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Lamda Calculus

For the following forms apply β-reduction and α-substitution to reduce to lowest form. Indicate at each step the rule you are applying.

1. $(\lambda x.x)(\lambda x.x)$

→ $(\lambda x.x)(\lambda y.y)$ // α-substitution: rename x to y

→ $(\lambda y.y)$ // β-reduction $[(\lambda y.y)/x]x$: replace x with λy.y

2. $(\lambda x.x\ x)(\lambda x.\lambda y.x\ x)$

→ $(\lambda x.\lambda y.x\ x)(\lambda x.\lambda y.x\ x)$

// β-reduction $[(\lambda x.\lambda y.x\ x)/x]x$: replace x with $\lambda x.\lambda y.x\ x$

→ $(\lambda x.\lambda y.x\ x)(\lambda u.\lambda y.u\ u)$ // α substitution: rename x to u

→ $(\lambda u.\lambda y.u\ u)(\lambda u.\lambda y.u\ u)$

// β-reduction $[(\lambda u.\lambda y.u\ u)/x]x$: replace x with $\lambda u.\lambda y.u\ u$

→ $(\lambda u.\lambda y.u\ u)(\lambda z.\lambda y.z\ z)$ // α-substitution: rename u to z

→ $\lambda y\ (\lambda z.\lambda y.z\ z)(\lambda z.\lambda y.z\ z)$

Non-terminating lamda expression.

3. $((\lambda x.(x\ y))\ (\lambda z.z))$

→ $((\lambda z.z)\ y)$ // β reduction: replace x with $\lambda z.z$

→ y // β reduction: replace z with y

4. $(\lambda z.z)(\lambda y.y\ y)(\lambda x.x\ a)$

→ $(\lambda y.y\ y)(\lambda x.x\ a)$ // β-reduction: replace z with $\lambda y.y\ y$

→ $(\lambda x.x\ a)(\lambda x.x\ a)$ // β-reduction: replace y with $\lambda x.x\ a$

→ $(\lambda x.x\ a)\ a$ // β-reduction: replace x with $\lambda x.x\ a$

→ $a\ a$ // β-reduction: replace x with a

5. $(\lambda z.z)\ (\lambda z.z\ z)(\lambda z.z\ y)$

→ $(\lambda z.z\ z)(\lambda z.z\ y)$ // β-reduction: replace z with $\lambda z.z\ z$

→ $(\lambda z.z\ y)(\lambda z.z\ y)$ // β-reduction: replace z with $\lambda z.z\ y$

→ $(\lambda z.z\ y)\ y$ // β-reduction: replace z with $\lambda z.z\ y$

→ $y\ y$ // β-reduction: replace z with y

6. $(\lambda x.\lambda y.x\ y\ y)(\lambda a.a)\ b$

→ $(\lambda y.(\lambda a.a)\ y\ y)\ b$ // β-reduction: replace x with $\lambda a.a$

→ $(\lambda a.a)\ b\ b$ // β-reduction: replace y with b

→ $b\ b$ // β-reduction: replace a with b

7. $(\lambda x.x\ x)(\lambda y.y\ x)\ z$

→ $(\lambda y.y\ x)(\lambda y.y\ x)\ z$ // β-reduction: replace x with $\lambda y.y\ x$

→ $(\lambda y.y\ x)\ x\ z$ // β-reduction: replace y with $\lambda y.y\ x$

→ $x\ x\ z$ // β-reduction: replace y with x

8. $(\lambda x.(\lambda y.(x\ y))\ y)\ z$
→ $(\lambda x.(\lambda u.(x\ u))\ y)\ z$ // α-substitution: rename y to u
→ $(\lambda u.(z\ u))\ y$ // β-reduction: replace x with z
→ $z\ y$ // β-reduction: replace u with y

9. $((\lambda x.x\ x)(\lambda y.y))(\lambda y.y)$
→ $((\lambda y.y)(\lambda y.y))(\lambda y.y)$
// β-reduction: replace x with $\lambda y.y$
→ $(\lambda y.y)(\lambda y.y)$
// β-reduction replace y with $\lambda y.y$
→ $\lambda y.y$ // β-reduction: replace y with $\lambda y.y$

10. $(((\lambda x.\lambda y.(x\ y))(\lambda y.y))\ \omega)$
→ $(((\lambda x.\lambda a.(x\ a))(\lambda y.y))\ \omega)$
// α-substitution: rename y to a

→ $((\lambda a.((\lambda y.y)\ a))\ \omega)$
// β-reduction: replace x with $\lambda y.y$

→ $(\lambda y.y)\ \omega$ // β-reduction: replace a with ω
→ ω // β-reduction: replace y with ω