Project 2

Due April 21.

Consider the $n \times n$ tridiagonal system of equations $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix}$$

and

$$\mathbf{b}^{\mathrm{T}} = \begin{bmatrix} 1 + \frac{1^2}{(n+1)^4} & \frac{2^2}{(n+1)^4} & \frac{3^2}{(n+1)^4} & \cdots & \frac{(n-1)^2}{(n+1)^4} & 6 + \frac{n^2}{(n+1)^4} \end{bmatrix}.$$

Write a program that takes a value for n and solves for $\mathbf{x}$ using the following two methods:

1. Crout factorization of A into LU where L is lower triangular and U is upper triangular. Your program should output the ∞-norm of the residual of your computed solution.

2. Gauss-Seidel iteration starting with $\mathbf{x}_0 = 0$ and terminating when the residual is less than 10^{-8} in ∞-norm. Your program should output the ∞-norm of the residual of your computed solution and the number of iterations used.

Neither of your programs should use an $n \times n$ array to store the matrix A or the factors L and U. You must implement a memory efficient version of the algorithms. For example, A can be stored using 3 arrays of length n.

Run your programs for $n = 10, 100, 1000, 10000$. Answer the following:

1. Compute ∞-norm backward errors of LU and Gauss-Seidel. How do they compare?

2. How does the accuracy depend on the size of the problem for each method?

3. How does the number of iterations required for Gauss-Seidel depend on the size of the problem?