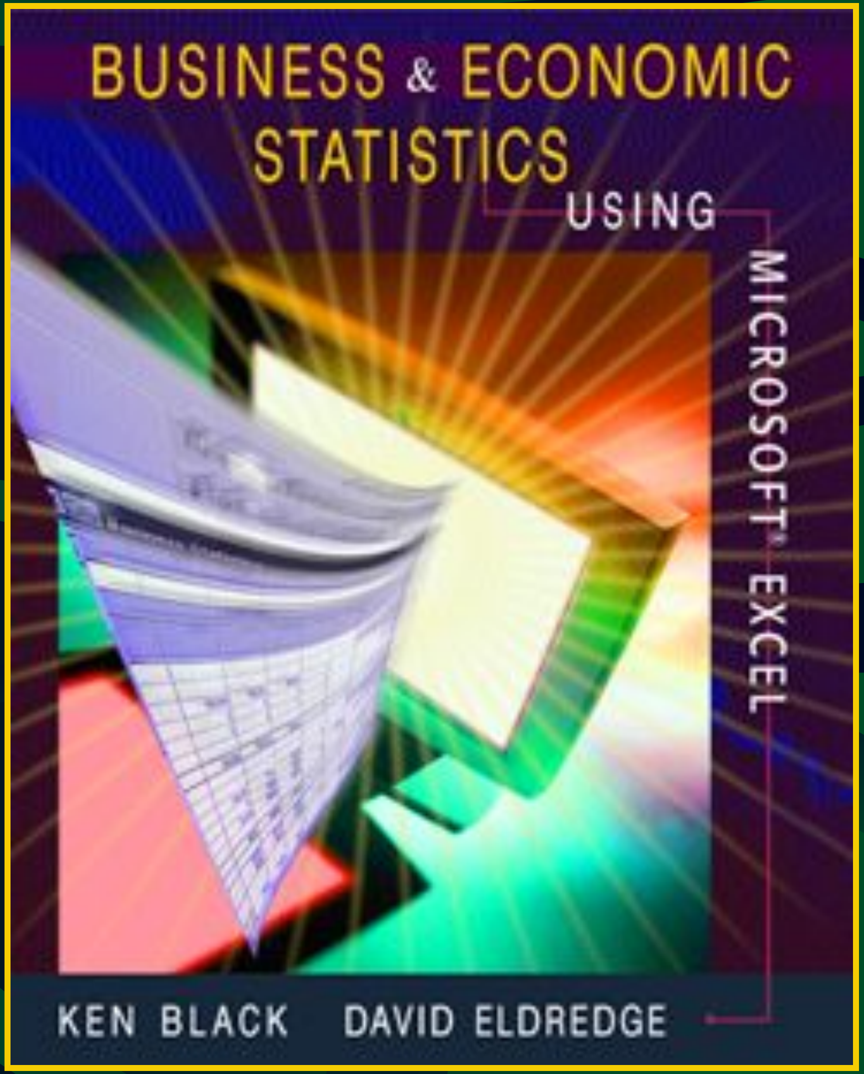


Chapter 3

Descriptive Statistics

Learning Objectives

- Distinguish between measures of central tendency, measures of variability, and measures of shape
- Understand the meanings of mean, median, mode, quartile, percentile, and range
- Compute mean, median, mode, percentile, quartile, range, variance, standard deviation, and mean absolute deviation

Learning Objectives -- Continued

- Differentiate between sample and population variance and standard deviation
- Understand the meaning of standard deviation as it is applied by using the empirical rule
- Understand box and whisker plots, skewness, and kurtosis

Measures of Central Tendency

- Measures of central tendency yield information about “particular places or locations in a group of numbers.”
- Common Measures of Location
  - Mode
  - Median
  - Mean
  - Percentiles
  - Quartiles

Mode

- The most frequently occurring value in a data set
- Applicable to all levels of data measurement (nominal, ordinal, interval, and ratio)

- Bimodal -- Data sets that have two modes
- Multimodal -- Data sets that contain more than two modes

Mode -- Example

- The mode is 44.
- There are more 44s than any other value.

35	41	44	45
37	41	44	46
37	43	44	46
39	43	44	46
40	43	44	46
40	43	45	48

Median (ΔΙΑΜΕΣΟΣ)

- Middle value in an ordered array of numbers.
- Applicable for ordinal, interval, and ratio data
- Not applicable for nominal data
- Unaffected by extremely large and extremely small values.

Median: Computational Procedure

- First Procedure
  - Arrange observations in an ordered array.
  - If number of terms is odd, the median is the middle term of the ordered array.
  - If number of terms is even, the median is the average of the middle two terms.

- Second Procedure
  - The median's position in an ordered array is given by (n+1)/2.

