

# Data Mining:

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## Concepts and Techniques

(3<sup>rd</sup> ed.)


### — Chapter 4 —

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# Chapter 4: Data Warehousing and On-line Analytical Processing

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- Data Warehouse: Basic Concepts 
- Data Warehouse Modeling: Data Cube and OLAP
- Data Warehouse Design and Usage
- Data Warehouse Implementation
- Data Generalization by Attribute-Oriented Induction
- Summary

# What is a Data Warehouse?

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- Defined in many different ways, but not rigorously.
  - A decision support database that is maintained **separately** from the organization's operational database
  - Support **information processing** by providing a solid platform of consolidated, historical data for analysis.
- "A data warehouse is a subject-oriented, integrated, time-variant, and nonvolatile collection of data in support of management's decision-making process."—W. H. Inmon
- Data warehousing:
  - The process of constructing and using data warehouses

# Data Warehouse—Subject-Oriented

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- Organized around major subjects, such as **customer, product, sales**
- Focusing on the modeling and analysis of data for decision makers, not on daily operations or transaction processing
- Provide **a simple and concise** view around particular subject issues by **excluding data that are not useful in the decision support process**

# Data Warehouse—Integrated

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- Constructed by integrating multiple, heterogeneous data sources
  - relational databases, flat files, on-line transaction records
- Data cleaning and data integration techniques are applied.
  - Ensure consistency in naming conventions, encoding structures, attribute measures, etc. among different data sources
    - E.g., Hotel price: currency, tax, breakfast covered, etc.
  - When data is moved to the warehouse, it is converted.

# Data Warehouse—Time Variant

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- The time horizon for the data warehouse is significantly longer than that of operational systems
  - Operational database: current value data
  - Data warehouse data: provide information from a historical perspective (e.g., past 5-10 years)
- Every key structure in the data warehouse
  - Contains an element of time, explicitly or implicitly
  - But the key of operational data may or may not contain “time element”

# Data Warehouse—Nonvolatile

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- A **physically separate store** of data transformed from the operational environment
- Operational **update of data does not occur** in the data warehouse environment
  - Does not require transaction processing, recovery, and concurrency control mechanisms
  - Requires only two operations in data accessing:
    - *initial loading of data* and *access of data*

# OLTP vs. OLAP

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	<b>OLTP</b>	<b>OLAP</b>
<b>users</b>	clerk, IT professional	knowledge worker
<b>function</b>	day to day operations	decision support
<b>DB design</b>	application-oriented	subject-oriented
<b>data</b>	current, up-to-date detailed, flat relational isolated	historical, summarized, multidimensional integrated, consolidated
<b>usage</b>	repetitive	ad-hoc
<b>access</b>	read/write index/hash on prim. key	lots of scans
<b>unit of work</b>	short, simple transaction	complex query
<b># records accessed</b>	tens	millions
<b>#users</b>	thousands	hundreds
<b>DB size</b>	100MB-GB	100GB-TB
<b>metric</b>	transaction throughput	query throughput, response

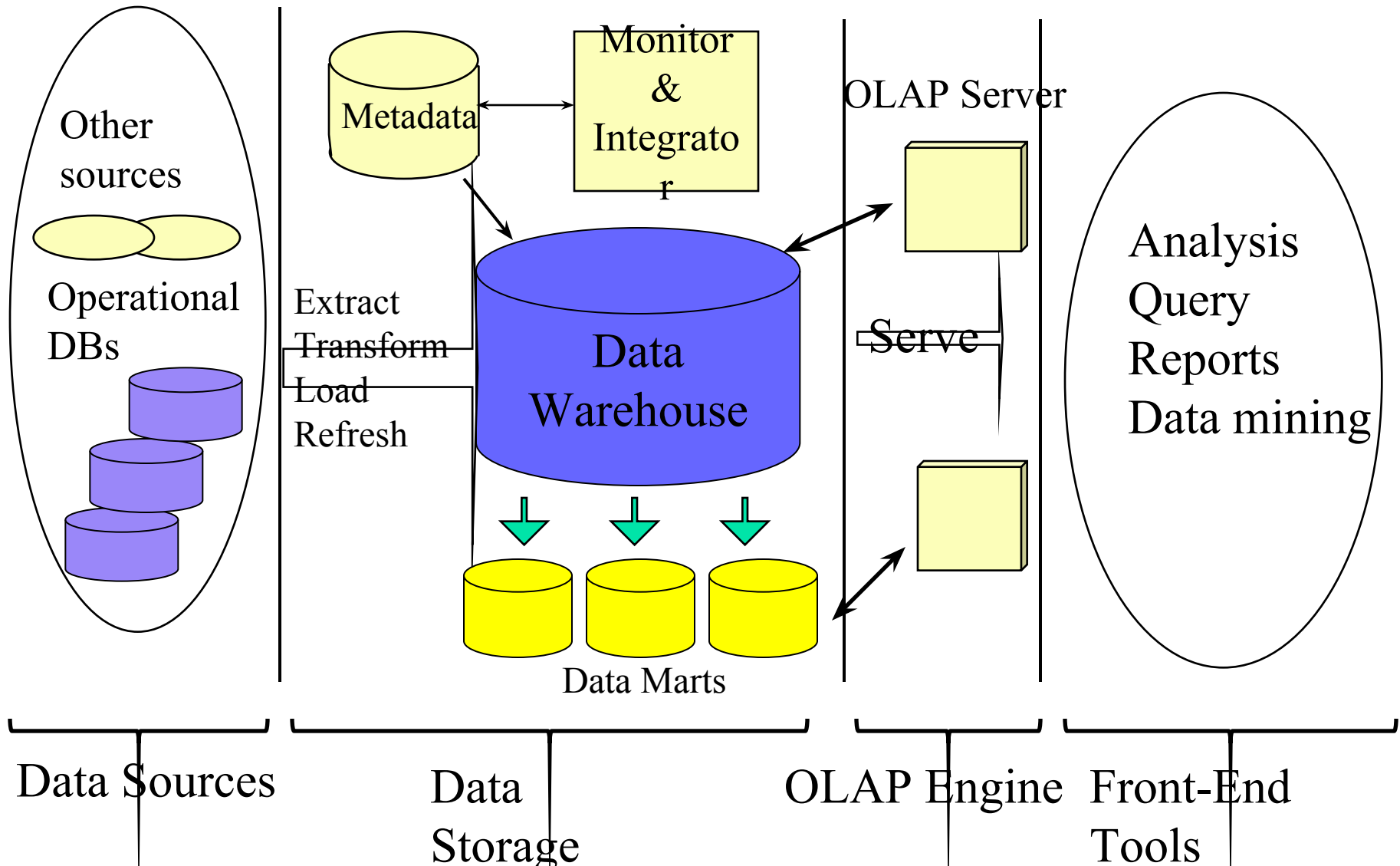


# Why a Separate Data Warehouse?

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- High performance for both systems
  - DBMS— tuned for OLTP: access methods, indexing, concurrency control, recovery
  - Warehouse—tuned for OLAP: complex OLAP queries, multidimensional view, consolidation
- Different functions and different data:
  - missing data: Decision support requires historical data which operational DBs do not typically maintain
  - data consolidation: DS requires consolidation (aggregation, summarization) of data from heterogeneous sources
  - data quality: different sources typically use inconsistent data representations, codes and formats which have to be reconciled
- Note: There are more and more systems which perform OLAP analysis directly on relational databases

# Data Warehouse: A Multi-Tiered Architecture



# Three Data Warehouse Models

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- **Enterprise warehouse**
  - collects all of the information about subjects spanning the entire organization
- **Data Mart**
  - a subset of corporate-wide data that is of value to a specific groups of users. Its scope is confined to specific, selected groups, such as marketing data mart
    - Independent vs. dependent (directly from warehouse) data mart
- **Virtual warehouse**
  - A set of views over operational databases
  - Only some of the possible summary views may be materialized

# Extraction, Transformation, and Loading (ETL)

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- **Data extraction**

- get data from multiple, heterogeneous, and external sources

- **Data cleaning**

- detect errors in the data and rectify them when possible

- **Data transformation**

- convert data from legacy or host format to warehouse format

- **Load**

- sort, summarize, consolidate, compute views, check integrity, and build indices and partitions

- **Refresh**

- propagate the updates from the data sources to the warehouse


# Metadata Repository

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- **Meta data** is the data defining warehouse objects. It stores:
- Description of the **structure** of the data warehouse
  - schema, view, dimensions, hierarchies, derived data defn, data mart locations and contents
- **Operational** meta-data
  - data lineage (history of migrated data and transformation path), currency of data (active, archived, or purged), monitoring information (warehouse usage statistics, error reports, audit trails)
- The **algorithms** used for summarization
- The **mapping** from operational environment to the data warehouse
- Data related to **system performance**
  - warehouse schema, view and derived data definitions
- **Business data**
  - business terms and definitions, ownership of data, charging policies

