PLSC 597: Modern Measurement

Item Response Theory, I

February 22, 2018

Item Response Theory ("IRT")

- Origins in psychometrics / testing
- Measurement *model*
- *Unidimensional*
- *Discrete* responses **Y**
- Equally descriptive and inferential

Basic Setup

$$Y^* = \text{ latent trait ("ability")}$$

$$Y = \text{observed measures}$$

- $i \in \{1, 2 \ldots N\}$ indexes *subjects* / *units*, and
- $j \in \{1, 2, \ldots J\}$ indexes *items* / *measures*.

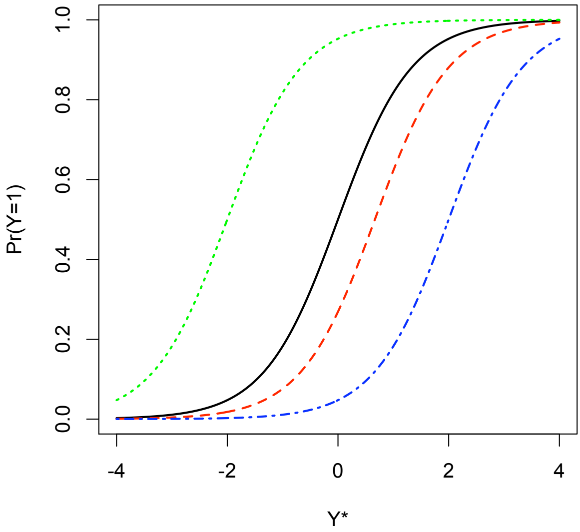
$$Y_{ij} = \begin{cases} 0 & \text{if subject } i \text{ gets item } j \text{ "incorrect,"} \\ 1 & \text{if subject } i \text{ gets item } j \text{ "correct."} \end{cases}$$

One-Parameter Logistic Model ("1PLM")

$$\Pr(Y_{ij} = 1) = \frac{\exp(\theta_i - \beta_j)}{1 + \exp(\theta_i - \beta_j)}$$

Here,

- θ_i = respondent i's *ability*,
- β_j = item j's *difficulty*.
- $\beta_j \equiv$ value of Y^* where $\Pr(Y_{ij} = 1) = 0.50$


1.0
0.8
0.6
0.4
0.2
0.0
Pr(Y=1)
-4
-2
0
2
4
Y*

1PLM

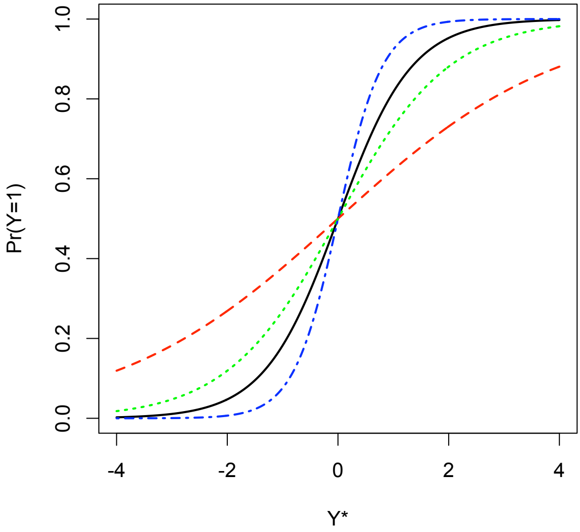
- a.k.a. "Rasch" model (Rasch 1960)
- Implicit "slope" $= 1.0$
- Implies items are equally "discriminating"
- If not...

Two-Parameter Logistic Model ("2PLM")

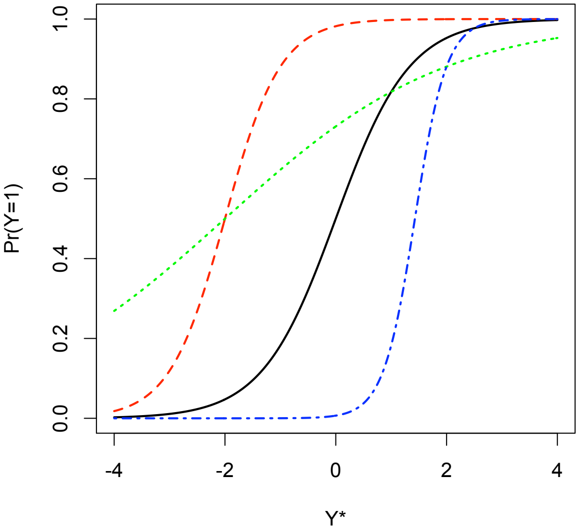
$$\Pr(Y_{ij} = 1) = \frac{\exp[\alpha_j(\theta_i - \beta_j)]}{1 + \exp[\alpha_j(\theta_i - \beta_j)]}$$

- θ_i = respondent i's *ability*,
- β_j = item j's *difficulty*,
- α_j = item j's *discrimination*.