Median: Example with an Odd Number of Terms

Ordered Array includes:
3 4 5 7 8 9 11 14 15 16 16 17 19 19 20 21 22

- There are 17 terms in the ordered array.
- Position of median = (n+1)/2 = (17+1)/2 = 9
- The median is the 9th term, 15.
- If the 22 is replaced by 100, the median remains at 15.
- If the 3 is replaced by -103, the median remains at 15.

Mean (ΜΕΣΟΣ)

- Is the average of a group of numbers
- Applicable for interval and ratio data, not applicable for nominal or ordinal data
- Affected by each value in the data set, including extreme values
- Computed by summing all values in the data set and dividing the sum by the number of values in the data set

Population Mean

$$\mu = \frac{\sum X}{N} = \frac{X_1 + X_2 + X_3 + \ldots + X_N}{N}$$

$$= \frac{24 + 13 + 19 + 26 + 11}{5}$$

$$= \frac{93}{5}$$

$$= 18.6$$

Sample Mean

$$\bar{X} = \frac{\sum X}{n} = \frac{X_1 + X_2 + X_3 + \ldots + X_n}{n}$$

$$= \frac{57 + 86 + 42 + 38 + 90 + 66}{6}$$

$$= \frac{379}{6}$$

$$= 63.167$$

Quartiles

Measures of central tendency that divide a group of data into four subgroups

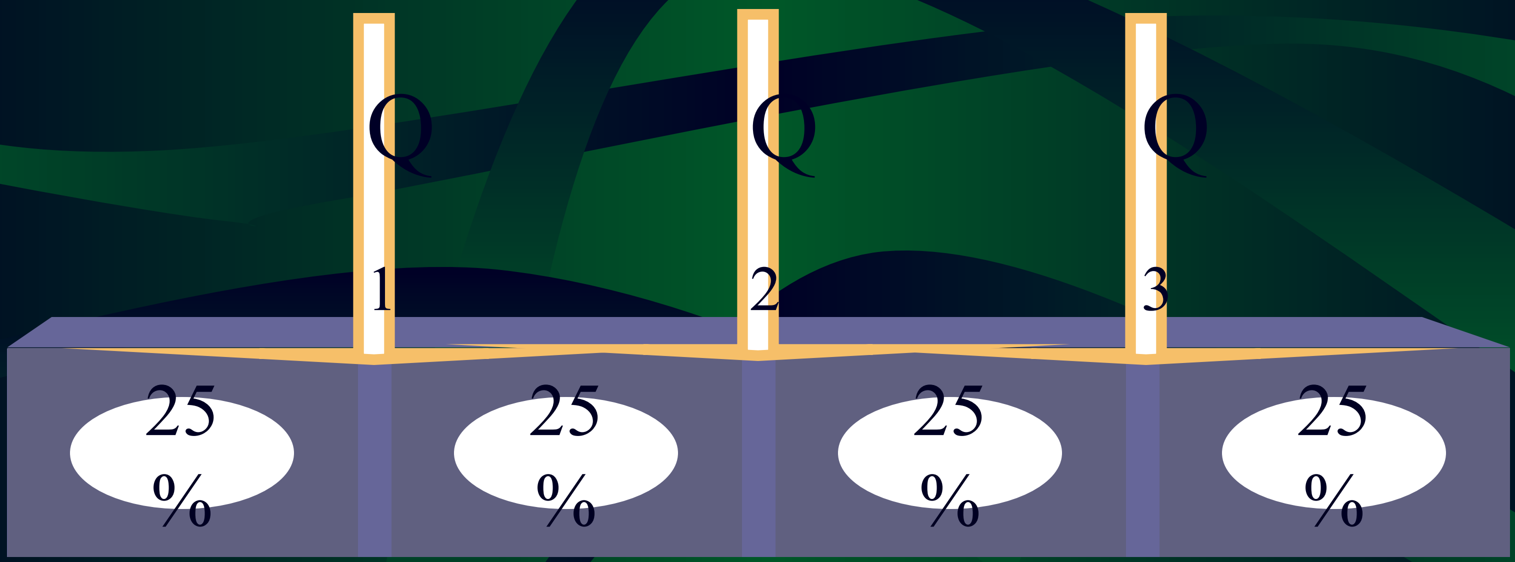
- Q_1: 25% of the data set is below the first quartile
- Q_2: 50% of the data set is below the second quartile
- Q_3: 75% of the data set is below the third quartile

Quartiles, *continued*

- Q_1 is equal to the 25th percentile
- Q_2 is located at 50th percentile and equals the median
- Q_3 is equal to the 75th percentile

