$$f(x+y+z,\ x^n+y^n+z^n)=0$$

UP

Practice

$$\Rightarrow \frac{x+y}{xy} + \frac{2}{z} = c_1 + c_2$$

$$\Rightarrow \frac{xy}{xy} + \frac{2}{1} = c_1 + c_2 \quad [\because x+y = xy,\ z = 1]$$

$$\Rightarrow 1 + 2 = c_1 + c_2$$

$$\therefore c_1 + c_2 = 3$$

Putting values in eqn (v),

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{z} = 3$$

$\therefore \frac{1}{x} + \frac{1}{y} + \frac{2}{z} = 3$ is the required integral surface.

(Ans)

Similarly,

By taking 2nd and 3rd part of eqn (ii),

$$\frac{dy}{y^2} = \frac{dz}{-z^2}$$

By integrating,

$$\int y^{-2}\,dy = -\int z^{-2}\,dz$$

$$\Rightarrow \int y^{-2}\,dy + \int z^{-2}\,dz = 0$$

$$\Rightarrow \frac{y^{-2+1}}{-2+1} + \frac{z^{-2+1}}{-2+1} = -c_2$$

$$\Rightarrow -y^{-1} - z^{-1} = -c_2$$

$$\Rightarrow y^{-1} + z^{-1} = c_2 \quad \text{——— (iv)}$$

By adding (iii) and (iv),

$$x^{-1} + z^{-1} + y^{-1} + z^{-1} = c_1 + c_2 \quad \text{——— (v)}$$

$$\Rightarrow x^{-1} + y^{-1} + 2z^{-1} = c_1 + c_2$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{2}{z} = c_1 + c_2$$

CamScanner

Math

Find the integral surface of $x^2p + y^2q + z^2 = 0$, which passes through the hyperbola $xy = x + y$, $z = 1$.

Solⁿ: Given,

$$x^2p + y^2q + z^2 = 0$$

$$\Rightarrow x^2p + y^2q = -z^2$$

By definition of Lagrange's auxiliary equation,

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{-z^2} \quad \text{——— (ii)}$$

Taking 1st and 3rd part ~~par~~ of eqⁿ (ii),

$$\frac{dx}{x^2} = \frac{dz}{-z^2}$$

By integrating —

$$\int \frac{dx}{x^2} = \int \frac{dz}{-z^2}$$

$$\Rightarrow \frac{x^{-2+1}}{-2+1} = -\left(\frac{z^{-2+1}}{-2+1}\right) + c_1$$

$$\Rightarrow -x^{-1} = +z^{-1} + c_1$$

$$\Rightarrow x^{-1} + z^{-1} = c_1 \quad \text{——— (iii)}$$

[$-c_1 = c_1$ as c_1 is arbitrate (unknown)]

Max Min transitive:

$$\cancel{\mu_R(x,y) \geq \bigvee_z}$$

$$\mu_R(x,y) \geq \bigvee_z \left[\mu_R(x,z) \wedge \mu_R(z,y)\right]$$

—x—

Class 8

Math

Partial Diff. Eqn:

Partial derivative মানে Diff.

2 ও তার অধিক independent variable মানে,

x y z --> Dependent

$x + y = z$

PDE এর 2 টি way মানে,

Formation of PDE:

① Elimination of Arbitrary Constants

② " " " Functions.

$$\frac{\partial z}{\partial x} = p \quad , \quad \frac{\partial z}{\partial y} = q$$

Indi

Arbitrary constant

(*) Find PDE of the following eqⁿ

$$\underbrace{(x-a)^2 + (y-b)^2}_{\text{arbitrary const}} = c - z^2 \quad \text{——— (i)}$$

যেহেতু constant
Independent variable
এর সাথে যদি এমন
Arbitrary constant

Solⁿ: Diff. (1) partially w.r.to x,

$$2(x-a) + 0 = 0 - 2z\frac{\partial z}{\partial x}$$

$$\Rightarrow \cancel{2}(x-a) = -\cancel{2}z\frac{\partial z}{\partial x}$$

$$\Rightarrow (x-a) = -zp \quad \text{——— (ii)}$$

Diff (1) partially w.r.to y,

$$0 + 2(y-b) = 0 - 2z\frac{\partial z}{\partial y}$$

$$\Rightarrow 2(y-b) = -2z\frac{\partial z}{\partial y} \qquad \left[\frac{\partial z}{\partial y} = q\right]$$

$$\Rightarrow \cancel{(y-b)} = \cancel{-2zq} \quad \text{——— (iii)}$$

$$\Rightarrow (y-b) = -zq$$

by using (ii), (iii) in (1),

$$(-zp)^2 + (-zq)^2 = c - z^2$$

$$\Rightarrow z^2p^2 + z^2q^2 = c - z^2$$

(Ans)

Arbitrary const

(#)

$$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2} \quad \text{——— (i)}$$

Diff (i) partially w.r.t. to x,

$$2\frac{\partial z}{\partial x} = \frac{1}{a^2}2x + 0$$

$$\Rightarrow p = \frac{1}{a^2}\cdot x \quad \text{——— (ii)}$$

Diff (i) partially w.r.t. to y,

$$2\frac{\partial z}{\partial y} = 0 + \frac{2y}{b^2}$$

$$\Rightarrow q = \frac{y}{b^2} \quad \text{——— (iii)}$$

by using (ii), (iii) in (i),

$$2z = px + qy \quad \text{(Ans)}$$

Elimination function

(*) $$z = y^2 + 2\boxed{f\left(\frac{1}{x} + \log y\right)} \longrightarrow \text{arbitrary function} \quad \text{(i)}$$

Diff. (1) partial w.r.to x,

$$\frac{\partial z}{\partial x} = 0 + 2f'\left(\frac{1}{x} + \log y\right)$$

$$\Rightarrow p = 2f'\left(\frac{1}{x} + \log y\right)\left(-x^{-2}\right) \longrightarrow \text{(ii)}$$

