Homework 1

1. Find the value of a and b so that the polynomial $x^3 + ax^2 + bx + 15$ is divisible by $x^2 + 2x - 15$.
2. The LCM and HCF of two numbers are 156 and 4 respectively. If one number is divided my 3, the quotient is 17 and remainder is 1. Find the other number.
3. Find the smallest number which, on being added 23 to it, is exactly divisible by 32, 36, 48 and 96.
4. In a wedding ceremony there are 60 kids, 84 boys and 108 girls respectively. Find the maximum number of room to spend a night where in a room there will be same numbers of guest. Find the total number of guests in each room.
5. Find the least number of soldiers in a regiment such that they stand in rows of 15, 20, 25 and form a perfect square.
6. Find HCF and LCM of $35(x^4 – 27x)$ and $40(2x^3 – 5x^2 -3x)$
7. Factorize :
    a. $144x^2 – y^2$
    b. $54x^4 + 27x^3a – 16x – 8a$
    c. $x^2 + 14x + 49$
    d. $x^3 + 10x^2 − 48x$
    e. $(a – 1)x^2 + a^2 xy + (a + 1)y^2$
8. The atmospheric pressure p on a balloon or an aircraft decreases with increasing height. This pressure, measured in millimeters of mercury, is related to the height h (in kilometers) above sea level by the formula

$$p = 760e^{-0.145h}$$

    a. Find the atmospheric pressure at the height of 5 kilometers.
    b. Find the height of a mountain if the atmospheric pressure is 517 millimeters of mercury.

9. A herd of 20 white-tailed deer is introduced to a coastal island where there had been no deer before. Their population is predicted to increase according to the logistic curve where A is the number of deer expected in the herd after t years.

$$A = \frac{100}{1 + 4e^{-0.14t}}$$

   a. How many deer will be present after 2 years? After 6 years? Round answers to the nearest integer.
   b. How many years will it take for the herd to grow to 50 deer? Round answer to the nearest integer.

10. This law states that the rate at which an object cools is proportional to the difference in temperature between the object and its surrounding medium. The temperature T of the object t hours later is given by

$$T = T_m + (T_0 - T_m)e^{-kt}$$

Where T_m is the temperature of the surrounding medium and T_o is the temperature of the object at t = 0. Suppose a bottle of wine at a room temperature of 72^0 is placed in the refrigerator to cool before a dinner party. If the temperature in the refrigerator is kept at 40^0 and k = 0.4, find the temperature of the wine, to the nearest degree, after 3 hours.