Chapter 2

HCF and LCM

Factors of algebraic expression is obtained and then common factor of the expression is collected. This common factors is the H.C.F.

To find the L.C.M. we find the common factors and then we multiply then by the left out factors.

Example 1. If P(x) = $(x^3 + 1)(x - 1)$ and Q(x) = $(x^2 - x + 1)(x^2 - 3x + 2)$, find the GCD and LCM of P(x) and Q(x).

Solution:

$$p(x) = (x^3 + 1)\ (x - 1)$$
$$= (x+1)(x^2 - x + 1(x - 1)$$

$$Q(x) = (x^2 - x + 1)\ (x^2 - 3x + 2)$$
$$= (x^2 - x + 1)(x^2 - 2x - x + 2)$$
$$= (x^2 - x + 1)(x - 1)(x - 2)$$

GCD $= (x - 1)\ (x^2 - x + 1)$

LCM $= (x - 1)(x^2 - x + 1(x + 1)(x - 2)$

$$= (x^3 + 1)(x^2 - 3x + 2)$$

Example 2. The LCM and GCD of two polynomials, P(x) and Q(x) are $2(x^4 - 1)$ and $(x + 1)(x^2 + 1)$ respectively. If P(x) = $x^3 + x^2 + x + 1$, find Q(x) .

Solution:

$$p(x) = x^3 + x^2 + x + 1$$
$$= x^2(x + 1) + 1(x + 1)$$
$$= (x + 1)(x^2 + 1)$$

$$Q(x) = \frac{LCM \times GCD}{P(x)}$$
$$= \frac{2(x^4 - 1)\ \ (x+1)(x^2+1)}{(x+1)(x^2+1)}$$
$$= 2(x^4 - 1) = 2x^4 - 2$$

Example 3. (x - k) Is the g.c.d. of $x^2 + x - 12$ and $2x^2 - kx - 9$. Find the value of k.

Solution:- Let $P(x) = x^2 + x - 12$ and $Q(x) = 2x^2 - kx - 9$

$x - k$ is the GCD. $\therefore\ x - k$ is

factor of P(x) and Q(x)

$\therefore\ P(k) = 0\ and\ Q(k) = 0$

Putting x = k in P(x) we get

$$k^2 + k - 12 = 0$$
$$or,\quad (k+4)(k-3) = 0$$
$$or,\quad k = -4, 3 - - - - - - - - -(1)$$

Putting x = k in Q(x) we get

$$2k^2 - k^2 - 9 = 0$$
$$Or,\quad k^2 - 9 = 0$$
$$Or,\quad k^2 = 9$$
$$Or,\quad k = \pm 3 - - - - - - -(2)$$

from (i) and (ii) we get k = 3.

Example 4. $(x^2 + x - 2)$ is the G.D.C. of the expression $(x - 1)(2x^2 + ax + 2)$ and $(x + 2)(3x^2 + bx + 1)$. Find the value of a and b.

Solution:- $x^2 + x - 2 = (x + 2)(x - 1)$

Let $P(x) = (x - 1)(2x^2 + ax + 2)$

and $Q(x) = (x + 2)(3x^2 + bx + 1)$

Let us take P(x), here we find that x + 2 is a factor of $2x^2 + ax + 2$.

i.e. $2(-2)^2 + a(-2) + 2 = 0$

or, $-2a + 10 = 0$

$\therefore$ $a = 5$

Similarly $x - 1$ is a factor of $3x^2 + bx + 1$

$\therefore$ $3(1)^2 + b.1 + 1 = 0$

$b = -4$

$\therefore$ $a = 5, b = -4$.

Example 5. Determine the value of m such that $(x - 5)$ is a factor of the polynomial

$f(x) = 3x^3 - 16x^2 + mx + 50$

Solution: $f(x) = 3x^3 - 16x^2 + mx + 50$

$(x - 5)$ is a factor of $f(x)$

$\therefore$ $f(5) = 0$

Or, $3(5)^3 - 16(5)^2 + m(5) + 50 = 0$

Or, $375 - 400 + 5m + 50 = 0$

Or, $5m + 25 = 0$

$\therefore$ $m = -5$

Example 6. Find the value of a and b so that the polynomial $x^3 + ax^2 + bx - 6$ is completely divisible by $x^2 - 4x + 3$.

Solution: $x^2 - 4x + 3 = x^2 - 3x - x + 3$

$= x(x - 3) -1(x - 3)$

$(x - 3)(x - 1)$

Let $f(x) = x^3 + ax + bx - 6$

as $x - 3$ is a factor of $f(x)$

$\therefore$ $f(3) = 0$

i.e. $3^3 + a(3)^2 + b(3) - 6 = 0$

Or, $27 + 9a + 3b - 6 = 0$

Or, $9a + 3b = -21$

Or, $3a + b = -7$ -------------(i)

Also, $x - 1$ is a factor of $f(x) \cdot f(1) = 0$

i. e. $1^3 + a(1)^2 + b(1) - 6 = 0$

Or, $a + b = 5$ ---------------(ii)

Subtracting (i) from (ii) we get $-2a = 12 \cdot a = -6$.