Diff (1) partial w.r.to y,

$$\frac{\partial z}{\partial x} = 2y + 2f'\left(\frac{1}{x} + \log y\right)\frac{1}{y}$$

$$\Rightarrow \frac{\partial z}{\partial x} - 2y = 2f'\left(\frac{1}{x} + \log y\right)\frac{1}{y} \longrightarrow \text{(iii)}$$

Dividing (ii) and (iii),

$$\frac{p}{q-2y} = \frac{-2f'\left(\frac{1}{x} + \log y\right)y}{2f'\left(\frac{1}{x} + \log y\right)x^2}$$

$$\Rightarrow px^2 = -qy + 2y^2$$

$x\,dx + y\,dy + z\,dz = 0$

Intnegating →

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c_1$$

Now taking multipliers $1, 1, 1 = \frac{l\,dx + m\,dy + n\,dz}{0}$

$$\frac{dx}{z-y} = \frac{dy}{x-z} = \frac{dz}{y-x} = \frac{dx + dy + dz}{z-y+x-z+y-x}$$

$$= \frac{dx + dy + dz}{0}$$

taking 1st and 4th,

$$\frac{dx}{z-y} = \frac{dx + dy + dz}{0}$$

$$\Rightarrow (z-y)(dx + dy + dz) = 0$$

$$\Rightarrow dx + dy + dz = 0$$

by intnegating

$$x + y + z = c_2$$

Solve: $(z-y)p + (x-z)q = y-x$ —— (i)

By def of LAE:

$$\frac{dx}{z-y} = \frac{dy}{x-z} = \frac{dz}{y-x} \quad \text{—— (ii)}$$

Choosing multipliers $l = x$, $m = y$, $n = z$.

$$\frac{\overset{①}{dx}}{z-y} = \frac{\overset{②}{dy}}{x-z} = \frac{\overset{③}{dz}}{y-x}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{(xz-xy)+(xy-zy)+(zy-xz)}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{0}$$

l, m, n

$$\frac{l\,dx + m\,dy + n\,dz}{lP + mQ + nR} = \frac{l\,dx + m\,dy + n\,dz}{0}$$

Taking 1st and 4th part of eqn (iii),

$$\frac{dx}{z-y} = \frac{x\,dx + y\,dy + z\,dz}{0}$$

$$\Rightarrow (z-y)(x\,dx + y\,dy + z\,dz) = 0$$

$(z-y) = 0$ হলে না পাওয়া z এবং x, y এর মধ্যে Operand আছে,

Taking first and third part of eq^n ①,

$$\frac{du}{y^2 z} = \frac{dz}{xy^2}$$

$$\Rightarrow \cancel{y^2} z\, dz = x\cancel{y^2}\, dx$$

$$\Rightarrow \frac{z^2}{2} = \frac{x^2}{2} + c_2$$

$$\Rightarrow z^2 - x^2 = 2c_2$$

Required complete Sol^n:

$$f(x^3 - y^3, z^2 - x^2) = 0$$

$$f(3c_1, 2c_2) = 0$$

CamScanner

Solve: $\frac{y^2 z}{x} p + xzq = y^2$

$$\Rightarrow \frac{y^2 z}{x} \frac{\partial z}{\partial x} + xz \frac{\partial z}{\partial y} = y^2$$

$$\Rightarrow \underbrace{\boxed{y^2 z}}_{P} p + \underbrace{\boxed{x^2 z}}_{Q} q = \underbrace{\boxed{xy^2}}_{R} \quad \text{——— (i)}$$

By def of LAE,

$$\frac{dx}{y^2 z} = \frac{dy}{x^2 z} = \frac{dz}{xy^2} \quad \text{——— (ii)}$$

Taking first 2 parts of eqn (ii),

$$\frac{dx}{y^2 \cancel{z}} = \frac{dy}{x^2 \cancel{z}}$$

z আছে যাতে এসে কাটা যায় না হয়, কাটা যেত না,

$$\Rightarrow \int x^2 dx = \int y^2 dy$$

$$\Rightarrow \frac{x^3}{3} = \frac{y^3}{3} + c_1$$

$$\Rightarrow \frac{x^3}{3} - \frac{y^3}{3} = c_1$$

$$\Rightarrow x^3 - y^3 = 3c_1 \qquad \boxed{u = a}$$

Class 09

Math

Lagranges Linear PDE:

Standard Form: $Pp + Qq = R$ — (i), where P, Q, R f^n of x, y, z & $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

Working Rule:

(1) Find Lagranges Auxiliary Eq^n, $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ — (ii)

(2) Solve LAE key → (i) Grouping method.
(ii) Method of Multipliers.
(iii) Both i & ii.

(3) Suppose $u = a$ & $v = b$ are two sol^{n}s of eq^n (i) then the complete solution is:

$$f(a, b) = 0, \text{ or } f(u, v) = 0$$

$$= \frac{D+3}{-2^2-9} \sin 2x \qquad [\text{Put } D^2 = -2^2]$$

$$= \frac{D+3}{-13} \sin 2x$$

$$= \frac{-1}{13}\left(D\sin 2x + 3\sin 2x\right)$$

$$= -\frac{1}{13}\left(2\cos 2x + 3\sin 2x\right)$$

General solⁿ,

$$y = CF + PI$$

$$= e^{-\frac{1}{2}x}\left(c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right) - \frac{1}{13}\left(2\cos 2x + 3\sin 2x\right)$$

(Ans)

(*)(*)(*)

sin, cos

Type: 3

$$\frac{1}{f(D)} \sin(ax) \text{ OR } \frac{1}{f(D)} \cos(ax)$$

$$\boxed{\text{Put, } D^2 = -a^2}$$

Solve: $(D^2+D+1)y = \sin 2x$

AE is,

$$m^2+m+1=0$$

$$\Rightarrow m = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$CF = e^{-\frac{1}{2}x}\left(c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right)$$

