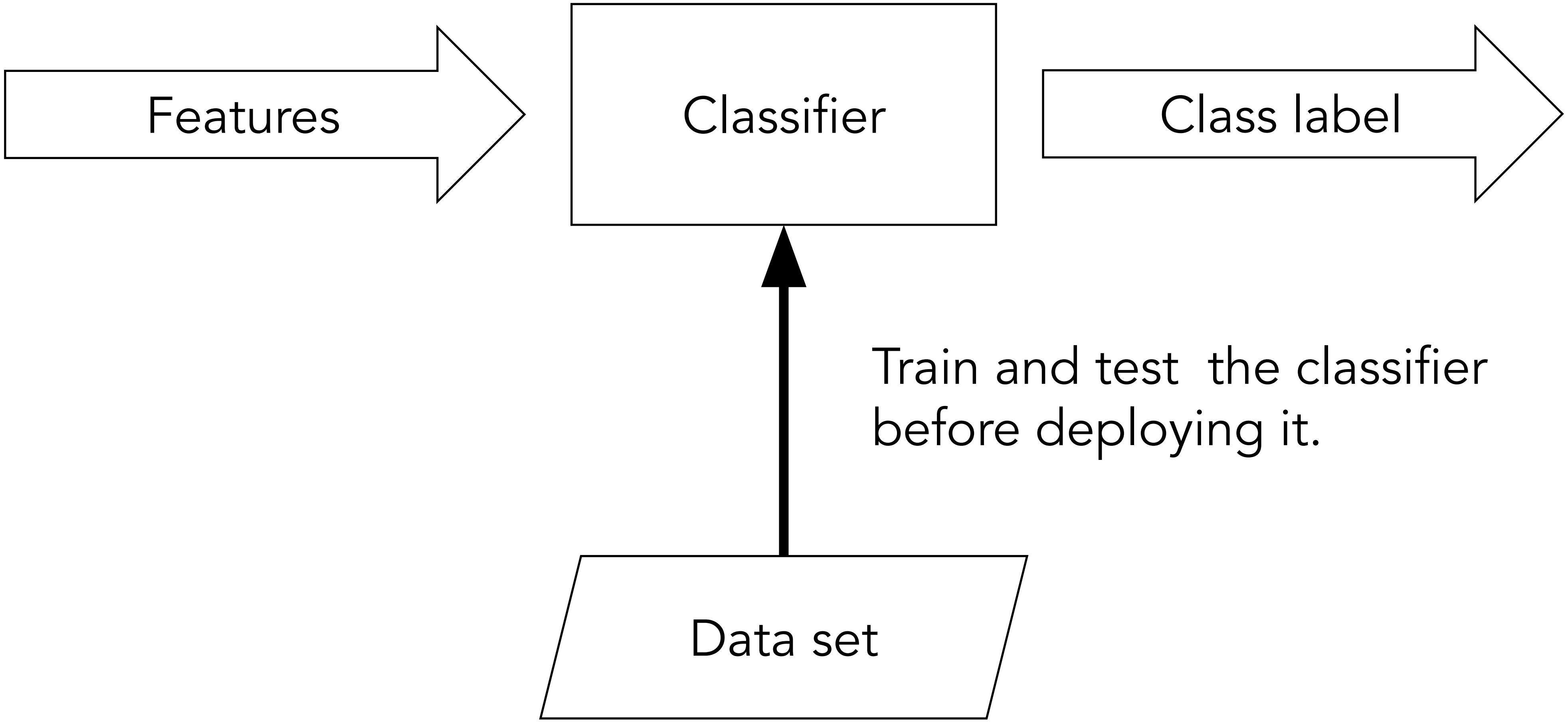
CSE-435 Pattern Recognition

Classification regions and discriminant functions

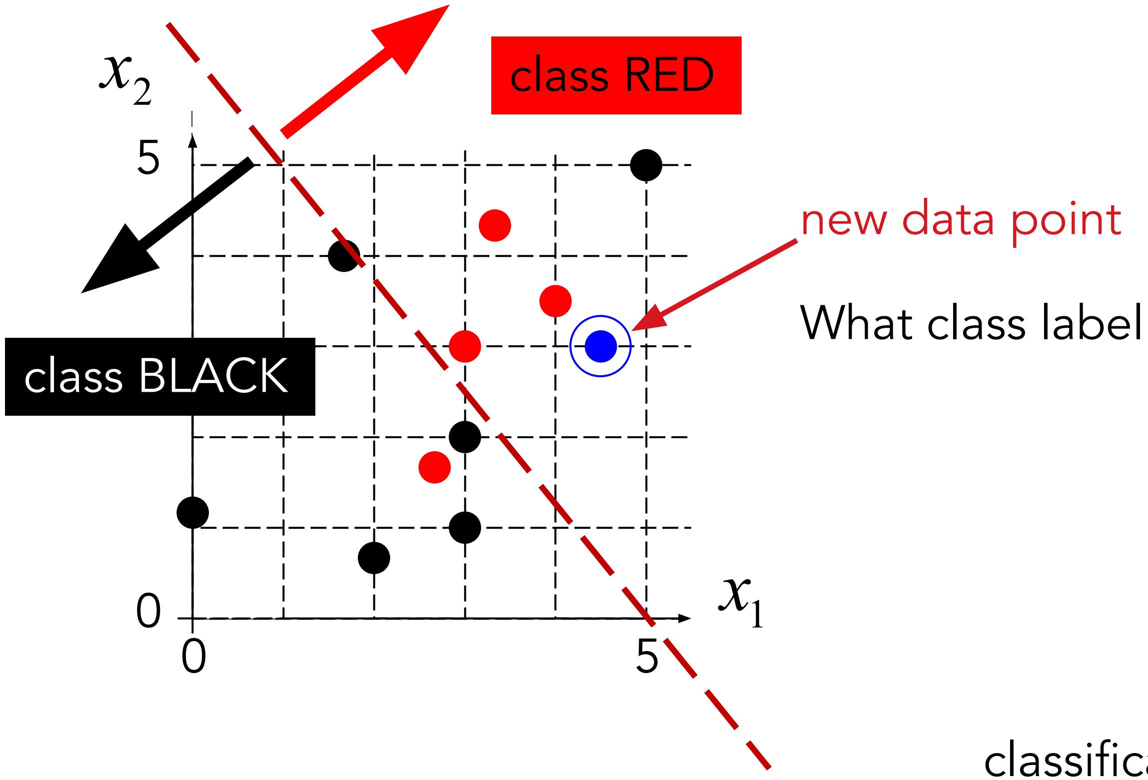
[redacted]

Classification regions

- What is a classifier?
- What is a classification region?
- Examples


Features
Classifier
Class label
Train and test the classifier
before deploying it.
Data set

A classifier is any function, method or algorithm that assigns a class label to any given object.

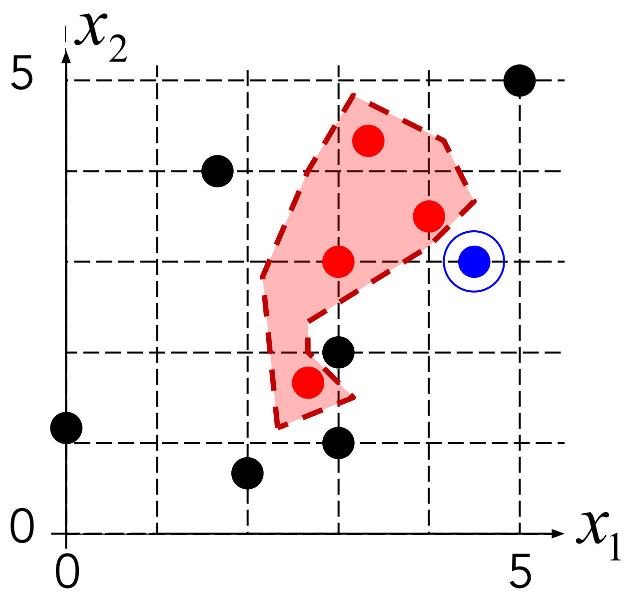


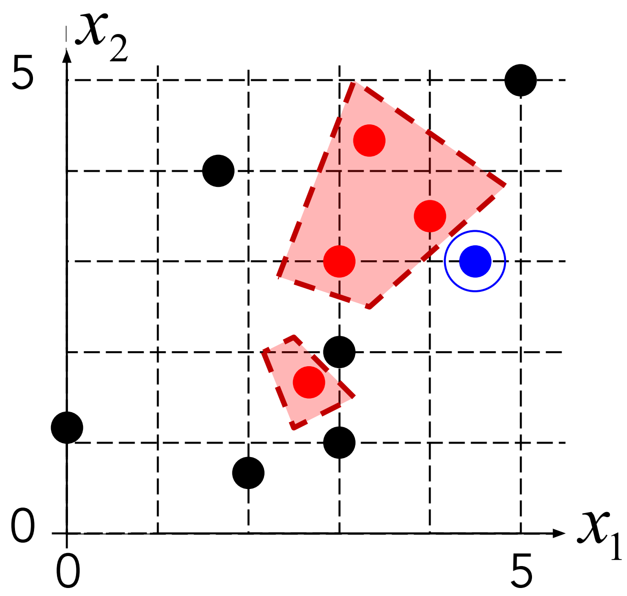
What class label should we assign?