Identical Difficulty, Different Discrimination



Different Difficulty & Discrimination



2PLM

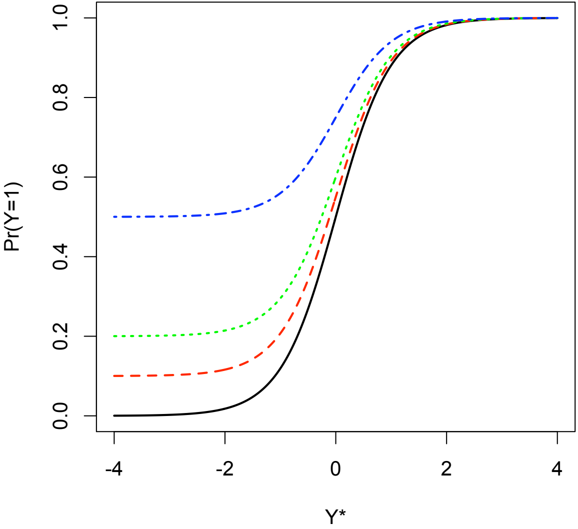
- Due to Birnbaum (1968)
- Similar to "typical" logit...
- Nests the 1PLM as a special case ($\alpha_j = 1 \; \forall \; j$)

Three-Parameter Logistic Model ("3PLM")

$$\Pr(Y_{ij} = 1) = \delta_j + (1 - \delta_j)\left\{\frac{\exp[\alpha_j(\theta_i - \beta_j)]}{1 + \exp[\alpha_j(\theta_i - \beta_j)]}\right\}$$

- θ_i = respondent *i*'s *ability*,
- β_j = item *j*'s *difficulty*,
- α_j = item *j*'s *discrimination*.
- δ_j = *lower asymptote* of $\Pr(Y_{ij} = 1)$ (incorrectly: "guessing" parameter).

3PLM, Constant α & β, Varying δ



The Two Big Assumptions

- *Unidimensionality*
- *Local Item Independence* ("No LID"):

$$\text{Cov}(Y_{ij}, Y_{ik} | \theta_i) = 0 \; \forall \; j \neq k$$

Estimation: Notation

$$P_{ij} = \Pr(Y_{ij} = 1),$$

$$\begin{aligned} Q_{ij} &= \Pr(Y_{ij} = 0) \\ &= 1 - \Pr(Y_{ij} = 1), \end{aligned}$$

$$\Psi = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_J \\ \alpha_1 \\ \vdots \\ \alpha_J \\ \delta_1 \\ \vdots \\ \delta_J \end{pmatrix}.$$

Estimation: Likelihoods

Known $\Psi = \alpha$, β, δ:

$$L(\mathbf{Y}|\Psi) = \prod_{j=1}^{J} P_{ij}^{Y_{ij}} Q_{ij}^{1-Y_{ij}}.$$

Known θ:

$$L(\mathbf{Y}|\theta) = \prod_{i=1}^{N} P_{ij}^{Y_{ij}} Q_{ij}^{1-Y_{ij}}.$$

Estimation: Likelihoods

$$L(\mathbf{Y}|\Psi, \theta) = \prod_{i=1}^{N} \prod_{j=1}^{J} P_{ij}^{Y_{ij}} Q_{ij}^{1-Y_{ij}}$$

$$\ln L(\mathbf{Y}|\Psi, \theta) = \sum_{i=1}^{N} \sum_{j=1}^{J} Y_{ij} \ln P_{ij} + (1 - Y_{ij}) Q_{ij}.$$

Parameterization

- $N + J$ parameters in the 1PLM,
- $N + 2J$ parameters in the 2PLM,
- $N + 3J$ parameters in the 3PLM.

But...

- NJ observations,
- Asymptotics as $N \to \infty$, $J \to \infty$...

Estimation: Conditional Likelihood

Total score is:

$$T_i = \sum_{j=1}^{J} Y_{ij} \;\in\; \{0, 1, ...J\}$$

$$L = \prod_{i=1}^{N} \frac{\exp[\alpha_j(\theta_t - \beta_j)]}{1 + \exp[\alpha_j(\theta_t - \beta_j)]}$$

θ_t are "score-group" parameters corresponding to the $J + 1$ possible values of T.

Estimation: Conditional Likelihood

- Equivalent to fitting a conditional logit model:

$$\Pr(Y_{ij} = 1) = \frac{\exp(\mathbf{Z}_{ij}\gamma)}{\sum_{j=1}^{J} \exp(\mathbf{Z}_{ij}\gamma)}$$

  with $\mathbf{Z}_{ij} =$ "item dummies."

- Useful only for 1PLM (since T_i is a sufficient statistic for θ_i).