Now,

$$P.I = \frac{1}{D^2+D+1} \sin 2x$$

$$= \frac{1}{-4+D+1} \sin 2x \quad \left[\text{Put } D^2 = a^2 \text{ or } D^2 = -2^2\right]$$

$$= \frac{1}{D-3} \sin 2x$$

$$= \frac{D+3}{(D+3)(D-3)} \sin 2x$$

$$\Rightarrow \frac{D+3}{D^2-9} \sin 2x$$

CamScanner

Solve: $(D^3+3D^2+3D+1)y=e^{-x}$

Solⁿ: A.E is,

$$(m^3+3m^2+3m+1)=0$$

$$\Rightarrow (m+1)^3=0$$

$$\therefore m=-1,-1,-1$$

$$\frac{d}{dx}(D+1)^3=3(D+1).1$$

$$CF=(c_1+c_2x+c_3x^2)e^{-x}$$

Now, $$PI=\frac{1}{(D+1)^3}\cdot e^{-x}$$

$$=x\cdot\frac{1}{3(D+1)^2}\cdot e^{-x} \quad [\text{Put } D=-1]$$

Again, $$PI=\frac{x^2\cdot 1}{6(D+1)}e^{-x} \qquad \frac{d}{dx}3(D+1)^2=6(D+1).1$$

Again, $$PI=x^3\cdot\frac{1}{6(1+0)}e^{-x}$$

$$=\frac{x^3}{6}\cdot e^{-x}$$

General solution,

$$y=CF+PI$$

$$y=(c_1+c_2x+c_3x^2)e^{-x}+\frac{x^3}{6}e^{-x}$$

(Ans)

Type-2:

Solve: $(D^2-3D+2)y=e^x$

Sol^n: A.E, $m^2-3m+2=0$

$\Rightarrow m^2-m-2m+2=0$

$\Rightarrow m(m-1)-2(m-1)=0$

$\Rightarrow (m-2)(m-1)=0$

$\Rightarrow m=2,1$

$CF=c_1e^{2x}+c_2e^x$

Now, $PI=\frac{1}{D^2-3D+2}\cdot e^x$

$=\frac{1}{(D-2)(D-1)}e^x$ $\quad [f(\alpha)=0,\ f(1)=0]$

$=x\cdot\frac{1}{2D-3}\cdot e^x$

$=x\cdot\frac{1}{2\times1-3}\cdot e^x$

$=-xe^x$

General Solution,

$y=CF+PI$

$y=c_1e^{2x}+c_2e^x-xe^x$

(Ans)

$$CF = C_1 e^{4x} + C_2 e^{x}$$

Now, Particular integral,

$$PI = \frac{1}{D^2 - 3D + 2} \cdot e^{4x}$$

$D^2 - 3D + 2$
$\Rightarrow D^2 - D - 2D + 2$
$\Rightarrow D(D-1) - 2(D-1)$
$\Rightarrow (D-2)(D-1)$ ✓

Put, $D = 4$ (as $a = 4$),

$$PI = \frac{1}{(D-2)(D-1)} e^{4x}$$

$$= \frac{1}{(4-2)(4-1)} e^{4x}$$

$$PI = \frac{1}{6} e^{4x}$$

General Soln,

$$y = CF + PI$$

$$= 4e^{2x} + C_2 e^{x} + \frac{1}{6} e^{4x}$$

(Ans)

Math

Particular Integral:

$$P_0 D^n y + P_1 D^{n-1} y + P_2 D^{n-2} y + \cdots + P_n y = \phi(x)$$

$$\Rightarrow (P_0 D^n + P_1 D^{n-1} y + \cdots + P_n) y = \phi(x)$$

$$f(D) \cdot y = \phi(x)$$

$$\Rightarrow y = \frac{1}{f(D)} \cdot \phi(x)$$

Particular integral (PI).

Type 01:

e^{ax} form $\Rightarrow \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$

where, $f(a) \neq 0$

Put $D = a$,

Solve: $(D^2 - 3D + 2) y = e^{4x}$

Sol^n: AE is, $m^2 - 3m + 2 = 0$

$$\Rightarrow m^2 - m - 2m + 2 = 0$$

$$\Rightarrow m(m-1) - 2(m-1) = 0$$

$$\Rightarrow (m-2)(m-1) = 0$$

$$\therefore m = 2 \mid m = 1$$

$$= \frac{1}{a^2 - 3 \times a + 2} e^{4x}$$

a

$$= \frac{1}{6} e^{4x}$$

General sol^n, $y = CF + PI$

$$\therefore y = c_1 e^{x} + c_2 e^{2x} + \frac{1}{6} e^{4x}$$

(Ans)

$$(*)\ \underbrace{(P_0 D^n + P_1 D^{n-1} + \cdots + P_n)}_{f(D)}\, y = X$$

$$\Rightarrow f(D)\, y = X$$

$$\therefore\ y = \frac{1}{f(D)} \cdot X$$

$y \rightarrow$ Particular Integral

$\boxed{\text{Type 1}}$

$$(*)\ (D^2 - 3D + 2)\, y = e^{4x}$$

A.E is,

$$m^2 - 3D + 2 = 0$$

$$\Rightarrow m^2 - 2m - m + 2 = 0$$

$$\Rightarrow m(m-2) - 1(m-2) = 0$$

$$\Rightarrow (m-1)(m-2) = 0$$

$$\therefore m = +1, +2$$

$$\therefore\ CF = c_1 e^{x} + c_2 e^{2x}$$

$$\frac{1}{f(D)}\, e^{4x}$$

$$PI = \frac{1}{f(D)}\, X$$

$$= \frac{1}{D^2 - 3D + 2} \cdot e^{4x}$$

$f(D)$ এ D এর স্থানে বসাব,

(4) <u>Solve:</u> $(D^4 - a^4)y = 0$

$m^4 - a^4$

$\Rightarrow (m^2 - a^2)(m^2 + a^2) = 0$

$\Rightarrow (m + a)(m - a)(m^2 + a^2) = 0$

$m = a, -a$ | $m^2 = -a^2$

real, distinct | $\Rightarrow m = \sqrt{-a^2}$

$= \sqrt{i^2 a^2}$

$= \pm ai$ → complex, distinct

$\boxed{\alpha = 0}$

$\boxed{\beta = a}$

Two roots are

$CF = c_1 e^{-ax} + c_2 e^{ax} + 1 \cdot (c_3 \cos ax + c_4 \sin ax)$

Here,

$$CF = e^{-\frac{1}{2}x}\left(c_1 \frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right)$$