classification regions for our linear classifier

The classification regions may consist of disjoined parts, and may be of any shape

piece-wise linear classifier



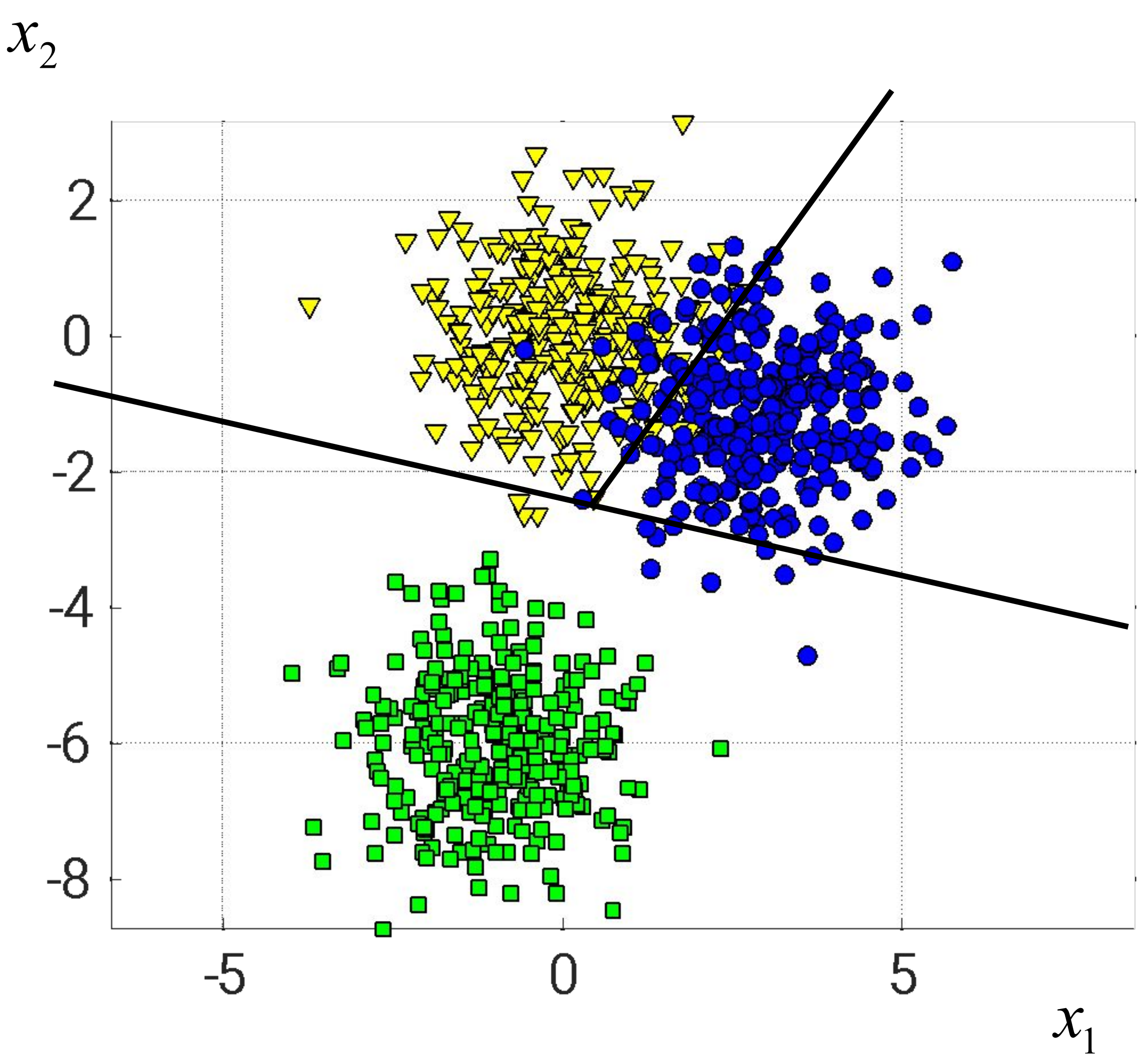


Notice the connection:

classifier = classification regions

(one determines the other)

Find classification regions (= find a classifier)

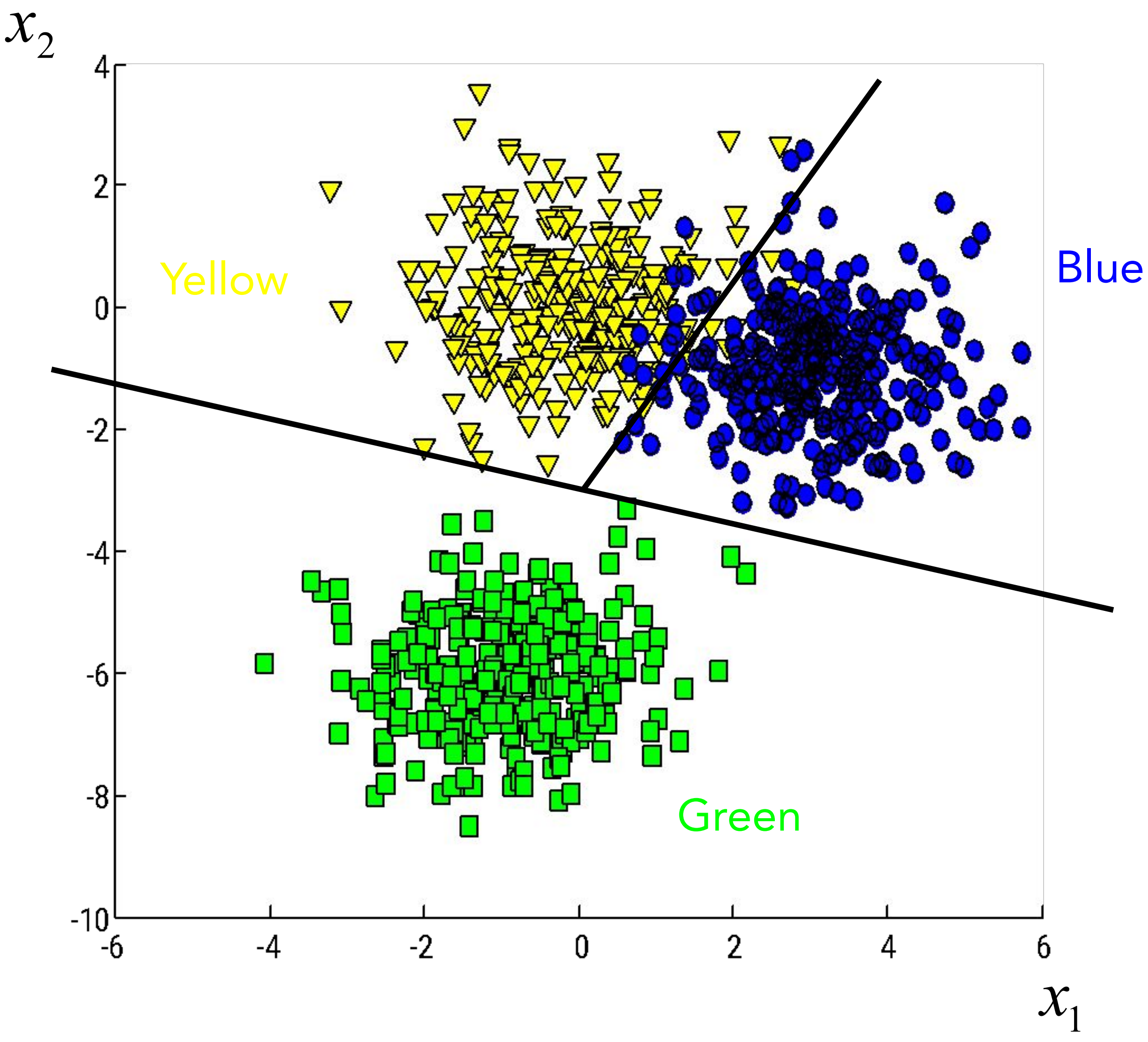


c = 3 classes
n = 2 features
N = 900 objects

linear classifier

EASY IN 2D!

