Data Communication

Analog to Digital Conversion

ANALOG TO DIGITAL CONVERSION

- **A digital signal is superior to an analog signal because it is more robust to noise and can easily be recovered, corrected and amplified.**

- **Therefore, the tendency today is to convert an analog signal to digital data. conversion technique.**

- **Pulse code modulation (PCM) is considered as the most common conversion technique.**

Pulse Code Modulation (PCM)

PCM consists of three steps to digitize an analog signal:

Sampling

- ✔ Converts signal into discrete format over time.
- ✔ The sampled signals can be generated without distortion.

Quantization

- ✔ Transforms signal discrete in amplitude.
- ✔ Efficient quantization process uses minimum number of bits subject to limited distortion.

Pulse Code Modulation (PCM)

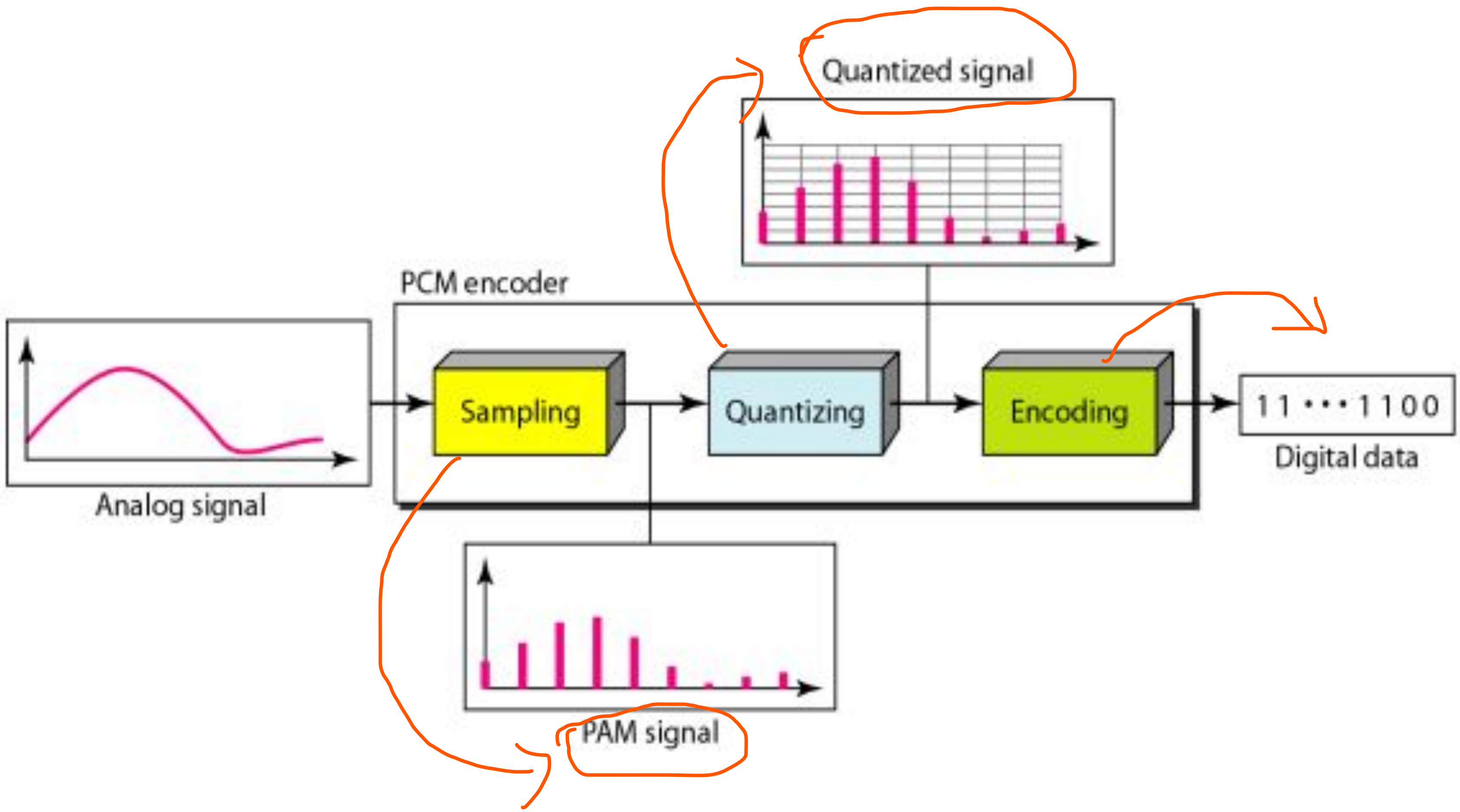
Encoding

- ✔ Compresses digital data and eliminates redundant information.
- ✔ Does not cause distortion.

