in $O(P)$ invalidations, where P is the number of processors.
 - **Comparison to Test-and-Set Lock:** The test-and-set lock generates one invalidation per waiting processor per test, leading to significantly higher interconnect traffic.
 - **$O(P^2)$ Interconnect Traffic:** If all processors have the lock cached, the interconnect traffic can be quadratic ($O(P^2)$). However, this is **still less than the traffic generated by the simple test-and-set lock**.
- **More Scalable (Due to Less Traffic):**
 - **Scalability:** The reduced interconnect traffic makes the **test-and-test-and-set lock more scalable**. It can handle an increasing number of processors more efficiently than the simple test-and-set lock.

- **Impact:** As the number of processors grows, the performance impact of the test-and-test-and-set lock remains more manageable, making it suitable for larger multiprocessor systems.
- **Storage Cost Unchanged (One Integer):**
 - **Storage Requirements:** The storage cost of the test-and-test-and-set lock remains the same as the simple test-and-set lock, requiring only one integer to represent the lock.
- **Still No Provisions for Fairness:**
 - **Fairness¹²:** The test-and-test-and-set lock does not include mechanisms to ensure fairness. This means that **some processors may experience longer waiting times to acquire the lock, leading to potential starvation.**
 - **Implications:** While the lock reduces interconnect traffic and improves scalability, the lack of fairness can still be a significant drawback in systems where equitable access to resources is important.

The **test-and-test-and-set locking mechanism** offers several advantages over the simple test-and-set lock, including **reduced interconnect traffic** and **improved scalability**. However, it introduces **slightly higher latency in uncontended cases** and **does not address fairness issues**. Overall, it is a more efficient and scalable locking solution for multiprocessor systems, though not without its limitations.

¹²**Fairness** in the context of lock mechanisms refers to the **equitable distribution of opportunities for multiple processors or threads to acquire the lock**. A fair locking mechanism ensures that all processors have a roughly equal chance of acquiring the lock, preventing any single processor from being starved of access to the critical section.

9 Heterogeneous Processing

Heterogeneous Processing involves using various types of processors to optimize performance for different workloads. The key components are:

- **Widely Parallelized Components**
 - These are tasks that can run many operations concurrently, maximizing efficiency.
 - **Example:** Image processing tasks that apply filters to many pixels simultaneously.
- **Wide SIMD Execution Components**
 - Stands for Single Instruction, Multiple Data (SIMD), where one instruction is applied to multiple data points at once (page 26).
 - **Example:** Vector computations in scientific simulations, where similar mathematical operations are performed on large data sets.
- **Predictable Data Access Components**
 - Data access patterns that are orderly and consistent, which allows for efficient prefetching and caching.
 - **Example:** Loading sequentially stored data from memory, such as processing linear arrays.
- **Difficult to Parallelize Components**
 - These tasks are challenging to execute concurrently due to dependencies between operations or complex algorithms.
 - **Example:** Recursive algorithms where each step depends on the previous one.
- **Divergent Control Flow Components**
 - These have branches and loops that cause different execution paths, making parallel execution less straightforward.
 - **Example:** Control structures in code with many conditional statements (`if-else`), each leading to different processing paths.
- **Unpredictable Access Components**
 - Data access patterns are irregular, making it hard to predict which data will be needed next, but can still benefit from caches in some cases.
 - **Example:** Accessing elements in a data structure like a hash table, where locations are spread unpredictably.

★ Main Idea

Efficiency is achieved by using the right type of processing unit (CPU, GPU, etc.) for each specific task within an application and matching the task to the most appropriate computing resource.

Therefore, the essence of heterogeneous processing is to use the most appropriate type of processor for each task to get the best performance. Think of it as using the right tool for the right job.

Some tasks are best handled by powerful CPUs, while others might benefit from GPUs, FPGAs, or other specialized processors. By combining these different processors, we can handle a wide range of tasks more efficiently than by using just one type.

❖ Practical Examples of Heterogeneous Processing

1. [“Skylake” Processors \(2015\)](#): early Intel processors that began incorporating multiple types of cores to handle different tasks efficiently.
2. [Alder Lake \(12th Generation Intel Processors\)](#): modern processors with improved heterogeneous capabilities, featuring powerful and efficient cores for various workloads.
3. Discrete GPUs for More Heterogeneity: adding separate graphics processing units (GPUs) to handle tasks like rendering graphics, which CPUs aren't optimized for.
4. Mobile Heterogeneous Processors: processors in mobile devices that integrate CPUs, GPUs, and other specialized cores for energy efficiency and performance.
5. GPU-Accelerated Supercomputing: using GPUs in supercomputers for tasks that require massive parallel computation, such as scientific simulations.
6. [Frontier Supercomputer \(2022\)](#): an example of a state-of-the-art supercomputer that leverages multiple types of processors to achieve top-tier performance.

9.1 Energy Constrained Computing

Energy efficiency is critical in computing, especially in scenarios where power is limited, such as mobile devices or data centers. Different types of processors consume varying amounts of power based on their energy efficiency and the nature of the tasks they're designed to handle.

The following formula shows that **Power Consumption** is the **product of performance** (operations per second) and **energy efficiency** (joules per operation):

$$\text{Power} = (\text{Ops/second}) \times (\text{Joules/Op}) \quad (14)$$

- **Ops (Operations)**: The number of tasks or operations performed.
- **Second (Time)**: The duration over which these operations are performed.
- **Joules (Energy)**: The unit of energy consumed per operation.
- **Ops/second**: Operations performed on a second.
- **Joules/Op**: Energy used for each operation.

It is important to leverage the specialization (using fixed function hardware) to achieve better energy efficiency.

General-Purpose vs Specialized Processors

Specialized processors are designed for specific tasks, making them more efficient compared to general-purpose processors.

- **General-Purpose Processors (GPPs)**: CPUs are designed to handle a wide range of tasks, making them versatile but not always energy-efficient. They consume more power because they are built to perform various operations reasonably well.
- **Specialized Processors (SPs)**: These include GPUs, FPGAs, and custom accelerators designed for specific tasks. Because they focus on particular workloads, they can be optimized for energy efficiency. For example:
 - **GPUs**: Excellent for parallel tasks like graphics rendering and matrix operations, consuming less power per operation in those tasks compared to CPUs.
 - **FPGAs**: Can be tailored to execute specific algorithms with high energy efficiency.

9.2 Compute Specialization

✓ Efficiency Benefits of Compute Specialization

✓ Throughput-Maximized Processor Architectures

- These architectures are **designed to maximize the number of operations performed per second**, leveraging the parallelism inherent in certain applications (for example the GPU cores).
- ⌚ Performance Gains: Approximately **10x improvement in performance per watt** compared to high-quality C code running on a CPU.
 - * **Conditions for Improvement:** This level of improvement is achieved when the code:
 1. Is compute-bound;
 2. Maps well to wide data-parallel execution.
 - * **Application Areas:** Image processing, machine learning, and scientific simulations where operations can be parallelized across multiple data points.

✓ Fixed-Function Application-Specific Integrated Circuits (ASICs)

- **Application-Specific Integrated Circuits (ASICs)** are custom-designed to perform specific tasks with maximum efficiency, tailored to the needs of a particular application.
- ⌚ Performance Gains: Can provide **100-1000x or greater improvement in performance per watt** compared to general-purpose CPUs (GPPs).
 - * **Conditions for Improvement:** This level of efficiency is achieved when the workload:
 1. Is compute-bound;
 2. Does not rely heavily on floating-point arithmetic.
 - * **Application Areas:** Cryptocurrency mining, specific types of data encryption/decryption, and certain signal processing tasks, where the same set of operations is repeated multiple times.

These benefits are derived from focusing on the strengths of dedicated hardware, allowing them to outperform general-purpose processors in their designated tasks.

Remark: When is a workload compute-bound?

When a workload is described as compute-bound, it means that the **performance of the task is primarily limited by the speed of the processor performing the computations**. In other words, the bottleneck in this case is the processor's ability to perform computations, as opposed to other factors such as data retrieval or memory access.

Therefore, when we say that the workload is compute-bound, we are

referring to **workloads that spend most of their time performing computations and arithmetic operations rather than other tasks** (such as accessing memory).

Tasks such as complex mathematical calculations, scientific simulations, and certain types of data processing (such as graphics rendering) are often compute-bound.

In contrast, a memory-bound workload would be limited by the speed of memory access, while an I/O-bound workload would be limited by the speed of input/output operations.

⌚ Complexity of Executing an Instruction on a Modern Processor

The steps involved are:

1. Read Instruction:

- Converts virtual addresses to physical addresses.
- Interacts with the instruction cache to fetch the instruction.
- Retrieves the instruction from the cache memory.

2. Decode Instruction:

- Converts the high-level instruction into micro-operations.
- Fetches the micro-operations from the uop cache (Micro-Operation Cache).

3. Check for Dependencies/Pipeline Hazards:

Ensures that there are no conflicts or dependencies that could cause delays in execution.

4. Identify Available Execution Resource:

Determines which execution units are free to perform the required operations.

5. Data Retrieval:

- Accesses necessary data from the register file SRAM.
- Transfers the retrieved data to the appropriate execution resource.

6. Perform Arithmetic Operation:

Executes the arithmetic or logical operation specified by the instruction.

7. Data Write-Back:

- Move Data from Execution Resource to Register File: Transfers the result back to the register file.
- Control Write to Register File: Ensures that the result is correctly written back to the register file SRAM.

The overhead distribution is:

- Instruction Supply: 42%
- Data Supply: 28%
- Clock and Control: 24%
- Arithmetic: 6%

This breakdown shows that a significant portion of the overhead comes from instruction and data supply, emphasizing the importance of efficient cache and memory management.

❷ How does SIMD execution reduce the overhead of certain types of computations?

SIMD (Single Instruction, Multiple Data) execution allows a single instruction to perform the same operation on multiple data points simultaneously. This reduces the overhead by minimizing repetitive fetch and decode cycles.

Computations that benefit from SIMD have data-parallel characteristics, meaning they can be broken down and executed in parallel on different data elements with the same operation.

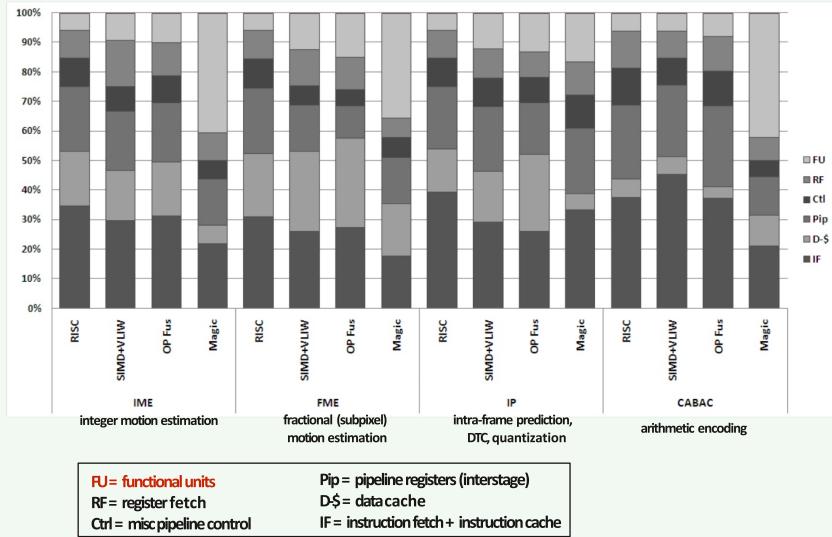
✓ Reducing Overhead

- **Fetch and Decode:** By fetching and decoding a single instruction that applies to multiple data points, SIMD reduces the repetitive overhead of fetching and decoding several instructions. This saves time and resources.
 - **Parallel Execution:** SIMD utilizes parallelism, executing the same operation across various data points in one go, which helps streamline the overall process.
- ✓ SIMD helps *simplify* the process by reducing the number of steps required to achieve the same result.** Instead of executing multiple instructions sequentially, SIMD groups these into a single instruction cycle, minimizing the overhead related to instruction supply and data supply.

This efficiency gain is crucial in workloads that are data-parallel and compute-bound, where performing the same operation on large datasets can benefit significantly from SIMD's parallel processing capabilities.

Example 1: H.264 Video Encoding Energy Consumption

This figure shows the energy consumption breakdown for various stages of H.264 video encoding.



Stages:

- **IME (Integer Motion Estimation):** Identifies block movement between frames.
- **FME (Fractional Motion Estimation):** Refines the movement estimation at a sub-pixel level.
- **Intra (Intra-frame Prediction, DCT, Quantization):** Predicts and transforms data within a single frame.
- **CABAC (Context-Adaptive Binary Arithmetic Coding):** A form of arithmetic encoding used for entropy coding.
- **Others:** Includes additional stages and tasks within the encoding process.

Energy Components:

- **FU (Functional Units):** Represents the energy consumed by the functional units.
- **RF (Register Fetch):** Energy used for fetching data from registers.
- **Ctrl (Misc Pipeline Control):** Control-related energy consumption.
- **Pip (Pipeline Registers):** Energy used for pipeline registers.

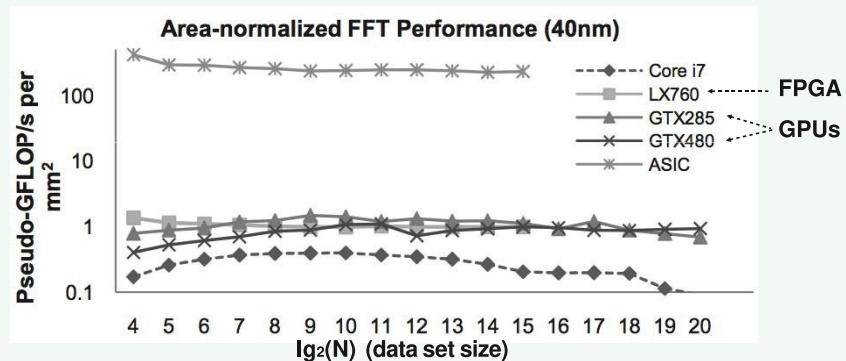
- **DS (Datacache):** Data cache energy consumption.
- **IF (Instruction Fetch + Instruction Cache):** Energy used for fetching instructions and accessing instruction cache.

The fraction of energy consumed by functional units (FU) is relatively small compared to other components, even when SIMD (Single Instruction, Multiple Data) instructions are used.

We want to emphasize the efficiency benefits of using specialized circuits for specific operations, as well as the importance of understanding the power distribution for complex tasks such as video encoding. **While specialized circuits can perform certain tasks more efficiently, overall power consumption must be considered to optimize performance.**

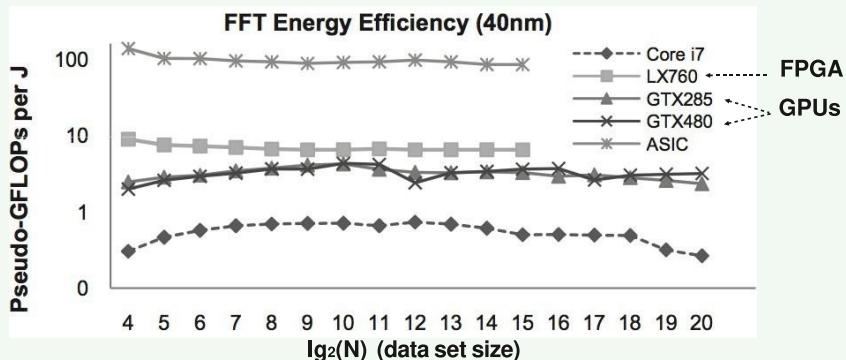
Example 2: Throughput and Energy Benefits of Specialization for Fast Fourier Transform (FFT)

The following graph compares the **performance per unit area** of different types of processors for executing the Fast Fourier Transform (FFT) using a 40nm semiconductor process.



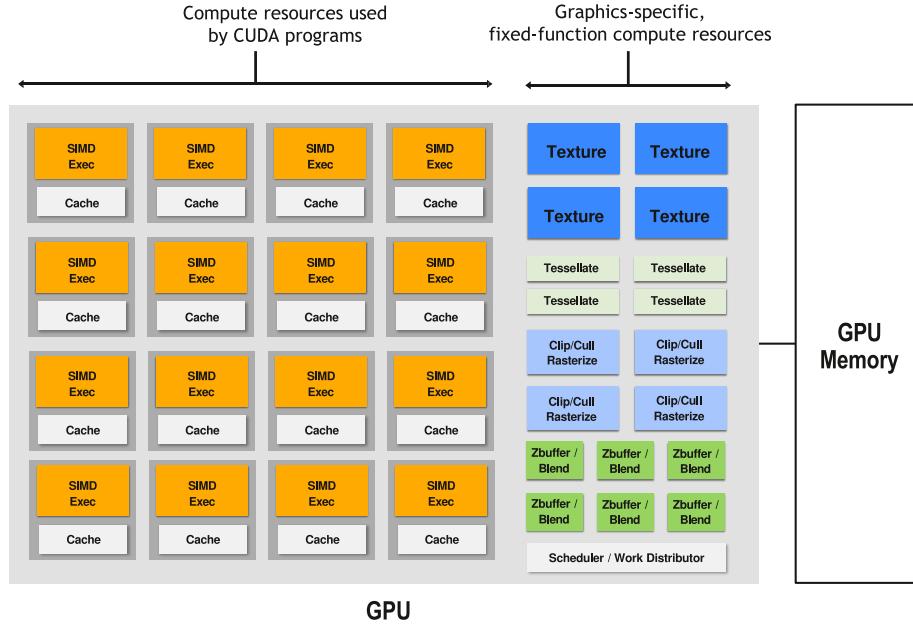
It illustrates the substantial benefits of using specialized hardware like ASICs and GPUs for FFT computations. ASICs, in particular, demonstrate remarkable area efficiency, making them ideal for applications where chip space is a critical constraint. GPUs also show considerable improvements, highlighting their role in high-performance, area-efficient computing tasks.

The following graph compares the **energy efficiency** of different processors for executing the Fast Fourier Transform (FFT) using a 40nm semiconductor process.



The graph illustrates the substantial energy-efficiency benefits of using specialized hardware like ASICs and GPUs for FFT computations. ASICs, in particular, offer remarkable energy savings, making them ideal for applications where power consumption is a critical constraint. GPUs also provide considerable improvements in energy efficiency, underscoring their role in high-performance, energy-efficient computing tasks.

Heterogeneous Multi-Core Architecture in GPUs



- **Diverse Core Types**

- **SIMD (Single Instruction, Multiple Data) Cores:** These cores are designed to execute the same instruction across multiple data points simultaneously. This structure excels in parallel processing tasks, such as matrix multiplications and vector operations, which are common in graphics and scientific computations.
- **Special-Purpose Cores:** GPUs include cores that are specifically tailored for tasks like texture mapping, shading, and rasterization. These cores handle operations that are unique to graphical rendering, giving GPUs their prowess in handling extensive graphical data more efficiently than CPUs.

- **Fixed-Function Units**

- **Texture Units:** These units perform operations like texture filtering and texture mapping, essential for applying texture to 3D models.
- **Tessellation Units:** These break down complex, high-order surfaces into simpler polygons, which are easier to render.
- **Rasterization Units:** These convert vector graphics (such as shapes and models) into a raster image (pixels or dots) for display on the screen.
- **Memory Controllers.** GPUs use unified memory architecture, where both CPU and GPU can access the same memory space. This architecture eliminates the need to copy data between CPU and GPU, reducing latency and increasing computational efficiency.

- **Scheduler and Work Distribution.** GPUs have sophisticated schedulers that dynamically allocate tasks to different processing units. These schedulers ensure optimal usage of the GPU's resources by balancing workloads across the multiple cores and units.

By **combining different types of cores and units, GPUs can handle a wide array of tasks concurrently**. For instance, while general SIMD cores might be processing large batches of data in parallel, texture units could be working on rendering images, and rasterization units converting those rendered images to pixels-all happening simultaneously, thanks to the architecture's heterogeneous nature.

In summary, the **heterogeneous multi-core architecture of GPUs allows them to excel at parallel processing and specialized graphics tasks**, making them exponentially faster and more efficient than CPUs for such workloads.

■ Digital signal processors (DSPs) - Qualcomm Hexagon DSP

Digital Signal Processors (DSPs) can be a type of processor used in the heterogeneous computing, specialized for signal processing tasks, handling audio, video, and communication data.

Example 3: Qualcomm Hexagon DSP

An example of a DSP is the Qualcomm Hexagon. Its role in heterogeneous systems is:

- **Specialized Processing Tasks:** The Hexagon DSP is designed to efficiently manage tasks like modem processing, audio processing, and image processing-tasks that would be less efficient if handled by a general-purpose CPU.
- **Parallelism and Efficiency:** By using complex instructions such as SIMD (Single Instruction, Multiple Data) and VLIW (Very Long Instruction Word), the Hexagon DSP can handle multiple operations per instruction cycle, improving speed and efficiency for specific types of data processing.
- **Integration with Other Processors:** In heterogeneous computing systems, the Hexagon DSP works alongside CPUs and GPUs. For example, in smartphones like the Google Pixel, the Hexagon DSP handles signal processing while the CPU manages general tasks, and the GPU handles graphical rendering.

The **benefits** of the Hexagon DSP in Heterogeneous Computing:

- ✓ **Performance Optimization:** Offloading specific tasks to the DSP frees up the CPU and GPU to focus on other tasks, leading to better overall system performance.
- ✓ **Energy Efficiency:** The Hexagon DSP's specialized design consumes less power for certain tasks compared to using a CPU, making the system more energy-efficient. This is crucial for battery-operated devices like smartphones.
- ✓ **Design Flexibility:** Incorporating the Hexagon DSP into a heterogeneous computing system allows for more flexible design options, catering to various workloads and processing requirements.

In devices like the Google Pixel phone, the Qualcomm Hexagon DSP showcases the practical application of heterogeneous computing. It performs tasks such as audio processing and image recognition, enhancing the device's overall capabilities and efficiency.

Example 4: Anton Supercomputer

The **Anton Supercomputer** is a specialized machine designed for molecular dynamics (MD) simulations, developed by [D. E. Shaw Research](#). It represents an excellent example of how specialized processors are integrated into heterogeneous computing systems to tackle specific scientific tasks.

Key features of the Anton Supercomputer:

- Anton is designed explicitly for simulating the time evolution of proteins and other biological macromolecules. Achieves substantially higher performance for MD simulations compared to general-purpose supercomputers.
- Anton's architecture consists of many ASICs tailored to handle particle-particle interactions efficiently. For example, Anton 2 contains 512 ASICs. This subsystem performs calculations of electrostatic and van der Waals forces using 32 deeply pipelined modules running at 800 MHz. Handles bond forces and fast Fourier transforms, using general-purpose Tensilica cores and specialized SIMD cores called geometry cores.
- Anton's custom network is designed to handle the communication patterns of N-body simulations with a three-dimensional torus topology. Each ASIC connects to a high-speed 3D torus network with significant bandwidth to manage data transfer.
- By integrating specialized hardware components and a custom communication network, Anton can perform MD simulations much faster than general-purpose supercomputers. Anton achieves simulation speeds of up to 17,000 nanoseconds per day for complex systems like protein-water.

Linking to Heterogeneous Computing:

- Anton exemplifies heterogeneous computing by using specialized processors (ASICs) for specific tasks. This approach contrasts with general-purpose processors used in homogeneous systems.
- Similar to how DSPs handle signal processing tasks in heterogeneous systems, Anton's ASICs are optimized for MD simulations, leading to increased computational efficiency and speed.
- In a heterogeneous system, different types of processors (CPUs, GPUs, DSPs, ASICs) collaborate to perform various tasks optimally. Anton's architecture integrates ASICs and custom communication networks to achieve this.
- Anton's use in molecular dynamics demonstrates how specialized hardware can advance scientific research by achieving simulations previously impossible with traditional supercomputers.

■ Field Programmable Gate Arrays (FPGAs)

Field Programmable Gate Arrays (FPGAs) are integrated circuits that can be configured by the user after manufacturing. FPGAs offer a middle ground between Application-Specific Integrated Circuits (ASICs) and general-purpose processors, providing a high degree of customization and flexibility.

The main features are:

- **Configurable Logic Blocks**

- FPGAs consist of an array of programmable logic blocks that can be configured to perform specific computational tasks.
- These logic blocks can be interconnected through programmable routing to form complex digital circuits.

- **High Parallelism**

- FPGAs are well-suited to tasks that benefit from parallel processing, such as signal processing, encryption, and machine learning.
- Multiple operations can be executed concurrently, making FPGAs highly efficient for certain workloads.

- **Reconfigurability**

- Unlike ASICs, which are permanently configured for specific tasks, FPGAs can be reprogrammed to adapt to different applications and workloads.
- This flexibility allows FPGAs to be used in prototyping, custom hardware design, and dynamic system updates.

- **Low Latency and High Throughput**

- FPGAs offer lower latency and higher throughput for data processing compared to CPUs and even GPUs in some scenarios.
- This makes them ideal for real-time applications and scenarios where data must be processed with minimal delay.

What is their role in heterogeneous computing?

- **Specialized Task Execution**

- In heterogeneous computing systems, FPGAs serve as accelerators for specific tasks that are highly parallel or require custom hardware.
- For example, signal processing tasks offloaded to an FPGA can be handled more efficiently than by a CPU or GPU.

- **Integration with Other Processors**

- Heterogeneous systems often include a combination of CPUs, GPUs, and FPGAs, each contributing its strengths to the overall performance.

- The CPU can manage general tasks and high-level control, while the FPGA handles specialized, compute-intensive functions.

- **Efficiency and Flexibility**

- FPGAs provide a balance of performance and versatility, allowing them to be reprogrammed for different applications as needed.
- This reconfigurability enhances the flexibility of heterogeneous systems, making them adaptable to changing requirements.

In the Google Pixel camera, FPGAs are used to accelerate image processing tasks, improving performance and efficiency. The Qualcomm Hexagon DSP, mentioned earlier, integrates FPGA-like features to handle audio, image, and signal processing tasks efficiently within a heterogeneous system.

In summary, FPGAs play a critical role in heterogeneous computing by providing customizable, efficient, and flexible solutions for specialized tasks. They work with other types of processors to improve overall system performance and adaptability.

Example 5: Project Catapult

Project Catapult is a Microsoft Research initiative aimed at enhancing cloud computing efficiency.

Project Catapult integrates Field-Programmable Gate Arrays (FPGAs) alongside traditional CPUs to boost performance and reduce latency for various datacenter workloads. The project initially targeted improving Bing's search engine performance by offloading parts of the document ranking process to FPGAs.

The system uses FPGAs as a compute layer that can act as a local accelerator, an inline processor, or a remote accelerator for distributed computing. Used in Microsoft's data centers, Project Catapult deployed FPGAs at a massive scale to improve not only Bing but other applications requiring high-speed computations.

The advantages of this project are:

- **Increased Performance:** By accelerating specific workloads, FPGAs provide enhanced computational power, reducing the time required for complex tasks.
- **Flexibility:** FPGAs offer a flexible and reprogrammable architecture, allowing them to adapt to different types of workloads.

Building on the success of Project Catapult, Microsoft launched Project Brainwave to enable real-time AI, further leveraging FPGA acceleration.

Project Catapult demonstrates how integrating FPGAs within heterogeneous computing systems can significantly enhance performance and flexibility, especially for intensive tasks like search engine optimization and AI.

Example 6: Amazon F1

Amazon EC2 FPGA, also known as **Amazon F1**, is a compute instance type within Amazon Web Services (AWS) that accelerates workloads by using custom hardware logic. With Amazon F1, developers can create and deploy custom FPGA designs to help accelerate tasks and increase performance.

F1 instances can significantly boost the performance of compute-intensive applications by leveraging the power of FPGAs. Custom FPGA designs reduce latency and improve throughput for demanding applications.

AWS provides a set of tools and libraries to develop, test, and deploy FPGA designs easily. The toolkit includes Amazon's FPGA Developer AMI (Amazon Machine Image) and AWS Management Console. F1 integrates seamlessly with other AWS services such as Lambda, S3, and RDS (Relational Database Service), allowing for comprehensive cloud-based solutions.