Quartile values are not necessarily members of the data set

Quartiles



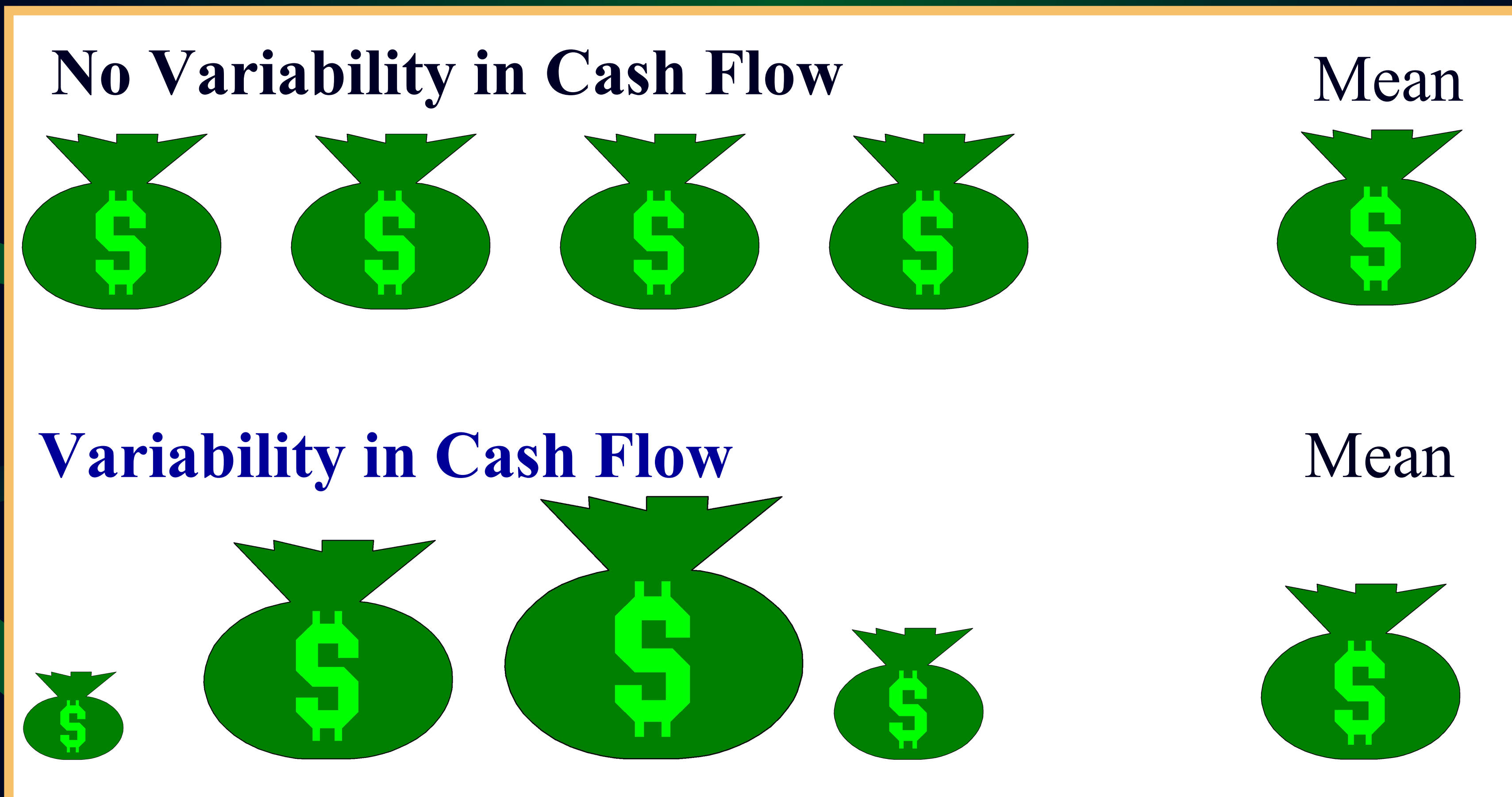
Quartiles: Example

- Ordered array: 106, 109, 114, 116, 121, 122, 125, 129
- Q_1: $$i = \frac{25}{100}(8) = 2 \qquad Q_1 = \frac{109+114}{2} = 111.5$$
- Q_2: $$i = \frac{50}{100}(8) = 4 \qquad Q_2 = \frac{116+121}{2} = 118.5$$
- Q_3: $$i = \frac{75}{100}(8) = 6 \qquad Q_3 = \frac{122+125}{2} = 123.5$$