Pulse Code Modulation (PCM)

- Before we sample, we have to filter the signal to limit the maximum frequency of the signal as it affects the sampling rate.

- Filtering should ensure that we do not distort the signal, i.e remove high frequency components that affect the signal shape.



Quantized signal
PCM encoder
Sampling
Quantizing
Encoding
1 1 • • • 1 1 0 0
Digital data
Analog signal
PAM signal

SAMPLING

- Analog signal is sampled every T_s secs.
- T_s is referred to as the sampling interval.
- $f_s = {}^1/_{T_s}$ is called the sampling rate or sampling frequency.

There are **3 sampling methods**:

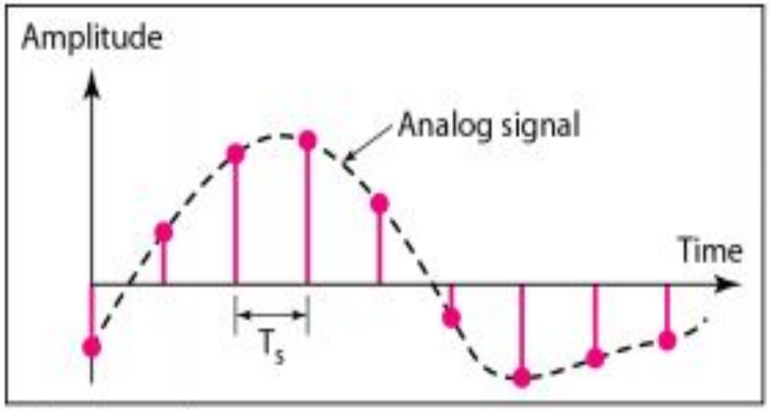
Ideal - an impulse at each sampling instant

Natural - a pulse of short width with varying amplitude

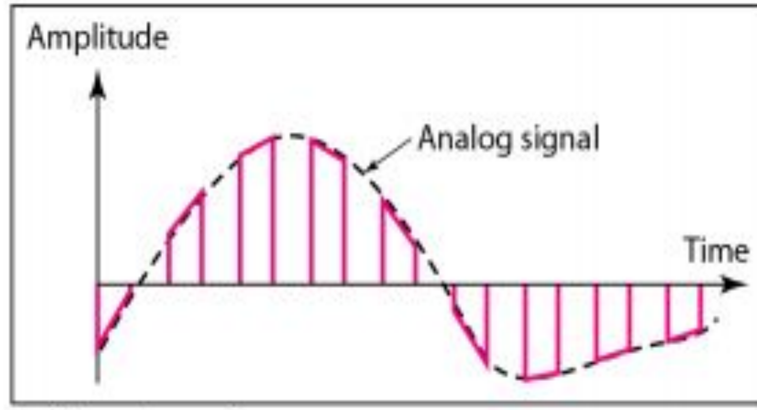
Flattop - sample and hold, like natural but with single amplitude value

The process is referred to as **pulse amplitude modulation (PAM)** and the outcome is a signal with analog (non integer) values

