LCM & HCF

Factors and Multiples: If a number 'm' divides another number 'n' exactly, then we say that 'm' is a factor of 'n' and that 'n' is a multiple of 'm'.

eg. 3 is a factor of 12 and therefore 12 is a multiple of 3.

Least Common Multiple (L.C.M.)

L.C.M. is the least non-zero number in common multiples of two or more numbers.

Multiple of 6 = 6, 12, 18, 24, 30, ...

Multiple of 8 = 8, 16, 24, 32, 40, ...

Common Multiple of 6 and 8 = 24, 48

....

Least Common Multiple = 24

Factorisation Method:

Find the L.C.M. of 12, 27 and 40

Factors of 12

= 2x2x3 = 2^2x3

Factors of 27

= 3x3x3 = 3^3

Factors of 40

= 2x2x2x5 = 2^3x5

2	12	3	27	2	40
2	6	3	9	2	20
3	3	3	3	2	10
	1		1	5	5
					1

∴ L.C.M. = 2^3 x 3^3 x5 =1080

SHORT CUT METHOD
(Division Method)

Find the L.C.M. of 12, 27, 40

2	12, 27, 40
2	6, 27, 20
3	3, 27, 10
	1, 9, 10

∴ L.C.M. = 2x2x3x9x10 = 1080

HIGHEST COMMON FACTOR (H.C.F)

The highest common factor of two or more numbers is the greatest number which divides each of them exactly.

eg. Find the H.C.F. of 24 and 56

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 56 = 1, 2, 4, 7, 8, 14, 28, 56

Common factors of 24 and 56 are 1, 2, 4, 8

∴ H.C.F. of 24 and 56 = 8

Factorisation Method: H.C.F. can be found by resolving the given numbers into prime factors and then taking the product of least powers of all common factors, that occur in these numbers.

Eg. Find H.C.F. of 48, 108, 140

Factors of 48

= 2x2x2x2x3

= 2^4x3

Factors of 108

= 2x2x3x3x3 = 2^2x3^3

2	48	2	108	2	140
2	24	2	54	2	70
2	12	3	27	5	35
2	6	3	9		7
	3		3		

Factors of 140

= 2x2x5x7 = 2^2 x 5 x 7

H.C.F. = 2^2 = 4

Division Method

Find the H.C.F. of 48, 108, 140

2	48, 108, 140
2	24, 54, 70
	12, 27, 35

H.C.F.=2x2= 4

QUICKER & SHORT CUT METHOD

Find the H.C.F. of 777 and 1147

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777) 1147 (1
     777
     ———
     370) 777(2
          740
          ———
          37) 370 (10
              370
              ———
                0
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H.C.F. of 777 and 1147 is 37

* The product of two given numbers is equal to the product of their H.C.F. and L.C.M.

L.C.M. of two numbers

$$= \frac{\text{Product of numbers}}{\text{H.C.F. of numbers}}$$

L.C.M. of given fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

H.C.F of given fractions

$$= \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

♦ The L.C.M of a given set of numbers would be either the highest or higher than the highest of the given numbers.

♦ The H.C.F. of a given set of numbers would be either the lowest or lower than the lowest.

Solved Examples

1. Find the L.C.M. of 125,64,8 and 3.

Ans : Given numbers are 5^3, 2^6, 2^3 and 3

∴ L.C.M. $5^3 \times 2^6 \times 3 = 24,000$

2. Find the L.C.M. of $\frac{1}{3}, \frac{5}{6}, \frac{5}{9}, \frac{10}{27}$?

Ans: L.C.M. of fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

L.C.M. of 1, 5 and 10 is 10

H.C.F of 3, 6, 9 and 27 is 3

L.C.M. of given fractions $= \dfrac{10}{3}$

3. Find the H.C.F. of $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$

Ans: H.C.F. of fractions

$$= \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

H.C.F. of 1, 3, 5, 7 and 9 is 1

L.C.M of 2, 4, 6, 8 and 10 is 120

H.C.F. of given fractions $= \dfrac{1}{120}$

4. The L.C.M. of two number is 2310. Their H.C.F. is 30. If one number is 210, the other is:

Ans: The other number

$$= \frac{\text{L.C.M.} \times \text{H.C.F.}}{\text{given number}} = \frac{2310 \times 30}{210} = 330$$

5. The H.C.F. and L.C.M. of two numbers are 44 and 264 respectively. If the first number is divided by 2, the quotient is 44, The other number is

Ans: First number = 2x44 = 88

$$\text{Second number} = \frac{44 \times 264}{88} = 132$$

6. The least square number which is divisible by 6, 8 and 15 is:

Ans: The least number divisible by 6, 8 and 15 is their L.C.M. which is 120

Now 120 = 2x2x2x3x5

To make it a perfect square, it mus multiplied by 2x3x5

∴ Required Number=120x2x3x5=3600

7. The least number of square tiles required to pave the ceiling of a room 15ı 17cm long and 9m. 2cm broad is:

Ans: Size of largest square tile

= H.C.F. of 1517 cm and 902 cm

= 41 cm.

∴ Least number of tiles required

$$= \frac{\text{Area of the room}}{\text{Area of one tile}}$$

$$= \frac{1517 \times 902}{41 \times 41} = 814$$

8. Find the least number which when divided separately by 15, 20, 36 and 48 leaves 3 as remainder in each case.

Ans : Required number

= L.C.M. of (15,20,36 and 48) +3

= 720 + 3 = 723

9. Find the greatest number that will divide 197 and 269 and leaves 5 as remainder in each case.

Required number = H.C.F. of [(197-5) and (269-5)]

= H.C.F. of (192 and 264) = 8

12. Five bells begin to toll together and toll respectively at intervals of 6,7,8,9 and 12 seconds. How many times they will toll together in one hour, excluding the one at the start?

Ans: L.C.M. of 6,7,8,9 and 12

= 2x2x3x7x2x3 = 504

ie, The bells will toll together after each 504 seconds. In one hour, they will toll together

$$= \frac{60 \times 60}{504} = 7 \text{ times}$$