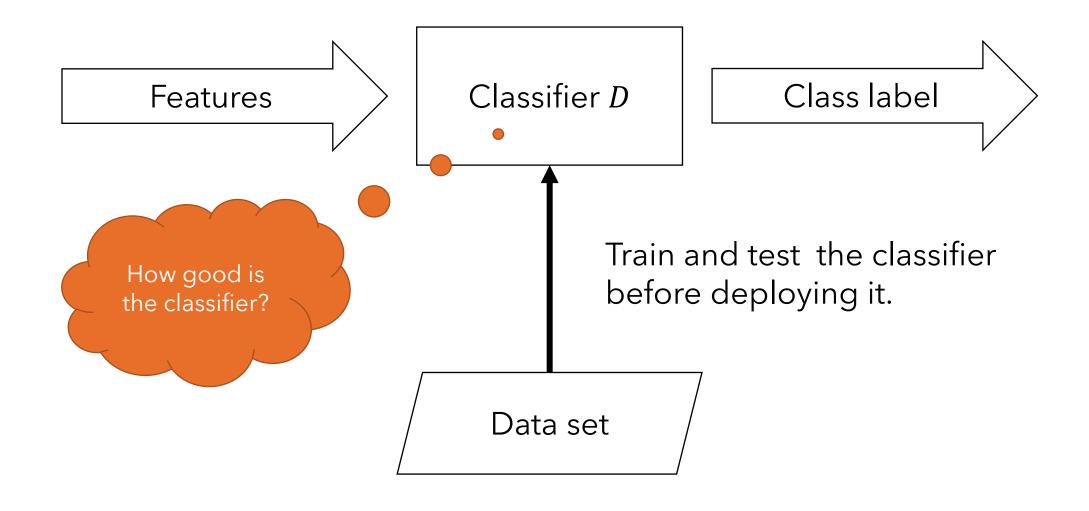


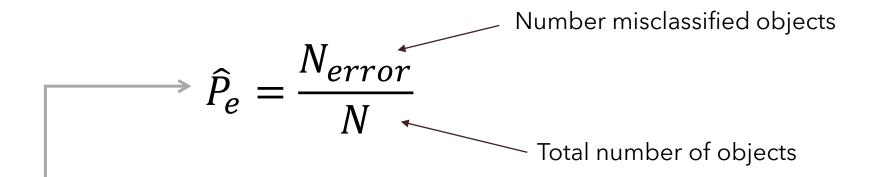
- Classification error
- What is the confusion matrix?
- Examples



How do we know how good classifier D is?

We can estimate **the probability of error** of *D* on unseen data.

Calculate this probability using a data set with N objects:



Notice the "hat". This means that we don't know the true value! We have only an estimate. The true value is usually denoted by P_e . (But we will be using P_e for our estimate for simplicity of notation.)

The probability of error P_e

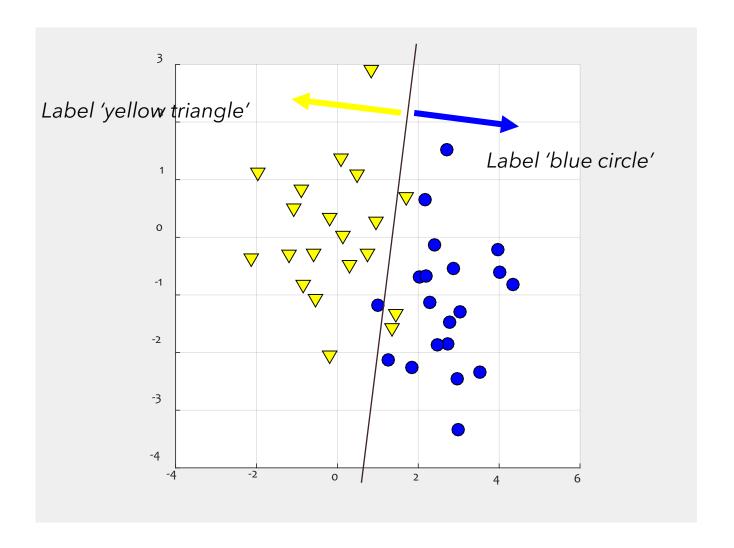
- **= error rate** of the classifier
- **= error** of the classifier