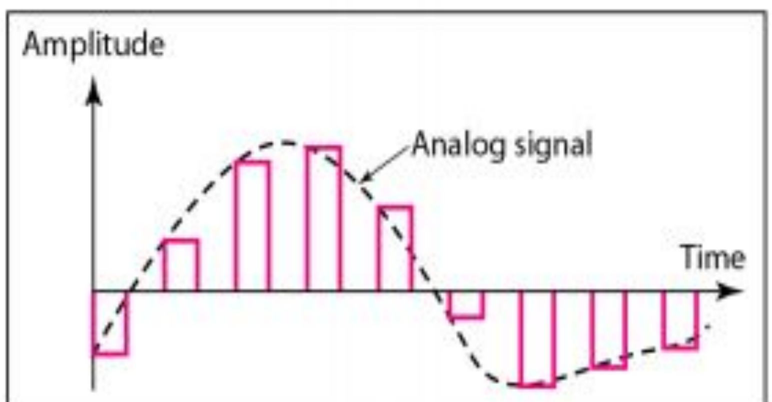
SAMPLING



a. Ideal sampling



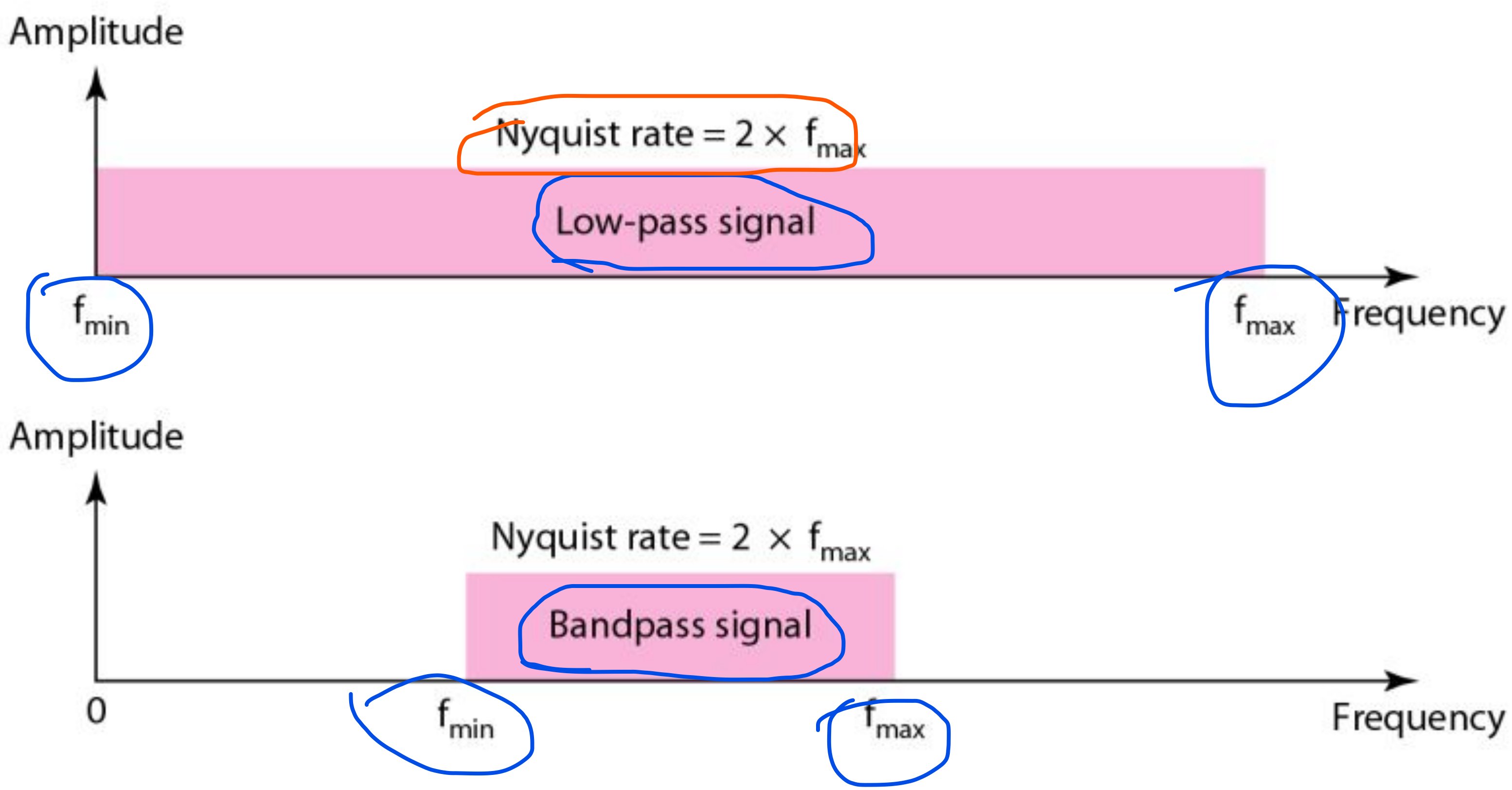
b. Natural sampling



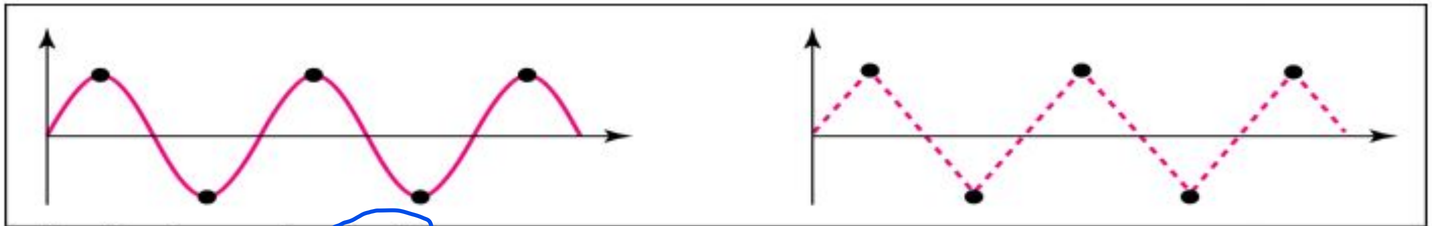
c. Flat-top sampling

SAMPLING

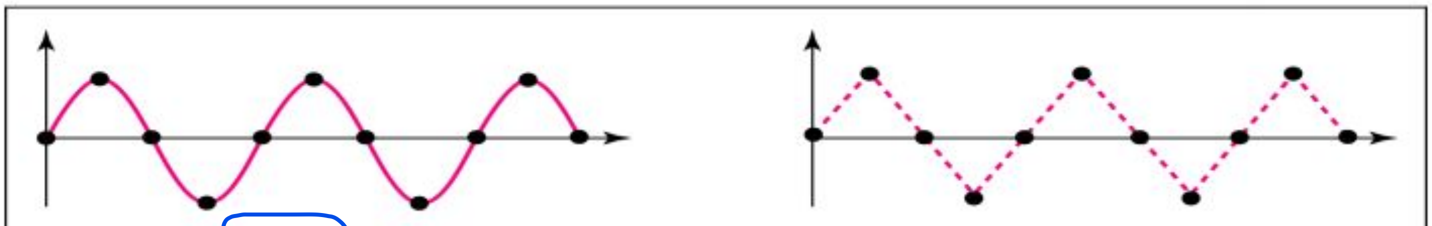
- According to the **Nyquist theorem**, the **sampling rate** must be at least **2 times the highest frequency** contained in the signal.