General solⁿ,

$$y = e^{-\frac{1}{2}x}\left(c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right)$$

(Ans)

Type 1 Way 1

$\alpha \pm i\beta$, $\alpha \pm i\beta \rightarrow$ 4 complex

$$CF = e^{\alpha x}\left[(c_1 + c_2 x)\cos\beta x + (c_3 + c_4 x)\sin\beta x\right]$$

Way 2 $\alpha \pm i\beta$, $\alpha \pm i\beta$, $\alpha \pm i\beta \rightarrow$ 6 complex

$$CF = e^{\alpha x}\left[(c_1 + c_2 x + c_3 x^2)\cos\beta x + (c_4 + c_5 x + c_6 x^2)\sin\beta x\right]$$

Type 3: Solve: $(D^2+D+1)y=0$

$$\boxed{ax^2+bx+c=0}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

A.E is –

$$m^2+m+1=0$$

$$\Rightarrow m=\frac{-1\pm\sqrt{-3}}{2}$$

$$=\frac{-1\pm\sqrt{3i^2}}{2}$$

$$\therefore m=\frac{-1\pm\sqrt{3}\,i}{2} \longrightarrow \text{ইহা মান (*)}$$

$$\therefore m=\frac{-1}{2}+\frac{\sqrt{3}}{2}i,\ \frac{-1}{2}-\frac{\sqrt{3}}{2}i$$

[D^3 থাকলে complex number আসবে না]

m value is distinct and

$$\boxed{\begin{array}{c}\alpha\pm i\beta\\ CF=e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)\end{array}}$$

যদি দুটি ভাঙ্গা আকারে $m=5\pm3i,\ 7\pm4i$ থাকে তাহলে CF = 2টির বেশি অপেক্ষক আসবে,

Class 6

Math

m এর মান কত
1, -1, বা তার কোনো
গুণিতক যদি মিলে হয়
=0 করে তাহলে
তাকে common নাও,

Solve: $(D^3 + 6D^2 + 11D + 6)y = 0$

$$m^3 + 6m^2 + 11m + 6 = 0$$

$$\Rightarrow m^3 + 3m^2 + 3m^2 + 9m + 2m + 6 = 0$$

$$\Rightarrow m^2(m+3) + 3m(m+3) + 2(m+3) = 0$$

$$\Rightarrow (m^2 + 3m + 2)(m+3) = 0$$

$$m^2 + 3m + 2 = 0$$

$$\Rightarrow m^2 + 2m + m + 2 = 0$$

$$\Rightarrow m(m+2) + 1(m+2) = 0$$

$$(m+1)(m+2) = 0$$

$$\therefore m = -1, -2$$

$$\therefore m = -3$$

Here root are real and distinct,

General solⁿ, CF, $= c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{-3x}$

General $x = 0$, ~~2m~~,

$$y = CF$$

apply type 2

(4) $(D^3 - 2D^2 - 4D + 8)y = 0$

A.E is,

$$m^3 - 2m^2 - 4m + 8 = 0$$

$$\Rightarrow m^3 + 2m^2 - 4m^2 - 8m + 4m + 8 = 0$$

$\therefore (m = -2, 2, 2)$

$$\Rightarrow m^2(m+2) - 4m(m+2) + 4(m+2) = 0$$

$$\Rightarrow (m+2)(m^2 - 4m + 4) = 0$$

$$\therefore m = -2, 2, 2.$$

Here, roots are real and similar.

$$CF = c_1 e^{-2x} + (c_2 + c_3 x) e^{2x}$$

For 3 টি ~~সমান~~ same হলে

$$\boxed{CF = (c_1 + c_2 x + c_3 x^2) e^{2x}}$$ → understand

General solⁿ;

$$\therefore y = c_1 e^{-2x} + (c_2 + c_3 x) e^{2x}$$ (Ans)

2 roots are similar and one is distinct.

Q: $(D^2 - 7D + 12)y = 0$

Sol^n: Auxilary equation, $\boxed{u = 0}$

$\hookrightarrow m^2 - 7m + 12 = 0$

$\Rightarrow$

step 2

$\Rightarrow m^2 - 3m - 4m + 12 = 0$

$\Rightarrow m(m-3) - 4(m-3) = 0$

$\Rightarrow (m-4)(m-3) = 0$

$\therefore m = 4, 3$ → Distinct and real value.

Type 1

Here roots are $\boxed{\text{real and distinct}}$.

$\therefore$ General Sol^n, $y = \underbrace{c_1 e^{3x} + c_2 e^{4x}}_{\text{complementary function}}$

Type 2

Root are real and similar.

$$\frac{d}{dx} \rightarrow D \quad \Big| \quad \int dx = \frac{1}{D}$$

$$\Rightarrow (P_0 D^n + P_1 D^{n-1} + \cdots + P_n) y = X$$

X এর মানের উপর Depend করে।

if $X = 0$, then General sol^n, $y = CF$ → complementary function

if $X \neq 0$, then " " , $y = CF + PI$ → Particular Integral

How to find CF?

Step-01: Find auxilary equation.

Step-02: " the root of A.E.

