## Formula Sheet

		not p	p and q	p or q	p xor q	p nand q	p nor q
p	q	$\sim p$	$(p \wedge q)$	$(p \lor q)$	$(p \oplus q)$	$\sim (p \land q)$	$\sim (p \lor q)$
F	F	Т	F	F	F	Т	Т
$\overline{F}$	Т	Т	F	Т	Т	Т	F
Т	F	F	F	Т	Т	Т	F
Т	Т	F	Т	Т	F	F	F
		\rightarrow	Height	4	#	Ů,	4

p	$\mid q \mid$	$p \to q$	$p \leftrightarrow q$
F	F	Т	Т
F	Т	Т	F
Т	F	F	F
Т	Т	Т	Т

Logical Equivalence ( $\equiv$ ) Laws

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Identity (I)	$(p \wedge T) \equiv p$	$(p \vee F) \equiv p$
	$(T \land p) \equiv p$	$(F \vee p) \equiv p$
Universal Bound (UB)	$(p \land F) \equiv F$	$(p \vee T) \equiv T$
	$(F \wedge p) \equiv F$	$(T \vee p) \equiv T$
Idempotent (ID)	$(p \land p) \equiv p$	$(p \lor p) \equiv p$
Commutative (COM)	$(p \land q) \equiv (q \land p)$	$(p \lor q) \equiv (q \lor p)$
Associative (ASS)	$(p \land (q \land r)) \equiv ((p \land q) \land r)$	$(p \lor (q \lor r)) \equiv ((p \lor q) \lor r)$
Distributive (DIST)	$(p \lor (q \land r)) \equiv ((p \lor q) \land (p \lor r))$	$(p \land (q \lor r)) \equiv ((p \land q) \lor (p \land r))$
	$((r \land q) \lor p) \equiv ((r \lor p) \land (q \lor p))$	$((r \lor q) \land p) \equiv ((r \land p) \lor (q \land p))$
Absorption (ABS)	$(p \lor (p \land q)) \equiv p$	$(p \land (p \lor q)) \equiv p$
	$((q \land p) \lor p) \equiv p$	$((q \lor p) \land p) \equiv p$
Negation (NEG)	$(p \land \sim p) \equiv F$	$(p \vee \sim p) \equiv T$
	$(\sim p \land p) \equiv F$	$(\sim p \lor p) \equiv T$
Double Negation (DNEG)	$\sim \sim p \equiv p$	
DeMorgan's (DM)	$\sim (p \land q) \equiv (\sim p \lor \sim q)$	$\sim (p \lor q) \equiv (\sim p \land \sim q)$
Definition of Exclusive OR (XOR)	$(p \oplus q) \equiv ((p \lor q) \land \sim (p \land q))$	$(p \oplus q) \equiv ((p \land \sim q) \lor (\sim p \land q))$
Definition of Implication (IMP)	$(p \to q) \equiv (\sim p \lor q)$	
Contrapositive (CONTP)	$(p \to q) \equiv (\sim q \to \sim p)$	
Definition of Biconditional (BIC)	$(p \leftrightarrow q) \equiv ((p \to q) \land (q \to p))$	$(p \leftrightarrow q) \equiv \sim (p \oplus q)$

Rules of Inference

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Modus Ponens: [M.PON]	$(p \to q)$	Modus Tollens: [M.TOL]	$(p \to q)$
	p		$\sim q$
	$\therefore q$		$\therefore \sim p$
Generalization: [GEN]	p	Specialization: [SPEC]	$(p \wedge q)$
	$\therefore (p \lor q)$		$\therefore p$
Conjunction: [CONJ]	p	Elimination: [ELIM]	$(p \lor q)$
	q		$\sim q$
	$\therefore (p \land q)$		$\therefore p$
Transitivity: [TRANS]	$(p \to q)$	Proof by cases: [CASE]	$(p \to r)$
	$(q \to r)$		$(q \to r)$
	$\therefore (p \to r)$		$\therefore ((p \lor q) \to r)$
Resolution: [RES]	$(p \lor q)$	Contradiction: [CONTD]	$(p \to F)$
	$(\sim p \lor r)$		$\therefore \sim p$
	$\therefore (q \lor r)$		
	$\therefore (q \lor r)$		

