1.a) $f(x) = \sqrt{x+1}$

We know,

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

$$= \frac{(\sqrt{x+h+1} - \sqrt{x+1})(\sqrt{x+h+1} + \sqrt{x+1})}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \frac{(\sqrt{x+h+1})^2 - (\sqrt{x+1})^2}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \frac{x+h+1-x-1}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$$

b) $f(x) = \dfrac{3x}{x-4}$

We know,

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{\dfrac{3(x+h)}{x+h-4} - \dfrac{3x}{x-4}}{h}$$

$$= \frac{\dfrac{3(x+h)(x-4) - 3x(x+h-4)}{(x+h-4)(x-4)}}{h}$$

$$= \frac{\dfrac{3x^2 - 12x + 3xh - 12h - 3x^2 - 3xh + 12x}{(x+h-4)(x-4)}}{h}$$

$$= \frac{-12h}{h(x+h-4)(x-4)}$$

$$= \frac{-12}{(x+h-4)(x-4)}$$

(Ans).

2\. a) $f(x) = 2x^2 - 3x$, $g(x) = \dfrac{4}{1-x}$

$$
\begin{aligned}
f \circ g &= f(g(x)) \\
&= f\left(\frac{4}{1-x}\right) \\
&= 2 \cdot \left(\frac{4}{1-x}\right)^2 - 3 \cdot \frac{4}{1-x} \\
&= \frac{32}{(1-x)^2} - \frac{12}{1-x} \\
&= \frac{32 - 12(1-x)}{(1-x)^2} \\
&= \frac{32 - 12 + 12x}{(1-x)^2} \\
&= \frac{20 + 12x}{(1-x)^2}
\end{aligned}
$$

$$g \circ f = g(f(x))$$

$$= g(2x^2 - 3x)$$

$$= \frac{4}{1 - (2x^2 - 3x)}$$

$$= \frac{4}{1 - 2x^2 + 3x}$$

(Ans).

b) $f(x) = \sqrt{x}$, $g(x) = x - 4$

$fog = f(g(x))$

$= f(x-4)$

$= \sqrt{x-4}$

$gof = g(f(x))$

$= g(\sqrt{x})$

$= \sqrt{x} - 4$

Ans.

c) $f(x) = \dfrac{x}{x-1}$, $g(x) = \dfrac{2x-1}{x}$

$$
\begin{aligned}
f \circ g &= f(g(x)) \\
&= f\left(\frac{2x-1}{x}\right) \\
&= \frac{\frac{2x-1}{x}}{\frac{2x-1}{x} - 1} \\
&= \frac{\frac{2x-1}{x}}{\frac{2x-1-x}{x}} \\
&= \frac{2x-1}{x-1}
\end{aligned}
$$

$$g\circ f = g(f(x))$$

$$= g\left(\frac{x}{x-1}\right)$$

$$= \frac{2\left(\frac{x}{x-1}\right) - 4}{\frac{x}{x-1}}$$

$$= \frac{\frac{2x - 4x + 4}{x-1}}{\frac{x}{x-1}}$$

$$= \frac{-2x + 4}{x}$$

(Ans)

3. a) $p(x) = \sqrt{\frac{2}{x-1}}$

$$= \frac{\sqrt{2}}{\sqrt{x-1}}$$

Here,

$$x - 1 > 0$$

or, $x > 1$

Domain: $(1, \alpha)$

Here,

$$y = \sqrt{\frac{2}{x-1}}$$

or, $y^2 = \frac{2}{x-1}$

or, $x - 1 = \frac{2}{y^2}$

or, $x = \frac{2}{y^2} + 1$

or, $x = \frac{2+y^2}{y^2}$

or, $f^{-1}(x) = \frac{2+x^2}{x^2}$

Range: $\mathbb{R} \setminus \{0\}$

b) $p(x) = \frac{-x}{\sqrt{-x-2}}$

Here,

$-x-2 > 0$

or, $-x > 2$

or, $x < -2$

Domain : $(-\alpha, -2)$

c) $f(x) = \begin{cases} 3 & \text{if } x \le -2 \\ -\frac{1}{3}x + \frac{7}{3} & \text{"} \quad -2 < x < 1 \\ -3x+5 & \text{"} \quad x \geq 1 \end{cases}$