The probability of correct classification P_c

= accuracy of the classifier

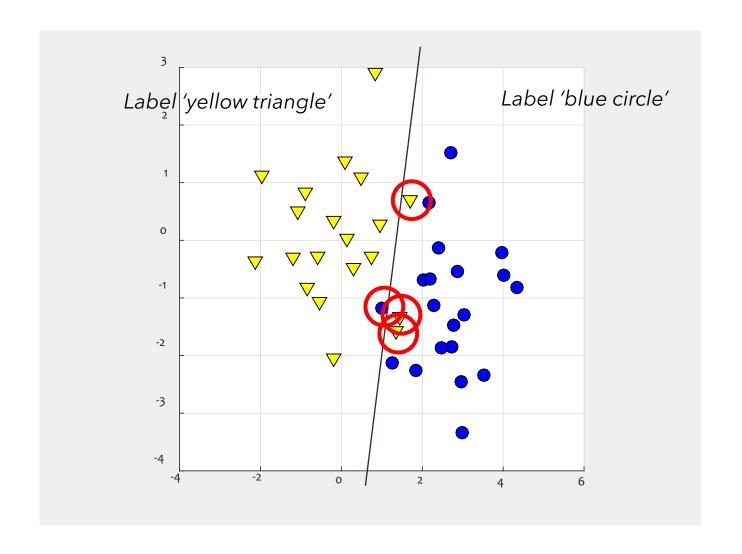
$$P_c = 1 - P_e = \frac{N_{correct}}{N} = 1 - \frac{N_{error}}{N}$$

Classifier D_1 defined by the linear discriminant function



How many errors does this classifier commit for the given data set?

There are 40 objects in total. What is the estimated error rate of this classifier?



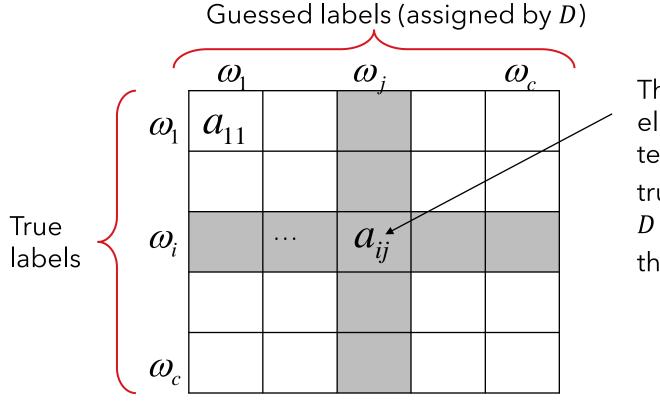
How many errors does this classifier commit for the given data set?

4

There are 40 objects in total. What is the estimated error rate of this classifier?

