

# Data Mining: Concepts and Techniques

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— Chapter 2 —

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# Chapter 2: Getting to Know Your Data

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- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary



# Types of Data Sets

- **Record**

- Relational records
- Data matrix, e.g., numerical matrix, crosstabs
- Document data: text documents: term-frequency vector
- Transaction data

- **Graph and network**

- World Wide Web
- Social or information networks
- Molecular Structures

- **Ordered**

- Video data: sequence of images
- Temporal data: time-series
- Sequential Data: transaction sequences
- Genetic sequence data

- **Spatial, image and multimedia:**

- Spatial data: maps
- Image data:
- Video data:

	team	coach	pla y	ball	score	game	Wi n	lost	timeout	season
Document										
1	Document 1	3	0	5	0	2	6	0	2	0
2	Document 2	0	7	0	2	1	0	0	3	0
3	Document 3	0	1	0	0	1	2	2	0	3

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

# Important Characteristics of Structured Data

## Dimensionality

- Curse of dimensionality
- Sparsity → ratio of missing to:
  - Only presence counts
- Resolution →  $\nu\nu$ 
  - Patterns depend on the scale
- Distribution → mean, mode
  - Centrality and dispersion

attribute

start

# ✓ Data Objects

- Data sets are made up of data objects.
- A data object represents an entity.
- Examples:
  - sales database: customers, store items, sales
  - medical database: patients, treatments
  - university database: students, professors, courses
- Also called *samples*, *examples*, *instances*, *data points*, *objects*, *tuples*.
- Data objects are described by **attributes**.  
*columns* →
- ✓ Database rows -> data objects; columns -> attributes.  
rows → data objects; columns → attributes.

# Attributes

- **Attribute (or dimensions, features, variables):** a data field, representing a characteristic or feature of a data object.
    - *E.g., customer\_ID, name, address*
  - **Types:**
    - Nominal
    - Binary
    - Numeric: quantitative
      - Interval-scaled
      - Ratio-scaled
- 
- The handwritten notes consist of three arrows drawn in blue ink. One arrow points from 'Nominal' to the word 'Nominal'. Another arrow points from 'Binary' to the word 'Binary'. A third arrow points from 'Numeric: quantitative' to the word 'Nominal'.

# Attribute Types

- **Nominal:** categories, states, or “names of things”
  - $\text{Hair\_color} = \{\text{auburn}, \text{black}, \text{blond}, \text{brown}, \text{grey}, \text{red}, \text{white}\}$
  - marital status, occupation, ID numbers, zip codes
- **Binary**
  - Nominal attribute with only 2 states (0 and 1)
  - Symmetric binary: both outcomes equally important
    - e.g., gender
  - Asymmetric binary: outcomes not equally important.
    - e.g., medical test (positive vs. negative)
    - Convention: assign 1 to most important outcome (e.g., HIV positive)
- **Ordinal**
  - Values have a meaningful order (ranking) but magnitude between successive values is not known.
  - $\text{Size} = \{\underline{\text{small}}, \underline{\text{medium}}, \underline{\text{large}}\}, \underline{\text{grades}}, \underline{\text{army rankings}}$

✓ A+, X, A-

# Numeric Attribute Types

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- Quantity (integer or real-valued)
- **Interval**
  - Measured on a scale of **equal-sized units**
  - Values have order
    - E.g., temperature in  $C^\circ$  or  $F^\circ$ , calendar dates
  - No true zero-point
- **Ratio**
  - Inherent **zero-point**
  - We can speak of values as being an order of magnitude larger than the unit of measurement ( $10\ K^\circ$  is twice as high as  $5\ K^\circ$ ).
    - e.g., *temperature in Kelvin, length, counts, monetary quantities*

# Discrete vs. Continuous Attributes

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- **Discrete Attribute**

- Has only a finite or countably infinite set of values
  - E.g., zip codes, profession, or the set of words in a collection of documents
- Sometimes, represented as integer variables
- Note: Binary attributes are a special case of discrete attributes

- **Continuous Attribute**

- Has real numbers as attribute values
  - E.g., temperature, height, or weight
- Practically, real values can only be measured and represented using a finite number of digits
- Continuous attributes are typically represented as floating-point variables

# Chapter 2: Getting to Know Your Data

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- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
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# Basic Statistical Descriptions of Data

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- Motivation
  - To better understand the data: central tendency, variation and spread
- Data dispersion characteristics
  - median, max, min, quantiles, outliers, variance, etc.
- Numerical dimensions correspond to sorted intervals
  - Data dispersion: analyzed with multiple granularities of precision
  - Boxplot or quantile analysis on sorted intervals
- Dispersion analysis on computed measures
  - Folding measures into numerical dimensions
  - Boxplot or quantile analysis on the transformed cube

# Measuring the Central Tendency

## Mean (algebraic measure) (sample vs. population):

Note:  $n$  is sample size and  $N$  is population size.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad \mu = \frac{\sum x}{N}$$

- Weighted arithmetic mean:
- Trimmed mean: chopping extreme values

$$\bar{x} = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}$$

3 5 7 9

## Median:

sort ✓

- Middle value if odd number of values, or average of the middle two values otherwise
- Estimated by interpolation (for *grouped data*): ✓

age	frequency
1–5	200
6–15	450
16–20	300
21–50	1500
51–80	700
81–110	44

## Mode

$$median = L_1 + \left( \frac{n/2 - (\sum freq)}{freq_{median}} \right) width$$

- Value that occurs most frequently in the data
- Unimodal, bimodal, trimodal
- Empirical formula:

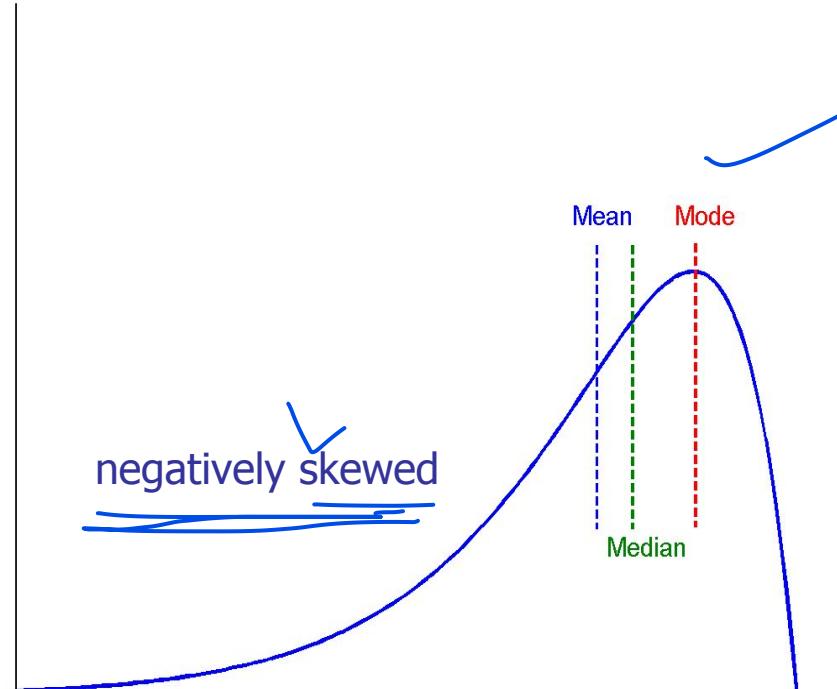
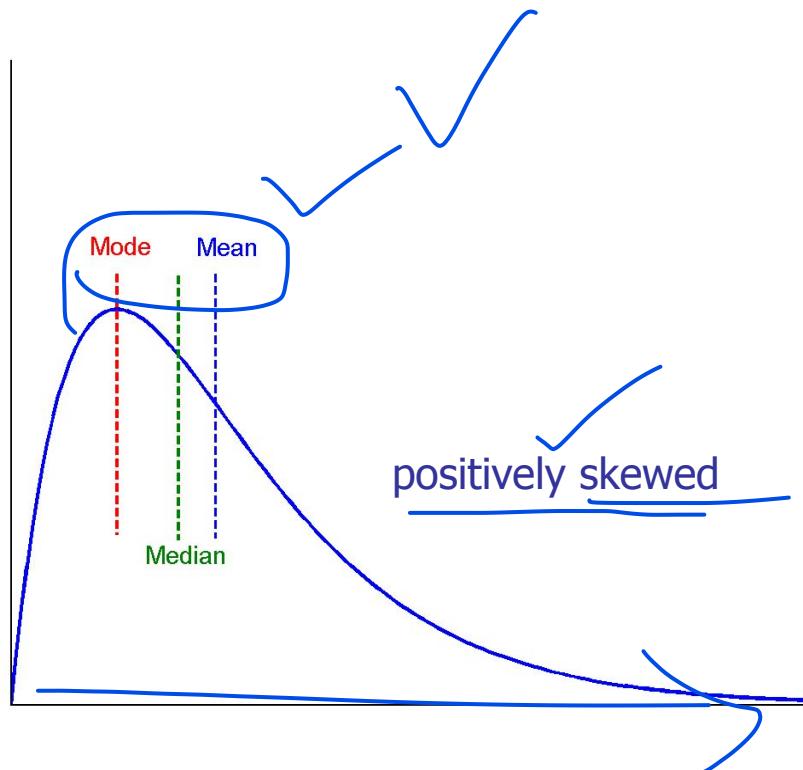
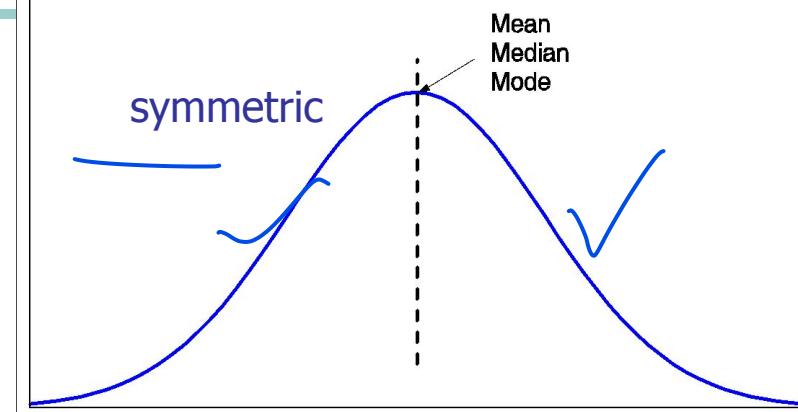
$$mean - mode = 3 \times (mean - median)$$

1 2 3 4 5 6 9

# Symmetric vs. Skewed Data

- Median, mean and mode of symmetric, positively and negatively skewed data

1 2 2 2 3 3 5



# Measuring the Dispersion of Data

- Quartiles, outliers and boxplots

- Quartiles:**  $Q_1$  ( $25^{\text{th}}$  percentile),  $Q_3$  ( $75^{\text{th}}$  percentile)

- Inter-quartile range:**  $IQR = Q_3 - Q_1$

- Five number summary:** min,  $Q_1$ , median,  $Q_3$ , max

- Boxplot:** ends of the box are the quartiles; median is marked; add whiskers, and plot outliers individually

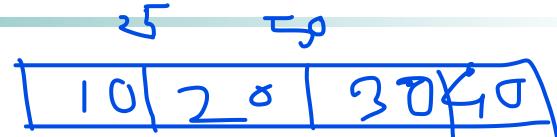
- Outlier:** usually, a value higher/lower than  $1.5 \times IQR$

- Variance and standard deviation (*sample: s, population:  $\sigma$* )

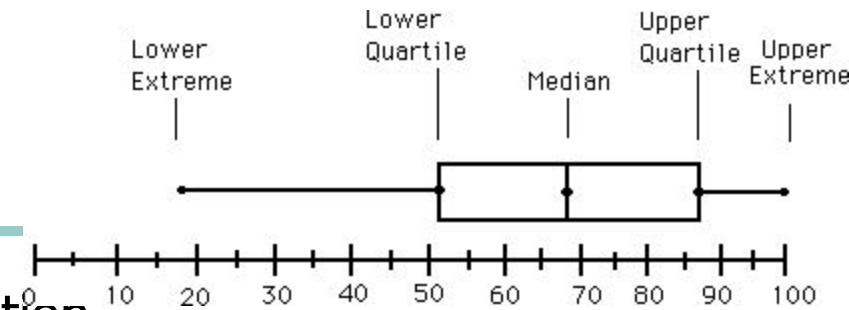
- Variance:** (algebraic, scalable computation)

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{1}{n-1} \left[ \sum_{i=1}^n x_i^2 - \frac{1}{n} (\sum_{i=1}^n x_i)^2 \right] \quad \sigma^2 = \frac{1}{N} \sum_{i=1}^n (x_i - \mu)^2 = \frac{1}{N} \sum_{i=1}^n x_i^2 - \mu^2$$

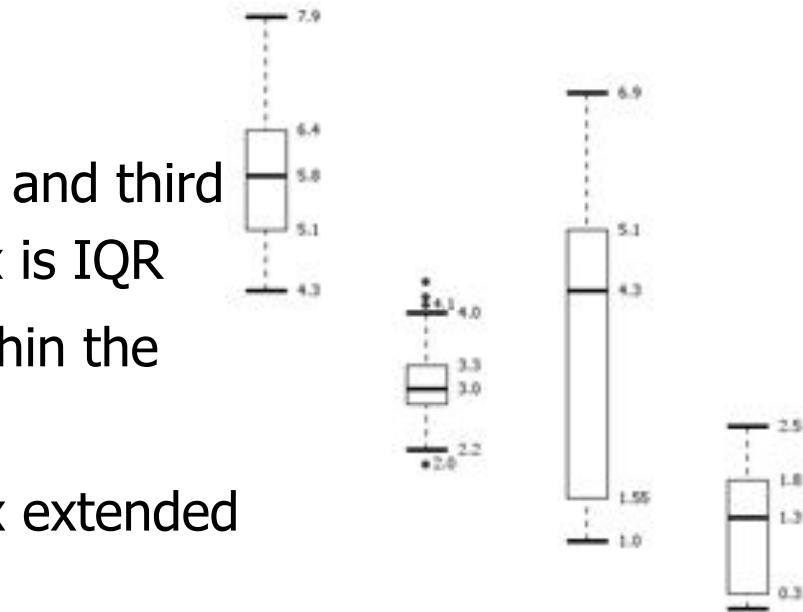
- Standard deviation s (or  $\sigma$ )** is the square root of variance  $s^2$  (or  $\sigma^2$ )



