# Three Dimensional Modeling Transformations

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- Methods for object modeling transformation in three dimensions are extended from two dimensional methods by including consideration for the z coordinate.

# 3D Point

- We will consider points as column vectors. Thus, a typical point with coordinates (x, y, z) is represented as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

# 3D Point Homogenous Coordinate

- A 3D point **P** is represented in homogeneous coordinates by a 4-dim. Vect:

$$\mathbf{P} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

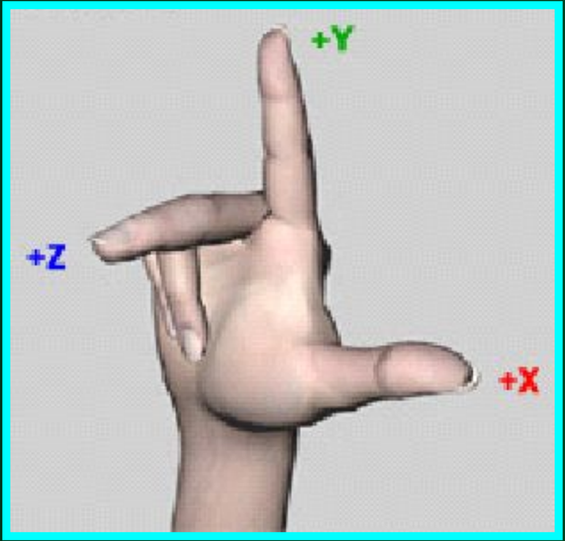
# 3D Point Homogenous Coordinate

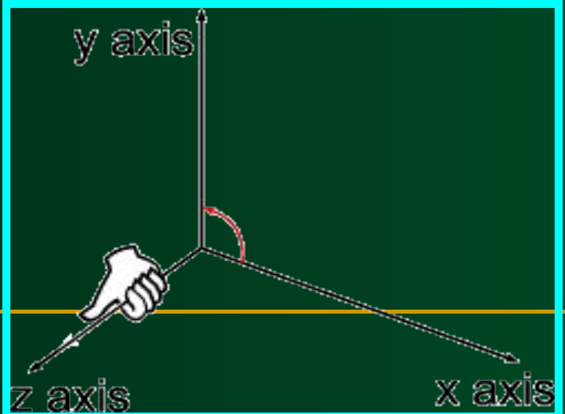
- We don't lose anything
- The main advantage: it is easier to compose translation and rotation
- Everything is matrix multiplication

$$\boxed{\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}$$

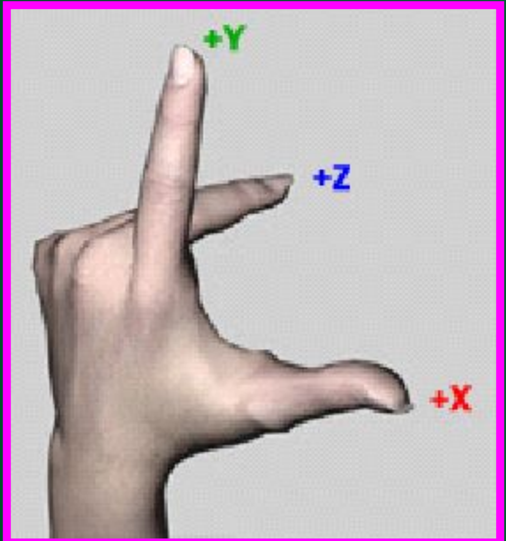
# 3D Coordinate Systems

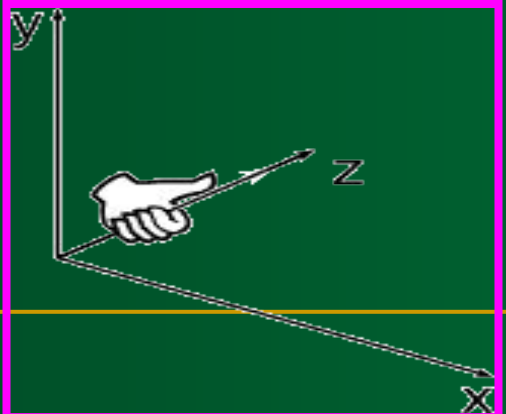
- ***Right Hand*** **coordinate system:**




