Ques: 01

Obtain the fourier expansion of a function $f(x)$ of period 2π such that -

$$f(x) = \begin{cases} 1, & -\pi < x < 0 \\ 0, & 0 < x < \pi \end{cases}$$

(Ans): $a_0 = 1, a_n = 0, b_n = \frac{-1}{n\pi}[1-(-1)^n]$

Ques: 02

Expand the Fourier series for periodic function $f(x)$ of period 2π such that -

$$f(x) = x^2 \text{ in the interval } -\pi < x < \pi$$

(Ans): $a_0 = \frac{2\pi^2}{3}, a_n = \frac{4}{n^2}(-1)^n, b_n = 0$

Ques:03

Expand the fourier series for periodic function $f(x)$ of period 2π such that -

$f(x) = x$ in the interval $-\pi < x < \pi$

(Ans): $a_0 = 0, a_n = 0, b_n = \frac{-2}{n}(-1)^n$

Ques:04

Let, $f(x)$ be a function of period 2π such that -

$$f(x) = \begin{cases} 0; & -\pi < x < 0 \\ x; & 0 < x < \pi \end{cases}$$

limit $-\pi$ to π

Find the fourier expansion of: $f(x)$.

(Ans): $a_0 = \frac{\pi}{2}, a_n = \frac{1}{n^2\pi}[(-1)^n - 1], b_n = \frac{-(-1)^n}{n}$

Ques: 05

Find the fourier series of $f(x) = x^2$ in $(0,2\pi)$ or $0 < x < 2\pi$.

(Ans): $a_0 = \frac{8\pi^2}{3}, a_n = \frac{4}{n^2}, b_n = \frac{-4\pi}{n}$

Ques:06

Find the fourier series of a function $f(x)$ of period 2π such that -

$$f(x) = \begin{cases} x; & 0 < x < \pi \\ 0; & \pi < x < 2\pi \end{cases}$$

(Ans): $a_0 = \frac{\pi}{2}, a_n = \frac{1}{n^2\pi}[(-1)^n - 1], b_n = \frac{-(-1)^n}{n}$

Ques:07

Find the fourier series of function $f(x) = \frac{x}{2}$ over the interval $0 < x < 2\pi$

(Ans): $a_0 = \pi, a_n = 0, b_n = \frac{-1}{n}$

Ques: 08

Find $F\{f(x)\}$ or $F(s)$ if -

FT

$$f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

and hence evaluate, $\int_{-\infty}^{\infty} \frac{s\cos(s) - \sin(s)}{s^3} \cos(s/2) ds$

(Ans): $F(s) = \frac{-4(s\cos(s) - \sin(s))}{s^3}$

and, $\int_{-\infty}^{\infty} \frac{s\cos(s) - \sin(s)}{s^3} \cos(s/2) ds = \frac{-3\pi}{8}$

Ques: 09

FS

Find the fourier expansion of x^2 in $(0, a)$.

[Hint → Type-03 → Limit $(0,2c)$, here $(2c = a)$]

(Ans: $a_0 = \frac{2a^2}{3}, a_n = \frac{a^2}{n^2\pi^2}, b_n = \frac{-a^2}{n\pi}$)

Ques: 10

FS

Obtain fourier expansion of $f(x) = x^2$ in $(-l, l)$

[Hint ⟼ Type:04 $(-c$ to c)]

(Ans: $a_0 = \frac{2l^2}{3}, a_n = \frac{4l^2(-1)^n}{(n\pi)^2}, b_n = 0$)

Ques: 11

FST

Find $F_s\{f(x)\}$ if $f(x) = \begin{cases} \cos x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$

(Ans): $\frac{1}{2}\left[\frac{1-(-1)^{1+s}}{1+s}+\frac{(-1)^{1-s}-1}{1-s}\right]$

[Hint: Apply FST] & [Try to use cos $(n\pi)$ value while replacing limits]

Ques: 12

Find $f(x)$ if its cosine trans form is e^{-as} [Hint: Apply. IFCT]

(Ans): $\frac{a}{2\pi(a^2+x^2)}$

Ques: 13

Find Fourier Cosine Transform of -

$$f(x) = e^{-ax}$$

[Hint: Apply FCT]

(Ans). $\frac{a}{a^2+s^2}$

Ques. 14:

Find Laplace Transform of -

i) $L\{5t^7 - \cos\ 5t + 7e^{-3t}\}$

ii) $L\{t\cos\ t + e^{2t}5t^3 - t^2\}$

iii) $L\{t^2\sin\ t\}$

iv) $L\{e^{2t}t\cos\ 7t - 5t\cos\ t\}$

(Ans):

i) $\frac{25200}{s^8} - \frac{s}{s^2+25} + \frac{7}{s+3}$

ii) $\frac{s^2-1}{(s^2+1)^2} + \frac{30}{(s-2)^4} - \frac{2}{s^3}$

iii) $\frac{2(s^2-1)}{(s^2+1)^3}$

iv) $\frac{(s-2)^2-49}{\{(s-2)^2+49\}^2} - \frac{5(s^2-1)}{(s^2+1)^2}$

***Ques*. 15**:

Find Inverse La Place Transform of -

i) $L^{-1}\left\{\frac{3}{(s-2)^2+9}\right\}$
(Ans): $e^{2t}\sin\ 3t$

ii) $L^{-1}\left\{\frac{s+3}{(s+3)^2+4}\right\}$
(Ans): $e^{-3t}\cos\ 2t$

iii) $L^{-1}\left\{\frac{\Gamma 7}{(s-5)^7}\right\}$
(Ans): $e^{5t}t^6$

iv) $L^{-1}\left\{\frac{1}{s^2-8s+16}\right\}$
(Ans): $e^{4t}t$

v) $L^{-1}\left\{\frac{s}{s^2-4s+13}\right\}$
(Ans): $e^{2t}\cos\ 3t + \frac{2e^{2t}\sin\ 3t}{3}$