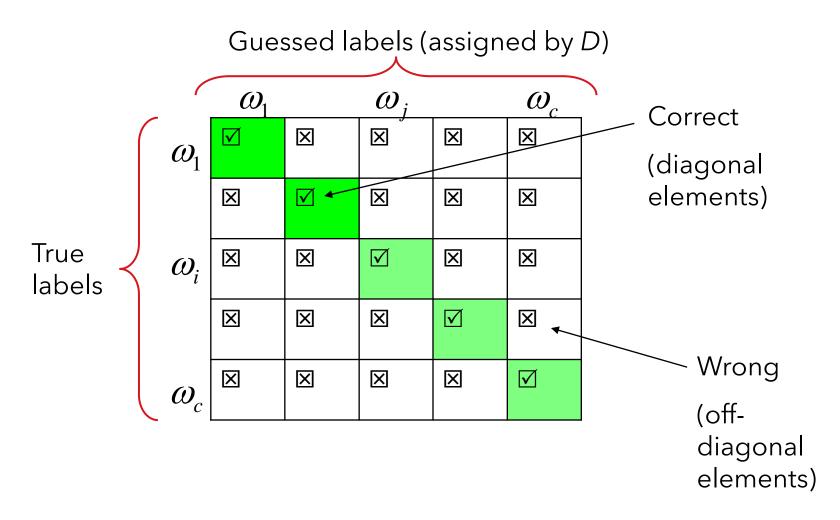
Error rate = 4/40 = 0.1



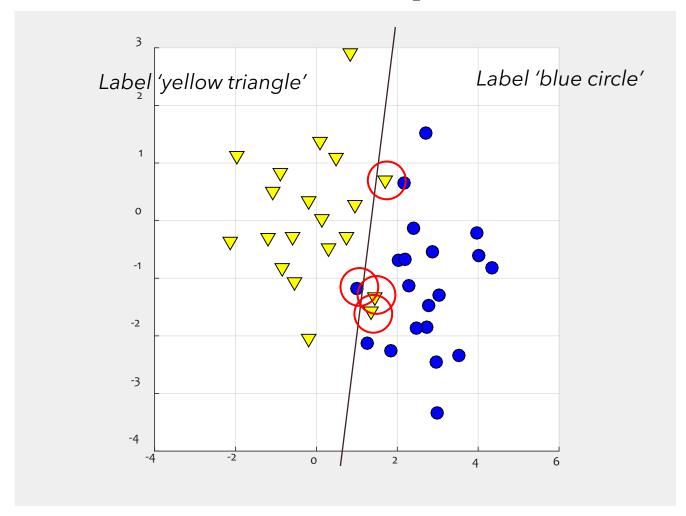


The number of elements of the testing set whose true label is ω_i and D has assigned them to ω_i









c = 2 classes



Blue

19

n = 2 features

N = 40 objects

(20 from each class)

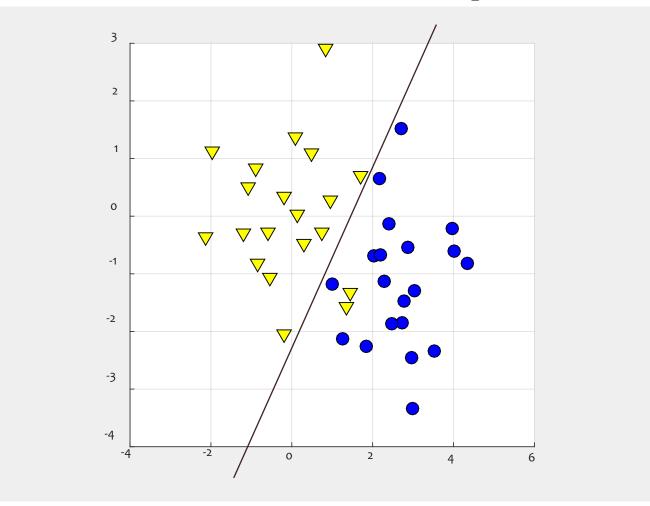
Assigned

D_1	Yellow	
Yellow	17	
Blue	1	

True

 $P_D = 4/40 = 0.10$ (probability of error)





c = 2 classes



n = 2 features

N = 40 objects

(20 from each class)