- ***Left Hand*** **coordinate system:**



# 3D Transformation

- In homogeneous coordinates, 3D transformations are represented by 4×4 matrixes:

$$\boxed{\begin{bmatrix} a & b & c & t_x \\ d & e & f & t_y \\ g & h & i & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

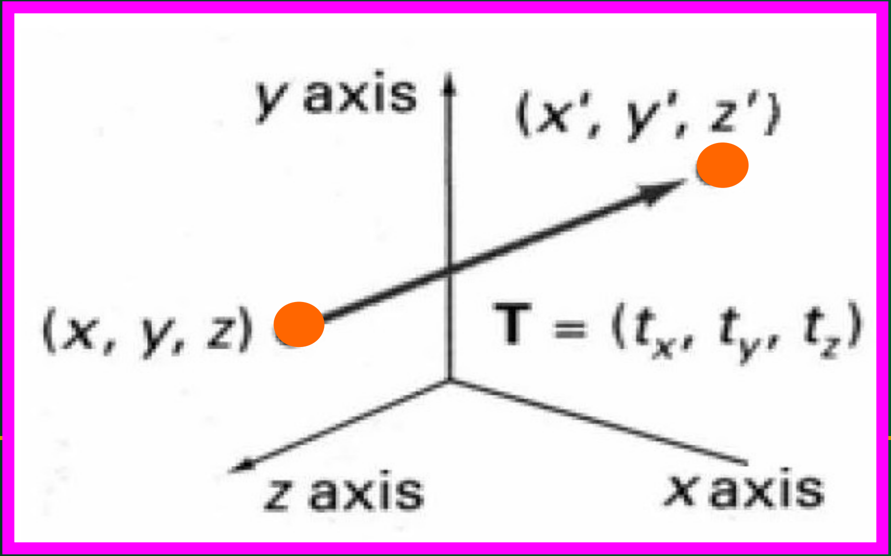
# 3D Translation

# 3D Translation

- **P** is **translated** to **P'** by:

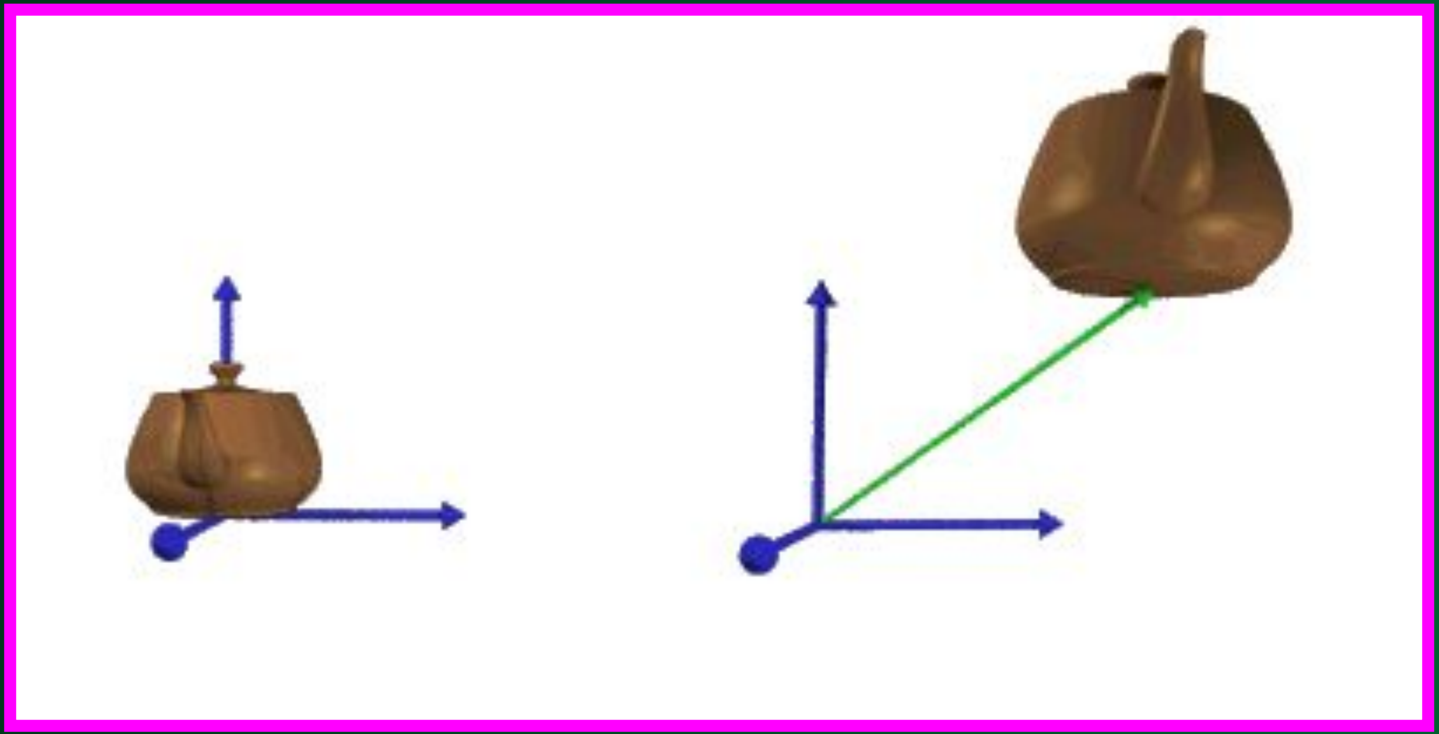
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{T} \cdot \mathbf{P}$$