Step-03: Nature of roots.

$$\Rightarrow \int \frac{1}{3x^2 y}\,dx + \int \frac{2}{3x}\,dx + \int -\frac{1}{3y}\,dy = c$$

$$\Rightarrow \frac{1}{3y}\int \frac{x^{-2+1}}{-2+1} + \frac{2}{3}\ln x - \frac{1}{3}\ln y = c$$

$$\Rightarrow -\frac{1}{3yx} + \frac{2}{3}\ln x - \frac{1}{3}\ln y = c$$

(Ans)

Exam

[Imp – Exact diff eqn]

Linear Diff eqn with constant co-efficients:

$$P_0 \frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + P_n y = X$$

$P_0, P_1, P_2, \ldots, P_n \longrightarrow$ fn of x or constants.

$X \longrightarrow$ fn of x

$$\left(P_0 \frac{d^n}{dx} + P_1 \frac{d^{n-1}}{dx^{n-1}} + P_2 \frac{d^{n-2}}{dx^{n-2}} + \cdots + P_n\right) y = X$$

$$\frac{y(xy+2x^2y^2)}{3x^3y^3}\,dx + \frac{x(xy-x^2y^2)}{3x^3y^3}\,dy = 0$$

$$\Rightarrow \underbrace{\left(\frac{1}{3yx^2}+\frac{2}{3x}\right)}_{M}dx + \underbrace{\left(\frac{1}{3xy^2}-\frac{1}{3y}\right)}_{N}dy = 0 \quad \text{——— (ii)}$$

Now,

$$\frac{\partial M}{\partial y} = \frac{1}{3}\cdot -1y^{-2}\cdot x^{-2} + 0$$

$$= -\frac{1}{3}y^{-2}x^{-2} = \frac{-1}{3x^2y^2}$$

$$\frac{\partial N}{\partial x} = -\frac{1}{3}\cdot x^{-2}\cdot y^{-2} - 0$$

$$= \frac{-1}{3x^2y^2}$$

It is exact.

Now,

$$\int M\,dx + \int N\,dy = c$$

$$\Rightarrow \int\left(\frac{1}{3xy^2}+\frac{2}{3x}\right)dx + \int -\frac{1}{3y}\,dy = c$$

Class 5

Math

Type 4: $\underbrace{y(xy + 2x^2y^2)}_{M}\,dx + \underbrace{x(xy - x^2y^2)}_{N}\,dy = 0$ (i)

Soln:

$$\frac{\partial M}{\partial y} = 2xy + 6x^2y^2$$

[It is not exact]

$$\frac{\partial N}{\partial x} = 2xy - y^2 3x^2$$

Other

$$\boxed{\begin{gathered} y f(x,y)\,dx + x g(x,y)\,dy = 0 \\ f(x,y) \neq g(x,y) \end{gathered}}$$

$$IF = \frac{1}{Mx - Ny} = \frac{1}{\cancel{x^2y^2} + 2x^3y^3 - \cancel{x^2y^2} + x^3}$$

$$= \frac{1}{3x^3y^3}$$

Multiplying eqn (1) by IF,

General solⁿ;

$$\int M\,dx + \int N\,dy = c$$

$$\Rightarrow \int -x^2 y^{-3}\,dx + \int y^{-1}\,dy = c$$

$$\Rightarrow -y^{-3}\int x^2\,dx + \int \frac{1}{y}\,dy = c$$

$$\Rightarrow \frac{-y^{-3}x^3}{3} + \ln y = c \quad \text{(Ans)}$$

Type-3: Solve, $x^2y\,dx - (x^3+y^3)\,dy = 0$ ——— (i)

$$\Rightarrow \frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$$

As eqn (1) is in Homogeneous form,

$$IF = \frac{1}{Mx + Ny} \qquad M\,dx + N\,dy = 0$$

$$= \frac{1}{x^3y - x^2y - y^4}$$

$$= \frac{1}{-y^4}$$

$$= -y^{-4}$$

Multiply

$$\underbrace{-x^2y^{-3}}_{M}\,dx - \underbrace{(-x^3y^{-4} - y^{-1})}_{N}\,dy = 0 \quad \text{——— (ii)}$$

$$\frac{\partial M}{\partial y} = -x^2 \cdot (-3) \cdot y^{-4} = 3x^2y^{-4}$$

$$\frac{\partial N}{\partial x} = y^{-4}\,3x^2 = 3x^2y^{-4}$$

CamScanner

Now, $\frac{\partial M}{\partial y} = 1 - 4y^{-3}$ | $\frac{\partial N}{\partial x} = 1 + 0 - 4y^{-3}$

$= 1 - 4y^{-3}$

(ii) eqn is exact,

General soln,

$$\int M\,dx + \int N\,dy = c$$

$$\Rightarrow \int (y + 2y^{-2})\,dx + \int 2y\,dy = c$$

$$\Rightarrow yx + 2y^{-2}x + 2\cdot\frac{y^2}{2} = c$$

$$\Rightarrow yx + 2y^{-2}x + y^2 = c$$

$$\Rightarrow (y + 2y^{-2})x + y^2 + c$$ (Ans)

$2(2y^2+1)$

Type 2: $\underbrace{(y^4+2y)}_{M}dx + \underbrace{(xy^3+2y^4-4x)}_{N}dy = 0$ —— (i)

Here,

$$\frac{\partial M}{\partial y} = 4y^3+2$$

$$\frac{\partial N}{\partial x} = y^3 - 4$$

$4y^3+2-y^3+4$

$(4y^3+6)$

$3(y^3+2)$

check

eqn (i) is not exact,

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{3y^3+6}{y} = \frac{3\cancel{(y^3+2)}}{y\cancel{(y^3+2)}}$$

$$= \frac{3}{y}$$

$$IF = e^{\int -\frac{3}{y}dy}$$

$$= e^{-3\ln y}$$

$$= e^{\ln y^{-3}}$$

$$= y^{-3}$$

$a\ln b = \ln b^a$

$\boxed{e^{\ln x} = x}$

Multiplying

$$\underbrace{(y+2y^{-2})}_{M}dx + \underbrace{(x+2y-4xy^{-3})}_{N}dy = 0 \quad \text{—— (ii)}$$