# Chapter 4: Data Warehousing and On-line Analytical Processing

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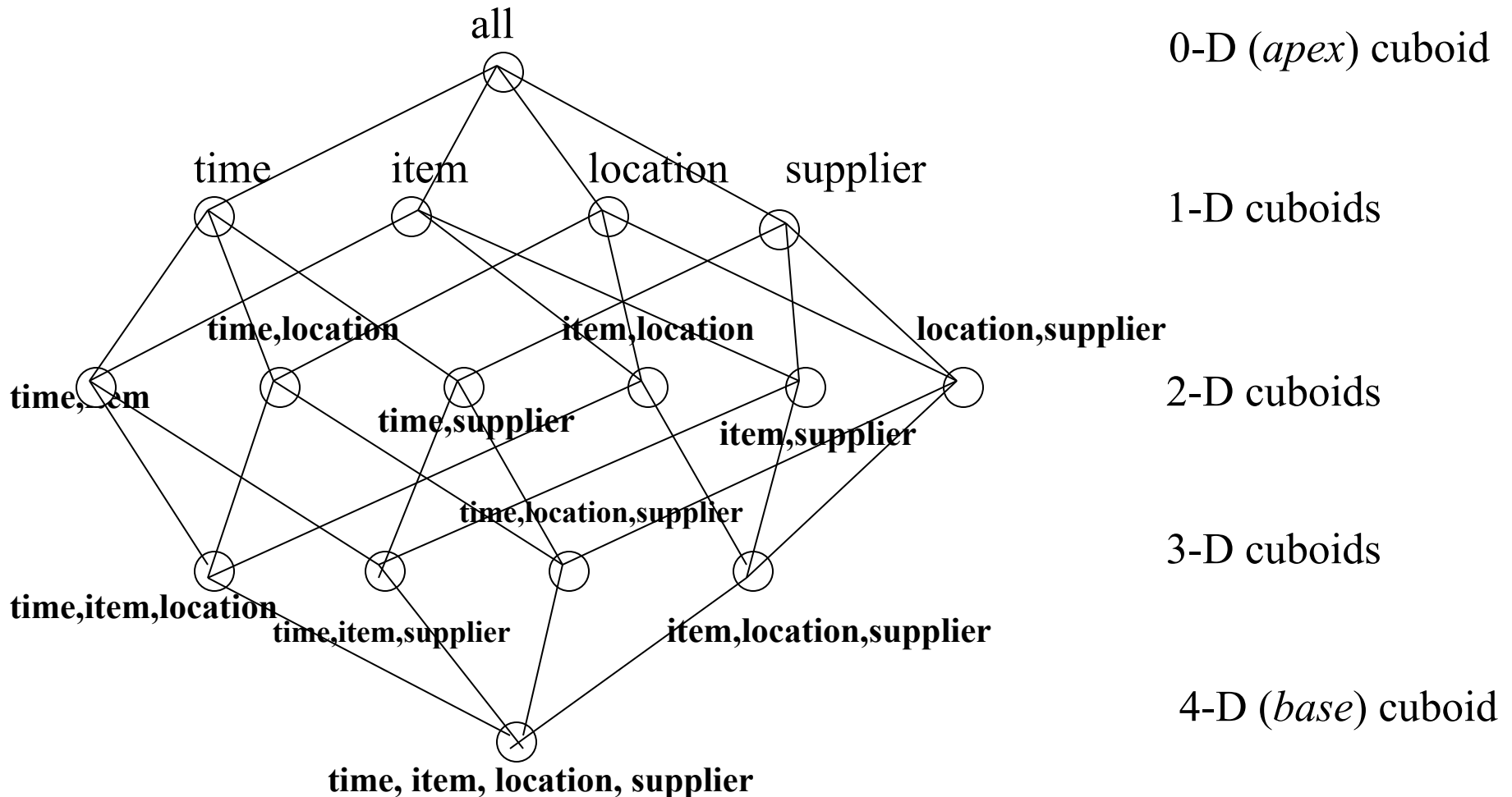
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# From Tables and Spreadsheets to Data Cubes

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- A **data warehouse** is based on a **multidimensional data model** which views data in the form of a data cube
- A data cube, such as **sales**, allows data to be modeled and viewed in multiple dimensions
  - **Dimension tables**, such as **item** (**item\_name**, **brand**, **type**), or **time**(**day**, **week**, **month**, **quarter**, **year**)
  - **Fact table** contains **measures** (such as **dollars\_sold**) and keys to each of the related dimension tables
- In data warehousing literature, an n-D base cube is called a **base cuboid**. The top most 0-D cuboid, which holds the highest-level of summarization, is called the **apex cuboid**. The lattice of cuboids forms a **data cube**.

# Cube: A Lattice of Cuboids



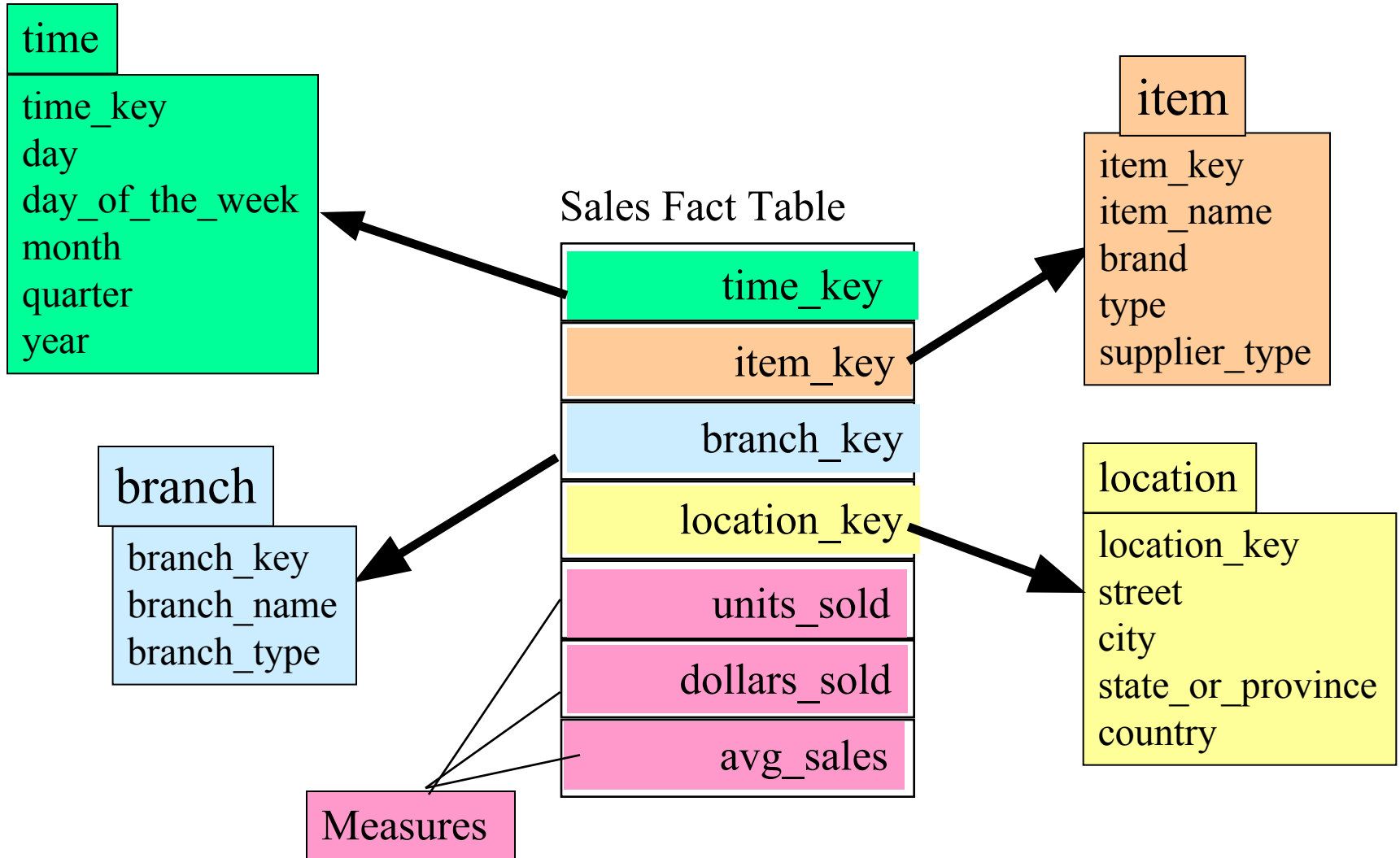


# Conceptual Modeling of Data Warehouses

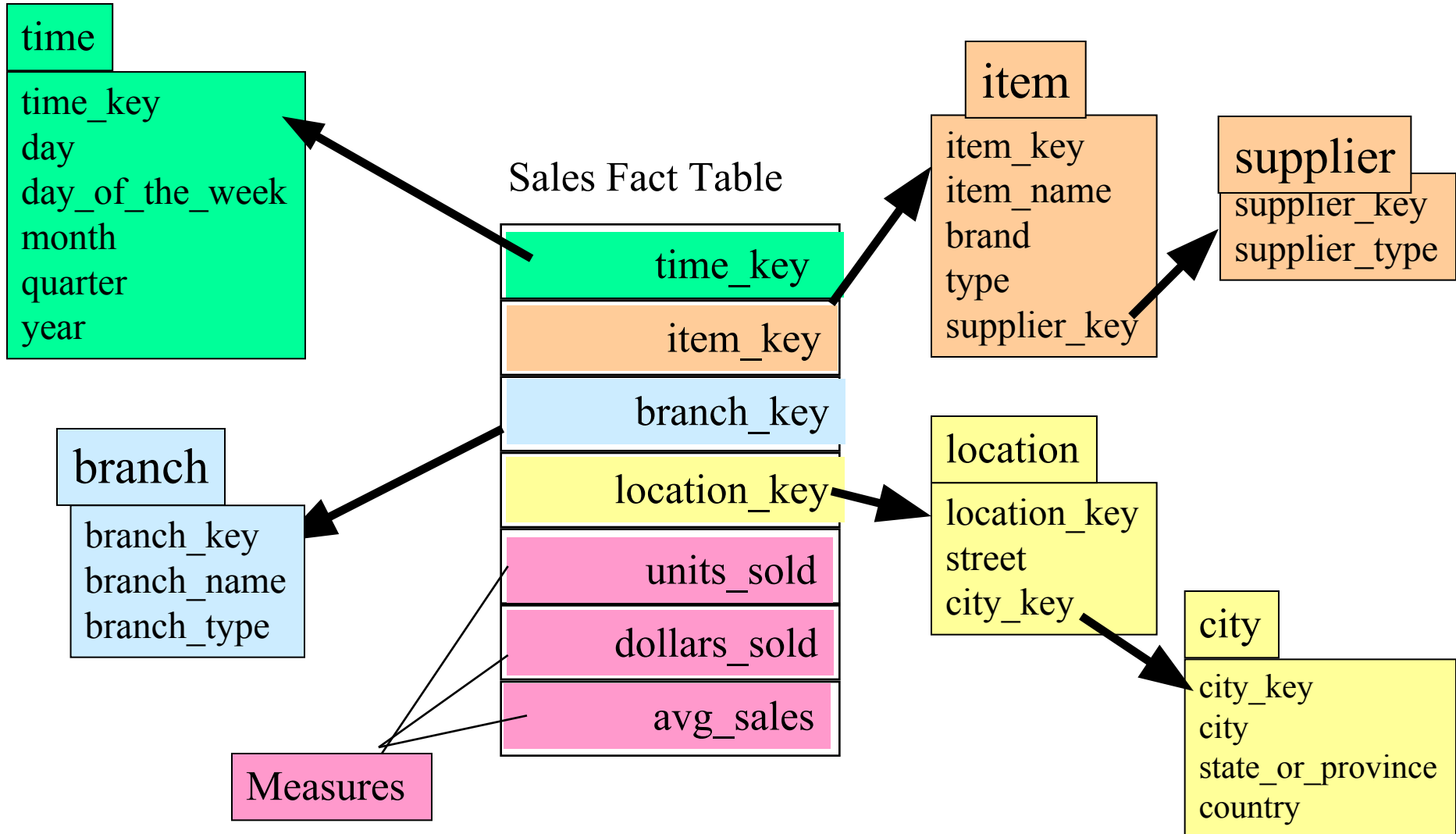
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- Modeling data warehouses: dimensions & measures
  - Star schema: A fact table in the middle connected to a set of dimension tables
  - Snowflake schema: A refinement of star schema where some dimensional hierarchy is **normalized** into a set of smaller dimension tables, forming a shape similar to snowflake
  - Fact constellations: Multiple fact tables share dimension tables, viewed as a collection of stars, therefore called **galaxy schema** or fact constellation