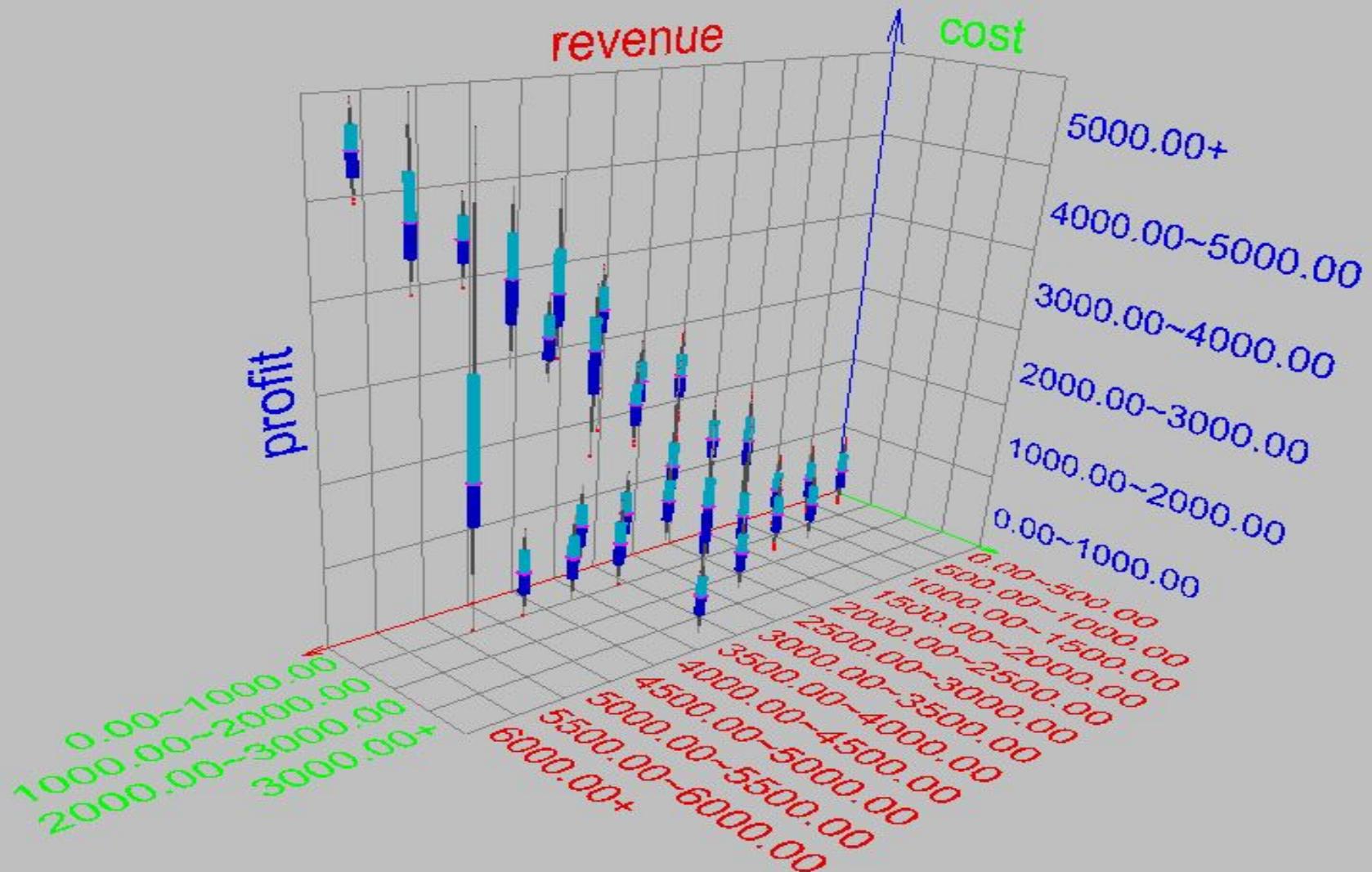
# ✓ Boxplot Analysis



- **Five-number summary** of a distribution
  - Minimum, Q1, Median, Q3, Maximum
- **Boxplot**
  - Data is represented with a box
  - The ends of the box are at the first and third quartiles, i.e., the height of the box is IQR
  - The median is marked by a line within the box
  - Whiskers: two lines outside the box extended to Minimum and Maximum
  - Outliers: points beyond a specified outlier threshold, plotted individually

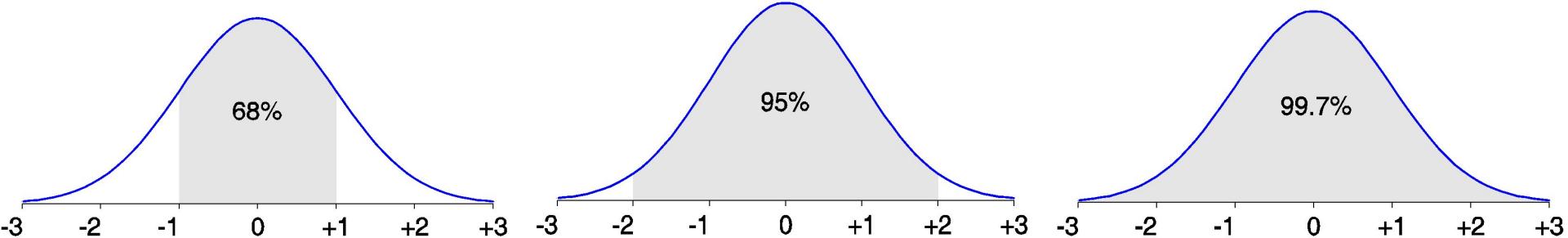


# Visualization of Data Dispersion: 3-D Boxplots



# Properties of Normal Distribution Curve

- The normal (distribution) curve
  - From  $\mu-\sigma$  to  $\mu+\sigma$ : contains about 68% of the measurements ( $\mu$ : mean,  $\sigma$ : standard deviation)
  - From  $\mu-2\sigma$  to  $\mu+2\sigma$ : contains about 95% of it
  - From  $\mu-3\sigma$  to  $\mu+3\sigma$ : contains about 99.7% of it

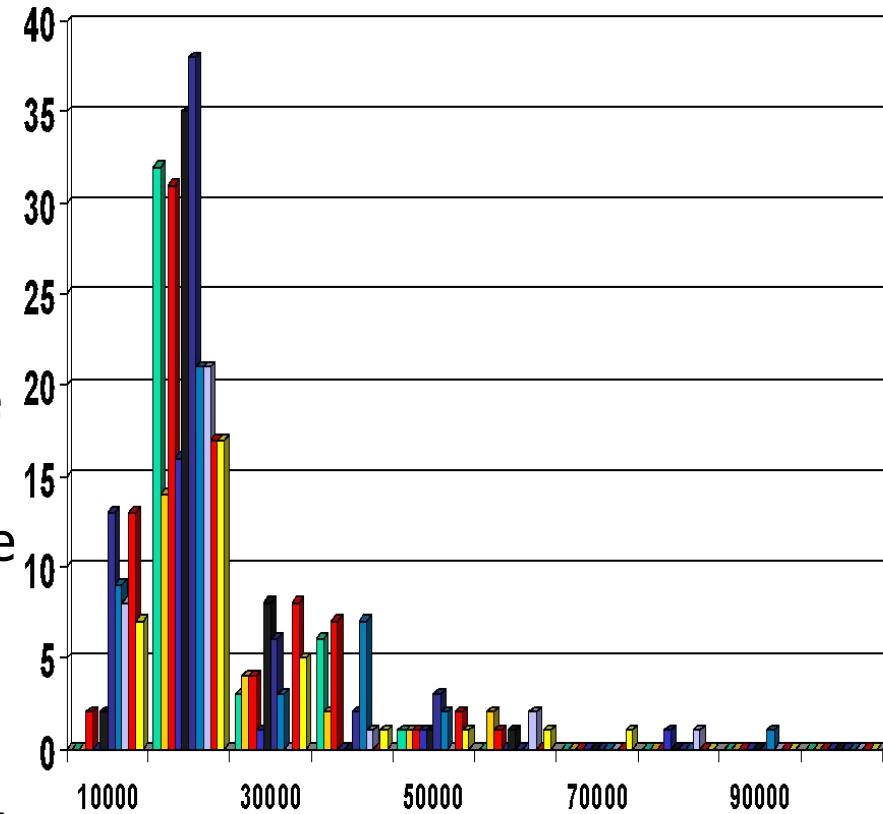


# ✓ Graphic Displays of Basic Statistical Descriptions

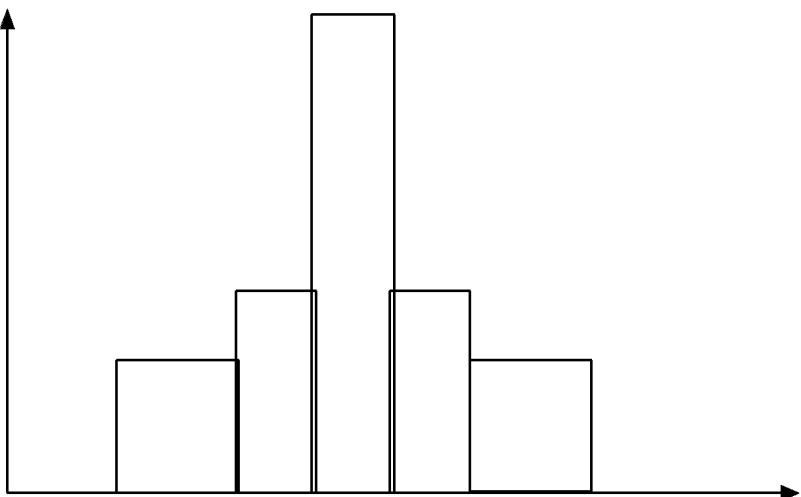
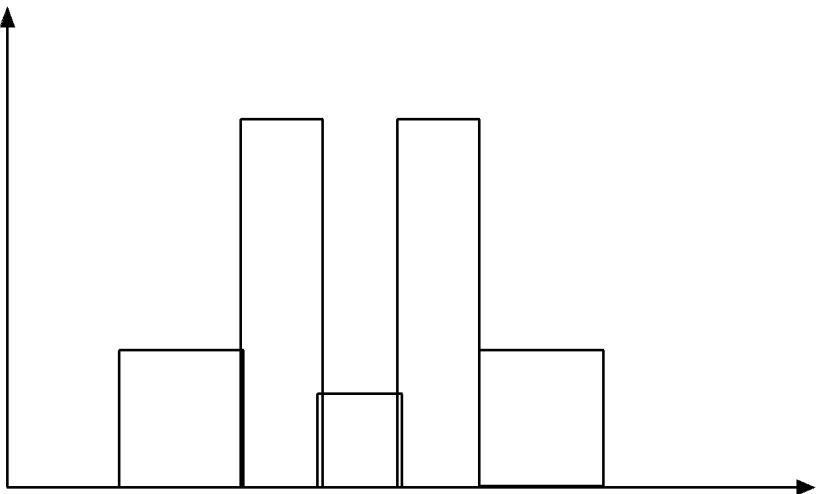
- **Boxplot:** graphic display of five-number summary
- **Histogram:** x-axis are values, y-axis repres. frequencies
- **Quantile plot:** each value  $x_i$  is paired with  $f_i$ , indicating that approximately  $100 f_i \%$  of data are  $\leq x_i$
- **Quantile-quantile (q-q) plot:** graphs the quantiles of one univariant distribution against the corresponding quantiles of another
- **Scatter plot:** each pair of values is a pair of coordinates and plotted as points in the plane

# Histogram Analysis

- Histogram: Graph display of tabulated frequencies, shown as bars
- It shows what proportion of cases fall into each of several categories
- Differs from a bar chart in that it is the *area* of the bar that denotes the value, not the height as in bar charts, a crucial distinction when the categories are not of uniform width
- The categories are usually specified as non-overlapping intervals of some variable. The categories (bars) must be adjacent



