(1) Here,

$$\text{Divisor} = x^2 + 2x - 15$$
$$= (x+5)(x-3)$$

$\therefore$ $x = -5$, or $x = 3$

If $x = -5$ then,

$$x^3 + ax^2 + bx + 15 = 0$$

or, $(-5)^3 + a(-5)^2 + b(-5) + 15 = 0$

or, $-125 + 25a - 5b + 15 = 0$

or, $25a - 5b = 110$

or, $5a - b = 22$ ——— (i)

If $x = -3$ then,

$$x^3 + ax^2 + bx\ 15 = 0$$

or, $(-3)^3 + a(-3)^2 + b(3) + 15 = 0$

or, $27 + 9a + 3b + 15 = 0$

or, $9a + 3b = -42$

or, $3a + b = -14$ ——— (ii)

① + ② ⇒

$8a = 8$

or, $a = 1$

From ① ⇒

$5 \times 1 - b = 22$

or, $-b = 17$

or, $b = -17$

(Ans)

2. Here,

Divisor = 3

Quotient = 17

Remainder = 1

So the number is $= (17 \times 3) + 1$

$= 52$

We know,

$$LCM \times HCF = a \times b$$

or, $$a = \frac{LCM \times HCF}{b}$$

$$= \frac{156 \times 4}{52}$$

$$= 12$$

(Ans).

3. LCM of 32, 36, 48 and 96 is

2	32, 36, 48, 96
2	16, 18, 24, 48
	8, 9, 12, 24

$\therefore$ L.C.M $= 2 \times 2 \times 8 \times 9 \times 12 \times 24$

$= 82944$

So the number is $= 82944 - 23$

$= 82921$

(Ans)

4. The number of guest in each room is the HCF of 60, 84 and 108

H.C.F = 12

∴ There are 12 guest in one room

Number of rooms $= \frac{60 + 84 + 108}{12}$

$= 21$

(Ans).

5.

5. The LCM of ~~12~~ 15, 20, 25 is,

$$5 \,\big|\, \underline{15,\ 20,\ 25}$$
$$3,\ 4,\ 5$$

LCM $= 5 \times 3 \times 4 \times 5$

$= 300$

Now,

$300 = 2 \times 2 \times 3 \times 5 \times 5$

To make is perfect square, it needs to be multiplied by 3

$\therefore$ Least number of soldiers in a regiment $= 300 \times 3$

$= 900$

(Ans).

6. $35(x^4 - 27x) = 5 \times 7 \times x \times (x^3 - 27)$

$= 5 \times 7 \times x \times (x-3)$

$(x^2 + 3x + 9)$

$40(2x^3 - 5x^2 - 3x) = 2 \times 2 \times 2 \times 5 \times x \times$

$(2x^2 - 5x - 3)$

$= 2 \times 2 \times 2 \times 5 \times x \times$

$(x-3)(2x+1)$

$\therefore$ H.C.F $= 5x(x-3)$

$\therefore$ L.C.M $= 5 \times 7 \times 2 \times 2 \times 2 \times x \times (x-3)$

$(x^2 + 3x + 9)(2x+1)$

$= 280x(x^3 - 27)(2x+1)$

(Ans).

7.e)

$$(a-1)x^2 + a^2xy + (a+1)y^2$$

Here,

$$a-1 = p$$
$$a+1 = q$$

(x) $a^2 - 1 = pq$

or, $a^2 = pq + 1$

So,

$$px^2 + (pq+1)xy + qy^2$$

$$= px^2 + pq\,xy + xy + qy^2$$

$$= x(px+y) + qy(px+y)$$

$$= (px+y)(x+qy)$$

$$= \{(a-1)x+y\}\{x+(a+1)y\}$$

$$= (ax-x+y)(x+ay+y)$$

(Ans).

8. a) Here,

Height, $h = 5$ km

We know,

The atmospheric pressure,

$$p = 760\, e^{-0.145h}$$

$$= 760\, e^{-0.145 \times 5}$$

$$= 368.09 \text{ millimeters}$$

b) We know,

$$p = 517 \text{ millimeters}$$

We have,

$$p = 760\, e^{-0.145h}$$

or, $517 = 760\, e^{-0.145h}$

or, $0.68 = e^{-0.145h}$

or, $\ln(0.68) = \ln e^{-0.145h}$

or, $h = \dfrac{\ln(0.68)}{-0.145} = 2.66$ km (Ans).

9. a) The number of deers after 2 years,

$$A = \frac{100}{1+4e^{-0.14t}}$$

$$= \frac{100}{1+4e^{-0.14\times 2}}$$

$$= 24.86 \approx 24$$

The number of deers after 6 years

$$A = \frac{100}{1+4e^{-0.14t}}$$

$$= \frac{100}{1+4e^{-0.14\times 6}}$$

$$= 36.67 \approx 36$$

b) We have,

$$A = 50$$

or, $$50 = \frac{100}{1 + 4e^{-0.14t}}$$

or, $$1 + 4e^{-0.14t} = 2$$

or, $$4e^{-0.14t} = 1$$

or, $$e^{-0.14t} = \frac{1}{4}$$

or, $$\ln e^{-0.14t} = \ln(0.25)$$

or, $$t = \frac{\ln(0.25)}{-0.14}$$

$$= 9.9 \approx 10 \text{ years.}$$

10. Here,

$T =$

$T_m = 40^\circ$

$T_0 = 72^\circ$

$k = 0.1$

$t = 3$ hours

We know,

$$T = T_m + (T_0 - T_m)e^{-kt}$$

$$= 40^\circ + (72^\circ - 40^\circ)e^{-0.1 \times 3}$$

$$= 40^\circ + 32^\circ \times e^{-1.2}$$

$$= 49.64^\circ \approx 50^\circ$$

(Ans).