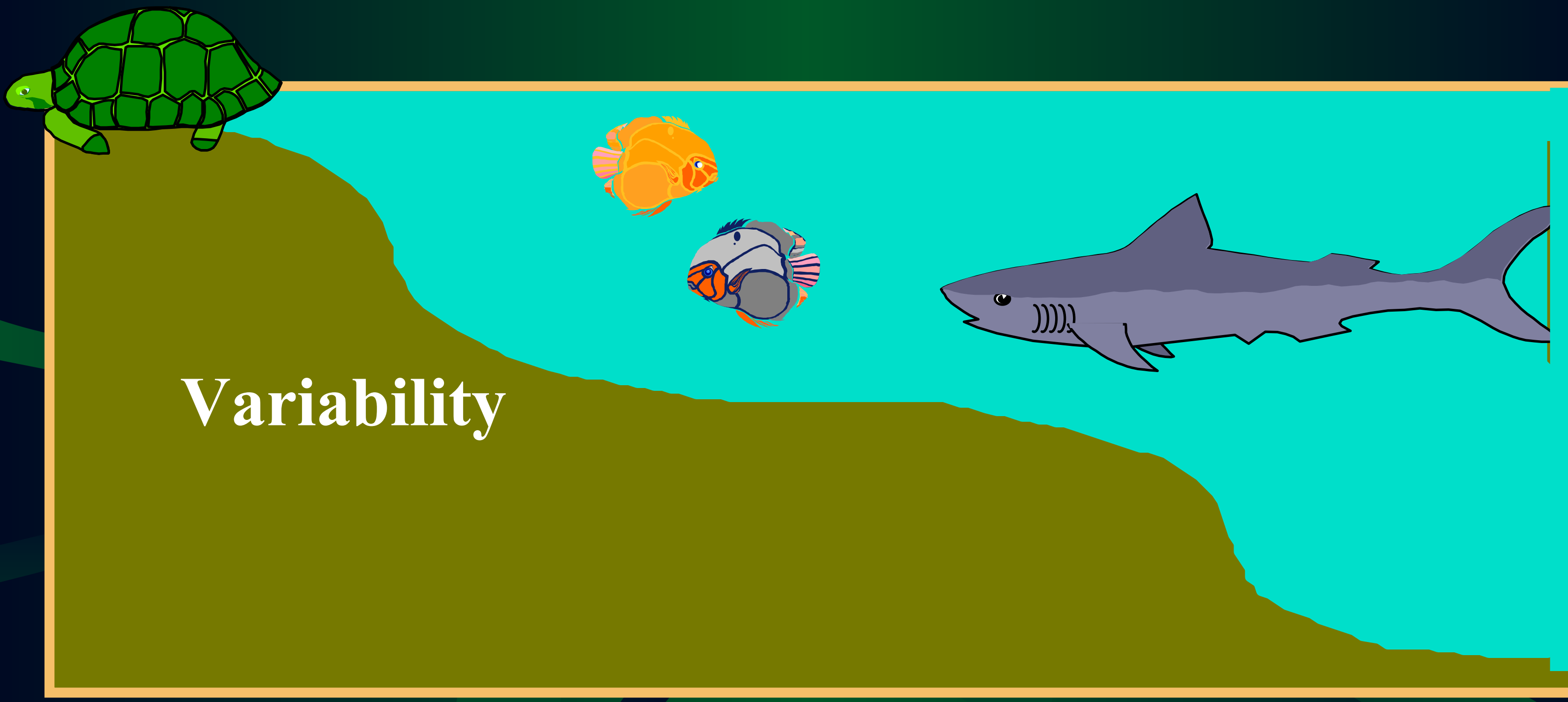
Measures of Variability

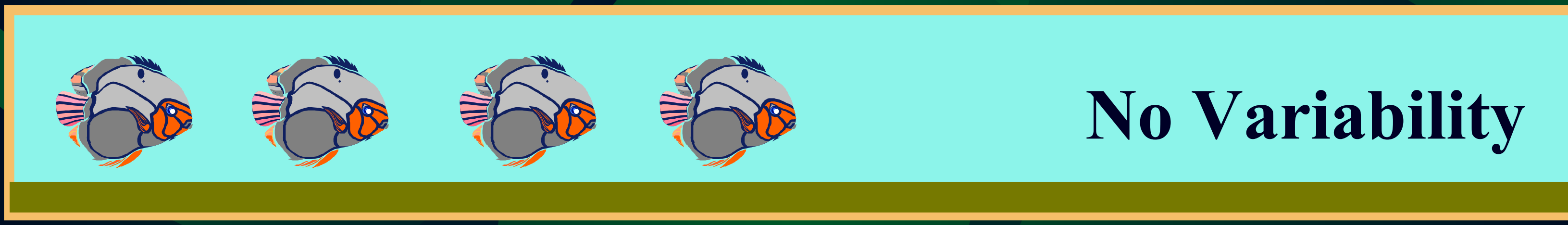
- Measures of variability describe the spread or the dispersion of a set of data.
- Common Measures of Variability
  - Range
  - Interquartile Range
  - Mean Absolute Deviation
  - Variance
  - Standard Deviation
  - Z scores
  - Coefficient of Variation

Variability



Variability





Range

- The difference between the largest and the smallest values in a set of data
- Simple to compute
- Ignores all data points except the two extremes
- Example: Range = Larg
  = 48 - 35 = 13

35	41	44	45
37	41	44	46
37	43	44	46
39	43	44	46
40	43	44	46
40	43	45	48

Interquartile Range

- Range of values between the first and third quartiles
- Range of the “middle half”
- Less influenced by extremes