Intregrating factor, $IF = e^{\int \frac{1}{x} dx}$

$= e^{\ln x}$

$= x$

যদি ভগ্নাংশ হয়

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \rightarrow IF\, e^{\int \square dx} / e^{\int \square dy}$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} \rightarrow IF\, e^{-\int \square dx} / e^{-\int \square dy}$$

Multiplying (1) by IF,

$$\underbrace{(x^3 + y^2 x + x^2)}_{M} dx + \underbrace{x^2 y}_{N} dy = 0 \quad \text{——— (ii)}$$

$$\Rightarrow \frac{\partial M}{\partial y} = 2xy, \quad \frac{\partial N}{\partial x} = 2xy$$

Therefore eqn (ii) is exact,

General Soln,

$$\int M dx + \int N dy = c$$

$$\Rightarrow \int (x^3 + y^2 x + x^2) dx + \int 0 dy = c$$

$$\Rightarrow \frac{x^4}{4} + y^2 \frac{x^2}{2} + \frac{x^3}{3} + 0 = c$$

$$\Rightarrow \frac{x^4}{4} + y^2 \frac{x^2}{2} + \frac{x^3}{3} = c$$

(Ans)

Class 9

Math

Exact

Reducible to Exact Diff. eqn:

Type-1: $\underbrace{(x^2+y^2+x)}_{M}dx + \underbrace{xy}_{N}dy = 0$ ——— (i)

∂ = Partial differenciat

Soln:

$$\frac{\partial M}{\partial y} = 0 + 2y + 0 = 2y$$

$$\frac{\partial N}{\partial x} = y$$

As, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ $\therefore$ eqn (1) is not exact.

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{\boxed{}} = \boxed{}$$

either a f^n of x or a f^n of y

M/N হলে

তখন M অথবা N যিনি ভাগ হবে, কিন্তু অবশ্যই ভাগ করা

Must be $f(x)$ / $f(y)$ type অবশ্যই হ

Here,

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{\boxed{N}} = \frac{y}{xy} = \frac{1}{x}$$

এখানে $f(x,y)$ কোনো আসবে না, Only $f(x)$ or $f(y)$ হবে,

Exact diff

(#) $x(x^2+y^2-a^2)\,dx + y(x^2-y^2-b^2)\,dy = 0$

(#) $\cancel{(y-x)\frac{dy}{dx} = (x+y)\frac{dy}{dx}}$

Homogeneous

(#) $y - x\frac{dy}{dx} = x + y\frac{dy}{dx}$

(#) $(1+e^{x/y})dx + e^{x/y}(1-x/y)\,dy = 0$

Sol^n

$$\frac{\partial M}{\partial y} = 0 + e^{x/y}\cdot x\cdot(-1)\,y^{-2}$$

$$= -e^{x/y}\cdot x\cdot y^{-2}$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(e^{x/y} - e^{x/y}\cdot x/y\right)$$

$$= e^{x/y}\cdot\frac{1}{y} - e^{x/y}\cdot\frac{1}{y} - \frac{x}{y}e^{x/y}\cdot\frac{1}{y}$$

$$= -e^{x/y}\,y^{-2}$$

দুইটি same হলে exact equation হবে,

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, eq^n (1) is exact.

General eq^n sol^n ⇒

$$\int M\,dx + \int N\,dy = c$$

$$\Rightarrow \int (1+e^{x/y})\,dx + \int 0\,dy = c$$

x free থাকা element লাগবে

Exact Diff. Eqn:

$M(x,y)\,dx + N(x,y)\,dy = 0$ is exact —— (i)

if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

if (i) is exact then—

General solⁿ is—

$$\int M\,dx + \int N\,dy = c$$

terms free of 'x'
(যেখানে x নাই সেগুলি নিব)

'y' is constant here

Explain:

$\frac{\partial M}{\partial y}$ এর ক্ষেত্রে x হবে constant. $\frac{\partial N}{\partial x}$ এর ক্ষেত্রে y হবে constant.

$$\Rightarrow x\frac{dv}{dx} = \frac{-v^2 - v + v^2}{1-v}$$

$$\Rightarrow x \cdot \frac{dv}{dx} = \frac{-v}{1-v}$$

$$\Rightarrow \frac{1}{x}\,dx = \frac{1-v}{-v}\cdot dv$$

Integrating

$$\Rightarrow \int \frac{1}{x}\,dx = \int \frac{1-v}{-v}\,dv$$

$$\Rightarrow \int \frac{1}{-v}(1-v)\,dv = \int \frac{1}{x}\,dx$$

$$\Rightarrow -\int \frac{1}{v}\,dv + \int \frac{v}{v}\,dv = \ln x + \ln c$$

c কে lnc লিখলে সহজ

$$\Rightarrow -\ln v + v = \ln x + \ln c$$

$$\Rightarrow v = \ln x + \ln v + \ln c$$

$$\Rightarrow v = \ln(xvc)$$

$$\Rightarrow e^{v} = e^{\ln(xvc)}$$

$$\Rightarrow e^{v} = xvc$$

$$\Rightarrow e^{y/x} = yc$$

(Ans)

$$\Rightarrow \frac{dv}{du} = (f(v) - v) \cdot \frac{1}{u}$$

$$\Rightarrow \int \frac{1}{f(v) - v}\, dv = \int \frac{1}{u}\, du$$

(4) $x(x-y)\, dy + y^2\, dx = 0$

$$\Rightarrow x(x-y)\, dy = -y^2\, dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{x(x-y)} \quad \text{——— (i)}$$