Some use cases are:

- Cryptographic Processing: Accelerating encryption and decryption tasks.
- Financial Modeling: Running complex financial algorithms.
- Genomics: Accelerating DNA sequencing and analysis.
- Image and Signal Processing: Enhancing image recognition and processing.

9.3 Challenges of heterogeneous designs

The challenges of heterogeneous computing designs are:

- **Choosing the Right Processor for Each Task.** In a heterogeneous system, one of the major challenges is identifying which processor (CPU, GPU, FPGA, DSP, etc.) is best suited for a particular task. Each type of processor has its strengths and optimal use cases, making the selection process essential for maximizing performance and efficiency.
- **Software Development Challenges:**
 - **Mapping Applications to Resources.** Developers need to design algorithms that decompose into components and map effectively onto the heterogeneous mix of processing units. This means picking the right tool for the job and ensuring each component works efficiently on its intended hardware.
 - **Complex Scheduling.** The scheduling problem becomes more complex in heterogeneous systems due to the need to dynamically allocate tasks across different processors. This requires advanced scheduling algorithms to balance workload distribution and avoid bottlenecks.
 - **Programming Complexity.** Heterogeneous systems often involve multiple programming languages and instruction set architectures (ISAs), increasing the complexity of development. Developers must be proficient in various programming models and tools to create optimized applications.
- **Hardware Design Challenges:**
 - **Resource Mixture Balance.** Hardware designers face the challenge of determining the appropriate mix of resources within the system. Allocating too few throughput-oriented resources can lower peak throughput for parallel workloads, while too few sequential processing resources can limit performance for sequential tasks.
 - **Specialized Function Allocation.** Designers must decide how much chip area should be dedicated to specific functions, such as video processing, AI inference, or data encryption. This allocation impacts the system's overall performance and efficiency.
- **Interoperability and Integration:**
 - **Interfacing Different Components.** Ensuring seamless communication and interoperability among different processors within a heterogeneous system can be challenging. Different processors may have distinct communication protocols and data formats, requiring sophisticated integration strategies.
 - **Unified Memory Access.** Achieving efficient shared memory access across various processors is crucial. Unified memory architectures help, but they require careful design to avoid performance penalties and ensure data consistency.

- **Optimization and Tuning:**

- **Performance Tuning.** Optimizing the performance of a heterogeneous system involves fine-tuning each component and the overall system architecture. This process can be time-consuming and requires deep knowledge of each processor's capabilities and limitations.
- **Energy Efficiency.** Balancing performance and energy efficiency is essential, especially for battery-powered devices. Designers need to optimize workloads to minimize power consumption without sacrificing performance.

- **Cost and Complexity:**

- **Development Costs.** The complexity of developing and maintaining heterogeneous systems can increase costs. This includes the need for specialized hardware, development tools, and skilled personnel.
- **Increased Complexity.** The inherent complexity of heterogeneous systems can lead to longer development cycles and more potential points of failure. Ensuring robust design and testing processes is critical to handle this complexity.

⚠ Pitfalls of Heterogeneous Designs

Heterogeneous computing systems are designed to handle diverse and complex workloads by utilizing various specialized processors. While this architecture offers significant advantages in terms of performance, efficiency, and flexibility, it also presents several notable pitfalls:

- **Under-Provisioning Specialized Components:**

- **Resource Allocation.** Allocating too few resources to specialized components like rasterization units can lead to bottlenecks.

Example 7

For example, if the rasterizer in a graphics pipeline is under-provisioned, the throughput of pixel fragments may not meet the demand of the SIMD cores, causing them to run at lower efficiency.

- **Efficiency Loss.** Under-provisioning can result in significant efficiency losses.

Example 8

For example, if the rasterizer throughput is only 1T when 1.2T is needed, programmable cores may only operate at 80% efficiency, leaving a substantial portion of the chip idle.

- **Over-Provisioning Components:**

- **Resource Waste.** Over-provisioning fixed-function components to avoid bottlenecks can lead to resource waste. Over-allocating chip area for components like the rasterizer minimizes the advantage of fixed-function units, as resources could have been used more effectively elsewhere.
- **Reduced Performance Gains.** Over-provisioning of fixed-function components can reduce the overall performance gains of the heterogeneous system because the added resources may not result in proportional improvements in throughput or efficiency.

- **Complexity in Load Balancing:**

- **Dynamic Workloads.** Balancing workloads dynamically across different processing units is complex. Changes in workload intensity can lead to uneven distribution, causing some units to become overburdened while others remain underutilized.
- **Scheduling Challenges.** Efficiently scheduling tasks to match the strengths of each processing unit requires sophisticated algorithms and can be challenging to implement correctly. Poor scheduling can result in suboptimal performance and increased latency.

- **Integration and Interoperability Issues:**

- **Interfacing Different Processors.** Ensuring seamless communication between heterogeneous processors can be challenging. Different processors have unique communication protocols, data formats, and memory access patterns that require careful design to avoid interoperability issues.
- **Data Consistency.** Maintaining data consistency across multiple processing units involves managing data transfer and synchronization efficiently. Inadequate handling of these aspects can lead to data corruption or delays.

- **Programming and Development Complexity:**

- **Multiple Programming Models.** Developing applications for heterogeneous systems often involves using various programming models and tools. This increases development complexity and requires specialized knowledge from developers.
- **Debugging and Optimization.** Debugging and optimizing heterogeneous systems can be more challenging due to the interactions between different processing units. Identifying performance bottlenecks and resolving issues requires deep insights into the behavior of each processor type.

9.4 Reducing energy consumption

There are two main idea to reducing energy consumption in heterogeneous computing systems: **Specialized Processing** and **Minimizing Data Movement**.

■ Specialized Processing

Specialized Processing involves **using the most energy-efficient processor for a specific task**. This idea leverages the unique strengths of different processor types (CPUs, GPUs, FPGAs, DSPs, etc.) to optimize performance while minimizing energy consumption.

A possible implementation might be:

- Assign workloads to processors best suited for them.

Example 9

For example, use:

- CPUs for general-purpose, sequential tasks.
- GPUs for parallel processing tasks like rendering and simulations.
- FPGAs for tasks requiring reconfigurable logic and custom hardware acceleration.
- DSPs for signal processing tasks.

- Deploy fixed-function units for repetitive tasks such as audio processing, video decoding/encoding, and image processing. These units consume less power compared to general-purpose processors.

The main advantages are:

- ✓ Using the right processor for specific tasks **reduces the overall energy consumption**.
- ✓ Specialized processors can perform tasks more efficiently, leading to **faster execution times** and lower power usage.

Minimizing Data Movement

Data Movement has a high energy cost. By **reducing the frequency and distance over which data is transferred**, significant energy savings can be achieved.

A possible implementation might be:

- **Minimize the amount of data moved** between memory and processors.
- **Structure algorithms to reuse data stored in local memory** as much as possible, reducing accesses to external memory.
- **Implement compression techniques** to reduce the volume of data transferred. This can involve using specialized hardware for efficient compression and decompression.

The main advantages are:

- ✓ Keeping data transfers to a minimum **reduces the energy required for memory access**.
- ✓ Reduced data movement leads to **lower latency and faster processing times**.

Example 10

Reading 64 bits of data from local SRAM (1mm away on-chip) consumes about 26 picoJoules, whereas reading the same data from low-power mobile DRAM (LPDDR) consumes around 1200 picoJoules. Thus, minimizing data movement to external memory significantly saves energy (26 pJ vs 1200 pJ).

★ Trends in Energy-Optimized Computing

The following trends collectively **aim to address the pressing need for more sustainable computing solutions**. As the demand for computing power continues to grow, optimizing energy consumption helps to reduce environmental impact and ensure that technological advances do not come at the expense of the planet.

1. **Compute Less!** Each computation costs energy, so by minimizing unnecessary operations, the energy efficiency of the system increases.

The parallel algorithms may perform more work than sequential ones, which sometimes makes them less desirable despite potentially faster execution times. The goal is to balance the speed and energy consumption.

2. **Specialize Compute Units.** This involves using specialized processing units tailored to specific tasks to reduce energy consumption:

- **Heterogeneous Processors:** Combining CPU-like cores (general-purpose) with GPU-like cores (optimized for parallel tasks) to use the best processor for each task.
- **Fixed-Function Units:** Utilizing units designed for specific tasks such as audio processing, video encoding/decoding, and image processing. These units consume less energy compared to general purpose processors for their respective tasks.
- **Specialized Instructions:** Incorporating specialized instructions like AVX for vector processing and AES-NI for encryption to perform tasks more efficiently.
- **Programmable Soft Logic (FPGAs):** Using FPGAs, which can be reprogrammed for different tasks, providing flexibility and energy efficiency.

3. **Reduce Bandwidth Requirements.** This trend focuses on minimizing data transfer to save energy.

Restructuring algorithms to maximize reuse of on-chip data, thereby reducing the need to transfer data from memory. Also, using additional computation to compress data before it is transferred to memory, reducing the amount of data moved. This approach helps reduce the power costs associated with data movement. For example, using fixed-function hardware to reduce the overhead of general-purpose data compression and decompression.

10 Patterns

10.1 Dependencies

Dependencies are critical when designing parallel programs as they directly influence the program's correctness and potential for parallelism.

Definition 1

A **dependency** arises when one operation depends on an earlier operation to complete and produce a result before this later operation can be performed.

Non-directly, dependencies **determine the order in which operations must be executed to maintain correctness**.

❸ Why is sequential consistency important?

Sequential Consistency ensures that the results of a parallel program's execution are as if the operations of all processors were executed in some sequential order, while preserving the program order for each processor. It is so important because:

- **Enforcing Dependencies:**

- **Ordered Execution.** Sequential consistency ensures that operations are executed in an order that respects their dependencies.
- **Data Integrity.** By maintaining the correct order of dependent operations, sequential consistency preserves the integrity of the data being manipulated in parallel programs.

- **Predictable Behavior:**

- **Program Order Preservation.** Dependencies define the required order of operations within a program. Sequential consistency ensures each processor's operations appear in sequence, respecting these dependencies.
- **Non-Interference.** Sequential consistency guarantees that operations from different processors can interleave their executions in many possible sequences, but the final results are consistent with some sequential execution that respects dependencies.

This type of model has already been discussed in Section 8.3 on page 226. However, we can summarize the guarantees it provides:

- ✓ **Simplified Reasoning.** Programmers can reason about parallel programs more easily, as if they were sequential.
- ✓ **Correctness.** It helps maintain the correctness of shared memory operations in concurrent programs.

💡 How to detect dependencies?

By identifying where dependencies occur, we can determine where operations need to be synchronized or reordered to ensure accurate results. But the real dilemma is, “*how do we know when two or more statements are interdependent?*”. The short answer is: it is not always that easy. It depends on the complexity of the code, but in general, the definitions are as follows:

- **Independent Statements:** two statements are independent if their **execution order does not affect the computation outcome**.

The implication is that the order of execution is interchangeable without changing the final state.

- **Dependent Statements:** statements are dependent if the **order of their execution impacts the computation result**.

The implication is that changing the order of execution alters the final outcome, so the sequence of operations matters.

☰ Type of dependencies

Dependencies can be categorized into several types:

- **Control Dependencies:** execution of one operation depends on the control flow (like `if` statements or loops).
- **Data Dependencies:** one operation requires the result of another operation:
 - **True Dependency** (Read After Write, RAW): a subsequent operation reads data after it's written.
 - **Anti-Dependency** (Write After Read, WAR): a write operation must occur after all preceding read operations.
 - **Output Dependency** (Write After Write, WAW): two write operations must occur in a particular sequence.

Example 1: Dependencies

Some dependency scenarios:

- **Independent Statements**

- Statement 1: `a=1;`
- Statement 2: `b=1;`

Since these two statements do not rely on each other's values, they are independent. Executing them in any order does not change the outcome.

- **True (flow) Dependency**

- Statement 1: `a=1;`
- Statement 2: `b=a;`

`S2` depends on the value of `a` from `S1`. This is a true (or flow) dependency, as `S2` requires `S1` to execute first to produce the correct result.

- **Output Dependency**

- Statement 1: `a=f(x);`
- Statement 2: `a=b;`

Both statements write to `a`, creating an output dependency. The order matters because both statements affect the value of `a`.

- **Anti-Dependency**

- Statement 1: `a=b;`
- Statement 2: `b=1;`

There is an anti-dependency where `S1` reads `b`, and `S2` writes to `b`. The execution order affects the value read by `S1`.

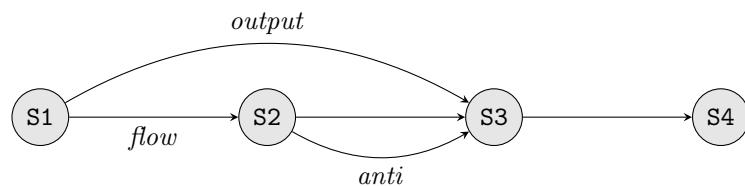
☒ Dependency Graph

Dependencies can be represented as a graph, also known as a **dependency graph**.

- **Nodes (statements).** Each node represents a statement in the program. For example:

- Statement 1: `a = 1;`
- Statement 2: `b = a;`
- Statement 3: `a = b + 1;`
- Statement 4: `c = a;`

The dependency graph is:



- **Edges (dependencies).** Arrows between nodes indicate dependencies between statements. Here are the key types of dependencies shown (in the previous dependency graph):

– **True (flow) dependency:**

$$S2 \quad \delta \quad S3$$

An arrow from $S2$ to $S3$ signifies that $S3$ depends on $S2$ for the value of a .

– **Output dependency:**

$$S1 \quad \delta^0 \quad S3$$

An arrow from $S1$ to $S3$ indicates that both $S1$ and $S3$ write to a .

– **Anti dependency:**

$$S2 \quad \delta^{-1} \quad S3$$

An arrow from $S2$ to $S3$ shows that $S2$ reads a , while $S3$ writes to a .

❷ Why do we compute dependencies?

Computing data dependencies is important for several reasons, such as optimizing code (compilers can reorder or optimize code better if they know which statements depend on each other), avoiding data hazards (in hardware design, understanding dependencies is essential to avoid hazards), debugging and maintenance (understanding dependencies helps debugging by showing how data flows through the program), etc.

Therefore, by **computing data dependencies**, we get a clear picture of how data is used and modified throughout the program. This systematic approach can lead to more efficient, reliable, and maintainable code.

❸ How do we compute dependencies?

Data dependency relationships can be found by comparing the IN and OUT sets of each node. The IN and OUT sets of a statement S are defined as:

- **$IN(S)$:** set of memory locations (**variables**) that may be **used** in S .
- **$OUT(S)$:** set of memory locations (**variables**) that may be **modified** by S .

In other words, $IN(S)$ and $OUT(S)$ sets represent **all possible memory locations** (**variables**) that **might be read from or written to** by a statement S .

Note that the IN and OUT sets are often larger than the actual number of locations used or modified in practice.

This conservativeness comes from a need to be cautious. When compiling or analyzing code, it is **better to consider more memory locations than to miss any, ensuring that dependencies are fully captured**. This way, no potential dependencies are overlooked, even if it means including some memory locations that may not be actively used in all scenarios.

Example 2: Simple analogy

Imagine we are listing all the books we might need for a project. We list every possible book we could *potentially* read, even if we end up only needing to read a few of them. This conservative approach ensures we won't miss a book we might actually need later.

② Properties when we compute data dependencies

Assuming that there is a path from S_1 to S_2 , the following shows interesting properties about intersection between the IN and OUT sets:

- **Flow Dependency (True Dependency):**

$$\text{out}(S_1) \cap \text{in}(S_2) \neq \emptyset \quad S_1 \delta S_2$$

Statement S_1 writes to a location that statement S_2 reads from later.

- **Anti Dependency:**

$$\text{in}(S_1) \cap \text{out}(S_2) \neq \emptyset \quad S_1 \delta^{-1} S_2$$

Statement S_1 reads from a location that statement S_2 writes to later.

- **Output Dependency:**

$$\text{out}(S_1) \cap \text{out}(S_2) \neq \emptyset \quad S_1 \delta^0 S_2$$

Both S_1 and S_2 write to the same location.

≡ Loop Dependencies

In a loop, we generally encounter two main types of dependencies:

- **Loop-Carried Dependencies.** These occur when an **iteration of a loop depends on data from a previous iteration**.

Example 3

In the following loop, each iteration depends on the result of the previous iteration, making it parallelizable only in a pipeline manner.

```
1 for (i=1; i<100; i++) {
2     a[i] = a[i-1] + 1;
3 }
```

- **Loop-Independent Dependencies:** These exist if **iterations do not rely on results from each other directly**.

Example 4

In the following loop, each iteration can be executed independently, allowing for parallelization.

```
1 for (i=0; i<100; i++) {
2     a[i] = i;
3 }
```

Furthermore, the dependency is:

- **Lexically Forward Dependency** occurs when the **source statement** (the one that produces the value) **appears before the target statement** (the one that consumes the value) within the same iteration of code. Essentially, it follows the natural top-to-bottom order of reading code.

Definition 2: Lexically Forward Dependency

Lexically Forward Dependency, when the source statement appears before the target statement in the loop body.

Example 5: Lexically Forward Dependency

In the following code, $a = b + 1$ (the source) comes before $c = a + 2$ (the target) within the same loop iteration.

```
1 for (i=0; i<5; i++) {
2     a = b + 1; // Source statement
3     c = a + 2; // Target statement
4 }
```

- **Lexically Backward Dependency** occurs when the source statement comes *after* the target statement within the same iteration. It's a bit counterintuitive since it involves looking back to an earlier point in the code to resolve a dependency.

Definition 3: Lexically Backward Dependency

Lexically Backward Dependency, when the source comes after the target.

Example 6: Lexically Backward Dependency

In the following code, `c = a + 2` (the target) relies on the value of `a` which is produced later by `a = b + 1` (the source) within the same iteration.

```

1 for (i=0; i<5; i++) {
2     c = a + 2; // Target statement
3     a = b + 1; // Source statement
4 }
```

Example 7: Lexical dependency analogy

Lexical dependencies are not an easy topic to understand. Here we offer a simple analogy for better understanding.

Think of a cooking recipe as a loop iteration. Imagine we have a recipe that lists steps to make a dish.

- **Lexically Forward Dependency:** This is like following a recipe where we need to boil water (*source step*) before we can add pasta to it (*target step*). The sequence flows naturally from top to bottom.
- **Lexically Backward Dependency:** Imagine we need to taste the pasta (*target step*) to decide if we should add more salt (*source step*), even though the step to add salt appears later in the list. This means we'd have to look back to the earlier part of the iteration to decide if we need to add more salt.

❓ I really should understand lexical dependencies?

Understanding these dependencies helps when we're optimizing loops for parallel execution. **Lexically forward dependencies are generally simpler to manage since they align with the natural execution order.** Lexically backward dependencies require careful handling to ensure correct results, as they involve "looking back" within the iteration.

Example 8: Loop dependencies and optimization

Consider the following codes:

- First double for loop iteration:

```

1 for (int i = 0; i < 100; i++)
2     for (int j = 1; j < 100; j++)
3         a[i][j] = f(a[i][j - 1]);

```

- Second double for loop iteration:

```

1 for (int j = 1; j < 100; j++)
2     for (int i = 0; i < 100; i++)
3         a[i][j] = f(a[i][j - 1]);

```

Remark: Cache

When optimizing for cache performance, the goal is to maximize data locality. Data locality can be broken into two types:

1. **Temporal Locality**: Reusing the same data within a short period.
2. **Spatial Locality**: Accessing data locations that are near each other.

We can make the following considerations:

✓ In the **first loop**, the inner loop iterates over *j* while *i* remains constant. This means that within each iteration of the outer loop, we access consecutive elements in the same row (*a[i][...]*). Since arrays are usually stored in row-major order (consecutive elements of a row are stored in consecutive memory locations), this loop has **good spatial locality**. Elements of *a[i][...]* are likely to be in the cache, reducing cache misses.

✗ In the **second loop**, the inner loop iterates over *i* while *j* remains constant. This means we're accessing elements in the same column across different rows (*a[...][j]*). Since rows are stored consecutively in memory, accessing elements across rows (column-wise access) leads to poor spatial locality. Each access to *a[i][j]* is more likely to cause a cache miss because it jumps to a different row.

Dependencies and Parallelism

- Achieving Parallelism with Dependencies

- **Concept:** Even when our code has dependencies, we can still achieve parallelism. The key is to identify parts of the code that can run independently and execute those in parallel.
- **Practice:** Look for independent computations that don't rely on each other's results and schedule them for parallel execution.
- **Example:** If some parts of our loop iterations don't depend on others, those can be parallelized while managing the dependent parts separately.

- Control Ordering of Events on Different Processors (Cores)

- **Concept:** Dependencies dictate how different events in our program must be ordered, especially when running on multiple processors or cores.
- **Practice:** Maintain a partial order of execution, which respects the dependencies and constraints imposed by the code.
- **Example:** If one computation depends on the result of another, ensure that the dependent computation only starts once the necessary data is available.

- Use Synchronization Mechanisms

- **Concept:** Running code concurrently requires synchronization to ensure that all dependencies and orderings are respected.
- **Practice:** Use synchronization tools such as locks, semaphores, or barriers to control access to shared resources and manage dependencies.
- **Example:** If multiple threads are updating a shared variable, synchronization mechanisms ensure that updates occur in the correct order, preventing race conditions.

10.2 Parallel Patterns

② What are Parallel Patterns?

Parallel Patterns are essentially **recurring combinations of task distribution and data access strategies that can be applied to solve specific problems** in parallel computing. These patterns act as a “vocabulary” for algorithm design, making it easier to communicate and understand various approaches to parallelism.

By applying these patterns, developers can achieve efficient and effective parallelism, ensuring that tasks are executed concurrently without conflicts and with optimal performance.

- ✓ **Simplicity**. They provide clear, reusable templates for designing parallel algorithms.
- ✓ **Efficiency**. They help optimize performance by utilizing parallelism effectively.
- ✓ **Universality**. They are applicable across different parallel programming systems and platforms.

In essence, **parallel patterns help streamline the process of designing and implementing parallel algorithms**, making it **easier to manage dependencies and achieve concurrent execution**.

Roughly speaking, parallel patterns are **similar to design patterns** in computer science, but they focus specifically on parallel computing. They provide reusable solutions to common problems in parallel programming, with the goal of simplifying and streamlining the development process.

≡ Types of Parallel Patterns

1. **Nesting Pattern**: Combining different parallel patterns within one another.
2. **Serial/Parallel Control Patterns**: Handling control flow in parallel and serial region of code.
3. **Serial/Parallel Data Management Patterns**: Managing data access and modifications in parallel and serial contexts.
4. Other Patterns:
 - Superscalar Sequences
 - Futures
 - Speculative Selection
 - Workpile
 - Search
 - Segmentation
 - Expand
 - Category Reduction
5. **Programming Model Support for Patterns**: Tools and models that support implementing these patterns.

10.2.1 Nesting Pattern

The **Nesting Pattern** is a compositional pattern¹³ that allows for the hierarchical composition of patterns. This is used in both serial and parallel algorithms. The main idea is to use “pattern diagrams”¹⁴ to visually show how tasks can be organized and composed in a hierarchy.

Key Points

- **Hierarchical Composition:** The Nesting Pattern allows patterns to be nested within each other, creating a hierarchy of patterns.
- **Task Blocks:** Each “task block” in a pattern diagram represents a location of general code in an algorithm.
- **Replaceability:** Any task block in the diagram can be replaced with a pattern that has the same input/output and dependencies.

In the following figure, we can see a diagram showing a multiple task blocks. Some of these blocks are highlighted in green to indicate that they can be replaced with other patterns. The goal is to illustrate how we can nest patterns to build more complex algorithms.

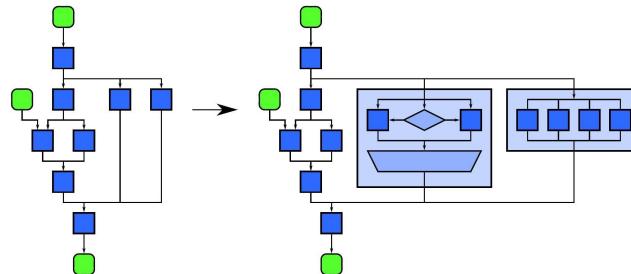


Figure 39: Graphical example of a nesting pattern.

¹³Compositional Patterns are design patterns that allow us to compose complex systems from simpler, reusable components. These patterns emphasize the importance of combining smaller pieces of functionality in a structured and predictable way. They are particularly useful in managing the complexity of large systems by breaking them down into manageable parts.

¹⁴Pattern Diagrams are visual tools used to represent and illustrate patterns, particularly in the context of software engineering and algorithm design. They help in understanding how different components or tasks fit together to form a complete system or process.

Example 9: Pattern Diagrams

Suppose we are designing a parallel algorithm to process large data sets. A pattern diagram can help us visualize the composition of different patterns, such as data partitioning, task scheduling, and data aggregation. Each of these patterns can be represented as a set of nodes and edges nested within a larger pattern that represents the entire algorithm.

10.2.2 Serial Control Patterns

Serial Control Patterns are fundamental programming constructs that dictate the sequence of execution in a program. They are the basic building blocks of any algorithm, defining how the flow of control moves through the program from start to finish. In other words, they determine the order in which tasks are performed.

② Why “serial” control patterns?

These patterns are called “serial” because the **tasks** they control are **executed one after the other**, in a specific order, without overlapping.

≡ Types

1. Sequence Pattern

≡ **Definition.** An ordered list of tasks that are executed in a specific order.

? **Assumption.** The program text ordering will be followed.

Example 10: Sequence Pattern

In the following example, each task must be completed before the next one begins.

```
1 int A = 10;
2 int T = foo(A);
3 int S = bar(T);
4 int B = delta(S);
```

2. Selection Pattern

≡ **Definition.** A condition is evaluated first; depending on the result, one of two tasks is executed.

? **Assumption.** Only one of the tasks will be executed, not both.

Example 11: Selection Pattern

In the following example, the function **a** is executed if condition **c** is true; otherwise, function **b** is executed.

```
1 if (c) {
2     a();
3 } else {
4     b();
5 }
```

3. Iteration Pattern

▀ Definition. A condition is evaluated, and if true, a task is executed repeatedly until the condition becomes false.

▲ Complication in Parallelizing. Dependencies may exist between iterations.

Example 12: Iteration Pattern

In the following example, function `a` is executed `n` times. The condition `i < n` is evaluated, and if true, the function `a` is repeated until `i < n` becomes false (then `i = n`).

```
1 for (int i = 0; i < n; ++i) {
2     a();
3 }
```

4. Recursion Pattern

▀ Definition. A dynamic form of nesting where functions call themselves.

★ Special Case. Tail recursion can be converted into iteration, which is important for functional languages.

Example 13: Recursion Pattern

```
1 int recursive_function(int n) {
2     if n > 0
3         return n * recursive_function(n - 1);
4     return 1; // base case
5 }
```

These patterns are the **foundation of structured serial programming**. Also, when parallelizing serial algorithm, it's crucial to understand these patterns to effectively manage dependencies and concurrency.

10.2.3 Parallel Control Patterns

Parallel Control Patterns are design patterns used in parallel computing to manage the execution flow of tasks that can run simultaneously. These patterns help in organizing and structuring parallel code to ensure efficient execution, resource management, and scalability. They are essential for leveraging the power of multi-core processors and distributed computing systems.

In essence, parallel control patterns provide the blueprint for designing and implementing parallel algorithms, ensuring that tasks are executed efficiently and effectively.

☰ Types

1. Fork-Join Parallel Pattern

☰ **Definition.** The Fork-Join pattern allows control flow to fork into multiple parallel flows and then rejoin later.

✖ **Implementation.** Languages like Cilk Plus use `spawn` and `sync` to implement this pattern. When a function is spawned, it runs in parallel with the caller. The caller then uses `sync` to wait for all spawned functions to complete before proceeding.

✖ **Difference from Barrier.** A “join” involves only one thread continuing, while a “barrier” allows all threads to continue.

2. Map Parallel Pattern

☰ **Definition.** The Map pattern applies a function to each element of a collection independently, producing a new collection of results.