Estimation: Marginal Likelihood

$$L(\mathbf{Y}|\Psi, \theta) = \prod_{i=1}^{N} \left[\int_{-\infty}^{\infty} \prod_{j=1}^{J} P_{ij}^{Y_{ij}} Q_{ij}^{1-Y_{ij}} d\theta \right]$$

- Analogous to "random effects"...
- Eliminates inconsistency as $N \rightarrow \infty$, *but*
- Requires *strong* exogeneity of θ and Ψ.

Estimation: Bayesian Approaches

- Place priors on θ, Ψ;
- Estimate via sampling from posteriors, via MCMC.
- Eliminates problems with $\hat{\alpha}$, $\hat{\beta}$, $\hat{\theta} = \infty$ (see below).
- Easily extensible to other circumstances (hierarchical/multilevel, etc.)

Identification

Two Issues:

- *Scale* invariance: $L(\hat{\Psi}) = L(\hat{\Psi} + c)$
- *Rotational* invariance: $L(\hat{\Psi}) = L(-\hat{\Psi})$

Fixes:

- Set one (arbitrary) $\beta_j = 0$, and another (arbitrary) $\beta_k > 0$, or
- Fix two θ_is at specific values.

Further (Potential) Concerns

- $Y_{ij} = 0/1 \; \forall \; i \;\rightarrow\; \beta_j = \pm\infty$.
- $Y_{ij} = 0/1 \; \forall \; j \;\rightarrow\; \theta_i = \pm\infty$.
- Separation / "empty cells" $\rightarrow\; \alpha_j = \pm\infty$.
- Problematic for joint and conditional approaches; more easily dealt with in the Bayesian framework.

- Estimates of $\hat{\alpha}$s, $\hat{\beta}$s, and/or $\hat{\delta}$s, plus $\hat{\theta}$s
- Associated s.e.s / c.i.s
- "Scale-free" quantities of interest...

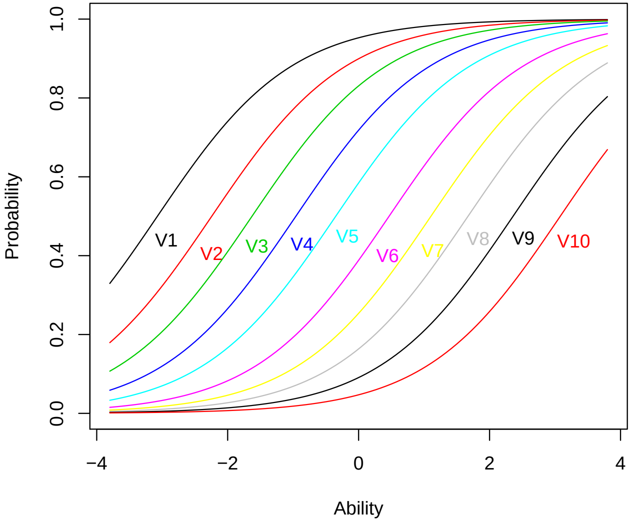
IRT Models in R

- Library `ltm` (marginal estimation)
  - `rasch` (1PLM)
  - `ltm` (2PLM)
  - `tpl` (3PLM)
- Library `MCMCpack` (Bayesian estimation)
  - 1 and 2PLM
  - Standard, hierarchical, dyamic, multidimensional
- `ideal` (in library `pscl`) (Bayesian estimation)
  - 1 and 2PLM
  - *k*-dimensional
  - takes a `rollcall` object
- Other packages: `eRm`, `irtoys`, `irtProb`, `MiscPsycho`, etc.

A Simulation: Rasch's Model

```
> N <- 1000
> K <- 10
> set.seed(7222009)
> Rasch1Data <- sim.irt(nvar=K,n=N,low=-3,high=3,
+                   a=1,c=0,d=NULL,mu=0,sd=1,
+                   mod="logistic")
> Rasch1<-rasch(Rasch1Data$items)
> summary(Rasch1)

Model Summary:
 log.Lik  AIC  BIC
   -4666 9354 9408

Coefficients:
            value std.err  z.vals
Dffclt.V1  -3.073   0.180 -17.082
Dffclt.V2  -2.243   0.134 -16.678
Dffclt.V3  -1.628   0.109 -14.924
Dffclt.V4  -0.958   0.089 -10.744
Dffclt.V5  -0.349   0.079  -4.402
Dffclt.V6   0.465   0.081   5.766
Dffclt.V7   1.104   0.093  11.866
Dffclt.V8   1.671   0.111  15.075
Dffclt.V9   2.359   0.140  16.856
Dffclt.V10  3.080   0.180  17.126
Dscrmn      0.977   0.045  21.678

Integration:
method: Gauss-Hermite
quadrature points: 21

Optimization:
Convergence: 0
max(|grad|): 0.009
quasi-Newton: BFGS
```