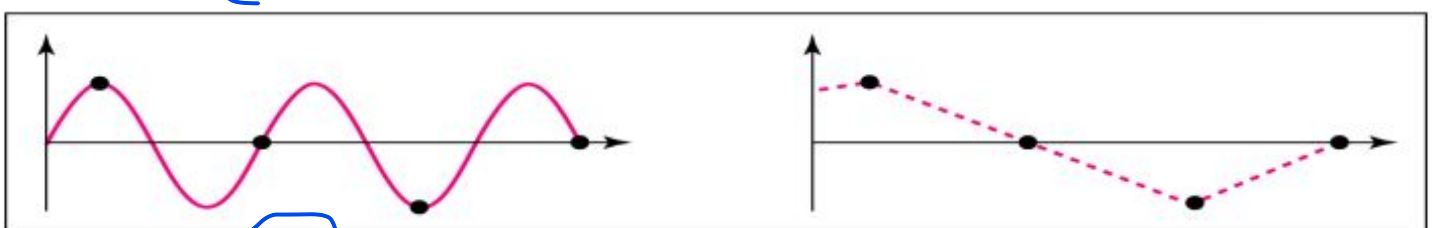
NYQUIST RATE



a. Nyquist rate sampling: $f_s = 2f$



b. Oversampling: $f_s = 4f$



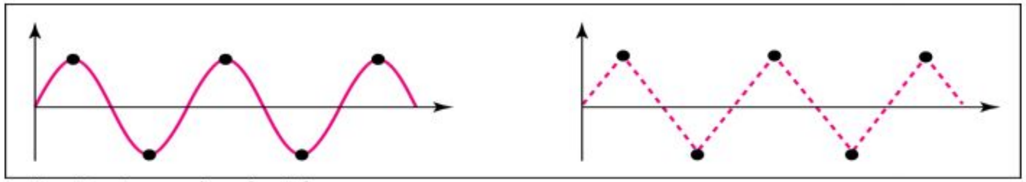
c. Undersampling: $f_s = f$

For an intuitive example of the Nyquist theorem, let us sample a sine wave at three sampling rates: f_s = 4f (2 times the Nyquist rate), f_s = 2f (Nyquist rate), and f_s = f (one-half the Nyquist rate).