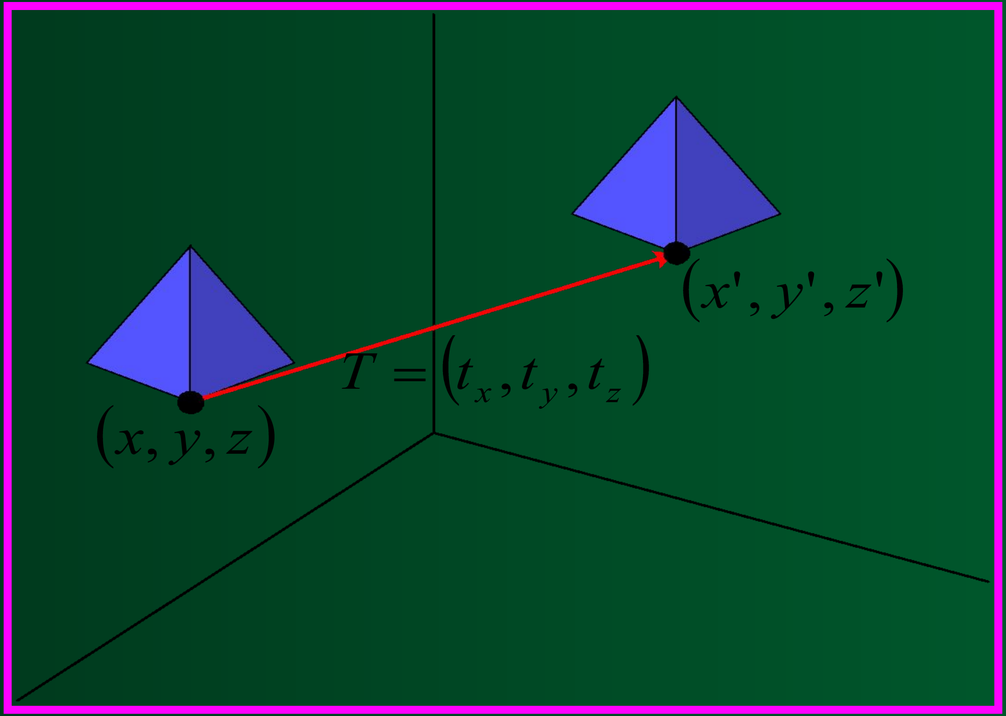
# 3D Translation

- An object is translated in 3D dimensional by transforming each of the defining points of the objects .