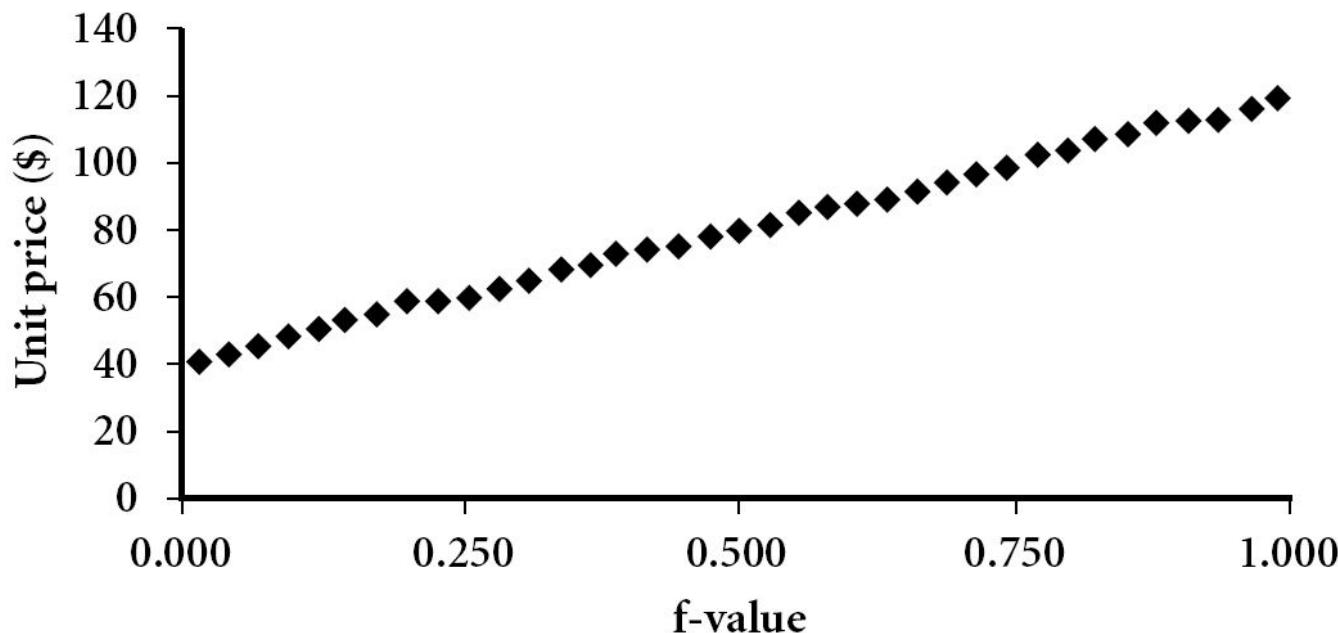
# Histograms Often Tell More than Boxplots



- The two histograms shown in the left may have the same boxplot representation
  - The same values for: min, Q1, median, Q3, max
- But they have rather different data distributions

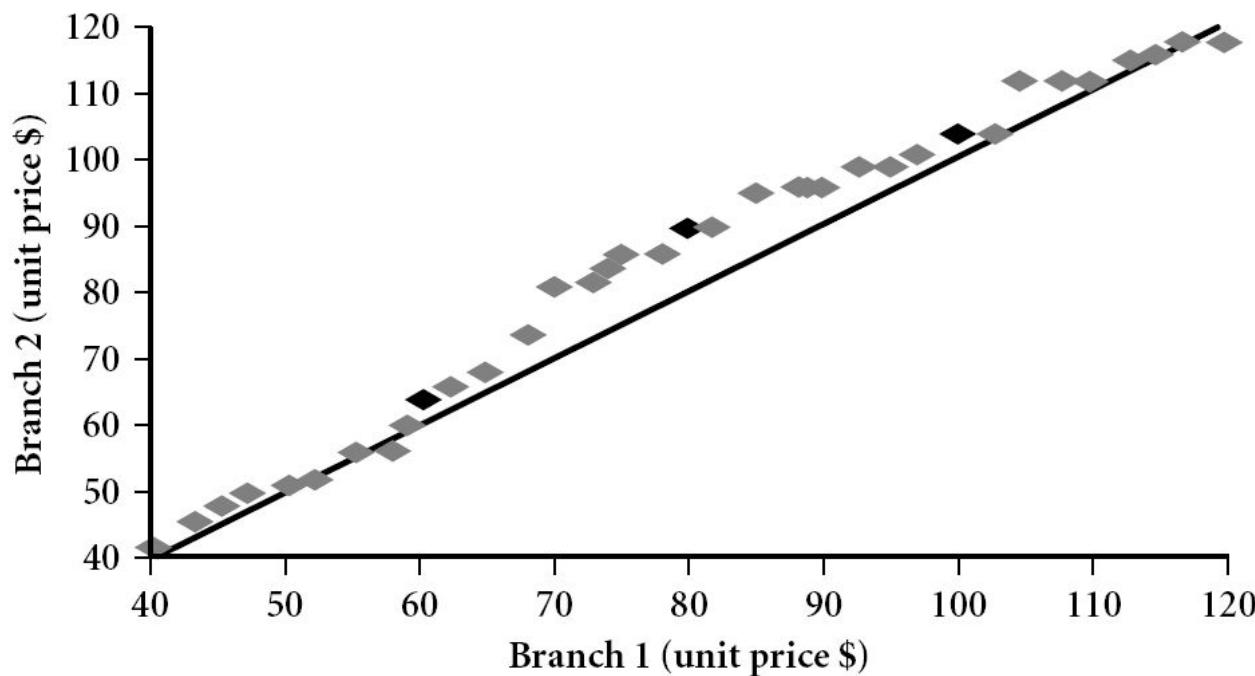
# Quantile Plot

- Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences)
- Plots **quantile** information
  - For a data  $x_i$ , data sorted in increasing order,  $f_i$  indicates that approximately  $100 f_i\%$  of the data are below or equal to the value  $x_i$



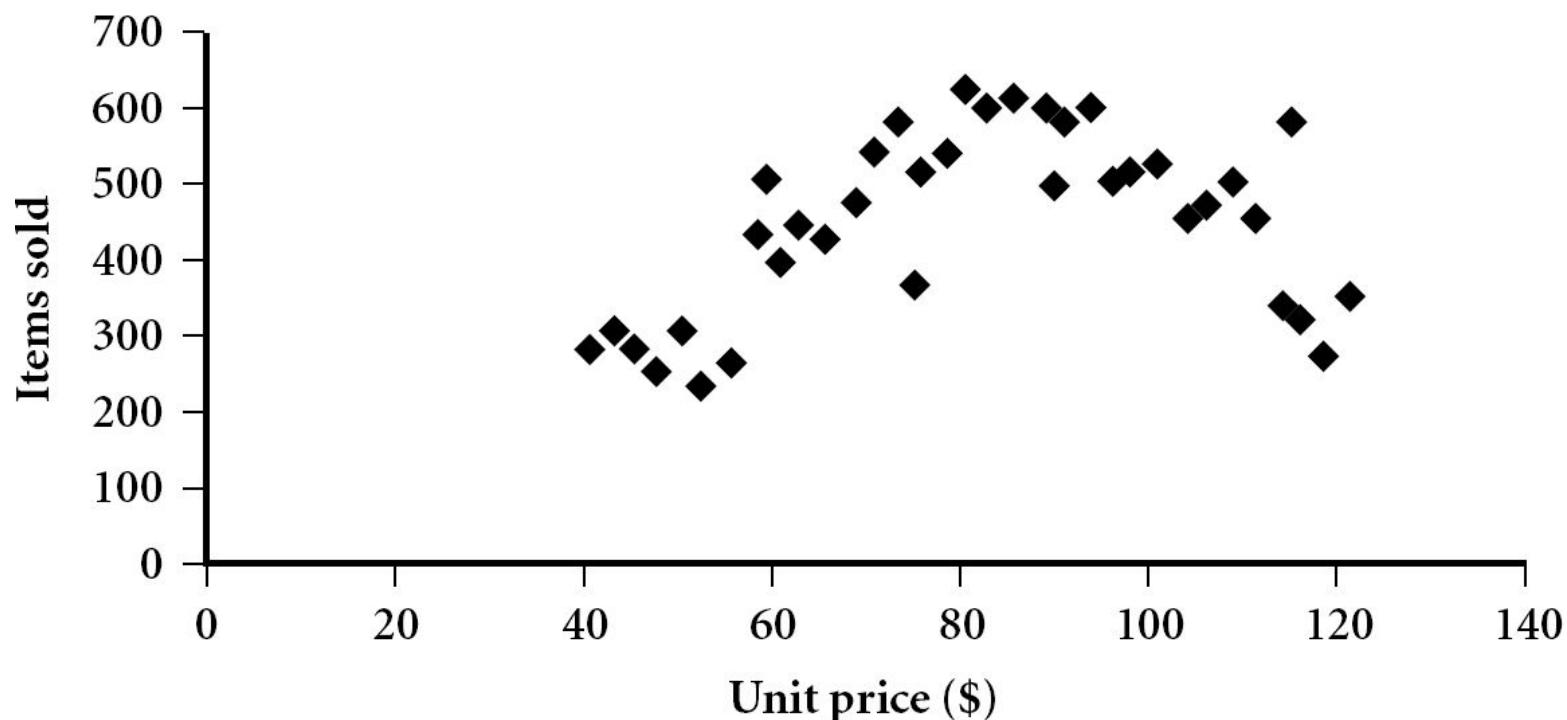
# Quantile-Quantile (Q-Q) Plot

- Graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- View: Is there is a shift in going from one distribution to another?
- Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2.

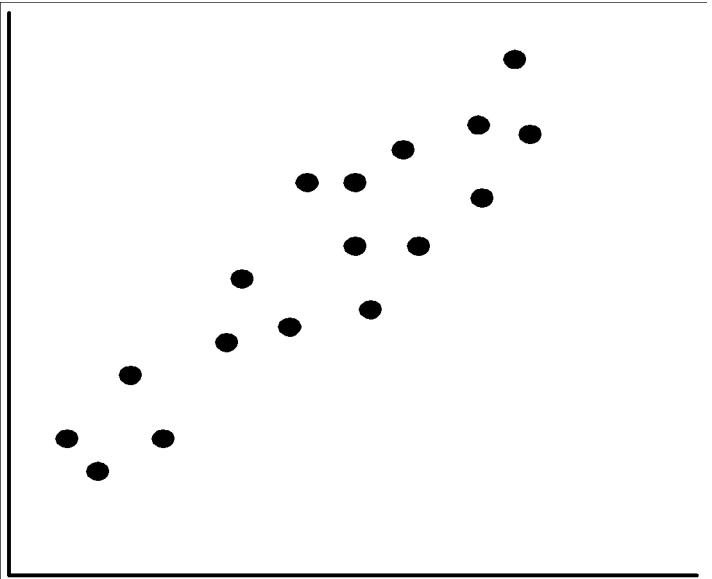


# Scatter plot

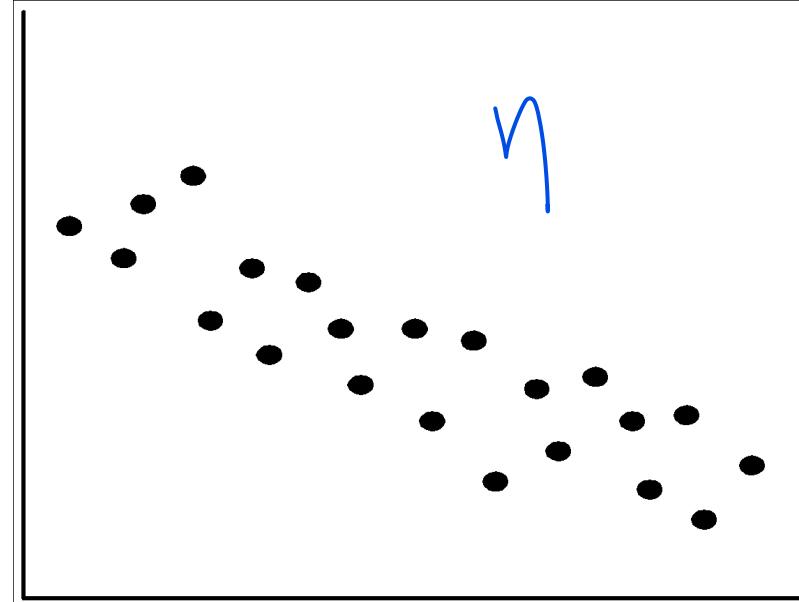
- Provides a first look at bivariate data to see clusters of points, outliers, etc
- Each pair of values is treated as a pair of coordinates and plotted as points in the plane



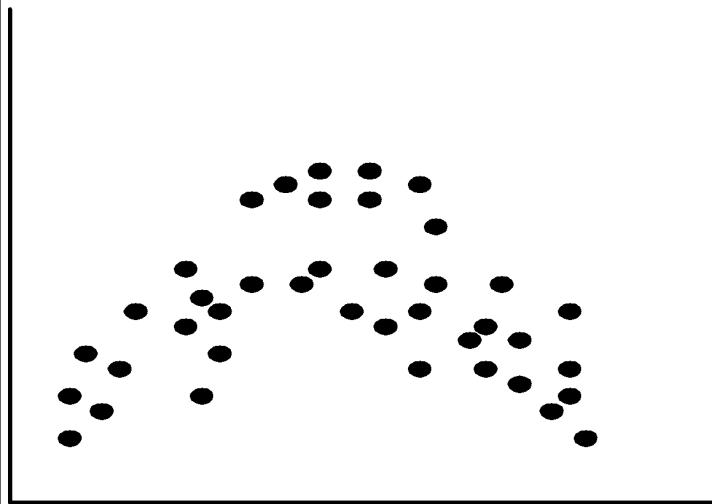
# Positively and Negatively Correlated Data