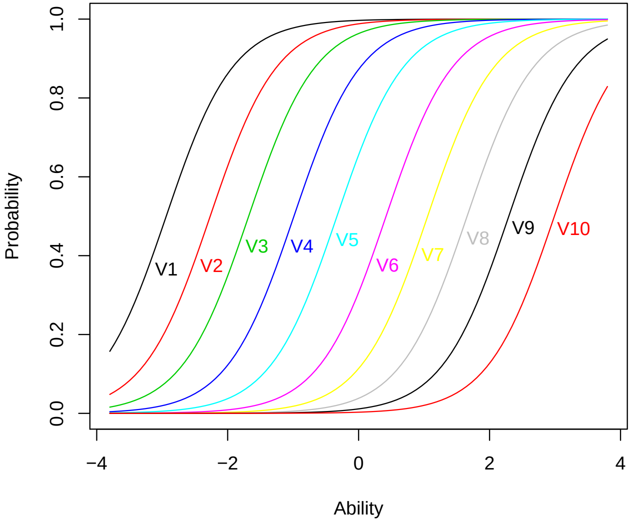
Simulation: Estimated (Rasch) IRFs



Discrimination Parameter $\neq 1.0$

```
> set.seed(7222009)
> RaschAltData <- sim.irt(nvar=K,n=N,low=-3,high=3,
+                         a=2,c=0,d=NULL,mu=0,sd=1,
+                         mod="logistic")
> RaschAlt<-rasch(RaschAltData$items)
> summary(RaschAlt)

Model Summary:
 log.Lik  AIC  BIC
   -3147 6315 6369

Coefficients:
            value std.err  z.vals
Dffclt.V1  -2.938   0.165 -17.811
Dffclt.V2  -2.267   0.111 -20.426
Dffclt.V3  -1.679   0.083 -20.238
Dffclt.V4  -0.988   0.063 -15.693
Dffclt.V5  -0.331   0.054  -6.152
Dffclt.V6   0.419   0.055   7.665
Dffclt.V7   1.054   0.065  16.239
Dffclt.V8   1.657   0.083  20.067
Dffclt.V9   2.294   0.112  20.459
Dffclt.V10  2.990   0.167  17.909
Dscrmn      1.948   0.079  24.764

Integration:
method: Gauss-Hermite
quadrature points: 21

Optimization:
Convergence: 0
max(|grad|): 0.00073
quasi-Newton: BFGS
```

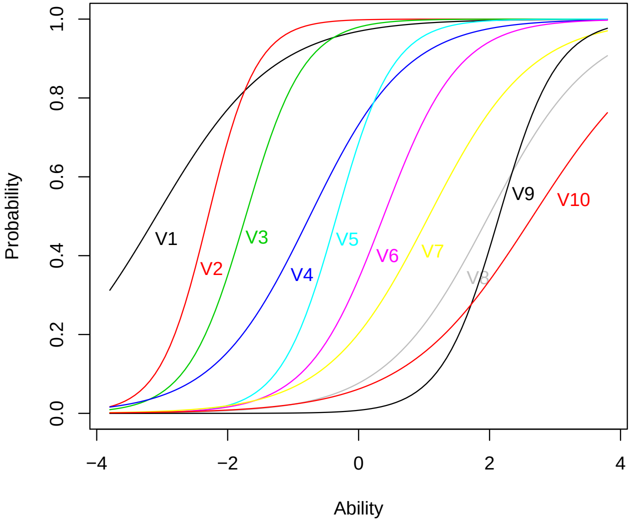
Simulation: Estimated IRFs



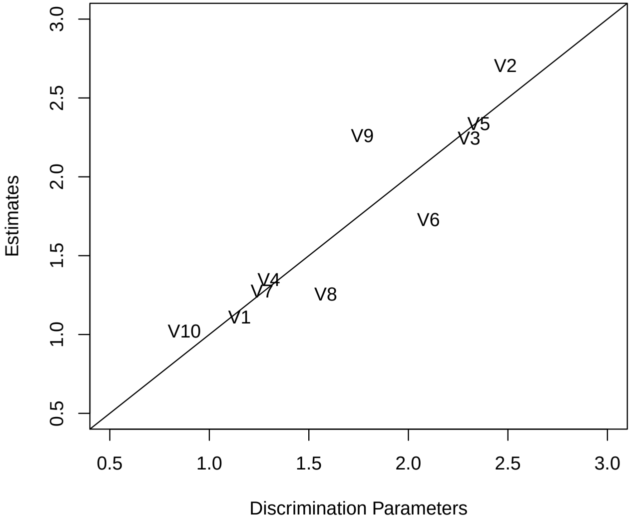
Simulation: 2PLM

```
> set.seed(7222009)
> Discrims <- runif(K,0.5,3)
> Rasch2Data <- sim.irt(nvar=K,n=N,low=-3,high=3,
+                           a=Discrims,c=0,d=NULL,mu=0,sd=1,mod="logistic")
> Rasch2<-ltm(Rasch2Data$items~z1)
> summary(Rasch2)

Coefficients:
             value std.err  z.vals
Dffclt.V1   -3.091   0.439  -7.037
Dffclt.V2   -2.297   0.203 -11.316
Dffclt.V3   -1.724   0.125 -13.775
Dffclt.V4   -0.746   0.081  -9.230
Dffclt.V5   -0.334   0.052  -6.432
Dffclt.V6    0.381   0.060   6.377
Dffclt.V7    1.073   0.103  10.457
Dffclt.V8    1.984   0.194  10.216
Dffclt.V9    2.145   0.185  11.627
Dffclt.V10   2.660   0.336   7.908
Dscrmn.V1    1.114   0.207   5.378
Dscrmn.V2    2.707   0.622   4.352
Dscrmn.V3    2.246   0.336   6.686
Dscrmn.V4    1.349   0.141   9.587
Dscrmn.V5    2.337   0.281   8.302
Dscrmn.V6    1.729   0.178   9.713
Dscrmn.V7    1.277   0.140   9.110
Dscrmn.V8    1.256   0.168   7.461
Dscrmn.V9    2.261   0.426   5.310
Dscrmn.V10   1.024   0.164   6.226
.
.
.
```