# 3D Translation

- An Object represented as a set of polygon surfaces, is translated by translate each vertex of each surface and redraw the polygon facets in the new position.




- **Inverse Translation:**

$$\boxed{T^{-1}(t_x, t_y, t_z) = T(-t_x, -t_y, -t_z)}$$

# 3D Rotation

# 3D Rotation

- In general, rotations are specified by a ***rotation axis*** and an ***angle***.
- Positive rotation angles produce counterclockwise rotations about a coordinate axis

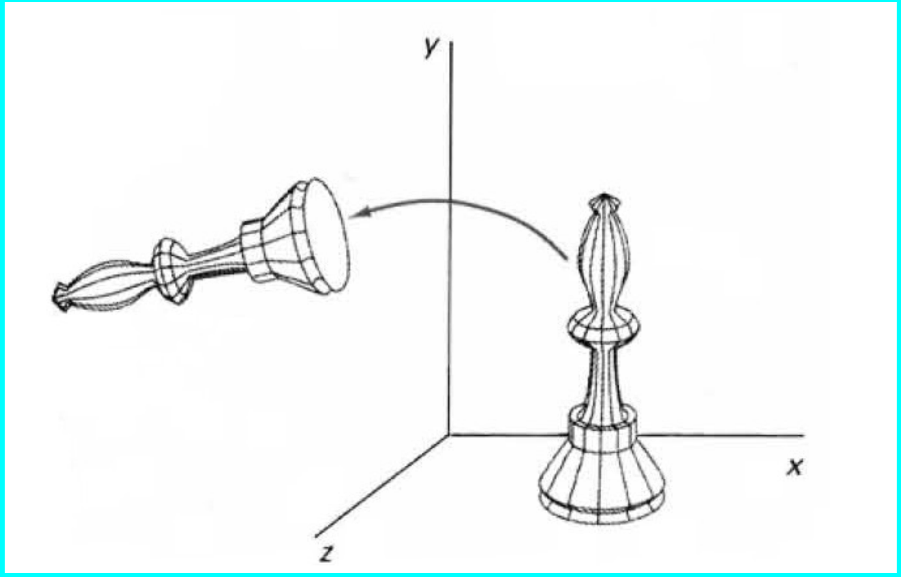
# Coordinate Axis Rotations

# Coordinate Axis Rotations

- **Z-axis rotation:** For z axis same as 2D rotation:

$$\boxed{\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}$$

$$\mathbf{P}' = \mathbf{R}_z(\theta) \cdot \mathbf{P}$$

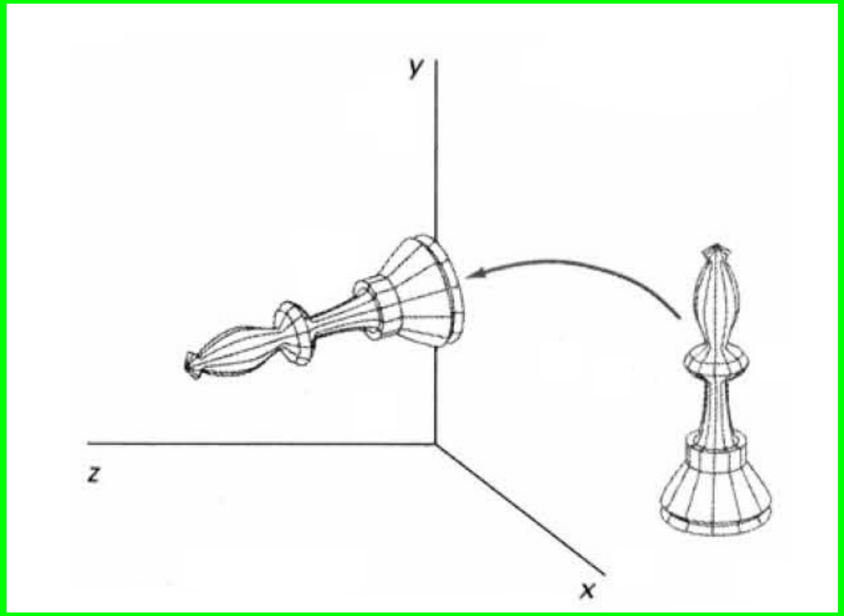


# Coordinate Axis Rotations

- **X-axis rotation:**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{R}_x(\theta) \cdot \mathbf{P}$$