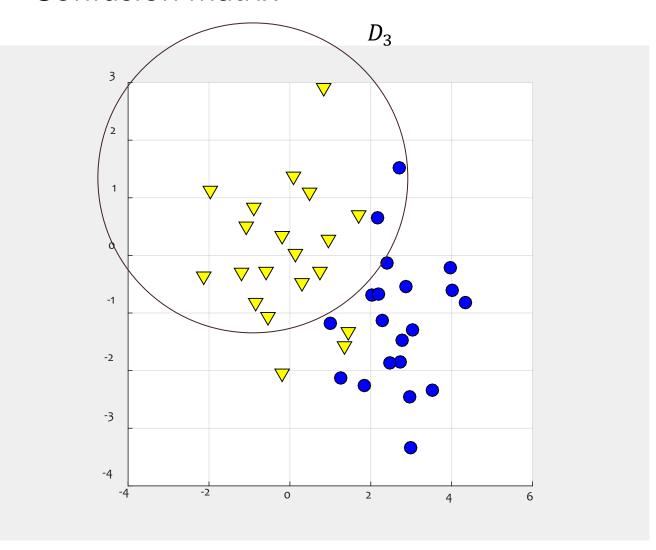
Assigned

True

D_2	Yellow	Blue	
Yellow	18	2	
Blue	0	20	

$$P_D = 2/40 = 0.05$$

(probability of error)



c = 2 classes



n = 2 features

N = 40 objects

(20 from each class)

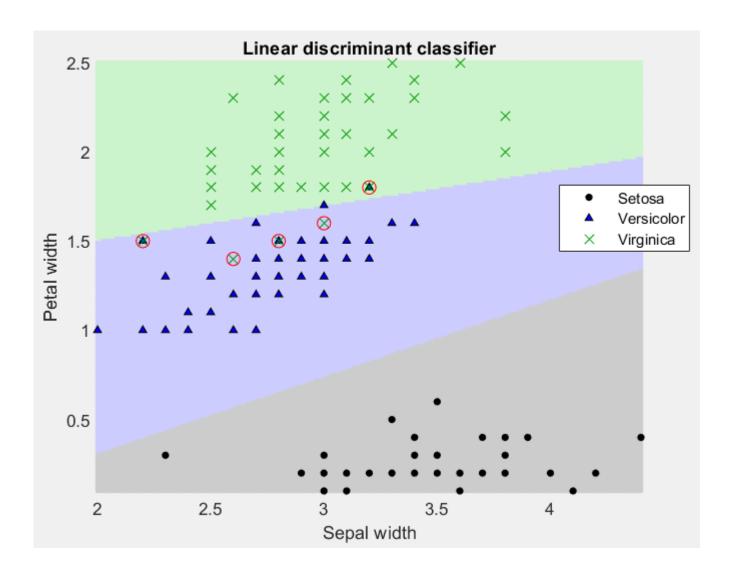
Assigned

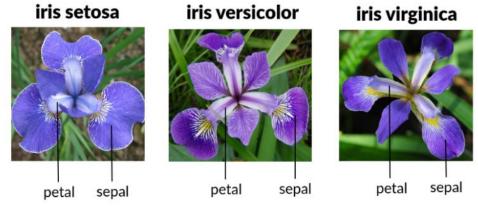
D_3	Yellow	Blue
Yellow	17	3
Blue	2	18

 $P_D = 5/40 = 0.125$ (probability of error)