✖ **Usage.** It replaces serial iterations where each iteration is independent.

✖ **Elemental Function.** The function applied to each element is called an “elemental function”.

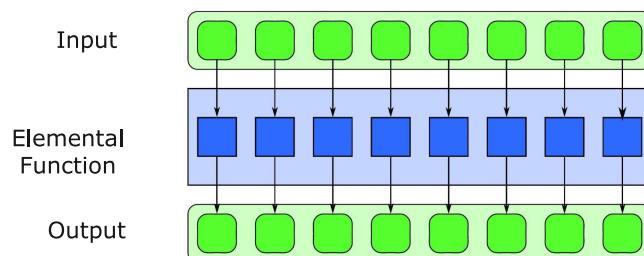


Figure 40: Visual representation of the Map Parallel Pattern.

Example 14: Map analogy

A good analogy is the `map` method used in Javascript. The difference is that the `map` in Javascript is done sequentially, instead the map pattern is done in parallel. However, as we can see from the example provided in the official documentation, a function is applied to each element of the input:

```

1 // Elemental Function
2 function elementalFunction(num) {
3   return num * 2;
4 }
5
6 // Example array
7 const array1 = [1, 4, 9, 16];
8
9 // Pass an elementalFunction to map
10 const map1 = array1.map((x) => elementalFunction(x));
11
12 console.log(map1);
13 // Expected output: Array [2, 8, 18, 32]

```

Here the official documentation:



3. Stencil Parallel Pattern

Definition. The Stencil pattern extends the Map pattern by allowing the elemental function to access neighboring elements.

Usage. Commonly used in iterative solvers or simulations where the state of an element depends on its neighbors.

Consideration. Boundary conditions must be handled carefully.

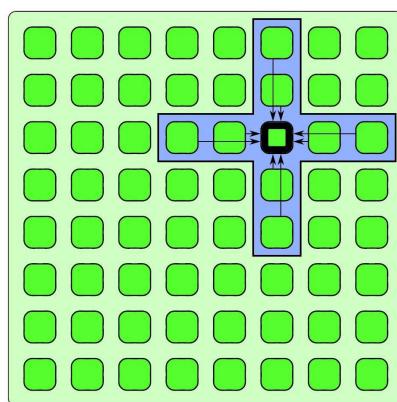


Figure 41: Visual representation of the Stencil Parallel Pattern.

4. Reduction Parallel Pattern

- Definition.** The Reduction pattern combines elements of a collection using an associative “combiner function” (e.g., addition, multiplication, max, min).
- Parallelism.** Different orderings of the reduction are possible due to the associative property, making it suitable for parallel execution.

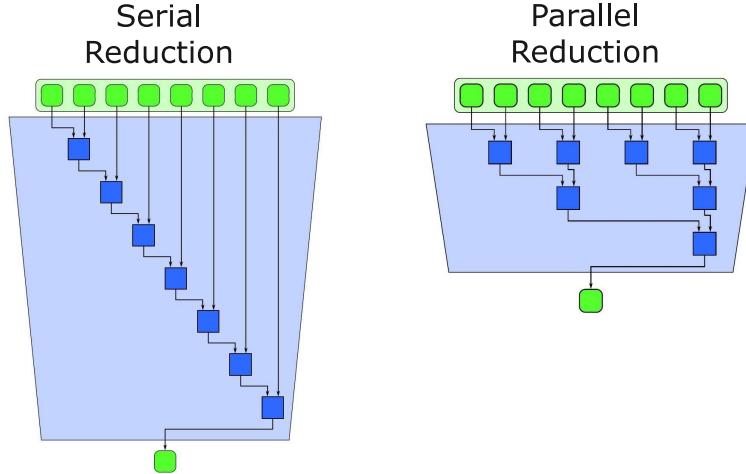


Figure 42: Visual representation of the Reduction Parallel Pattern.

Example 15: Reduction analogy

A good analogy is the `reduce` method used in Javascript. The difference is that the `reduce` in Javascript is done sequentially, instead the reduce pattern is done in parallel.

```

1  function combinerFun(acc, curVal) {
2      return acc + curVal;
3  }
4
5 // Example array
6 const arrayToReduce = [1, 2, 3, 4];
7
8 // 0 + 1 + 2 + 3 + 4
9 // steps:
10 // 0: 0 + 1
11 // 1: 1 + 2
12 // 2: 3 + 3
13 // 3: 6 + 4
14 // result = 10
15 const initialValue = 0;
16 const sumOfArray = arrayToReduce.reduce(
17     (acc, curVal) => combinerFun(acc, curVal),
18     initialValue
19 );
20
21 console.log(sumOfArray);
22 // Expected output: 10

```

Here the official documentation:



5. Scan Parallel Pattern

Definition. The Scan pattern computes all partial reductions of a collection.

Usage. For each element in the output collection, it computes a reduction of the input elements up to that point.

Parallelization. Although it has dependencies, the associative nature of the function allows parallelization.

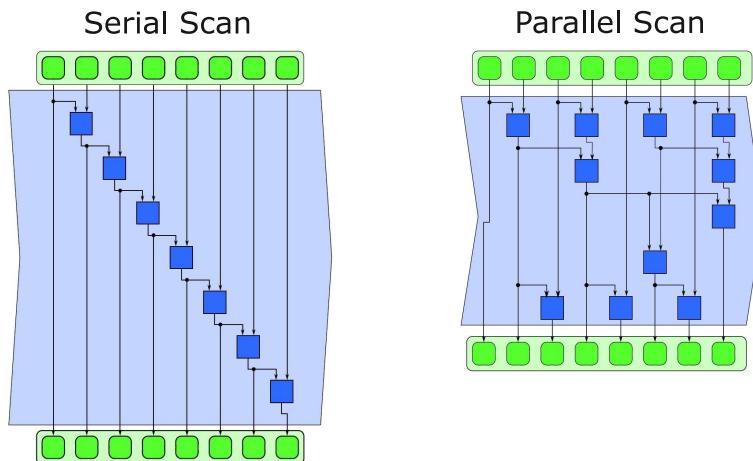


Figure 43: Visual representation of the Scan Parallel Pattern.

Example 16: Scan analogy

```

1 // JavaScript doesn't provide a native implementation of
2 // the scan,
3 // but we can build it manually
4 // * array - input
5 // * fn - reduction to apply to input values
6 // * initialValue - initial value of the accumulator
7 function scan(array, fn, initialValue) {
8     // create the output array
9     const result = [];
10    // The scan method is a partial reduction,
11    // so we need to apply the partial reduce
12    array.reduce((acc, value, index) => {
13        // combiner function to apply
14        const newAcc = fn(acc, value);
15        // save the calculated value
16        result.push(newAcc);
17    }, initialValue);
18    return result;
19 }

```

```

15     // at the i-th iteration
16     result[index] = newAcc;
17     // returns the calculated value,
18     // since it will be the new accumulator
19     return newAcc;
20 }, initialValue);
21 // return the output
22 return result;
23 }
24
25 // combiner function of the reduction pattern
26 function combinerFunc(accumulator, currentValue) {
27   return accumulator + currentValue;
28 }
29
30 // Example array
31 const arr = [1, 2, 3, 4];
32 // Scan
33 const sumRes = scan(
34   arr, (a, b) => combinerFunc(a, b), 0
35 );
36
37 console.log(sumRes);
38 // Expected output: [1, 3, 6, 10]

```

To test the execution, here there is a free console (or just open the browser console):



6. Recurrence Parallel Pattern

☰ Definition. The Recurrence pattern is a more complex version of Map, where elements can depend on the outputs of adjacent elements.

✖ Usage. Similar to Map, but requires a serial ordering of elements for computability.

10.2.4 Serial Data Management Patterns

Serial Data Management Patterns refer to the various **methods and strategies** used by serial programs to **manage data**. These patterns deal with how data is allocated, shared, read, written, and copied in a program. Understanding these patterns is crucial for writing efficient and maintainable code.

≡ Types

1. Random Read and Write Pattern

Definition. Memory locations are accessed using addresses, often through pointers. This allows for random access to data, meaning any memory location can be read from or written to at any time.

Challenges. Aliasing (uncertainty about whether two pointers refer to the same memory location) can cause problems when converting serial code to parallel code.

Example 17: Random Read and Write Pattern

```
1 #include <iostream>
2
3 using namespace std;
4
5 int main() {
6     int array[10];
7
8     // Write to random positions
9     array[3] = 5;
10    array[7] = 10;
11
12    // Read from random positions
13    cout << "Value at position 3: " << array[3] << endl;
14    cout << "Value at position 7: " << array[7] << endl;
15
16    return 0;
17 }
```

2. Stack Allocation Pattern

- Definition.** Stack allocation is used for dynamically allocating data in a Last-In-First-Out (LIFO) manner. This is very efficient because allocation and deallocation can be done in constant time.
- Benefits.** Preserves locality, making data access faster. In parallel programming, each thread usually has its own stack, preserving thread locality.

Example 18: Stack Allocation Pattern

```

1 #include <iostream>
2
3 using namespace std;
4
5 void functionB() {
6     int b = 10; // Stack allocation
7     cout << "Function B, value of b: " << b << endl;
8 }
9
10 void functionA() {
11     int a = 5; // Stack allocation
12     cout << "Function A, value of a: " << a << endl;
13     functionB();
14 }
15
16 int main() {
17     functionA();
18     return 0;
19 }
```

3. Heap Allocation Pattern

- Definition.** Used when data cannot be allocated in a LIFO manner. Heap allocation allows for dynamic memory allocation at any time.

- Challenges.** Slower and more complex than stack allocation. In parallel programming, a parallelized heap allocator should be used to keep separate pools for each parallel worker.

Example 19

```

1 #include <iostream>
2
3 using namespace std;
4
5 int main() {
6     int* ptr = new int; // Heap allocation
7     *ptr = 10;
8     cout << "Value in heap: " << *ptr << endl;
9     delete ptr; // Free the allocated memory
10    return 0;
11 }
```

4. Objects Pattern

█ Definition. Objects are language constructs that associate data with code to manipulate and manage that data. They can have member functions and belong to a class of objects.

? **Usage.** In parallel programming, objects are often generalized in various ways to support concurrent execution.

Example 20

```
1 #include <iostream>
2
3 using namespace std;
4
5 class MyObject {
6 public:
7     int value;
8
9     void display() {
10         cout << "Value: " << value << endl;
11     }
12 };
13
14 int main() {
15     MyObject obj; // Stack allocation of an object
16     obj.value = 15;
17     obj.display();
18
19     MyObject* objPtr = new MyObject(); // Heap
20     allocation of an object
21     objPtr->value = 20;
22     objPtr->display();
23     delete objPtr; // Free the allocated memory
24
25     return 0;
}
```

10.2.5 Parallel Data Management Patterns

Parallel Data Management Patterns are strategies used to manage data in parallel computing environments. These patterns ensure that data is properly managed, avoiding issues like race conditions and ensuring efficient data locality. They are crucial for optimizing the performance of parallel programs by managing how data is allocated, shared, accessed, and manipulated across multiple parallel workers.

☰ Types

1. Pack Parallel Pattern

☰ **Description.** Pack is used to eliminate unused space in a collection by discarding elements marked as false and placing the remaining elements in a contiguous sequence.

_QMARK **Usage.** Commonly used with the Map pattern.

▢ **Inverse Operation.** Unpack is the inverse operation, placing elements back in their original locations.

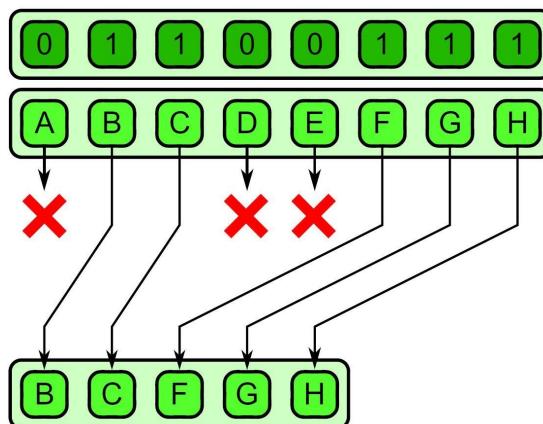


Figure 44: Visual representation of the Pack Parallel Pattern.

2. Pipeline Parallel Pattern

☰ **Description.** Pipeline connects tasks in a producer consumer manner. Tasks are organized in a linear sequence where the output of one task becomes the input of the next.

_QMARK **Usage.** Useful when combined with other patterns to multiply available parallelism. Pipelines can also be structured as Directed Acyclic Graphs (DAGs).

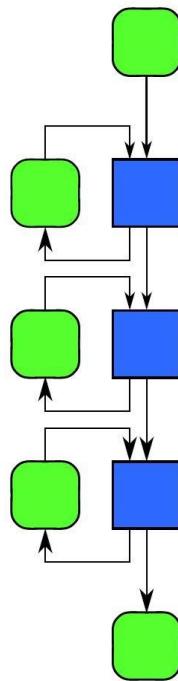


Figure 45: Visual representation of the Pipeline Parallel Pattern.

3. Geometric Decomposition Parallel Pattern

☰ Description. Geometric Decomposition arranges data into subcollections, which can be overlapping or non-overlapping.

❓ Usage. This pattern provides a different view of the data without necessarily moving it. It's commonly used in grid-based computations like finite element analysis or image processing.

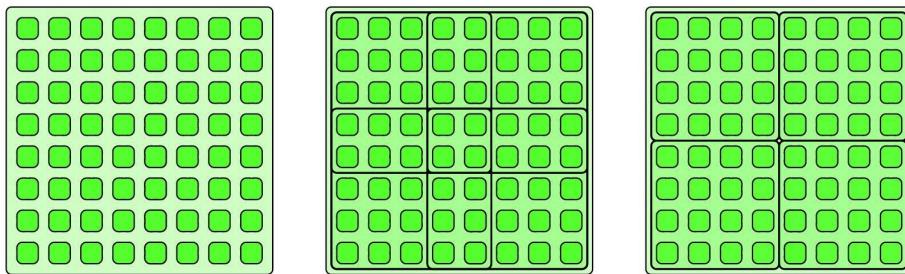


Figure 46: Visual representation of the Geometric Decomposition Parallel Pattern.

4. Gather Parallel Pattern

Description. Gather reads a collection of data given a collection of indices, similar to a combination of Map and random serial reads.

Usage. The output collection shares the same type as the input collection but has the same shape as the indices collection.

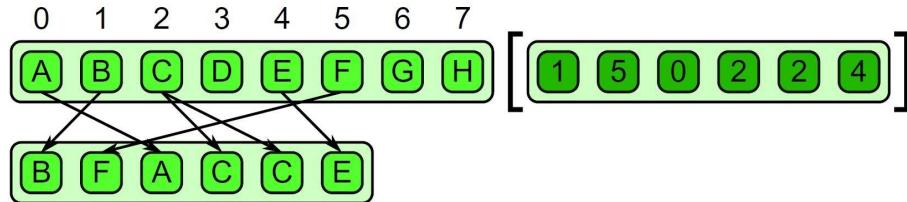


Figure 47: Visual representation of the Gather Parallel Pattern.

5. Scatter Parallel Pattern

Description. Scatter is the inverse of Gather. It writes each element of the input to the output at the given index specified by a collection of indices.

Usage. Race conditions can occur when multiple writes target the same location, so care must be taken to avoid conflicts.

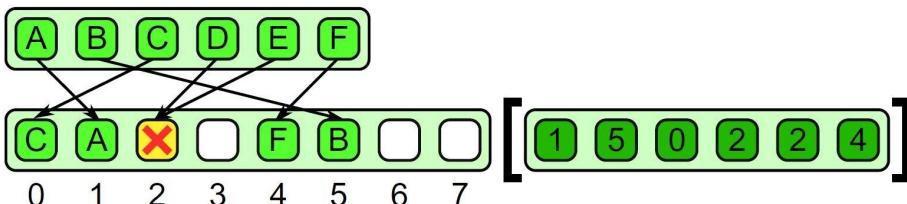


Figure 48: Visual representation of the Scatter Parallel Pattern.

10.2.6 Other Parallel Patterns

There are other important parallel patterns that we don't introduce. The following lists some of them with a brief description.

- **Superscalar Sequences Parallel Pattern**

Description. Write a sequence of tasks, ordered only by dependencies.

Usage. Helps in organizing tasks such that they can be executed out of order as long as dependencies are respected, increasing parallelism.

- **Futures Parallel Pattern**

Description. Similar to fork-join, but tasks do not need to be nested hierarchically.

Usage. Allows asynchronous execution of tasks, where a “future” represents a value that will be computed and available in the future (a similar logic exists in Python: [official documentation](#)).

- **Speculative Selection Parallel Pattern**

Description. A general version of serial selection where the condition and both outcomes can all run in parallel.

Usage. Used when it's beneficial to execute multiple branches in parallel and discard the results of the non-selected branch once the condition is evaluated.

- **Workpile Parallel Pattern**

Description. A general map pattern where each instance of the elemental function can generate more instances, adding to the “pile” of work.

Usage. Efficient for dynamic task generation, where new tasks are created as the algorithm progresses.

- **Search Parallel Pattern**

Description. Finds some data in a collection that meets specific criteria.

Usage. Utilized in searching algorithms where multiple search operations can run in parallel to speed up the process.

- **Segmentation Parallel Pattern**

Description. Operations on subdivided, non-overlapping, non-uniformly sized partitions of 1D collections.

Usage. Effective for processing large datasets by dividing them into manageable segments that can be processed in parallel.

- **Expand Parallel Pattern**

❑ **Description.** A combination of pack and map patterns.

❑ **Usage.** Allows elements to be expanded and processed in parallel, useful for operations that generate variable amounts of output from each input element.

- **Category Reduction Parallel Pattern**

❑ **Description.** Given a collection of elements each with a label, find all elements with the same label and reduce them.

❑ **Usage.** Common in data aggregation tasks where elements are grouped by a key (label) and then reduced (aggregated) by some operation.

10.3 Map Pattern

10.3.1 What is a Map?

The **Map Pattern** involves applying a function to each element in a collection independently (without knowledge of neighbors), producing a new collection of results. This pattern is **highly parallelizable** because each element is processed independently of the others.

Key Points

- **Independence.** Each iteration of the map operation is independent of the others, making it highly parallelizable.

More precisely, assuming an infinite number of processors ∞ available, the operation takes constant time $O(1)$, but the overhead for managing these parallel tasks grows logarithmically with input size $O(\log n)$; however, even with the implementation overhead, the time complexity remains very efficient and grows slowly with input size.

Modifying shared state breaks parallelism independence, and the violation can cause some problems: non-determinism, data races, undefined behavior, segmentation faults.

- **Unary Maps.** Maps can operate over a single collection (one input, one output).

Example 21: Unary Map

```

1 // Unary Map takes a single value
2 // (a single position of the input array)
3 // and returns a single value
4 // (the new value of the output array)
5 function elementalFunction(val) {
6     return val * 2;
7 }
8
9 // input
10 const inArr = [1, 2, 3, 4];
11
12 // apply unary map
13 const outArr = inArr.map((inVal) => elementalFunction(
14     inVal));
15 // expected output: [2, 4, 6, 8]
16 console.log(outArr);

```

- **N-ary Maps.** Maps can also operate with multiple inputs and outputs, such as performing element-wise operations on two arrays.

Example 22: N-ary Map

0	1	2	3	4	5	6	7	8	9		
3	7	0	1	4	0	0	4	5	3	1	0
2	4	2	1	8	3	9	5	5	1	2	1
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
5		2	2		3	9	9		4	3	1

```

1 int twoToOne(int x, int y) {
2     return x + y;
3 }
```

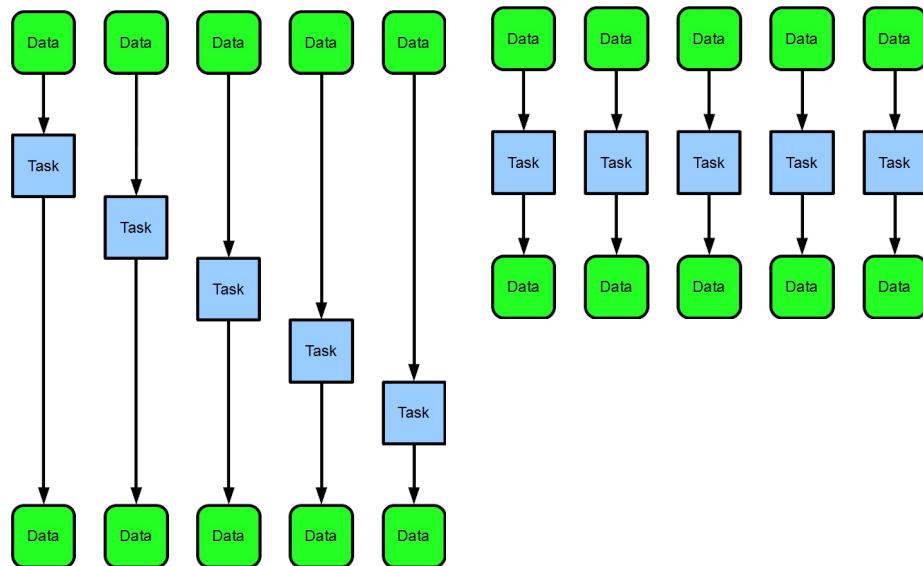


Figure 49: A sequential Map on the left and a parallel Map on the right.

✓ Advantages

- ✓ **Easy to Parallelize.** Since each element is processed independently, map operations can be easily parallelized to improve performance.
- ✓ **Local State.** Each element has its own input and output, avoiding shared state issues.

10.3.2 Optimizations

10.3.2.1 Sequences of Maps

Sequences of Maps refer to a series of map operations that are applied one after another in a pipeline. Each operation in the sequence takes the output of the previous operation as its input. This pattern is common in tasks involving vector math, where multiple small operations like additions and multiplications are applied in sequence to elements of an array or collection.

Key Points

- **Independent Operations.** Each map operation is independent and can be parallelized. However, when several maps are applied in sequence, each intermediate result might be written to memory.
- **Vector Math.** Often, tasks in vector math consist of multiple small operations applied as maps, such as $a[i] = f(a[i])$ followed by $a[i] = g(a[i])$, and so on.
- ✖ **Memory Inefficiency.** A naïve implementation that writes each intermediate result to memory can waste memory bandwidth and potentially overwhelm the cache.
- 💡 **How to Improve Performance?** In the following pages, we introduce two techniques, code fusion and cache fusion, that can be used to reduce memory bandwidth usage and improve cache efficiency.

Example 23: Sequences of Maps

Imagine we have two map operations to apply to an array: one to double each element and another to add 3 to each element.

```

1 for (int i = 0; i < n; ++i) {
2     a[i] = a[i] * 2;
3 }
4
5 for (int i = 0; i < n; ++i) {
6     a[i] = a[i] + 3;
7 }
```

In this example, each element of the array a is first doubled, and then 3 is added to each element.

10.3.2.2 Code Fusion

Code Fusion is an **optimization technique** used to improve the performance of programs, particularly in parallel computing. It involves **combining multiple operations into a single, more efficient operation**. By doing so, code fusion increases arithmetic intensity, reduces memory/cache usage, and enhances overall computational efficiency.

Key Points

- **Combining Operations.** Code fusion *fuses* together multiple operations so that they are performed simultaneously, rather than sequentially.

Example 24: Code Fusion

Instead of performing a series of separate operations on an array, we combine them into one loop, reducing the number of passes over the data.

- **Increasing Arithmetic Intensity.** Arithmetic intensity refers to the ratio of computational operations to memory operations. By fusing operations, we increase this ratio, meaning **more calculations are done per memory access**.
- **Benefit.** Higher arithmetic intensity leads to better utilization of the CPU and other computational resources.
- **Reducing Memory/Cache Usage.** By reducing the number of intermediate results that need to be stored in memory, code fusion **minimizes memory access and optimizes cache usage**.
- **Use of Registers.** Ideally, **operations can be performed using registers alone**, which are the fastest form of storage in a CPU. By keeping data in registers and reducing memory access, code fusion achieves maximum computational efficiency.

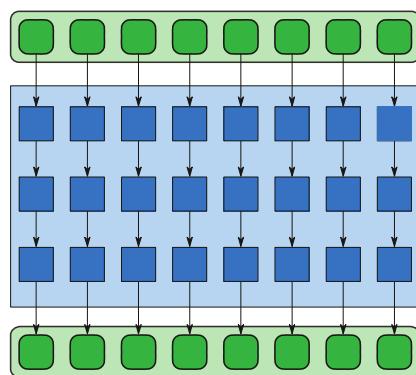


Figure 50: Graphical example of a code fusion.

10.3.2.3 Cache Fusion

Cache Fusion is an optimization technique that improves the efficiency of memory access by ensuring that data is accessed and processed in a way that takes full advantage of the CPU cache. This is particularly important in parallel computing, where efficient use of cache can significantly enhance performance.

Key Points

- **Optimizing Cache Usage.** Cache fusion involves structuring code and data access patterns to maximize the cache's effectiveness. This means minimizing cache misses and ensuring that data stays in the cache as long as needed.
- **Reducing Memory Bandwidth Usage.** By optimizing how data is accessed, cache fusion reduces the number of memory accesses needed, thus decreasing memory bandwidth usage.

How does it work?

1. **Breaking Operations into Smaller Blocs.** Instead of processing large blocks of data all at once, cache fusion breaks these operations into smaller blocks that fit into the cache.
 - ✓ This ensures that data accessed remains in the cache, reducing the need to repeatedly access main memory.
2. **Efficient Data Movement.** The smaller blocks are processed one at a time, with each block being loaded into the cache, processed, and then written back to memory.
 - ✓ This minimizes the movement of data between the cache and main memory, improving overall performance.

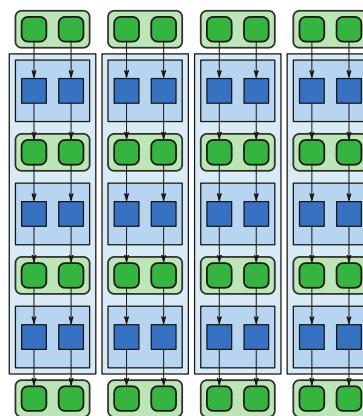


Figure 51: Graphical example of a cache fusion.

10.3.3 Related Patterns

Related Patterns are design patterns that share similarities in structure, function, or purpose. They often address similar problems or solutions in different ways, complementing each other within a broader context of software or algorithm design. Understanding related patterns helps in selecting the most appropriate pattern for a specific task and recognizing how different patterns can be combined or interchanged to achieve optimal results.

☰ Patterns related to Map

- **Stencil Pattern related to Map Pattern**

☰ **Description.** Each instance of the map function accesses neighbors of its input, offset from its usual input.

In other words, it is used when we need to **apply a function to a collection of elements, but each function's input may also involve neighboring elements.**

_QMARK **Usage.** Common in tasks like image processing or solving partial differential equations (PDEs), where the value of each element depends not only on itself but also on its surrounding elements.

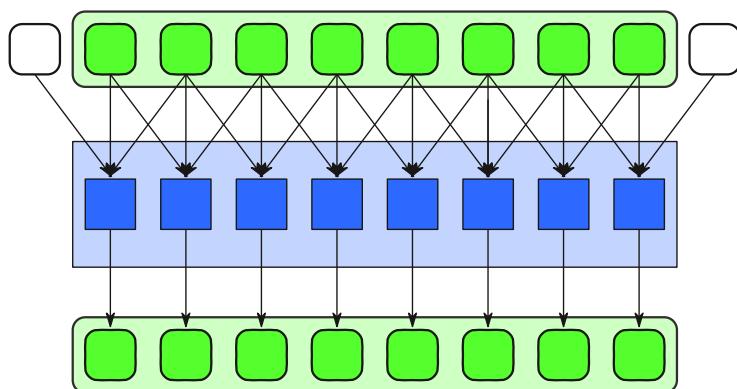


Figure 52: Visual representation of the Stencil Pattern.

Example 25: Stencil Pattern related to Map Pattern

In a 2D grid, the function applied to each cell may also read values from its neighboring cells (e.g., for edge detection in image processing).