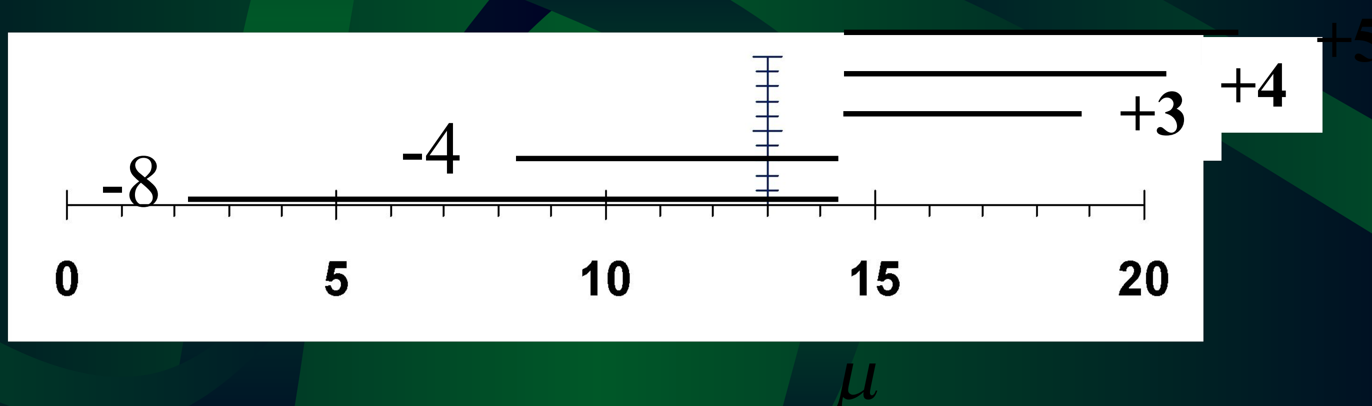
$$\boxed{Interquartile\,Range = Q_3 - Q_1}$$

Deviation from the Mean

- Data set: 5, 9, 16, 17, 18
- Mean:

$$\mu = \frac{\sum X}{N} = \frac{65}{5} = 13$$

- Deviations from the mean: -8, -4, 3, 4, 5



Mean Absolute Deviation

- Average of the absolute deviations from the mean

X	$X-\mu$	$\lvert X-\mu \rvert$
5	-8	+8
9	-4	+4
16	+3	+3
17	+4	+4
18	+5	+5
	0	24

$$M.A.D. = \frac{\sum \lvert X-\mu \rvert}{N} = \frac{24}{5} = 4.8$$

Population Variance

- Average of the squared deviations from the arithmetic mean

X	$X-\mu$	$(X-\mu)^2$
5	-8	64
9	-4	16
16	+3	9
17	+4	16
18	+5	25
	0	130

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N} = \frac{130}{5} = 26.0$$

Population Standard Deviation

- Square root of the variance

X	$X-\mu$	$(X-\mu)^2$
5	-8	64
9	-4	16
16	+3	9
17	+4	16
18	+5	25
	0	130

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N} = \frac{130}{5} = 26.0$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{26.0} = 5.1$$

Empirical Rule

- Data are normally distributed (or approximately normal)

Distance from the Mean	Percentage of Values Falling Within Distance
$\mu \pm 1\sigma$	68
$\mu \pm 2\sigma$	95
$\mu \pm 3\sigma$	99.7

Sample Variance

- Average of the <u>squared</u> deviations from the arithmetic mean

X	$X-\overline{X}$	$(X-\overline{X})^2$
2,398	625	390,625
1,844	71	5,041
1,539	-234	54,756
1,311	-462	213,444
7,092	0	663,866

$$S^2 = \frac{\sum (X-\overline{X})^2}{n-1} = \frac{663{,}866}{3} = 221{,}288.67$$

Sample Standard Deviation

- Square root of the sample variance

X	$X - \overline{X}$	$(X - \overline{X})^2$
2,398	625	390,625
1,844	71	5,041
1,539	-234	54,756
1,311	-462	213,444
7,092	0	663,866

$$S^2 = \frac{\Sigma(X - \overline{X})^2}{n-1} = \frac{663{,}866}{3} = 221{,}288.67$$

$$S = \sqrt{S^2} = \sqrt{221{,}288.67} = 470.41$$

Coefficient of Variation

- Ratio of the standard deviation to the mean, expressed as a percentage
- Measurement of <u>relative</u> dispersion