Powers of 2

						WCID O										
$2^{16}$	$2^{15}$	$2^{14}$	$2^{13}$	$2^{12}$	$2^{11}$	$2^{10}$	$2^{9}$	$2^{8}$	$2^{7}$	$2^{6}$	$2^5$	$2^{4}$	$2^3$	$2^2$	$2^1$	$2^{0}$
65,536	32,768	16,384	8,192	4,096	2,048	1,024	512	256	128	64	32	16	8	4	2	1

Binary Representation

						11141 5 10
$x_3$	$x_2$	$x_1$	$x_0$	HEX	unsigned	signed
0	0	0	0	0	0	0
0	0	0	1	1	1	1
0	0	1	0	2	2	2
0	0	1	1	3	3	3
0	1	0	0	4	4	4
0	1	0	1	5	5	5
0	1	1	0	6	6	6
0	1	1	1	7	7	7

$x_3$	$x_2$	$x_1$	$x_0$	HEX	unsigned	signed
1	0	0	0	8	8	-8
1	0	0	1	9	9	-7
1	0	1	0	A	10	-6
1	0	1	1	В	11	-5
1	1	0	0	C	12	-4
1	1	0	1	D	13	-3
1	1	1	0	E	14	-2
1	1	1	1	F	15	-1

Domains

$\mathbb{Z}$	Integers	$\{\cdots, -2, -1, 0, 1, 2, \cdots\}$	$\mathbb{Q}$	Rational Numbers	$\left\{ \frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{Z}^* \right\}$
$\mathbb{Z}^+$	Positive Integers	$\{x \in \mathbb{Z}, x > 0\}$	Q	Irrational Numbers	$\{x \in \mathbb{R}, x \notin \mathbb{Q}\}$
$\mathbb{Z}^*$	Non-zero Integers	$\{x \in \mathbb{Z}, x \neq 0\}$	$\mathbb{N}_0$	Natural Numbers	$\{0, 1, 2, 3, \cdots\}$
$\mathbb{R}$	Real Numbers	$\{\cdots, -\frac{20}{6}, 0, 1, \sqrt{2}, \pi, \cdots\}$	$\mathbb{N}_1$		$\{1,2,3,\cdots\}$

**Set Operations** 

	<b>-</b>
S contains a	$a \in S$
S does not contains a	$\sim (a \in S) \equiv a \notin S$
A is a subset of B	$A \subseteq B \equiv \forall x \in \mathcal{U}, x \in A \to x \in B$
A is not a subset of B	$A \nsubseteq B \equiv \exists x \in \mathcal{U}, x \in A \land x \notin B$
A is a proper subset of B	$A \subset B \equiv \forall x \in \mathcal{U}, (x \in A \to x \in B) \land \exists y \in \mathcal{U}, y \in B \land y \notin A$
A is equal to B	$A = B \equiv \forall x \in \mathcal{U}, x \in A \leftrightarrow x \in B$
A union B	$C = A \cup B = \{x \in \mathcal{U}   x \in A \lor x \in B\}$
A intersection B	$C = A \cap B = \{x \in \mathcal{U}   x \in A \land x \in B\}$
B minus A	$C = B - A = \{x \in \mathcal{U}   x \in B \land x \notin A\}$
Complement of A	$C = A^c = \{ x \in \mathcal{U}   x \notin A \}$

$$\mathbf{Big} \ \mathbf{O()}$$

$$f \in O(g) \Leftrightarrow \exists c \in \mathbb{R}^+, \exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n \ge n_0 \to f(n) \le c \cdot g(n)$$

$$O(1) < O(\log n) < O(n) < O(n\log n) < O(n^2) < O(2^n) < O(n!)$$

**Regular Expressions** 

	1
	Matches any character
[xy]	Matches one character from those listed
[x-z]	Matches one character from the range of characters listed
$[\hat{x}y]$	Matches one character from those not listed
	Matches one element from those separated by pipes
*	Matches the previous element 0 or more times
+	Matches the previous element 1 or more times
?	Matches the previous element 0 or 1 time
$\{m,n\}$	Matches the preceding element from m to n times
$\setminus s$	Matches a whitespace character
$\setminus d$	Matches a digit, same as [0-9]
$\setminus w$	Matches an alphanumeric character and underscore, same as [A-Za-z0-9_]