- **Workpile Pattern related to Map Pattern**

☰ **Description.** Workpile pattern allows work items to be added to the map while it is in progress, from inside map function instances.

_QMARK **Usage.** The work grows and is consumed by the map. It terminates when no more work is available.

Example 26: Workpile Pattern related to Map Pattern

Useful in tasks like graph traversal or dynamic task generation where new tasks are created during the execution.

- **Divide-and-Conquer Pattern related to Map Pattern**

▀ Description. This pattern applies if a problem can be divided into smaller subproblems recursively until a base case is reached that can be solved serially.

? Usage. Common in algorithms like mergesort, quicksort, and solving mathematical problems such as the Fibonacci sequence or matrix multiplication.

Example 27: Divide-and-Conquer Pattern related to Map Pattern

Divide an array into smaller subarrays, sort them individually, and then merge the sorted subarrays.

10.3.4 Scaled Vector Addition (SAXPY)

SAXPY stands for **Single-Precision A*X Plus Y**. It is a fundamental operation in linear algebra and is part of the Basic Linear Algebra Subprograms (BLAS) library. The operation is defined as:

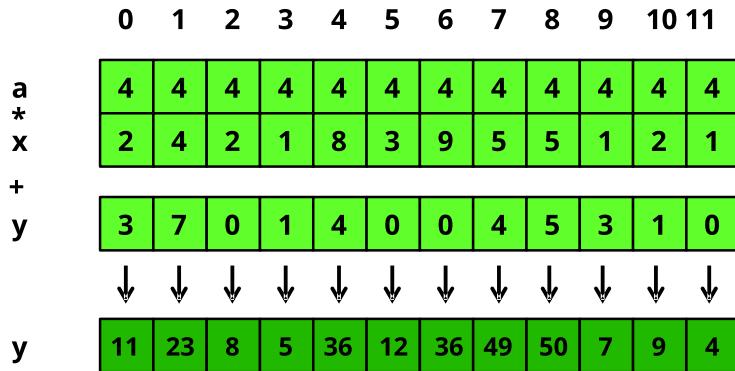
$$y = \alpha x + y \quad (15)$$

Where:

- α is a scalar (a single value).
- x and y are vectors (arrays of numbers).

The main observations that we can make are: each element in vector x is scaled by α ; the scaled elements of x are added to the corresponding elements of y ; but the most important thing is that each element in the vectors x and y is processed independently, making SAXPY **highly parallelizable**.

Maybe, a visual representation, can be useful to identify which pattern can be applied:



❖ Implementations

- A serial implementation of SAXPY written:

```

1 void saxpy_serial(
2     size_t n,           // the number of elements in the vectors
3     float a,            // scale factor
4     const float x[], // the first input vector
5     float y[]          // the output vector and second input
                           // vector
6 ) {
7     for (size_t i = 0; i < n; ++i)
8         y[i] = a * x[i] + y[i];
9 }
```

- **Threading Building Blocks** (TBB, parallel) implementation:

```

1 void saxpy_tbb(
2     int n,           // the number of elements in the vectors
3     float a,         // scale factor
4     float x[],      // the first input vector
5     float y[]       // the output vector and second input vector
6 ) {
7     tbb::parallel_for(
8         tbb::blocked_range<int>(0, n),
9         [&](tbb::blocked_range<int> r) {
10            for (size_t i = r.begin(); i != r.end(); i++)
11                y[i] = a * x[i] + y[i];
12        }
13    );
14 }
```

Where:

- `tbb::parallel_for(range, function)`
 - * **range**: Specifies the range of indices to be processed (0 to `n`).
 - * **function**: A lambda function that defines the operation for each subrange.
- Lambda function: `[&](tbb::blocked_range<int> r)`
 - * `[&]`: Capture clause that captures all variables by reference.
 - * `(tbb::blocked_range<int> r)`: Parameter list; in this case, the range `r` to be processed.
 - * Function body:

```

1 for (size_t i = r.begin(); i != r.end(); i++)
2     y[i] = a * x[i] + y[i];
```

Each element `y[i]` is updated independently within the subrange.

It works like this:

- **Range Splitting**: `tbb::blocked_range` divides the index range into subranges, which are processed in parallel.
- **Dynamic Load Balancing**: TBB's scheduler dynamically assigns subranges to available threads, ensuring efficient load balancing.
- **Data Independence**: Each update to `y[i]` is independent of other elements, making it safe for parallel execution.

- Cilk Plus (parallel) implementation:

```

1 void saxpy_cilk(
2     int n,           // the number of elements in the vectors
3     float a,         // scale factor
4     float x[],      // the first input vector
5     float y[]       // the output vector and second input vector
6 ) {
7     cilk_for (size_t i = 0; i < n; ++i)
8         y[i] = a * x[i] + y[i];
9 }
```

It works like this:

- **Automatic Parallelization:** `cilk_for` divides the loop iterations among available threads, balancing the workload dynamically.
- **Data Independence:** Each update to `y[i]` is independent of other elements, making it safe for parallel execution.
- **Simplified Syntax:** `cilk_for` provides a simple and familiar loop syntax for parallel loops, making it easy to convert serial loops into parallel loops.

- OpenCL (parallel) implementation:

```

1 __kernel void saxpy_opencl(
2     __constant float a,
3     __constant float* x,
4     __constant float* y
5 ) {
6     int i = get_global_id(0);
7     y[i] = a * x[i] + y[i];
8 }
```

It works much like CUDA.

- OpenMP (parallel) implementation:

```

1 void saxpy_openmp(
2     int n,           // the number of elements in the vectors
3     float a,         // scale factor
4     float x[],      // the first input vector
5     float y[]       // the output vector and second input vector
6 ) {
7     #pragma omp parallel for
8     for (int i = 0; i < n; ++i)
9         y[i] = a * x[i] + y[i];
10 }
```

10.4 Collectives operations

Collective operations are fundamental to parallel computing. They involve communication patterns that enable data to be distributed among multiple processes, combined, or transformed in various ways.

In essence, collectives are operations where all processes in a parallel program participate.

≡ Collective Patterns

- **Reduce**: Combines data from all processes into a single result.
- **Scan**: Computes cumulative operations (such as prefix sum) across processes.
- **Partition**: Divides data into subsets and distributes them among processes.
- **Scatter**: Distributes distinct pieces of data from one process to all other processes.
- **Gather**: Collects distinct pieces of data from all processes into one process.

Collectives are used in parallel algorithms to ensure efficient and effective data handling across multiple processes. They help in achieving tasks such as data aggregation, distribution, synchronization, and more.

10.4.1 Reduce (or Reduction) Pattern

The **Reduce Pattern** (or commonly called **Reduction Pattern**) is used to combine a collection of elements into a summary value using a **combiner function**. This pattern has already been introduced on page 287.

The **combiner function** combines elements pairwise. It must be associative to be parallelizable. Some common combiner functions include addition, multiplication, and finding the maximum or minimum value.

💡 Is it really important that the combiner function must be associative?

It is not important, it is essential. Before answering, we need to recall a bit of theory.

A **binary operation** (like addition) is associative if the order in which the operations are performed does not change the result.

Therefore, for a reduction operation to be parallelizable, the combiner function must be associative. This is because parallel reduction involves combining elements in pairs, often in different order, across multiple operations. If the operation is associative, we can divide the work between processes and combine the results in any order without changing the final result.

In summary, the combiner function must be associative to ensure that the end result is the same regardless of how the elements are grouped or combined in parallel.

If the reader is still not convinced, we present the following example as proof.

Example 28: Why associative property is important

It is well known that the addition is associative because:

$$(A + B) + C = A + (B + C)$$

This allows us to parallelize addition across multiple processes. The order in which the sums are performed doesn't matter, because the property guarantees that the result is always the same. Instead, subtraction is non-associative because:

$$(A - B) - C \neq A - (B - C)$$

This means that we cannot parallelize subtraction in the same way. In this scenario, different executions lead to different execution orders and different results.

≡ Types of Reduction

1. **Serial Reduction.** Combines elements in a linear fashion, one after the other.

Example 29: Serial Reduction Pattern

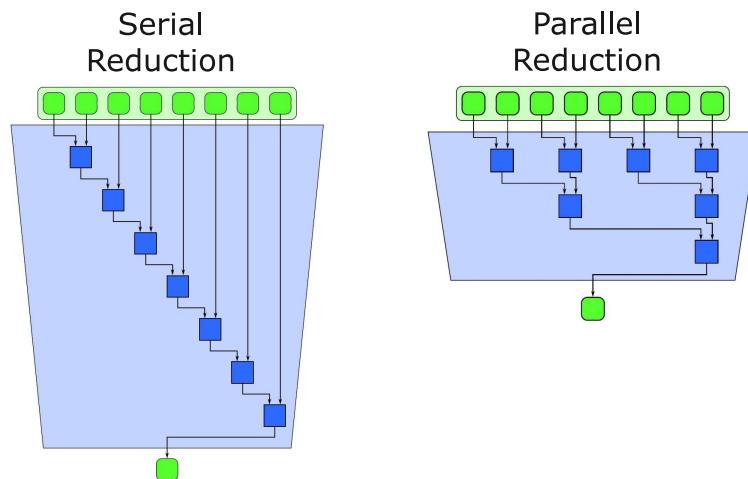
Summing elements [A, B, C, D] would be proceed as:

$$((A + B) + C) + D$$

2. **Parallel Reduction.** Combines elements in a tree-like structure, allowing multiple operations to be performed simultaneously.

Example 30: Parallel Reduction Pattern

Summing elements [A, B, C, D] in parallel could proceed as (A + B) and (C + D) in one step, followed by the final sum of these results.



Vectorization

In general, **Vectorization** refers to the process of converting an algorithm from operating on single elements at a time to operating on a set of elements simultaneously using vectorized instructions.

Modern processors have special vector processing units (such as SIMD - Single Instruction, Multiple Data) that can handle multiple data elements in a single instruction.

The Reduction Pattern combines elements in pairs until a single result is obtained. Vectorization can be applied to this process to combine multiple pairs simultaneously.

Example 31: Reduce Pattern Vectorization

Suppose we have an array of numbers that we want to sum. Instead of summing each pair one by one, we can use vectorized instructions to sum several pairs at once. This is especially beneficial for large datasets.

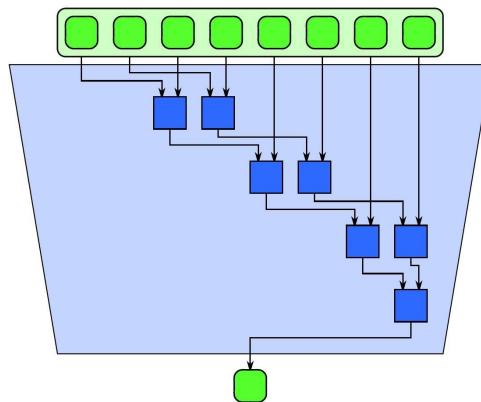


Figure 53: Graphical example of the vectorization technique.

Advantages

- ✓ **Speed.** Vectorization can greatly speed up the reduction process by making better use of CPU resources.
- ✓ **Parallelism.** It enhances the inherent parallelism of the reduction operation, further improving performance.

Tiling

Tiling (also known as blocking) is a technique where the workload is divided into smaller, more manageable chunks or “tiles”. Each tile can be processed independently, and the results can then be combined. We have already seen this topic in the CUDA environment, on page 136.

★ **Goal.** The main goal of tiling is to improve cache efficiency and reduce memory access latency by keeping frequently accessed data within the faster levels of the memory hierarchy (like the CPU cache).

The tiling in the reduction pattern is applied as follows:

1. **Divide Data.** The entire dataset is divided into smaller tiles. Each tile contains a subset of the data.
2. **Process Tiles.** Each tile is processed independently. For the reduction pattern, this means performing the reduction operation on the elements within each tile.
3. **Combine Results.** Once all tiles have been processed, the results from each tile are combined to obtain the final result.

Example 32: Tiling with Reduce Pattern

Imagine we have an array of numbers that we want to sum using tiling:

- Divide the array into smaller tiles (e.g., blocks of 100 elements each).
- Perform the summation within each tile in parallel.
- Combine the sums of all tiles to get the final result.

Advantages

- ✓ **Cache Efficiency.** By working on smaller tiles, the data can fit into the CPU cache, reducing the need to fetch data from slower main memory.
- ✓ **Parallelism.** Each tile can be processed in parallel, making it easier to distribute the workload across multiple processes or threads.
- ✓ **Scalability.** Tiling allows for better scalability, especially when dealing with large datasets.

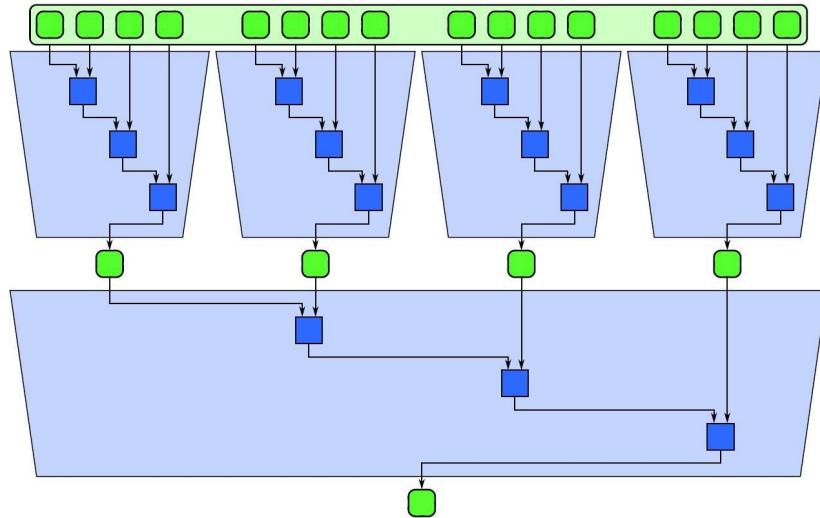


Figure 54: Graphical example of the tiling technique applied to the reduce pattern.

⚠ Limitations

Unfortunately, the reduce pattern has two major limitations: precision issues and order floating-point dependency.

✖ **Precision Issues.** Small rounding errors occur because some numbers cannot be represented exactly. For example, the number 0.1 cannot be represented precisely in binary floating-point format.¹⁵

When performing a reduction operation (e.g., summing a large array of floating-point numbers), these **small rounding errors can accumulate**. Each addition introduces a tiny error, and the more additions we perform, the larger the cumulative error becomes.

Example 33: Precision Issues

Suppose we sum the numbers [0.1, 0.2, 0.3, 0.4]. Ideally, the result should be 1.0. However, due to rounding errors, we might get something like 0.999999999999999 instead.

In large datasets, this effect can be more pronounced, leading to significant deviations from the expected result.

¹⁵Floating-point numbers are represented in a way that allows for a wide range of values but introduces some imprecision. This is because they are stored in a fixed number of bits, which can only approximate most real numbers.

✖ **Order Floating-Point Dependency.** Floating-point addition is not associative. This means that the order in which we add the numbers can affect the result.

In a parallel reduction, elements are combined in different orders depending on how they are distributed across processes. **Different orders of combination can lead to different results due to the non-associative nature of floating-point addition.**

Example 34: Order Floating-Point Dependency

Suppose we have three numbers $1e20$, $-1e20$, and 1.0 . If we add them as:

$$(1e20 + -1e20) + 1.0 = 0.0 + 1.0 = 1.0$$

But if we add them as:

$$1e20 + (-1e20 + 1.0) \approx 1.0$$

Due to the large magnitude difference between $1e20$ and 1.0 .

These problems can be partially solved using some intelligent techniques such as:

- Kahan Summation Algorithm:
- Higher Precision:



- Pairwise Summation:
- Consistent Order:



However, in this course we don't delve into these topics.

10.4.2 Scan Pattern

The **Scan Pattern** is used to **compute all partial results of an input sequence using a binary associative operation**. It generates a new sequence where each element is the result of applying the operation to all preceding elements in the input sequence.

☰ Types of Scans

- **Inclusive Scan Pattern**

☰ Definition. Each element in the result sequence includes the value of the current element in the input sequence.

☷ Result. The **result for each position is the combination** (using the associative operation) **of all elements up to and including the current element**.

✓ Use Case. It is useful when the result must contain the current element at every position. It can be used in parallel algorithms where each step depends on all previous steps, including the current one.

Example 35: Inclusive Scan Pattern

For an input array [A, B, C, D], an inclusive scan with an associative operation + results in:

[A, A + B, A + B + C, A + B + C + D]

For numbers:

- Input: [1, 2, 3, 4]
- Output: [1, 3, 6, 10]

- **Exclusive Scan Pattern**

☰ Definition. Each element in the result sequence excludes the value of the current element from the input sequence. Instead, it includes the combination of all prior elements.

☷ Result. The **result for each position is the combination** (using the associative operation) **of all elements before the current element**.

✓ Use Case. It is useful for scenarios where the result at each position should exclude the current element and provide the cumulative effect of all previous elements. It's often used in algorithms that require a shift in position, such as calculating running sums starting from the next element.

Example 36: Exclusive Scan Pattern

For an input array [A, B, C, D], an exclusive scan with an associative operation + results in:

$$[\times, A, A + B, A + B + C]$$

Where \times is the identity element of the operation. For numbers:

- Input: [1, 2, 3, 4]
- Output: [0, (1), (1 + 2), (1 + 2 + 3)] = [0, 1, 3, 6]

❷ Techniques for parallelizing Scan Pattern

- **Prefix Maximum Algorithm with Up and Down Sweep**

Definition. Prefix Maximum Algorithm with Up and Down Sweep is a special implementation of the scan pattern, designed to find the maximum value within an array using parallel processing.

It calculates the maximum value at the i -th index of the input array and contains the maximum value of the array at the last position.

For example, given the array:

$$[1, 4, 0, 2, 7, 2, 4, 3]$$

It returns:

$$[1, 4, 4, 4, 7, 7, 7, 7]$$

Where indicates that at position 2, the sub-array [1, 4, 0], the maximum value that can be found is 4.

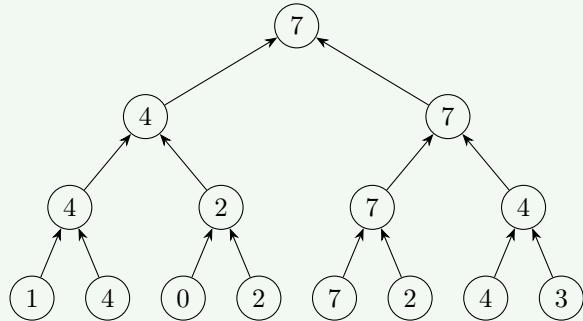
It consists of two main phases: Up-Sweep (Reduce) Phase and Down-Sweep Phase.

❖ Algorithm

1. **Up-Sweep (Reduce) Phase**. Create a balanced binary tree of partial results, focusing on finding the maximum value.
 - (a) **Pairwise Combination**: Begin by combining elements in pairs to find the maximum of each pair.
 - (b) **Level-by-Level Combination**: Continue combining results at each level of the tree until a single result (the maximum value) is obtained.

Example 37: Up-Sweep (Reduce) technique

We follow a tree approach to manually compute the up-sweep. We divide the input into pairs and evaluate each pair with the max operator. The result is the next node. For an array [1, 4, 0, 2, 7, 2, 4, 3]:



The comparison made are:

(a) Level 1:

* Pairs:

- $\max(1, 4)$
- $\max(0, 2)$
- $\max(7, 2)$
- $\max(4, 3)$

* Result: [4, 2, 7, 4]

(b) Level 2:

* Pairs:

- $\max(4, 2)$
- $\max(7, 4)$

* Result: [4, 7]

(c) Level 3:

* Pair: $\max(4, 7)$

* Result: [7]

At this point, the Up-Sweep phase has found the maximum value of the entire array.

2. **Down-Sweep Phase.** Use the partial results from the Up-Sweep phase to **compute the final scan results for all elements in the array**.
 - (a) **Initialization:** Begin with an initial value, typically the **identity element** (for maximum, it could be negative infinity $-\infty$).

- (b) **Propagate Results:** Propagate the maximum values back down the tree, updating the results for each element.

Example 37 (continue): Down-Sweep technique

Continuing from the Up-Sweep phase result [7] and the original array [1, 4, 0, 2, 7, 2, 4, 3] (page 317).

The main rule followed during the down-sweep technique is the following: the left leaf always inserts the value of the parent node, while the right leaf inserts the value calculated after applying the `max` operation to the value of the parent node and the left side just removed (which we find on the up-sweep graph).

On the last level of the tree, there is an exception: before inserting the left leaf directly, the `max` operation of the inherited value and the input value must be performed.

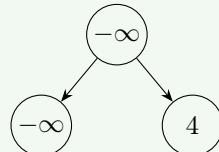
The steps are detailed:

- (a) Replace the maximum value on the root (7) and insert the identity element, in our case the $-\infty$.



- (b) Level 1:

- * Insert the parent node on the left leaf, so $-\infty$.
- * On the right leaf, insert the performed operation between the parent's value and the left leaf of the up-sweep tree. So the values to compare are $-\infty$ and 4: $\max(-\infty, 4) = 4$



(c) Level 2 ((from left to the right)):

* Vertex $-\infty$:

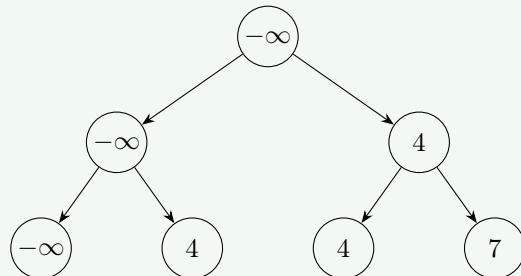
- Insert the parent node on the **left leaf**, so $-\infty$. It's not the last leaf of the tree, so we don't do any checking.
- The **right leaf** comparison is between $-\infty$ (the father's value) and the left leaf on the up-sweep tree (the value just replaced by the $-\infty$), so 4. Again, the comparison is:

$$\max(-\infty, 4) = 4$$

* Vertex 4:

- Insert the parent node on the **left leaf**, so 4.
- The **right leaf** comparison is between 4 (the father's value) and the left leaf on the up-sweep tree (the value just replaced by 4), so 7. The comparison is:

$$\max(4, 7) = 7$$



(d) Level 3 (from left to the right):

* Vertex $-\infty$:

- **Left leaf.** Since this is the last level, we need to compute an extra check. We need to compute:

$$\max(\text{inherited val}, \text{input val}) \quad (16)$$

In our case, the inherited value is the $-\infty$, but the input value on the first position is 1. Therefore:

$$\max(-\infty, 1) = 1$$

- **Right leaf.** We compare the value we just removed, 1, to the value of the parent: $-\infty$.

$$\max(1, -\infty) = 1$$

The result should be 1 BUT we are at the last level of the tree. So we need to do an extra check between the inherited value (what we want to place) and the input value on the second position (our case 4):

$$\max(1, 4) = 4$$

* Vertex 4:

- **Left leaf.** We apply formula 2d on page 319, the idea is the same as before:

$$\max(4, 0) = 4$$

- **Right leaf.** We compare the value just removed, 0, to the value of parent: 4.

$$\max(0, 4) = 4$$

And the value obtained with the input value on the array:

$$\max(4, 2) = 4$$

* Vertex 4:

- **Left leaf.** We apply formula 2d on page 319, the idea is the same as before:

$$\max(4, 7) = 7$$

- **Right leaf.** We compare the value just removed, 7, to the value of parent: 4.

$$\max(7, 4) = 7$$

And the value obtained with the input value on the array:

$$\max(7, 2) = 7$$

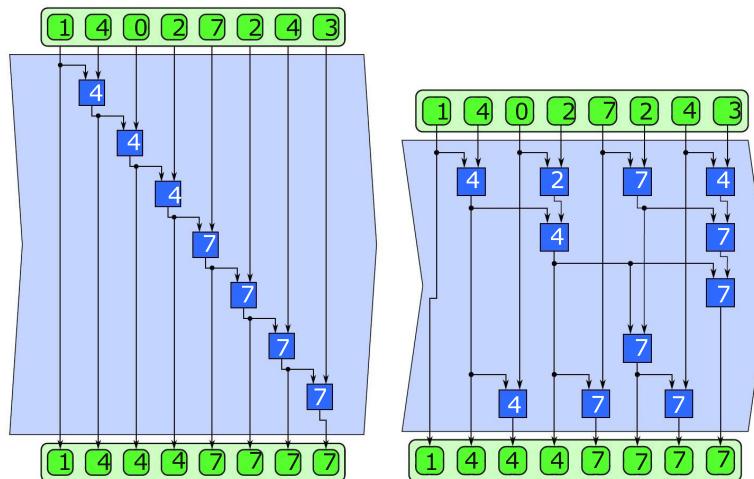
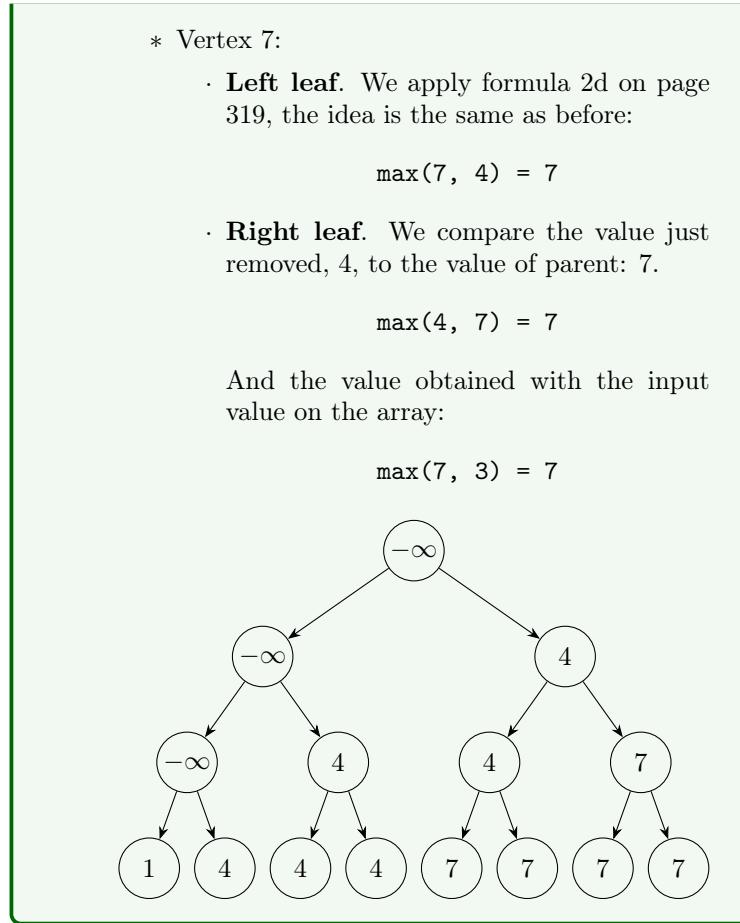


Figure 55: Graphical example of the Maximum Algorithm with Up and Down Sweep. On the left a sequential version and on the right a parallel version. The parallel version was analyzed on the previous pages.

- **Three-Phase Scan with Tiling**

 **Definition.** Three-Phase Scan with Tiling is a technique that divides the input array into smaller tiles and processes them in three distinct phases to efficiently perform the scan operation. The main goal is to improve parallelism and cache efficiency by splitting the input array into smaller tiles and processing them independently.

 **Algorithm**

1. **Tiling and Local Scan.** Divide the input array into smaller tiles and perform the scan operation on each tile independently.
 - (a) **Divide the Input Array:** Break the input array into smaller blocks or tiles.
 - (b) **Local Scan (Inclusive):** Compute the scan for each tile independently.

Example 38: Tiling and Local Scan

For an array [1, 2, 3, 4, 5, 6, 7, 8] with a tile size of 4:

- * Tiles: [1, 2, 3, 4] and [5, 6, 7, 8]
- * Local Scans:
 - First tile: [1, 3, 6, 10]
 - Second tile: [5, 11, 18, 26]

2. **Scan of Tile Results.** Compute the scan of the final elements of each tile to handle dependencies between tiles.
 - (a) **Extract Final Elements:** Take the last element of each tile.
 - (b) **Global Scan:** Perform a scan operation on these final elements to propagate the results across tiles.