Simulation: Estimated 2PLM IRFs



Estimation of Discrimination Parameters



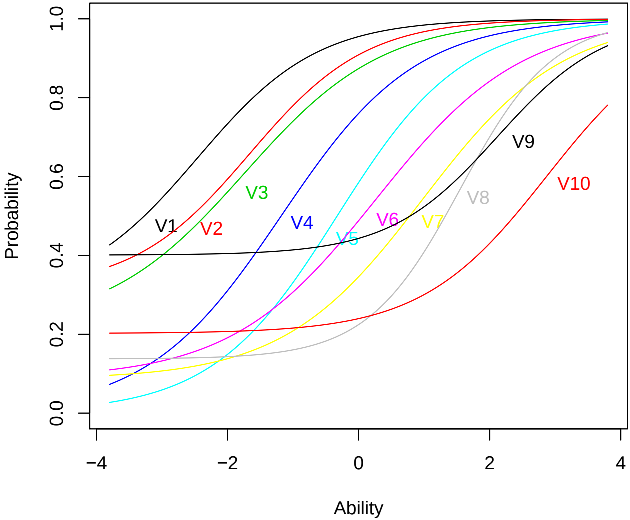
Simulation: 3PLM

```
> N <- 10000
> set.seed(7222009)
> GuessThresh <- round(rbeta(K,1,8),digits=1)
> Rasch3Data <- sim.irt(nvar=K,n=N,low=-3,high=3,
+                       a=1,c=GuessThresh,d=NULL,mu=0,sd=1,mod="logistic")

> Rasch3 <- tpm(Rasch3Data$items)
Warning message:
In tpm(Rasch3Data$items) :
  Hessian matrix at convergence is not positive definite; unstable solution.

> summary(Rasch3)

Coefficients:
            value std.err  z.vals
Gussng.V1   0.294     NaN     NaN
Gussng.V2   0.315   0.233   1.353
Gussng.V3   0.218   0.353   0.618
Gussng.V4   0.001   0.016   0.066
Gussng.V5   0.001   0.021   0.070
Gussng.V6   0.087   0.096   0.905
Gussng.V7   0.086   0.052   1.638
Gussng.V8   0.137   0.020   6.795
Gussng.V9   0.401   0.025  16.302
Gussng.V10  0.202   0.022   9.309
.
.
.
```

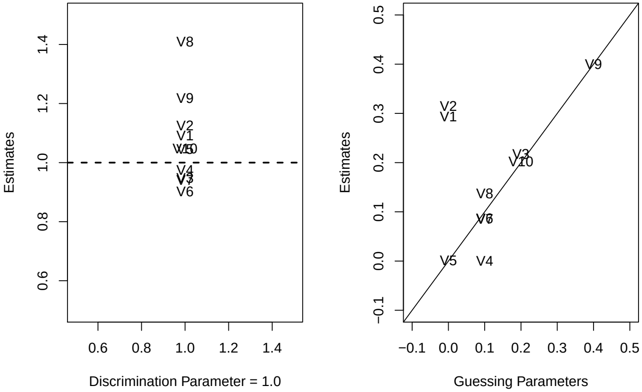
Simulation: 3PLM (continued)

```
.
.
.
Dffclt.V1  -2.461     NaN     NaN
Dffclt.V2  -1.665   0.591  -2.818
Dffclt.V3  -1.740   0.899  -1.937
Dffclt.V4  -1.181   0.061 -19.316
Dffclt.V5  -0.326   0.056  -5.850
Dffclt.V6   0.270   0.273   0.988
Dffclt.V7   0.979   0.123   7.930
Dffclt.V8   1.551   0.073  21.245
Dffclt.V9   2.114   0.214   9.858
Dffclt.V10  2.872   0.419   6.856
Dscrmn.V1   1.093     NaN     NaN
Dscrmn.V2   1.126   0.168   6.685
Dscrmn.V3   0.948   0.173   5.462
Dscrmn.V4   0.976   0.049  19.812
Dscrmn.V5   1.046   0.055  18.969
Dscrmn.V6   0.903   0.162   5.588
Dscrmn.V7   0.941   0.158   5.957
Dscrmn.V8   1.410   0.250   5.628
Dscrmn.V9   1.218   0.432   2.819
Dscrmn.V10  1.048   0.378   2.769

Integration:
method: Gauss-Hermite
quadrature points: 21

Optimization:
Optimizer: optim (BFGS)
Convergence: 0
max(|grad|): 0.47
```