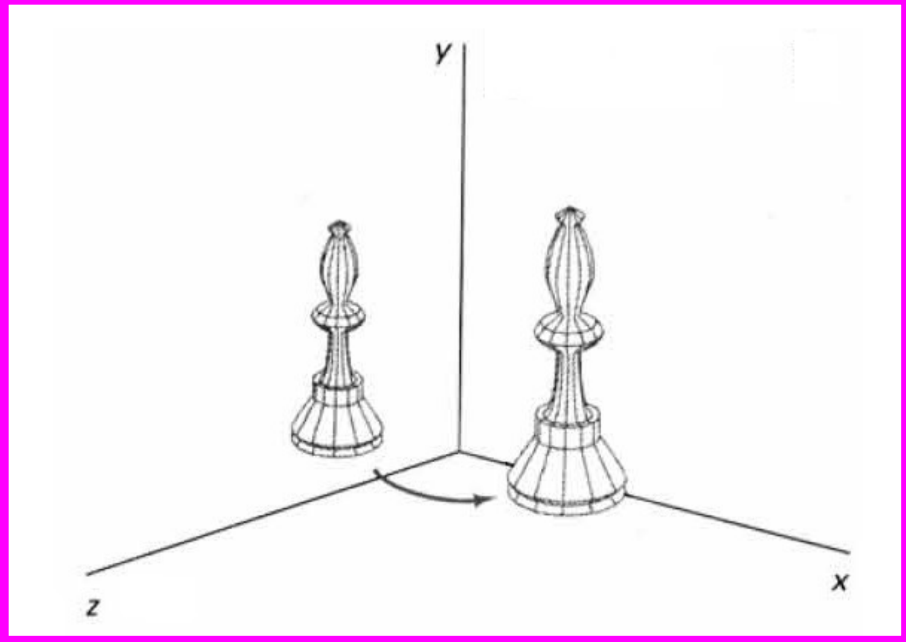


# Coordinate Axis Rotations

- **Y-axis rotation:**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{R}_y(\theta) \cdot \mathbf{P}$$



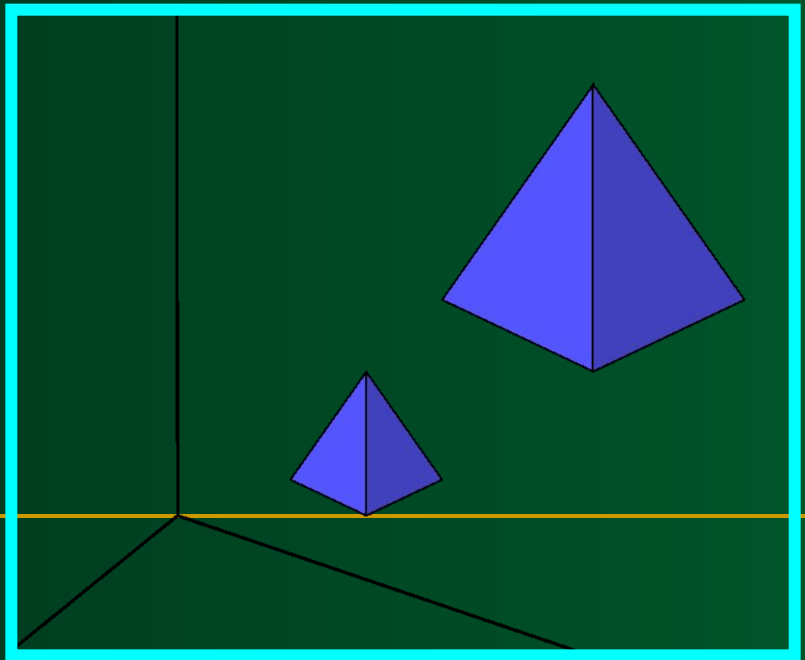
# 3D Scaling

# 3D Scaling

- **About origin:** Changes the size of the object and repositions the object relative to the coordinate origin.