Let,

$$y/x = v$$

$$\Rightarrow y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Putting (i) in,

$$v + x\frac{dv}{dx} = \frac{-v^2 x^2}{x^2(1-v)}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{-v^2}{(1-v)}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-v^2}{1-v} - v$$

$$\boxed{\begin{aligned} x &= \lambda x \\ y &= \lambda y \end{aligned}}$$

$$f(x,y) = x^3 - 2x^2y + y^3$$

$$\Rightarrow f(\lambda x, \lambda y) = \lambda^3 x^3 - 2\lambda^2 x^2 y \lambda + \lambda^3 y^3$$

$$= \lambda^3 (x^3 - 2x^2y + y^3)$$

$$= \lambda^3 f(x,y)$$

$$\boxed{n = 3}$$

First order differenciate eqn of Homogeneous form:

$$\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)} \longrightarrow \text{both are homogeneous } f^n \text{ of same order.}$$

$$\frac{dy}{dx} = f(y/x) \quad \text{——— (i)}$$

Let, $y/x = v$

$$\Rightarrow y = vx \longrightarrow (uv)$$

$$\Rightarrow \frac{dy}{dx} = v \cdot 1 + x \cdot \frac{dv}{dx}$$

Putting values in (1),

$$v + x\frac{dv}{dx} = f(v)$$

$$\Rightarrow x\frac{dv}{dx} = f(v) - v$$

Ques

* PNN structure.
* Smoothing Parameter কীভাবে effect করে,
* Advantage and disadv of PNN.

Class 3

Math

Homogeneous function:

$f(x, y)$ is homogeneous

if, $f(\lambda x, \lambda y) = \lambda^n f(x, y)$

$n \rightarrow$ order of homogeneous f^n

$f(x, y) = x^2 + 2xy + y^2$

$f(\lambda x, \lambda y) = \lambda^2 x^2 + 2\lambda^2 xy + \lambda^2 y^2$

$= \lambda^2 (x^2 + 2xy + y^2)$

$= \lambda^n f(x, y)$

$\boxed{n = 2}$

CamScanner

$$\Rightarrow -2x = \frac{dz}{dx}$$

$$\Rightarrow -2x\,dx = dz$$

Putting values in (iv),

$$ve^{z} = \int -ze^{z}\,dz + c$$

$$\Rightarrow ve^{z} = -\left(ze^{z} - 1 \cdot e^{z}\right) + c$$

$$\Rightarrow ve^{z} = e^{z}(1 - z) + c$$

$$\Rightarrow y^{-2}e^{-x^2} = e^{-x^2}(1 + x^2) + c$$

(Ans)

(*) $\frac{dy}{dx} + \frac{2}{x}y = \frac{y^3}{x^3}$ ⟶ Bernole

(*) $\frac{dy}{dx} + 4y = x$ ⟶ simple

(*) $x^4\frac{dy}{dx} + x^3y = -\sec(xy)$ ⟶ S

imp

$$\Rightarrow y^{-3}\frac{dy}{dx} = \frac{-1}{2}\frac{dv}{dx}$$

Putting values in (ii),

$$-\frac{1}{2}\frac{dv}{dx} + xv = x^3$$

$$\Rightarrow \frac{dv}{dx} - 2xv = -2x^3 \quad \text{[-2 দিয়া গুণ]} \quad \text{——— (iii)}$$

Determine IF, —2

$$IF = e^{\int p\,dx}$$

$$= e^{\int -2x\,dx}$$

$$= e^{-2\cdot\frac{x^2}{2}}$$

$$= e^{-x^2}$$

[y এর coefficient use করা]

General solution →

$$v\,IF = \int Q\,IF\,dx$$

$$\Rightarrow v.e^{-x^2} = \int -2x^3 . e^{-x^2}\,dx + c \quad \text{——— (iv)}$$

Let, $-x^2 = z$

$$\Rightarrow y\sqrt{1-x^2} = \sin^{-1}x + c$$ (Ans)

Bernoullie's Form:

$$\frac{dy}{dx} + Py = Qy^n$$

Step-1: Divide the eq^n by y^n

Step-2: Assume v = coefficient of P.

Step-3: Find IF

Step-4: General Sol^n

(*) Solve: $\frac{dy}{dx} = x^3y^3 - xy$

$$\Rightarrow \frac{dy}{dx} + xy = x^3y^3 \quad \text{——— (i)}$$

$$\Rightarrow y^{-3}\frac{dy}{dx} + xy^{-2} = x^3 \quad \text{——— (ii)}$$ [y^3 fu[illegible]]

Let, $y^{-2} = v$

$$\Rightarrow -2y^{-3}\frac{dy}{dx} = \frac{dv}{dx}$$ differentiate with respect to x

Differentiate

$$\int \frac{f'(x)}{f(x)} = \ln f(x)$$

$$= e^{\int \frac{-x}{1-x^2}\,dx}$$

$$1-x^2 = -2x$$

$$= e^{\frac{1}{2}\int \frac{-2x}{1-x^2}\,dx}$$

$$= e^{\frac{1}{2}\ln(1-x^2)}$$

$$= e^{\ln(1-x^2)^{\frac{1}{2}}}$$

$$\boxed{\ln e / e^{\ln} / \ln^{e} = 1}$$

$$= 1\cdot(1-x^2)^{\frac{1}{2}}$$

$$= \sqrt{1-x^2}$$

General Solution is -

$$y\,IF = \int q\,IF\,dx + c$$

$$\Rightarrow y\left(\sqrt{1-x^2}\right) = \int \frac{1}{1-x^2}\cdot\left(\sqrt{1-x^2}\right)dx + c$$

$$\Rightarrow y\sqrt{1-x^2} = \int \frac{1}{\sqrt{(1-x^2)^2}}\left(\sqrt{1-x^2}\right)dx + c$$

$$\Rightarrow y\sqrt{1-x^2} = \int \frac{1}{\sqrt{(1-x^2)}}\,dx + c$$

Integration

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x+c$$

(Ans)

LDE:

$$\frac{dy}{dx} + Py = Q$$

IF = Integrating Factor.