NYQUIST RATE

It can be seen that sampling at the Nyquist rate can create a good approximation of the original sine wave (part a).

Oversampling in part b can also create the same approximation.



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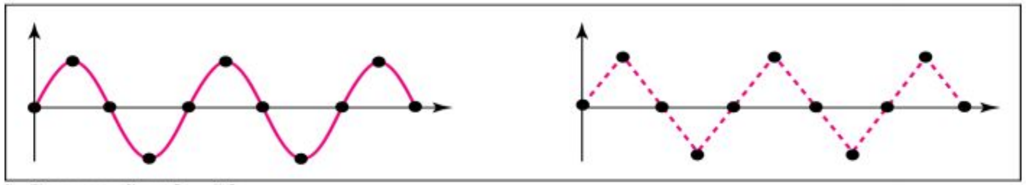

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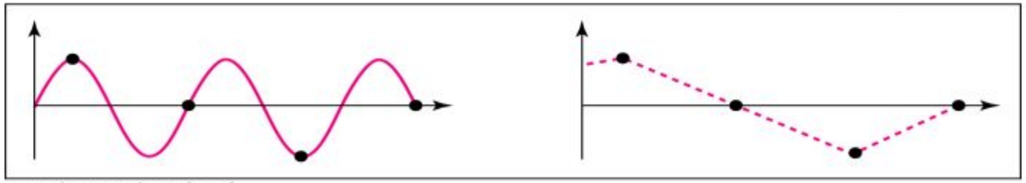


c. Undersampling: $f_s = f$

Sampling below the Nyquist rate (part c) does not produce a signal that looks like the original sine wave.

Telephone companies digitize voice by assuming a maximum frequency of 4000 Hz. The sampling rate therefore is 8000 samples per second.

MATHEMATICAL PROBLEMS

Q1: A complex bandpass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

Solution:

We cannot find the minimum sampling rate in this case because we do not know where the bandwidth starts or ends. Because the maximum frequency in the signal is unknown.

MATHEMATICAL PROBLEMS

Q2: A complex low-pass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

MATHEMATICAL PROBLEMS

Q3: A signal, $x(t) = \sin(2\pi 20t) + \sin(2\pi 50t) + \sin(2\pi 100t)$ is given. For successful reconstruction of the signal, what should be the minimum sampling frequency?

f_{max}

Solution:

In this mathematical problem, the existing frequencies are $f_1 = 20Hz$, $f_2 = 50Hz$ and $f_3 = 100Hz$. Thus, f_m =100Hz.

According to Nyquist theorem, $f_s = 2 \times f_m = 200\ Hz$

Therefore, minimum allowable sampling frequency for the successful reconstruction of the signal is $200\ Hz$

Quantization

- Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.

- The amplitude values are infinite between the two limits.

- We need to map the *infinite* amplitude values onto a finite set of known values.