Example 39: Scan of Tile Results

- * Final Elements: 10 (from the first tile) and 26 (from the second tile)
- * Global Scan: [10, 36] (assuming addition and identity element 0)

3. **Distribution of Tile Results.** Distribute the results of the scanned tile results to all elements in their respective tiles.
 - (a) **Distribute Results:** Add the scan results of the previous tiles to the elements of the current tile.

Example 40: Distribution of Tile Results

- * First Tile Remains the Same: [1, 3, 6, 10]
 - * Second Tile: add the scan result 10 from the first tile's last element:
 - $5 + 10 = 15$
 - $11 + 10 = 21$
 - $18 + 10 = 28$
 - $26 + 10 = 36$
- [15, 21, 28, 36]

Combining the results of the two tiles, we get:

[1, 3, 6, 10, 15, 21, 28, 36]

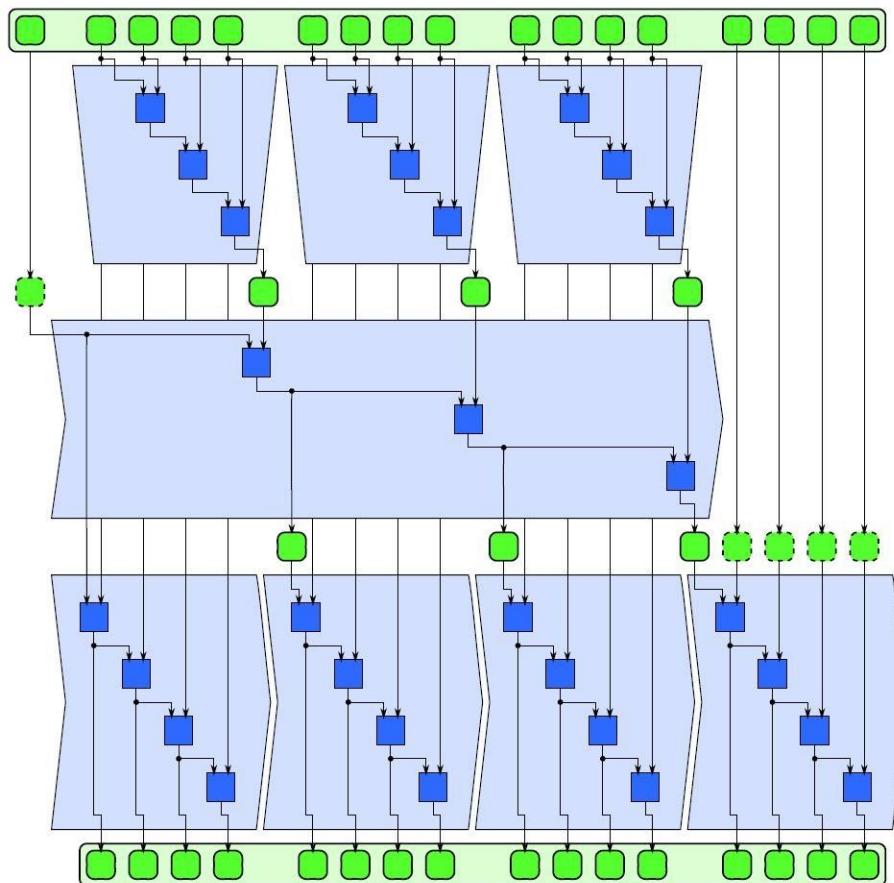


Figure 56: Graphical example of the Three-Phase Scan with Tiling.

- **Fusion of Map Pattern with Scan Pattern**

 **Definition.** Combine the transformation capabilities of the map pattern with the cumulative operations of the scan pattern to **achieve more complex and efficient parallel computations**.

When we fuse the map and scan patterns, we **first apply the map function to transform each element in the input sequence, and then apply the scan operation to the transformed sequence.**

 **Algorithm**

1. **Apply Map Function:** Transform each element in the input sequence using the map function.
2. **Apply Scan Operation:** Perform the scan operation on the transformed sequence to compute cumulative results.

Example 41: Fusion of Map Pattern with Scan Pattern

Let's consider a practical example where we want to compute the prefix sums of the squares of an input array [1, 2, 3, 4].

1. **Map Function**

- (a) Define the map function as $f(x) = x^2$.
- (b) Apply the map function: [1, 4, 9, 16] (squares of the input elements),

2. **Scan Operation**

- (a) Perform an inclusive scan on the transformed sequence [1, 4, 9, 16].
- (b) Inclusive scan result: [1, 5, 14, 30].

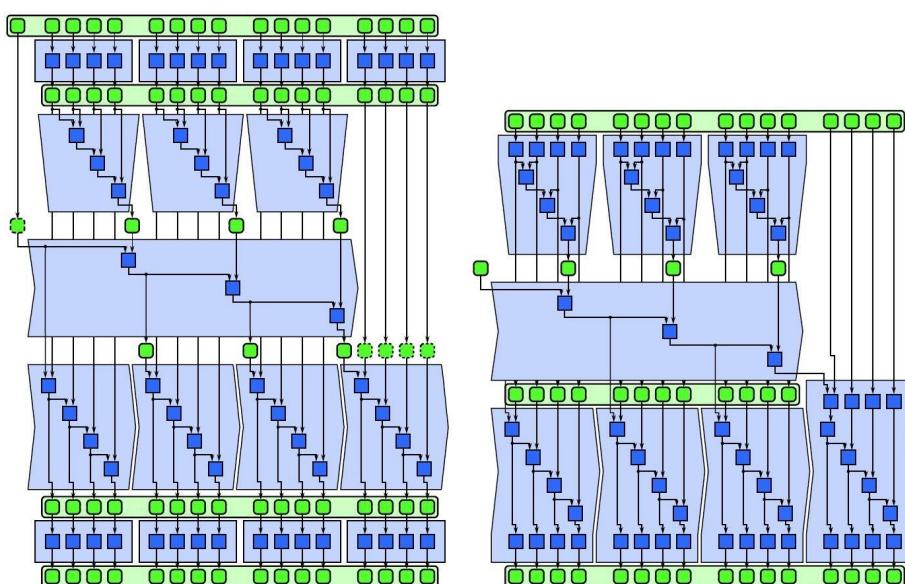


Figure 57: Graphical example of the Fusion of Map Pattern with Scan Pattern.

10.5 Gather Pattern

10.5.1 What is a Gather?

⌚ Why is the need for the gather pattern born?

There are two main reasons for this:

1. Data Movement.

❓ **Problem.** Data movement is often more costly than computation, especially when transferring data across memory layers or networks. This includes factors like:

- Locality optimization. Reducing data access times by keeping frequently accessed data close to the processing unit.
- Efficient data access. Improving performance by minimizing the cost of accessing scattered data locations.

✓ **Solution.** The gather pattern addresses these issues by efficiently collecting data from multiple, scattered memory locations into a contiguous structure.

2. Parallel Data Reorganization.

❓ **Problem.** To achieve performance gains in parallel algorithms, data often needs to be reorganized efficiently. This involves:

- Parallel data movement. Moving large datasets simultaneously rather than sequentially.
- Consistency management. Ensuring data integrity while reorganizing in parallel.
- Intermediate data structures. Holding temporary results during reorganization.

✓ **Solution.** The gather pattern is crucial for parallel data reorganization because it:

- ✓ Enables parallel data movement, allowing multiple data points to be gathered simultaneously from different sources.
- ✓ Maintains data consistency, ensuring that the reorganization process doesn't introduce errors.
- ✓ Supports intermediate structures, making it easier to manage data during complex transformations.

▀ What is a Gather Pattern?

The **Gather Pattern** is a data movement operation that **creates a new (*source*) collection of data by reading elements from an existing (*input*) data collection**. It is commonly used in parallel computing to efficiently **reorganize** or **extract data based on specific indices**. At first it might seem complicated, but with some simple examples it will be very easy.

In general, it works like this:

1. **Read** data from the input collection at positions specified.
2. **Write** the gathered data to the output collection in the same order as the indexes specified in Input.
3. Returns an output collection that has the same shape (or dimensionality) as the index collection.

It can be seen as a **combination** of `map` and `random read` operations because it **performs multiple reads from non-contiguous memory locations**.

☒ Serial Implementation

The following pseudocode shows a serial implementation of the gather pattern:

```

1 template<typename Data, typename Idx>
2 void gather(
3     size_t n,      // number of elements in data collection
4     size_t m,      // number of elements in index collection
5     Data a[],      // input data collection (n elements)
6     Data A[],      // output data collection (m elements)
7     // used to modify the output in place
8     Idx idx[]     // input index collection (m elements)
9 ) {
10    // Iterate over index collection
11    for (size_t i = 0; i < m; ++i) {
12        // Get the i-th index
13        size_t j = idx[i];
14        // Ensure index is within bounds;
15        // It avoids buffer overflow,
16        // and ensure memory safety
17        assert(0 <= j && j < n);
18        // Perform random read and write to output
19        A[i] = a[j];
20    }
21 }
```

The pseudo-signature of the function is:

- Input:
 - `a[]`: original data collection (size `n`).
 - `idx[]`: index collection specifying which elements to gather (size `m`).
- Output:
 - `A[]`: output collection to store gathered elements (size `m`, trivial since we iterate over the input indices `idx`)

The process performs the following operations:

1. **Loop through each index i** in the index collection ($\text{idx}[]$).
2. **Access the data** at position $\text{idx}[i]$ in $\text{a}[]$.
3. **Assertion** ensures that the index is valid (within the bounds of $\text{a}[]$).
4. **Copy the data** from $\text{a}[\text{idx}[i]]$ to the corresponding position $\text{A}[i]$ in the output collection.

It is interesting to note that there are good **opportunities to parallelize** the code thanks to the for loop. In fact, **each iteration is independent** of the others (no data dependencies between iterations). More precisely, **each thread writes to a unique position in the output array** (collection) A . Also, **each position is valid** thanks to the assert boundary check.

Example 42: Gather Pattern

Let the following arguments:

- Source Array, contains the original data elements:

```
A = [A, B, C, D, E, F, G, H]
```

Indices 0 through 7, so the n argument is 8.

- Index Array, specifies which elements from the source array to retrieve:

```
idx = [1, 5, 0, 2, 2, 4]
```

Indices 0 through 5, so the m argument is 6.

- Output Array, will store the gathered elements in the order of the indices idx :

```
B = []
```

Empty initially, will be filled based on idx .

The function calculates:

1. First iteration:
 - (a) Get 0-th index: $\text{idx}[0] = 1$
 - (b) Boundary check? $0 \leq \text{idx}[0] \wedge \text{idx}[0] < 8$ ✓
 - (c) Perform random read: $\text{B}[0] = \text{A}[\text{idx}[0]] = \text{A}[1] = \text{B}$
2. Second iteration:
 - (a) Get 1-th index: $\text{idx}[1] = 5$
 - (b) Boundary check? $0 \leq \text{idx}[1] \wedge \text{idx}[1] < 8$ ✓
 - (c) Perform random read: $\text{B}[1] = \text{A}[\text{idx}[1]] = \text{A}[5] = \text{F}$

3. Third iteration:
- Get 2-th index: $\text{idx}[2] = 0$
 - Boundary check? $0 \leq \text{idx}[2] \wedge \text{idx}[2] < 8$ ✓
 - Perform random read: $B[2] = A[\text{idx}[2]] = A[0] = A$
4. Fourth iteration:
- Get 3-th index: $\text{idx}[3] = 2$
 - Boundary check? $0 \leq \text{idx}[3] \wedge \text{idx}[3] < 8$ ✓
 - Perform random read: $B[3] = A[\text{idx}[3]] = A[2] = C$
5. Fifth iteration:
- Get 4-th index: $\text{idx}[4] = 2$
 - Boundary check? $0 \leq \text{idx}[4] \wedge \text{idx}[4] < 8$ ✓
 - Perform random read: $B[4] = A[\text{idx}[4]] = A[2] = C$
6. Sixth iteration:
- Get 5-th index: $\text{idx}[5] = 4$
 - Boundary check? $0 \leq \text{idx}[5] \wedge \text{idx}[5] < 8$ ✓
 - Perform random read: $B[5] = A[\text{idx}[5]] = A[4] = E$

The returned output collection is:

$$B = [B, F, A, C, C, E]$$

Some interesting observations we can make are:

- **Same dimensionality:** output data collection (array B) has the same number of elements as the number of indexes in the index collection (value m).
- **Same types:** elements of the output collection are of the same type as the input data collection.

10.5.2 Shift

The **Shift operation** is a specialized case of the gather pattern where **data is moved uniformly left or right in memory**. Unlike random gathers (already seen in section 10.5.1, page 326), shifts involve **regular, predictable data movement with fixed offsets**.

★ Key Features

- **Data Movement.** Elements are shifted by a fixed distance.
- **Offset Access.** Data accesses are offset uniformly, making the operation predictable.

▲ Boundary Conditions

With the shift operation, the main problem we can encounter is mainly one: boundary conditions.

When shifting data, **handling the edge elements** (out-of-bounds data) is crucial. There are several strategies:

- ✓ **Default Value.** Fill the vacant position with a default (e.g., 0 or null).

Example 43: Default Boundary Condition

Input array:

```
A = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

Shift with default boundary condition:

- Left shift:

```
A = [2, 3, 4, 5, 6, 7, 8, 9, 10, 0]
```

- Right shift:

```
A = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

- ✓ **Duplicate.** Replicate the edge element to fill the gap.

Example 44: Duplicate Boundary Condition

Input array:

```
A = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

Shift with duplicate boundary condition:

- Left shift:

```
A = [2, 3, 4, 5, 6, 7, 8, 9, 10, 10]
```

- Right shift:

```
A = [1, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

- ✓ **Rotate**. Wrap around the array, moving the last element to the first position (circular shift).

Example 45: Rotate Boundary Condition

Input array:

```
A = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

Shift with rotate boundary condition:

- Left shift:

```
A = [2, 3, 4, 5, 6, 7, 8, 9, 10, 1]
```

- Right shift:

```
A = [10, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

💡 Advantages & Benefits

- Shifts are highly efficient with **vector instructions** due to their regularity.
- They allow **parallel shifting** of multiple elements simultaneously.
- Good **data locality** enhances performance, as adjacent elements are accessed together.

10.5.3 Zip

The **Zip operation** is a special case of the gather pattern where **two (or more)** arrays are **combined by interleaving their elements**. It functions like a zipper, **taking one element from each array in sequence to form a new combined array**. It is important to note that it **works with different types**, so we can zip elements of different types, like integers and floats, or even complex objects.

❖ How does it work?

The operation **takes an element from the first array**, then **one from the second array**, another from the third, and so on, and **repeats the process**. The **output is the combined sequence**.

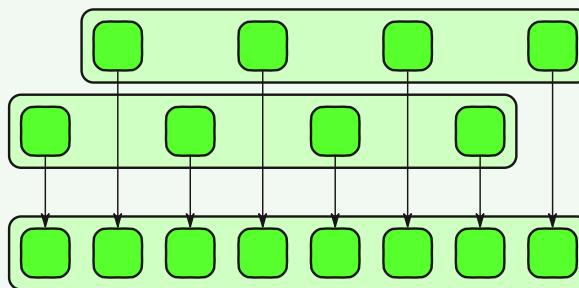
Example 46: zip operation

Consider two arrays:

1. Real Parts: contains real numbers.
2. Imaginary Parts: contains imaginary numbers.

The output is a combined sequence like:

`[Real0, Imag0, Real1, Imag1, Real2, Imag2, ...]`



❖ Parallelism

Each pair of elements (one from each array) can be **combined independently**. This independence allows **parallel execution** since there's no dependency between the operations for different pairs.

10.5.4 Unzip

The **Unzip operation** is essentially the **reverse of the zip operation**. While `zip` combines multiple arrays into one by interleaving their elements, `unzip` separates a single interleaved array back into its original components.

💡 How does it work?

The operation **extracts sub-arrays at regular offsets (strides) to separate the elements into their original groups**. The input is a combined sequence (e.g., alternating real and imaginary parts).

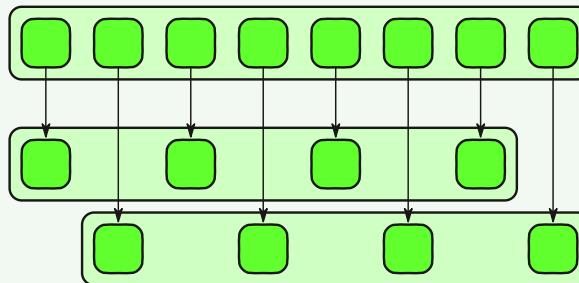
Example 47: unzip operation

Consider the input array:

```
[Real0, Imag0, Real1, Imag1, Real2, Imag2, ...]
```

The output are:

- Array of Real Parts: [Real0, Real1, Real2]
- Array of Imaginary Parts: [Imag0, Imag1, Imag2]



⚡ Parallelism

Each element extraction is independent. This allows for **parallel data access** because there's no dependency between different extractions.

However, the `unzip` operation is an **efficient data extraction because it takes advantage of stride-based memory access**, which can be optimized for performance.

10.6 Scatter Pattern

10.6.1 What is a Scatter?

The **Scatter Pattern** is a data movement pattern where **elements from a source array are distributed (or scattered) to various locations in a destination array based on specified indices**. Essentially, it's about writing data to random locations.

★ Key Features

In general, the **input** of the scatter pattern is:

- **Source data array**: value to be written.
- **Index array**: specifies where each value should be written in the destination.

Each element from the source is written to the position in the target array specified by the corresponding index. The **output is a target array where the data is scattered across the positions**.

⚠ Gather vs. Scatter

The main differences between gather and scatter are:

- **Gather** (read focused): Involves **random reads** where the **read locations are provided as input**.
- **Scatter** (write focused): Involves **random writes** with **write locations provided as input**, which can lead to race conditions.

☒ Serial implementation

The following pseudocode shows a serial implementation of the scatter pattern:

```

1 template<typename Data, typename Idx>
2 void scatter(
3     size_t n,           // Number of elements
4                     // in the output data collection
5     size_t m,           // Number of elements
6                     // in the input data and index collection
7     Data a[],          // Input data collection (m elements)
8     Data A[],          // Output data collection (n elements)
9     Idx idx[]          // Input index collection (m elements)
10 ) {
11     // Loop through each input element
12     for (size_t i = 0; i < m; ++i) {
13         // Get the target index from the index array
14         size_t j = idx[i];
15         // Ensure the index is within output array bounds
16         assert(0 <= j && j < n);
17         // Perform the scatter: write a[i] to A[j]
18         A[j] = a[i];
19     }
20 }
```

The method is characterized by:

- **Loop (for)**: Iterates through each element in the input array $a[]$ (length m).
- **Indexing ($j = \text{idx}[i]$)**: Determines **where to place $a[i]$** in the output array $A[]$.
- **Boundary Check (assert)**: Ensures the index j is **valid** (avoids out-of-bounds errors).
- **Write Operation ($A[j] = a[i]$)**: **Scatters** the data from $a[i]$ to $A[j]$.

The possibilities to parallelize the code are interesting, but instead of the gather pattern, here we have a problem with race conditions on write operations. So the code can be parallelized (**parallelize over for loop to perform random write**), but we **need to find some strategies to avoid race conditions**.

⚠ Race Conditions in Scatter

Unfortunately, the scatter pattern can lead to race conditions because of the way it handles writes to memory. As we know, a race condition occurs when two or more operations try to access and modify the same data at the same time, and the final result depends on the order in which the operations are performed.

In the scatter pattern, we're performing random writes to different locations in memory. The **problem arises when multiple write operations target the same location at the same time**. Because the **writes happen in parallel**, we can't predict:

1. Which write will happen first
2. Which value will be stored last

Example 48: Race Condition in Scatter

Consider this:

- Source Data:

$$A = [A, B, C, D, E, F]$$
- Index Array (write locations):

$$\text{idx} = [1, 5, 0, 2, 2, 4]$$

This means that the two threads that will handle the 3rd and 4th positions on the index array will suffer from a race condition. Because if they try to write at the same time, the result will depend on which thread finishes last:

- If thread A (writing D) finishes last, position 2 will be equal to D.
- If thread B (writing E) finishes last, position 2 will be equal to E.

2. Optimizing Performance: Converting Scatter to Gather

Scatter operations are generally more expensive than gather operations because of how data is written to memory:

- ✗ **Cache Line Issues.** Writing data to memory can cause **cache conflicts**, making memory access slower.
- ✗ **False Sharing Problem.** If **multiple CPU cores write to different parts of the same cache line**, they interfere with each other, slowing down performance.

So we can switch to gather pattern to improve performance. Unfortunately, this speedup can **only be achieved if we know the memory addresses in advance**.

⌚ So it is only a low-level knowledge? Let's get this straight.

✓ When we know the memory addresses in advance

- It is a scenario where the scatter operation follows a **fixed, predictable pattern**; or the indices where data will be written are statically determined (at compile time) or precomputed before execution. Also, if the same scattering pattern is reused multiple times, precomputing the addresses is worthwhile.

Example 49: when we know the memory addresses in advance

Suppose we're implementing matrix transposition, and each element moves to a fixed new location. The scatter indices can be precomputed once, and instead of writing directly, we gather the elements and write them sequentially.

✗ When we don't know the memory addresses in advance

- It is a scenario where the **target indexes depend on runtime data** (e.g., calculation results or user input); or the distribution pattern is dynamic and unpredictable, changing with each execution. Also, when data distribution is affected by real-time conditions, such as load balancing in parallel computing.

Example 50: when we don't know the memory addresses in advance

Suppose we are sorting numbers in parallel using a scatter operation, but the final positions depend on comparisons performed at runtime. Since the scatter addresses are computed on the fly, we can't precompute an equivalent gather operation.

So **if the pattern is known in advance, we can convert scatter to gather to improve performance**. But if the **pattern is dynamic**, we have to deal with unpredictable memory writes, leading to potential performance **problems like cache contention and atomic conflicts**.

10.6.2 Avoid race conditions

10.6.2.1 Atomic Scatter

Atomic Scatter is a **collision resolution strategy** used when multiple threads attempt to write to the same memory location simultaneously. It is based on one of the most common and famous topic: **atomic operations**.

❷ How does it work?

✓ **Atomic Writes.** Each write is **atomic**, meaning it either **completes fully or doesn't happen at all**. No partial writes.

✗ **Non-Deterministic Outcome.** Since it relies solely on atomic writes, this doesn't mean that there is a mechanism to check which thread should write before another. When a collision occurs, the outcome depends on which write completes first. There's no predefined rule for determining which value to store.

Example 51: Atomic Scatter

Consider this:

- Source Data:

$$A = [A, B, C, D, E, F]$$

- Index Array (write locations):

$$idx = [1, 5, 0, 2, 2, 4]$$

Notice that each thread is assigned to every position of the `idx` array. This means that threads #4 and #5 must write to the same position on the output array. However, we adopt the atomic strategies so that what happens depends on the order of writing.

A possible scenario:

1. Thread #4 asks to do an atomic write to position 2 of the output array. The operation is marked atomic and can be done safely. The value written to `A[2]` is D.
2. Then thread #5 performs the same steps and overwrites the value written by thread #4 and writes the value E to `A[2]`.

This is non-deterministic, so this is a possible scenario, but it could also be the other way around and the final value could be D.

✓ Advantages

- ✓ **Simple to implement.** No need for complex locking mechanisms.
- ✓ **Fast.** Atomic operations are optimized in hardware, making them efficient for parallel execution.

✗ Disadvantages

- ✗ **Unpredictable results.** Non-determinism can be problematic for algorithms that require consistent outputs.
- ✗ **Data loss.** One of the colliding values will be lost without any mechanism to combine or preserve both.

② When to use Atomic Scatter?

Atomic scatter is suitable when:

- **Collisions are rare:** so the occasional data loss or non-determinism is acceptable.
- **We don't care which value is kept:** for example, in random sampling or approximate algorithms.

10.6.2.2 Permutation Scatter

Permutation Scatter is a special type of scatter pattern where **collisions are strictly prohibited**. This means that **every input element must be placed in a unique output position**, ensuring that the output is a permutation (reordering) of the input.

★ Key Features

- **Collisions are illegal.** Unlike atomic scatter, where collisions are resolved non-deterministically, **permutation scatter ensures that no two elements try to write to the same location.**
- **Output is a permutation of the input.** The output array consists of the same elements as the input but in a different order, **without any duplicates or missing values.**
- **Collision checking in advance.** To **guarantee uniqueness**, a **pre-check must be done** to detect any duplicate indices **before executing the scatter operation.**

❓ What if there is a collision during the pre-check?

If collisions exist, scatter cannot proceed as-is and may **need to be transformed into a gather operation** instead.

❓ Why convert a scatter to a gather?

Instead of letting multiple elements write to the same index in parallel (scatter), we can:

1. Detect collisions beforehand by checking for duplicate indices in the index array.
2. Reformulate the operation as a gather, where **each output index reads from a unique input location**, instead of multiple inputs writing to the same index.
3. Ensure correctness by enforcing a one-to-one mapping between input and output locations.

So it is clear that the **gather ensures that each output location reads from only one input, eliminating write conflicts.**

Example 52: Convert Scatter to Gather

Image we have an input array and an index array:

- Input: `A = [A, B, C, D]`
- Index: `idx = [1, 3, 3, 2]`

Here we have a conflict at index 3 because both B and C want to write here. Therefore, the pre-check reports this conflict; the operation has been marked as undefined behavior; we convert the operation from a scatter to a gather.

Now the approaches we can make are two: we **convert the whole operation** into a gather and we do that in parallel (to achieve high performance); or the **position where the permutation cannot guarantee the atomic write, are passed to a gather function to guarantee atomicity**.

The implementation depends on the programmer and the behavior we want. What we need to know is that: **if a conflict is detected during permutation, the conversion to gather is the best choice that we can make.**

Permutation scatter requires a one-to-one mapping between input and output indices. If collisions exist, scatter must be turned into gather so that instead of multiple inputs writing to the same location, each output position retrieves its correct value without conflicts.

⚠️ Permutation vs. Atomic Scatter

- **Collision**

- **Atomic**: Allowed, resolved arbitrarily.
- **Permutation**: Not allowed.

- **Write conflicts**

- **Atomic**: Handled using atomic operations.
- **Permutation**: Avoided by pre-checking

- **Output structure**

- **Atomic**: Can overwrite values.
- **Permutation**: Always a permutation of input.

- **Use Cases**

- **Atomic**: Hash tables, parallel memory writes.
- **Permutation**: Matrix transposition, FFT scrambling.

10.6.2.3 Merge Scatter

The **Merge Scatter** strategy is a way to resolve write conflicts in a scatter operation by applying a merge operator that combines values instead of discarding or randomly selecting one.

❖ How does it work?

Instead of letting one thread overwrite another's data, Merge Scatter applies a merge function to combine values when a conflict occurs. The function must be:

- **Associative**, order of grouping doesn't change result:

$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

- **Commutative**, order of values doesn't change result:

$$a \oplus b = b \oplus a$$

💡 Why these properties? Because parallel execution means that scatter operations happen in arbitrary order, so the merge result must be independent of the order in which values arrive. In other words, it guarantees deterministic behavior (because we always know what the result will be).

Example 53: Merge Scatter

Suppose we have the following data:

Input	[2, 3, 1, 1, 5, 6]
Indices	[1, 5, 0, 2, 2, 4]

Positions 3 and 4 have multiple writes, causing collisions. Therefore, we use the merge scatter strategy and we define the merge function as: addition.

So instead of overwriting, we sum the values in case of a conflict:

$$[1, 2, 6, _, 6, 3]$$

At the second position, the index where there was a collision, the merge function was applied between the value 1 and 5 ($1 + 5 = 6$).

✓ Advantages

- ✓ **Preserves Data.** Unlike atomic scatter (where some values are lost), merge scatter ensures all data contributes to the final output.
- ✓ **Parallel-Friendly.** Works well in parallel settings since order doesn't affect the result.

- ✓ **Useful in some scenario:** histogram computation, sparse matrix operations, parallel reductions.