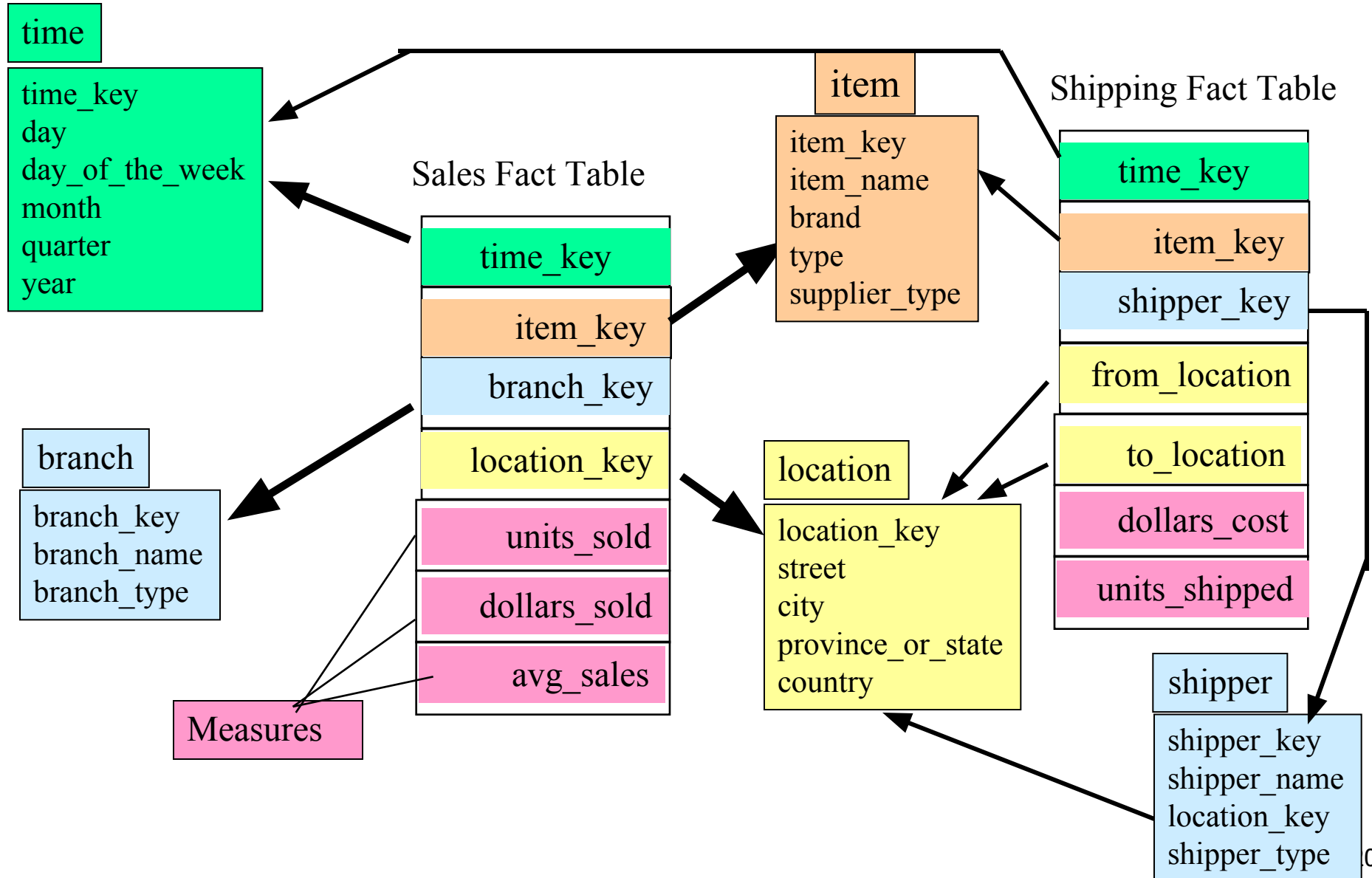
# Example of Star Schema



# Example of Snowflake Schema

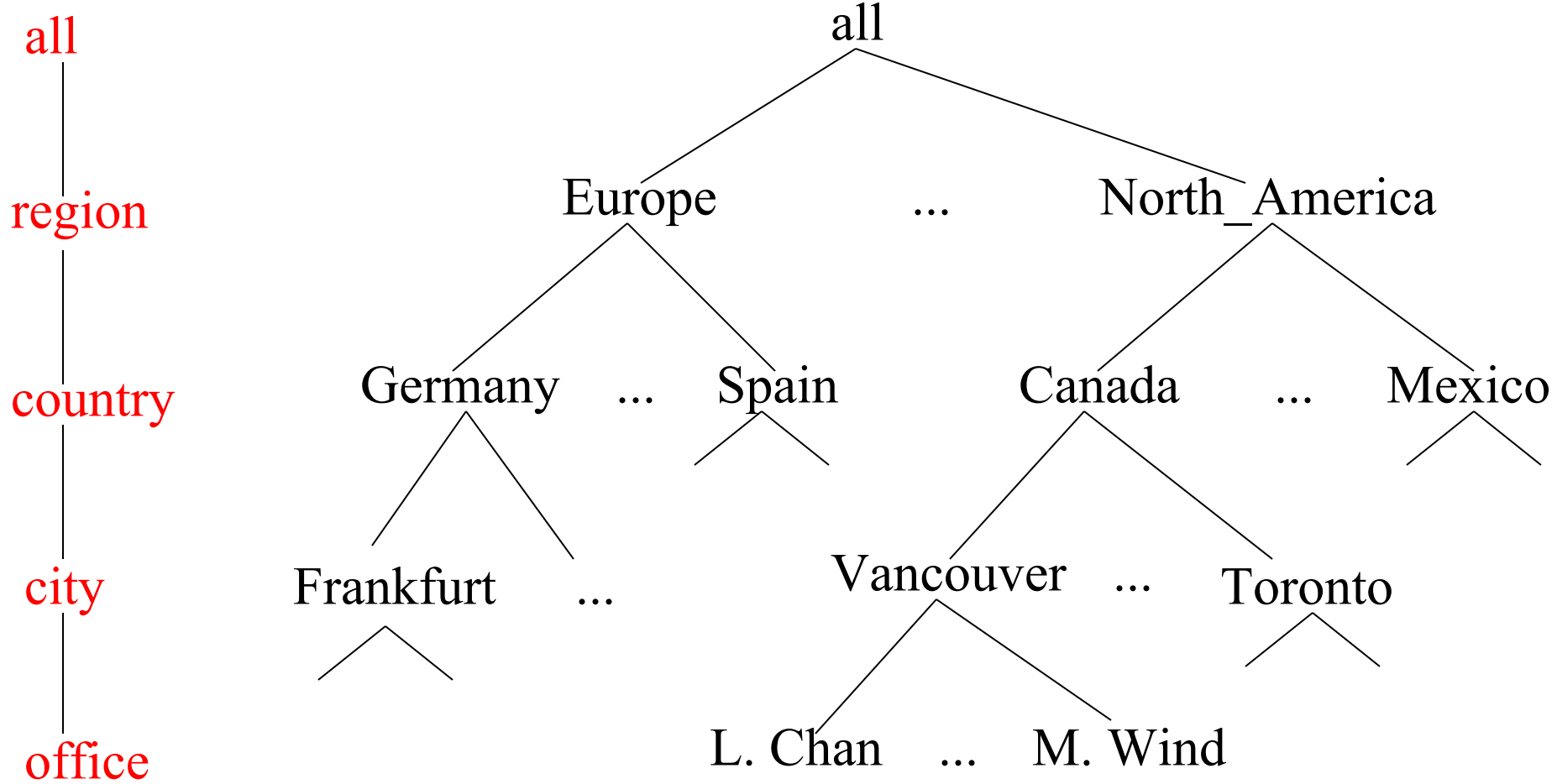


# Example of Fact Constellation



# A Concept Hierarchy: Dimension (location)

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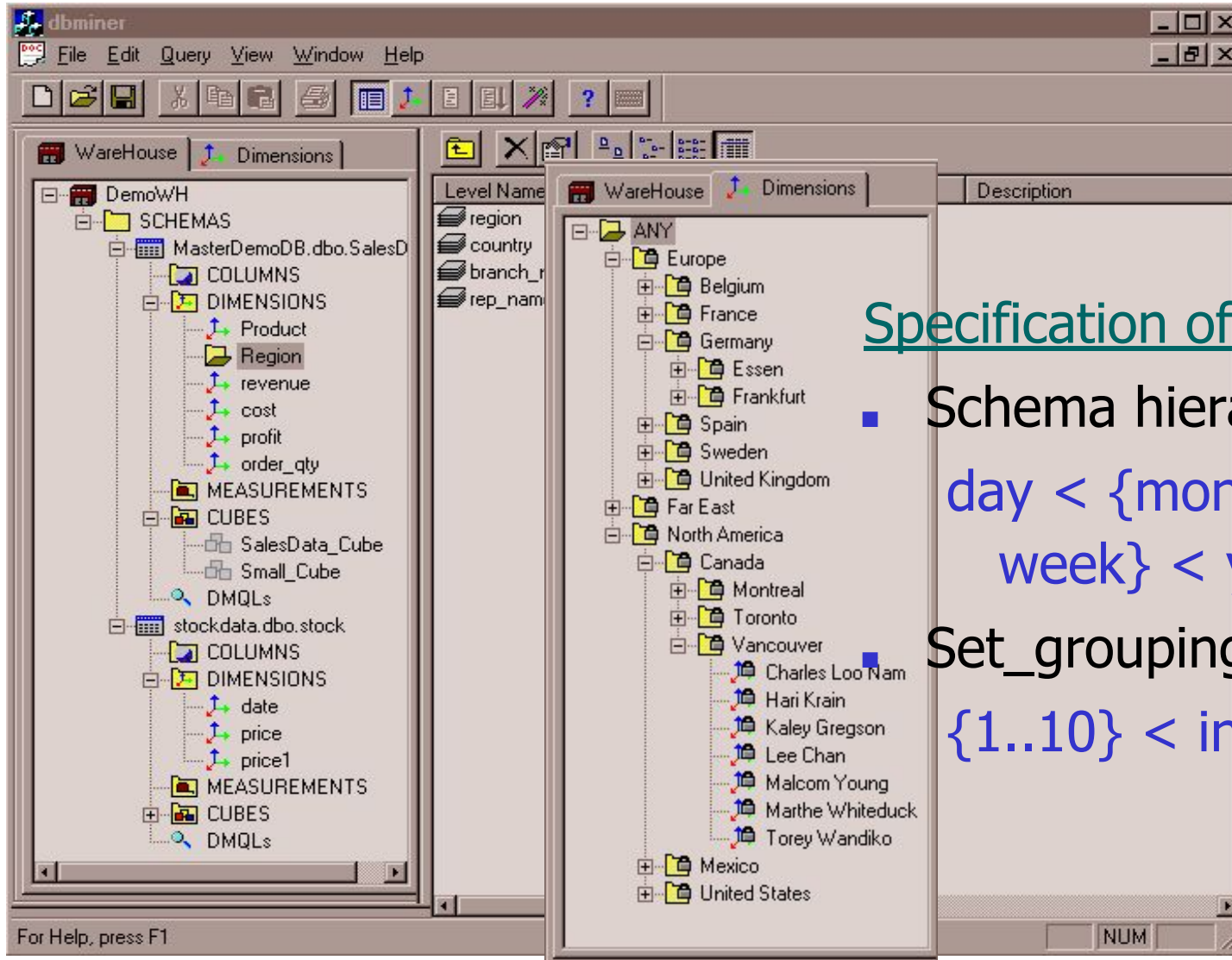


# Data Cube Measures: Three Categories

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- Distributive: if the result derived by applying the function to  $n$  aggregate values is the same as that derived by applying the function on all the data without partitioning
  - E.g., `count()`, `sum()`, `min()`, `max()`
- Algebraic: if it can be computed by an algebraic function with  $M$  arguments (where  $M$  is a bounded integer), each of which is obtained by applying a distributive aggregate function
  - E.g., `avg()`, `min_N()`, `standard_deviation()`
- Holistic: if there is no constant bound on the storage size needed to describe a subaggregate.
  - E.g., `median()`, `mode()`, `rank()`