Putting $a = -6$ in (ii) we get

$b = 5 - b = 5 - (-6)$

$= 5 + 6 = 11$

$\therefore\ a = -6, b = 11$

Exercise - 4

1. Find the HCF of the following:

1. $2x^4 - 2y^4$ and $3x^3 + 6x^2y - 3xy^2 - 6y^3$
2. $12(x^3 + x^2 + x + 1)$ and $18(x^4 - 1)$
3. $x^3 + 2x^2 - 3x$ and $2x^3 + 5x^2 - 3x$
4. $2(x^4 - y^4)$ and $3(x^3 + 2x^2y - xy^2 - 2y^3)$
5. $18(x^3 - x^2 + x - 1)$ and $12(x^4 - 1)$
6. $45(x^4 - x^3 - x^2)$ and $75(8x^5 + x^2)$
7. $36(3x^4 + 5x^3 - 2x^2)$ and $54(27x^4 - x)$
8. $42(2x^3 - 5x^2 - 3x)$ and $60(8x^4 + x)$
9. $4(x^4 - 1)$ and $6(x^3 - x^2 - x + 1)$

2. Find the LCM of the following polynomials:

1. $35(x^4 - 27x)$ and $40(2x^3 - 5x^2 - 3x)$
2. $20(2x^3 + 3x^2 - 2x)$ and $45(x^4 + 8x)$
3. $15(4x^3 - 4x^2 + x)$ and $35(2x^2 - 7x + 3)$
4. $25(x^2 + 7x + 12)$ and $15x(x^2 - 16)$
5. $x(8x^3 + 27)$ and $2x^2 (2x^2 + 9x + 9)$

3. Find the GCD and LCM of the polynomials P(x) and Q(x), where

$P(x) = (x^3 - 27) (x^2 - 3x + 2)$ and

$Q(x) =(x^2 + 3x + 9) (x^2 - 5x + 6)$

4. The LCM and GCD of two polynomials, P(x) and Q(x) are $56(x^4 + x)$ and $4(x^2 - x + 1)$ respectively. If $P(x) = 28(x^3 + 1)$, find Q(x) .

5. For what value of k, the g.c.d. of $x^2 + x - (2k + 2)$ and $2x^2 + kx - 12$ is $x + 4$?

6. Find the value of K for which the g.c.d. of $x^2 - 2x - 24$ and $x^2 - kx - 6$ is $x - 6$.

7. $(x^2 - x - 6)$ is the GCD of the expression $(x + 2)(2x^2 + ax + 3)$ and $(x - 3)(3x^2 + bx + 8)$. Find the value of a and b.

8. $(x + 1)(x - 4)$ is the g.c.d. of the polynomials $(x - 4)(2x^2 + x - a)$ and $(x + 1)(2x^2 + bx - 12)$ find a and b .

9. $(x - 3)$ is the g.c.d. of $(x^3 - 2x^2 + px + 6)$ and $(x^2 - 5x + q)$. Find $(6p + 5q)$.

10. Find the value of a and b so that the polynomials P(x) and Q(x) have $(x - 1)(x + 4)$ as their HCF:

$$P(x) = (x^2 - 3x + 2)(x^2 + 7x + a)$$

$$Q(x) = (x^2 + 5x + 4)(x^2 - 5x + b)$$

11. Find the value of a and b so that the polynomial $x^3 + ax^2 + bx + 15$ is divisible by $x^2 + 2x - 15$.

12. Find the value of p and q so that the polynomial $f(x) = px^3 + 2x^2 - 19x + 9$ is divisible by $x^2 + x - 6$.

13. Determine the value of k such that $x + 3$ is a factor of the polynomial

$$f(x) = kx^3 + x^2 - 22x - 21$$

14. If $x - 2$ is a factor of $x^2 + ax - 6 = 0$ and $x^2 - 9x + b = 0$, find the value of a and b.

Answers

1. (i) $x^2 - y^2$ $(x^2 + 1)$ (ii) $6(x + 1)$
(iii) $x(x + 3)$ (iv) $x^2 - y^2$
(v) $6(x - 1)(x^2 + 1)$ (vi) $15x^2(2x + 1)$
(vii) $18x(3x - 1)$ (viii) $6x(2x + 1)$
(ix) $2(x^2 - 1)$

3. G. C. D. = $(x^3 - 27)(x - 2)$

2. (i) $280x(x - 3)(2x + 1)(x^2 + 3x + 9)$
(ii) $180x(x + 2)(2x - 1)(x^2 - 2x + 4)$
(iii) $105x((x - 3)(2x - 1)^2$
(iv) $75x(x^2 - 16)(x + 3)$
(v) $2x^2(8x^2 + 27)(x + 3)$

4. $8(x^3 - x^2 + x)$

6. K = 5

LCM = (x - 1) (x - 2) (x - 3) (x^2 + 3x + 9)

5. K = 5

7. a = -7, b = 10

9. 0

11. a = 1, b = -17

13. K = 2

8. a = 1, b= -5

10. a = 12, b = 4

12. p = 3, q = 6

14. a = 1, b = 14