$$C.V. = \frac{\sigma}{\mu}(100)$$

Coefficient of Variation

$$\mu_1 = 29$$

$$\sigma_1 = 4.6$$

$$C.V._{1} = \frac{\sigma_1}{\mu_1}(100)$$

$$= \frac{4.6}{29}(100)$$

$$= 15.86$$

$$\mu_2 = 84$$

$$\sigma_2 = 10$$

$$C.V._{2} = \frac{\sigma_2}{\mu_2}(100)$$

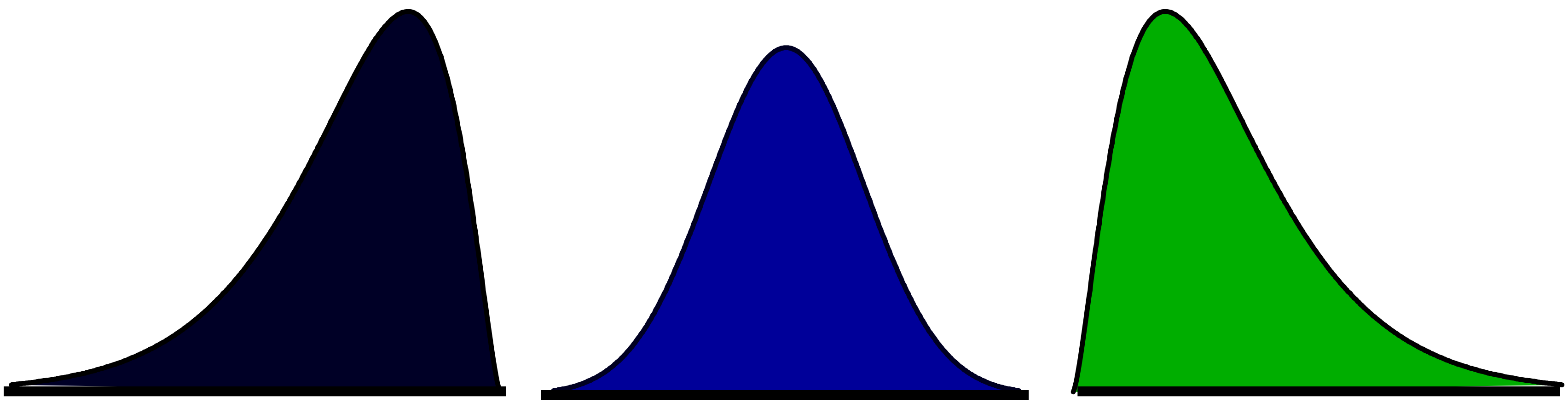
$$= \frac{10}{84}(100)$$

$$= 11.90$$

Measures of Shape

- **Skewness**
  – Absence of symmetry
  – Extreme values in one side of a distribution
- **Kurtosis**
  – Peakedness of a distribution
- **Box and Whisker Plots**
  – Graphic display of a distribution
  – Reveals skewness