# View of Warehouses and Hierarchies



## Specification of hierarchies

- Schema hierarchy

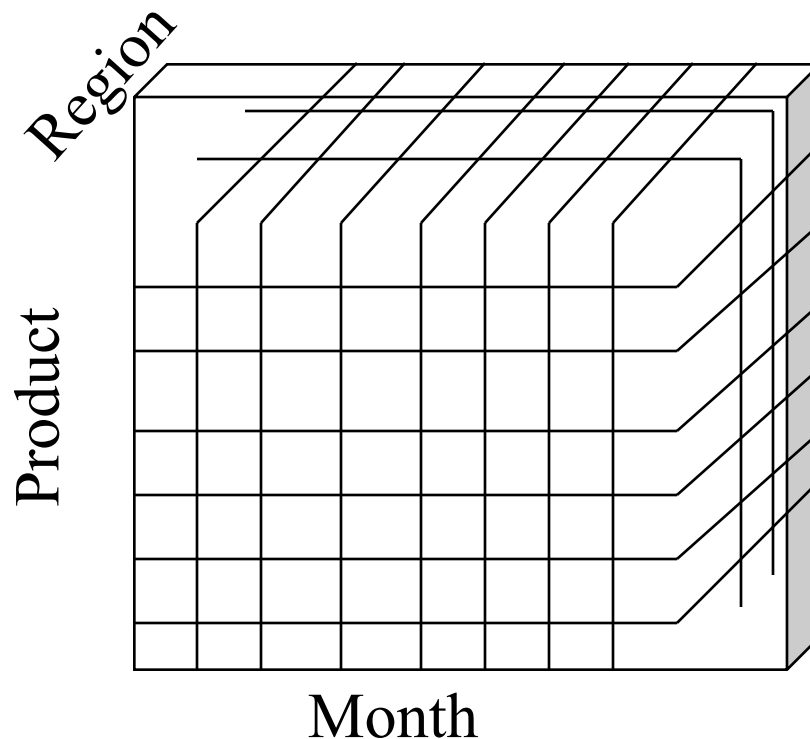
day < {month < quarter;  
week} < year

- Set\_grouping hierarchy

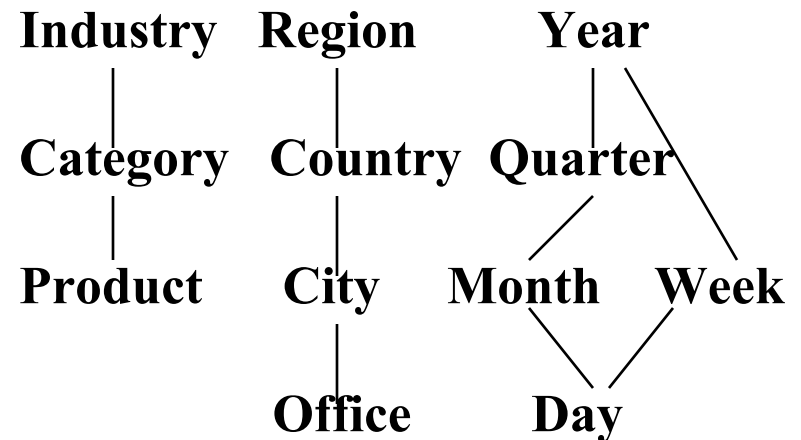
{1..10} < inexpensive

# Multidimensional Data

- Sales volume as a function of product, month, and region