Accuracy = (17+18)/40 = 0.875

True





	Setosa	Versicolor	Virginica
Setosa	50	0	0
Versicolor	0	49	1
Virginica	0	4	46





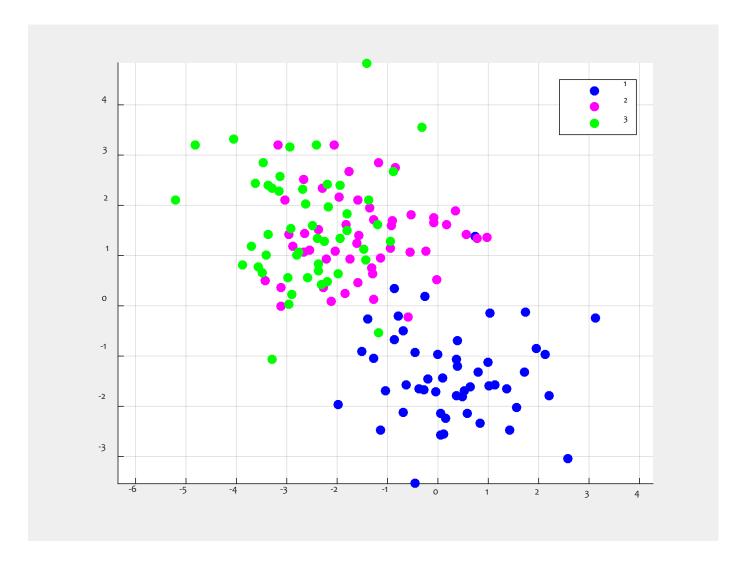
Assigned

rue

	1	2	3
1	10	1	2
2	0	8	2
3	2	2	9

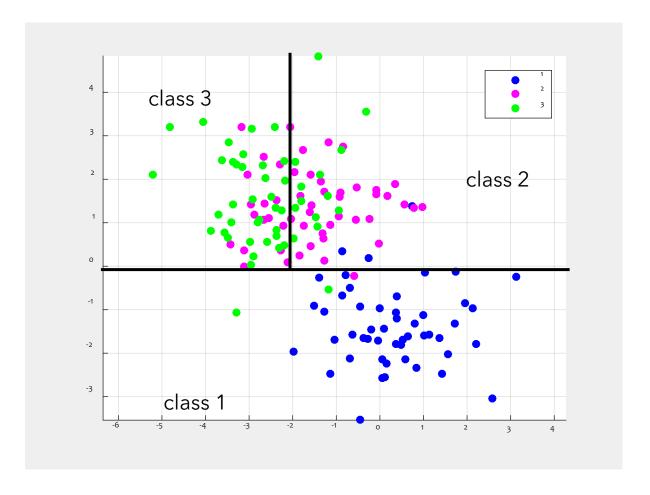
- Q1. How many points are there in Z? (N = ?)
- Q2. How many points from Z have label Class 2?
- Q3. Estimate the accuracy of the classifier.
- Q4. If the classifier predicts class label 2, what is the probability that this label is correct?

Q5. Write Python code to create three Gaussian classes (see Lab 1), two of which are highly overlapping, and the third is away from both as in the figure below:



Q6. Based on the way you picked the class means, program a rule-based classifier which splits the 2D feature space as shown below:

Note that the exact positions of the horizontal and the vertical lines depend on YOUR chosen class means



Q7. Write a Python function called confusion_matrix which will take two input arguments: the true labels and the assigned labels, and will output the confusion matrix.

Q8. Subsequently, apply your function to obtain the confusion matrix for your data and classifier. Calculate the <u>accuracy</u> and the <u>error rate of your classifier</u>.

Q9. Generate a second random data set from the same distribution as in Problem 5. Classify the data according to your rule-based classifier. Calculate the confusion matrix, the accuracy and the error of your classifier with the new data set.

Answers to some questions:

Q1. Add all elements.
$$N = |Z| = 36$$

Q2. Sum up row # 2 (true labels = 2).
$$0 + 8 + 2 = 10$$

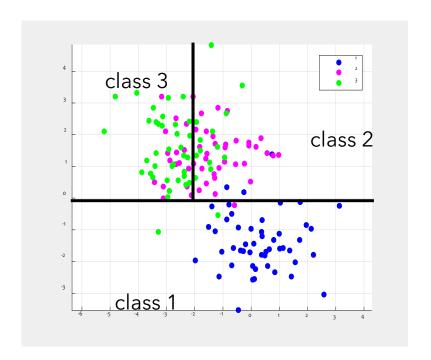
Q3. Sum up the diagonal entries and divide by N.

$$P_{acc} = \frac{10+8+9}{36} = \frac{27}{36} = 0.75$$

Q4. Conditional probability. The condition is "assigned label is 2". Total outcomes favourable for this event is the sum of column #2: 1 + 8 + 2 = 11. Then the probability of being correct is number of correctly recognised objects from class 2 divided by this total:

$$P(Correct|Assigned\ label\ is\ 2) = \frac{8}{11} = 0.73$$

Q6. In any programming language:



If
$$x_2 < -0.1$$
 then class 1

Else if $x_1 > -2$ then class 2

Else, class 3