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# Project: A stochastic simulation system for protein aggregation

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## 1 Introduction

Diseases like Alzheimer and Parkinson disease are the result of proteins aggregating into large fractal structures that hinder the cell function or even destroy them. Understanding how aggregates are formed and change over time is important to understand when they become harmful and how maybe treatments affect aggregate formation.

The goal of this Internship is to work on the implementation of a simulation system required to study aggregation between proteins. This work is performed in collaboration with the Switch lab in the KU Leuven, who have an extensive expertise in studying aggregation and related diseases.

## 2 First 2 weeks (12th-23rd August)

(Maybe add the administrative part, introduction to the team members and all...) During the first two weeks of the intership, I researched the subject and mostly did a of state of the around the subject to have a baseline knowledge around the subject. !!!!!!!!!!!!!!! TO ADD MORE DETAILS !!!!!!!!!!!!!!!

## 3 Third week (26th-31st August)

Now having a basic understanding around the subject, I could refine my reasearch and focus on the specific papers discussing the problem in order to have a deeper understanding of the theory before starting any kind of implementation. !!!!!!!!!!!!!!! TO ADD MORE DETAILS !!!!!!!!!!!!!!!

## 4 Fourth week (2nd-6th September)

The goal of this week was : To implement a simple prototype (in python for now) of an exact numerical simulation method to simulate trajectories of discrete, stochastic systems.

### 4.1 Step 1 : Mathematical Descriptions of Chemical Processes

A coupled system of chemical reactions of the form :  $X_1 + X_2 \rightarrow X_3 + \dots$  states that one molecule from substance  $X_1$  reacts with one molecule of substance  $X_2$  to give one molecule of substance  $X_3$

#### 4.1.1 Hypothesis :

The solution is well mixed  $\rightarrow$  nonreactive collisions occur far more than reactive collisions  $\rightarrow$  fast dynamics of motion can be neglected  $\rightarrow$  Can use the number of each kind of molecule to represent the system.

**Theorem 1.** *The probability that a certain reaction  $\mu$  will take place in the next instant of time  $dt$  is given by  $a_\mu dt + o(dt)$ , where  $a_\mu$  is independent of  $dt$ .*

## 4.2 Step 2 : The stochastic framework

1. We have a set of reactions



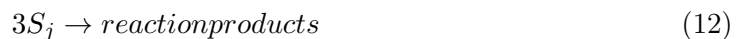
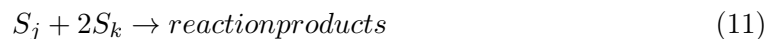
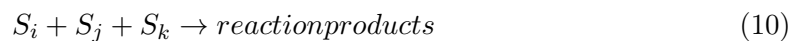
2. The propensities of the reactions are given by  $k_1, k_2, \dots, k_5$
3. The probability that a given molecule  $A$  reacts with a given molecule  $B$  in a small time  $dt$  is  $k_1 dt + o(dt)$ .

## 4.3 Step 3 : Gillespie's stochastic framework of chemical kinetics

The principle task is to develop a method for simulation the time evolution of the  $N$  quantities  $\{X_i\}$ , knowing only their initial values  $\{X_i^{(0)}\}$ , the form of the  $M$  reactions  $\{R_\mu\}$  and the values of the reaction parameters  $\{c_\mu\}$ .

**Definition 2.** *Problem definition :* We are given a volume  $V$  containing molecules of  $N$  chemically active species  $S_i (i = 1, \dots, N)$ . Let  $X_i \equiv$  current number of molecules of chemical species  $S_i \in V, (i = 1, 2, \dots, N)$  and let  $R_\mu (\mu = 1, \dots, M)$  be the chemical reactions in which the species  $S_i$  can participate where each reaction  $R_\mu$  is characterized by a numerical reaction parameter  $c_\mu$ .

