1.

The standard treatment for a particular disease has a $\frac{2}{5}$ probability of success. A certain doctor has undertaken research in this area and has produced a new drug which has been successful with 11 out of 20 patients. The doctor claims that the new drug represents an improvement on the standard treatment.

Test, at the 5% significance level, the claim made by the doctor.

2.

A polling organisation claims that the support for a particular candidate is 35%. It is revealed that the candidate will pledge to support local charities if elected. The polling organisation think that the level of support will go up as a result. It takes a new poll of 50 voters.

3.

Over a long period of time it has been found that in Enrico's restaurant the ratio of non-vegetarian to vegetarian meals is 2 to 1. In Manuel's restaurant in a random sample of 10 people ordering meals, 1 ordered a vegetarian meal. Using a 5% level of significance, test whether or not the proportion of people eating vegetarian meals in Manuel's restaurant is different to that in Enrico's restaurant.

Kurtosis

The Kurtosis of a given set of data values can be calculated using the formula,

$$\textbf{Kurtosis} = \sum(X_i - \bar{X})^4 / n\sigma^4.$$

- $\bar{X}$ denotes the mean.
- σ denotes the standard deviation.

The Kurtosis of distribution gives us some idea about the shape of the distribution. Depending on the value of the Kurtosis we can classify our data as leptokurtic, mesokurtic, and platykurtic.

Types of kurtosis

Distributions can be categorized into three groups based on their kurtosis:

	Category		
	Mesokurtic	**Platykurtic**	**Leptokurtic**
Tailedness	Medium-tailed	Thin-tailed	Fat-tailed
Outlier frequency	Medium	Low	High
Kurtosis	Moderate (3) ✓	Low (< 3) ✓	High (> 3) ✓
Excess kurtosis	0	Negative	Positive
Example distribution	Normal	Uniform	Laplace

Example 1: Kurtosis for Ungrouped Data

Consider the following set of ungrouped data values.

23, 34, 38, 47, 59, 63, 84.

We first calculate the values of the mean and the standard deviation.

Mean $\bar{X} = \Sigma X_i/n = (23+34+38+47+59+63+84)/7 = 348/7 = 49.7143$.

X_i	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$(X_i - \bar{X})^3$	$(X_i - \bar{X})^4$
	$(X_i - 49.7143)$	$(X_i - 49.7143)^2$	$(X_i - 49.7143)^3$	$(X_i - 49.7143)^4$
23	-26.714	713.653	-19065	509301
34	-15.714	246.939	-3880.5	60978.8
38	-11.714	137.225	-1607.5	18830.6
47	-2.7143	7.3673	-19.997	54.2778
59	9.2857	86.2245	800.656	7434.66
63	13.2857	176.51	2345.06	31155.9
84	34.2857	1175.51	40303.2	1381824
TOTAL = 348	TOTAL = 0	TOTAL = 2543.43	TOTAL = 18876.2	TOTAL = 2009579

$$\text{Standard Deviation } \sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$$

Standard Deviation $\sigma = \sqrt{(\Sigma(X_i - \bar{X})^2/n)} = \sqrt{(2543.3/7)} = 19.0617$.

We now calculate the Kurtosis using the formula,

$$\text{Kurtosis } = \frac{\sum(x_i - \bar{x})^4}{n\sigma^4}$$

Kurtosis $= \Sigma(X_i - \bar{X})^4/n\sigma^4 = 2009579/(7*19.0617^4) = 2.1745$.

Example 2: Grouped Data Kurtosis Calculation

Consider the following set of data values given in the form of a grouped frequency distribution table.

Class Intervals	Frequency
0-5	2
5-10	3
10-15	1
15-20	4
20-25	5
25-30	9
30-35	6
35-40	12
40-45	8
45-50	7

We calculate the values required to calculate the mean and the standard deviation,

Classes	Class Mark X_i	f_i	f_i*X_i	$(X_i-\bar{X})$	$f_i(X_i-\bar{X})^2$	$f_i(X_i-\bar{X})^4$
0 – 5	2.5	2	5	-28.86	1665.76	1387376
5-10	7.5	3	22.5	-23.86	1707.85	972249
10-15	12.5	1	12.5	-18.86	355.686	126513
15 – 20	17.5	4	70	-13.86	768.36	147594
20 – 25	22.5	5	112.5	-8.8596	392.467	30806.1
25 – 30	27.5	9	247.5	-3.8596	134.072	1997.26
30 – 35	32.5	6	195	1.1404	7.8024	10.1462
35 – 40	37.5	12	450	6.1404	452.447	17059
40 – 45	42.5	8	340	11.1404	992.859	123221
45 – 50	47.5	7	332.5	16.1404	1823.58	475062
		n=57	Σf_iX_i=1787.5	TOTAL =0	$\Sigma f_i(X_i-\bar{X})^2$ =8300.877	$\Sigma f_i(X_i-\bar{X})^4$ =3281887

$$\text{Mean} = \Sigma f_iX_i/\Sigma f_i = \Sigma f_iX_i/n = 1787.5/57 = 31.3596.$$

$$\text{Standard Deviation } \sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n}}$$

Standard Deviation $\sigma = \sqrt{(\Sigma f_i(X_i - \bar{X})^2/n)} = \sqrt{(8300.877/57)} = 12.0677$.

The formula for calculating kurtosis for a set of grouped data values is as follows,

Kurtosis $= \Sigma f_i(X_i - \bar{X})^4/n\sigma^4 = 3281887.0786/(57*12.0677^4) = 2.7149$.