Find classification regions (= find a classifier)

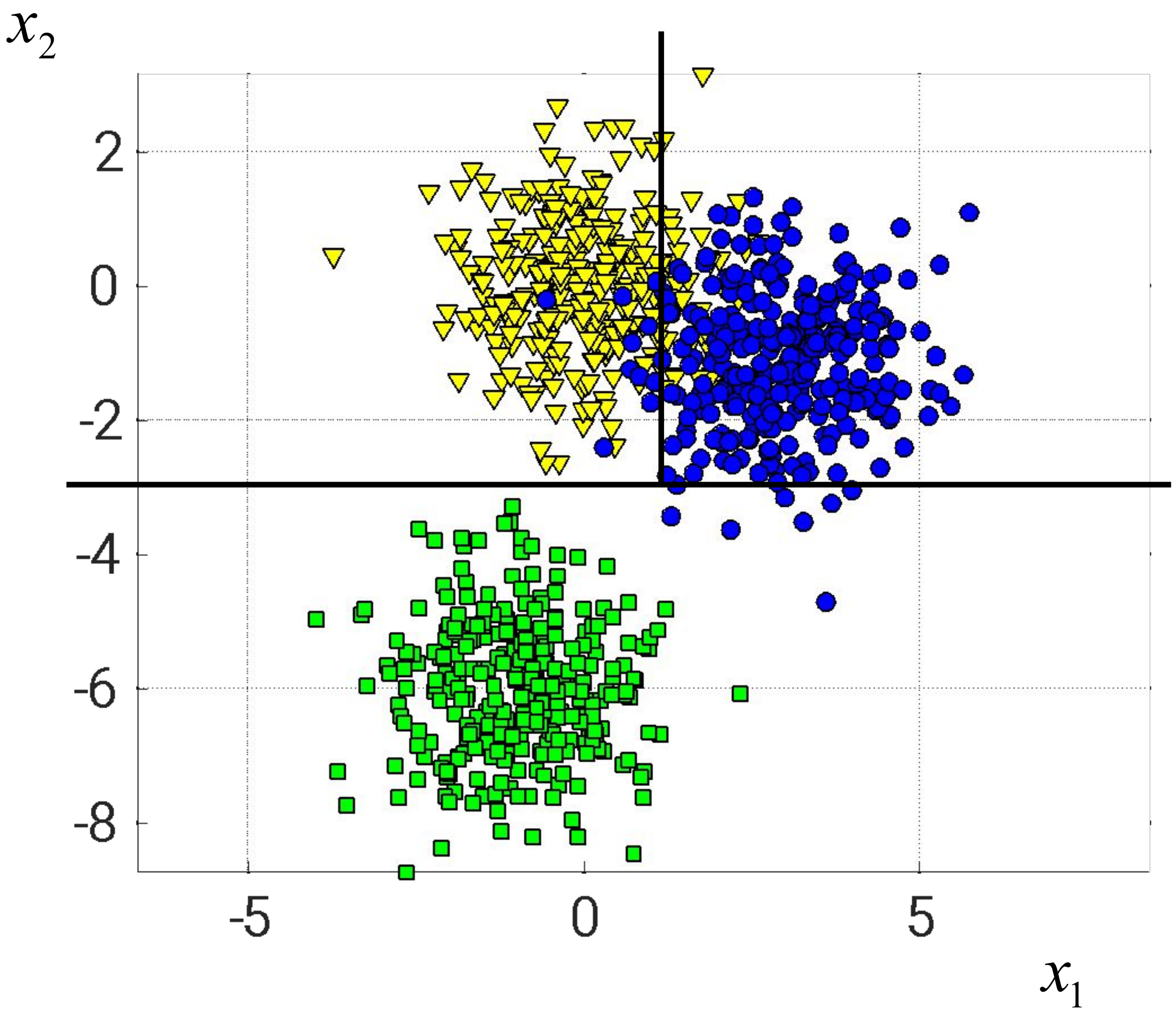


c = 3 classes
n = 2 features
N = 900 objects

linear classifier

classification
regions

Find classification regions (= find a classifier)

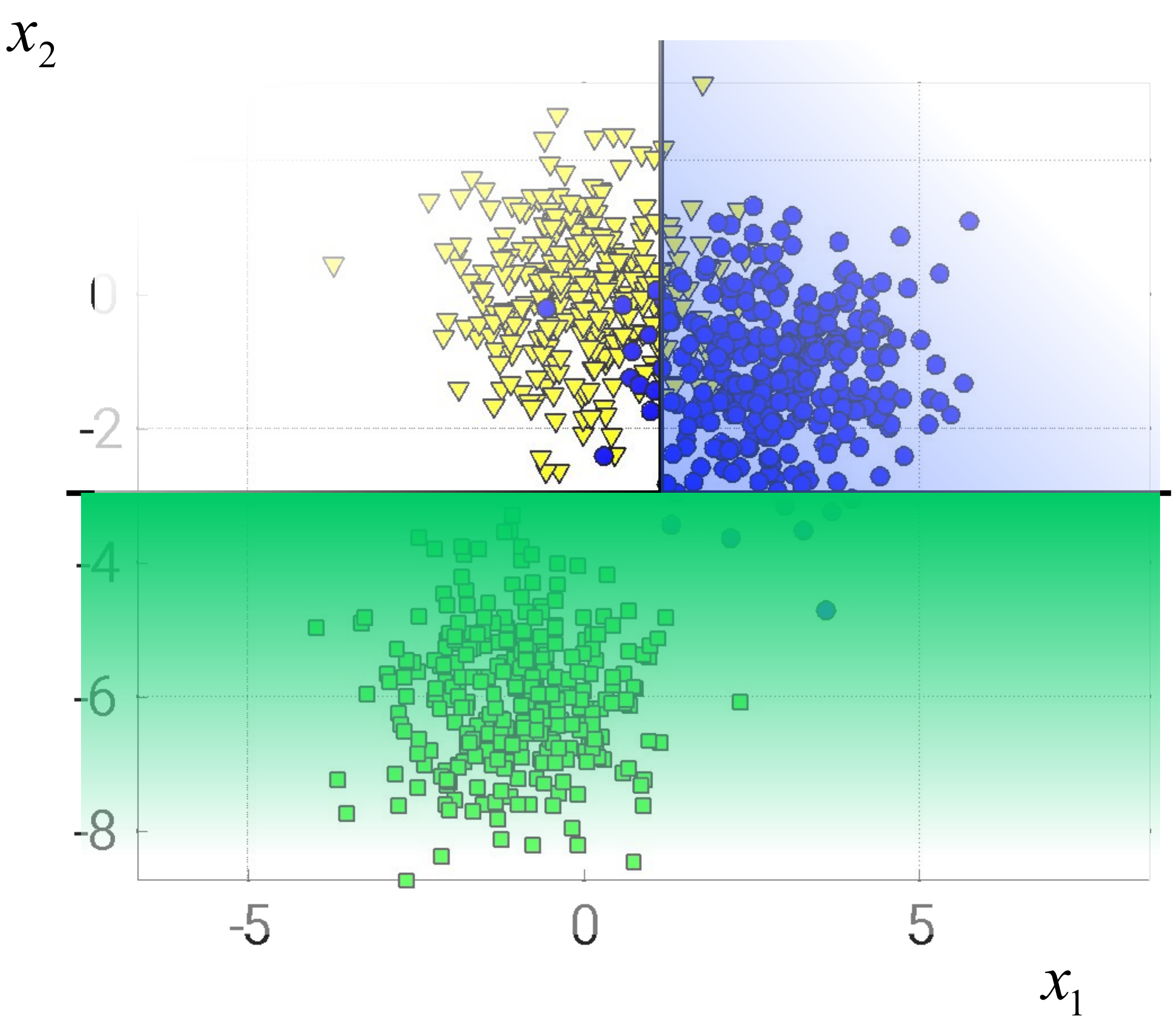


One possible classifier:

If $x_2 < -3.5$ then GREEN
else
    if $x_1 < 1.5$ then
    YELLOW
    else
    BLUE
End

rule-based classifier

Find classification regions (= find a classifier)



One possible classifier:

If $x_2 < -3.5$ then GREEN
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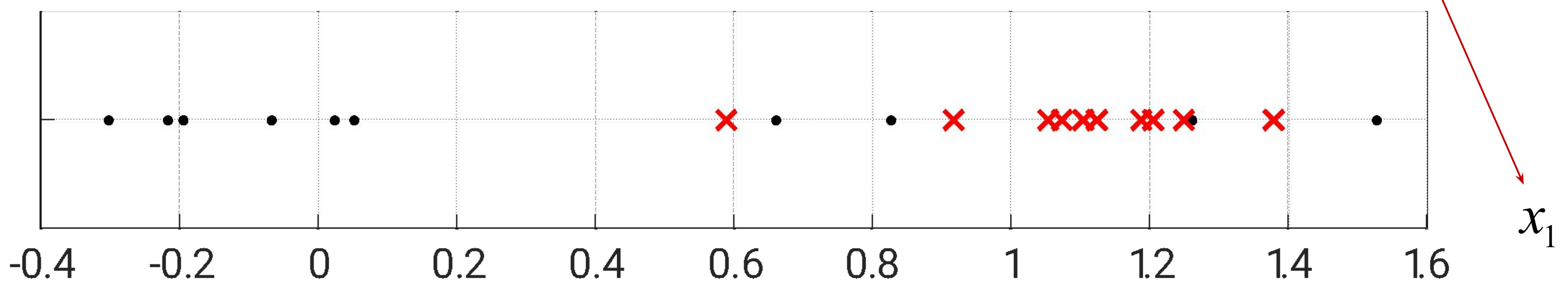
End

