equarcl

Force each discretized coil arc has the same length to eliminate the non-uniqueness of Fourier harmonics.

[called by: denergy.]

contents

1.1 Introductiom

The Fourier harmonics depends on the selection of Fourier angel, which is not unique. Thus, it makes the solution for coil optimization not unique and optimizors applying Newton method would fail to find the minimum of cost functions. To eliminate the non-uniqueness of Fourier harmonics, one general way is to add an additional spectral constraints. Details can be found in Spec:bb00aa. Here, what we used is just simply forcing each discretized coil arc has the same length.

1.2 Equal arc constraint

1. The arc length for a discretized coil can be represented as,

$$dl = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \tag{1}$$

2. Assuming that all arcs in a coil have the same length. Then, the coil length is $L = 2\pi * dl$, where 2π is the discretization number.

3. Then our equal-arc constraint can be written in,

$$eqarc = \int_{coils} \int_{icoil} \frac{1}{2} (\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} - \frac{L_i}{2\pi})^2 dt \tag{2}$$

After normalized,

$$eqarc = \int_{coils} \int_{icoil} \frac{1}{2} (\frac{2\pi dl}{L_i} - 1)^2 dt \tag{3}$$

1.3 First derivatives

1. The first derivatives for equal-arc constraint are,

$$\begin{aligned} \frac{\partial eqarc}{\partial x^i_m} &= \int_{icoil} (\frac{2\pi dl}{L_i} - 1)(\frac{2\pi}{L_i} \frac{\partial dl}{\partial x^i_m} - \frac{2\pi dl}{{L_i}^2} \frac{\partial L_i}{\partial x^i_m}) && (4) \\ \frac{\partial dl}{\partial x^i_m} &= \frac{\dot{x}}{dl} \frac{\partial \dot{x}}{\partial x^i_m} && (5) \\ \frac{\partial L_i}{\partial x^i_m} &= \int \frac{\partial dl}{\partial x^i_m} dt && (6) \\ & && (7) \end{aligned}$$

1.4 Second derivatives

1. The second derivatives for equal-arc constraint are,

$$\frac{\partial^2 eqarc}{\partial x^i_m \partial x^i_n} = \int_{icoil} (\frac{2\pi}{L_i} \frac{\partial dl}{\partial x^i_m} - \frac{2\pi dl}{{L_i}^2} \frac{\partial L_i}{\partial x^i_m})(\frac{2\pi}{L_i} \frac{\partial dl}{\partial x^i_n} - \frac{2\pi dl}{{L_i}^2} \frac{\partial L_i}{\partial x^i_n}) + (\frac{2\pi dl}{L_i} - 1) \left(\frac{2\pi}{L_i} \frac{\partial^2 dl}{\partial x^i_m \partial x^i_n} - \frac{2\pi}{{L_i}^2} \frac{\partial dl}{\partial x^i_m} \frac{\partial L_i}{\partial x^i_n} - \frac{2\pi}{{L_i}^2} \frac{\partial dl}{\partial x^i_n} \frac{\partial L_i}{\partial x^i_m} - \frac{2\pi dl}{{L_i}^2} \frac{\partial^2 L_i}{\partial x^i_m \partial x^i_n} + \frac{4\pi dl}{{L_i}^3} \frac{\partial L_i}{\partial x^i_m} \frac{\partial L_i}{\partial x^i_n} \right) \tag{8}$$

1.5 Notes

For unfixed bugs with normalized version and also for the consideration of changing minimum condition, I only constructed the version without normalization. The normalization may be added sometime later.