Exponent

9. $\dfrac{4^n-1}{2^n-1}=2^n+1$

10. $\dfrac{2^{2p+1}\cdot 3^{2p+q}\cdot 5^{p+q}\cdot 6^p}{3^{p-2}\cdot 6^{2p+2}\cdot 10^p\cdot 15^q}=\dfrac{1}{2}$

11. $\left(\dfrac{a^l}{a^m}\right)^n\cdot\left(\dfrac{a^m}{a^n}\right)^l\cdot\left(\dfrac{a^n}{a^l}\right)^m=1$

12. $\dfrac{a^{p+q}}{a^{2r}}\times\dfrac{a^{q+r}}{a^{2p}}\times\dfrac{a^{r+p}}{a^{2q}}=1$

13. $\left(\dfrac{x^a}{x^b}\right)^{\frac{1}{ab}}\cdot\left(\dfrac{x^b}{x^c}\right)^{\frac{1}{bc}}\cdot\left(\dfrac{x^c}{x^a}\right)^{\frac{1}{ca}}=1$

14. $\left(\dfrac{x^a}{x^b}\right)^{a+b}\cdot\left(\dfrac{x^b}{x^c}\right)^{b+c}\cdot\left(\dfrac{x^c}{x^a}\right)^{c+a}=1$

15. $\left(\dfrac{x^p}{x^q}\right)^{p+q-r}\cdot\left(\dfrac{x^q}{x^r}\right)^{q+r-p}\cdot\left(\dfrac{x^r}{x^p}\right)^{r+p-q}=1$

16. If $a^x=b$, $b^y=c$ and $c^z=a$, then show that, $xyz=1$

17. $4^x=8$

18. $2^{2x+1}=128$

19. $(\sqrt{3})^{x+1}=(\sqrt[3]{3})^{2x-1}$

20. $2^x+2^{1-x}=3$

21. $P=x^a$, $Q=x^b$ and $R=x^c$

1) Find the values of $P^{bc}\cdot Q^{-ca}$.

2) Find the values of $\left(\dfrac{P}{Q}\right)^{a+b}\times\left(\dfrac{Q}{R}\right)^{b+c}\div 2(RP)^{a-c}$

3) Show that, $\left(\dfrac{P}{Q}\right)^{a^2+ab+b^2}\times\left(\dfrac{Q}{R}\right)^{b^2+bc+c^2}\times\left(\dfrac{R}{P}\right)^{c^2+ca+a^2}=1$

22. $X=(2a^{-1}+3b^{-1})^{-1}$, $Y=\sqrt[pq]{\dfrac{x^p}{x^q}}\times\sqrt[qr]{\dfrac{x^q}{x^r}}\times\sqrt[rp]{\dfrac{x^r}{x^p}}$

and $Z=\dfrac{5^{m+1}}{(5^m)^{m-1}}\div\dfrac{25^{m+1}}{(5^{m-1})^{m+1}}$, where $x,p,q,r>0$

1) Find the value of X

2) Show that, $Y+\sqrt[3]{81}=5$

3) Show that, $Y\div Z=25$

Logarithms

2. Find the value of x :

1) $\log_5 x = 3$ 2) $\log_x 25 = 2$ 3) $\log_x \frac{1}{16} = -2$

3. Show that,

1) $5\log_{10}5 - \log_{10}25 = \log_{10}125$
2) $\log_{10}\frac{50}{147} = \log_{10}2 + 2\log_{10}5 - \log_{10}3 - 2\log_{10}7$
3) $3\log_{10}2 + 2\log_{10}3 + \log_{10}5 = \log_{10}360$

4. Simplify:

1) $7\log_{10}\frac{10}{9} - 2\log_{10}\frac{25}{24} + 3\log_{10}\frac{81}{80}$
2) $\log_7(\sqrt[5]{7} \cdot \sqrt{7}) - \log_3\sqrt[3]{3} + \log_4 2$
3) $\log_e\frac{a^3b^3}{c^3} + \log_e\frac{b^3c^3}{d^3} + \log_e\frac{c^3d^3}{a^3} - 3\log_e b^2c$

5. $x = 2, y = 3, z = 5, w = 7$

1) What is the log of $\sqrt{y^3}$ to the base 3.
2) Find the value of $w\log\frac{xz}{y^2} - x\log\frac{z^2}{x^2y} + y\log\frac{y^4}{x^4z}$
3) Show that, $\frac{\log\sqrt{y^3} + y\log x - \frac{y}{x}\log(xz)}{\log(xy) - \log z} = \log_y\sqrt{y^3}$

6. A model for the number N of people in a college community who have heard a certain rumor is

$$N = P\,(1 - e^{-0.15d})$$

where P is the total population of the community and d is the number of days that have elapsed since the rumor began. In a community of 1000 students, how many students will have heard the rumor after 3 days?

7. Between 12:00 pm and 1:00 pm, cars arrive at Citibank's drive-thru at the rate of 6 cars per hour (0.1 cars per minute). The following formula from probability can be used to determine the probability that a car will arrive within t minutes of 12:00 pm.

$$F = 1 - e^{-0.1t}$$

(a) Determine the probability that a car will arrive within 10 minutes of 12:00 pm (that is, before 12:10 pm).

(b) Determine the probability that a car will arrive within 40 minutes of 12:00 pm (before 12:40 pm).

8. Suppose that a student has 500 vocabulary words to learn. If the student learns 15 words after 5 minutes, the function

$$L(t) = 500(1 - e^{-0.0061t})$$

approximates the number of words L that the student will have learned after t minutes.

(a) How many words will the student have learned after 30 minutes?

(b) How many words will the student have learned after 60 minutes?