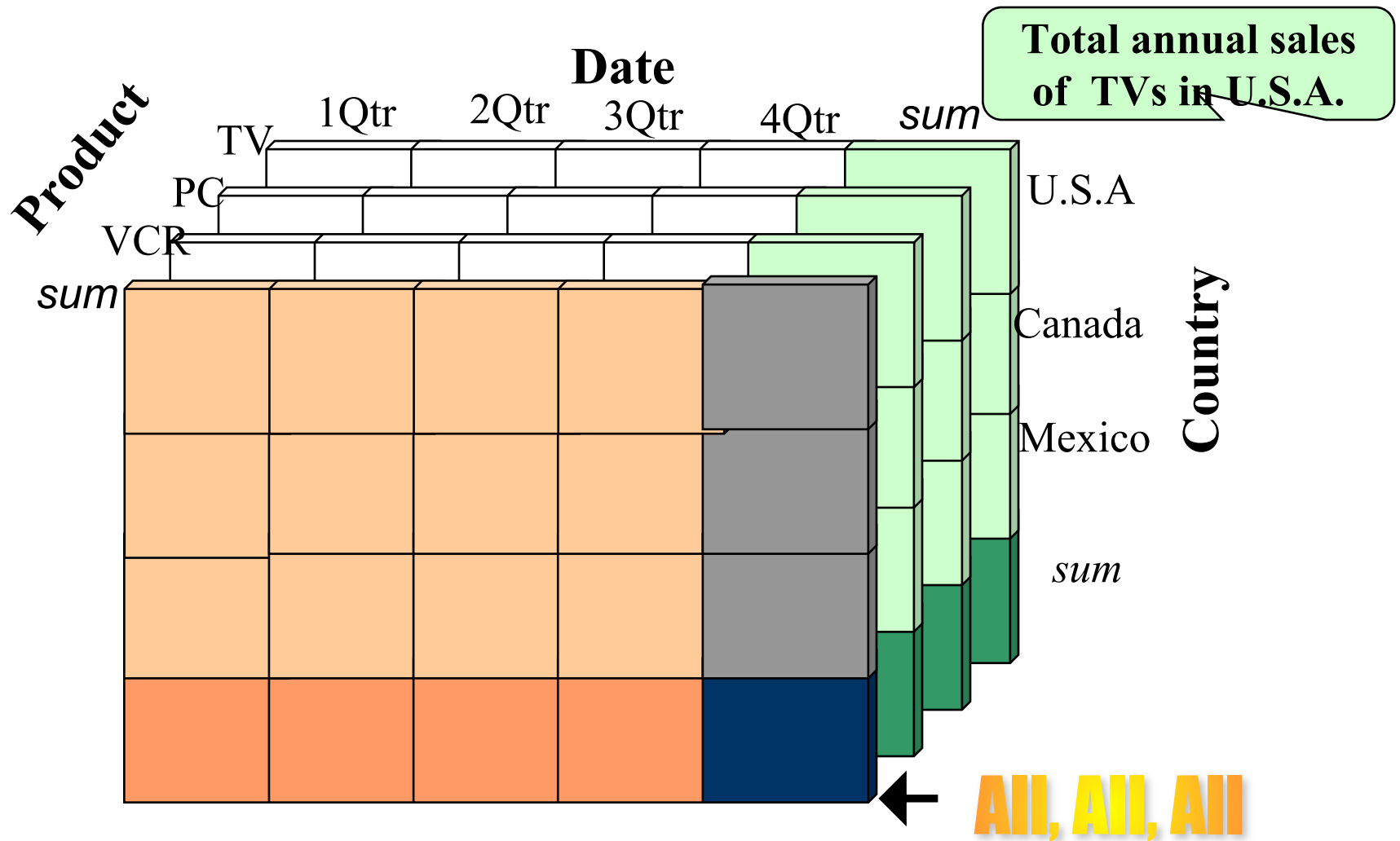


**Dimensions:** *Product, Location, Time*  
**Hierarchical summarization paths**



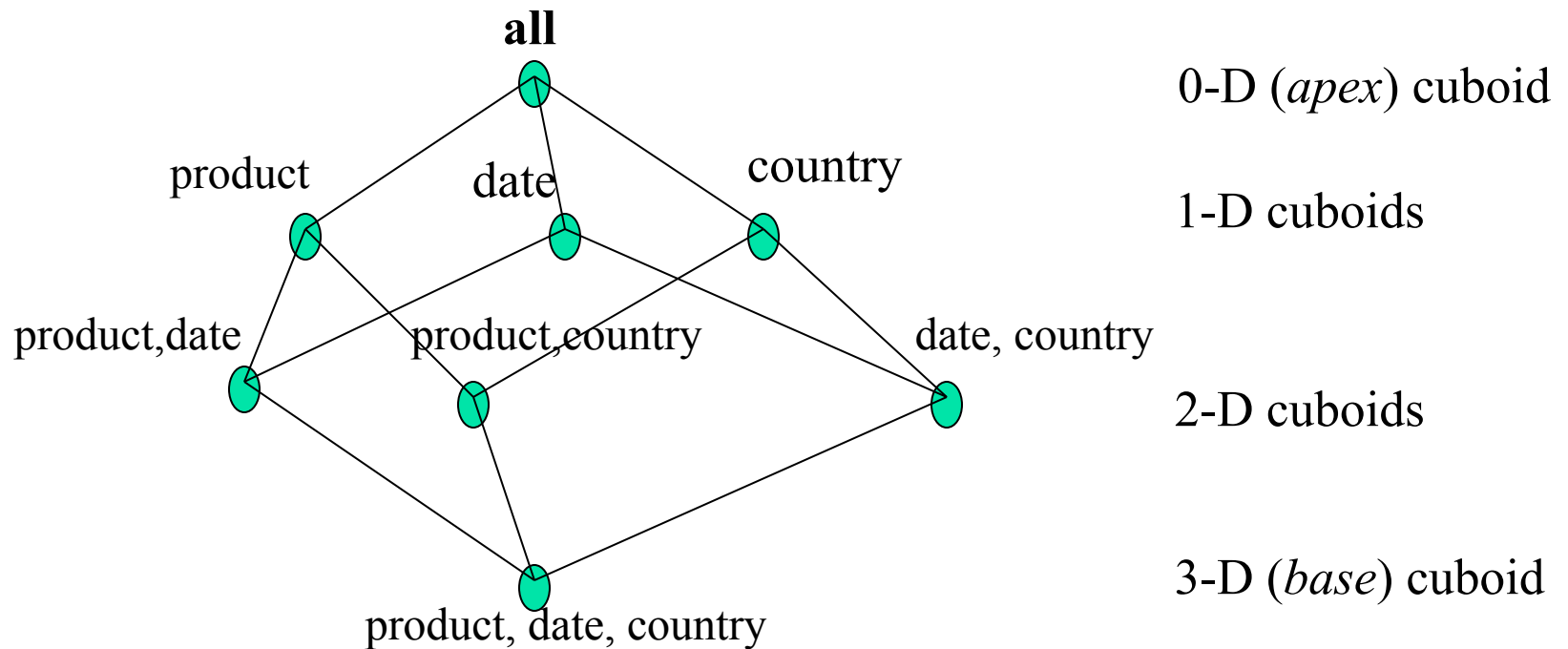


# A Sample Data Cube



# Cuboids Corresponding to the Cube

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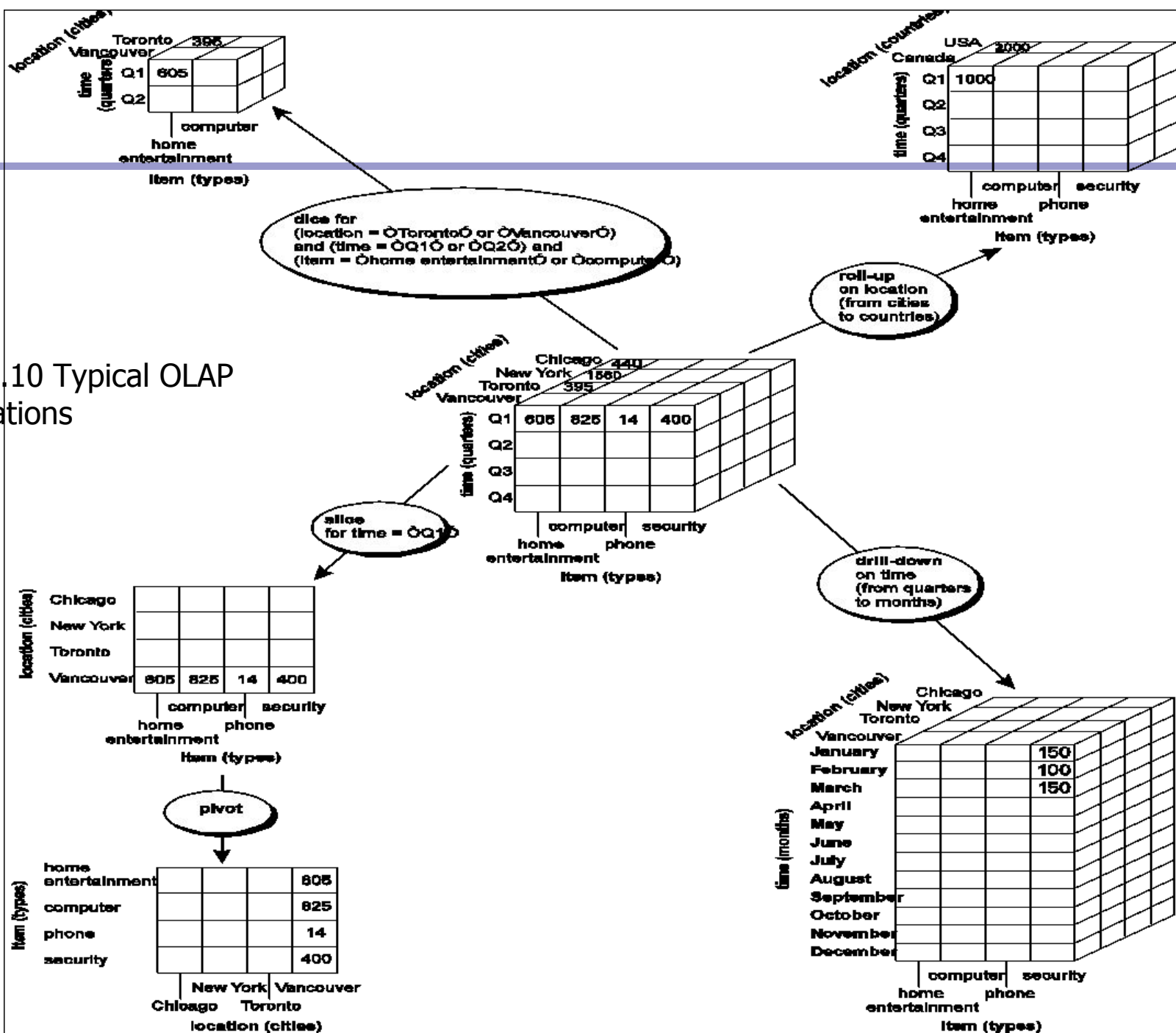