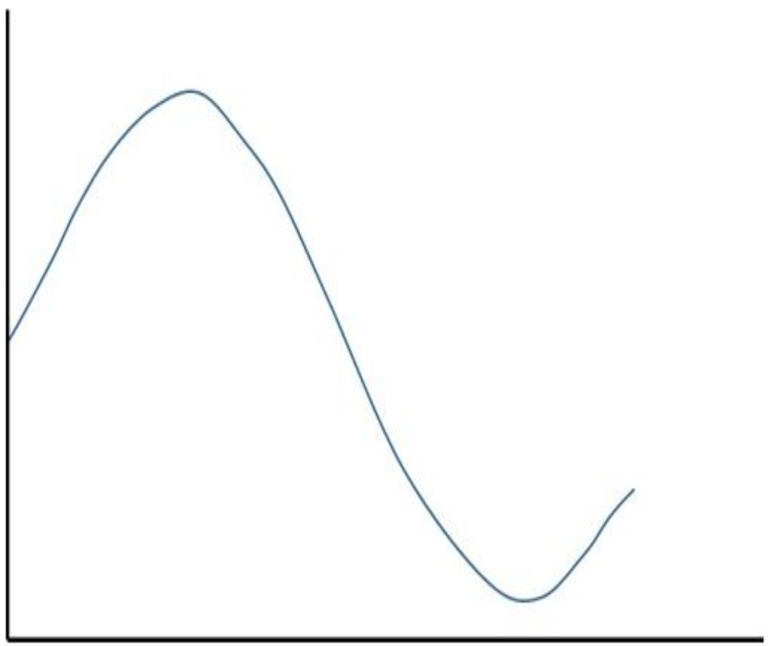
Quantization

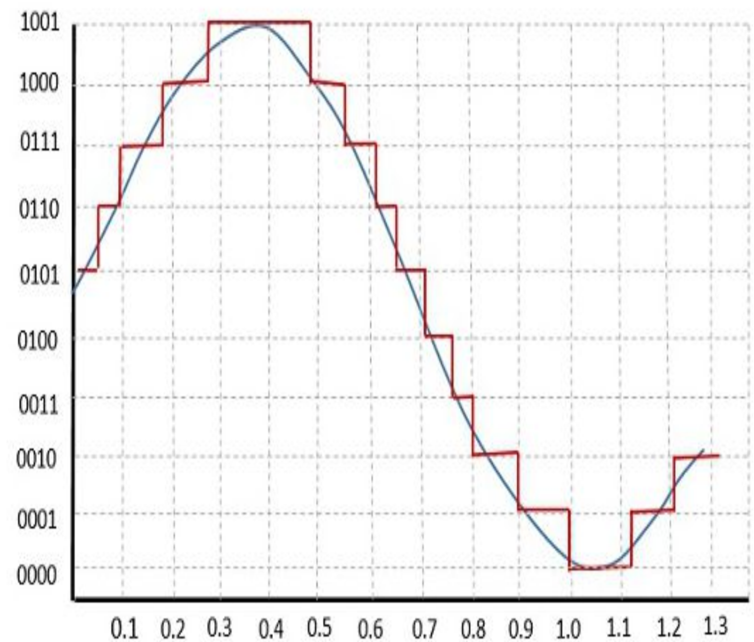
- This is achieved by dividing the distance between min and max into L zones, each of height Δ.

$$\boxed{\Delta = (max - min)/L}$$

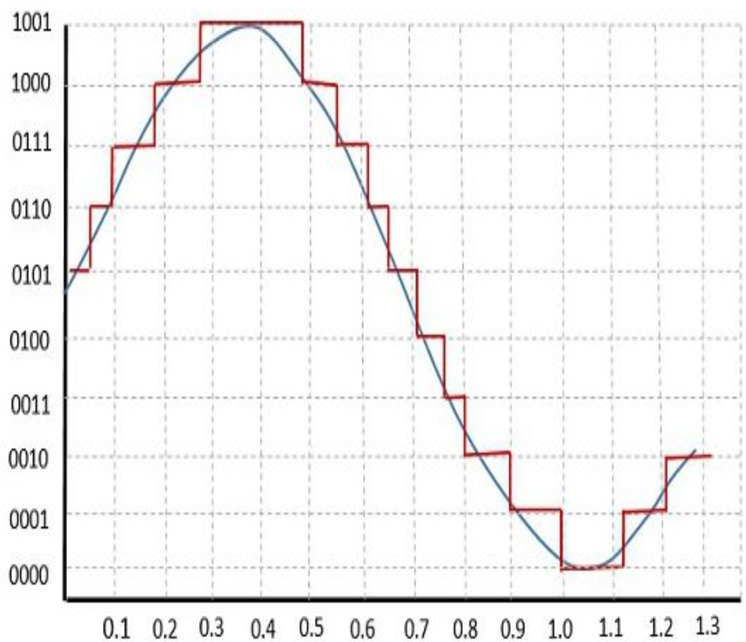
- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.