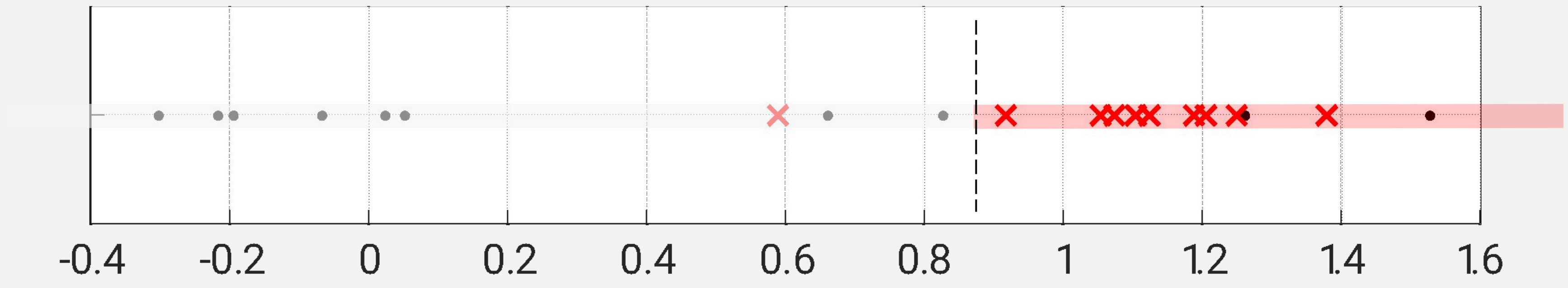
rule-based classifier

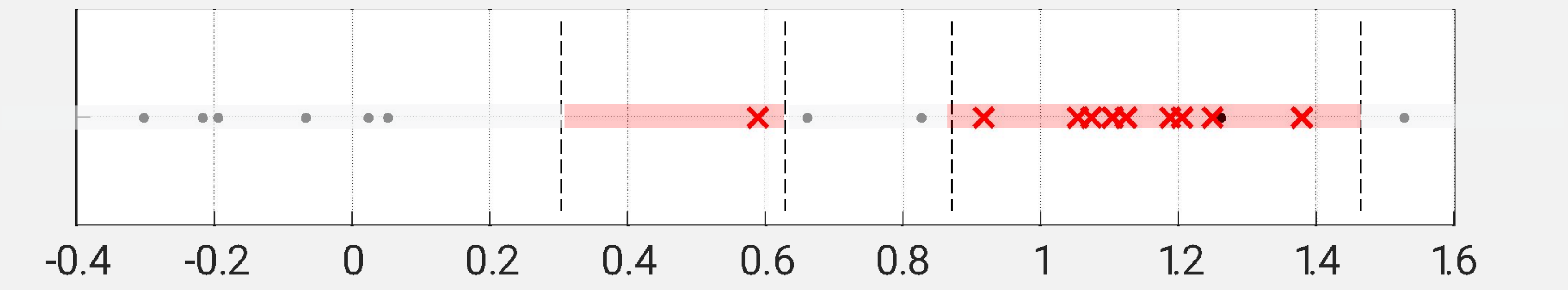
Find classification regions (= find a classifier)

One feature!



Two possible classifiers = different classification regions

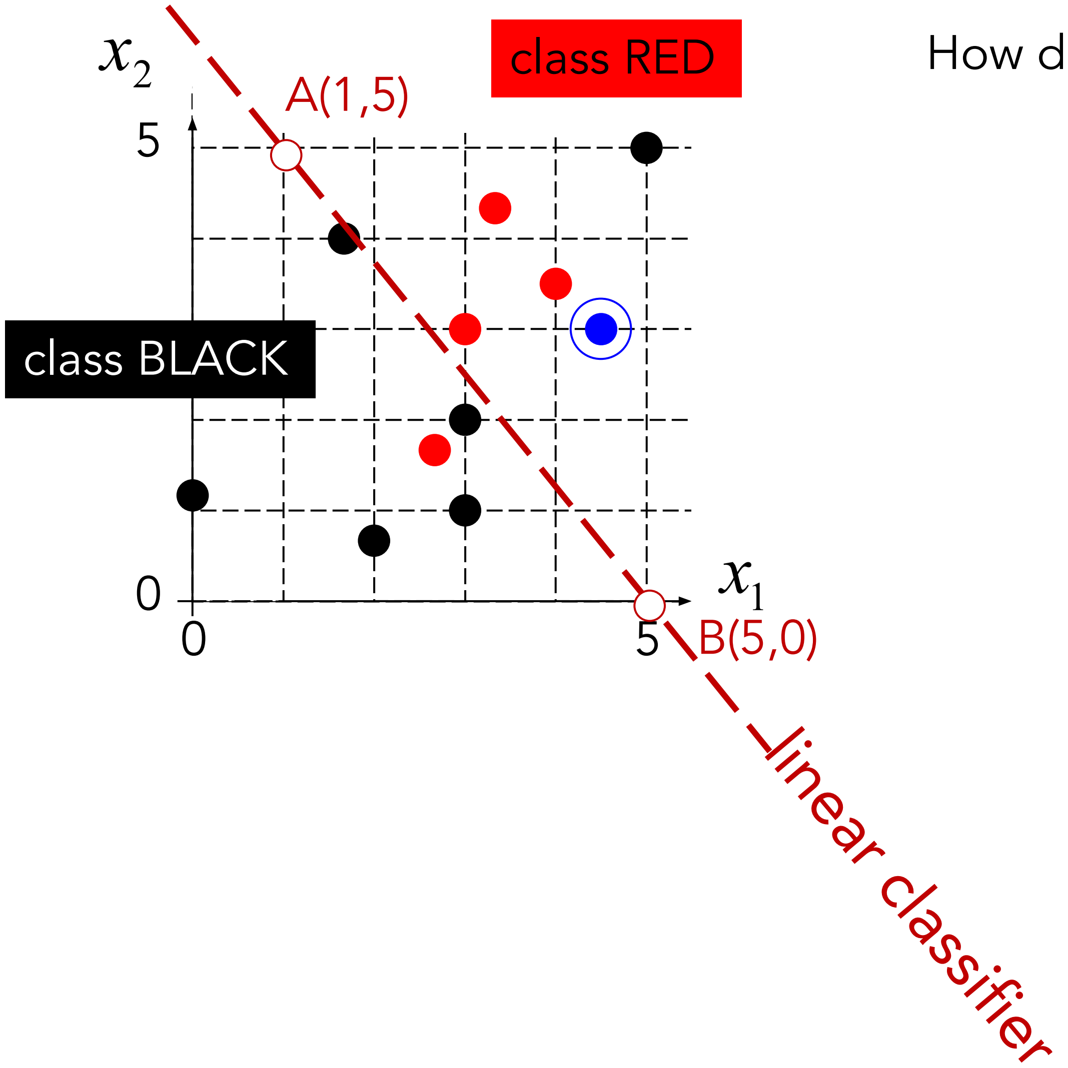




Discriminant functions

- What is a discriminant function?
- How do we calculate a linear discriminant function in 2D?
- The canonical model of a classifier

Recall: A **classifier** is any function, method or algorithm that assigns a class label to any given object.



How do we "design" the classifier?

Pick points A and B and find the line through them.