Skewness

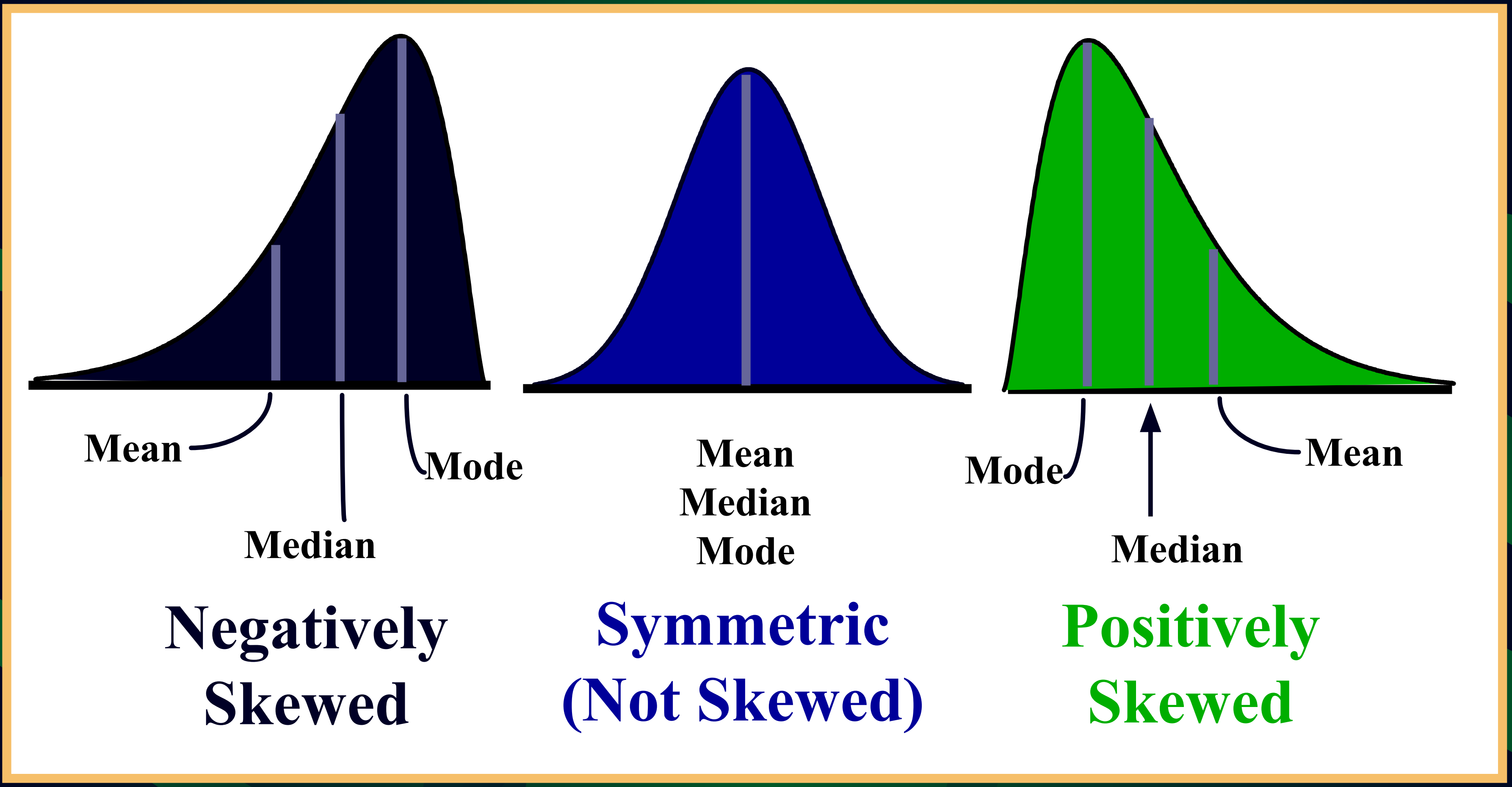


Negatively Skewed

Symmetric (Not Skewed)

Positively Skewed

Skewness



Mean
Median
Mode
Negatively Skewed
Mean
Median
Mode
Symmetric (Not Skewed)
Mode
Median
Mean
Positively Skewed

Coefficient of Skewness

- Summary measure for skewness

$$S = \frac{3(\mu - M_d)}{\sigma}$$

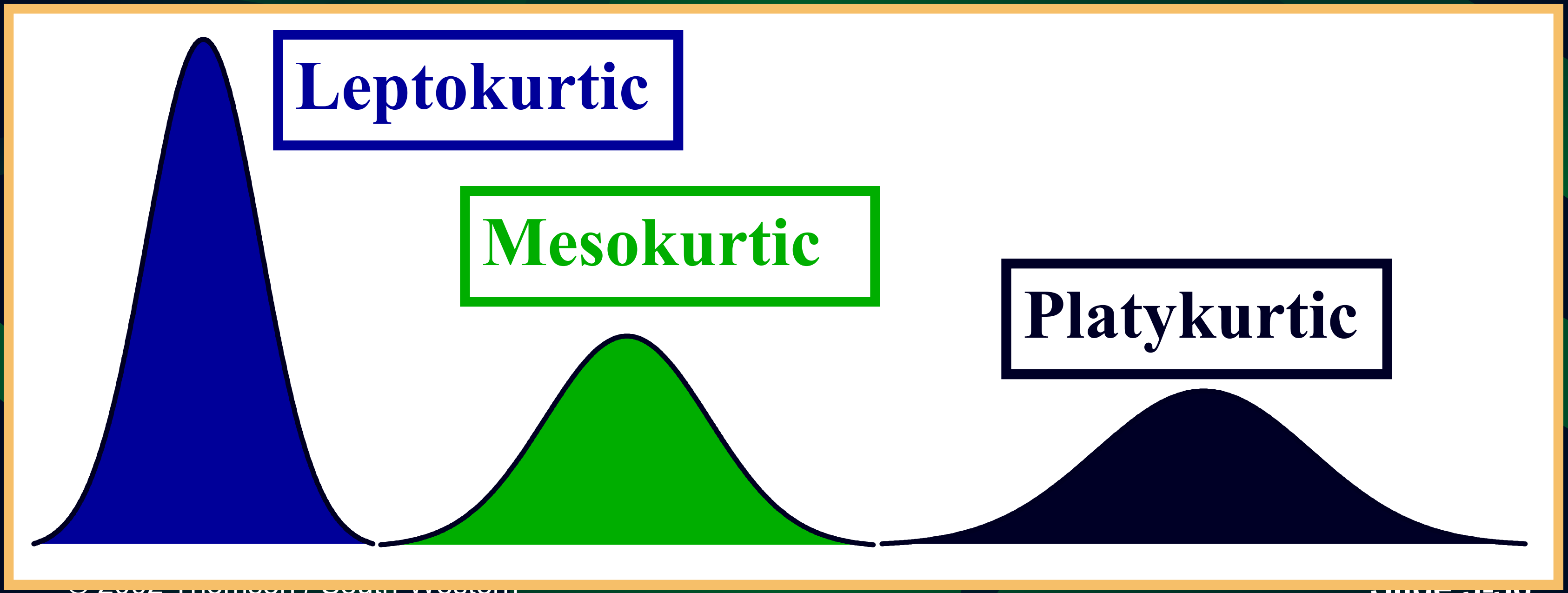
- If S < 0, the distribution is <u>negatively skewed</u> (skewed to the left).
- If S = 0, the distribution is <u>symmetric</u> (not skewed).
- If S > 0, the distribution is <u>positively skewed</u> (skewed to the right).