D

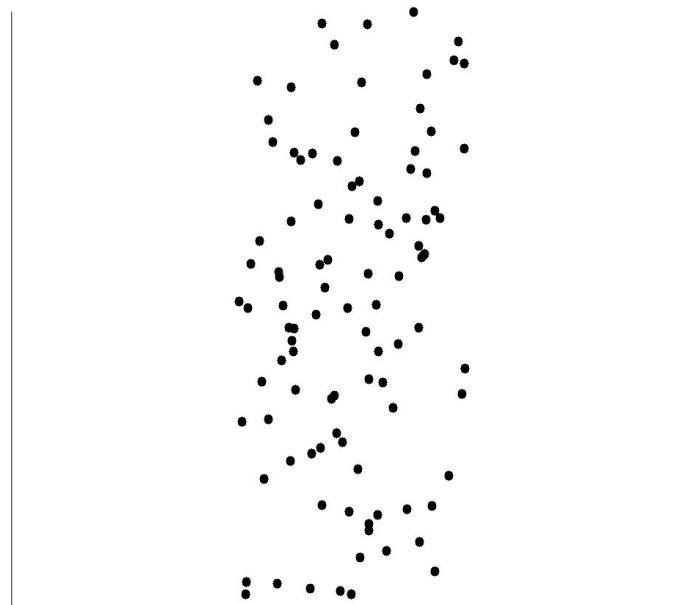
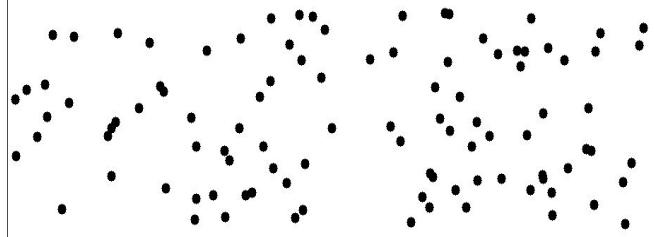
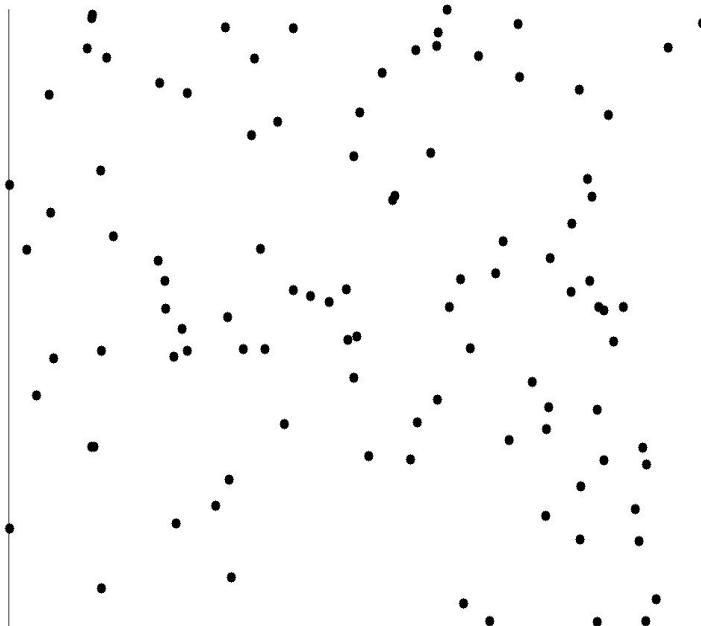


N



- The left half fragment is positively correlated
- The right half is negative correlated

## Uncorrelated Data



# Chapter 2: Getting to Know Your Data

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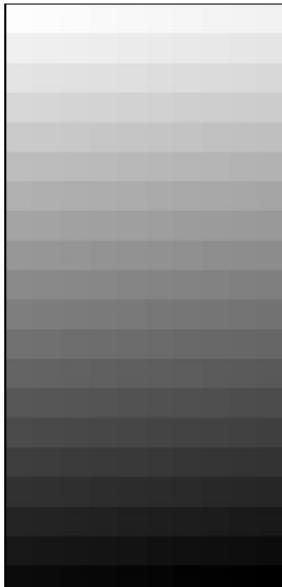
# Data Visualization

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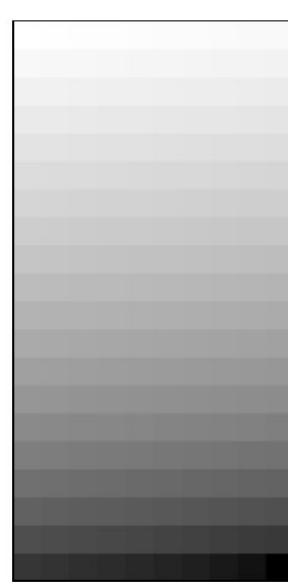
- Why data visualization?
  - Gain insight into an information space by mapping data onto graphical primitives
  - Provide qualitative overview of large data sets
  - Search for patterns, trends, structure, irregularities, relationships among data
  - Help find interesting regions and suitable parameters for further quantitative analysis
  - Provide a visual proof of computer representations derived
- Categorization of visualization methods:
  - Pixel-oriented visualization techniques
  - Geometric projection visualization techniques
  - Icon-based visualization techniques
  - Hierarchical visualization techniques
  - Visualizing complex data and relations

# Pixel-Oriented Visualization Techniques

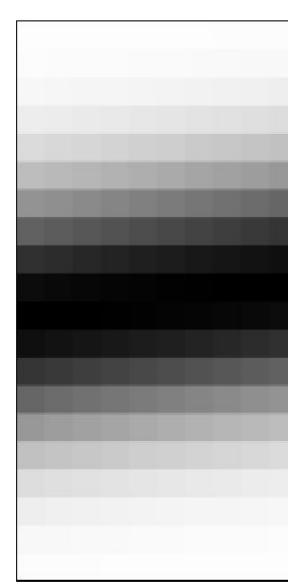
- For a data set of  $m$  dimensions, create  $m$  windows on the screen, one for each dimension
- The  $m$  dimension values of a record are mapped to  $m$  pixels at the corresponding positions in the windows
- The colors of the pixels reflect the corresponding values



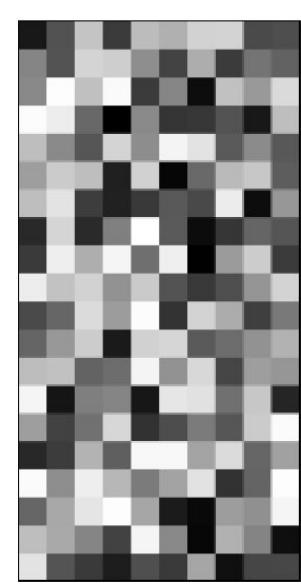
(a) Income



(b) Credit Limit



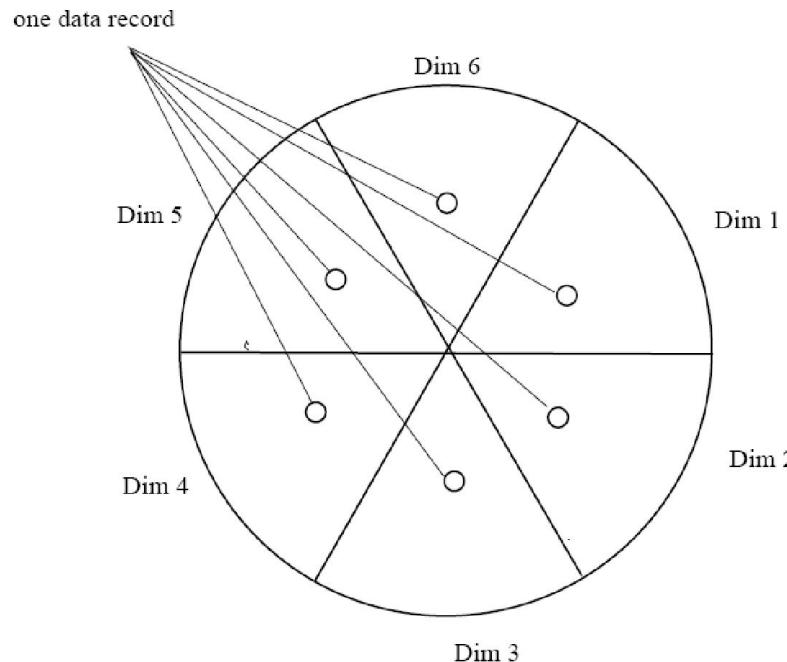
(c) transaction volume



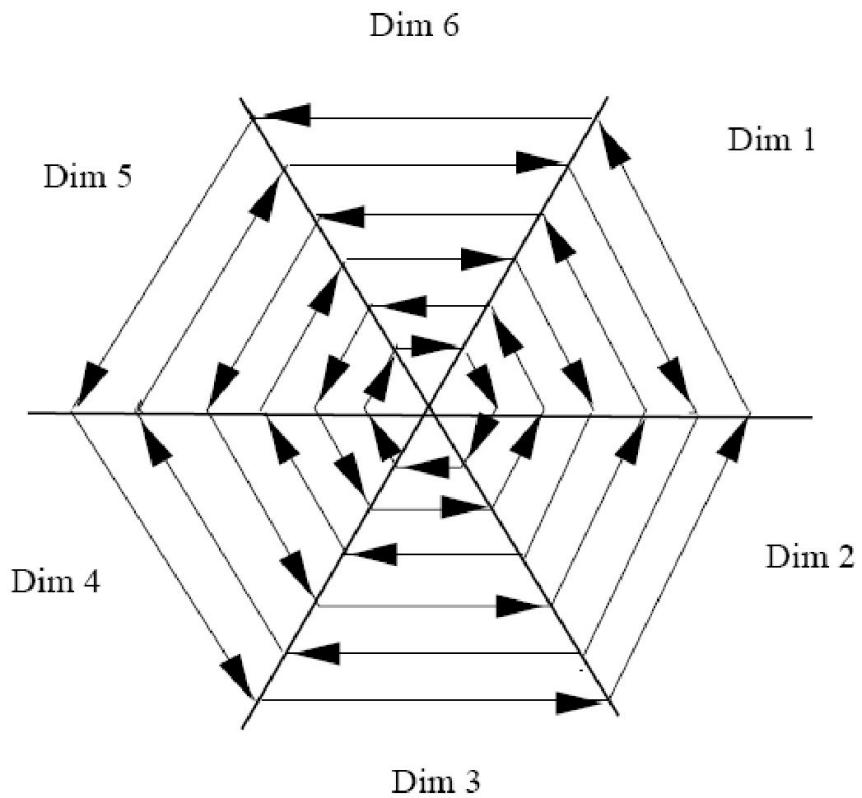
(d) age

# Laying Out Pixels in Circle Segments

- To save space and show the connections among multiple dimensions, space filling is often done in a circle segment



(a) Representing a data record in circle segment



(b) Laying out pixels in circle segment

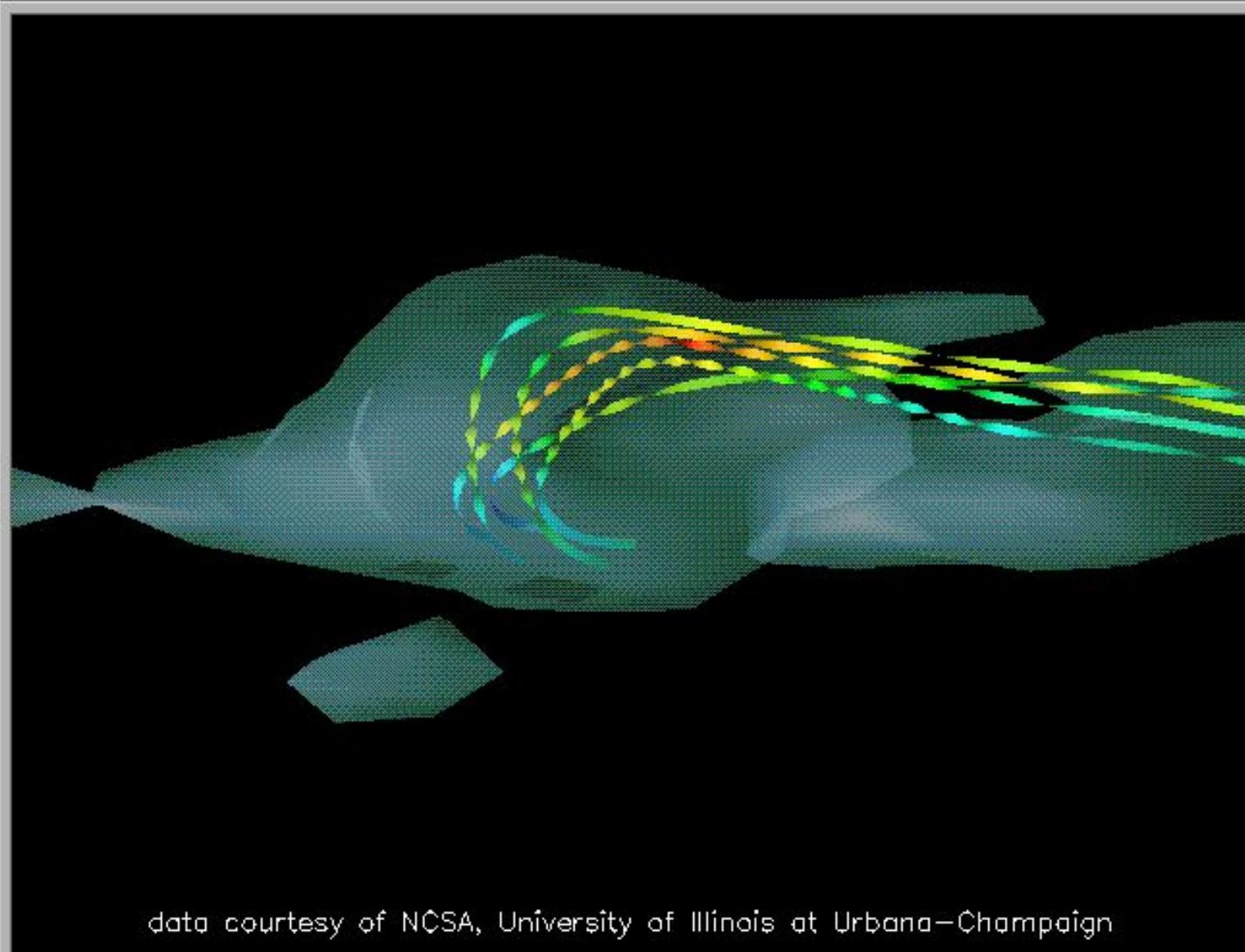
# Geometric Projection Visualization Techniques