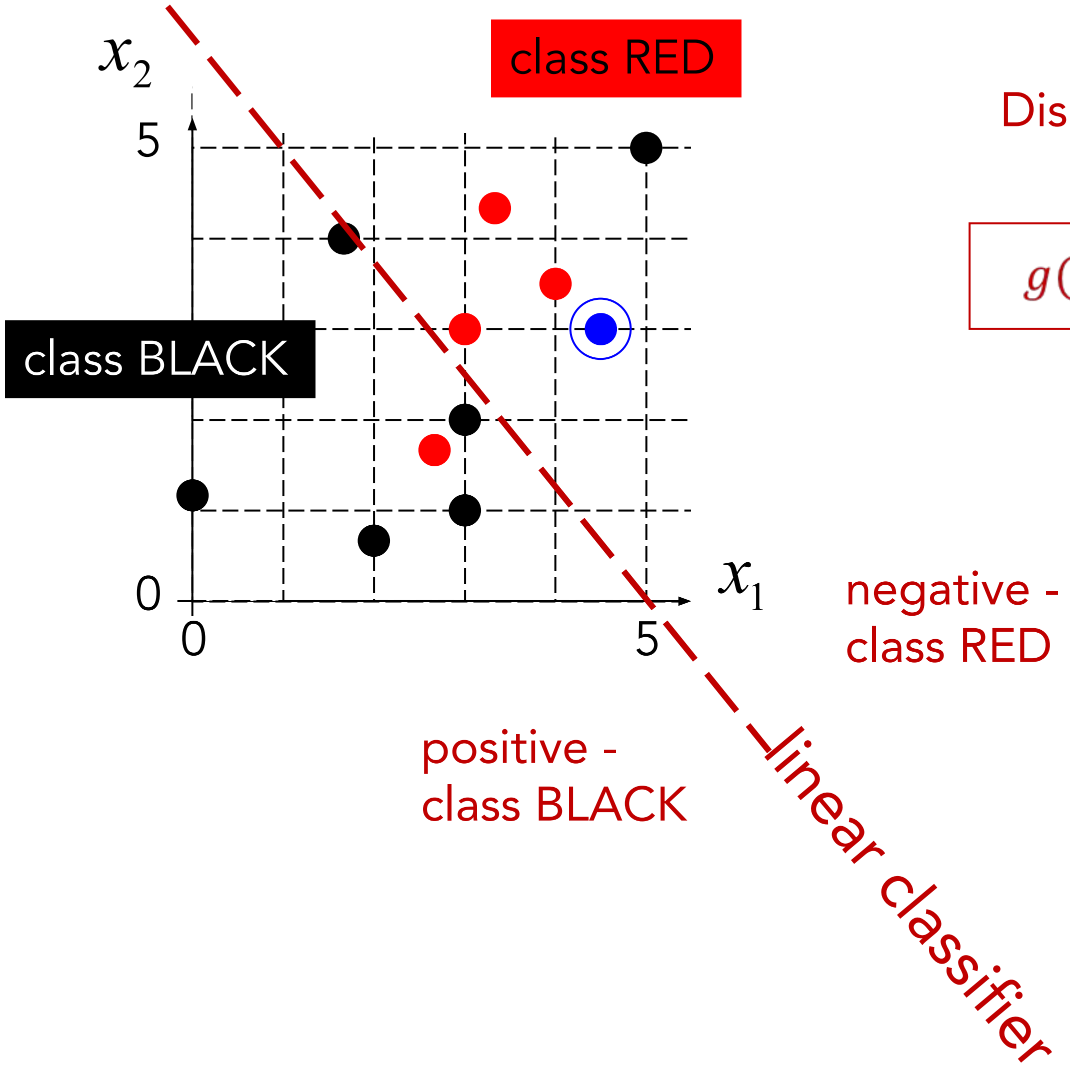
$$\frac{x-1}{5-1} = \frac{y-5}{0-5}$$

$$-5(x-1) = 4(y-5)$$

$$-5x + 5 - 4y + 20 = 0$$

$$-5x - 4y + 25 = 0$$

A discriminant function takes the feature values for an object and outputs a value that is used to determine the class label of the object.



Discriminant function

$$\boxed{g([x_1, x_2]^T) = -5x_1 - 4x_2 + 25}$$

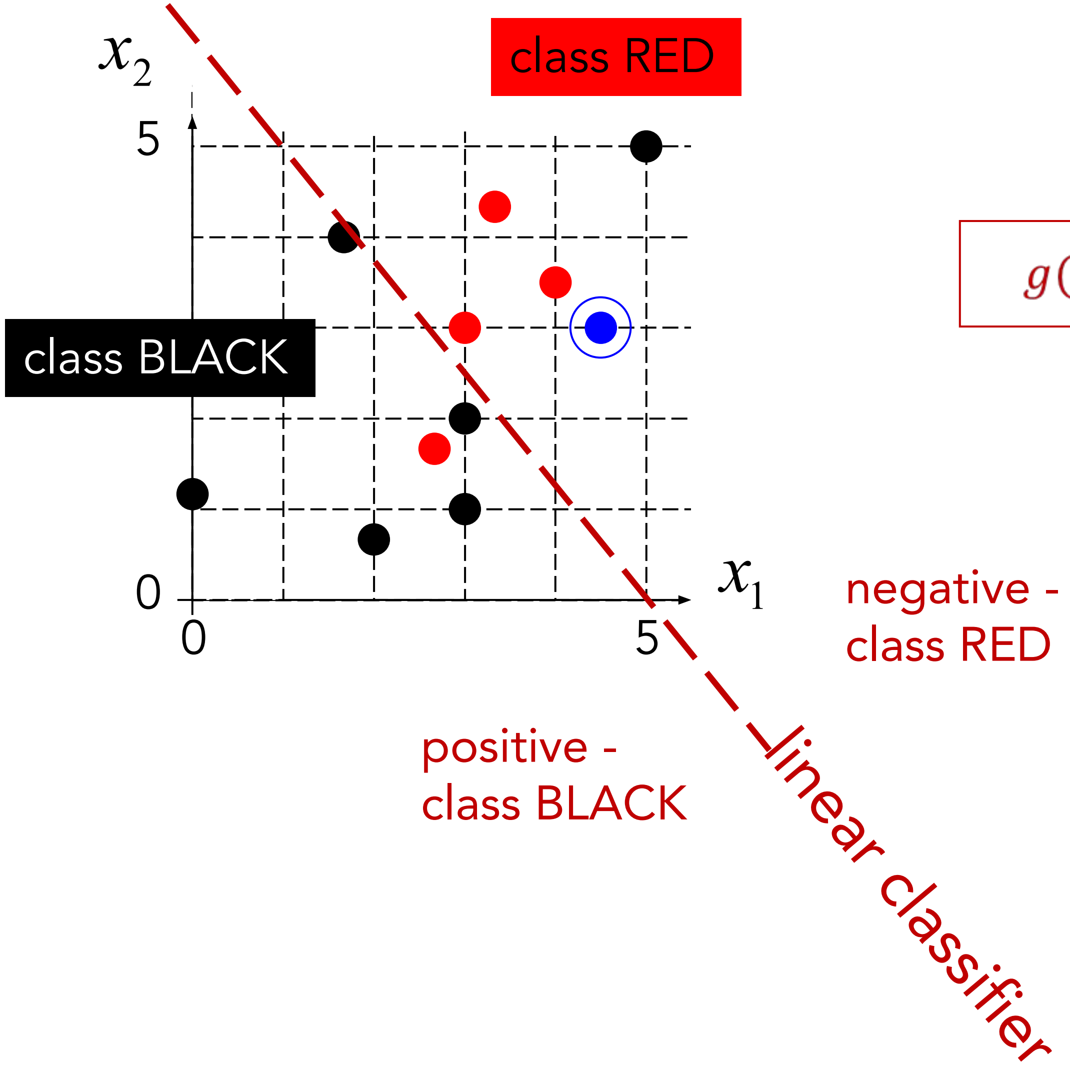
same as

$$g(\mathbf{x}) = -5x_1 - 4x_2 + 25$$

because

$$\mathbf{x} = [x_1, x_2]^T$$

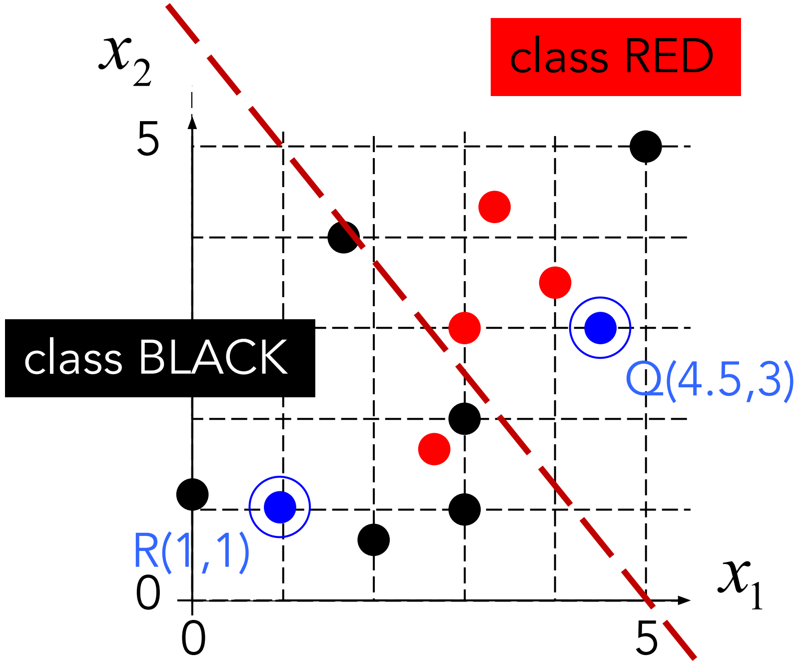
A discriminant function takes the feature values for an object and outputs a value that is used to determine the class label of the object.



$$g([x_1, x_2]^T) = -5x_1 - 4x_2 + 25$$

For two classes ($c = 2$) we can use ONE discriminant function, and determine the class label by its sign.