Domain : $(-\alpha, -2] \cup (-2, 1) \cup [1, \alpha)$

$= (-\alpha, \alpha)$

Range : $\{3\} \cup (3, 2) \cup [2, \alpha)$

$= \boxed{\cancel{[2, \alpha)}} \quad (-\alpha, 3]$

d) $f(x) = \begin{cases} \frac{2}{3}x + 4 & \text{if } x < 0 \\ -\frac{1}{2}x + 3 & \text{"} \quad 0 < x < 2 \\ -\frac{1}{2}x & \text{"} \quad x > 2 \end{cases}$

Domain $= (-\alpha, 0) \cup (0, 2) \cup (2, \alpha)$

Range $= (-\alpha, 4) \cup (3, 2) \cup (-1, -\alpha)$

$= (-\alpha, 4)$

1. a) $f(x) = \dfrac{2x-1}{x}$

or, $y = \dfrac{2x-1}{x}$

or, $xy = 2x - 1$

or, $2x - xy = 1$

or, $x(2-y) = 1$

or, $x = \dfrac{1}{2-y}$

or, $f^{-1}(x) = \dfrac{1}{2-x}$

b) $g(x) = \frac{4}{1-x}$

or, $y = \frac{4}{1-x}$

or, $y - xy = 4$

or, $xy = y - 4$

or, $x = \frac{y-4}{y}$

or, $f^{-1}(x) = \frac{x-4}{x}$

5. a) $f(x) = \dfrac{2x}{x-2}$

Here, $f\left(\dfrac{1}{2}\right) = \dfrac{2 \cdot \frac{1}{2}}{\frac{1}{2} - 2}$

$= \dfrac{1}{\frac{-3}{2}}$

$= -\dfrac{2}{3}$

So, the point $\left(\dfrac{1}{2}, -\dfrac{2}{3}\right)$ is on the graph.

b) $f(4) = \dfrac{2 \cdot 4}{4-2}$

$= \dfrac{8}{2}$

$= 4$

The point $(4, 4)$ is on the graph.

c) Here,

$$\frac{2x}{x-2} = 1$$

or, $2x = x-2$

or, $x = -2$

So the point $(-2, 1)$ is on the graph.

d) To get x-intercept we will take $y = 0$.

So, $\frac{2x}{x-2} = 0$

or, $2x = 0$

or, $x = 0$

So there is no x-intercept.

c) To get y-intercept we will take

$x = 0$.

$$y = \frac{2x}{x-2}$$

$$= \frac{2 \cdot 0}{0-2}$$

$$= 0$$

So there is no y-intercept.

6. $f(x) = 2x^2 + 3$, $g(x) = 4x^3 + 1$

a) ∴ We know,

$(f+g)(x) = f(x) + g(x)$

$= 2x^2 + 3 + 4x^3 + 1$

$= 4x^3 + 2x^2 + 4$

$\therefore (f+g)(5) = 4 \times (5)^3 + 2(5)^2 + 4$

$= 554$

b) ∴ We know,

$(f \times g)(x) = f(x) \times g(x)$

$= (2x^2+3)(4x^3+1)$

$= 8x^5 + 2x^2 + 12x^3 + 3$

$= 8x^5 + 12x^3 + 2x^2 + 3$

7. a) $p(x) = \frac{-x}{\sqrt{-x-2}}$

To check ~~whe~~ whether it is an even function or odd function or neither replace x by $-x$.

$$p(-x) = \frac{-(-x)}{\sqrt{-(-x)-2}}$$

$$= \frac{x}{\sqrt{x-2}}$$

$p(-x) \neq p(x)$ so it is not an even function.

$p(x) \neq -p(x)$ so it is not an odd function.

b) $f(x) = 4x^3 + 1$

Replace x by $-x$

$f(x) = 4(-x)^3 + 1$

$= -4x^3 + 1$

$f(-x) \neq f(x)$ so it is not an even function.

$f(-x) \neq -f(x)$ so it is not an odd function.