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- Visualization of geometric transformations and projections of the data
- Methods
  - Direct visualization
  - Scatterplot and scatterplot matrices
  - Landscapes
  - Projection pursuit technique: Help users find meaningful projections of multidimensional data
  - Prosection views
  - Hyperslice
  - Parallel coordinates

# Direct Data Visualization

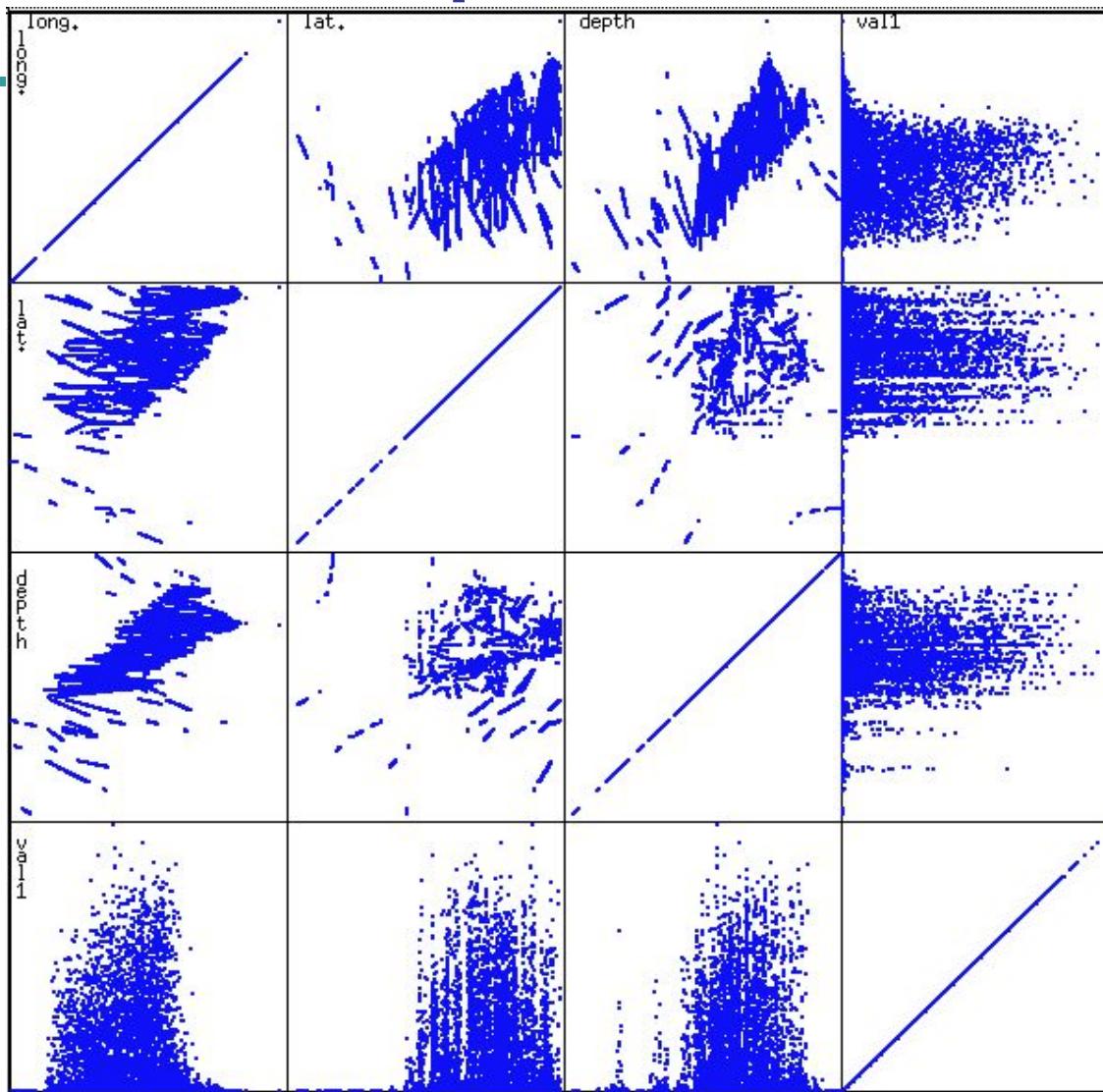
Ribbons with Twists Based on Vorticity



data courtesy of NCSA, University of Illinois at Urbana-Champaign

# Scatterplot Matrices

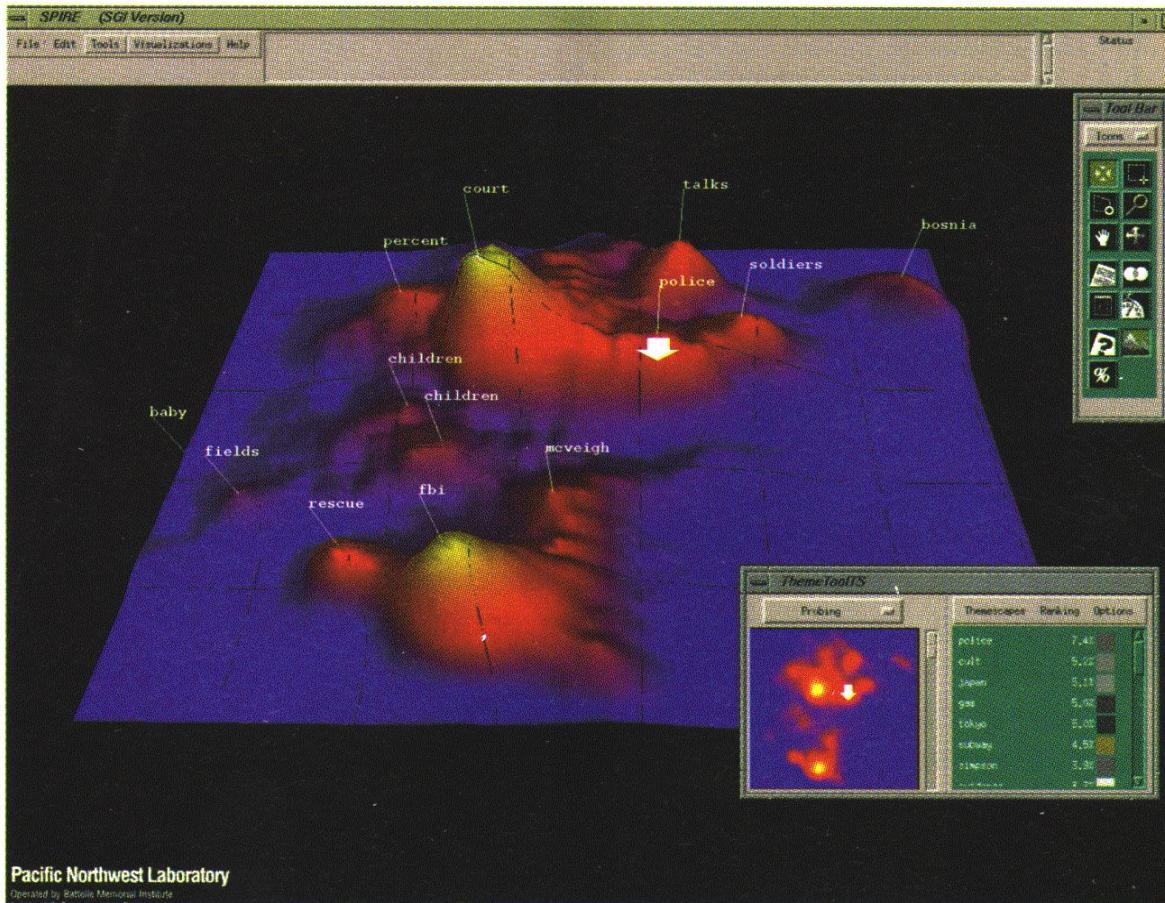
Used by permission of M. Ward, Worcester Polytechnic Institute



Matrix of scatterplots (x-y-diagrams) of the k-dim. data [total of  $(k^2/2-k)$  scatterplots]

# Landscapes

Used by permission of B. Wright, Visible Decisions Inc.

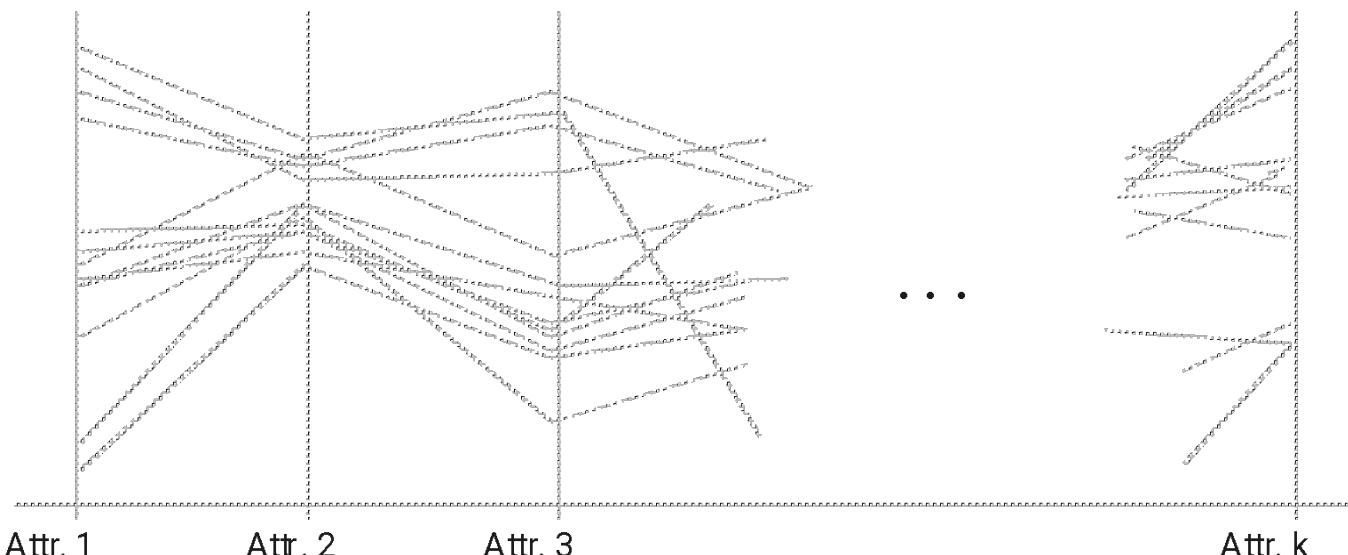


news articles  
visualized as  
a landscape

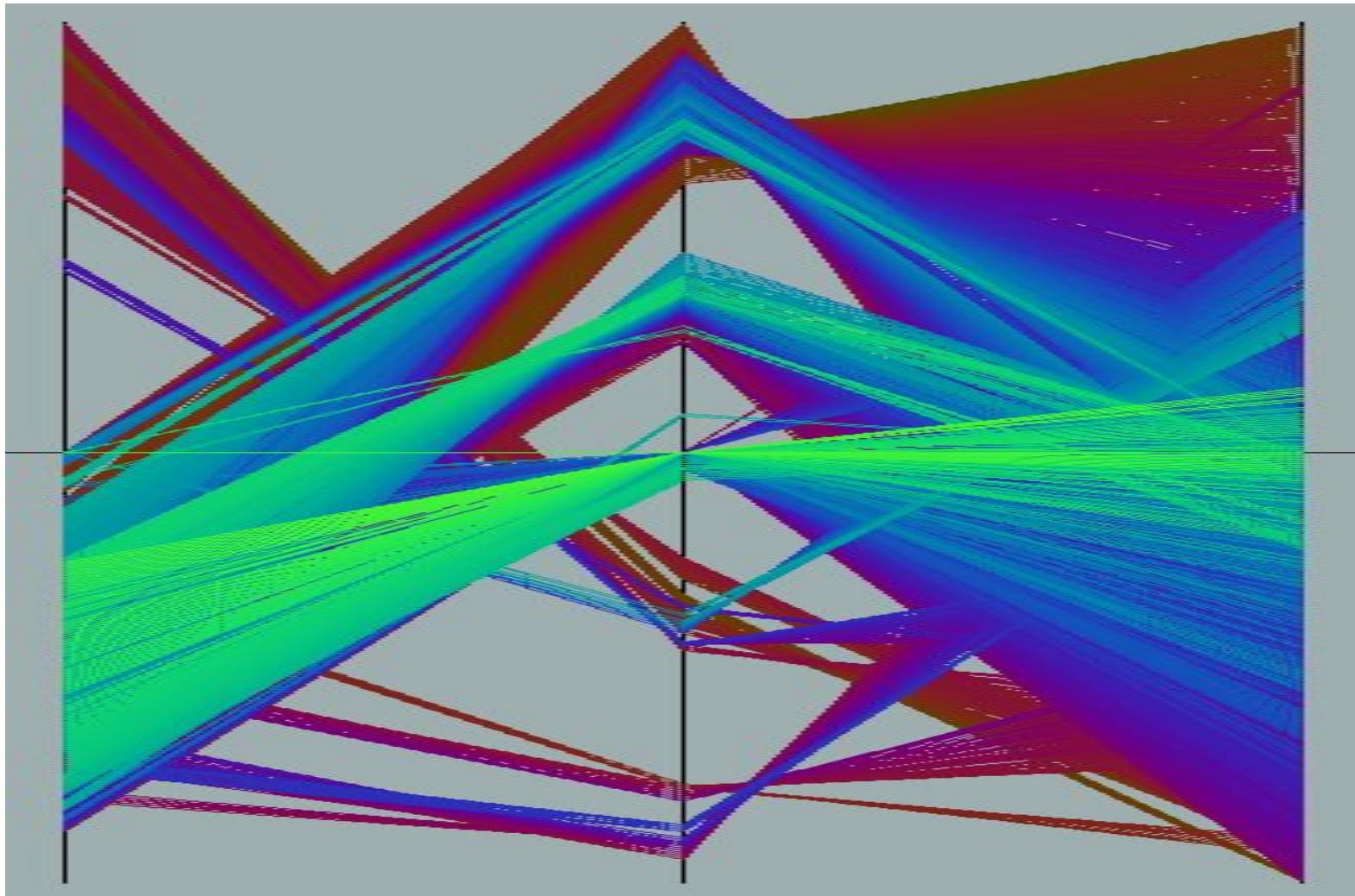
- Visualization of the data as perspective landscape
- The data needs to be transformed into a (possibly artificial) 2D spatial representation which preserves the characteristics of the data