A discriminant function takes the feature values for an object and outputs a value that is used to determine the class label of the object.



$$g([x_1, x_2]^T) = -5x_1 - 4x_2 + 25$$

Check with the discriminant function

$$g(Q) = -5 \times 4.5 \quad -4 \times 3 \quad +25$$
$$= -22.5 - 12 + 25$$
$$= -9.5$$

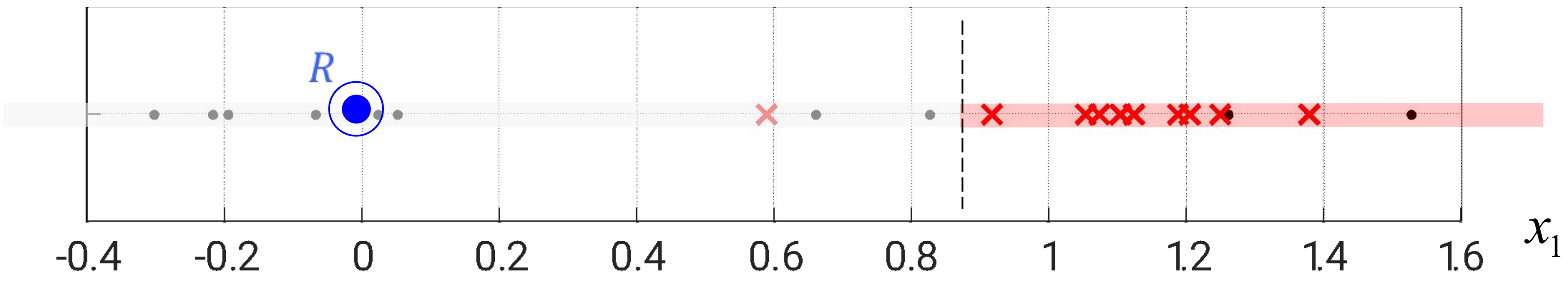
negative - class RED

Check with the discriminant function

$$g(R) = -5 \times 1 \quad -4 \times 1 \quad +25$$
$$= -9 + 25$$
$$= 16$$

positive - class BLACK

A discriminant function takes the feature values for an object and outputs a value that is used to determine the class label of the object.



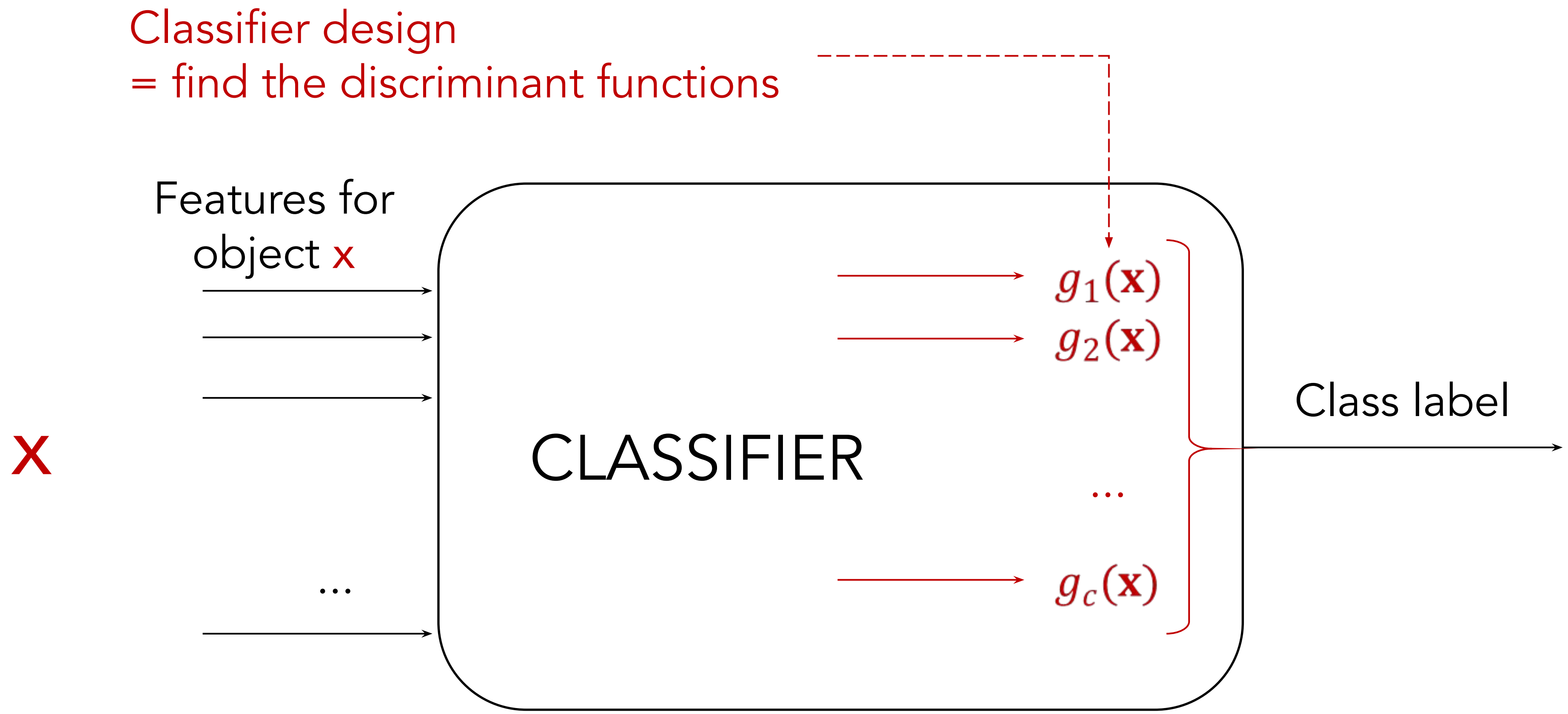
What would be the discriminant function for this classifier (prepare the function so that it takes positive values for class black and negative values for class red)?

$$g(x_1) = 0.88 - x_1$$

Check with the discriminant function

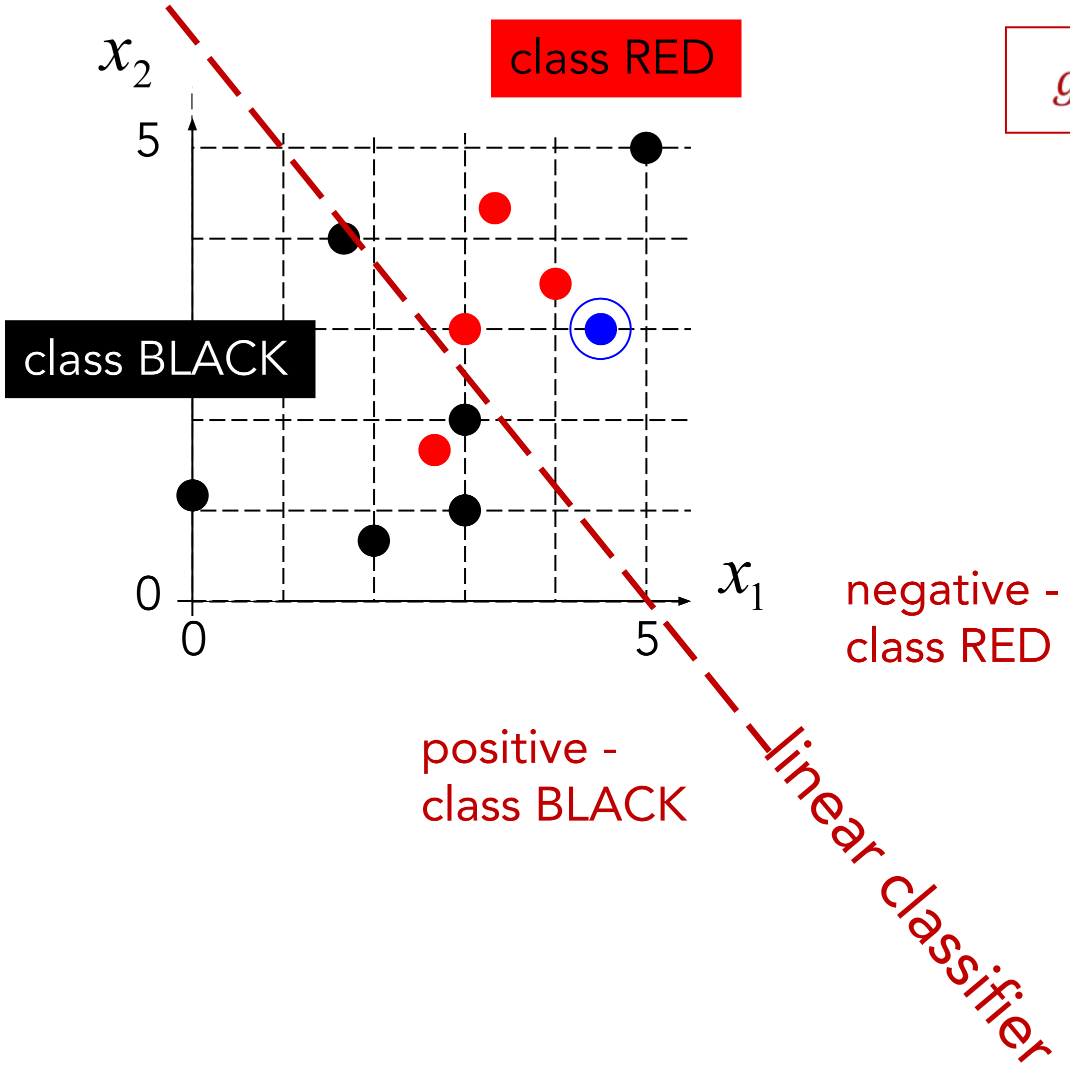
$g(R) = 0.88 - 0 = 0.88$ positive - class BLACK