# Typical OLAP Operations

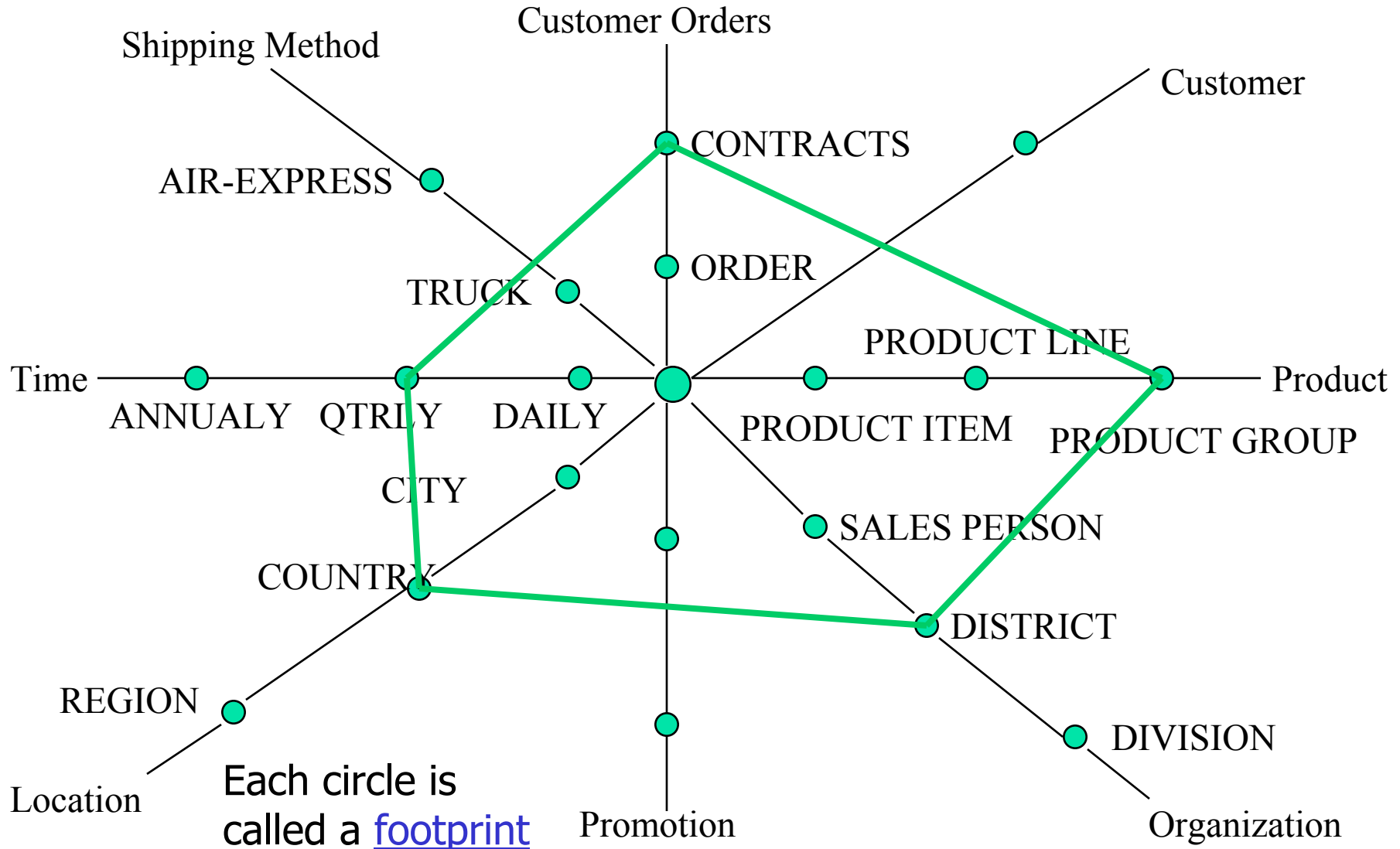
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- **Roll up (drill-up):** summarize data
  - *by climbing up hierarchy or by dimension reduction*
- **Drill down (roll down):** reverse of roll-up
  - *from higher level summary to lower level summary or detailed data, or introducing new dimensions*
- **Slice and dice:** *project and select*
- **Pivot (rotate):**
  - *reorient the cube, visualization, 3D to series of 2D planes*
- Other operations
  - **drill across:** *involving (across) more than one fact table*
  - **drill through:** *through the bottom level of the cube to its back-end relational tables (using SQL)*

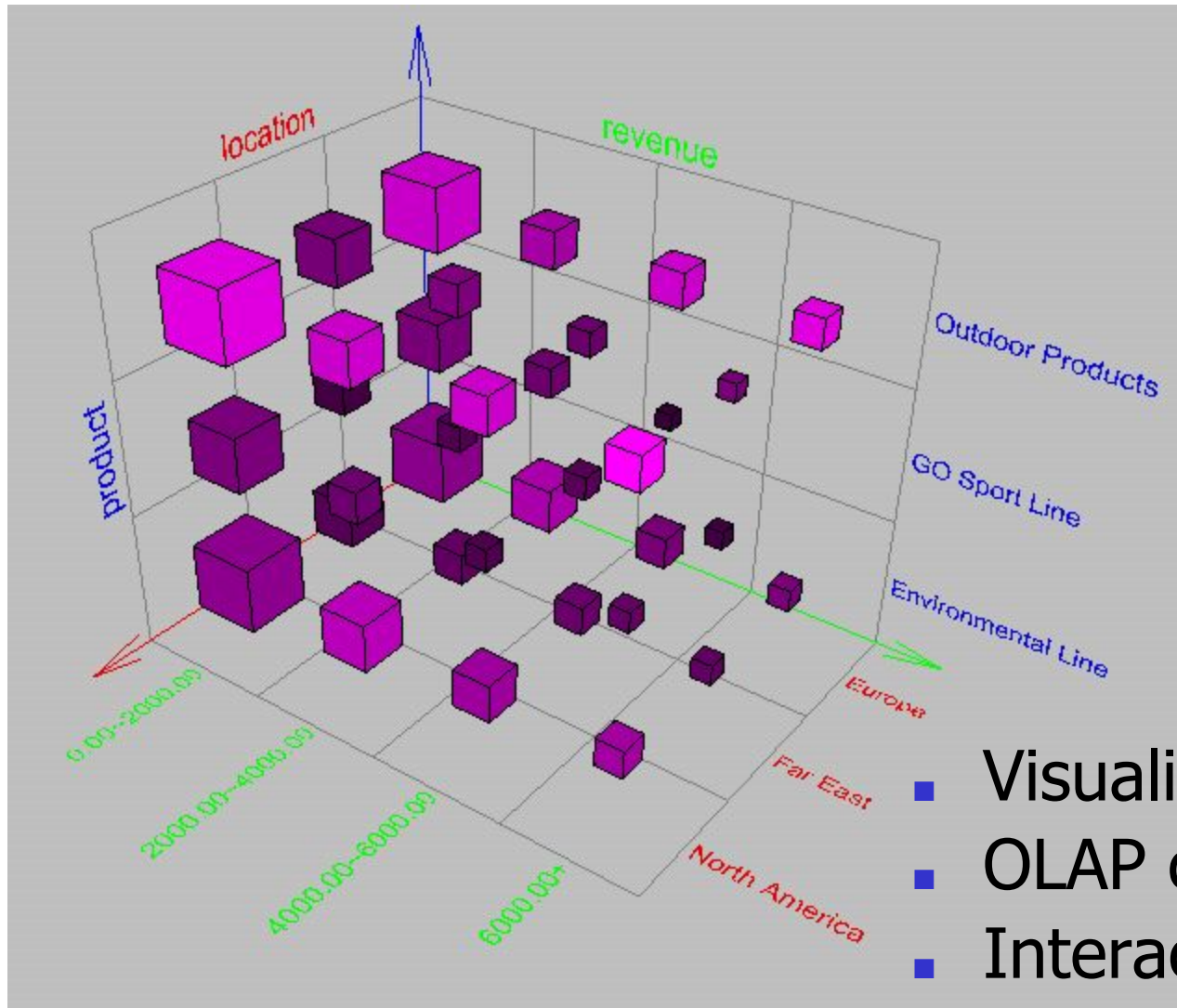
Fig. 3.10 Typical OLAP Operations



# A Star-Net Query Model




# Browsing a Data Cube



- Visualization
- OLAP capabilities
- Interactive manipulation

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# Design of Data Warehouse: A Business Analysis Framework

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- Four views regarding the design of a data warehouse
  - **Top-down view**
    - allows selection of the relevant information necessary for the data warehouse
  - **Data source view**
    - exposes the information being captured, stored, and managed by operational systems
  - **Data warehouse view**
    - consists of fact tables and dimension tables
  - **Business query view**
    - sees the perspectives of data in the warehouse from the view of end-user