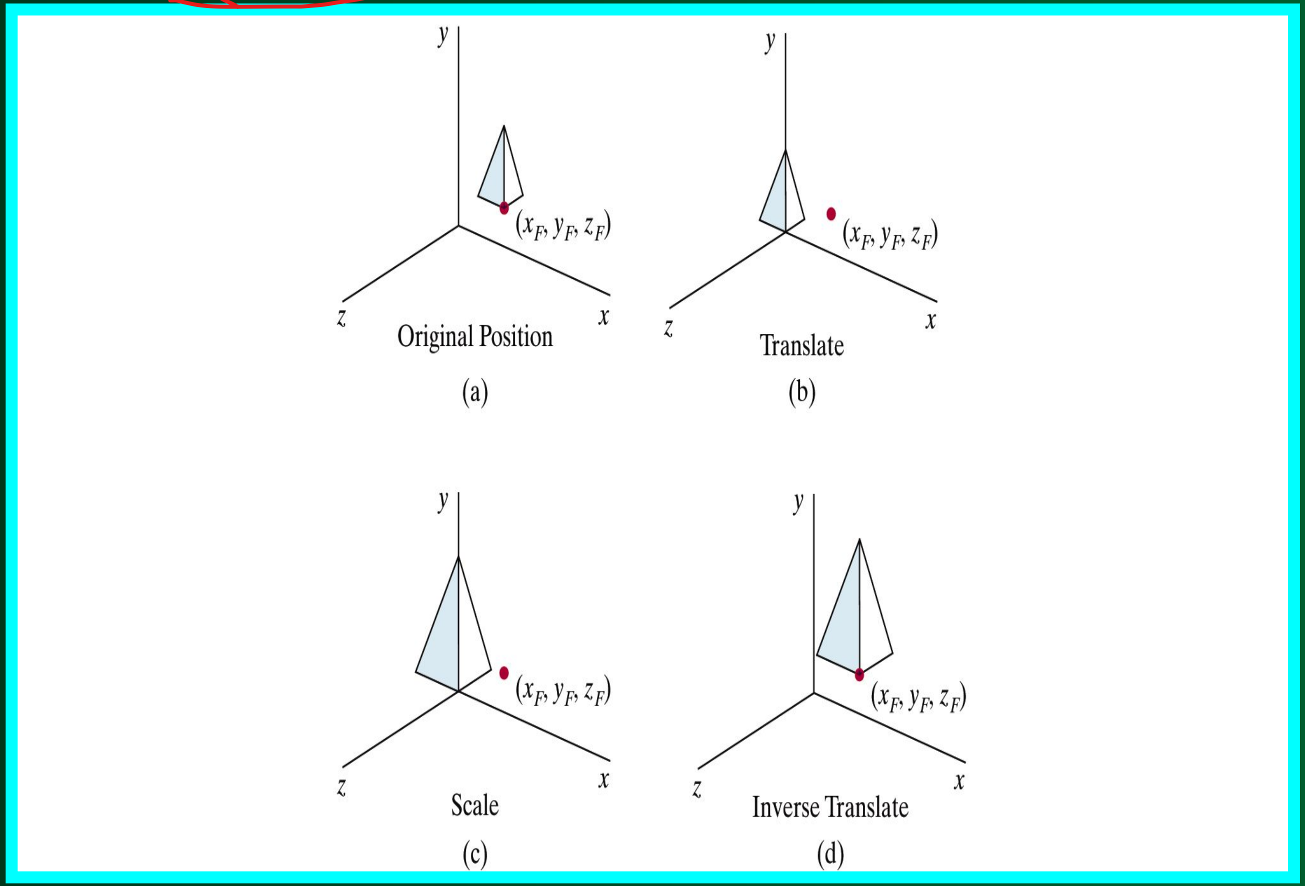
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{S} \cdot \mathbf{P}$$