Canonical model of a classifier



The class label we assign to x is the tag of the LARGEST discriminant function

We can transform the SINGLE discriminant function into TWO discriminant functions to conform with the canonical mode of the classifier.

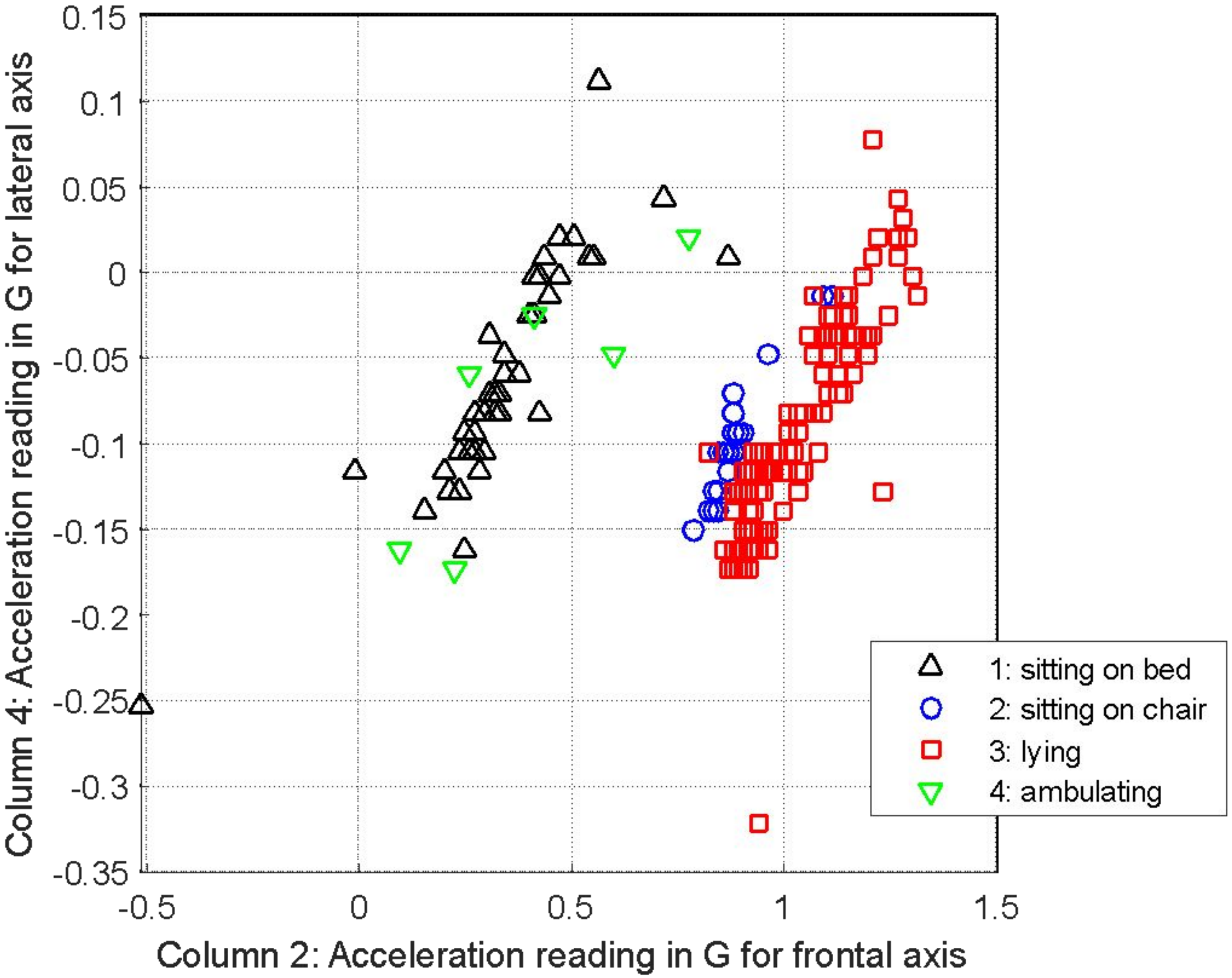


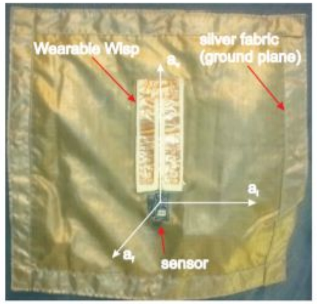
$$g([x_1, x_2]^T) = -5x_1 - 4x_2 + 25$$

$g_1(\mathbf{x}) = -5x_1 - 4x_2 + 25$ black

$g_2(\mathbf{x}) = 0$ red

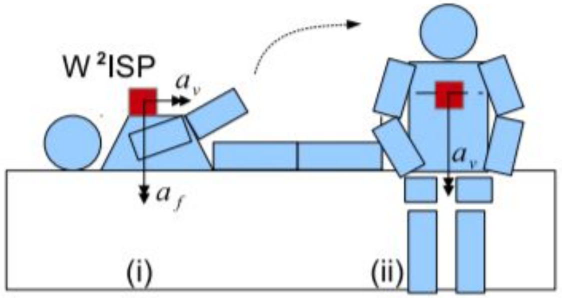
A real-life data example – activity recognition for older people





(a)

(b)



Q1. Draw the class boundaries and shade classification regions of classifier D given by the following discriminant functions (class 1 positive, class 2 negative): x $= [-1, 2]$T

$$g_1(\mathbf{x}) = 3x_1 + 2x_2 - 7$$

$$g_2(\mathbf{x}) = 0$$

Q2. Draw the class boundaries and shade classification regions of classifier D given by the following discriminant function (class 1 positive, class 2 negative):

$$g(x) = -x^2 + 3x - 1$$

(Notice that there is only one variable, therefore the regions are on the x-axis only.)

Not all classifiers are represented through discriminant functions.