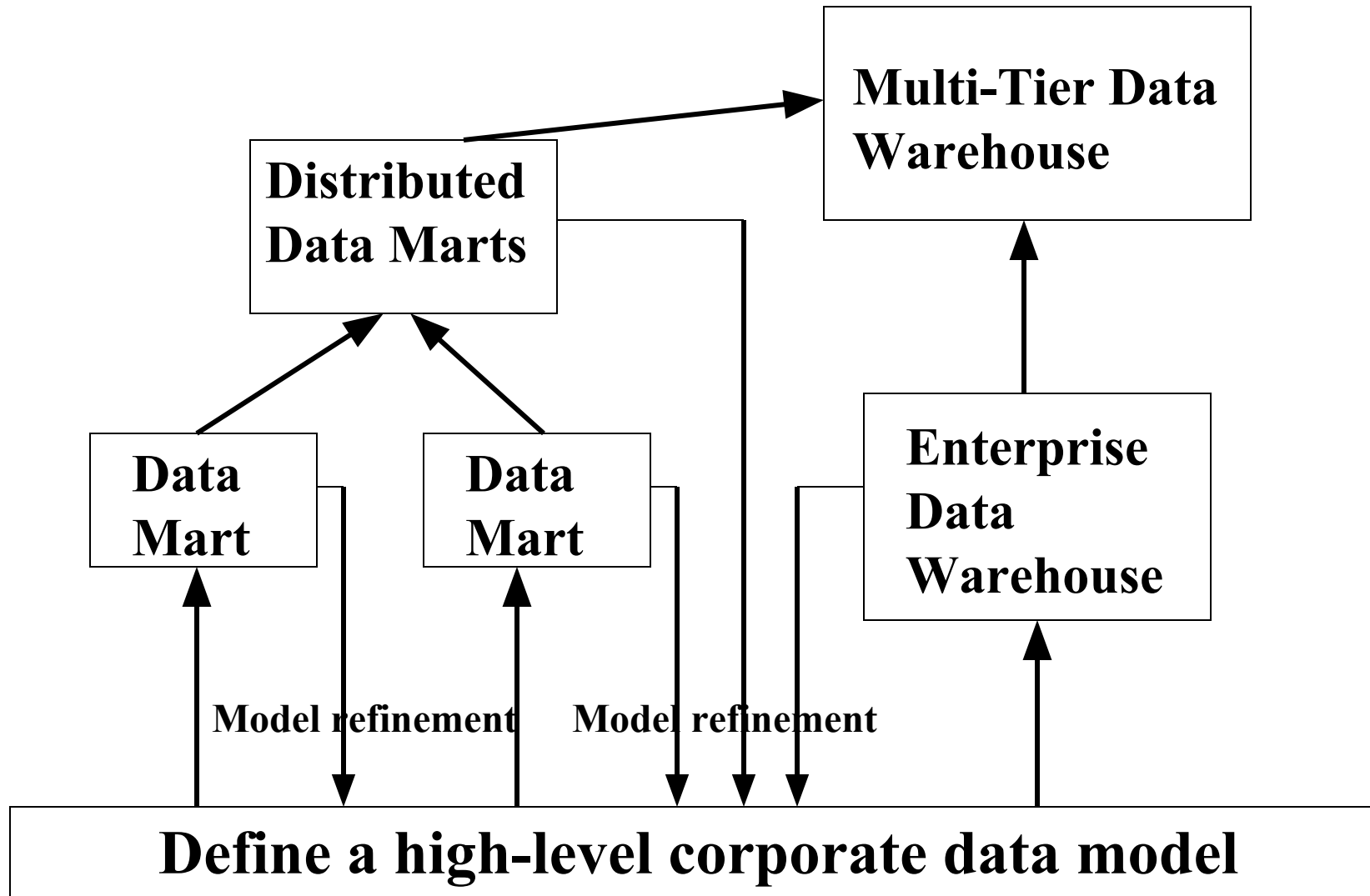
# Data Warehouse Design Process

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- **Top-down, bottom-up approaches or a combination** of both
  - Top-down: Starts with overall design and planning (mature)
  - Bottom-up: Starts with experiments and prototypes (rapid)
- **From software engineering point of view**
  - Waterfall: structured and systematic analysis at each step before proceeding to the next
  - Spiral: rapid generation of increasingly functional systems, short turn around time, quick turn around
- **Typical data warehouse design process**
  - Choose a **business process** to model, e.g., orders, invoices, etc.
  - Choose the **grain (atomic level of data)** of the business process
  - Choose the **dimensions** that will apply to each fact table record
  - Choose the **measure** that will populate each fact table record

# Data Warehouse Development: A Recommended Approach

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# Data Warehouse Usage

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- Three kinds of data warehouse applications
  - Information processing
    - supports querying, basic statistical analysis, and reporting using crosstabs, tables, charts and graphs
  - Analytical processing
    - multidimensional analysis of data warehouse data
    - supports basic OLAP operations, slice-dice, drilling, pivoting
  - Data mining
    - knowledge discovery from hidden patterns
    - supports associations, constructing analytical models, performing classification and prediction, and presenting the mining results using visualization tools


# From On-Line Analytical Processing (OLAP) to On Line Analytical Mining (OLAM)

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- Why **online analytical mining**?
  - High quality of data in data warehouses
    - DW contains integrated, consistent, cleaned data
  - Available information processing structure surrounding data warehouses
    - ODBC, OLEDB, Web accessing, service facilities, reporting and OLAP tools
  - OLAP-based exploratory data analysis
    - Mining with drilling, dicing, pivoting, etc.
  - On-line selection of data mining functions
    - Integration and swapping of multiple mining functions, algorithms, and tasks

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# Efficient Data Cube Computation

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- Data cube can be viewed as a lattice of cuboids
  - The bottom-most cuboid is the base cuboid
  - The top-most cuboid (apex) contains only one cell
  - How many cuboids in an n-dimensional cube with L levels?

$$T = \prod_{i=1}^n (L_i + 1)$$

- Materialization of data cube
  - Materialize every (cuboid) (**full materialization**), none (**no materialization**), or some (**partial materialization**)
  - Selection of which cuboids to materialize
    - Based on size, sharing, access frequency, etc.

# The “Compute Cube” Operator

- Cube definition and computation in DMQL

**define cube** sales [item, city, year]: sum (sales\_in\_dollars)

**compute cube** sales

- Transform it into a SQL-like language (with a new operator **cube by**, introduced by Gray et al.'96)

SELECT item, city, year, SUM (amount)

FROM SALES

**CUBE BY** item, city, year

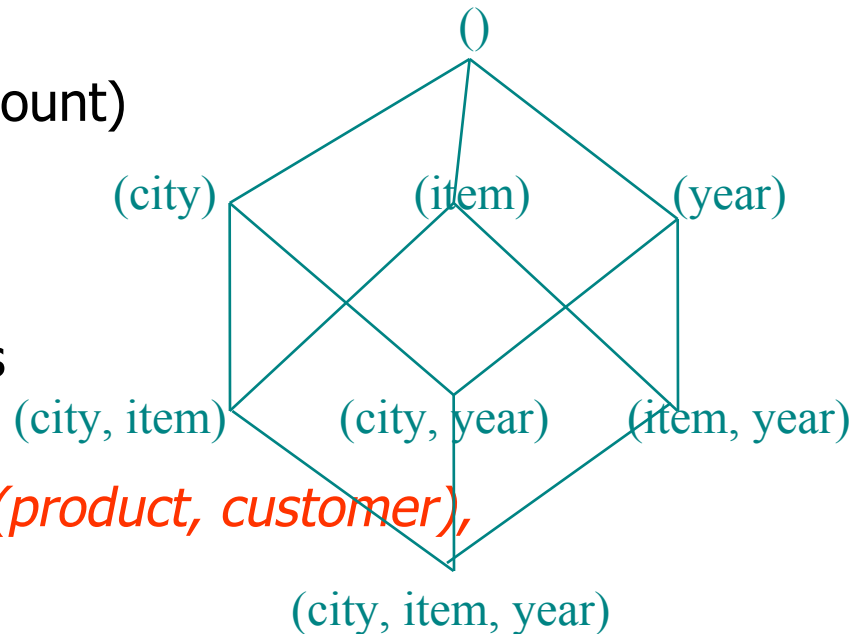
- Need compute the following Group-Bys

*(date, product, customer),*

*(date, product), (date, customer), (product, customer),*

*(date), (product), (customer)*

*()*



# Indexing OLAP Data: Bitmap Index

- Index on a particular column
- Each value in the column has a bit vector: bit-op is fast
- The length of the bit vector: # of records in the base table
- The  $i$ -th bit is set if the  $i$ -th row of the base table has the value for the indexed column
- not suitable for high cardinality domains
- A recent bit compression technique, Word-Aligned Hybrid (WAH), makes it work for high cardinality domain as well [Wu, et al. TODS'06]

**Base table**

Cust	Region	Type
C1	Asia	Retail
C2	Europe	Dealer
C3	Asia	Dealer
C4	America	Retail
C5	Europe	Dealer

**Index on Region**

RecID	Asia	Europe	America
1	1	0	0
2	0	1	0
3	1	0	0
4	0	0	1
5	0	1	0

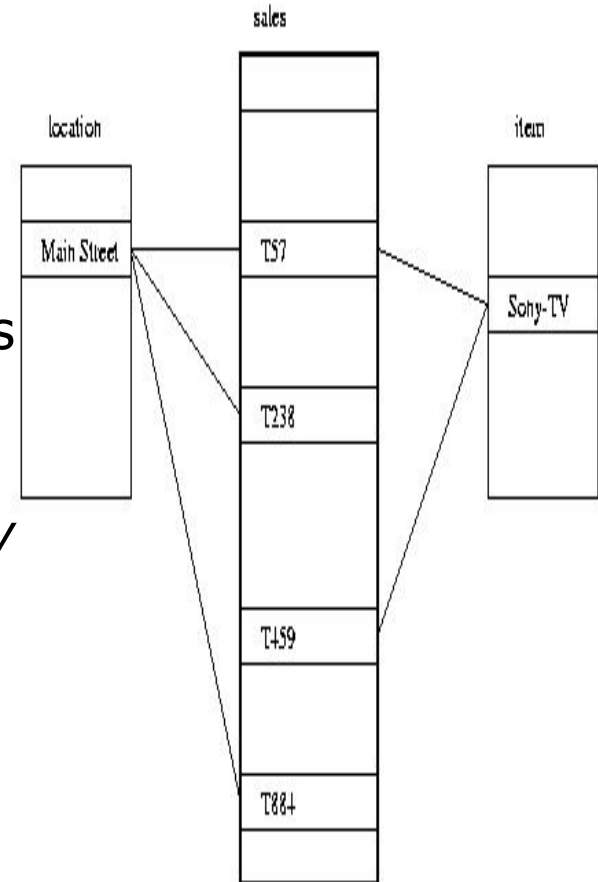
**Index on Type**

RecID	Retail	Dealer
1	1	0
2	0	1
3	0	1
4	1	0
5	0	1



# Indexing OLAP Data: Join Indices

- Join index:  $JI(R\text{-id}, S\text{-id})$  where  $R (R\text{-id}, \dots)$   $\square\square$   
 $S (S\text{-id}, \dots)$
- Traditional indices map the values to a list of record ids
  - It materializes relational join in JI file and speeds up relational join
- In data warehouses, join index relates the values of the dimensions of a star schema to rows in the fact table.
  - E.g. fact table: *Sales* and two dimensions *city* and *product*
    - A join index on *city* maintains for each distinct city a list of R-IDs of the tuples recording the Sales in the city
  - Join indices can span multiple dimensions