# 3D Scaling

- About any fixed point:




$$\mathrm{T}(x_f, y_f, z_f) \cdot \mathrm{S}(s_x, s_y, s_z) \cdot \mathrm{T}(-x_f, -y_f, -z_f) = \begin{bmatrix} s_x & 0 & 0 & (1-s_x)x_f \\ 0 & s_y & 0 & (1-s_y)y_f \\ 0 & 0 & s_z & (1-s_z)z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Composite

# 3D Transformations

# Composite 3D Transformations

- **Same way as in two dimensions:**
  - Multiply matrices
  - Rightmost term in matrix product is the first transformation to be applied

# 3D Reflections

# 3D Reflections

- **About an axis:** equivalent to 180°rotation about that axis

# 3D Reflections

## About a plane:

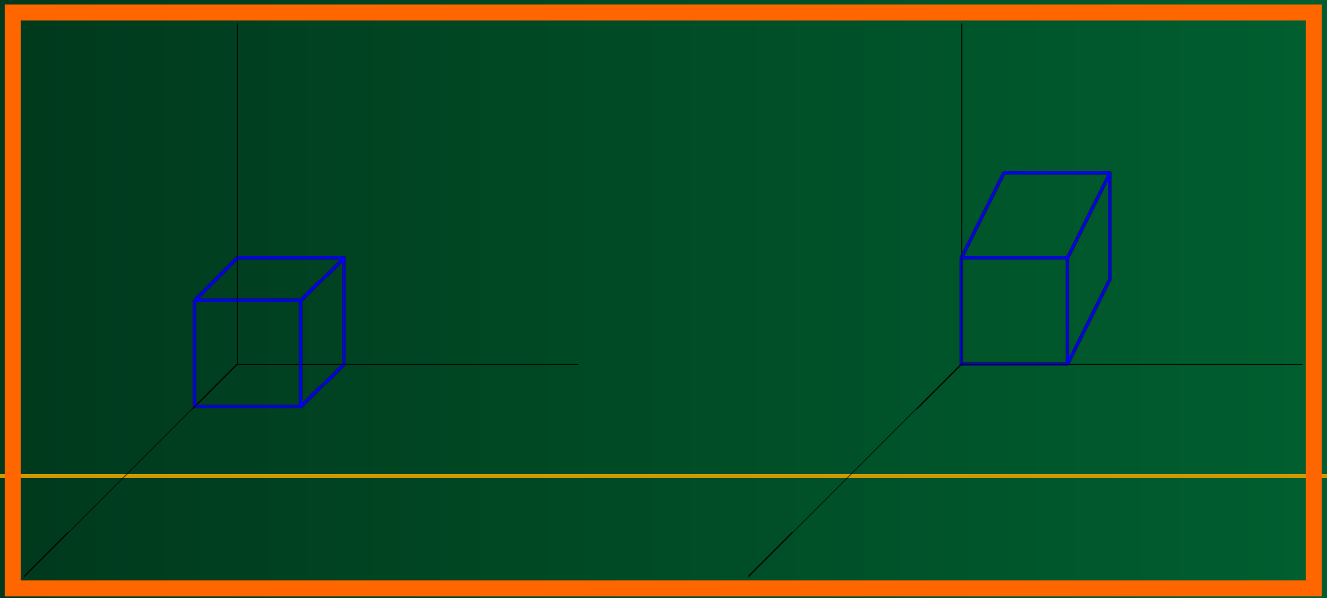
- A reflection through the **xy** plane:

$$\begin{bmatrix} x \\ y \\ -z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# 3D Shearing

# 3D Shearing

- **Modify object shapes**
- **Useful for perspective projections:**
  - E.g. draw a cube (3D) on a screen (2D)
  - Alter the values for **x** and **y** by an amount proportional to the distance from $z_{ref}$



# 3D Shearing

$$M_{zshear} = \begin{bmatrix} 1 & 0 & sh_{zx} & -sh_{zx} \cdot z_{ref} \\ 0 & 1 & sh_{zy} & -sh_{zy} \cdot z_{ref} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$