There are severl subroutines in this file. The main purpose is to calculate the Fourier harmonics of Bn on the plasma surface. Right now, the Fourier decompositions are carried out using normal Fourier Transformation in polar coordinates. In the future, FFT and flux coordinates capability will be included.

[called by: costfun.]

0.1 Background

The width of magnetic island is proportional to the square root of resonant perturbation amplitude. Directly optimizing Bn harmonics (with well-chose sensitivity matrix) rather than minimizing the surface integration would be a better idea. Especially for designing RMP coils in tokamaks.

0.2 0-order cost function

In the flux coordinate (ψ, θ, ϕ), the normal magnetic field perturbation can be written as,

$$B_n(\theta, \phi) = \vec{B} \cdot \nabla \psi = \sum_{m,n} \Delta_{mn} e^{-i(m\theta - n\phi)} \ . \tag{1}$$

If we define the cost function $\mathbf{H} = \mathbf{bharm}$ as,

$$\begin{aligned} H &= \frac{1}{2} \sum_{m,n} w_{mn} \left(\Delta_{mn} - \Delta^{o}_{mn}\right)^2 \\ &= \frac{1}{2} \sum_{m,n} w_{mn} \left[(\Delta^{c}_{mn} - \Delta^{c,o}_{mn})^2 + (\Delta^{s}_{mn} - \Delta^{s,o}_{mn})^2 \right] \end{aligned} \tag{2}$$

Using trigonometric functions,

$$\Delta^{c}_{mn} = \frac{1}{2\pi^2} \int_0^{2\pi} \int_0^{2\pi} Bn \ \cos(m\theta - n\phi) d\theta d\phi \tag{3}$$

$$\Delta^{s}_{mn} = \frac{1}{2\pi^2} \int_0^{2\pi} \int_0^{2\pi} Bn \ \sin(m\theta - n\phi) d\theta d\phi \tag{4}$$

0.3 1st-order derivatives

The derivatives of H with respect to coil parameters can be calculated as,

$$\frac{\partial H}{\partial x} = \sum_{m,n} w_{mn} \left[(\Delta^{c}_{mn} - \Delta^{c,o}_{mn}) \frac{\partial \Delta^{c}_{mn}}{\partial x} \ + \ (\Delta^{s}_{mn} - \Delta^{s,o}_{mn}) \frac{\partial \Delta^{s}_{mn}}{\partial x} \right] \tag{5}$$

$$\frac{\partial \Delta^{c}_{mn}}{\partial x} = \frac{1}{2\pi^2} \int_0^{2\pi} \int_0^{2\pi} \left[\frac{\partial B_x}{\partial x} n_x + \frac{\partial B_y}{\partial y} n_y + \frac{\partial B_z}{\partial z} n_z \right] \ \cos(m\theta - n\phi) d\theta d\phi \tag{6}$$

$$\frac{\partial \Delta^{s}_{mn}}{\partial x} = \frac{1}{2\pi^2} \int_0^{2\pi} \int_0^{2\pi} \left[\frac{\partial B_x}{\partial x} n_x + \frac{\partial B_y}{\partial y} n_y + \frac{\partial B_z}{\partial z} n_z \right] \ \sin(m\theta - n\phi) d\theta d\phi \tag{7}$$