# Hopfield net

## Training Algo

For storing a set of binary patterns $s(P)$, $P = 1$ to $P$
where $s(P) = (s_1(P), s_2(P), \cdots, s_n(P))$, weight matrix is given by:

$$w_{ij} = \sum_{p=1}^{P} [2s_i(P)-1][2s_j(P)-1], \text{ for } i \neq j$$

for Bipolar input patterns:

$$w_{ij} = \sum_{p=1}^{P} s_i(P)\, s_j(P), \text{ for } i \neq j$$

& weight have no self connection, $w_{ij} = 0$.

## Testing Algo.

step 0: Initialize weights to store patterns.

step 1: When activations of net are not converged, perform step 2-8.

$[1 \;\; 1 \;\; 1 \; 0$

Step 2: For each input vector $x$, perform steps 3-7.

$x \rightarrow [0 \;\; 0 \;\; 1 \;\; 0]$

Step 3: Make initial activation of equal to external input vector $x$:

$$y_i = x_i \;(i = 1 \text{ to } n)$$

$Y = [0 \;\; 0 \;\; 1 \;\; 0]$

Step 4: Perform steps 5-7 for each unit $y_i$ (Unit updated is random order)

Step 5: Calculate net input of network

$$y_{in} = x_i + \sum y_j w_{ji}$$

Q. Construct a auto associative discrete hopfield network with input vector [1 1 1 -1]. Test discrete hopfield network with missing entries in first & second components of stored vector.

Sol^n

Input vector is $x = [1 \ 1 \ 1 \ -1]$

weight matrix.

$$w = \sum s^T(P)\, t(P) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} [1 \ 1 \ 1 \ -1]$$

$$= \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

weight matrix with no self connection

$$w = \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix}$$

Binary representation for given input vector is [1 1 1 0]. We carry out asynchronous updation of weights here let it be..

$y_1, y_4, y_3, y_2$

for input vector will 2 missing entries is first and second components of stored vector [0 0 1 0]

<u>step 0</u> : weight are initialized to store pattern.

$$W = \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix}$$

step 1: Input vector is $x = [0 \quad 0 \quad 1 \quad 0]$

step 2: Input vector $y = [0 \quad 0 \quad 1 \quad 0]$

<u>Step 3</u>: Choosing unit $y_1$ for updating its activation

$$y_{in} = x_1 + \sum_{j=1}^{4} y_i w_{ji}$$

$$= 0 + [0 \quad 0 \quad 1 \quad 0] \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix} = 0 + 1 = 1$$

applying activation, $y_{in_1} > 0 \Rightarrow y_1 = 1$. Broadcasting $y$ to all other units, we get
$y = [1 \quad 0 \quad 1 \quad 0]$

Choosing unit $y_4$ for updating its activation

$$y_{in4} = x_4 + \sum_{j=1}^{4} y_j w_{j4}$$

$$= 0 + [1\ 0\ 1\ 0]\begin{bmatrix}-1\\-1\\-1\\0\end{bmatrix}$$

$$= 0 - 2 = -2 < 0$$

$y_{in4} < 0 \Rightarrow y_4 = 0$, therefore, $y = [1\ 0\ 1\ 0]$

↳ No convergence.

Step 5 Choosing unit $y_3$ for updating its activation

$$y_{in3} = x_3 + \sum_{j=1}^{4} y_i w_{i3}$$

$$= 1 + [1\ 0\ 1\ 0]\begin{bmatrix}1\\1\\0\\-1\end{bmatrix}$$

$$= 1 + 1 = 2$$

$y_{in3} > 0 \Rightarrow y_3 = 1$. therefore. $y = [1\ 0\ 1\ 0]$

↳ No convergence.

Step 6:

Choosing unit $Y_2$ for updating its activation

$$y_{in2} = x_2 + \sum_{j=1}^{4} y_j w_{j2}$$

$$= 0 + [1\ 0\ 1\ 0] \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

$$= 0 + 2$$

$y_{in2} > 0 \Rightarrow y_2 = 1$, therefore, $y = [1\ 1\ 1\ 0]$