✖ Disadvantages

✖ **Increased Computational Overhead.** Instead of a simple write operation, we now have to perform a **merge operation** (e.g., addition, `max`, `min`). This adds **extra computational cost**, especially if the merge function is complex. Also, some architectures may require atomic operations or locks to ensure safe merging, which can slow down execution.

In summary, the **more conflicts there are, the more overhead is introduced by the merge function.**

✖ **Not Always Deterministic.** If the merge function is not associative and commutative, the result can depend on the order of execution, which is unpredictable in parallel settings.

Note that this is very common in real-world scenarios; for **example**, when using floating-point addition, different execution orders can result in small numerical inconsistencies due to floating-point precision issues.

✖ **Possible Loss of Information.** If the merge function is not a lossless operation, some original data might be lost.

Example 54: Loss of Information

For example, if the merge function is `max()`, only the largest value survives, and all others are discarded. This means we cannot reconstruct the original inputs from the output.

✖ **Limited Applicability.** Merge Scatter is only applicable when a meaningful merge function exists. Some applications require exact values, and merging (e.g., summing or taking max) may not be acceptable.

Example 55: Limited Applicability

For example, if each thread writes a different pixel color in an image processing task, merging colors with sum/max may produce unintended artifacts.

So we can summarize when we need to avoid merge scatter:

- ✖ When we need **exact values** without modification.
- ✖ When the merge function is **not associative or commutative**.
- ✖ When merging **significantly increases computational complexity**.

10.6.2.4 Priority Scatter

Priority Scatter is a collision resolution strategy where **each element in the input array is assigned a priority value** based on its position (or another criterion). When multiple threads attempt to write to the same memory location, the **element with the highest priority wins**, and the others are discarded.

★ Key Features & Advantages

- **Predefined Priority System.** Each element has a priority, which could be based on position, timestamp, depth, or another factor.
- ✓ **Deterministic Resolution.** Unlike Atomic Scatter, where race conditions make the final result unpredictable, Priority Scatter ensures a well-defined winner.
- ✓ **Common in 3D Graphics.** In 3D rendering, when multiple objects overlap at the same pixel, the object closest to the camera (highest priority) is displayed.

✖ Disadvantages

- ✖ **Loss of Information.** Lower-priority elements are discarded entirely.
- ✖ **Predefined Priority Needed.** Requires an external ranking mechanism.
- ✖ **Not Always Fair.** Some data might consistently dominate, suppressing important contributions.

⚠ Comparison Atomic vs. Merge vs. Priority Scatter

Strategy	Resolution Mechanism	Result Type
Atomic	First write wins (random)	Non-deterministic
Merge	Merges values (e.g. sum/max)	Aggregated
Priority	Highest-priority wins	Deterministic

Table 8: Differences between Scatter strategies.

10.7 Pack Pattern

10.7.1 What is a Pack?

The **Pack Pattern** is used to remove unused elements from a collection while keeping the retained elements contiguous in memory. The goal is to improve performance by reducing memory fragmentation and increasing data locality, which benefits cache efficiency.

❖ How the Pack Algorithm works

1. Convert the input array into Boolean values. Each element is mapped to 1 (*keep*) or 0 (*discard*).
2. Perform an exclusive scan (prefix sum) on the Boolean array. This step computes the new positions (offsets) of the retained elements.
3. Use the computed offsets to move retained elements to their new locations. The elements corresponding to 1's in the Boolean array are copied to their new positions.

Example 56: Pack Pattern

We have an input array and a mask that tells us which elements to keep (1) and which to remove (0):

```
Index   = [0, 1, 2, 3, 4, 5, 6, 7]
Input   = [A, B, C, D, E, F, G, H]
Keep (Mask) = [0, 1, 1, 0, 0, 1, 1, 1]
```

We want to create a new array with only the elements marked 1, but we don't yet know *where* to place them.

The exclusive scan (prefix sum) on the `Keep` array calculates the new indices for the elements we want to keep.

We compute the scan using addition (starting from 0):

```
Index   = [0, 1, 2, 3, 4, 5, 6, 7]
Keep (Mask) = [0, 1, 1, 0, 0, 1, 1, 1]
Exclusive Scan (New Index) = [0, 0, 1, 2, 2, 2, 3, 4]
```

The scan tells us:

- B goes to index 0
- C goes to index 1
- F goes to index 2
- G goes to index 3
- H goes to index 4

[B, C, F, G, H]

💡 Why is Packing Useful?

- Reduces **memory waste** by eliminating unnecessary elements.
- Improves **data locality** (sequential memory access is faster than scattered access).
- Boosts **cache efficiency**, reducing cache misses and improving parallel performance.

💡 What is the inverse of the Pack pattern?

The **Unpack Pattern** is the inverse of packing. It **restores the packed elements to their original positions** in an expanded array, reintroducing gaps where necessary.

🛠 How Unpacking Works

Given the same Boolean mask used in packing, unpacking spreads elements back to their original locations, **leaving empty slots where elements were previously removed**.

1. Start with the packed array and the original mask.
2. Read the mask to decide where elements should go:
 - If 1, we take the next packed element.
 - If 0, we insert a placeholder (empty slot).
3. Reconstruct the original array.

Example 57: Unpack Pattern

We have:

1. The packed array: [B, C, F, G, H]
2. The original mask: [0, 1, 1, 0, 0, 1, 1, 1]

Now we will restore elements using the mask:

- If **Keep (Mask) = 1**, we copy the next element from the packed array into that position.
- If **Keep (Mask) = 0**, we insert an empty slot (or default value).

Finally, we place elements back into their original positions using the mask:

Index = [0, 1, 2, 3, 4, 5, 6, 7]

Restored Value = [_, B, C, _, _, F, G, H]

🔗 Fusion of Map and Pack: Efficient Filtering of Results

The **fusion of map and pack patterns** is an optimization technique that improves efficiency when many elements in a map operation are discarded (see map pattern on page 298)

The key idea is that **Map pattern applies a function to each element** (e.g., checking for collisions) and **Pack pattern removes unnecessary elements**, storing only meaningful results.

This fusion ensures that **only relevant results are stored and transmitted**, reducing unnecessary computations and memory usage.

1. **Map checks for collisions.** Each element is processed using a mapping function, which determines whether the element should be kept or discarded. Some elements fail the check.
2. **Pack stores only actual collisions.** Only elements that passed the check (e.g., B, C, F, G, H) are stored in the output. Elements that failed the check (e.g., A, D, E) are removed. The output contains only relevant elements, avoiding unnecessary storage.
3. **Reduces output bandwidth.** The output size depends on valid results, not on the total number of inputs. This is **beneficial when most of the input elements are discarded**.
4. **Selective output.** Unlike map-only operations, where each input produces an output, **this fusion allows selective output**.

10.7.2 Split

The **Split operation** is an extension of the Pack pattern, but instead of removing elements, it **reorganizes them into exactly two groups while preserving all data** (If there are more than two groups, it is a bin operation, see page 349).

❖ How does Split work?

1. Each element has a corresponding state (0 or 1).
2. Instead of discarding elements, as in the normal Pack operation, elements are **moved into two separate regions** in the output array.
3. Elements with one state (e.g., 0) go to the first half, while elements with the other state (e.g., 1) go to the second half.

Example 58: Split operation

We have:

```
Input elements = [A, B, C, D, E, F, G, H]
Binary Condition (Mask) = [1, 0, 0, 1, 1, 0, 0, 0]
```

What we do is:

1. Classify elements:
 - Elements corresponding to 0: [B, C, F, G, H]
 - Elements corresponding to 1: [A, D, E]
2. Place Elements in two groups:
 - Upper half (0s first): [B, C, F, G, H]
 - Lower half (1s second): [A, D, E]

And the result is:

```
[B, C, F, G, H, A, D, E]
```

⚠ Differences between Pack and Split

Feature	Pack	Split
Removes elements?	✓	✗
Reorganizes elements?	✓	✓
Maintains all data?	✗ (discards elements)	✓ (all data kept)

10.7.3 Unsplit

The **Unsplit operation** is the inverse of Split. Instead of separating elements into two groups, it **reconstructs the original ordering from the split version**.

❖ How does Split work?

1. Start with the output formed by Split.
2. Use the original mask (0s and 1s) to determine where elements should go.
3. Reconstruct the original sequence by placing elements back in their original positions.

Example 59: Unsplit operation

We have:

```
Output from Split = [B, C, F, G, H, A, D, E]
Binary Condition (Mask) = [1, 0, 0, 1, 1, 0, 0, 0]
```

What we do is:

1. Place elements back according to the mask:
 - First position (1 in the mask) → Take from Lower half: A
 - Next (0 in the mask) → Take from Upper half: B
 - Next (0 in the mask) → Take from Upper half: C
 - Next (1 in the mask) → Take from Lower half: D
 - Next (1 in the mask) → Take from Lower half: E
 - Next (1 in the mask) → Take from Lower half: F
 - Next (0 in the mask) → Take from Upper half: G
 - Next (0 in the mask) → Take from Upper half: H

And the result is (original order restored):

```
[A, B, C, D, E, F, G, H]
```

⚠ Differences between Split and Unsplit

Feature	Split	Unsplit
Operation type	Reorders into two groups	Reconstructs original order
Loses information?	✗	✗
Involves a mask?	✓	✓
Data organization	Elements grouped	Elements placed back

10.7.4 Bin

The **Bin operation** is a generalization of the **Split operation**, where elements are categorized into **more than two groups (bins)** instead of just two.

❖ How does Split work?

1. Each element is assigned a category (bin number).
2. Elements are grouped based on their bin numbers.
3. The final arrangement collects all elements belonging to the same bin together.

Example 60: Bin operation

We have:

$$\begin{aligned} \text{Elements} &= [\text{A, B, C, D, E, F, G, H}] \\ \text{Mask (Categories/Bins)} &= [2, 0, 1, 3, 3, 1, 1, 2] \end{aligned}$$

What we do is:

1. Identify the Unique Bins: the mask contains four different categories (0, 1, 2, 3), so we create 4 bins.
2. Distribute Elements into Bins:
 - Bin (category) 0 → Elements Assigned: B
 - Bin (category) 1 → Elements Assigned: C, F, G
 - Bin (category) 2 → Elements Assigned: A, H
 - Bin (category) 3 → Elements Assigned: D, E
3. Arrange output by Bins: elements are grouped together in bin order:

$$[\text{B, C, F, G, A, H, D, E}]$$

⚠ Differences between Split and Bin

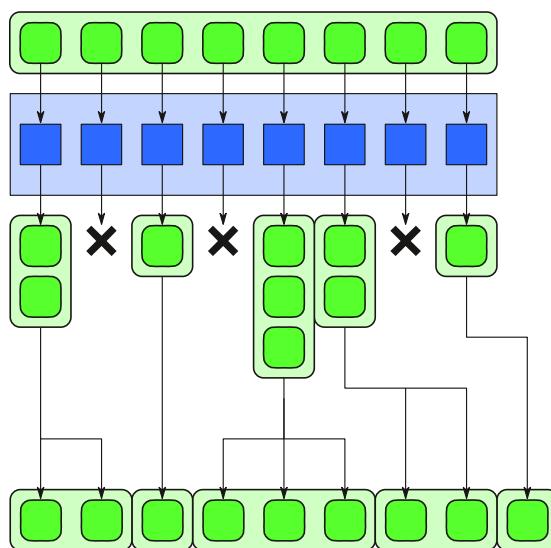
Feature	Split (Binary)	Bin (Generalized Split)
Categories	Only 2	Greater than 2 (minimum 3)
Grouping	Two groups	Multiple bins

10.7.5 Expand

The **Expand pattern** is a flexible generalization of the Pack pattern where each input element can produce multiple output elements, instead of at most one. In other words, it dynamically increases the number of outputs per input. It is widely used in parallel processing to efficiently handle variable-length results.

★ Key Idea

1. Unlike Pack, which filters elements, Expand **duplicates or expands elements**.
2. **Each element can generate any number of outputs** (zero, one, or multiple).
3. Useful when processing elements **produces variable-length results**.



❓ Why is this useful?

- **Adaptive Computation:** Expands based on conditions, useful in graph algorithms and database queries.
- **Efficient Storage:** Dynamically handles variable-length outputs without wasting memory.
- **Parallel Processing:** Enables dynamic parallel work distribution, where different inputs produce varying workloads.

10.8 Partitioning Data

Partitioning is a strategy used to parallelize computations by dividing data into independent, non-overlapping sections. This allows different processing units (threads or cores) to work independently, avoiding conflicts and improving performance.

❖ How does partitioning work?

1. **Divide the Computational Domain.** Data is divided into non-overlapping, equal-sized regions.

Importance of Non-Overlapping Regions: Prevents race conditions (simultaneous writes to the same memory); avoids unnecessary synchronization overhead.

2. **Work on Sections Individually (Parallel Execution).** Each section is processed independently by a separate thread or core. Synchronization may be needed if sections share dependencies. Techniques include:

- Divide-and-Conquer (e.g., Merge Sort)
- Fork-Join Model (parallel task execution)
- Map-Reduce (parallel data processing)

3. **Combine the Results.** The computed results from each section are merged into a final output. Strategies depend on the problem:

- Reduction → Summing partial results (e.g., parallel prefix sum).
- Gathering → Collecting elements into a final array.
- Merging → Combining sorted sections (e.g., parallel merge sort).

To summarize:

1. **Divide:** Break the data into chunks.
2. **Process:** Compute in parallel, ensuring minimal conflicts.
3. **Combine:** Merge results efficiently.

❷ Why is Partitioning important?

- ✓ **Efficient Load Balancing:** ensures each processing unit gets an equal workload.
- ✓ **Avoids Conflicts:** prevents threads from accessing the same memory location.
- ✓ **Scalability:** allows parallel execution without excessive synchronization overhead.

10.9 AoS vs. SoA

This section explain two major ways of organizing data in memory for parallel processing and optimization: AoS and SoA.

Array of Structures (AoS)

Array of Structures (AoS) is a data structure where **each element in the array is a structure (object) containing multiple fields**.

Example 61: C/C++ AoS Representation

```

1 struct Particle {
2     float x, y, z;      // Position
3     float vx, vy, vz;   // Velocity
4 };
5
6 Particle particles[N]; // Array of structures

```

Pros

- ✓ **Better cache utilization** when data is accessed **randomly** (good for scattered reads).
- ✓ **Easier to manage** as a single object-oriented entity.

Cons

- ✗ **Less efficient for vectorization** (SIMD operations).
- ✗ **More cache misses** if accessing only one field (e.g., reading all x-coordinates).

Table 9: Pros and cons of AoS.

Structure of Arrays (SoA)

Structure of Arrays (SoA) is a data structure where **each field of the structure is stored in separate arrays** instead of combined in a single structure.

Example 62: C/C++ SoA Representation

```

1 struct Particles {
2     float x[N], y[N], z[N];      // Position arrays
3     float vx[N], vy[N], vz[N];   // Velocity arrays
4 };
5
6 Particles particles; // Structure of arrays

```

✓ Pros	✗ Cons
<ul style="list-style-type: none"> ✓ Better for vectorization and SIMD operations (can process multiple x-coordinates at once). ✓ Avoids false sharing, when multiple threads modify different fields of the same structure. 	<ul style="list-style-type: none"> ✗ Harder to manage if working with complex structures. ✗ Random access may be a bit slower due to separate memory locations.

Table 10: Pros and cons of SoA.

📘 Layout Variations

The following data layouts are **different memory representations of the same structure**, but they differ in how padding is handled in both AoS and SoA.

More specifically, each variant represents a way of organizing and aligning data in memory that can affect performance, vectorization, and cache efficiency.

1. **Array of Structures (AoS) with Padding at the End.** All structures are stored sequentially in memory. Extra padding is added at the end of the array to ensure proper memory alignment. Padding at the end helps to avoid cache conflicts when the array is processed.

Example 63: AoS with Padding at the End

```

1 struct Particle {
2     float x, y, z, vx, vy, vz; // 6 floats
3 };
4 Particle particles[N];      // AoS layout

```

Where is the padding? If `sizeof(Particle)` is not a multiple of the memory alignment requirement (e.g., 16 or 32 bytes for SIMD), then the compiler may add extra padding at the end of the array to ensure efficient access.

Array of Structures (AoS), padding at end



2. **Array of Structures (AoS) with Padding After Each Structure.** Padding is inserted inside each structure, not just at the end. Helps align each structure according to CPU memory constraints. Ensures that each structure is memory-aligned, avoiding performance penalties in SIMD/vectorized processing.

Example 64: AoS with Padding After Each Structure

```

1 struct Particle {
2     float x, y, z;           // 3 floats
3     float vx, vy, vz;       // 3 floats
4     float padding[2];        // Ensuring proper memory
5     alignment
6 };
7 Particle particles[N]; // AoS layout with padding

```

Array of Structures (AoS), padding after each structure



3. **Structure of Arrays (SoA) with Padding at the End.** All fields are stored in separate arrays. Extra padding is added at the end of each array. Ensures memory alignment for vectorized processing, and padding prevents cache conflicts when working with large datasets.

Example 65

```

1 struct Particles {
2     float x[N], y[N], z[N];    // Position arrays
3     float vx[N], vy[N], vz[N]; // Velocity arrays
4     float padding[N];         // Added padding for
5     alignment
6 };

```

Structure of Arrays (SoA), padding at end



4. **Structure of Arrays (SoA) with Padding After Each Component.** Padding is added between each component, not just at the end. Useful for architectures that require strict memory alignment per field. Prevents memory access penalties when CPU requires aligned data loads and can improve performance in SIMD/vectorized operations.

Example 66

```

1 struct Particles {
2     float x[N];             // Position x
3     float padding_x[N];    // Padding
4     float y[N];             // Position y
5     float padding_y[N];    // Padding
6     float z[N];             // Position z
7     float padding_z[N];    // Padding
8 };

```

Structure of Arrays (SoA), padding after each component



When to use AoS or SoA?

- Use **AoS** when working with **random access patterns** (or small objects).
- Use **SoA** when performing **vectorized computations** or **parallel processing over large datasets**.

Example 67: AoS implementation

Here is an example of an AoS implementation:

```

1 struct node {
2     float x, y, z;
3 };
4
5 struct node NODES[1024];
6
7 float dist[1024];
8
9 for(int i = 0; i < 1024; i += 16) {
10     float x[16], y[16], z[16], d[16];
11
12     x[:] = NODES[i:16].x;
13     y[:] = NODES[i:16].y;
14     z[:] = NODES[i:16].z;
15
16     d[:] = sqrtf(x[:] * x[:] + y[:] * y[:] + z[:] * z[:]);
17
18     dist[i:16] = d[:];
19 }
```

✓ **Most logical representation.** Data is stored as an array of structures, making it **intuitive to access entire objects at once**.

✗ **Bad for vectorization.** Each structure contains multiple fields (x, y, z), so when processing multiple elements, the **data for different fields is scattered in memory**.

✗ **Poor cache efficiency.** SIMD vector operations **prefer contiguous memory access**, but here, x, y , and z are interleaved across structures, requiring scatter/gather operations, which are expensive.

✗ **Vectorization** engines (e.g., AVX) struggle because accessing x, y , and z together **does not align well with SIMD memory patterns**.

Example 68: SoA implementation

Here is an example of a SoA implementation:

```

1 struct node1 {
2     float x[1024], y[1024], z[1024];
3 };
4
5 struct node1 NODES1;
6
7 float dist[1024];
8
9 for(int i = 0; i < 1024; i += 16) {
10     float x[16], y[16], z[16], d[16];
11
12     x[:] = NODES1.x[i:16];
13     y[:] = NODES1.y[i:16];
14     z[:] = NODES1.z[i:16];
15
16     d[:] = sqrtf(x[:] * x[:] + y[:] * y[:] + z[:] * z[:]);
17
18     dist[i:16] = d[:];
19 }
```

- ✓ **Each field has its own contiguous array.** Instead of structuring data as objects, we structure it as separate arrays for each attribute ($x[]$, $y[]$, $z[]$).
- ✓ **Much better for vectorization.** When **applying SIMD operations**, the processor can efficiently load 16 (or more) values of $x[]$, $y[]$, and $z[]$ in one go, **avoiding scatter/gather penalties**.
- ✓ **Cache-friendly.** Since all x -values are contiguous, prefetching and memory locality are improved, **reducing cache misses**.
- ✓ **HPC.** This layout is widely used in high-performance computing, graphics (e.g., GPUs), and physics simulations because it **maximizes vectorization efficiency**.

In summary, **AoS** is easier to understand, but **SoA** is better for performance-critical applications due to cache efficiency and vectorization.

10.10 Stencil Pattern

10.10.1 What is a Stencil?

A **Stencil Pattern** is a computational pattern used in parallel computing where each point in a grid (or data structure) is updated based on a fixed set of neighboring points. It's commonly used in scientific computing, numerical simulations, and image processing.

❖ How the Stencil works

1. **Define a Grid.** The data is usually structured in a 1D, 2D, or 3D grid (e.g., an image, matrix, or mesh).
2. **Select a Stencil shape.** We choose a pattern defining how each point depends on its neighbors. Common shapes are:
 - 1D: [-1, 0, +1] (left, center, right).
 - 2D: + (cross) or x (diagonal) patterns.
 - 3D: cube-based neighborhoods.
3. **Apply the Stencil to each grid point.** For each element, compute the new value based on its neighbors. For example, the average of neighboring values.
4. (optional) **Handle Boundaries.** Ghost cells, mirroring, or skipping edges to prevent out-of-bounds errors.
5. **Iterate (Time-Stepping, if needed).** Some stencil computations run for multiple iterations, updating the grid repeatedly.

Example 69: How Convolution Uses the Stencil Pattern

1. **Grid Representation.** An image is a 2D grid of pixel values.
2. **Stencil Shape.** A small matrix called kernel (or filter) moves over the image. A common kernel can be blur, or edge detection, or sharpening.
3. **Applying the Stencil.** Each pixel's new value is computed using a weighted sum of its neighbors.
4. **Boundary Handling.** Pixels on the edges require special treatment (padding, mirroring, or ignoring).

For example, we can use the following kernel:

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Where each pixel of the image is replaced by the average of itself and its 8 neighbors.

10.10.2 Implementing stencil with shift

In the previous section we gave a brief description of what a stencil pattern is (page 357). Now in this chapter we want to show one of the most common patterns (variations) of the stencil pattern: Stencil with shift.

Stencil with Shift is a possible implementation of the Stencil pattern. The main features are:

- Instead of iterating through neighbors explicitly using loops (which may be inefficient), we **shift the input data by different offsets**.
- **Each shifted version of the array represents a different part of the stencil window.**
- This method is **commonly used in SIMD** (Single Instruction Multiple Data) architectures **to enable vectorization**.

✓ Why Apply Shift? Advantages

It is an obvious question. Why should we add a shift to an operation that is already working? The answer is efficiency:

- ✓ **Efficient Vectorization.** Shifting allows all elements of a vector register to be loaded together. Instead of accessing neighbors one by one (which causes memory divergence), we perform multiple aligned loads.

Note: **does not directly reduce memory traffic**, but allows vectorization. Therefore, **vectorization reduces the number of instructions, which improves performance**. So it is a consequence and not a direct effect.

- ✓ **Better Memory Access Pattern.** Instead of scattered memory accesses (which are slow), **shifting ensures contiguous memory accesses**, improving cache performance.
- ✓ **Simplifies Computation.** The stencil computation reduces to element-wise operations on shifted versions of the input array.

Note that shifting **works well only when applied along the dimension where elements are stored consecutively in memory**. Otherwise, the computation becomes significantly slower due to inefficient memory access (for example, if we apply a shift to a column-major data structure).

💡 How does it work?

1. Start with the Original Array.
2. Create Shifted Copies.
3. Apply a stencil formula (such as average).

Example 70: Stencil with Shift

1. Start with the original array:

```
[1, 2, 3, 4, 5, 6, 7]
```

2. Create shifted copies:

- Left shift by 1:

```
[2, 3, 4, 5, 6, 7, 8]
```

- Right shift by 1:

```
[0, 1, 2, 3, 4, 5, 6]
```

3. Apply the Stencil Formula. For example, compute 3-point average:

```
1 numerator = left_shift[i] + center[i] + right_shift[i];
2 result[i] = numerator / 3;
```

10.10.3 Cache optimizations

Cache optimizations in stencil computations focus on **reducing cache misses** and **maximizing data reuse** to improve performance. Since stencil operations involve accessing neighboring elements in a structured way, we can optimize memory access patterns to reduce unnecessary memory traffic.

💡 Why do we need cache optimizations?

- **Cache Misses.** When accessing memory that is not in the cache, the processor must fetch data from main memory, which is slow. **Stencil computations often involve non-contiguous memory accesses**, leading to many cache misses.
- **False Sharing.** If **multiple cores write to the same cache line**, it causes cache invalidations and slows down computation.
- **Redundant Memory Accesses.** Without optimization, different **cores may load the same data multiple times**, wasting bandwidth.

💡 Key cache optimization techniques

1. **Data Layout Awareness.** If rows are stored contiguously in memory, accessing horizontal neighbors is efficient. Accessing vertical neighbors (different rows) can cause cache misses because they are stored in different memory locations.

It is not a proper optimization technique but rather a recommendation. If we know the memory layout, such as row-major order, we can ensure better cache utilization and achieve higher cache hit rates.

2. **Assigning Work to Cores Efficiently.** The core allocation strategies are mainly two:

- (a) **Assigning Rows to Cores.** Each core processes an entire row.

- ✓ Maximizes horizontal data locality because rows are stored consecutively in memory.
- ✓ Minimizes cache misses for horizontal accesses (e.g., left and right neighbors in a stencil).
- ✗ Vertical offsets (e.g., top and bottom neighbors in a stencil) require cores to redundantly load adjacent rows, which can increase memory traffic.

- (b) **Assigning Columns to Cores.** Each core processes an entire column.

- ✗ Columns are not stored contiguously in row-major memory layouts, so **every access incurs a cache miss**.
- ✗ Creates **false sharing** when multiple cores write to the same cache line, causing cache invalidations and performance degradation.

In other words, choosing to assign rows to cores is the better choice.

3. **Strip-Mining**. It is a way of assigning work to cores. Instead of assigning full rows or columns, **divide the array into *strips***. Each **strip's width is chosen as a multiple of the cache line size**, **ensuring that each core loads only what it needs**.

It prevents redundant reads and **keeps frequently accessed data in cache**.

4. **Loop Blocking (Tiling)**. Instead of processing the entire array at once, **process small *tiles* that fit in cache**. This keeps active data in fast cache memory, minimizing slow memory accesses.

10.10.4 Communication optimizations

In parallel computing, when a stencil computation is distributed across multiple processing elements (PEs), we **need** to make some special considerations **to handle communication and boundary dependencies**.

★ Key techniques & concepts

The key techniques and concepts involved in optimizing stencil communication are as follows:

1. **Distributed Data and Ghost Cells.** When data is distributed among processing elements, we also copy ghost cells. **Ghost cells** are additional copies of boundary data from neighboring processing elements. These cells are essential to perform stencil computations involving shared boundaries without accessing the actual neighbor's data directly.

After each iteration, a processing element must exchange updated boundary data (ghost cells) with its neighbors to prepare for the next iteration.

2. **Importance of Replicating Ghost Cells.** Instead of accessing shared memory for boundaries, we made a replication of ghost cells in local memory for each processing element because it is more efficient.

This minimizes fine-grained sharing, which often leads to high communication overhead. **After each iteration**, ghost cells are swapped and updated through communication.

3. **Halo Regions.** A **Halo** is the set of all ghost cells required for stencil computation. The halo ensures all necessary neighbor data is available for one iteration.

Deep Halo is an extending of the halo size that allows multiple stencil iterations to be computed *without* inter-processing element communication.

This improves independence between processing elements, but it also increases redundant computations and requires more memory.

4. **Latency Hiding.** To maximize computational efficiency, we can try to hide latency overlapping communication with computation.

While waiting for ghost cell updates, processing elements can compute the interior of the stencil (data independent of ghost cells). This ensures that communication delays do not completely block computation progress.