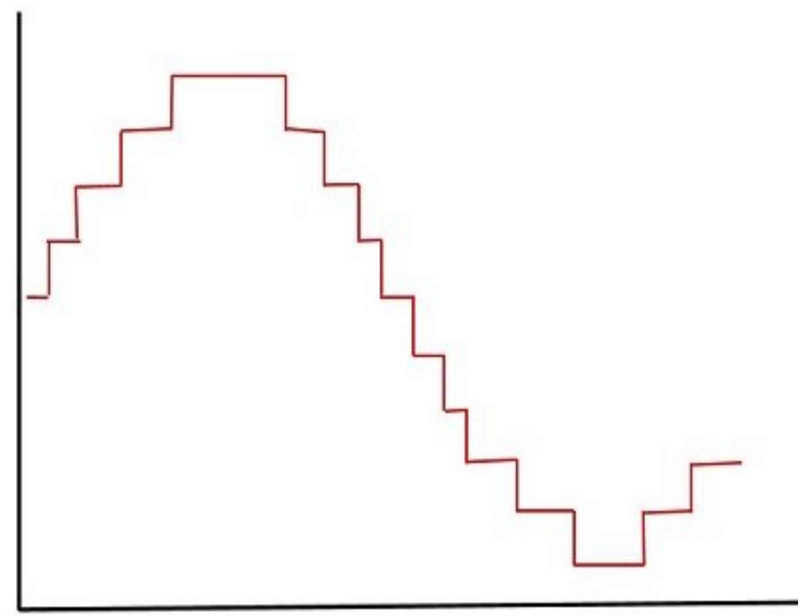
Quantizing an Analog Signal



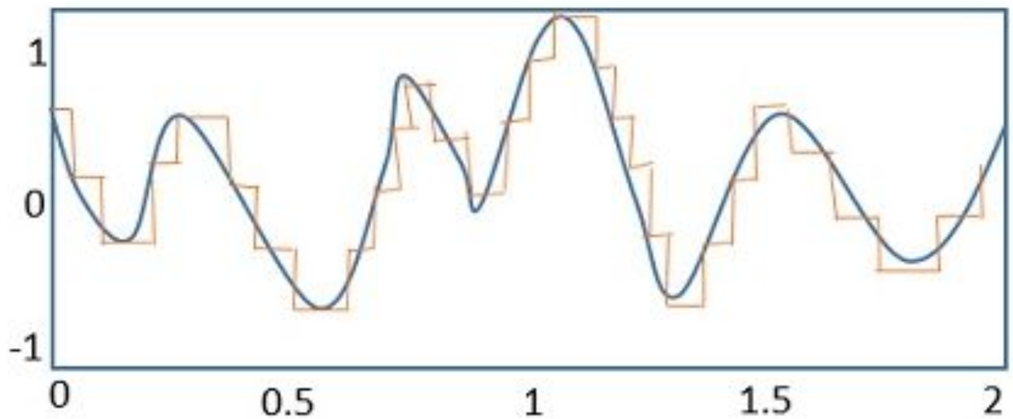


Quantizing an Analog Signal

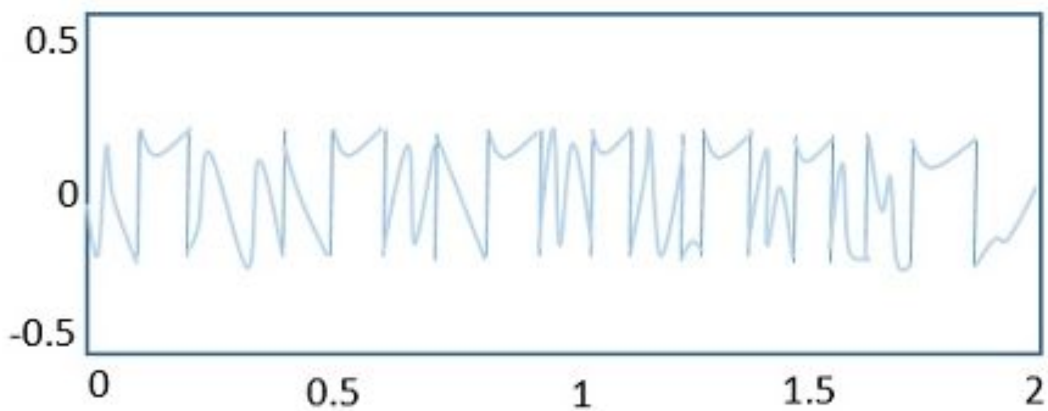




Quantizing an Analog Signal



Original
and
Quantized
Signal



Quantization
Error

Quantization Zones

- Assume we have a voltage signal with amplitudes V_{min}=-20V and V_{max}=+20V.

- We want to use L=8 quantization levels. Zone width $\Delta = (20 - -20)/8 = 5$

- The 8 zones are: -20 to -15, -15 to -10, -10 to -5, -5 to 0, 0 to +5, +5 to +10, +10 to +15, +15 to +20

- The midpoints are: -17.5, -12.5, -7.5, -2.5, 2.5, 7.5, 12.5, 17.5

Assigning Codes to Zones : Encoding

- Each zone is then assigned a binary code.
- The number of bits required to encode the zones, or the number of bits per sample as it is commonly referred to, is obtained as follows:

$$\boxed{n_b = \log_2 L}$$

- Given our example, $n_b = 3$
- The 8 zone (or level) codes are therefore: 000, 001, 010, 011, 100, 101, 110, and 111
- Assigning codes to zones:
  - 000 will refer to zone -20 to -15
  - 001 to zone -15 to -10, etc.