# Parallel Coordinates

- n equidistant axes which are parallel to one of the screen axes and correspond to the attributes
- The axes are scaled to the [minimum, maximum]: range of the corresponding attribute
- Every data item corresponds to a polygonal line which intersects each of the axes at the point which corresponds to the value for the attribute



# Parallel Coordinates of a Data Set



# Icon-Based Visualization Techniques

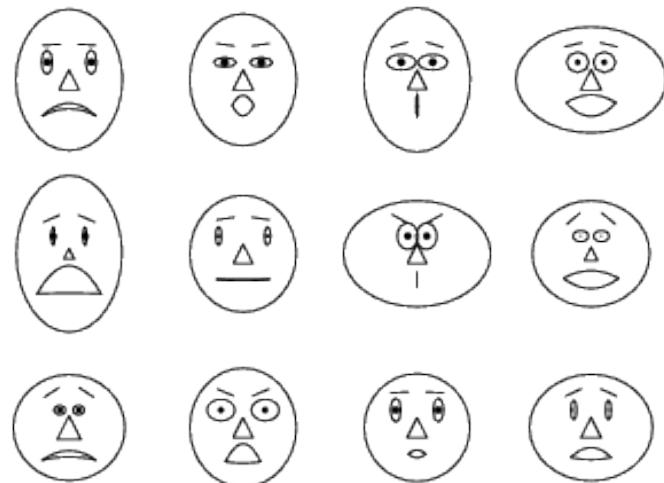
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- Visualization of the data values as features of icons
- Typical visualization methods
  - Chernoff Faces
  - Stick Figures
- General techniques
  - Shape coding: Use shape to represent certain information encoding
  - Color icons: Use color icons to encode more information
  - Tile bars: Use small icons to represent the relevant feature vectors in document retrieval

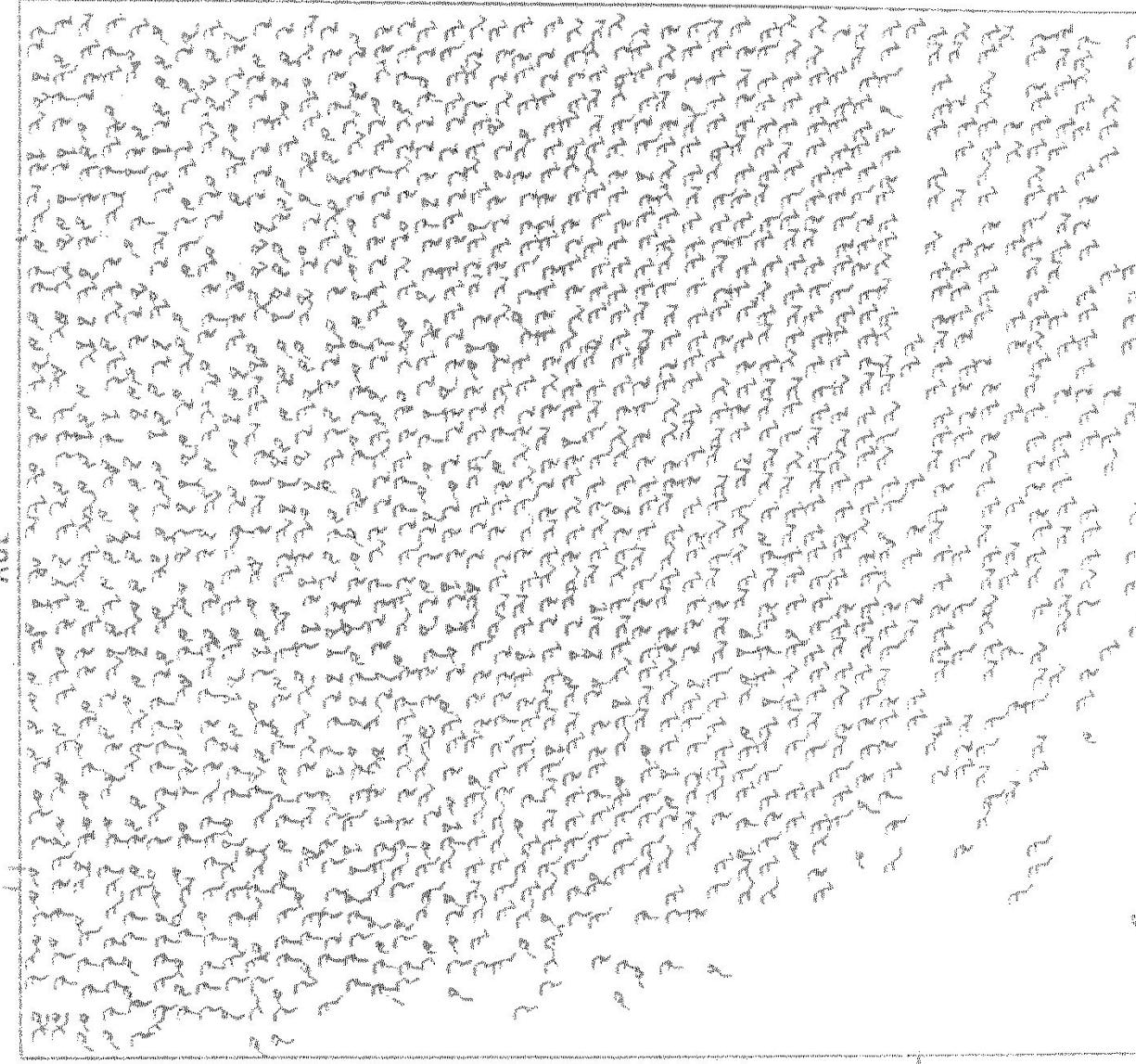
# Chernoff Faces

- A way to display variables on a two-dimensional surface, e.g., let  $x$  be eyebrow slant,  $y$  be eye size,  $z$  be nose length, etc.
- The figure shows faces produced using 10 characteristics--head eccentricity, eye size, eye spacing, eye eccentricity, pupil size, eyebrow slant, nose size, mouth shape, mouth size, and mouth opening): Each assigned one of 10 possible values, generated using *Mathematica* (S. Dickson)

- REFERENCE: Gonick, L. and Smith, W. *The Cartoon Guide to Statistics*. New York: Harper Perennial, p. 212, 1993
- Weisstein, Eric W. "Chernoff Face." From *MathWorld--A Wolfram Web Resource*.  
[mathworld.wolfram.com/ChernoffFace.html](http://mathworld.wolfram.com/ChernoffFace.html)



# Stick Figure



A census data figure showing age, income, gender, education, etc.

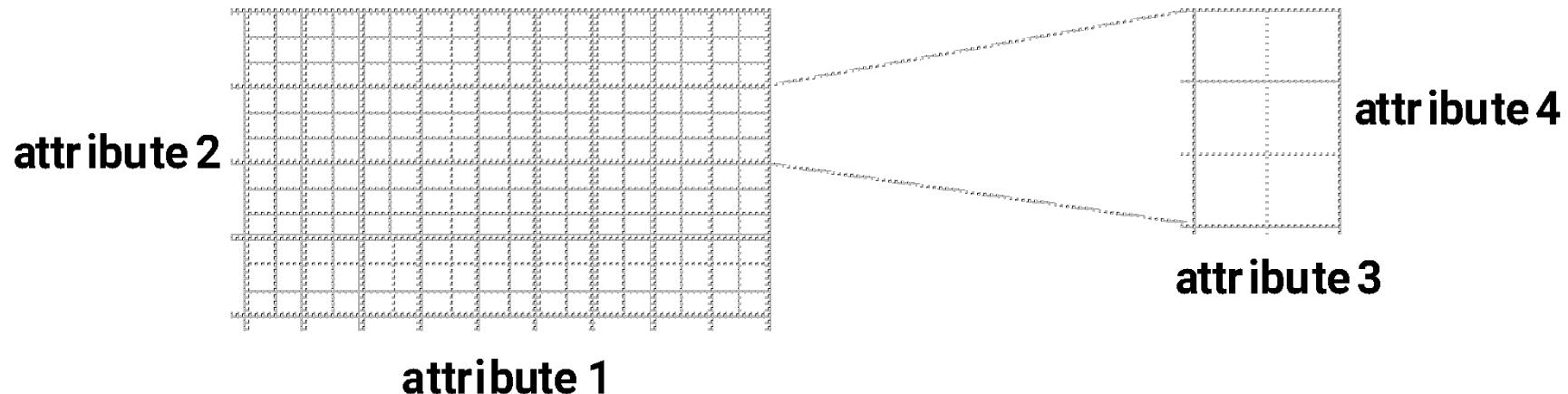
A 5-piece stick figure (1 body and 4 limbs w. different angle/length)

# Hierarchical Visualization Techniques

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- Visualization of the data using a hierarchical partitioning into subspaces
- Methods
  - Dimensional Stacking
  - Worlds-within-Worlds
  - Tree-Map
  - Cone Trees
  - InfoCube

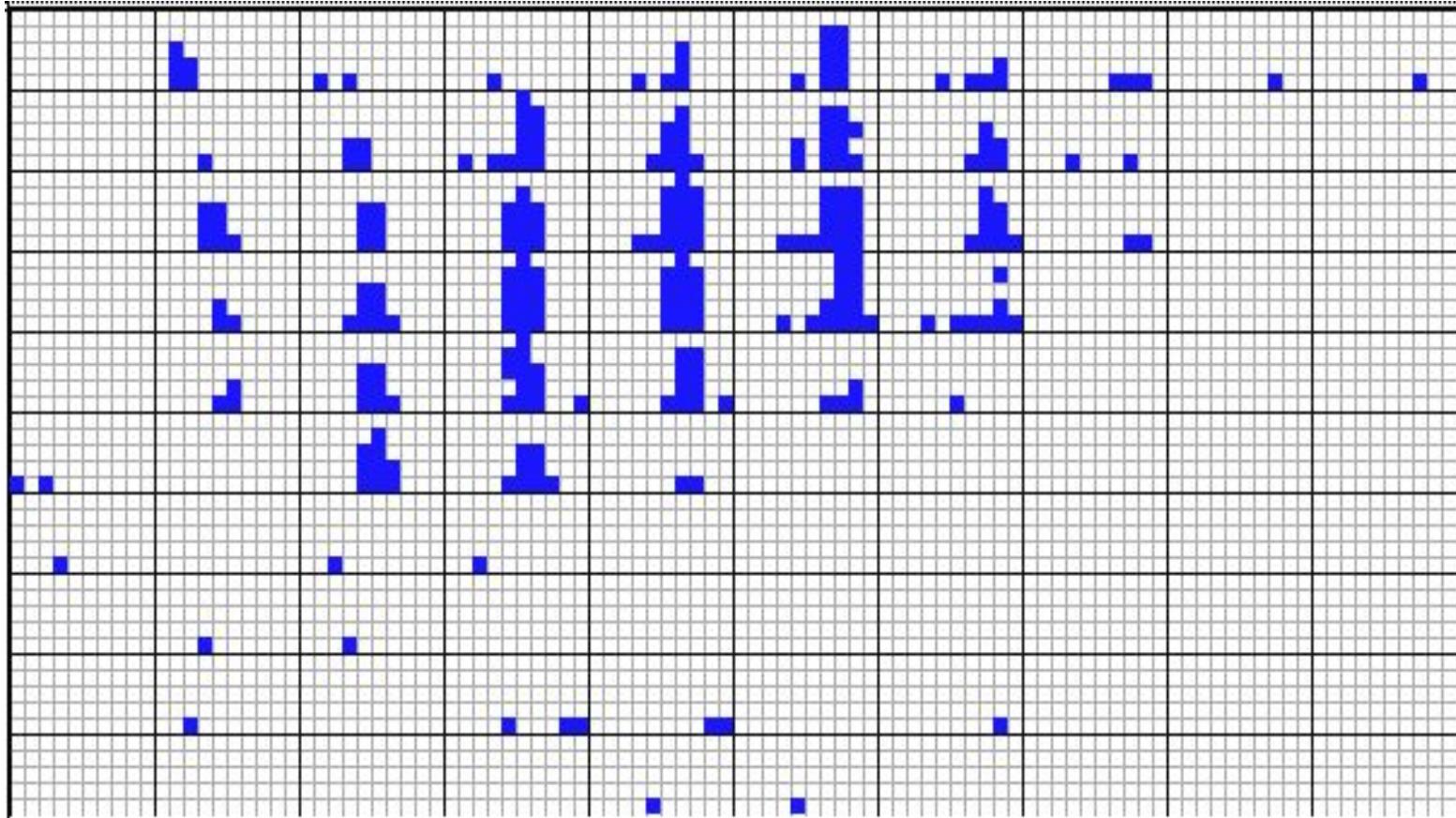
# Dimensional Stacking



- Partitioning of the n-dimensional attribute space in 2-D subspaces, which are 'stacked' into each other
- Partitioning of the attribute value ranges into classes. The important attributes should be used on the outer levels.
- Adequate for data with ordinal attributes of low cardinality
- But, difficult to display more than nine dimensions
- Important to map dimensions appropriately