5. **Key Trade-offs.** Stencil and communication optimizations involve balancing the following:

(a) Larger Halos:

- ✓ Fewer communication steps and more independence.
- ✗ Increased memory use and redundant computations.

(b) Smaller Halos:

- ✓ Reduced memory overhead.
- ✗ More frequent communication required, potentially reducing efficiency.

11 Parallel Patterns in OpenMP and CUDA

11.1 OpenMP

OpenMP provides efficient implementations of many parallel programming patterns. Here are some of the key patterns that we have already discussed:

1. Map Pattern on OpenMP

Characteristics

- **No dependencies** between operations.
- Simple and **ideal for SIMD** (Single Instruction, Multiple Data) execution.
- Executes independently on different data elements.

Example 1: Map Pattern on OpenMP

```
1 #pragma omp parallel for
2 for (int i = 0; i < N; i++) {
3     result[i] = operation(data[i]);
4 }
```

2. Reduction Pattern on OpenMP

Characteristics

- **Combines results from multiple threads into a single value** using a specified operation (e.g., sum, min, max).
- **Supported natively** in OpenMP (see 5.3.1.1, page 66). It is an easy integration for operations that aggregate data across threads.

Example 2: Reduction Pattern on OpenMP

```
1 #pragma omp parallel for reduction(+:sum)
2 for (int i = 0; i < N; i++) {
3     sum += array[i];
4 }
```

3. Workpile Pattern on OpenMP

Characteristics

- Handles irregular work distribution that may change dynamically at runtime.
- Assumes all tasks are independent.
- Commonly used in applications like tree searches or recursive computations.
- Leverages OpenMP's tasking feature for dynamically generated workloads.

Example 3: Workpile Pattern on OpenMP

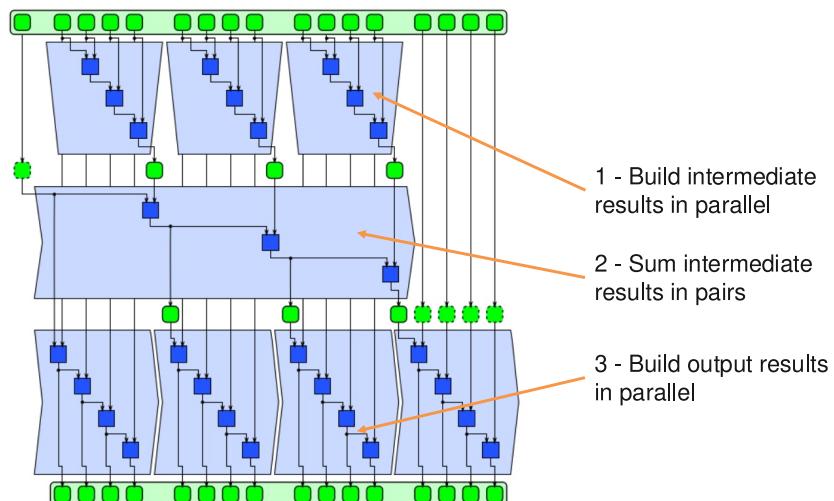
```

1 #pragma omp task
2 void process_task() {
3     // Perform a unit of work
4 }
```

4. Scan Pattern on OpenMP

Characteristics

- Efficiently performs prefix-sum or similar operations over an array.
- Widely used for cumulative computations and inclusive/exclusive scans.
- OpenMP 5.0 introduced dedicated support for this pattern.
- Three-Phase Approach:
 - (a) Build intermediate results in parallel.
 - (b) Combine intermediate results in pairs.
 - (c) Build final output in parallel.



Example 4: Scan Pattern on OpenMP

Example with a SIMD reduction clause:

```

1 #pragma omp simd reduction(inscan, +:scan_a)
2 for (int i = 0; i < N; i++) {
3     simd_scan[i] = scan_a;
4     #pragma omp scan exclusive(scan_a)
5     scan_a += array[i];
6 }
```

11.2 Histogram Pattern

The **Histogram Pattern** is a fundamental and widely used computational method for **analyzing large datasets by aggregating values into predefined bins**. It is used in applications such as feature extraction, fraud detection, and speech recognition.

★ Characteristics

█ **Definition.** For each data element, a specific *bin counter* is identified and incremented.

❖ Applications

- **Feature extraction:** Identifying key characteristics in images or data.
- **Fraud detection:** Analyzing transactional data for anomalies.
- **Speech recognition:** Identifying patterns in audio signals.

⌚ Main Challenges

⌚ **Output Interference:** Avoiding concurrent writes to the same bin counter in a parallel implementation.

⌚ **Memory Access Efficiency:** Ensuring memory coalescence for better bandwidth utilization.

Example 5: Histogram for Letter Count in Strings

Given an input string, the goal is to count the frequency of letters grouped into bins. For instance: the string “**p**rogramm**i**ng mass**i**ve **p**arallel p**ro**cessors” could be grouped into 4-letter bins like {a-d, e-h, i-l, ...}.

The output is:

```
1 a-d: 5
2 e-h: 5
3 i-l: 6
4 m-p: 10
5 q-t: 10
6 u-x: 1
7 y-z: 1
```

✖ Sequential Implementation in C

```

1 #define ALPHABET_LENGTH 26
2
3 sequential_Histogram(char *data, int length, int *histo) {
4     for (int i = 0; i < length; ++i) {
5         int alphabet_position = data[i] - 'a';
6         if (alphabet_position >= 0 &&
7             alphabet_position < ALPHABET_LENGTH) {
8             histo[alphabet_position / 4]++;
9         }
10    }
11 }
```

⌚ Parallel Implementation

To parallelize, we must:

1. **Partition Input:** Divide the input dataset into sections.
2. **Thread Processing:** Assign each thread a section to process independently.

So we need to choose how to split the input and which thread to assign to each piece of data.

- **Simplest parallel version.** The simplest parallel implementation divides the input into sections. The **number of sections created is equal to the number of threads available**. For example, if the input are 28 words long and we have 4 threads, each section length will be $28 \div 4 = 7$ words per thread. In parallel, **each thread iterates through a section**.

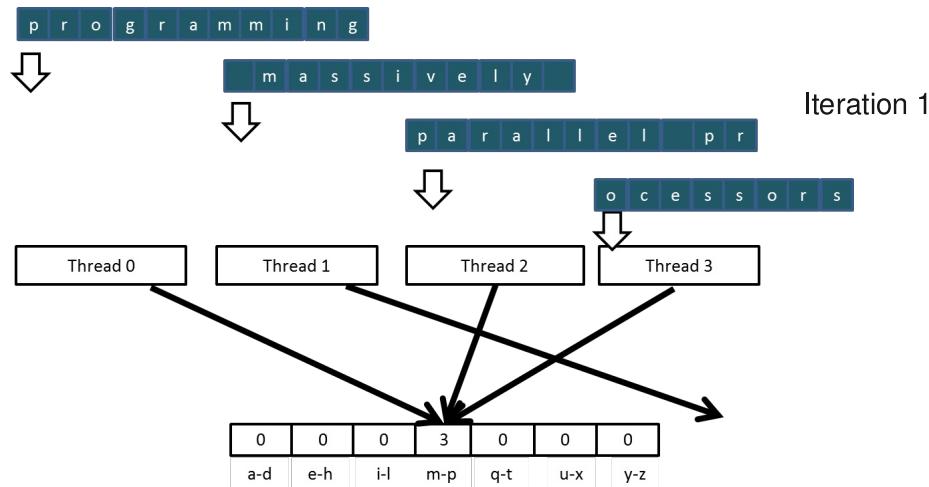


Figure 58: The simplest parallel implementation of the histogram pattern.

✖ Simplest parallel version: Inefficient memory access

Each thread works on its assigned contiguous section of the input. And this might be good, but we don't consider that **memory accesses across threads are not contiguous!** So at each iteration, each thread requests memory locations not contiguous on memory.

- ✖ **Memory Access Efficiency.** Adjacent threads work on non-adjacent memory sections. While each thread accesses contiguous memory, the threads themselves are **not accessing contiguous memory as a group.**
- ✖ **DRAM Bandwidth.** Accesses from different threads are not coalesced, meaning **memory requests are spread out across the memory space**, leading to inefficient use of bandwidth. If the memory addresses are in the same location, the memory requests are grouped together.

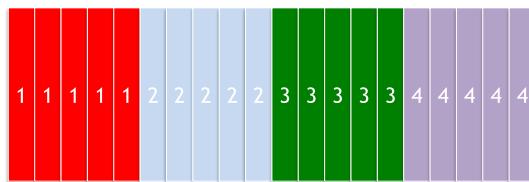


Figure 59: Memory allocation between threads on the simplest parallel version.

- **Parallel version with interleaved partitioning.** The interleaved partitioning, **threads work on interleaved indices**, so their **access are distributed across contiguous memory locations.**

⚡ Interleaved partitioning: the best memory option

In this case, **memory accesses from different threads are closer together in memory**, which aligns better with how memory is organized in hardware (cache lines and DRAM bursts). As a result:

- ✓ **Memory Coalescing.** Threads access memory in a way that aligns with cache lines and DRAM bursts.
- ✓ **Bandwidth Utilization.** Memory requests are grouped together, maximizing the DRAM bandwidth and improving overall performance.

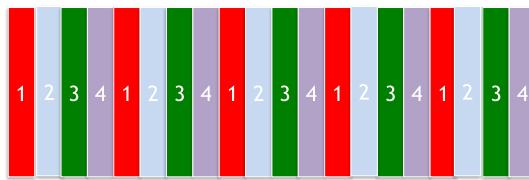


Figure 60: Memory allocation between threads on the interleaved partitioning version.

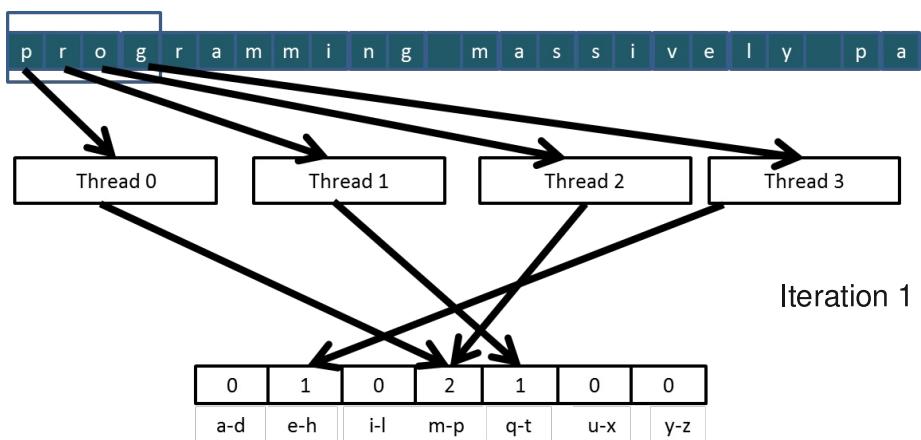


Figure 61: The parallel version with interleaved partitioning.

💡 Why Interleaved Partitioning is Better

In the simplest version, the threads work independently on large contiguous chunks. While this avoids conflicts between threads, it leads to **scattered memory access** patterns at a hardware level. **DRAM modules are optimized** for coalesced accesses, which occur **when requests are close together**.

In interleaved partitioning, threads' accesses are distributed such that memory requests from different threads are **closer together in memory**. This aligns with how caches and DRAM are designed to handle access patterns efficiently, improving speed and reducing memory latency.

⚠️ Implementations suffer from race condition

- ⌚ **What happened?** In this parallel implementation, **multiple threads** update a shared histogram **simultaneously**.

For example, in the figure 59 (page 368), we can see that threads #0, #2, and #3 are trying to increment the same bin at the same time. The correct result is 3, but this can **lead to data corruption because these operations are not atomic**. So we got lucky.

- ⌚ **Why this happens.** Incrementing a histogram bin is typically a three-step process:

1. Read the current value of the bin.
2. Increment the value.
3. Write the new value back to the bin.

In parallel, **these steps can interleave among threads, causing incorrect results**.

✓ How to avoid race conditions

Exists two technique to use to avoid race conditions on the histogram pattern:

- **Atomic Operations.** We already discussed what atomic operations are and how they can solve (not the best way we can) the race conditions on page 337.

But here we are talking about implementation. In **CUDA**, **atomic operations are implemented as hardware-supported functions** that perform a **read-modify-write operation as a single instruction** on a memory address. For the histogram pattern, atomic operations help resolve race conditions by ensuring only one thread modifies a memory location at any given time.

Some of the most common APIs provided by CUDA include:

- **atomicAdd:** **Adds** a value to a memory address atomically. [Doc.](#)
- **atomicSub:** **Subtract** operation (atomically). [Doc.](#)
- **atomicMin:** **Find minimum** operation (atomically). [Doc.](#)
- **atomicMax:** **Find maximum** operation (atomically). [Doc.](#)
- **atomicCAS (Compare-And-Swap):** Compares a value at an address and swaps it conditionally. [Doc.](#)

✖️ Implementation of Atomic Operations in CUDA

In this implementation, atomic operations ensure that multiple threads updating the same histogram bin don't cause data corruption.

```

1  __global__
2 void histo_kernel(
3     unsigned char *buffer,
4     long size,
5     unsigned int *histo
6 ) {
7     // Unique thread ID
8     int tid = threadIdx.x + blockIdx.x * blockDim.x;
9     // Stride for processing chunks
10    int stride = blockDim.x * gridDim.x;
11
12    for (unsigned int i = tid; i < size; i += stride) {
13        // Calculate bin index
14        int alphabet_position = buffer[i] - 'a';
15        if (alphabet_position >= 0 && alphabet_position < 26)
16        {
17            // Atomic update to avoid race conditions
18            atomicAdd(&(histo[alphabet_position / 4]), 1);
19        }
20    }
21 }
```

- **tid:** Thread's unique ID, ensures threads process distinct elements (explained on page 185).
- **stride:** Ensures each thread processes non-overlapping data chunks.
- **atomicAdd:** Performs the addition operation atomically, preventing race conditions when multiple threads update the same histogram bin.

⌚ Performance Considerations & Recommendations

- **Memory Type:**
 - ⌚ **Global memory (DRAM):** High latency (over 1000 cycles per atomic operation).
 - ⌚ **L2 Cache:** Approximately 1/10th the latency of DRAM.
 - ⌚ **Shared Memory:** Lowest latency and is private to each thread block.
- **Throughput Impact:**
 - * **Atomic operations significantly reduce throughput** when multiple threads access the same location because threads are serialized.
 - * Access patterns and contention have a major impact on overall performance.

The recommendations are:

- ✓ **Use Shared Memory:** Reduce global memory access latency by leveraging shared memory for intermediate computation within thread blocks.
- ✓ **Reduce Contention:**
 - * **Partition data** so threads update different bins.
 - * **Minimize the number of atomic updates** to the same location (every atomic operation introduces potential overhead).

- **Privatization.** **Privatization** is a technique used to avoid race conditions in parallel computing by **providing private copies of shared resources to threads or blocks**.

❓ What it does

Privatization **creates private copies of shared resources** (e.g., histograms) **for each thread or block**. Instead of multiple threads contending to access and modify a shared resource, **each thread or block updates its own private copy independently**. The **operation performed on the data must be associative and commutative** (e.g., addition for histograms).

✅ Benefits

- ✓ **Reduces Contention.** Threads do not compete to access shared memory resources during intermediate computations, significantly reducing contention.
- ✓ **Improves Throughput.** Since atomic operations on shared memory are faster than on global memory, privatization improves performance, especially when using shared memory for private copies.

⚠ Challenges

- **Overhead.** **Allocating and initializing private copies adds computational overhead.** Combining or reducing private copies into a final shared copy at the end introduces additional steps.
- **Memory Fit.** The private data must fit into shared memory, **which limits the size of privatized resources**.

❓ How it works in combination with a histogram pattern

1. **Private Copies Initialization.** Each block of threads initializes a local histogram in shared memory (e.g., `histo_s[]`).
2. **Local Updates.** **Threads within a block update their private histogram using atomic operations** in shared memory. This step ensures no inter-block contention since the operations are confined to the shared memory of each block.
3. **Reduction Step.** At the end of processing, **private histograms are merged into a final shared histogram in global memory using atomic operations**.

❓ Why Private Copies in Shared Memory?

- Each block gets its own private copy of the histogram in **shared memory**, which is **fast and local to the block**.
- **Threads within the same block can update their shared histogram without interfering with threads in other blocks** because each block **works on its own local copy**.

- Key Benefit: **no contention between blocks**, as there is no shared resource across blocks.

ⓘ Why no inter-block contention?

- Since blocks do not access the same global memory histogram during local computations, there is **no need for atomic operations between blocks during this stage**.
- Threads within a block may still use **atomic operations**, but these are **limited to shared memory**, which is **much faster than global memory**.

ⓘ The reduction phase is not so slow?

Only after all blocks have finished updating their private histograms do they write their results back to the global histogram. At this stage, atomic operations are required, but **the volume of these operations is much smaller**. In fact, instead of all threads updating the global histogram during each update, there is **only one write per bin per block**.

Example 6: Histogram Patterns Privatization Analogy

Image we have 10 groups of workers (*blocks*), each responsible for counting objects (*bins*) in their own room (*shared memory*).

- ✖ **Without privatization:** All workers from all groups try to update a single global counter in a central office (*global memory*). They must wait in line to update the counter.
- ✓ **With privatization:** Each group has its own private counter in their room. They work independently without competing. At the end, a manager (*reduction step*) collects and sums up the results from each room.

❖ Implementation of Privatization in CUDA

```

1  __global__
2  void histogram_privatized_kernel(
3      unsigned char* input,
4      unsigned int* bins,
5      unsigned int num_elements,
6      unsigned int num_bins
7  ) {
8      // 1. Initialization
9      unsigned int tid = blockIdx.x * blockDim.x + threadIdx.x;
10     extern __shared__ unsigned int histo_s[];
11
12     for(
13         unsigned int binIdx = threadIdx.x;
14         binIdx < num_bins;
15         binIdx += blockDim.x
16     ) {
17         // Initialize private histogram
18         histo_s[binIdx] = 0;
19     }
20     __syncthreads();
21
22
23     // 2. Local Histogram Updates
24     for (
25         unsigned int i = tid;
26         i < num_elements;
27         i += blockDim.x * gridDim.x
28     ) {
29         int alphabet_position = buffer[i] - 'a';
30         if (alphabet_position >= 0 && alphabet_position < 26)
31         {
32             atomicAdd(&(histo_s[alphabet_position]), 1);
33         }
34     }
35     __syncthreads();
36
37     // 3. Reduction to Global Memory
38     for (
39         unsigned int binIdx = threadIdx.x;
40         binIdx < num_bins;
41         binIdx += blockDim.x
42     ) {
43         atomicAdd(&(histo[binIdx]), histo_s[binIdx]);
44     }
45 }
```

11.3 Reduction Pattern

The **Reduction Pattern** is designed to combine elements from a dataset to produce a single output (e.g., sum, max, min). This pattern was discussed on page 309.

Remark: Binary Tree

A **Binary Tree** is a **hierarchical data structure** in which **each node has at most two children**, referred to as the left child and the right child. Binary trees are widely used in computer science for various applications such as searching, sorting, and hierarchical organization of data. Key features of a binary tree are:

- **Root Node:** The topmost node of the binary tree. It is the starting point of the tree structure.
- **Children:** Each node can have up to two children: left child (left branch of the tree) and right child (right branch of the tree).
- **Leaf Nodes:** Nodes that have no children. These are at the “bottom” of the tree.
- **Parent Node:** A node that has at least one child.
- **Depth or Level:** The number of edges from the root node to a specific node. The root node is at level 0.
- **Height:** The maximum depth of any node in the tree.
- **Subtrees:** A binary tree can be divided into two smaller binary trees called the left subtree and the right subtree.

The binary tree has some special conditions:

- Number of Children: a node can have at most two children (left and right).
- Structure: it is a strictly hierarchical, with nodes divided into a left and a right child.
- Special Constraints: it has specific rules for arranging nodes:
 - **Full Binary Tree:** Every node has either 0 or 2 children.
 - **Complete Binary Tree:** All levels are fully filled except possibly the last level, which is filled from left to right.
 - **Perfect Binary Tree:** All internal nodes have two children, and all leaf nodes are at the same level.
 - **Balanced Binary Tree:** The height difference between the left and right subtrees of any node is at most 1.
 - **Degenerate (or Skewed) Binary Tree:** Each node has only one child (either left or right), forming a structure similar to a linked list.

¶ Parallel Reduction Tree

The parallel reduction operates by **pairing elements in a binary tree structure**. In each step, pairs of **elements are combined using an associative** and optionally **commutative operation**. The complexity is:

- Sequential reduction: $O(N)$.
- Parallel reduction: $O(\log_2(N))$.

This method is **work-efficient but not resource-efficient due to redundant operations across threads**.

☒ Simple Parallel Implementation

This implementation uses $\frac{N}{2}$ threads initially and operates in $\log_2(N)$ steps. At each step:

- Threads operate on pairs of elements.
- The number of **active threads is reduced by half after each step**.

Reduction is performed in-place using shared memory to reduce global memory traffic. The efficient memory usage is critical, as N must fit within the shared memory of a block.

The **Thread-to-Data Mapping** defines how threads in CUDA are assigned to process the elements of the input array during the reduction process.

- Each thread maps to an even index in the partial sums array.
- The first operand remains at the current thread's index.
- The second operand comes from progressively farther away (e.g., offset 1, 2, 4, ...).

🏃 Baseline CUDA implementation

The baseline CUDA implementation for the parallel reduction algorithm demonstrates a simple, straightforward way to implement reduction using shared memory and thread synchronization.

- **Shared Memory Allocation**

```
1 __shared__ float partialSum[SIZE];
```

Shared memory (`partialSum`) is declared to **store partial results of the reduction within a block**. Each thread in the block writes its corresponding value into `partialSum`.

- **Initialization**

```
1 partialSum[threadIdx.x] = X[
2     blockIdx.x * blockDim.x + threadIdx.x
3 ];
```

Each thread calculates its **unique global memory position** based on the block and thread indices, and **loads data into the shared memory**.

- **Reduction Loop**

```
1 for (unsigned int stride = 1; stride < blockDim.x; stride *=
2) {
3     __syncthreads();
4     if (t % (2 * stride) == 0) {
5         partialSum[t] += partialSum[t + stride];
6     }
}
```

The reduction process works by iteratively summing pairs of elements:

- **Stride**: Determines the “step size” for pairing elements in each iteration.
- **Synchronization**: Ensures that all threads have completed their work before the next iteration starts.
- **Thread Filtering**: Only threads whose index `t` satisfies the `if` condition participate in the addition. This creates two groups:
 1. **Active threads**: Perform the summation.
 2. **Inactive threads**: Sit idle but still consume resources.

⚠ Performance Issues

- ✖ **Warp Inefficiency**: Many threads become idle after the first few iterations, wasting computational resources.
- ✖ **Control Divergence**: Threads that meet the `if` condition execute different code paths compared to those that don’t, causing **warp divergence** (note the theory explained on page 151).

Improved CUDA implementation

This improved CUDA implementation optimizes the reduction process by addressing issues like thread divergence and leveraging shared memory for efficient computation.

- ✓ **Reduction of Divergence.** In the baseline implementation, threads often face divergence due to conditional checks based on `stride`. This version **modifies the indexing to ensure threads perform similar operations**, reducing divergence.

The change in indexing is possible because **reduction operators** (e.g., addition) are **commutative and associative**. So the **order of operation does not affect the result**.

- ✓ **Use of Shared Memory.** Partial sums are stored in shared memory (`__shared__ float partialSum[SIZE];`), which is **much faster than accessing global memory**.

Each thread writes its corresponding input value to shared memory, reducing the need for repeated global memory access during the reduction phase.

```

1  __shared__ float partialSum[SIZE];
2
3 // Each thread loads its input value into shared memory
4 partialSum[threadIdx.x] = X[blockIdx.x * blockDim.x + threadIdx.x];
5
6 // 't' is the thread index
7 unsigned int t = threadIdx.x;
8
9 /**
10 * Perform the reduction in shared memory.
11 *
12 * The 'stride' starts at half the block size and
13 * reduces by half in each iteration (bitwise shift).
14 */
15 for (
16     unsigned int stride = blockDim.x / 2;
17     stride >= 1;
18     stride = stride >> 1
19 ) {
20     // Ensure all threads complete their current step
21     __syncthreads();
22
23     // Only threads with index t < stride participate in this step
24     if (t < stride) {
25         // Add the value from the neighboring thread
26         partialSum[t] += partialSum[t + stride];
27     }
28 }
29
30 // At the end of the loop,
31 // partialSum[0] contains the total reduction result
32 // for that block

```

11.4 Scan Pattern

The scan pattern, also known as prefix sum, is a computational pattern where **each element in an output array is the sum of all previous elements up to the current index in the input array.**

🏃 Sequential Implementation

```

1 y[0] = x[0];
2 for (int i = 1; i < Max_i; ++i)
3     y[i] = y[i-1] + x[i];

```

The complexity is $O(N)$ for N elements. The pros are **computationally efficient** and **only one additional operation** compared to reduction. Unfortunately, the **limitation** is the **sequential** execution; cannot fully utilize parallel resources.

ParallelGroup Implementation with Interleaved Reduction Trees

1. **Input Reading:** Copy the input into shared memory.
2. **Iterative Computation:** In each iteration, thread j adds elements j and $j - \text{stride}$, storing the result back in j . Finally, double the stride and repeat until the entire array is processed.

🏃 Work-inefficient CUDA implementation

```

1 __shared__ float XY[SECTION_SIZE];
2 int i = blockIdx.x * blockDim.x + threadIdx.x;
3
4 if (i < InputSize) {
5     XY[threadIdx.x] = X[i];
6 }
7
8 // perform iterative scan on XY
9 for (
10     unsigned int stride = 1;
11     stride < blockDim.x;
12     stride *= 2
13 ) {
14     __syncthreads();
15     if (stride <= threadIdx.x) {
16         XY[threadIdx.x] += XY[threadIdx.x - stride];
17     }
18 }

```

The previous CUDA implementation is inefficient due to several factors:

✗ Work Inefficiency

- The implementation requires $\log_2(N)$ iterations for a data size of N , where each iteration involves progressively fewer additions (e.g., $(N - 1)$, $(N - 2)$, etc.).

- The total number of additions scales as $O(N \cdot \log_2(N))$, which is significantly higher than the sequential version's $O(N)$.

For example, with $N = 1024$, this results in a 10x computational overhead compared to the sequential version, as the hardware resources are not used optimally.

- ✖ **High Energy Consumption.** Due to the inefficiency of the work distribution, many hardware resources remain inactive during computation. This leads to energy wastage, as resources that could be utilized for other tasks are idling.
- ✖ **Shared Memory and Synchronization Overhead.** The implementation relies on shared memory to store intermediate results for each block. Synchronization barriers (`__syncthreads()`) are required at each iteration to ensure threads work correctly on shared data. This adds latency and further reduces efficiency.
- ✖ **Unbalanced Load Distribution.** As the number of elements to process reduces with each iteration, some threads become idle earlier than others. This unbalanced workload distribution leads to under-utilization of the GPU's parallel processing capabilities.
- ✖ **Lack of Intermediate Result Reuse.** Intermediate sums computed during the scan are not reused effectively in subsequent steps. This leads to redundant computations, further increasing inefficiency.

⌚ Efficient CUDA implementation

The improved CUDA implementation of the scan pattern involves two phases: **Reduction** and **Distribution**, both of which are designed to ensure work efficiency and minimize execution time.

- **Reduction Phase**

≡ **Goal.** Build a prefix sum (or partial reduction) **tree in shared memory**.

💡 **Key Idea.** Each **thread works on shared memory**:

`XY[2*BLOCK_SIZE]`

And **accumulates results in a binary tree fashion**.

✖ Process

1. The threads perform summation for strides increasing as powers of 2.

For example, for `stride = 1`, thread at index 1 adds values from index 0; for `stride = 2`, thread at index 3 adds values from index 1, and so on.

2. This builds a binary tree structure in memory, with the final prefix sum stored at the highest index.

⌚ **Synchronization.** The `__syncthreads()` call ensures all threads finish their stride operation before the next begins.