Coefficient of Skewness

$\mu_1 = 23$	$\mu_2 = 26$	$\mu_3 = 29$
$M_{d1} = 26$	$M_{d2} = 26$	$M_{d3} = 26$
$\sigma_1 = 12.3$	$\sigma_2 = 12.3$	$\sigma_3 = 12.3$
$S_1 = \dfrac{3(\mu_1 - M_{d1})}{\sigma_1}$	$S_2 = \dfrac{3(\mu_2 - M_{d2})}{\sigma_2}$	$S_3 = \dfrac{3(\mu_3 - M_{d3})}{\sigma_3}$
$= \dfrac{3(23-26)}{12.3}$	$= \dfrac{3(26-26)}{12.3}$	$= \dfrac{3(29-26)}{12.3}$
$= -0.73$	$= 0$	$= +0.73$

Kurtosis

- Peakedness of a distribution
  - Leptokurtic: high and thin
  - Mesokurtic: normal in shape
  - Platykurtic: flat and spread out



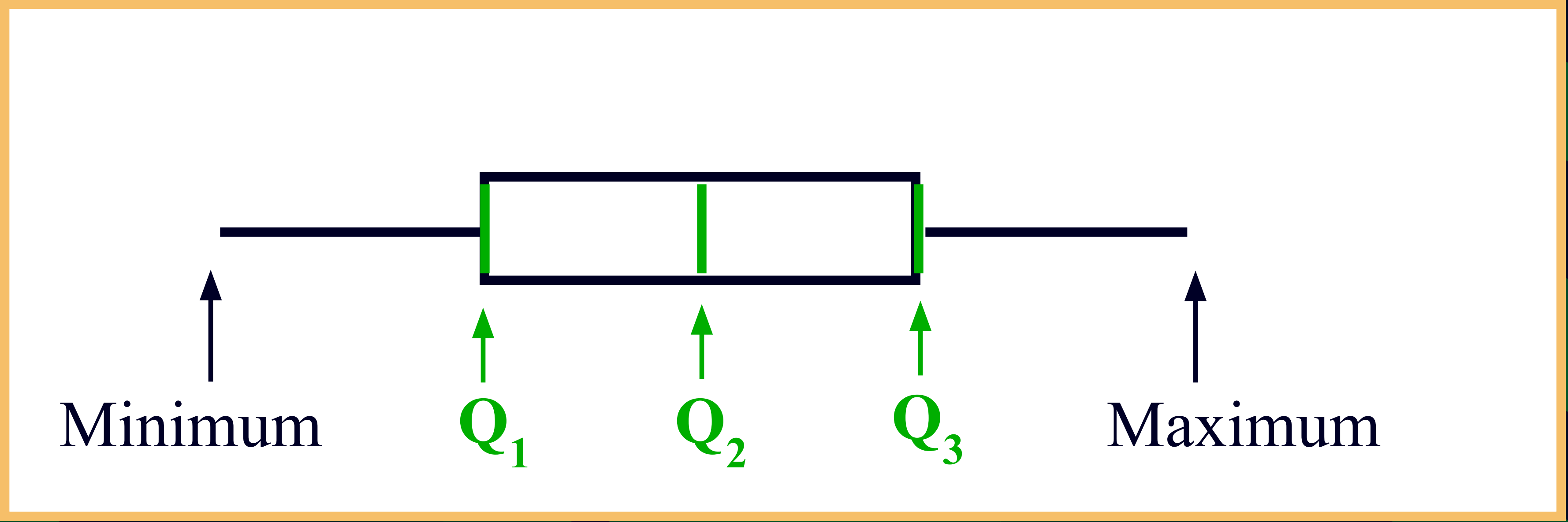
Box and Whisker Plot

- Five specific values are used:
  - Median, Q_2
  - First quartile, Q_1
  - Third quartile, Q_3
  - Minimum value in the data set
  - Maximum value in the data set

Box and Whisker Plot, *continued*

- **Inner Fences**
  - $IQR = Q_3 - Q_1$
  - Lower inner fence = $Q_1 - 1.5\ IQR$
  - Upper inner fence = $Q_3 + 1.5\ IQR$
- **Outer Fences**
  - Lower outer fence = $Q_1 - 3.0\ IQR$
  - Upper outer fence = $Q_3 + 3.0\ IQR$

Box and Whisker Plot



Skewness: Box and Whisker Plots, and Coefficient of Skewness