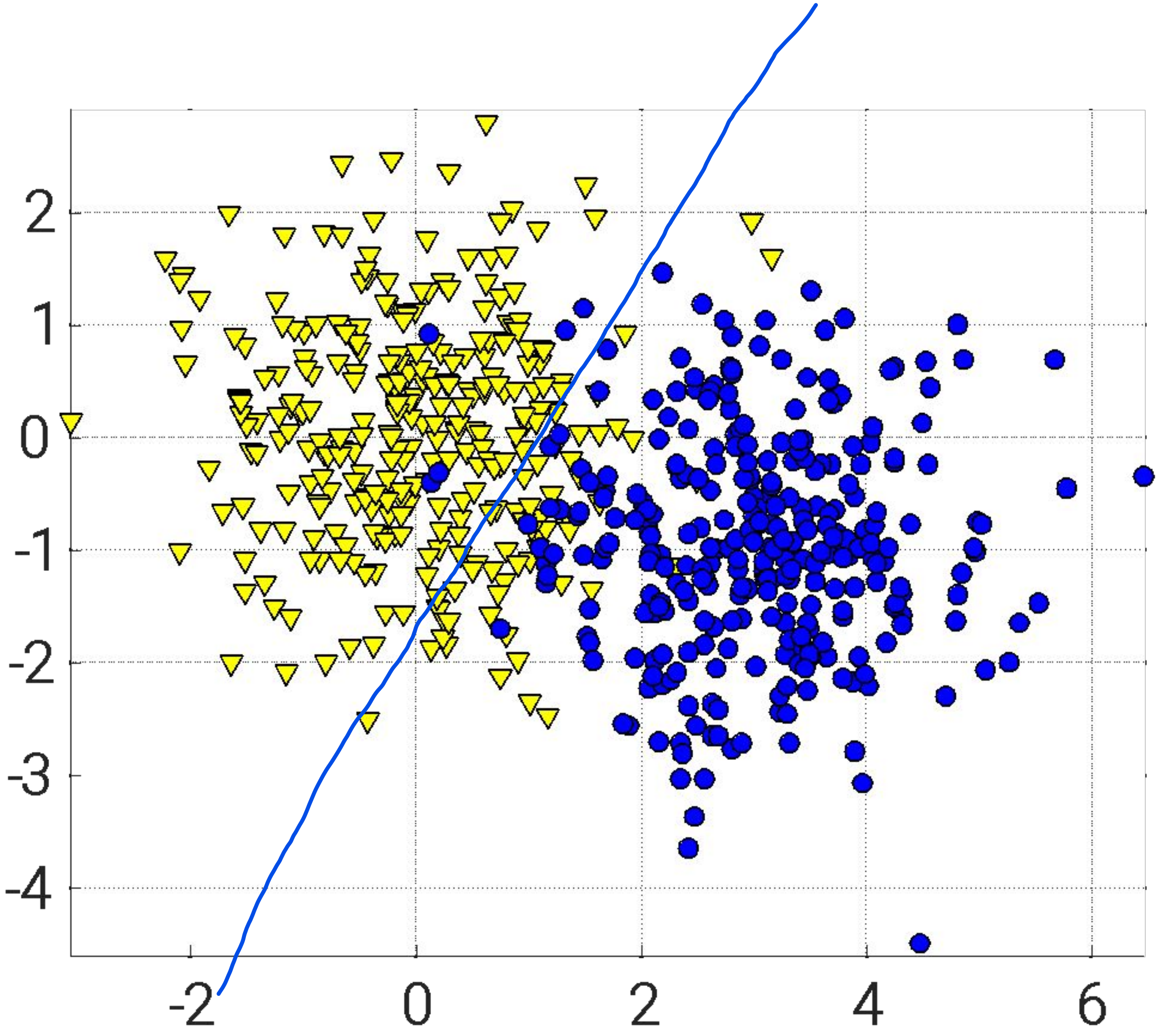
Q3. ~~Draw the class boundaries~~ and shade the classification regions of classifier D given by the following rule:

If (($x > 3$ and $y < 9$) and ($x < 7$ and $y > 2$)) or

if (($x > 5$ and $y < 8$) and ($x < 12$ and $y > -2$)),

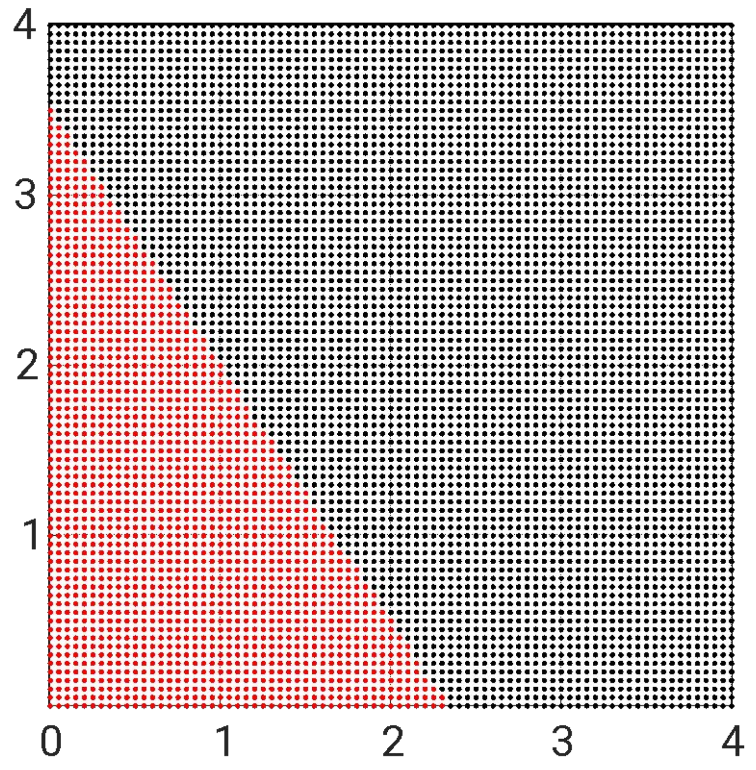
then class 1, else class 2.

Q4. Find the discriminant function of a linear classifier for the classification problem in the figure:

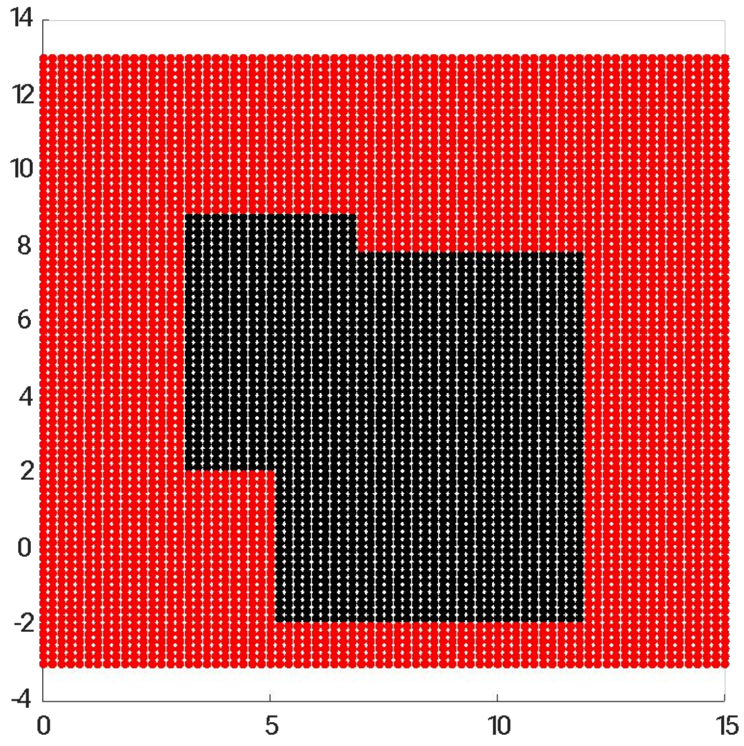


Answers

Q1



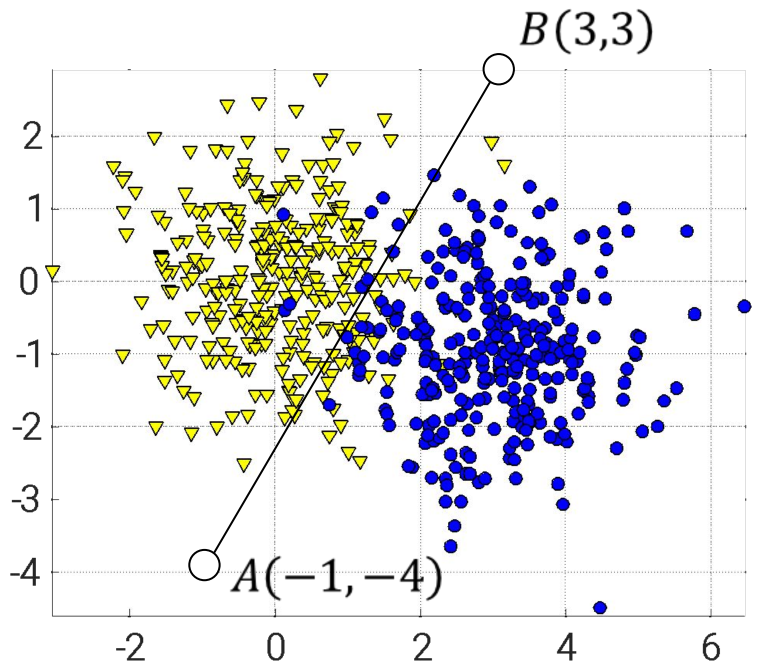
Q3



Q2

black – class1
red – class 2

Q4



$$\frac{x-(-1)}{3-(-1)} = \frac{y-(-4)}{3-(-4)}$$

$$7(x+1) = 4(y+4)$$

$$7x - 4y - 9 = 0$$

Discriminant function

$$g(\mathbf{x}) = 7x_1 - 4x_2 - 9$$

Check with point $R(0,0)$ from the yellow class.

$g(R) = -9$ yellow class should be assigned for negative values of the discriminant function.