✓ **Efficiency.** It executes $\log_2(N)$ iterations (with N being the input size). Total operations are $N - 1$.

- **Distribution Phase**

≡ **Goal.** Transform the prefix sum tree into the final scan result array.

💡 **Key Idea.** Threads traverse the binary tree in reverse (top-down) to propagate results downwards.

✖ Process

1. Strides start at half the block size (`BLOCK_SIZE / 2`) and decrease by powers of 2.
2. Each thread adds values from higher indices to lower indices in the tree to compute the final scan result.
3. The results are written to the output array (`Y[i]`).

⌚ **Synchronization.** Threads use `__syncthreads()` before and after each stride to ensure all updates are visible across threads.

✓ **Efficiency.** It executes $\log_2(N)$ iterations with a reduced number of operations. Total operations are no more than $(N - 1)$ in each phase.

The reason why it is improved are:

- ✓ **Work Efficiency.** Both phases together execute a maximum of $2(N - 1)$ operations. This is much better compared to the naïve implementation, which can perform up to $O(N \cdot \log(N))$ operations.
- ✓ **Parallelization.** Each thread handles fewer redundant operations, and shared memory minimizes memory latency.
- ✓ **Hardware Utilization.** With enough CUDA cores and shared memory, the overhead of extra work is compensated by reduced execution time.

```

1  ****
2  * Reduction Phase *
3  ****
4 // XY[2 * BLOCK_SIZE] is in shared memory
5
6 for (
7     unsigned int stride = 1;
8     stride <= BLOCK_SIZE;
9     stride *= 2
10) {
11    int index = (threadIdx.x + 1) * stride * 2 - 1;
12    if (index < 2 * BLOCK_SIZE) {
13        XY[index] += XY[index - stride];
14    }
15    __syncthreads();
16}
17
18 ****
19 * Distribution Phase *
20 ****
21 for (
22     unsigned int stride = BLOCK_SIZE / 2;
23     stride > 0;
24     stride /= 2
25) {
26    __syncthreads();
27    int index = (threadIdx.x + 1) * stride * 2 - 1;
28    if (index + stride < 2 * BLOCK_SIZE) {
29        XY[index + stride] += XY[index];
30    }
31 }
32 __syncthreads();
33 if (i < InputSize) {
34     Y[i] = XY[threadIdx.x];
35 }
```

12 Heterogeneous Computing - DSLs and HLS

12.1 Introduction to Heterogeneous Computing

Heterogeneous Computing (or **Heterogeneous Processing**) refers to systems that use multiple types of processors or accelerators to handle different workloads more efficiently.

- In contrast to traditional homogeneous systems (which only use CPUs), heterogeneous systems combine different processing units such as CPUs, GPUs, DSPs, and FPGAs.
- The goal is to match the right processor to the right task, achieving higher performance and energy efficiency.

Example 1: Heterogeneous Processing

A self-driving car requires CPUs for decision-making, GPUs for image recognition, and FPGAs for real-time sensor fusion.

2 Energy-Efficient Computing Strategies

When designing a heterogeneous system, performance isn't the only goal; **energy efficiency is just as critical**. Given a fixed power budget, simply **increasing performance without considering power constraints is inefficient**. Specialized hardware (e.g., FPGAs, ASICs) achieves better performance per watt than general-purpose processors.

There are two main strategies for improving energy efficiency:

1. **Use Specialized Processors.** CPUs are not energy-efficient due to instruction decoding, branch handling, and pipeline management overhead. Specialized hardware (FPGAs, ASICs) reduces overhead, leading to more computations per joule.

$$\text{Power} = \frac{\text{Op}}{\text{second}} \times \frac{\text{Joules}}{\text{Op}}$$

2. **Minimize Data Movement.** Memory access consumes more energy than computation! Optimizing data locality reduces power consumption. For example, moving computation closer to memory (e.g., using tensor core inside GPUs) significantly reduces energy cost.

12.2 Heterogeneous parallel programming

⚠ Challenges of Writing Portable and Efficient Parallel Code

Writing parallel programs for heterogeneous systems is difficult due to the following reasons:

1. **Diverse Hardware Architectures.** A CPU, GPU, and FPGA all have different programming models. **Code written for one hardware type may not perform well on another.**
2. **Performance vs. Productivity Trade-offs.**
 - **Performance:** Low-level programming (e.g., CUDA, OpenCL, Verilog) allows fine-tuned optimizations but **is hard to program**.
 - **Productivity:** High-level abstractions (e.g., OpenMP, DSLs) improve productivity but **may introduce performance overhead**.
3. **Memory Management.** Different memory models (shared vs. distributed) require different optimizations. Data movement between CPU and GPU memory can be costly if not handled efficiently.
4. **Scalability Issues.** Some **programs scale well on GPUs but poorly on CPUs** due to synchronization and memory bandwidth limitations.

✓ The Ideal Parallel Programming Language

An ideal parallel programming model should provide a balance of:

- ✓ **Performance.** Optimized execution across different hardware.
- ✓ **Productivity.** Easy to use and develop.
- ✓ **Generality.** Works across different architectures.

However, **most existing languages optimize only one or two** of these factors, leading to trade-offs.

Approach	Performance	Productivity	Generality
CUDA/OpenCL	✓ High	✗ Low	✗ Low
OpenMP (CPU)	✓ High	✓ Medium	✗ Low
MPI (Distributed)	✓ High	✗ Low	✓ High
FPGA/Verilog/VHDL	✓ Very High	✗ Very Low	✗ Low
High-Level Synthesis	✓ High	✓ Medium	✗ Low

⌚ Why is this important?

If we want **portable parallel programs**, we need **new high-level abstractions** like Domain-Specific Languages (DSLs), which will be covered in the next section.

12.3 DSLs and Halide

❓ What are Domain-Specific Languages (DSLs)?

A **Domain-Specific Language (DSL)** is a specialized programming language designed for a specific application domain. The main characteristics of DSLs are:

- **Restricted expressiveness** (focused on a single domain)
- **High-level, declarative syntax** (easier than general purpose languages)
- **Optimized performance** for the target domain
- **May be standalone or embedded** in another language

DSL Name	Target Domain	Key Benefits
Halide	Image Processing	Separates algorithm from scheduling for optimized execution.
TensorFlow	Machine Learning	Optimized computation graphs for AI workloads.
SQL	Databases	Declarative queries for efficient data retrieval.
Verilog/VHDL	Hardware Design	Describes digital circuits for synthesis.

Table 11: Examples of DSLs.

💡 Embedded vs. External DSLs

DSLs can be classified as:

- **External** DSLs:
 - ❑ Have their own *syntax* and *compiler/interpreter*.
 - ❑ **Example:** SQL, Halide, Verilog.
 - ❑ **Advantages:** can be **more optimized** but require custom compilers.
- **Embedded** DSLs:
 - ❑ Built inside another general-purpose language.
 - ❑ **Example:** TensorFlow (embedded in Python).
 - ❑ **Advantages:** benefit from **integration** with the host language

⚡ DSL Use Case: Halide for Image Processing

Halide is a Domain-Specific Language (DSL) for high-performance image processing.

❓ Why does image processing need DSLs?

- ➕ Image processing is **data-intensive** and **requires high performance**.
- 👎 Traditional solutions (C++, CUDA, OpenCV) **require manual optimizations**.
- 😩 Optimizing code for **parallelism and memory efficiency** is **difficult**.

❓ Why Halide?

- ✖ Separates “*what*” is computed from “*how*” it is executed.
- leftrightarrow Expresses computations at a high level, leaving optimizations to the compiler.
- 🌐 Portable across CPUs, GPUs, and FPGAs.

❖ How Halide Works: Separating Algorithm from Schedule

In Halide, a **key feature** is the separation of **what** a program computes (*computation/algorithm*) from **how** it executes (*schedule*). This means:

- The **algorithm** specifies **what operations should be performed**. In other words, specifies **what to compute** (like a mathematical formula).
- The **schedule** defines **how those operations should be executed efficiently** on the hardware. In other words, specifies **how to execute the computation** (parallelism, memory layout, vectorization).

Now we see the difference between the traditional approach we have always used and the halide approach:

- **Traditional Approach (C++, CUDA, OpenCV)**. In traditional programming languages (e.g., C++, OpenCV, CUDA), the **algorithm and execution strategy are mixed together**. This means that if we want to **change parallelization or memory access optimizations**, we must **rewrite parts of the algorithm itself**. This makes it *hard to experiment* with different optimizations.

⚠ Problems with the Traditional Approach

1. If we want to optimize for vectorization, parallel execution, or memory layout, we **must modify the algorithm itself**.
2. The **same code cannot easily be reused** for different architectures (e.g., CPU, GPU, FPGA).

Example 2: Problems with the Traditional Approach

```

1 void box_blur(const Image &in, Image &out) {
2     for (int y = 1; y < in.height() - 1; y++) {
3         for (int x = 1; x < in.width() - 1; x++) {
4             out(x, y) = (
5                 in(x-1, y) + in(x, y) + in(x+1, y)
6             ) / 3;
7         }
8     }
9 }
```

The problem here is that we need to specify the “*what*”, i.e. what operations should be performed, but also “*how*” these operations should be performed.

- **Halide’s Approach: Separate Algorithm from Execution.** Halide splits the computation into two parts:

1. **Computation (Algorithm) - *What to Compute***
 - Defines the mathematical computation.
 - Remains unchanged across different hardware targets.
2. **Schedule - *How to Execute Efficiently***
 - Controls memory layout, parallelization, and optimization.
 - Can be changed without modifying the algorithm.

Example 3: Box Blur in Halide

The computation part is separated from the scheduling! So a change in the algorithm can be made without affecting the control of the execution. Therefore, we define simply:

1. Computation (Algorithm), stays the same:

```

1 Var x, y;
2 Func blurx, blury;
3
4 // First pass: horizontal blur
5 blurx(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))
6             / 3;
7
8 // Second pass: vertical blur
9 blury(x, y) = (blurx(x, y-1) + blurx(x, y) + blurx(
10           x, y+1)) / 3;
```

This part **only describes the math, not how** it should run.

2. Schedule, controls execution (can be changed easily):

```

1 blury.tile(x, y, xi, yi, 256, 32)
2     // Vectorized execution for SIMD
3     .vectorize(xi, 8)
4     // Parallel execution over y-dimension
5     .parallel(y);
```

```
6  
7 blurx.compute_at(blury, x) // Compute blurx only  
     when needed by blury  
8     .vectorize(x, 8);
```

This part **controls execution strategy** but does **not modify the algorithm**. The same algorithm can now **run efficiently on different hardware architectures** just by changing the schedule.

✓ Why DSLs Matter for Performance and Productivity: Advantages

- ✓ **Performance Optimization.** A Halide program **can be better than hand-optimization C++ code**. Scheduling decisions affect parallel execution, memory locality, and vectorization.
- ✓ **Productivity.** Instead of manually optimizing, **Halide allows rapid exploration of different schedules**. Easier to **port to different architectures** (CPU, GPU, FPGA).

In conclusion, DSLs like Halide **automate low-level optimizations**, enabling **faster and more efficient** code for specialized domains.

12.4 Scheduling & Performance Optimization in Halide

This section focuses on how different scheduling strategies in Halide affect performance and how to choose the best schedule for different hardware architectures.

⌚ Why Scheduling Matters

Scheduling is the **key to optimizing performance in Halide**.

- A poorly scheduled program can be $10\times$ slower than an optimized one.
- Memory access, cache locality, parallelism, and vectorization all depend on scheduling.

As we have already seen, in traditional languages (C++, CUDA, OpenMP), scheduling decisions must be hard-coded into the algorithm. In Halide, the schedule is separate and can be changed without changing the algorithm.

⌚ How Scheduling Affects Performance

Different **scheduling strategies** impact how the computation is executed on hardware:

Scheduling Strategy	Impact
Serial Execution	Simple, but slow. No parallelism.
Parallel Execution	Uses multiple CPU cores. Good for multi-core CPUs.
Vectorization (SIMD)	Uses wide registers for efficiency (e.g., AVX).
Tiling	Improves cache locality by processing data in chunks.
Compute-at	Controls when intermediate results are computed.
Store-at	Controls where intermediate results are stored.

☰ Scheduling Strategies in Halide

Let's take the Box Blur Algorithm as an example. The **Box Blur** is a simple **image processing technique used for smoothing or blurring an image**. It performs two main steps:

1. Each pixel in the output image is computed as the **average of its neighboring pixels**.
2. It applies a **moving average filter** over a **small window** (e.g., 3×3 or 5×5 pixels).

It is used to reduce noise in images, to create a smooth, blurry effect and, above all, because it is fast and efficient, since it involves only two operations: adding and dividing.

However, we will now analyze the algorithm implemented in Halide using different strategies:

✗ Default Serial Execution (Slow). Without scheduling, Halide will execute in **serial order** (one pixel at a time).

```

1 Var x, y;
2 Func blurx, blury;
3
4 blurx(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y)) / 3;
5 blury(x, y) = (blurx(x, y-1) + blurx(x, y) + blurx(x, y+1)) /
   3;
```

✗ Problems

- ✗ No parallelism or vectorization.
- ✗ Poor memory access patterns.
- ✗ Slow execution.

㉑ Parallel Execution. We can **add parallelism** to use multiple CPU cores:

```
1 blury.parallel(y);
```

✓ Advantages

- ✓ Halide automatically splits the work across CPU cores.
- ✓ Useful for multi-threaded execution on CPUs.

㉒ Vectorization (SIMD). Modern CPUs support **SIMD instructions** (e.g., AVX, NEON) to process multiple pixels at once:

```
1 blury.vectorize(x, 8);
```

✓ Advantages

- ✓ Uses **SIMD registers** for faster execution.
- ✓ Works best for **data-parallel workloads** like image processing.

㉓ Tiling for Better Cache Performance. Instead of processing the whole image at once, we **divide it into smaller tiles**:

```
1 blury.tile(x, y, xi, yi, 256, 32);
```

✓ Advantages

- ✓ Each tile fits better in cache, **reducing memory latency**.

✓ Improves locality of reference (less cache thrashing).

⌚ Optimized Schedule: Combining Techniques. We can combine multiple scheduling strategies for maximum performance:

```

1 // Process in 256x32 tiles
2 blurry.tile(x, y, xi, yi, 256, 32)
3     // Vectorized execution
4     .vectorize(xi, 8)
5     // Parallel execution across CPU cores
6     .parallel(y);
7
8 // Compute blurx only when needed
9 blurx.compute_at(blurry, x)
10    .vectorize(x, 8);

```

✔ Advantages

- ✓ Breaks image into tiles for cache efficiency.
- ✓ Uses SIMD vectorization for fast execution.
- ✓ Runs in parallel on multiple CPU cores.
- ✓ Intermediate results (blurx) are computed only when needed.

⌚ Trade-offs in Scheduling

But *how do we choose the right scheduling?* Well, we need to find a good trade-off. Different scheduling choices affect **performance trade-offs**:

Scheduling Strategy	Performance Impact
Parallel Execution	Increases throughput, uses multiple cores.
Vectorization (SIMD)	Improves performance on CPUs/GPUs.
Tiling	Improves cache locality, reduces memory overhead.
Compute-at	Avoids redundant computations.
Store-at	Reduces memory footprint but increases re-computation.

Table 12: Performance trade-offs in Scheduling.

✓ Conclusion

In conclusion, the key takeaways are:

- Scheduling is the key to performance in Halide.
- Parallel execution, vectorization, and tiling significantly improve performance.
- Halide's flexibility allows quick experimentation with different schedules.
- The right schedule depends on hardware constraints (CPU, GPU, FPGA).

12.5 Introduction to HLS

② What is High-Level Synthesis (HLS)?

High-Level Synthesis (HLS) is a process that converts high-level software code (C/C++/Python) into hardware designs (Verilog/VHDL). It automates hardware generation, allowing developers to describe behavior in software-like code while the tool generates optimized circuits.

▣ Traditional FPGA/ASIC Design Flow (Without HLS)

The traditional FPGA or ASIC design flow is divided into three main steps:

1. Write Register-Transfer Level (RTL) code in Verilog/VHDL.
2. Manually optimize for timing, area, power.
3. Run logic synthesis, place & route, and fabrication.

The main **problems** with this approach are:

- ✗ **Time-consuming** (designing hardware manually takes months).
- ✗ **Error-prone** (low-level bugs are hard to debug).
- ✗ **Difficult to modify** (small changes require rewriting RTL code).

Aspect	Traditional RTL	High-Level Synthesis
Design Level	Low-level: gates, registers	High-level (C++, Python)
Productivity	✗ Time-consuming	✓ Faster development
Optimizations	Manual pipeline and control logic	Automated scheduling
Reusability	✗ Difficult to modify	✓ Easily reusable code
Learning Curve	Steep (hardware expertise needed)	Easier (similar to software programming)

Table 13:  Differences Between HLS and RTL-Based Hardware Design.

We can conclude that **HLS allows software engineers to efficiently design hardware without deep knowledge of Verilog/VHDL.**

✓ Why use HLS instead of traditional RTL design? Benefits of HLS

- ✓ **Faster Development Cycle.** Designers can write C++/Python instead of Verilog, reducing design time.
- ✓ **Automated Optimizations.** HLS compilers automatically optimize parallelism, pipelining, and resource allocation.
- ✓ **Easier HW/SW Co-Design.** Enables rapid prototyping of hardware accelerators.
- ✓ **Technology Independence.** The same C++ code can be compiled for different FPGA/ASIC platforms.

✗ Challenges of HLS

- Not all software code can be efficiently translated into hardware.
- HLS tools must explore a huge design space (parallelism, pipelining, resource constraints).
- Requires careful optimization of memory access and data flow.

✓ Conclusion

In conclusion, the key takeaways are:

- HLS translates high-level software (C/C++/Python) into hardware designs (Verilog/VHDL).
- It automates the hardware design process, reducing time and complexity.
- HLS enables software engineers to design hardware accelerators without deep RTL expertise.
- Challenges include optimizing memory access, parallelism, and scheduling.

12.6 HLS Workflow

This section explains **how HLS converts high-level code (C/C++/Python) into hardware** (Verilog/VHDL) and the key steps in the HLS design flow.

Inputs to an HLS Compiler

To generate hardware from high-level code, an **HLS tool** requires three main inputs:

1. **High-Level Code** (C, C++, or Python)
 - Describes the **algorithm's behavior**.
 - Written similarly to software but **optimized for hardware**.
2. **Library of Characterized Modules**
 - **Predefined hardware** building blocks (e.g., adders, multipliers, memory units).
 - Helps the compiler understand available resources.
3. **Constraints & Optimization Directives**
 - Designer-defined constraints such as:
 - **Area constraints** (how much hardware can be used).
 - **Timing constraints** (desired clock speed).
 - **Memory hierarchy** (external vs. on-chip memory).
 - Optional **HLS pragmas** (e.g., loop unrolling, pipelining) to fine-tune performance.

HLS Objectives

The goal of **HLS** is to generate an efficient hardware design based on the following objectives:

- ✓ **Minimize Area**: Uses fewer functional units, registers, and interconnects.
- ✓ **Maximize Speed**: Reduces the number of clock cycles (latency) and increases throughput.
- ✓ **Optimize Power**: Reduces energy consumption for embedded systems.

However, the main trade-offs must be:

- **Optimizing for speed** ⇒ may increase hardware area.
- **Reducing hardware area** ⇒ may increase execution time.

HLS Compilation Flow

HLS follows **three main steps**:

- **Front-End (Parsing and IR Generation)**. Converts C++/Python code into an **Intermediate Representation (IR)**. IR is similar to software compiler representations (e.g., LLVM IR). In other words, it breaks the program down into basic operations.
- **Middle-End (Optimization and Scheduling)**. At this point, the HLS tools performed three important operations:
 1. It analyzes data dependencies and determines execution order.
 2. It performs **scheduling** (decides when each operation runs).
 3. It allocates **hardware resources** (multipliers, memory, registers).

Example 4: Scheduling Choices

Scheduling Strategy	Impact
As Soon As Possible	Minimizes latency, but may use more hardware.
As Late As Possible	Reduces hardware usage, but may increase delay.
Loop Unrolling	Increases parallelism, but requires more area.
Pipelining	Allows overlapping computations to improve throughput.

- **Back-End (Hardware Generation)**. Finally, the tools made two steps:
 1. Converts optimized IR into Verilog/VHDL.
 2. Generates a Finite State Machine with Datapath (FSMD) representation.

The result is a **synthesizable hardware description** ready for FPGA/ASIC implementation.

Therefore, a **final output** of HLS tools is:

- Verilog/VHDL code (for FPGA/ASIC synthesis).
- Datapath and Controller Design.
- Simulation files for verification.

✓ Conclusion

In conclusion, the key takeaways are:

- **HLS automates hardware design** from high-level code (C/C++/Python).
- The workflow involves **parsing, optimization, scheduling, and hardware generation**.
- **Performance tuning** requires adjusting **scheduling, pipelining, and parallelism**.
- **HLS outputs Verilog/VHDL**, which can be synthesized on FPGAs/A-SICs.

12.7 HLS Compilation & Optimization

This section explains how **High-Level Synthesis (HLS)** transforms **high-level code (C/C++/Python)** into **optimized hardware circuits** and the key **optimization techniques** used to improve performance.

➡ High-Level Code Transformation into Hardware

In HLS, the **software-like code** is translated into **hardware logic**, consisting of:

1. **Datapath** performs operations (e.g., arithmetic, logic).
2. **Controller (FSM - Finite State Machine)** directs execution flow.

For example, the following code:

```
1 int add(int a, int b) {
2     return a + b;
3 }
```

HLS compiles this into a hardware circuit that includes:

- Registers for inputs (**a**, **b**) and output (**sum**).
- An ALU (**Adder**) for computation.
- A Finite State Machine (FSM) to control execution.

✳️ Key Optimization Techniques in HLS

HLS provides **various optimizations** to improve area, performance, and power consumption.

1. **Loop Unrolling**. **Duplicates loop iterations** to **increase parallel execution**. Reduces loop control overhead.

⚠ Problem

The following loop runs **one iteration at a time** (sequential):

```
1 for (int i = 0; i < 4; i++) {
2     sum += arr[i];
3 }
```

✔ Solution

The following iterations **execute in parallel**, making it 4× faster:

```
1 #pragma HLS unroll
2 for (int i = 0; i < 4; i++) {
3     sum += arr[i];
4 }
```

2. **Loop Pipelining.** Allows overlapping loop iterations, improving throughput. Reduces idle time between operations.

⚠ Problem

Each iteration waits for the previous one to finish (sequential):

```
1 for (int i = 0; i < 4; i++) {
2     arr[i] = arr[i] * 2;
3 }
```

✓ Solution

Each iteration starts before the previous one finishes, increasing speed:

```
1 #pragma HLS pipeline
2 for (int i = 0; i < 4; i++) {
3     arr[i] = arr[i] * 2;
4 }
```

3. **Resource Sharing.** Reuses hardware components to reduce circuit area. Trades performance for lower hardware cost.

⚠ Problem

Uses two separate adders, increasing area:

```
1 y1 = x1 + x2;
2 y2 = x3 + x4;
```

✓ Solution

Both additions use the same adder, reducing area:

```
1 #pragma HLS ALLOCATION instances=add limit=1
2 y1 = x1 + x2;
3 y2 = x3 + x4;
```

4. **Memory Optimization.** Reduces memory bottlenecks by using on-chip memory (BRAM, LUTRAM). Uses burst memory access to reduce external memory latency.

⚠ Problem

Each iteration accesses memory separately, slowing execution:

```
1 for (int i = 0; i < N; i++) {
2     arr[i] = read_from_memory(i);
3 }
```

✓ Solution

Splits memory into smaller blocks, allowing parallel access:

```

1 #pragma HLS array_partition variable=arr type=block factor=2
2 for (int i = 0; i < N; i++) {
3     arr[i] = read_from_memory(i);
4 }
```

✗ Challenges in Hardware Mapping

Even with optimizations, **not all code translates efficiently into hardware.**

Challenge	Solution
Too many loops (low parallelism)	Apply loop unrolling/pipelining.
Excessive memory access	Use on-chip memory and burst mode.
High area consumption	Enable resource sharing.
Dependency issues in loops	Apply loop transformations.

✓ Conclusion

In conclusion, the key takeaways are:

- **HLS converts high-level code into hardware logic**, optimizing execution.
- **Loop unrolling and pipelining improve parallelism.**
- **Memory and resource optimizations reduce area and latency.**
- **Careful scheduling is required** to balance speed and resource usage.

12.8 Scheduling in HLS

This section explores how **scheduling** affects performance in HLS, including key **scheduling techniques** like **ASAP**, **ALAP**, **list-based scheduling**, and **constrained scheduling**.

💡 What is Scheduling in HLS?

Scheduling in HLS determines **when each operation in the high-level code is executed** while considering **hardware resource constraints**.

The goal is to **minimize execution time (latency)** while using **minimal hardware resources (area)**. Scheduling maps operations to clock cycles, ensuring dependencies are met.

💡 Scheduling Techniques

Different **scheduling algorithms** can be used based on constraints (area, speed, power):

- **ASAP (As Soon As Possible) Scheduling.** Tries to execute operations **as early as possible**. Minimizes latency, but may increase hardware resource usage.

✓ Pros

- ✓ Good for high-speed designs.

✗ Cons

- ✗ Can use too many resources if multiple operations execute simultaneously.

Example 5: ASAP Scheduling

```

1 Cycle 1: x1 = a * b; // Multiplication
2 Cycle 2: x2 = c * d;
3 Cycle 3: y = x1 + x2; // Addition

```

- **ALAP (As Late As Possible) Scheduling.** Delays operations until the latest possible cycle. Reduces resource usage, but may increase latency.

✓ Pros

- ✓ Minimizes resource usage (fewer active units per cycle).

✗ Cons

- ✗ May introduce unnecessary delays.

Example 6: ALAP Scheduling

```

1 Cycle 1: x1 = a * b; // Multiplication
2 Cycle 2: (idle)      // Delays execution
3 Cycle 3: x2 = c * d;
4 Cycle 4: y = x1 + x2; // Addition

```

- **List-Based Scheduling.** Uses priority-based selection of operations. Balances latency vs. resource constraints.

❖ Steps

1. Construct a priority list based on dependencies.
2. Schedule operations by selecting the highest-priority task that fits resource constraints.
3. Continue until all operations are scheduled.

✓ Pros

- ✓ Efficient trade-off between area and speed.
- ✓ Used in most HLS tools (e.g., Vitis HLS, Bambu HLS).

- **Constrained Scheduling.** Used when hardware resources (adders, multipliers, memory) are limited. Tries to minimize execution time given a fixed number of resources.

✓ Pros

- ✓ Uses minimal hardware resources.

✗ Cons

- ✗ May increase total latency due to resource sharing.

Example 7: Constrained Scheduling

Clock Cycle	Adder (1 avail.)	Multiplier (1 avail.)
1	-	x1 = a * b
2	-	x2 = c * d
3	y = x1 + x2	-

Example 8: Scheduling a Basic Block

Consider the following code:

```

1 tmp1 = a * b;
2 tmp2 = c * d;
3 x1 = tmp1 * tmp2;
4 x2 = a * d;
5 x3 = x2 - x1;
6 x4 = x2 + x3;
```

Choosing the right schedule depends on whether the goal is to minimize execution time or minimize resource usage. However, here we have different scheduling results grouped by the method chosen.

Scheduling	Total Clock Cycles	Hardware Used
ASAP	⌚ 4 Cycles	High resource usage
ALAP	⚡ 6 Cycles	Low resource usage
List-Based	⌚ Balanced	Medium resource usage
Constrained	☒ Depends (constraints)	Minimum resources

✓ Conclusion

In conclusion, the key takeaways are:

- Scheduling determines **when operations execute**, impacting performance.
- **ASAP** scheduling **minimizes latency** but may use **more resources**.
- **ALAP** scheduling **minimizes resource usage** but may **increase latency**.
- List-based scheduling balances speed and area constraints.
- Constrained scheduling optimizes execution within hardware limits.

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