# Dimensional Stacking

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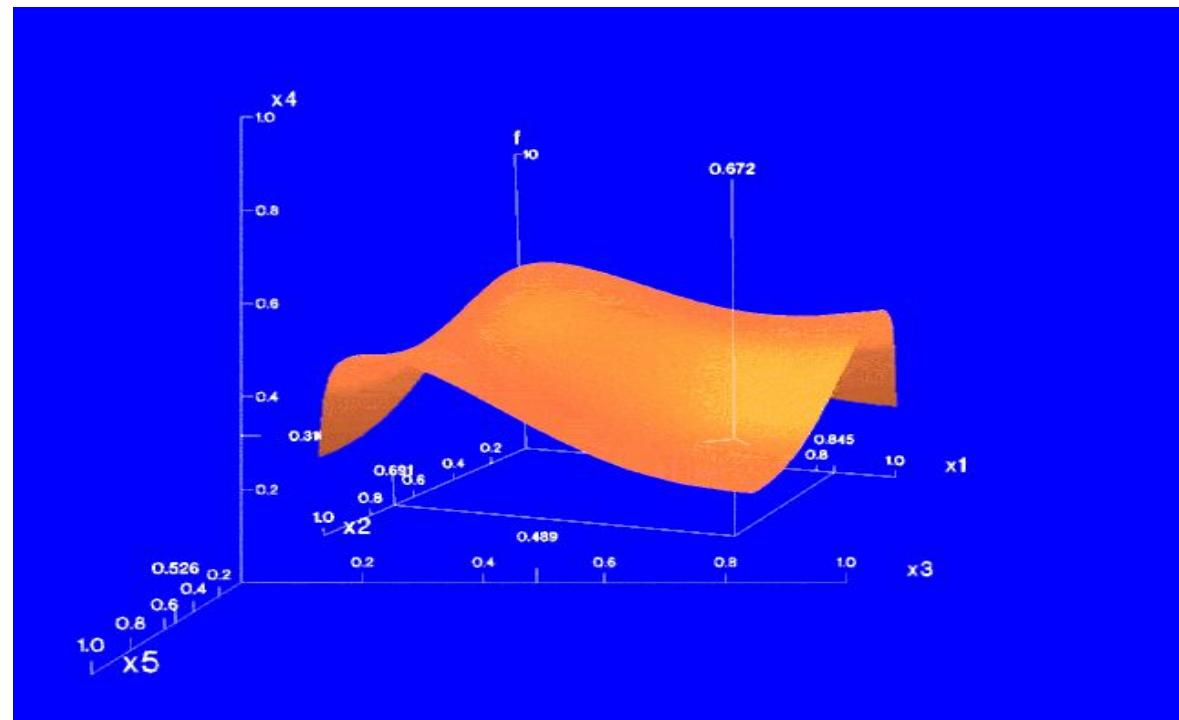


Visualization of oil mining data with longitude and latitude mapped to the outer x-, y-axes and ore grade and depth mapped to the inner x-, y-axes

# Worlds-within-Worlds

- Assign the function and two most important parameters to innermost world
- Fix all other parameters at constant values - draw other (1 or 2 or 3 dimensional worlds choosing these as the axes)
- Software that uses this paradigm

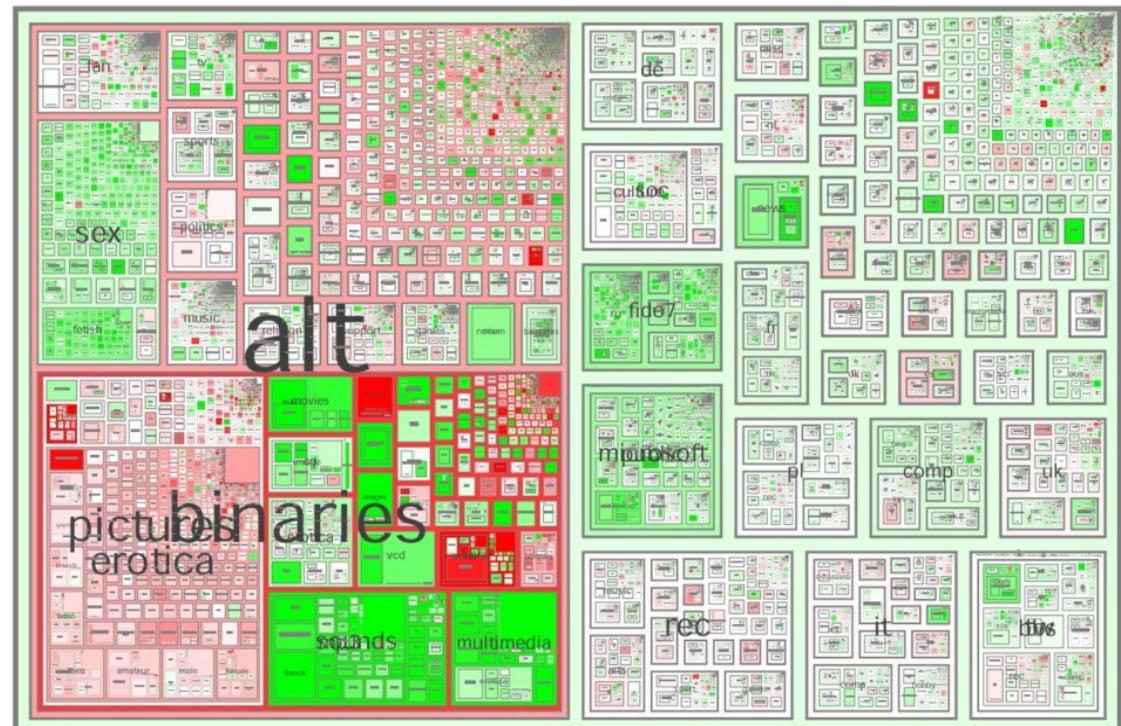
- N-vision: Dynamic interaction through data glove and stereo displays, including rotation, scaling (inner) and translation (inner/outer)
- Auto Visual: Static interaction by means of queries



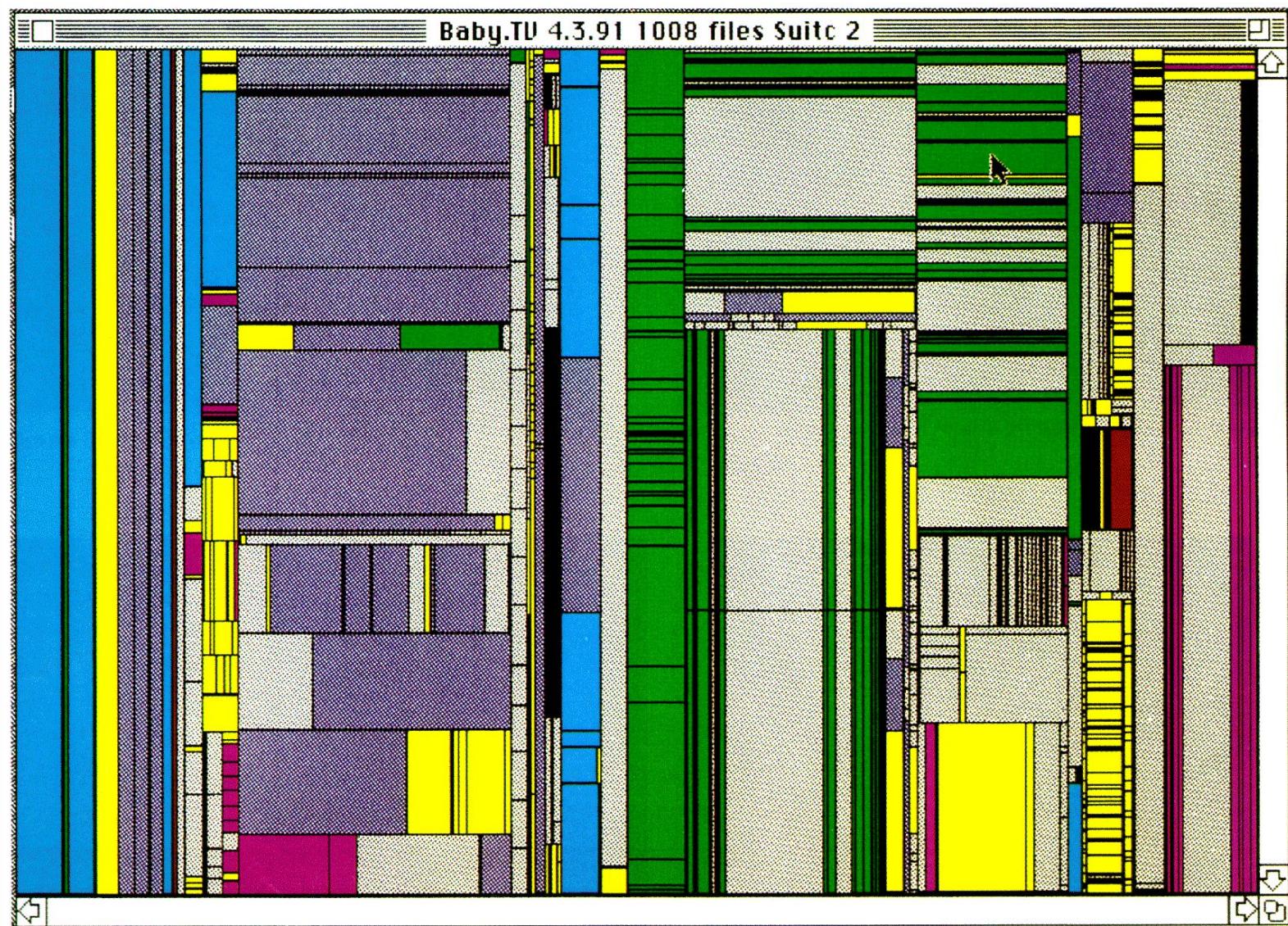
# Tree-Map

- Screen-filling method which uses a hierarchical partitioning of the screen into regions depending on the attribute values
  - The x- and y-dimension of the screen are partitioned alternately according to the attribute values (classes)

# MSR Netscan Image



# Tree-Map of a File System (Schneiderman)



# InfoCube

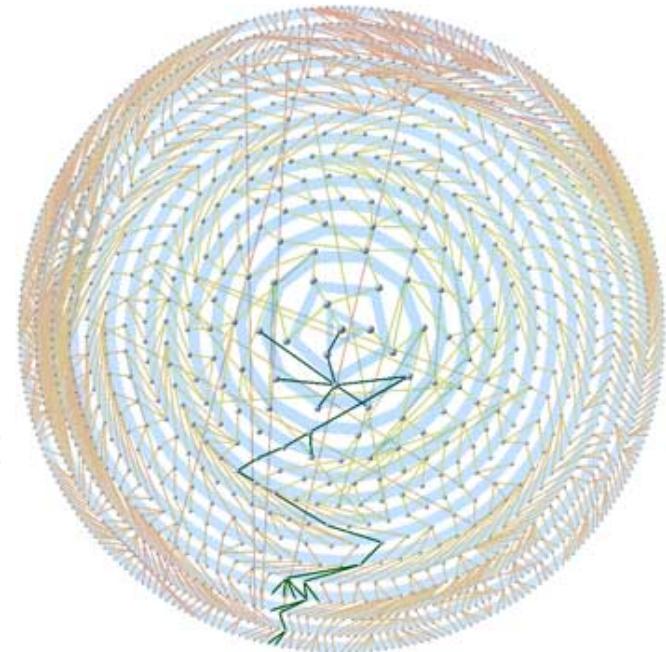
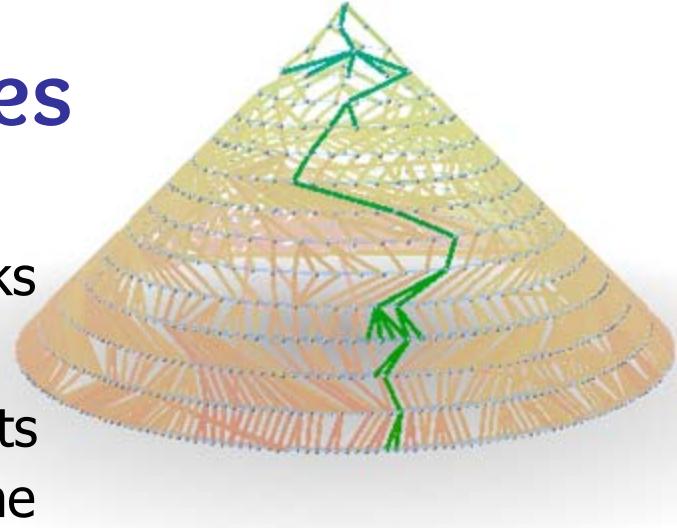
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- A 3-D visualization technique where hierarchical information is displayed as nested semi-transparent cubes
- The outermost cubes correspond to the top level data, while the subnodes or the lower level data are represented as smaller cubes inside the outermost cubes, and so on



# Three-D Cone Trees

- *3D cone tree* visualization technique works well for up to a thousand nodes or so
- First build a *2D circle tree* that arranges its nodes in concentric circles centered on the root node
- Cannot avoid overlaps when projected to 2D
- G. Robertson, J. Mackinlay, S. Card. "Cone Trees: Animated 3D Visualizations of Hierarchical Information", *ACM SIGCHI'91*
- Graph from Nadeau Software Consulting website: Visualize a social network data set that models the way an infection spreads from one person to the next