Simulation: Estimated 3PLM IRFs



Estimation of Discrimination Parameters



Example: SCOTUS Voting, 1994-2004

```
> summary(SCOTUS)

       id            Rehnquist        Stevens         OConnor          Scalia
 Min.   :   1   Min.   :0      Min.   :0      Min.   :0      Min.   :0
 1st Qu.: 377   1st Qu.:0      1st Qu.:0      1st Qu.:0      1st Qu.:0
 Median : 753   Median :0      Median :1      Median :0      Median :0
 Mean   : 753   Mean   :0      Mean   :1      Mean   :0      Mean   :0
 3rd Qu.:1129   3rd Qu.:1      3rd Qu.:1      3rd Qu.:1      3rd Qu.:1
 Max.   :1505   Max.   :1      Max.   :1      Max.   :1      Max.   :1
                NA's   :49     NA's   :51     NA's   :55     NA's   :41
    Kennedy          Souter          Thomas         Ginsburg         Breyer
 Min.   :0      Min.   :0      Min.   :0      Min.   :0      Min.   :0
 1st Qu.:0      1st Qu.:0      1st Qu.:0      1st Qu.:0      1st Qu.:0
 Median :0      Median :1      Median :0      Median :1      Median :1
 Mean   :0      Mean   :1      Mean   :0      Mean   :1      Mean   :1
 3rd Qu.:1      3rd Qu.:1      3rd Qu.:0      3rd Qu.:1      3rd Qu.:1
 Max.   :1      Max.   :1      Max.   :1      Max.   :1      Max.   :1
 NA's   :32     NA's   :37     NA's   :44     NA's   :39     NA's   :61
```

1PLM Using rasch

```
> # 1PLM / Rasch Model:
> require(ltm)
> OnePLM<-rasch(SCOTUS[c(2:10)])
> summary(OnePLM)

Model Summary:
 log.Lik   AIC   BIC
   -5529 11079 11132

Coefficients:
                 value std.err z.vals
Dffclt.Rehnquist  0.46   0.040   11.5
Dffclt.Stevens   -0.59   0.030  -19.8
Dffclt.OConnor    0.14   0.030    4.6
Dffclt.Scalia     0.52   0.041   12.5
Dffclt.Kennedy    0.21   0.032    6.5
Dffclt.Souter    -0.36   0.027  -13.1
Dffclt.Thomas     0.60   0.043   13.8
Dffclt.Ginsburg  -0.37   0.027  -13.4
Dffclt.Breyer    -0.26   0.027   -9.9
Dscrmn            3.74   0.130   28.9

Integration:
method: Gauss-Hermite
quadrature points: 21

Optimization:
Convergence: 0
max(|grad|): 0.0027
quasi-Newton: BFGS
```

Converted to $\widehat{\Pr(Y_i = 1|\theta_i} = 0)$

```
> # Convert to probabilities given theta=0
>
> coef(OnePLM, prob=TRUE, order=TRUE)
          Dffclt Dscrmn P(x=1|z=0)
Stevens    -0.59    3.7      0.900
Ginsburg   -0.37    3.7      0.797
Souter     -0.36    3.7      0.791
Breyer     -0.26    3.7      0.729
OConnor     0.14    3.7      0.373
Kennedy     0.21    3.7      0.311
Rehnquist   0.46    3.7      0.151
Scalia      0.52    3.7      0.126
Thomas      0.60    3.7      0.096
```

Alternative Model Constraining $\alpha = 1.0$

```
> AltOnePLM<-rasch(IRTData, constraint=cbind(length(IRTData)+1,1))
> summary(AltOnePLM)

Model Summary:
 log.Lik   AIC   BIC
   -6452 12923 12971

Coefficients:
                 value std.err z.vals
Dffclt.Rehnquist  1.26   0.073   17.3
Dffclt.Stevens   -1.07   0.071  -15.1
Dffclt.OConnor    0.56   0.069    8.1
Dffclt.Scalia     1.37   0.074   18.6
Dffclt.Kennedy    0.72   0.069   10.4
Dffclt.Souter    -0.58   0.068   -8.6
Dffclt.Thomas     1.53   0.075   20.3
Dffclt.Ginsburg  -0.61   0.068   -8.9
Dffclt.Breyer    -0.40   0.068   -5.9
Dscrmn            1.00      NA     NA
```

