In this subroutine, we calculate the potential between the coil set and an arbitrary surface, which could be the plasma boundary. By minimizing the potential, the distance between the coils and the "prevent" surface will be increased. Due to the singularity of the formula, the coils would not be able to have intersections with the prevent surface.

[called by: solvers.]

contents

Interface

(reserved for future documentation)

Prevent surface

By default, the "prevent" surface is the target plasma boundary. In following updates, it will be modified to enable an arbitrary toroidal surface, which could be a surface away from the plasma with certain distance.

Potential calculation

For each coil, the potential energy between coil and the prevent surface is calculated as

$$E(\mathcal{C},\mathcal{S}) \equiv \oint_{\mathcal{S}} \int_{\mathcal{C}} \frac{ds\ dl}{|\mathbf{x}_c - \mathbf{x}_s|^q} \ , \tag{1}$$

where $\mathcal{C}$ denotes the coil, $\mathcal{S}$ denotes the prevent surface and q is an user-specified exponential factor. The total energy for the entire coil set is then a summation over the coils,

$$f_S(\mathbf{X}) = \sum_{i=1}^{Ncoils} E(\mathcal{C}_i,\mathcal{S}) \ . \tag{2}$$

By minimizing f_S, the coil set would be driven away from the prevent surface. The exponential factor q can accelerate/decelerate the "preventing" effect. Because of the singularity, the coils cannot cross the prevent surface. In such way, the coils will be well-separated from the plasma boundary. Of course, this requires the initial guesses have no intersections with the surface.

The discretized expression for Eq.(1) is

$$E(\mathcal{C},\mathcal{S}) \equiv \sum_{icoil=1}^{Ncoils} \sum_{jzeta=0}^{Nzeta-1} \sum_{iteta=0}^{Nteta-1} \frac{\sqrt{g}\Delta\theta\Delta\zeta\sqrt{{x'_c}^2 + {y'_c}^2 + {z'_c}^2}\Delta t}{\left(\sqrt{(x_c - x_s)^2 + (y_c - y_s)^2 + (z_c - z_s)^2}\right)^q} \ . \tag{3}$$

First derivatives

The first order functional derivatives are

$$\frac{\delta E}{\delta x_c} = \frac{1}{P}\left(-\frac{q(x_c-x_s)\left({x'_c}^2+{y'_c}^2+{z'_c}^2\right)^2}{(x_s-x_c)^2+(y_s-y_c)^2+(z_s-z_c)^2} + \frac{qx'_c\left({x'_c}^2+{y'_c}^2+{z'_c}^2\right)((x_c-x_s)x'_c+(y_c-y_s)y'_c+(z_c-z_s)z'_c)}{(x_s-x_c)^2+(y_s-y_c)^2+(z_s-z_c)^2} \right.$$
$$\left. -x''_c\left({x'_c}^2+{y'_c}^2+{z'_c}^2\right) + x'_c\left(x'_c x''_c+y'_c y''_c+z'_c z''_c\right)\right) , \quad (4)$$

$$\frac{\delta E}{\delta y_c} = \frac{1}{P}\left(-\frac{q(y_c-y_s)\left({x'_c}^2+{y'_c}^2+{z'_c}^2\right)^2}{(x_s-x_c)^2+(y_s-y_c)^2+(z_s-z_c)^2} + \frac{qy'_c\left({x'_c}^2+{y'_c}^2+{z'_c}^2\right)((x_c-x_s)x'_c+(y_c-y_s)y'_c+(z_c-z_s)z'_c)}{(x_s-x_c)^2+(y_s-y_c)^2+(z_s-z_c)^2} \right.$$
$$\left. -y''_c\left({x'_c}^2+{y'_c}^2+{z'_c}^2\right) + y'_c\left(x'_c x''_c+y'_c y''_c+z'_c z''_c\right)\right) , \quad (5)$$

$$\frac{\delta E}{\delta z_c} = \frac{1}{P}\left(-\frac{q(z_c-z_s)\left({x'_c}^2+{y'_c}^2+{z'_c}^2\right)^2}{(x_s-x_c)^2+(y_s-y_c)^2+(z_s-z_c)^2} + \frac{qz'_c\left({x'_c}^2+{y'_c}^2+{z'_c}^2\right)((x_c-x_s)x'_c+(y_c-y_s)y'_c+(z_c-z_s)z'_c)}{(x_s-x_c)^2+(y_s-y_c)^2+(z_s-z_c)^2} \right.$$
$$\left. -z''_c\left({x'_c}^2+{y'_c}^2+{z'_c}^2\right) + z'_c\left(x'_c x''_c+y'_c y''_c+z'_c z''_c\right)\right) , \quad (6)$$

where $P = [(x_s-x_c)^2+(y_s-y_c)^2+(z_s-z_c)^2]^{q/2}[{x'_c}^2+{y'_c}^2+{z'_c}^2]^{3/2}$. In above equations, only the integrads are written out.

Second derivatives

(reserved for future development)

Comments and notes

1. In the numerator of Eq.(1), the coil length dl is presented. Thus, minimizing f_S could lead to reducing coil lengths. Thinking about if the coil becomes one single point and $dl = 0$, $f_S = 0$. Fortunately, minimizing f_B will normally drive coils to be longer and f_L quadrtic term can keep the coil length to be around a constant value. In addition, the exponential factor q (`cssep_factor`) could determine the piority between pushing coils away or condensate coils. It's recommend to keep $q >= 2$.
2. A brief document about the function of cssep can be seen at google doc.