**Definition 3.** *Type of reactions*



**Hypothesis 4.** *Fundamental Hypothesis* The reaction parameter  $c_\mu$  can be defined as follows :

**Definition 5.**  $c_\mu \delta t \equiv$  average probability that a particular combination of  $R_\mu$  reactant molecules will react accordingly in the next time interval  $\delta t$  (first order)

!!!!!!!!!!!!!!!!!!!!!! TO DO : FINISH THE DETAILS AND EXPLANATION OF EACH FORMULA !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

**Definition 6.** State of the system is defined by the number of molecules of each species and changes discretely whenever one of the reactions is executed. The probability that a certain reaction  $\mu$  will take place in the next instant of time is given by  $a_\mu dt + o(dt)$

**Example 7.** Let  $S$  be the set of states i.e  $S = (\#A, \#B, \#C, \#D, \#E, \#F, \#G)$ ,  $S$  will change to  $S' = (\#A - 1, \#B - 1, \#C + 1, \#D, \#E, \#F, \#G)$  if Reaction 1 is executed. The probability of this occurrence is given by :  $P(S', t + dt | S, t) = a_1 dt + o(dt)$

**Example 8.** Generating a single sample trajectory of a chemical process in the stochastic framework

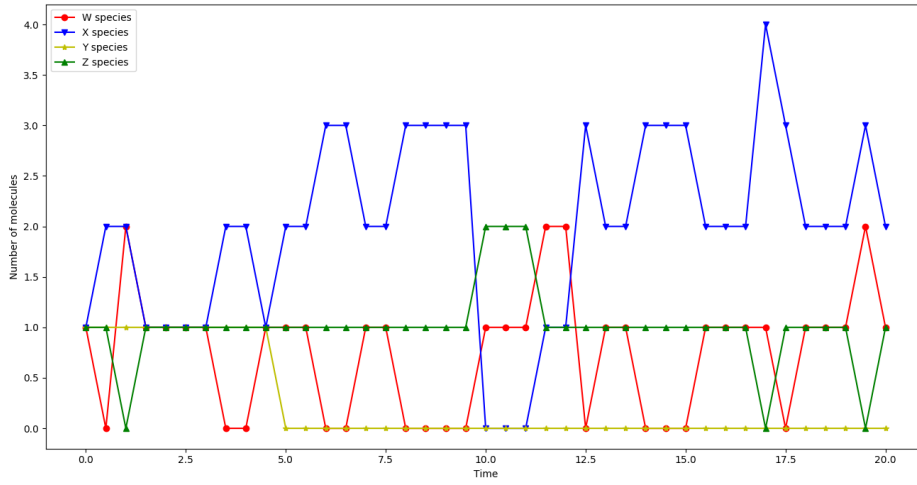


Figure 1: The x-axis denotes the time used and the y-axis denotes the number of molecules

#### 4.3.1 Gillespie's Direct Method

1. Probability density  $P(\mu, \tau)$  that the next reaction is  $\mu$  and it occurs at time  $\tau \rightarrow P(\mu, \tau)d\tau = a_\mu \exp(-\tau \sum_j a_j)d\tau$ .
2. Probability for reactions  $\rightarrow Pr(\text{Reaction} = \mu) = a_\mu / \sum_j a_j$ .
3. Probability distribution for times  $\rightarrow P(\tau)d\tau = (\sum_j a_j) \exp(-\tau \sum_j a_j)d\tau$

## 5 Conclusion

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**Algorithm 1** Gillespie's Direct Method

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**Input:****Output:**

```
while !(simulation time exceeded) do
  1. Initialization: Set initial number of molecules in the system, set  $t \leftarrow 0$ .
  2. Calculate the propensity function,  $a_i \forall i$ .
  3. Choose  $\mu$  according to the distribution in eq 5.
  4. Choose  $\tau$  according to an exponential with parameter  $\sum_j a_j$  (as in eq 6).
  5. Update the number of molecules to reflect execution of reaction  $\mu$ . Set  $t \leftarrow t + \tau$ .
  6. Go to step 2.
end while
```

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**References**