# Visualizing Complex Data and Relations

- Visualizing non-numerical data: text and social networks
- Tag cloud: visualizing user-generated tags
  - The importance of tag is represented by font size/color
- Besides text data, there are also methods to visualize relationships, such as visualizing social networks

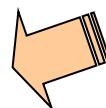


Newsmap: Google News Stories in 2005

# Chapter 2: Getting to Know Your Data

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- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary



# Similarity and Dissimilarity

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- **Similarity**
  - Numerical measure of how alike two data objects are
  - Value is higher when objects are more alike
  - Often falls in the range [0,1]
- **Dissimilarity** (e.g., distance)
  - Numerical measure of how different two data objects are
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- **Proximity** refers to a similarity or dissimilarity

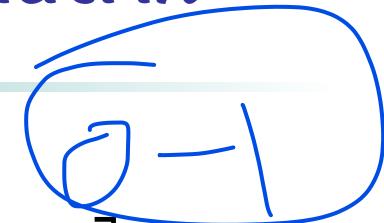


# Data Matrix and Dissimilarity Matrix

## ■ Data matrix

- n data points with p dimensions
- Two modes

$$\begin{bmatrix} x_{11} & \dots & x_{1f} & \dots & x_{1p} \\ \dots & \dots & \dots & \dots & \dots \\ x_{i1} & \dots & x_{if} & \dots & x_{ip} \\ \dots & \dots & \dots & \dots & \dots \\ x_{n1} & \dots & x_{nf} & \dots & x_{np} \end{bmatrix}$$



## ■ Dissimilarity matrix

- n data points, but registers only the distance
- A triangular matrix
- Single mode

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \dots & \dots & 0 \end{bmatrix}$$



## Proximity Measure for Nominal Attributes

- Can take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)
- Method 1: Simple matching
  - $m$ : # of matches,  $p$ : total # of variables
- Method 2: Use a large number of binary attributes
  - creating a new binary attribute for each of the  $M$  nominal states

Attribute

$$d(i, j) = \frac{p - m}{p}$$

# Proximity Measure for Binary Attributes

- A contingency table for binary data

		Object <i>j</i>		
		1	0	sum
Object <i>i</i>	1	<i>q</i>	<i>r</i>	<i>q + r</i>
	0	<i>s</i>	<i>t</i>	<i>s + t</i>
sum		<i>q + s</i>	<i>r + t</i>	<i>p</i>

- Distance measure for **symmetric**

binary variables:

$$d(i, j) = \frac{r + s}{q + r + s + t}$$

- Distance measure for **asymmetric**

binary variables:

$$d(i, j) = \frac{r + s}{q + r + s}$$

- Jaccard coefficient (*similarity* measure for **asymmetric** binary variables):

$$\text{sim}_{\text{Jaccard}}(i, j) = \frac{q}{q + r + s}$$

- Note: Jaccard coefficient is the same as "coherence":

$$\text{coherence}(i, j) = \frac{\text{sup}(i, j)}{\text{sup}(i) + \text{sup}(j) - \text{sup}(i, j)} = \frac{q}{(q + r) + (q + s) - q}$$

# Dissimilarity between Binary Variables

## ■ Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

- Gender is a symmetric attribute
- The remaining attributes are asymmetric binary
- Let the values Y and P be 1, and the value N 0

$$d(jack, mary) = \frac{0 + 1}{2 + 0 + 1} = 0.33$$

$$d(jack, jim) = \frac{1 + 1}{1 + 1 + 1} = 0.67$$

$$d(jim, mary) = \frac{1 + 2}{1 + 1 + 2} = 0.75$$

# Standardizing Numeric Data

- Z-score: 
$$z = \frac{x - \mu}{\sigma}$$
  - X: raw score to be standardized,  $\mu$ : mean of the population,  $\sigma$ : standard deviation
  - the distance between the raw score and the population mean in units of the standard deviation
  - negative when the raw score is below the mean, “+” when above
- An alternative way: Calculate the mean absolute deviation

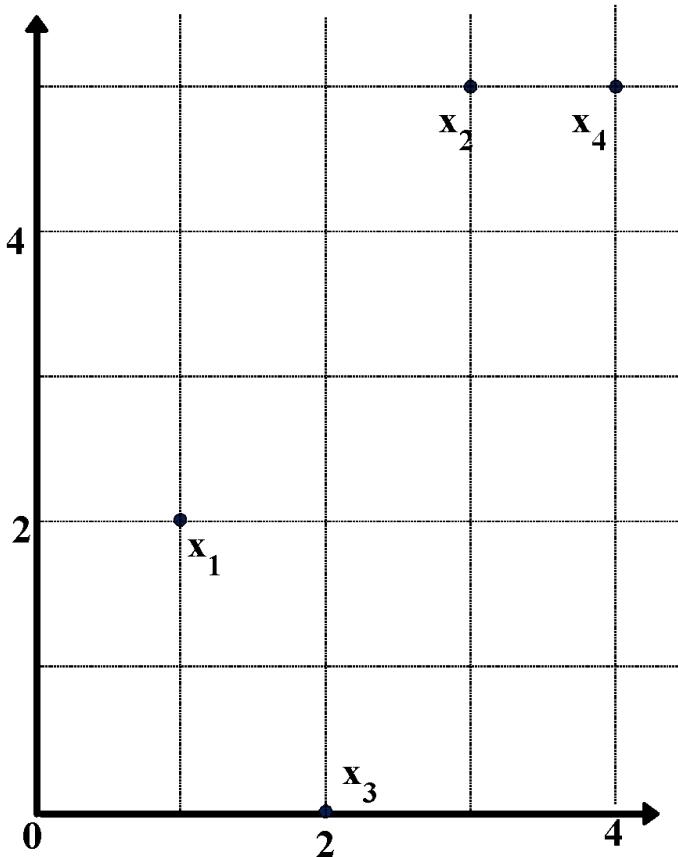
where 
$$s_f = \frac{1}{n}(|x_{1f} - m_f| + |x_{2f} - m_f| + \dots + |x_{nf} - m_f|)$$

$$m_f = \frac{1}{n}(x_{1f} + x_{2f} + \dots + x_{nf}).$$

- standardized measure (z-score): 
$$z_{if} = \frac{x_{if} - m_f}{s_f}$$
- Using mean absolute deviation is more robust than using standard deviation

# Example:

## Data Matrix and Dissimilarity Matrix



**Data Matrix**

point	attribute1	attribute2
$x_1$	1	2
$x_2$	3	5
$x_3$	2	0
$x_4$	4	5

**Dissimilarity Matrix**

(with Euclidean Distance)

	$x_1$	$x_2$	$x_3$	$x_4$
$x_1$	0			
$x_2$	3.61	0		
$x_3$	5.1	5.1	0	
$x_4$	4.24	1	5.39	0

## Distance on Numeric Data: Minkowski Distance

- *Minkowski distance*: A popular distance measure

$$d(i, j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}$$

where  $i = (x_{i1}, x_{i2}, \dots, x_{ip})$  and  $j = (x_{j1}, x_{j2}, \dots, x_{jp})$  are two  $p$ -dimensional data objects, and  $h$  is the order (the distance so defined is also called L- $h$  norm)

- Properties
  - $d(i, j) > 0$  if  $i \neq j$ , and  $d(i, i) = 0$  (Positive definiteness)
  - $d(i, j) = d(j, i)$  (Symmetry)
  - $d(i, j) \leq d(i, k) + d(k, j)$  (Triangle Inequality)
- A distance that satisfies these properties is a **metric**

# Special Cases of Minkowski Distance

- ✓  $h = 1$ : Manhattan (city block,  $L_1$  norm) distance
  - E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i, j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \dots + |x_{ip} - x_{jp}|$$

- $h = 2$ : ( $L_2$  norm) Euclidean distance

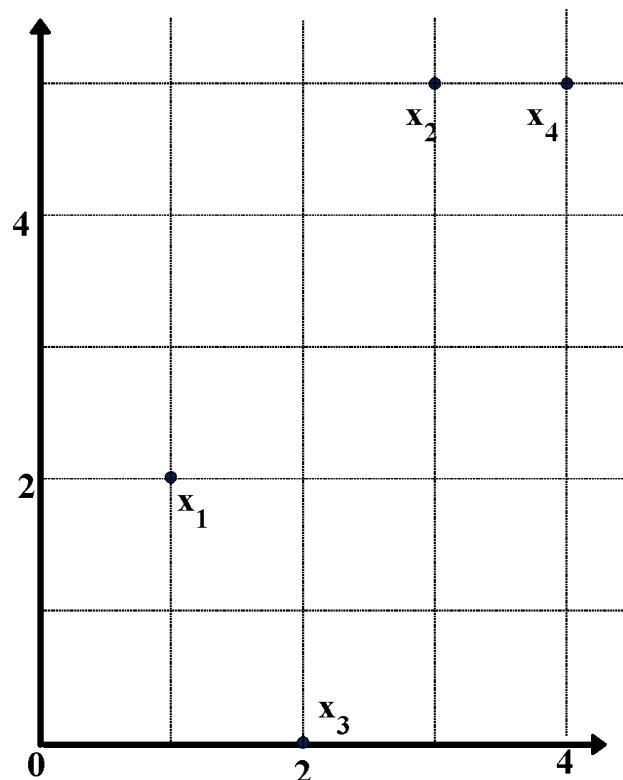
$$d(i, j) = \sqrt{(|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + \dots + |x_{ip} - x_{jp}|^2)}$$

- $h \rightarrow \infty$ . “supremum” ( $L_{\max}$  norm,  $L_\infty$  norm) distance.
  - This is the maximum difference between any component (attribute) of the vectors

$$d(i, j) = \lim_{h \rightarrow \infty} \left( \sum_{f=1}^p |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_f^p |x_{if} - x_{jf}|$$

# Example: Minkowski Distance

point	attribute 1	attribute 2
x1	1	2
x2	3	5
x3	2	0
x4	4	5



## Dissimilarity Matrices

### Manhattan ( $L_1$ )

L	x1	x2	x3	x4
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

### Euclidean ( $L_2$ )

L2	x1	x2	x3	x4
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

### Supremum

$L_\infty$	x1	x2	x3	x4
x1	0			
x2	3	0		
x3	2	5	0	
x4	3	1	5	0

# Ordinal Variables

- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank
- Can be treated like interval-scaled
  - replace  $x_{if}$  by their rank  $r_{if} \in \{1, \dots, M_f\}$
  - map the range of each variable onto  $[0, 1]$  by replacing  $i$ -th object in the  $f$ -th variable by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

- compute the dissimilarity using methods for interval-scaled variables

# ✓ Attributes of Mixed Type

- A database may contain all attribute types
  - Nominal, symmetric binary, asymmetric binary, numeric, ordinal
- One may use a weighted formula to combine their effects

$$\checkmark d(i,j) = \frac{\sum_{f=1}^p \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^p \delta_{ij}^{(f)}}$$

- $f$  is binary or nominal:  
 $d_{ij}^{(f)} = 0$  if  $x_{if} = x_{jf}$ , or  $d_{ij}^{(f)} = 1$  otherwise
- $f$  is numeric: use the normalized distance
- $f$  is ordinal
  - Compute ranks  $r_{if}$  and
  - Treat  $z_{if}$  as interval-scaled

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

# Cosine Similarity

- A **document** can be represented by thousands of attributes, each recording the *frequency* of a particular word (such as keywords) or phrase in the document.

Document	team	coach	hockey	baseball	soccer	penalty	score	win	loss	season
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

- Other vector objects: gene features in micro-arrays, ...
- Applications: information retrieval, biologic taxonomy, gene feature mapping, ...
- Cosine measure: If  $d_1$  and  $d_2$  are two vectors (e.g., term-frequency vectors), then

$$\cos(d_1, d_2) = (d_1 \cdot d_2) / \|d_1\| \|d_2\|,$$

where  $\cdot$  indicates vector dot product,  $\|d\|$ : the length of vector  $d$

# Example: Cosine Similarity

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- $\cos(d_1, d_2) = (d_1 \cdot d_2) / \|d_1\| \|d_2\|$ ,  
where  $\cdot$  indicates vector dot product,  $\|d\|$ : the length of vector  $d$
- Ex: Find the **similarity** between documents 1 and 2.

$$d_1 = (5, 0, 3, 0, 2, 0, 0, 2, 0, 0)$$

$$d_2 = (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)$$

$$d_1 \cdot d_2 = 5*3 + 0*0 + 3*2 + 0*0 + 2*1 + 0*1 + 0*1 + 2*1 + 0*0 + 0*1 = 25$$

$$\|d_1\| = \sqrt{(5*5 + 0*0 + 3*3 + 0*0 + 2*2 + 0*0 + 0*0 + 2*2 + 0*0 + 0*0)}^{0.5} = (42)^{0.5} \\ = 6.481$$

$$\|d_2\| = \sqrt{(3*3 + 0*0 + 2*2 + 0*0 + 1*1 + 1*1 + 0*0 + 1*1 + 0*0 + 1*1)}^{0.5} = (17)^{0.5} \\ = 4.12$$

$$\cos(d_1, d_2) = 0.94$$

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# Summary

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- Data attribute types: nominal, binary, ordinal, interval-scaled, ratio-scaled
- Many types of data sets, e.g., numerical, text, graph, Web, image.
- Gain insight into the data by:
  - Basic statistical data description: central tendency, dispersion, graphical displays
  - Data visualization: map data onto graphical primitives
  - Measure data similarity
- Above steps are the beginning of data preprocessing.
- Many methods have been developed but still an active area of research.

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