* n = log2L
* L= 2^n

Quantization Error

- When a signal is quantized, we introduce an error - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and coded value (midpoint) is referred to as the quantization error.
- The more zones, the smaller Δ which results in smaller errors.
- But, the more zones the more bits required to encode the samples -> higher bit rate

PCM Decoder

- To recover an analog signal from a digitized signal we follow the following steps:
  - We use a hold circuit that holds the amplitude value of a pulse till the next pulse arrives.
  - We pass this signal through a low pass filter with a cutoff frequency that is equal to the highest frequency in the pre-sampled signal.
- The higher the value of L, the less distorted a signal is recovered.

↑ L ↓ Distortion

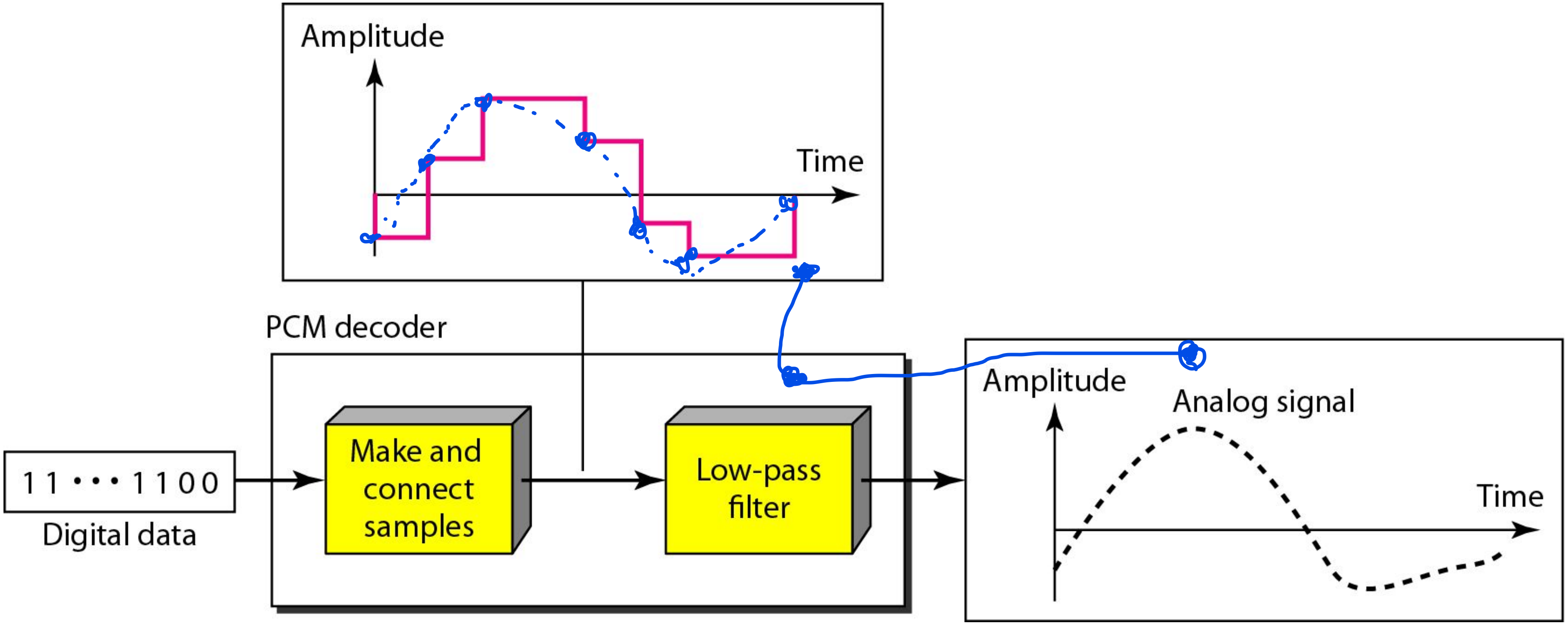


Figure 4.27 ***Components of a PCM decoder***