2PLM

```
> TwoPLM<-ltm(IRTData ~ z1)
> summary(TwoPLM)

Coefficients:
                  value std.err z.vals
Dffclt.Rehnquist   0.44   0.035   12.3
Dffclt.Stevens    -0.63   0.038  -16.7
Dffclt.OConnor     0.14   0.026    5.6
Dffclt.Scalia      0.59   0.042   14.1
Dffclt.Kennedy     0.20   0.028    7.2
Dffclt.Souter     -0.27   0.025  -10.7
Dffclt.Thomas      0.68   0.044   15.2
Dffclt.Ginsburg   -0.29   0.025  -11.8
Dffclt.Breyer     -0.24   0.025   -9.6
Dscrmn.Rehnquist   4.77   0.377   12.7
Dscrmn.Stevens     2.46   0.165   14.9
Dscrmn.OConnor     4.14   0.341   12.1
Dscrmn.Scalia      2.82   0.188   15.0
Dscrmn.Kennedy     4.74   0.448   10.6
Dscrmn.Souter      6.69   0.535   12.5
Dscrmn.Thomas      2.84   0.190   14.9
Dscrmn.Ginsburg    5.83   0.439   13.3
Dscrmn.Breyer      3.76   0.253   14.9
```

2PLM: Probabilities and Testing

```
> coef(TwoPLM, prob=TRUE, order=TRUE)

          Dffclt Dscrmn P(x=1|z=0)
Stevens    -0.63    2.5       0.82
Ginsburg   -0.29    5.8       0.85
Souter     -0.27    6.7       0.86
Breyer     -0.24    3.8       0.71
OConnor     0.14    4.1       0.35
Kennedy     0.20    4.7       0.28
Rehnquist   0.44    4.8       0.11
Scalia      0.59    2.8       0.16
Thomas      0.68    2.8       0.13
```

```
> anova(OnePLM, TwoPLM)

 Likelihood Ratio Table
         AIC   BIC log.Lik   LRT df p.value
OnePLM 11079 11132   -5529
TwoPLM 10882 10978   -5423 212.7  8  <0.001
```

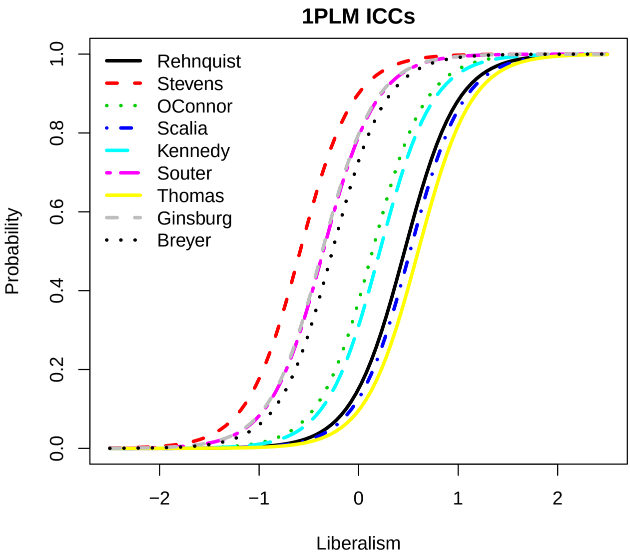
3PLM

```
> ThreePLM<-tpm(IRTData)
> summary(ThreePLM)

Coefficients:
                   value  std.err  z.vals
Gussng.Rehnquist  0.049    0.008   6.260
Gussng.Stevens    0.000    0.001   0.018
Gussng.OConnor    0.043    0.013   3.415
Gussng.Scalia     0.097    0.011   9.119
Gussng.Kennedy    0.071    0.014   5.162
Gussng.Souter     0.011    0.029   0.386
Gussng.Thomas     0.087    0.010   8.900
Gussng.Ginsburg   0.000    0.000   0.009
Gussng.Breyer     0.000    0.000   0.004
Dffclt.Rehnquist  0.716    0.030  23.511
Dffclt.Stevens   -0.630    0.038 -16.434
Dffclt.OConnor    0.340    0.040   8.537
Dffclt.Scalia     0.759    1.766   0.430
Dffclt.Kennedy    0.500    0.041  12.170
Dffclt.Souter    -0.294    0.063  -4.642
Dffclt.Thomas     0.808   10.610   0.076
Dffclt.Ginsburg  -0.329    0.030 -10.970
Dffclt.Breyer    -0.232    0.031  -7.439
Dscrmn.Rehnquist  8.735    4.259   2.051
Dscrmn.Stevens    2.577    0.181  14.214
Dscrmn.OConnor    3.979    0.439   9.068
Dscrmn.Scalia    26.537  578.889   0.046
Dscrmn.Kennedy    4.408    0.588   7.498
Dscrmn.Souter     6.698    1.416   4.731
Dscrmn.Thomas    34.074 2779.161   0.012
Dscrmn.Ginsburg   5.800    0.509  11.394
Dscrmn.Breyer     3.538    0.231  15.335
```