# Efficient Processing OLAP Queries

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- **Determine which operations** should be performed on the available cuboids
  - Transform *drill*, *roll*, etc. into corresponding SQL and/or OLAP operations, e.g., *dice* = selection + projection
- **Determine which materialized cuboid(s)** should be selected for OLAP op.
  - Let the query to be processed be on  $\{brand, province\_or\_state\}$  with the condition "*year = 2004*", and there are 4 materialized cuboids available:
    - 1)  $\{year, item\_name, city\}$
    - 2)  $\{year, brand, country\}$
    - 3)  $\{year, brand, province\_or\_state\}$
    - 4)  $\{item\_name, province\_or\_state\}$  where *year = 2004*Which should be selected to process the query?
- Explore indexing structures and compressed vs. dense array structs in MOLAP


# OLAP Server Architectures

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- Relational OLAP (ROLAP)
  - Use relational or extended-relational DBMS to store and manage warehouse data and OLAP middle ware
  - Include optimization of DBMS backend, implementation of aggregation navigation logic, and additional tools and services
  - Greater scalability
- Multidimensional OLAP (MOLAP)
  - Sparse array-based multidimensional storage engine
  - Fast indexing to pre-computed summarized data
- Hybrid OLAP (HOLAP) (e.g., Microsoft SQLServer)
  - Flexibility, e.g., low level: relational, high-level: array
- Specialized SQL servers (e.g., Redbricks)
  - Specialized support for SQL queries over star/snowflake schemas

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# Attribute-Oriented Induction

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- Proposed in 1989 (KDD '89 workshop)
- Not confined to categorical data nor particular measures
- How it is done?
  - Collect the task-relevant data (*initial relation*) using a relational database query
  - Perform generalization by attribute removal or attribute generalization
  - Apply aggregation by merging identical, generalized tuples and accumulating their respective counts
  - Interaction with users for knowledge presentation

# Attribute-Oriented Induction: An Example

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Example: Describe general characteristics of graduate students in the University database

- Step 1. Fetch relevant set of data using an SQL statement, e.g.,  
**Select** \* (i.e., name, gender, major, birth\_place, birth\_date, residence, phone#, gpa)  
**from** student  
**where** student\_status in {"Msc", "MBA", "PhD" }
- Step 2. Perform attribute-oriented induction
- Step 3. Present results in generalized relation, cross-tab, or rule forms

# Class Characterization: An Example

**Initial Relation**

Name	Gender	Major	Birth-Place	Birth_date	Residence	Phone #	GPA
Jim Woodman	M	CS	Vancouver,BC, Canada	8-12-76	3511 Main St., Richmond	687-4598	3.67
Scott Lachance	M	CS	Montreal, Que, Canada	28-7-75	345 1st Ave., Richmond	253-9106	3.70
Laura Lee	F	Physics	Seattle, WA, USA	25-8-70	125 Austin Ave., Burnaby	420-5232	3.83
...	...	...	...	...	...	...	...
<b>Removed</b>	<b>Retained</b>	<b>Sci,Eng, Bus</b>	<b>Country</b>	<b>Age range</b>	<b>City</b>	<b>Removed</b>	<b>Excl, VG,...</b>

**Prime Generalized Relation**

Gender	Major	Birth region	Age range	Residence	GPA	Count
M	Science	Canada	20-25	Richmond	Very-good	16
F	Science	Foreign	25-30	Burnaby	Excellent	22
...	...	...	...	...	...	...

Birth_Region Gender	Canada	Foreign	Total
M	16	14	30
F	10	22	32
Total	26	36	62

# Basic Principles of Attribute-Oriented Induction

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- Data focusing: task-relevant data, including dimensions, and the result is the *initial relation*
- Attribute-removal: remove attribute  $A$  if there is a large set of distinct values for  $A$  but (1) there is no generalization operator on  $A$ , or (2)  $A$ 's higher level concepts are expressed in terms of other attributes
- Attribute-generalization: If there is a large set of distinct values for  $A$ , and there exists a set of generalization operators on  $A$ , then select an operator and generalize  $A$
- Attribute-threshold control: typical 2-8, specified/default
- Generalized relation threshold control: control the final relation/rule size



# Attribute-Oriented Induction: Basic Algorithm

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- InitialRel: Query processing of task-relevant data, deriving the *initial relation*.
- PreGen: Based on the analysis of the number of distinct values in each attribute, determine generalization plan for each attribute: removal? or how high to generalize?
- PrimeGen: Based on the PreGen plan, perform generalization to the right level to derive a “prime generalized relation”, accumulating the counts.
- Presentation: User interaction: (1) adjust levels by drilling, (2) pivoting, (3) mapping into rules, cross tabs, visualization presentations.

# Presentation of Generalized Results

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- Generalized relation:
  - Relations where some or all attributes are generalized, with counts or other aggregation values accumulated.
- Cross tabulation:
  - Mapping results into cross tabulation form (similar to contingency tables).
  - Visualization techniques:
    - Pie charts, bar charts, curves, cubes, and other visual forms.
- Quantitative characteristic rules:
  - Mapping generalized result into characteristic rules with quantitative information associated with it, e.g.,

$grad(x) \wedge male(x) \Rightarrow$   
 $birth\_region(x) = "Canada"[t:53\%] \vee birth\_region(x) = "foreign"[t:47\%].$

# Mining Class Comparisons

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- Comparison: Comparing two or more classes
- Method:
  - Partition the set of relevant data into the target class and the contrasting class(es)
  - Generalize both classes to the same high level concepts
  - Compare tuples with the same high level descriptions
  - Present for every tuple its description and two measures
    - support - distribution within single class
    - comparison - distribution between classes
  - Highlight the tuples with strong discriminant features
- Relevance Analysis:
  - Find attributes (features) which best distinguish different classes

# Concept Description vs. Cube-Based OLAP

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## ■ **Similarity:**


- Data generalization
- Presentation of data summarization at multiple levels of abstraction
- Interactive drilling, pivoting, slicing and dicing

## ■ **Differences:**

- OLAP has systematic preprocessing, query independent, and can drill down to rather low level
- AOI has automated desired level allocation, and may perform dimension relevance analysis/ranking when there are many relevant dimensions
- AOI works on the data which are not in relational forms

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# Summary

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- **Data warehousing**: A **multi-dimensional model** of a data warehouse
  - A data cube consists of *dimensions* & *measures*
  - Star schema, snowflake schema, fact constellations
  - **OLAP** operations: drilling, rolling, slicing, dicing and pivoting
- **Data Warehouse Architecture, Design, and Usage**
  - Multi-tiered architecture
  - Business analysis design framework
  - Information processing, analytical processing, data mining, **OLAM** (Online Analytical Mining)
- **Implementation**: Efficient computation of data cubes
  - Partial vs. full vs. no materialization
  - Indexing OLAP data: Bitmap index and join index
  - OLAP query processing
  - OLAP servers: ROLAP, MOLAP, HOLAP
- **Data generalization**: Attribute-oriented induction

# References (I)

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- S. Agarwal, R. Agrawal, P. M. Deshpande, A. Gupta, J. F. Naughton, R. Ramakrishnan, and S. Sarawagi. On the computation of multidimensional aggregates. VLDB'96
- D. Agrawal, A. E. Abbadi, A. Singh, and T. Yurek. Efficient view maintenance in data warehouses. SIGMOD'97
- R. Agrawal, A. Gupta, and S. Sarawagi. Modeling multidimensional databases. ICDE'97
- S. Chaudhuri and U. Dayal. An overview of data warehousing and OLAP technology. *ACM SIGMOD Record*, 26:65-74, 1997
- E. F. Codd, S. B. Codd, and C. T. Salley. Beyond decision support. *Computer World*, 27, July 1993.
- J. Gray, et al. Data cube: A relational aggregation operator generalizing group-by, cross-tab and sub-totals. *Data Mining and Knowledge Discovery*, 1:29-54, 1997.
- A. Gupta and I. S. Mumick. *Materialized Views: Techniques, Implementations, and Applications*. MIT Press, 1999.
- J. Han. Towards on-line analytical mining in large databases. *ACM SIGMOD Record*, 27:97-107, 1998.
- V. Harinarayan, A. Rajaraman, and J. D. Ullman. Implementing data cubes efficiently. SIGMOD'96
- J. Hellerstein, P. Haas, and H. Wang. Online aggregation. SIGMOD'97

# References (II)

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- C. Imhoff, N. Galemme, and J. G. Geiger. Mastering Data Warehouse Design: Relational and Dimensional Techniques. John Wiley, 2003
- W. H. Inmon. Building the Data Warehouse. John Wiley, 1996
- R. Kimball and M. Ross. The Data Warehouse Toolkit: The Complete Guide to Dimensional Modeling. 2ed. John Wiley, 2002
- P. O'Neil and G. Graefe. Multi-table joins through bitmapped join indices. *SIGMOD Record*, 24:8–11, Sept. 1995.
- P. O'Neil and D. Quass. Improved query performance with variant indexes. SIGMOD'97
- Microsoft. OLEDB for OLAP programmer's reference version 1.0. In <http://www.microsoft.com/data/oledb/olap>, 1998
- S. Sarawagi and M. Stonebraker. Efficient organization of large multidimensional arrays. ICDE'94
- A. Shoshani. OLAP and statistical databases: Similarities and differences. PODS'00.
- D. Srivastava, S. Dar, H. V. Jagadish, and A. V. Levy. Answering queries with aggregation using views. *VLDB'96*
- P. Valduriez. Join indices. *ACM Trans. Database Systems*, 12:218-246, 1987.
- J. Widom. Research problems in data warehousing. CIKM'95
- K. Wu, E. Otoo, and A. Shoshani, Optimal Bitmap Indices with Efficient Compression, *ACM Trans. on Database Systems (TODS)*, 31(1): 1-38, 2006



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# Surplus Slides

# Compression of Bitmap Indices

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- Bitmap indexes must be compressed to reduce I/O costs and minimize CPU usage—majority of the bits are 0's
- Two compression schemes:
  - Byte-aligned Bitmap Code (BBC)
  - Word-Aligned Hybrid (WAH) code
- Time and space required to operate on compressed bitmap is proportional to the total size of the bitmap
- Optimal on attributes of low cardinality as well as those of high cardinality.
- WAH out performs BBC by about a factor of two