or,

$$\frac{dx}{dy} + Px = Q$$

Step-1: Determine $IF = e^{\int P\,dx}$ / $e^{\int P\,dy}$

Step-2: General Sol^n:

$$y \cdot IF = \int Q\, IF\, dx + c$$

or,

$$x \cdot IF = \int Q\, IF\, dy + c$$

Solve: $(1-x^2)\dfrac{dy}{dx} - xy = 1$

$$\Rightarrow \frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{1}{1-x^2} \quad \text{——— (i)}$$

Here,

$$IF = e^{\int \frac{-x}{1-x^2}dx}$$

Math

Class 2

(II) Reducible to variable Separation Method:

$$\frac{dy}{dx} = f(ax+by+c)$$

$f(x) = x^n$ → domain

(II) Solve:

$$\frac{dy}{dx} = (4x+y+1)^2 \quad \text{——— (1)}$$

Ordinary diff eqn

Let, $4x+y+1 = v$ ——— (ii)

$$\Rightarrow 4 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 4$$

Derivid

Putting values in (1),

$$\frac{dv}{dx} - 4 = v^2$$

$$\Rightarrow \frac{dv}{dx} = v^2 + 4$$

$$\Rightarrow \frac{1}{v^2+4}\, dv = dx$$

$$\int \frac{1}{x^2+a^2}\, dx = \frac{1}{a}\tan^{-1}(x/a) + c$$

Intregration of,

$$\int \frac{1}{v^2+4}\, dv = \int 1\, dx$$

$$\Rightarrow \frac{1}{2}\tan^{-1}(v/2) = x + c$$

Solve this:

$$(x^2 - yx^2)\,dy + (y^2 + xy^2)\,dx = 0$$

$$\Rightarrow x^2(1-y)\,dy + y^2(1+x)\,dx = 0$$

$$\Rightarrow y^2(1+x)\,dx = -x^2(1-y)\,dy$$

$$\Rightarrow -\frac{(1+x)}{x^2}\,dx = \frac{(1-y)}{y^2}\,dy \quad \text{——— (i)}$$

Intregrate (i) eqn;

$$\int -\frac{1}{x^2}\,dx - \int \frac{1}{x}\,dx = \int \frac{1}{y^2}\,dy - \int \frac{1}{y}\,dy$$

$$\Rightarrow -\frac{x^{-2+1}}{-2+1} - c_1 - \log x - c_2 = \frac{y^{-2+1}}{-2+1} + c_3 - \log y - c_4$$

$$\Rightarrow x^{-1} - \log x = -y^{-1} - \log y + \log c$$

$$\Rightarrow x^{-1} + y^{-1} = \log x - \log y + \log c$$

$$\Rightarrow \log e^{(x^{-1}+y^{-1})} = \log\left(\frac{xc}{y}\right)$$

$$\Rightarrow e^{x^{-1}+y^{-1}} = \frac{xc}{y}$$

First order ODE: → Ordinary Diff

① Seperation of variables:

→ 1 independent variable and 1st order

$$\frac{dy}{dx} = f(x, y) = f_1(x) \cdot f_2(y)$$

$$\frac{x^2}{5y}$$ → applicable

$\boxed{\text{Apply}}$

$$\frac{dy}{dx} = f_1(x) \cdot f_2(y)$$

$$\Rightarrow \frac{1}{f_2(y)}\, dy = f_1(x) \cdot dx \quad \text{—— (1)}$$

Integrating (1),

$$\int \frac{1}{f_2(y)}\, dy = \int f_1(x)\, dx$$

$$\frac{d^2y}{dx^2} = 2x(-A\cos x^2)\,2x - B(\sin x^2).2x) + (-A\sin x^2 + B\cos x^2).2$$

$$\left(\frac{d}{dx}uv = u\frac{d}{dx}(v) - \right.$$

$$\frac{d^2y}{dx^2} = -4x^2(A\cos x^2 + B\sin x^2) + (-A\sin x^2 + B\cos x^2).2$$

$$\frac{d^2y}{dx^2} = -4x^2y + \frac{dy}{dx}\cdot\frac{1}{x}$$

(Ans)

Hw

$$y = c_1\cos 2x + c_2\sin 2x$$

(Ans) / $\frac{d^2y}{dx^2} = -4y$

$$\left(\frac{d^2y}{dx^2}\right)^4 - 7\frac{dy}{dx} + \left(\frac{d^3y}{dx^3}\right)^2 = 0$$

Ordinary D.E.

Partial
↳ more than

$\frac{dy}{dx}$ → dependent
→ Independent

Order → Highest no of derivative = 3

Degree → Power of Highest order derivative = 2

$$\sqrt{\left(\frac{d^2y}{dx^2}\right)^4 - 7\frac{dy}{dx}} = \left(\frac{d^3y}{dx^3}\right)^2$$

Order → 3

degree → 4 $\left(\frac{d^3y}{dx^3}\right)^4$

Find diff eqn of –

$$y = A\cos x^2 + B\sin x^2 \quad \text{——— (i)}$$

differentiate (1) wrt. to x,

$$\frac{dy}{dx} = A(-\sin x^2)\cdot 2x + B\cos(x^2)\cdot 2x$$

$$\frac{dy}{dx} = 2x(-A\sin x^2 + B\cos x^2) \quad \text{——— (ii)}$$

Class 1

Math

Ordinary diff eqn

DE

ODE

PDE → Partial diff. eqn

'1' Independent variable

'1' or more dependent variable

more than '1' independent variables

$$\frac{d}{dx}(\sin x) \rightarrow \text{(derivative of } \sin x\text{)}$$

$\frac{d}{dx}$ → differencial operator

$$\frac{dx}{dy} + x = 0$$

$$f\left(x, y, \frac{dx}{dy}, \frac{d^2y}{dx}, \frac{d^3x}{dy^3}, \frac{d^4y}{dx^4} \cdots\right)$$

Ordinary → 1 ही independent variable.