3PLM: Testing

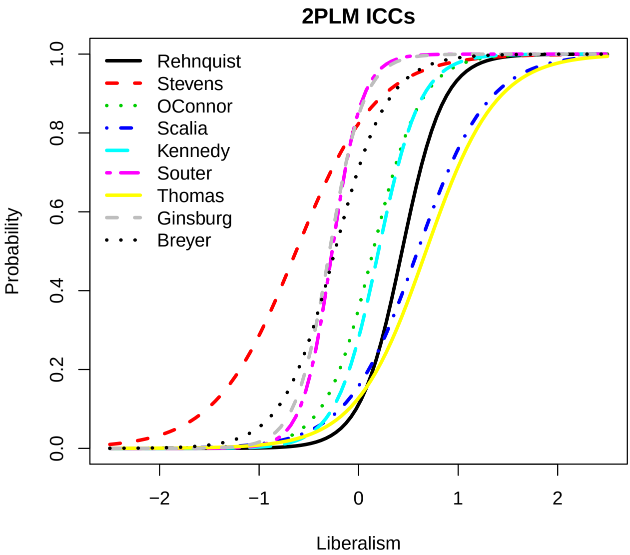
```
> anova(TwoPLM, ThreePLM)

 Likelihood Ratio Table
           AIC   BIC log.Lik    LRT df p.value
TwoPLM   10882 10978   -5423
ThreePLM 10737 10881   -5342 162.94  9  <0.001
```

Cool Plots, I

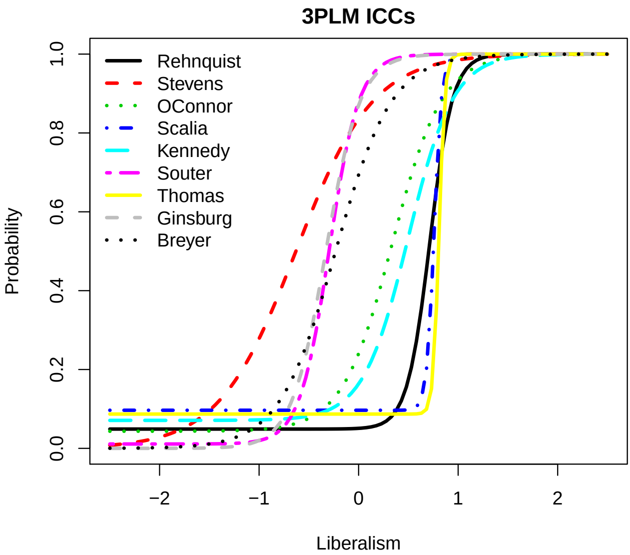


1PLM ICCs
Rehnquist
Stevens
OConnor
Scalia
Kennedy
Souter
Thomas
Ginsburg
Breyer
Probability
1.0
0.8
0.6
0.4
0.2
0.0
−2
−1
0
1
2
Liberalism

Cool Plots, II



Cool Plots, III



Miscellaneous Things, I: Dimensionality

- Usually, *unidimensional*

- Sometimes, *two-dimensional*

- Tests:
    - Tetrachoric correlations among items
    - DIMTEST (Stout & Zhang, etc.)
    - Yen's Q_3
    - 1-D vs. 2-D comparisons (LR tests, etc.)

Miscellaneous Things, II: "DIF"

- *Differential item functioning*
- Formally,

$$\Pr(Y_{ij} = 1) = \Lambda[\alpha_j(\theta_i - \mathbf{X}_i\beta_j)].$$

- $\rightarrow$ violates *local item independence*

Extensions

- Nominal/Multinomial Y
- Ordinal Y:
  - *Graded response model* ("GRM") (Samejima 1969)
  - *Partial credit model* (Masters 1982)
  - *Generalized partial credit model* (Muraki 1992)
- Models for mixed response types (Thissen and Wainer 2001, 2003)
- Hierarchical IRT models (e.g. Bolt and Kim 2005)
- Models with covariates (e.g., DeBoeck and Wilson 2004)

Further Reading / Useful References

Hambleton, Ronald K., H. Swaminathan, and H. Jane Rogers. 1991. *Fundamentals of Item Response Theory*. Newbury Park CA: Sage Publications.

de Ayala, R. J. 2008. *The Theory and Practice of Item Response Theory*. New York: The Guilford Press.

Fahrmeier, L., and G. Tutz. 2000. *Multivariate Statistical Modelling Based on Generalized Linear Models*. Berlin: Springer-Verlag.

De Boeck, Paul, and Mark Wilson, Eds. 2004. *Explanatory Item Response Models: A Generalized Linear and Nonlinear Approach.* New York: Springer.